

On the intrinsic paradox of the geometric point definition (solved using the Included Middle Logic) as the main cause of Euclid's postulate "inaccuracy", allowing the existence not only of non-Euclidean geometries but also of a new "t-metamathematics" used to redefine the basics of General relativity, Quantum field theory, Superstring theories and M-theory

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1st Motto: „The shortest distance between two points is under construction. ([Leo Aikman](#), American - Journalist (b.1908-d.1978); [[URL](#)])

2nd Motto: „The geometric Point aka <<the Circle with Zero radius>>, which I have named the indefinable God of geometry, the It „which is what It is”: the greatest mystery of all mathematics, quantum physics and General Relativity, the paradoxical nothingness from which all geometry was born and on which it stands; the Point brings mathematics so close to a religion (a dual religion of The Zero and The Infinite) as both are essentially based on a mystery by definition... maybe the same mystery of how <<God created the world from nothingness>> and how <<God is more what He is not (more potentiality) than what He is (reality)>>. In other words, all mathematics and physics and be regarded as as triple metaphysics of the point, zero and infinity."([Andrei-Lucian Drăgoi](#) ^{[[URL-CV](#)]}, MD pediatrician specialist and independent researcher and reviewer)

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Abstract (with abbreviations and wiki-like URLs)

This paper brings to attention the intrinsic paradox of the [geometric point \(GP\)](#) definition, a [paradox](#) solved in this paper by using [Stéphane Lupasco's Included Middle Logic \(IML\)](#) (which was stated by [Basarab Nicolescu](#) as one of the three pillars of [transdisciplinarity \[TD\]](#)) and its extended form: based on IML, a new “t-metamathematics” ([tMM](#)) (including a t-metageometry[[tMG](#)]) is proposed, which may explain the main cause of [Euclid's parallel postulate \(EPP\)](#) “inaccuracy”, allowing the existence not only of [non-Euclidean geometries \(nEGs\)](#), but also the existence of new EPP variants. tMM has far reaching implications, including the help in redefining the basics of [Einstein's General relativity theory \(GRT\)](#), [quantum field theory \(QFT\)](#), [superstring theories \(SSTs\)](#) and [M-theory \(MT\)](#).

Keywords (including a list of main abbreviations): [geometric point \(GP\)](#); [Stéphane Lupasco's Included Middle Logic \(IML\)](#); [Basarab Nicolescu](#), [transdisciplinarity \(TD\)](#); “t-metamathematics” ([tMM](#)); t-metageometry ([tMG](#)); [Euclid's parallel postulate \(EPP\)](#); [non-Euclidean geometries](#); new EPP variants; [Einstein's General relativity theory \(GRT\)](#); [quantum field theory \(QFT\)](#); [superstring theories \(SSTs\)](#); [M-theory \(MT\)](#);

Important note (1). This atypical [URL](#)-rich paper (which maximally exploits the layer of hyperlinks in this document), chooses to use Wikipedia links for all the important terms used. **The first (main) motivation** for this approach was that each Wikipedia web-article contains all the main reference (included as endnotes) on the most important terms used in this paper: it simply the most practical way to cite entire collections of important articles/books without using an overwhelming list of footnote/endnote references. **The second motivation** (for using Wikipedia hyperlinks directly included in keywords) was to assure a “click-away“ distance to short encyclopedic monographs on all the (important) terms used in this paper, so that the flow of reading to be minimally interrupted. **The third motivation** would be that it is an ideal format for a digital online (not printed) scientific paper in my point of view. Written science implies a mesh of interconnected concepts so that the html-like “URL-rich” format is an ideal tool: the standard format of the present published scientific papers is rigid and mainly adapted for offline printed format; however, more and more people prefer to read directly in digital format (PCs, laptops, tablets, phones, e-books, e-readers etc) which offers the possibility to access all the facilities of such format (including URLs).

Important note (2). This paper also exploits the advantages of the hierarchic tree-like model of presenting informational content which is very easy to be kept updated and well organized.

