

# **Time-Independent Solution of Navier-Stokes Equations**

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## **ABSTRACT**

In Navier-Stokes equations, we discover time-independent solutions by Newton potential.

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## 1. Introduction

We discover the time-independent solutions in Navier-Stokes equation by Newton potential.

According NASA's Navier-Stokes Equations(3-dimensional-unsteady),

Coordinate:  $(x, y, z)$ , Time:  $t$ , Pressure:  $\rho$ , Heat Flux:  $q$

Density:  $\rho$ , Stress:  $\tau$ , Reynolds Number:  $R_e$ ,

Velocity Components:  $(u, v, w)$  Total Energy:  $E_t$  Prandtl Number:  $P_r$

$$\text{Continuity: } \frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \quad (1)$$

$$\begin{aligned} \text{X-Momentum: } & \frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z} \\ & = -\frac{\partial \rho}{\partial x} + \frac{1}{R_e} \left[ \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} \right] \end{aligned} \quad (2)$$

$$\begin{aligned} \text{Y-Momentum: } & \frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} + \frac{\partial(\rho vw)}{\partial z} \\ & = -\frac{\partial \rho}{\partial y} + \frac{1}{R_e} \left[ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} \right] \end{aligned} \quad (3)$$

$$\begin{aligned} \text{Z-Momentum: } & \frac{\partial(\rho w)}{\partial t} + \frac{\partial(\rho uw)}{\partial x} + \frac{\partial(\rho vw)}{\partial y} + \frac{\partial(\rho w^2)}{\partial z} \\ & = -\frac{\partial \rho}{\partial z} + \frac{1}{R_e} \left[ \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} \right] \end{aligned} \quad (4)$$

$$\begin{aligned} \text{Energy: } & \frac{\partial E_t}{\partial t} + \frac{\partial(E_t u)}{\partial x} + \frac{\partial(E_t v)}{\partial y} + \frac{\partial(E_t w)}{\partial z} = -\frac{\partial(\rho u)}{\partial x} - \frac{\partial(\rho v)}{\partial y} - \frac{\partial(\rho w)}{\partial z} \\ & + \frac{1}{R_e P_r} \left[ \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} \right] \\ & + \frac{1}{R_e} \left[ \frac{\partial}{\partial x} (u\tau_{xx} + v\tau_{xy} + w\tau_{xz}) + \frac{\partial}{\partial y} (u\tau_{xy} + v\tau_{yy} + w\tau_{yz}) + \frac{\partial}{\partial z} (u\tau_{xz} + v\tau_{yz} + w\tau_{zz}) \right] \end{aligned} \quad (5)$$

## 2. Time-Independent solutions

For we solve equations, we use Newton potential. If we think the solution of Laplace equation,

$$u = \frac{C_1}{r^3} x, \quad v = \frac{C_1}{r^3} y, \quad w = \frac{C_1}{r^3} z, \quad r = \sqrt{x^2 + y^2 + z^2}, \quad \rho = \rho_0 \quad (6)$$

In this case, we solve Eq(1).

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = \rho_0 \left[ -\frac{3C_1}{r^5} (x^2 + y^2 + z^2) + \frac{3C_1}{r^3} \right] = 0 \quad (7)$$

Second point, in Eq(2)

$$\tau_{xx} = \frac{C_2}{r^6} x^2, \quad \tau_{xy} = \frac{C_2}{r^6} xy, \quad \tau_{xz} = \frac{C_2}{r^6} xz, \quad \dots, \tau_{ij} = \frac{C_2}{r^6} x^i x^j \quad (8)$$

$$\begin{aligned} & \frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z} \\ &= \rho_0 \left[ -\frac{6C_1^2}{r^8} (x^3 + xy^2 + xz^2) + \frac{C_1^2}{r^6} (2x + x + x) \right] = -\rho_0 \frac{2C_1^2}{r^6} x \\ &= -\frac{\partial \rho}{\partial x} + \frac{1}{R_e} \left[ \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} \right] \\ &= -\frac{\partial \rho}{\partial x} + \frac{1}{R_e} \left[ -\frac{6C_2}{r^8} (x^3 + xy^2 + xz^2) + \frac{C_2}{r^6} (2x + x + x) \right] \\ &= -\frac{\partial \rho}{\partial x} - \frac{1}{R_e} \frac{2C_2}{r^6} x \end{aligned} \quad (9)$$

Hence, in Eq(2), in Eq(3) and in Eq(4),

$$\begin{aligned} \frac{\partial \rho}{\partial x} &= \frac{2x}{r^6} \left( \rho_0 C_1^2 - \frac{C_2}{R_e} \right) = \frac{2C_3}{r^6} x, \quad C_3 = \rho_0 C_1^2 - \frac{C_2}{R_e} \\ \frac{\partial \rho}{\partial y} &= \frac{2y}{r^6} \left( \rho_0 C_1^2 - \frac{C_2}{R_e} \right) = \frac{2C_3}{r^6} y, \quad \frac{\partial \rho}{\partial z} = \frac{2z}{r^6} \left( \rho_0 C_1^2 - \frac{C_2}{R_e} \right) = \frac{2C_3}{r^6} z \end{aligned} \quad (10)$$