I. INTRODUCTION

1. **The definition of the geometric point (GP)**. In [Euclidean geometry](#), the [geometric point \(GP\)](#) is a [geometrical primitive](#), as it is not defined based on previously defined concepts, but GP is “only motivated informally, usually by an appeal to [intuition](#) and everyday experience” ^[URL]. GP is a fundamental concept upon which the geometry is built. GP is defined only by some axioms, that it must satisfy: one of these states that any GP has 0 dimensions (**0D**), so that it has zero length/area/3D-volume/n-volume (n dimensional [hyper]volume): GP is essentially defined to simply capture the notion of a unique location in [Euclidean space](#) ^[URL]. GP is one of the most fundamental objects of [Euclidean geometry](#) and was originally defined by [Euclid](#) "**that which has no part**" (in the sense of an indivisible/[irreducible](#) object). Euclid's GP postulation was neither complete nor definitive, as he occasionally assumed facts about GPs that did not follow directly from his axioms, such as the ordering of points on the line or the existence of specific points: however, modern expansions of geometry removed these assumptions. There are several (non-equivalent) definitions of the [mathematical/physical dimension of a space \(MDS\)](#) ([vector space dimension](#), [topological dimension](#), [Hausdorff dimension](#)) which is generally defined as **the minimum number of coordinates needed to specify the exact position of any GP within that (mathematical/physical) space**: in all of these common definitions, GP is 0D ^[URL]. The GP notion is strongly (and deeply) related to the concepts of [infinity](#), [infinitesimal](#) and [zero](#) (as also explained in this paper).
2. **Geometrical systems (GSs) without GPs**. Although GP is generally considered a fundamental notion in mainstream geometry (geometrical system) and topology, there are some [geometrical systems \(GSs\)](#) (like [noncommutative geometry](#) and [pointless topology](#)) which don't use GPs but use [pointless/pointfree space](#)" (**PLS**), which PLS is not defined as a [set](#) of GPs, but via some algebraic or logical structure (an algebra of [continuous functions](#) or sets, which looks like a well-known [function space](#) on the set): such structures generalize well-known [function spaces](#) in a way that the operation "take a value at this point" may not be defined. respectively.
3. **The point-like masses and electromagnetic (EM) charges used in the Quantum Field Theory**. In [classical electromagnetism \(cEM\)](#) but also in [quantum field theory \(QFT\)](#), electrons (and all the other QFT-theorized elementary particles [**EPs**] respectively) are idealized as GPs with possible non-zero rest energies/masses/EM-charges: the physicist [Paul Dirac](#) was the first to introduce his ([Dirac](#)) [delta function \(DF\)](#) (aka "**δ function**") which is essentially a [generalized function](#) on the [real \(number\) line](#) that takes the value 0 everywhere except for the 0 argument, with an integral equal to 1 over the entire real line. DF is usually regarded as an infinitely high and thin spike at the 0-origin (of the real line), with total area (the integral of DF) equal to 1 under the spike, which spike physically represents an idealized point-like EP with possible [rest mass/energy](#), [EM charge](#), [weak charge](#) or [color charge](#). ^[URL].

II. ON THE INTRACTABLE INTRINSIC PARADOX OF THE GEOMETRIC POINT DEFINITION

1. **Preliminary discussion on generic infinitesimals**. This paper proposes a subtle (although conventional) distinction between the generic [infinitesimal](#) positive [integer number](#) (*from \mathbb{Z}^+ set*) $\boxed{1/\infty_{\mathbb{Z}^+} > 0}$, the generic infinitesimal positive [rational number](#) (*from \mathbb{Q}^+ set*) $\boxed{1/\infty_{\mathbb{Q}^+} > 0}$ and the generic infinitesimal positive [real number](#) (*from \mathbb{R}^+ set*) $\boxed{1/\infty_{\mathbb{R}^+} > 0}$ so that: $\boxed{0 < (1/\infty_{\mathbb{R}^+}) < (1/\infty_{\mathbb{Z}^+}) < (1/\infty_{\mathbb{Q}^+)}$ and

$(0/\infty_{\mathbb{R}+}) = (0/\infty_{\mathbb{Z}+}) = (0/\infty_{\mathbb{Q}+}) = 0$. As $1/\infty_{\mathbb{R}+} (> 0)$ is conventionally established (by this paper) to be “the closest to zero generic infinitesimal” (closer to zero than any other generic positive integer or rational infinitesimal), it shall be briefly noted as $[1/\infty > 0]$, with the important mention that only the generic (real, rational, integer and natural at the same time) infinitesimal $0/\infty$ reaches zero, such as $[0/\infty = 0 \cdot (1/\infty) = 0]$.

- 2. Definitions (and notations) of two major GP types.** (1) A “zero-dimensional (0D) GP” (abbreviated as **zGP**) is defined as having zero length $L_{zGP} (= 0)$ and zero surface/volume/n-volume implicitly. (2) An “infinitesimal GP” with an infinitesimal length (abbreviated as **iGP**) is defined to have a non-zero length $L_{iGP} (= 1/\infty) \begin{matrix} \text{(def.} \\ > 0 \end{matrix}$.

- 3. The (intractable) intrinsic paradox of the GP definition (IP-GP).** IP-GP launched in this paper briefly states that:

- a. “(1) If GP is defined as a zGP with zero length ($L_{zGP} = 0$) (as all conventional GP definitions state), one CANNOT build a segment (seg) with a (non-zero) length $L_{seg} > 0$ EVEN WHEN using an infinite number of zGPs ($N_{zGPs(seg)} = \infty$) <<glued>> to one another, because

$$[N_{zGPs(seg)} \cdot L_{zGP} = \infty \cdot 0 = 0] < L_{seg}$$

(self-contradiction in those geometries which define (n>0)-dimensional objects [like 1D lines, 2D surfaces, 3D volumes and n-volumes] to be “composed” from zGPs).

- b. (2) Alternatively, if GP is defined as a iGP with an infinitesimal length ($L_{iGP} = 1/\infty$) then the number of GP (Euclidean) dimensions of that iGP (ND_{iGP}) would be larger than zero ($ND_{iGP} > 0$) which implies that iGP would actually be either (at least) a 1D segment (seg) with infinitesimal length $L_{seg} = 1/\infty$ or a fractal dust (aka [Cantor dust/set](#)) (composed of zGPs) with [fractal dimension](#) (D_{iGP}) equal to ND_{iGP} ($D_{iGP} = ND_{iGP}$), which D_{iGP} can have a value in the open interval (0,1) or even a non-integer value $D_{iGP} > 1$ ($\overline{1.x}D, \overline{2.x}D \dots \overline{i.x}D$ Cantor dusts) (self-contradiction, as a GP cannot be an infinite set of zGPs).”

- c. **Cantor dust - important redefinition.** Based on IP-GP (which is essentially a GP ambiguity “born from” the zGP-iGP duality), even Cantor dusts can be separated in two major classes:
- z-Cantor dusts (zCDs)**, which are stated to be composed from zGPs (so that they correspond to the “standard” CDs);
 - i-Cantor dusts (iCDs)**, which are stated to be composed from iGPs as if they are a kind of “incomplete” (incompletely “splitted”) CDs.

- d. **Remark (1) – the zero-infinitesimal duality(/polarity) (ZID).** As one may easily notice, IP-GP is essentially generated by the fact that, on the [real number line](#), there are actually an [infinite](#) number of distinct [reals](#) between the real number 0 ([zero](#)) and the positive real infinitesimal $1/\infty$, so that: $[1/\infty > 0] \Rightarrow [1/\infty \neq 0]$, even if $[1/\infty \cong 0]$ and [asymptotic limit](#) $\lim_{n \rightarrow \infty} (1/n) = 0$. This polarity

between zero (0) and the real infinitesimal $1/\infty (> 0)$ shall be called the **zero-infinitesimal duality(/polarity) (ZID)**. Clearly enough, IP-GP is also deeply connected to [Zeno’s Dichotomy paradox](#) (but also to other paradoxes proposed by the [pre-Socratic](#) Greek philosopher [Zeno of Elea](#),

- c. 490–430 BC) which states that: “**That which is in locomotion must arrive at the half-way stage before it arrives at the goal. Suppose Homer wishes to walk to the end of a path. Before he can get there, he must get halfway there. Before he can get halfway there, he must get a quarter of the way there. Before traveling a quarter, he must travel one-eighth; before an eighth, one-sixteenth; and so on: this description requires one to complete an infinite number of tasks, which is an impossibility.**” (as recounted by [Aristotle](#) in his [Physics](#) VI:9, 239b10). ZID additionally means that “**even the (real) infinity isn’t sufficient to turn an infinite number of zGPs into an continuous segment-like iGP, or into a segment, line, plane etc**”: this paper proposes another type of logic (including an alternative type of zero, of infinity and of GP) to solve this paradox, as presented next.
- e. **Remark (2).** As zGP is 0D, it is clearly an imaginary/virtual object that **exists and non-exists in the same time**, because “something” that has no-length/area/volume/n-volume has all the conditions of non-existence in both imaginary or real space (the conditions of a “non-object”): however, its non-existence doesn’t prevent any human being to work with such a virtual non-object as if it was a virtual object. The same with the 1D line and the 2D plane which are objects and non-objects as the same time, as they both exist and non-exist, because something with zero width or zero thickness cannot exist (even if it has non-zero length OR non-zero length plus non-zero width; this happens because a zero-length dimension nullifies all the others by multiplication: $L \cdot 0 = 0$ [for a zero-width line], $L \cdot l \cdot 0 = 0$ [for a zero-thick plane]). **The simultaneous existence and non-existence** of GP contradicts the “standard”/classical [excluded middle principle \(EMP\)](#) and invokes the modern [included middle principle \(IMP\)](#) ^[URL2] (aka “**the included third principle [ITP]**”) firstly formulated by the Romanian philosopher [Stefan Lupasçu](#) (also known as “[Stéphane Lupasco](#)”, as he lived for many years in France and published there) in his book “The Principle of Antagonism and the Logic of Energy” (first published in 1951). In his book called “Logic and contradiction” (“Logique et contradiction, P. U. F., Paris, 1947), Lupasco initially proposed the essential [Asymptoticity Principle \(conjecture\) \(AP\)](#) ^[URL2,URL3,URL4] which states that: “**Given any \boxed{A} defined as the representation of any real physical entities (objects, processes, events) and non-physical entities, NO real process implying \boxed{A} goes to the idealized, abstract limits of (binary) classical logic**”. Using AP, Lupasco essentially changed the classical affirmation(A)-negation (absolute) duality (the pair of conjugates A and $\neg A$) to the actualization-potentialization (relative) duality (the pair of conjugates $\boxed{A^{\approx}} \neq \boxed{A}$ and $\boxed{\neg(A^{\approx})} \neq \boxed{\neg A}$) (so that the “true” concept noted as \boxed{A} is replaced by the “actual” concept $\boxed{A^{\approx}}$ [alternative sign proposed by the author] and the “false” concept noted as $\boxed{\neg A}$ is replaced by the “potential” concept $\boxed{\neg(A^{\approx})}$ [alternative sign proposed by the author]). AP can also be formulated as: “**No process of actualization or potentialization of any generic element \boxed{A} goes to 100% completeness [in none of the possible/imaginable spaces/times/spacetimes of that process associated with that element \boxed{A}].**” Lupasco was inspired by [Heisenberg's uncertainty principle \(HUP\)](#) (which was first proposed by the German physicist [Werner Heisenberg](#) in 1927). **AP alternative formulation** Every real phenomenon, element or event \boxed{e} is always associated with an anti-phenomenon, anti-element or anti-event $\boxed{\neg e}$, such that the actualization of \boxed{e} entails the potentialization of $\boxed{\neg e}$ and vice versa, alternatively, without either ever disappearing completely. Aspects of phenomena that are generally considered independent can thus be understood as being in the dynamic opposition: one is “actualized” AND the other is “potentialized”. **The included middle logic (IML)**. IMP was based on AP and both AP and IMP were the starting points of a new included middle logic (IML) which implies 3 states of the same assertion A: (1) A (the assertion) or the “actual” concept