Therefore,

$$\rho = \frac{1}{2} \frac{C_3}{r^4}, \quad C_3 = \rho_0 C_1^2 - \frac{C_2}{R_e} \quad (11)$$

In Eq(5), if

$$E_t = E_0, \quad a_x = \frac{C_4}{r^3} x, \quad a_y = \frac{C_4}{r^3} y, \quad a_z = \frac{C_4}{r^3} z, \quad (12)$$

$$\begin{aligned} & \frac{\partial E_t}{\partial t} + \frac{\partial(E_t u)}{\partial x} + \frac{\partial(E_t v)}{\partial y} + \frac{\partial(E_t w)}{\partial z} = 0 \\ & -\frac{\partial(\rho u)}{\partial x} - \frac{\partial(\rho v)}{\partial y} - \frac{\partial(\rho w)}{\partial z}, \quad (\rho v, \rho u, \rho w) = \frac{1}{2} \frac{C_1 C_3}{r^7} (x, y, z) \\ &= 7 \frac{C_3 C_1}{2r^9} (x^2 + y^2 + z^2) - 3 \frac{C_3 C_1}{2r^7} = 2 \frac{C_1 C_3}{r^7} \end{aligned} \quad (13)$$

$$\frac{1}{R_e P_r} \left[ \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} \right] = 0 \quad (14)$$

$$\begin{aligned} \tau_{ij} &= \frac{C_2}{r^6} x^i x^j, \quad (\tau_{xj}v, \tau_{yj}u, \tau_{zj}w) = \frac{C_1 C_2}{r^9} (x^2, y^2, z^2) x^j \\ \frac{1}{R_e} &\left[ \frac{\partial}{\partial x} (u\tau_{xx} + v\tau_{xy} + w\tau_{xz}) + \frac{\partial}{\partial y} (u\tau_{xy} + v\tau_{yy} + w\tau_{yz}) + \frac{\partial}{\partial z} (u\tau_{xz} + v\tau_{yz} + w\tau_{zz}) \right] \\ &= \frac{1}{R_e} \left[ -9 \frac{C_1 C_2}{r^{11}} x^2 (x^2 + y^2 + z^2) + \frac{C_1 C_2}{r^9} (3x^2 + y^2 + z^2) \right. \\ &\quad - 9 \frac{C_1 C_2}{r^{11}} y^2 (x^2 + y^2 + z^2) + \frac{C_1 C_2}{r^9} (x^2 + 3y^2 + z^2) \\ &\quad \left. - 9 \frac{C_1 C_2}{r^{11}} z^2 (x^2 + y^2 + z^2) + \frac{C_1 C_2}{r^9} (x^2 + y^2 + 3z^2) \right] \\ &= \frac{1}{R_e} \left[ -9 \frac{C_1 C_2}{r^7} + 5 \frac{C_1 C_2}{r^7} \right] = -\frac{1}{R_e} \frac{2C_1 C_2}{r^7} \end{aligned} \quad (15)$$

Hence, Eq(5) is

$$\begin{aligned} 0 &= 2 \frac{C_1 C_3}{r^7} - \frac{1}{R_2} \frac{2C_1 C_2}{r^7}, \quad C_3 = \frac{C_2}{R_e} \\ C_3 &= \rho_0 C_1^2 - \frac{C_2}{R_e} = \frac{C_2}{R_e}, \quad C_2 = \rho_0 \frac{R_e}{2} C_1^2 \\ \rho &= \frac{1}{2} \frac{C_3}{r^4} = \frac{1}{2} \frac{C_2}{R_e} \frac{1}{r^4} = \rho_0 \frac{C_1^2}{4r^4} \end{aligned} \quad (16)$$

### 3. Conclusion

Therefore, solutions of Navier-Stokes equations(3-dimensional-unsteady),

$$\text{Pressure: } \rho = \rho_0 \frac{C_1^2}{4r^4}, \text{ Heat Flux: } q_x = \frac{C_4}{r^3} x, q_y = \frac{C_4}{r^3} y, q_z = \frac{C_4}{r^3} z$$

$$\text{Density: } \rho = \rho_0,$$

$$\text{Stress: } \tau_{ij} = \frac{C_2}{r^6} x^i x^j = \rho_0 \frac{R_e}{2r^6} C_1^2 x^i x^j$$

$$\text{Reynolds Number: } R_e,$$

$$\text{Velocity Components: } (u, v, w) = \frac{C_1}{r^3} (x, y, z)$$

$$\text{Total Energy: } E_t = E_0$$

## Reference

[1]<http://www.grc.nasa.gov>