$\boxed{A^\approx} \neq \boxed{A}$, (2) non-A (the negation of assertion A) or the “potential” concept $\boxed{\neg(A^\approx)} \neq \boxed{\neg A}$

AND (3) the “T” state defined as
$$T = \boxed{A \wedge (\neg A)} \stackrel{IMP}{=} \boxed{A^\approx \wedge (\neg A^\approx)} \stackrel{IMP}{=} \begin{cases} 1, & \text{if } A = 1 \\ 0, & \text{if } A = 0 \end{cases}$$
 (the so-

called “included middle/third” labeled “T” from “third”; “1” and “0” notations stand for the logical attributes “true” and “false” respectively) in which A (or A^\approx) and non-A (or $\neg(A^\approx)$) can be **true at the same time OR false at the same time**, so that $T \in \{0,1\}$, which puts IMP in contrast with

the classical EMP which states that
$$T = \boxed{A \wedge (\neg A)} \stackrel{EMP}{=} \begin{cases} 0, & \text{if } A = 1 \\ 0, & \text{if } A = 0 \end{cases}$$
. **The T-state meaning and**

the levels of reality, T is not a “middle/third” in the sense of being “between” A and $\neg A$, BUT rather in the sense that there is a 3rd position, another superior level of reality (alias “**reality level**” [RL]) which contains both A and $\neg A$: in other words, A and $\neg A$ exist at one inferior RL and T exists at another superior RL so that at the level of A and $\neg A$, there are only the two contradictory possibilities, BUT at a higher RL, there is a larger domain where both elements could be possible.

- f. **Remark (3).** IMP was further developed by the French-born American chemist and philosopher [Joseph E. Brenner](#) ^[URL2] and by the Romanian theoretical physicist [Basarab Nicolescu](#) ^[URL2,URL3-French-wiki,URL4-Romanian-wiki,URL5-work]. The continued the work of Lupasco and defined the concept of [TransDisciplinarity \(TD\)](#) ^[URL2,URL3-French wiki] [1,2,3] as supported by three different (BUT closely related) major “pillar”-concepts: (A) [Complexity](#); (B) [Levels of reality](#) (alias [reality-levels \[RLs\]](#)) (C) [IML](#) (which is quite a “model of thinking” for TD). **Note (1).** IML overcomes binary [dualism](#) and simple [linear causality](#), revealing a complex and multi-dimensional reality based on [non-linear causality](#): IML is actually a robust logic having properties of both determinacy/indeterminacy, universal/ particular, part/whole and actuality/possibility (potentiality). **Note (2).** IML is essentially an analytic approach of the “[Unity of opposites](#)” (UO), which is a central category of [dialectics](#) deeply related to the notion of [non-duality](#) (aka nondualism): UO defines a situation in which “**the existence or identity of a thing (or situation) depends on the co-existence of at least two conditions which are opposite to each other, yet dependent on each other and presupposing each other, within a field of tension.**” ^[URL]. **Cite from** the American philosopher and economic theorist (at the New School for Social Research in New York) [Melanie Swan](#). “**Included Middle is a position of greater complexity and possibility for addressing any situation. Conceiving of a third space that holds two apparent contradictions of a problem is what the Included Middle might bring to contemporary challenges in consciousness, artificial intelligence, disease pathologies, and unified theories in physics and cosmology.**” ^[URL]
- g. **Remark (4).** IMP was also supported by [Werner K. Heisenberg](#): Heisenberg noticed that EMP (which seemed to hold at large/macro length scales of our universe) doesn’t hold at the lowest (known) length scales (of our universe) and he pointed out how EMP has to be modified in [quantum mechanics \(QM\)](#), to accommodate the [quantum superposition principle \(QSP\)](#) (which states that “**any two or more [quantum states \[Qs\]](#) can be added together (“superposed”) and the result will be another valid quantum QS; and conversely, that every QS can be represented as a sum of two or more other distinct Qs.**”), the [quantum probability \(QP\)](#) and the [wave-particle duality \(WPD\)](#) (which states that: “**every particle or quantum entity may be partly described in terms not only of particles, but also of waves**”: WPD essentially states the inability of the classical concepts “particle” or “wave” to fully describe the behavior of quantum entities); an additional logic term was thus needed to describe this third possible situation, hence the **Included Middle**.

- h. **Remark (5).** The author of this paper has also extended IML by generalization and proposed a “generalized IML (GIML)” based on a “generalized IMP (GIMP)” [4,5]. IMP had already been applied in a variety of scientific domains and has a substantially wider potential applications.
- i. **Remark (6).** In a specific sense, IMP also appears to be applicable on the [symmetry breaking phenomenon](#) in physics, in which the [physical energy](#) magnitude levels (which are actually physical [information](#) density or [entropy](#) magnitude levels) can be regarded as RLs, so that the (quantum) elementary particles (EPs) of the [Standard model \(of particle physics\)](#) (SM) appear different at an inferior-rank RL, but are strongly related to one another by being the “faces”(“fragments”/“crops”/“puzzle-pieces”) of the same unified EP at a superior-rank RL. **For example**, the [photon](#) (a super-light EP with theoretical/predicted zero rest mass and only having non-zero relativistic energy/mass) and the [W/Z bosons](#) (very heavy EPs with quite large non-zero rest masses) all appear with quite “opposite” properties at some specific energy level (corresponding to a specific RL), BUT unify in a “super”-EP (the [electroweak field/force/interaction quanta](#)) at some superior energy level (superior-rank RL), as initially predicted by the electroweak theory (proposed by physicists [Sheldon Glashow](#), [Abdus Salam](#) and [Steven Weinberg](#)) and then experimentally demonstrated (in 1973) by: (1) the existence of [neutral currents](#) of Z-bosons (in neutrino scattering experiments conducted in the heavy liquid bubble chamber detector of CERN called “[Gargamelle](#)”, in operation between 1970 and 1979); (2) the discovery of the [W and Z gauge bosons](#) (in 1983) in proton–antiproton collisions by [UA1](#) and [UA2](#) collaborations at the converted [Super Proton Synchrotron](#).

j. **CHECKPOINT CONCLUSION.** IP-GP is essentially a GP definition ambiguity (generated by the zGP-iGP duality/polarity), with zGP and iGP being in “asymptotic opposition” and being potentially unifiable in a 3rd “T-state”, which shall be abbreviated as “TGP” (according to AP and IMP of IML)

- i. All the geometrical primitives (abbreviated as “g-primitives”) can be redefined as based on TGP and will be briefly named “tg-primitives”, including the TGP-based Cantor dust which is abbreviated as “tCD”.
- k. **Explanation (1).** This paper proposes that IP-GP (which is essentially the manifestation of TGP as a zGP-iGP ambiguity and asymptotic opposition), may actually explain the impossibility to demonstrate [Euclid’s \[parallel\] postulate](#) (EPP) (the 5th axiom from the list of [Euclidian axioms](#), which EPP does not specifically talk about [parallel lines](#), but it is only a postulate related to [parallelism](#); Euclid gave the definition of parallel lines in Book I, Definition 23, just before the five postulates ^[URL]). EPP is widely known in its equivalent formulation proposed by the Scottish mathematician [Jown Playfair](#), which states that: “**In a [2D] plane, given a line and a point not on it, at most one line parallel to the given line can be drawn through that point.**”. (known as [Playfair's axiom](#) [PA]).

- l. **zMPA - A proposed modified Playfair's axiom (MPA) based on zGP.** Based on $\boxed{1/\infty > 0} \Rightarrow \boxed{1/\infty \neq 0}$ (which is also the essence of IP-GP) and on the zGP definition, this paper proposes a “z” modified PA (zMPA) such as: “*In any 2D plane, given a line L_0 and a zGP “A” not on it (both localized in that given 2D plane), at least two lines L_1 and L_2 parallel to the given line L_0 can be drawn through that zGP “A”: the angle between L_1 and L_0 would be $\boxed{Ang_1 = \sphericalangle(L_1, L_0) = \arctan(1/\infty) > 0}$ AND the angle between L_2 and L_0 would be $\boxed{Ang_2 = \sphericalangle(L_2, L_0) = \arctan(0) = 0}$: it is obvious that L_1 and L_2 would not meet L_0 to infinity, so that they would be parallels to L_0 but they would also be distinct to each other, as $\boxed{Ang_1 \neq Ang_2}$ ”*

- m. **iMPA - A proposed modified Playfair's axiom (MPA) based on iGP (iMPA).** iMPA proposed in this paper states that: “*In any 2D plane, given a line L_0 and an iGP “A” not on it (both localized in that given 2D plane). (1) If that iGP “A” is interpreted as a 1D infinitesimal-length segment (S), at least two lines L_1 and L_2 parallel to the given line L_0 can be drawn through that iGP “A”: the angle between L_1 (containing S) would be $\boxed{Ang_1 = \sphericalangle(L_1, L_0) = \arctan(1/\infty) > 0}$ AND the angle between L_2 (also containing S) and L_0 would be $\boxed{Ang_2 = \sphericalangle(L_2, L_0) = \arctan(0) = 0}$; it is obvious that L_1 and L_2 would not meet L_0 to infinity, so that they would be parallels to L_0 but they would also be distinct to each other, as $\boxed{Ang_1 \neq Ang_2}$. (2) If that iGP “A” is interpreted as a Cantor dust (composed from an infinite number of zGPs), an infinite number of line-pairs parallel to the given line L_0 can be drawn through that iGP “A”: each of these line-pairs is stated to be composed of two distinct L_1 and L_2 parallel lines (both of these lines containing each zGP in part from that Cantor dust identified with that given iGP “A”) with $\boxed{Ang_1 = \sphericalangle(L_1, L_0) = \arctan(1/\infty) > 0}$ and $\boxed{Ang_2 = \sphericalangle(L_2, L_0) = \arctan(0) = 0}$.”*

- n. **Explanation (2).** This paper also proposes that it is this unified TGP concept (with its secondary zMPA and iMPA) that actually permits the imaginary projection of all [non-Euclidean geometries](#) into the 2D (flat) plane. The TGP-based iMPA retrodicts the existence of both [hyperbolic geometry](#) (aka [Bolyai–Lobachevskian geometry](#)) and the [elliptic geometry](#) (in 2D [2-space], 3D [3-space] or any other [n-space](#)), BUT also predicts the possibility of those non-Euclidean objects to be projected into the Euclidean (flat) 2D plane. **Cite from Henri Poincaré:** “The Geometry of [Lobatschewsky](#) [a name translated in English as “[Nikolai Lobachevsky](#)”]. —If it were possible to deduce [Euclid](#)’s [\[parallel\] postulate](#) from the several axioms, it is evident that by rejecting the postulate and retaining the [other \[Euclidian\] axioms](#) we should be led to contradictory consequences. It would be, therefore, impossible to find on those premises a coherent geometry. Now, this is precisely what Lobatschewsky has done. **He assumes at the outset that “several parallels may be drawn through a point to a given straight line”, and he retains all the other axioms of Euclid.** From these hypotheses he deduces a series of theorems between which it is impossible to find any contradiction, and he constructs a geometry as impeccable in its logic as Euclidean geometry.” ([Henri Poincaré, extract from pages 43-44](#) of the English translation variant of his book “[Science and hypothesis](#)” first published in 1905)
- o. **Observation (1).** zGP-iGP duality (and its TGP unified state) resembles the [particle-wave duality](#) of any [elementary particle \(EP\)](#) (also called “wavicle”=“wav[e]”+“[part]icle” as stated in [quantum mechanics](#): zGP would be the “particle”-state of the TGP and iGP would be the “wave”-state of the same TGP.

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4. The management of both zero-infinitesimal duality(/polarity) (ZID) and IP-GP.

- a. To manage the [zero-infinitesimal duality\(/polarity\) \(ZID\)](#) (between $1/\infty$ and 0), this paper proposes **three major distinct types of mathematics (each distinct type being actually a [metamathematics](#) -- the study of mathematics itself using mathematical methods):**
- i. the “**dualistic mathematics**” abbreviated as “**d-mathematics**” (all part of a “**d-metamathematic**”) in which $\boxed{1/\infty > 0} \Rightarrow \boxed{1/\infty \neq 0}$: d-metamathematic is based on the standard definition of the real infinite ∞ and of the real infinitesimal $1/\infty (> 0)$.

ii. the “**monadic mathematics**” abbreviated as “**m-mathematics**” (all part of a “**m-metamathematic**”) based on an “**imaginary**” (**im.**) (real) infinite called here “**im-infinite**”/ “**iminfinite**”/ “**iminfinity**” (or briefly “**imfinite**” / “**imfinity**” and abbreviated as “**imf**”), noted as ∞_{im} (so that “**im**” subscript won’t be confused with the imaginary part $i = \sqrt{-1}$) of [complex numbers](#)) and definable in at least two major distinct ways, both implying IML (in trying to solve ZID):

1. **imf1** is actually a set noted as ∞_{im1} and defined as infinite 1D matrix $\infty_{im1} = \{\infty_{im1(0)}, \infty_{im1(1)}, \infty_{im1(2)}, \infty_{im1(3)}, \dots, \infty_{im1(x)}\}$ (with $x \in \mathbb{R}$) such as $x / \infty_{im1(x)} = 0 (= 0 / \infty)$, for any $x \in \mathbb{R}$, so that $x = 0 \cdot \infty_{im1(x)}$, for any $x \in \mathbb{R}$, which is in clear contrast with $x / \infty > (0 / \infty) > 0$, for any $x(> 0) \in \mathbb{R}^+$ and also in contrast with $x / \infty \neq 0$, for any $x \in \mathbb{R}^*$. **Note that** all the elements of the ∞_{im1} set are “**distinct AND identical to each other at the same time**”, so that imf1 is an IML-based notion by definition.
2. **imf2** is a more “pretentious” imf alternative noted as ∞_{im2} which is not a set itself but is defined using a (necessary) new imaginary concept of **a multiple-valued real zero (mvRZ) (or an imaginary [im.] zero)** $0_{im} = \{0_0, 0_1, 0_2, \dots, 0_x\}$, with $x \in \mathbb{R}$ (with 0_{im} being also an infinite 1D matrix) so that: $x / \infty_{im2} = 0_x$, for any $x \in \mathbb{R}$, which is equivalent to $x = 0_x \cdot \infty_{im2}$, for any $x \in \mathbb{R}$ (for example: $0 / \infty_{im2} = 0_0$, $1 / \infty_{im2} = 0_1$, ..., $x / \infty_{im2} = 0_x$), with $0_0 \stackrel{def.}{=} 0$ (also by **mvRZ definition**) and the “imaginary integer part of 0_x of 0_x ” $im\ int(0_x) \stackrel{def.}{=} 0$ (defined as a new type of imaginary [integer-valued function](#) applicable on mvRZ only). **Note that** Imf2 definition is also in clear contrast with $x / \infty > (0 / \infty) > 0$, for any $x(> 0) \in \mathbb{R}^+$ and also in contrast with $x / \infty \neq 0$, for any $x \in \mathbb{R}^*$. All the elements of the 0_{im} set are stated to be “**non-zero (distinct to each other) AND zero (identical to each other) at the same time**”: that is why Imf2 is also an IML-based concept.
3. **Note.** Both imf1 and imf2 are notions similar to the infinity used in [complex analysis](#) ($\infty_{\mathbb{C}}$ or simply ∞), which has the property that $\infty_{\mathbb{C}} = z / 0$, with $z \in \mathbb{C}$ ^[URL]. However, Imf1 is defined as a set and Imf2 has a clear distinction to $\infty_{\mathbb{C}}$, which is mvRZ.

b. To manage IP-GP, this paper also proposes **three major distinct types of geometries (each distinct type being actually a [metageometry](#) concept, which may be considered a branch of [metamathematics](#) -- the study of mathematics itself using mathematical methods):**

i. “**z-geometries**” (all part of a “**z-metageometry**”) based on zGPs;

1. In z-metageometry, zGP is considered a geometrical primitive notion (abbreviated as “**g-primitive**”): all the non-zGP geometrical shapes (lines, planes, volumes, n-volumes) are also considered g-primitives independent to (thus NOT “based on”) zGP and to each other, because they are not reducible to zGPs (as they cannot be

splitted to zGPs) but only to iGPs: all these non-zGP g-primitives could be reducible to zGP only if z-metageometry would be “injected” with the (IML-based) *imf1* and *imf2* notions.

2. In z-metageometry, only z-Cantor dusts (**zCDs**) are reducible to an infinite number of zGPs: at the same time, remember that zCDs were stated to be indistinguishable from an iGP.
3. in z-metageometry, [calculus](#) is also “fatally stroked” by IP-GP, because zGPs cannot compose any curve/surface/volume **AND** a curve cannot be splitted into zGPs neither, BUT zGPs can only be splitted in iGPs, which iGPs don't allow a “true” unique [derivative](#) in a “unique GP”, because an iGP is: (1) either an infinitesimal-length 1D (straight) line-segment (which doesn't allow any derivatives) OR (2) a zCD/iCD composed from an infinite number of zGPs/iGPs (which implicitly allows an infinite number of derivatives OR none if considered that a iGP is a discontinuous iCD or zCD which doesn't allow any derivatives at all)

ii. “i-geometries” (all part of an “**i-metageometry**”) based on iGPs;

1. In i-metageometry, iGP is considered a g-primitive to which all the other non-iGP geometrical shapes (lines, planes, volumes, n-volumes) can be reduced to (by infinite splitting to infinitesimal parts), so that they all may be considered iGP-based shapes (with i-GP being the only g-primitive of this i-metageometry).
2. In i-metageometry, only i-Cantor dusts (**iCDs**) are reducible to an infinite number of iGPs: at the same time, remember that zCDs were stated to be indistinguishable from an iGP.
3. However, in i-metageometry, [calculus](#) is also “fatally stroked” by IP-GP, because even if shapes can be splitted into iGPs, iGPs don't allow a “true” unique [derivative](#) in a “unique GP”, because an iGP is: (1) either an infinitesimal-length 1D (straight) line-segment (which doesn't allow any derivatives) OR (2) a zCD/iCD composed from an infinite number of zGPs/iGPs (which implicitly allows an infinite number of derivatives OR none if considered that a iGP is a discontinuous iCD or zCD which doesn't allow any derivatives at all)

iii. AND (hybrid) “t-geometries” (all part of a “**t-metageometry**”) based on TGP based on IML (due to TGP being previously defined as an IML-based concept);

1. In t-metageometry, TGP is considered a g-primitive notion (abbreviated as “**g-primitive**”): all the non-TGP geometrical shapes (lines, planes, volumes, n-volumes) are also considered g-primitives “**independent-and-non-independent**” to TGP (at the same time, as allowed by IML), because they are “**reducible-and-not-reducible**” (at the same time, as allowed by IML) to TGPs, by using (IML-based) *imf1* and *imf2* (previously defined) notions.
2. In i-metageometry, only t-Cantor dusts (**tCDs**) are also “**reducible-and-not-reducible**” (at the same time, as allowed by IML) to an infinite number of TGPs.
3. “**Happily**”, only in t-metageometry [calculus](#) is truly allowed, because TGPs can compose any TGP-based curve/surface/volume **AND** any such shape can be also splitted into TGPs: such a “**zero-and-non-zero dimensional**” TGP may allow a “unique-and-non-unique” [derivative](#) in a “unique-and-non-unique” TGP.

c. Note(1). T-metageometry partially contradicts Poincare when he argues that geometry is actually: (1) just a convenient... convention inspired from the various shapes of the solids in nature (solids on which we can “rest our eyes” and of which we build our measurement tools) [see the **Addendum section** of this paper, for extensive citing from [Henri Poincaré](#)]; (2) and geometric axioms are

actually conventional definitions...in disguise (ideas expressed in his book "[Science and hypothesis](#)", Chapter III, in the last subchapter called "**On the nature of axioms**").

- i. This present paper argues that mathematics in general is not just a complex "convention", but is essentially a **quantitative metaphysics** which should permanently "struggle" to eliminate as many self-contradictions as possible: in this view, t-metamathematics may be also regarded as a "more clean" metamathematics, from which one more paradox has been "erased" with this important transdisciplinary tool called "**Included Middle Logic**" (IML).
- ii. T-metageometry essentially states and predicts that all conventions (including the conventional parts of geometry, physics or other sciences) are "doomed" to (gross or subtle) self-contradictions, tautologies etc (that will be discovered sooner or later) and that IML is an essential "ingredient" for those conventions to fully eliminate their inner self-contradictions.
- iii. **Conjecture (and prediction) (1).** This paper also predicts and conjectures (with the previous arguments) that IML (including the generalized IML [GIML], proposed by the author in past papers) may be an essential "ingredient" for any "mature" mathematics (including any "mature" geometry) and metamathematics (including any "mature" metageometry).
- iv. **Conjecture (and prediction) (2).** This paper also conjectures (and predicts) that both infinity and zero cannot be defined without self-contradiction unless using GIML[4].

5. Discussion (1). As both are essentially based on the imperfect definition of infinity (which infinity is conjectured by this paper to can only be defined using the **Included Middle Logic [IML]**), Note that IP-GP has some similarities to the notorious class of [omnipotence paradoxes](#) (OPs) (also related to the [irresistible force paradox](#)), with its most well-known "**paradox of the stone**" (named "**stone**"-OP in this paper) stating that: "**Could God create a stone so heavy that even He could not lift it?**" Answers:

- a. (1) If God can create that (infinitely heavy) stone, then he is not omnipotent (because He cannot lift that infinitely heavy stone that He had created);
- b. (2) if God can create the stone, then he is not omnipotent either (because He cannot create that infinitely heavy stone)
- c. **Note.** This specific "stone" OP is also considered vulnerable to objections based on the physical nature of [gravity](#) (such as how the [weight](#) of an object depends on what the local gravitational field is). There are alternative OP-variants that do not involve such difficulties:
 - i. **The "geometric" OP:** "If given the axioms of Euclidean geometry, can an omnipotent being create a triangle whose angles do not add up to exactly 180 degrees?"
 - ii. **The "prison" OP:** "Can God create a prison so secure that he cannot escape from it?"

d. **Two possible solutions to OPs.** This paper proposes two major distinct possible ways to solve this paradox:

- i. **OP-solution no.1 (using the "infinity" concept and IML).** The concept of "God" or "omnipotent being" can be redefined so that God would be not as a single entity, but an infinite-hierarchy entity defined as "**a set of Gods(i)**" (with various ascendant ranks indexed with an integer index i, from 1 to infinity), in which each God(i) can create a very heavy stone which cannot be lifted by that God(i) (who created it) but can be lifted by any other God(j>i) (which Gods(j>i) are all part of the same "set of Gods(i)") and so on: on the other hand (the alternative interpretation), a God(i) cannot create a stone which He cannot lift, BUT any God(j>i) can create a stone that cannot be lifted by that God(i). **An IML-based alternative formulation of the "stone"-OP.** In other words, "**God can-and-cannot create a stone which he can-and-cannot lift.**": a analogous formulation can be created for the other OP variants too. This redefinition was also proposed in one of past paper published by the author as "**The Infinite (number of) (inter)-Nested Creator-Minds Conjecture (INCM-C)**" [5]

- ii. **OP-solution no.2 (using both the “zero”/“nullification” and “infinity” concepts and IML).** God can be redefined so that **God himself may “fully” turn into an infinitely heavy unliftable stone/inescapable prison/paradoxical geometry etc, so that He wouldn’t no more be able to lift that stone, as He wouldn’t no more exist as a God (but only as a stone):** in other words, God can be defined so that **He may be able to nullify his omnipotence** and fully convert that omnipotence into some specific created object (which object, by its maximal weight/security etc would be another form of omnipotence); this solution also implies **an alternative definition of omnipotence** (redefined as **“the capacity of an omnipotent being to fully turn from one form to any other form”**) using both “zero” and “infinity” concepts and also based on IML, because this type of (redefined God) would also “be-able-and-not-be-able” to lift the stone (into which He fully transformed, nullifying any “lifting” capacity/prerogative)
- iii. **Comment.** It is clear that all these types of paradoxes are based on the classical [Excluded Middle Principle](#) (EMP), so that they can be easily absorbed by the more general IML and GIML.

6. **Discussion (2).** TGP, zMPA, iMPA OP-solutions (given in this paper) can all be considered “pieces” of the t-metamathematics (tMM) “puzzle”: moreover, the author of this paper also launched in 2017 a Vertical (generalization) of the Binary Goldbach Conjecture (BGC) (abbreviated as [“VBGC”](#)) which can be also considered a piece of tMM, more specifically a piece of t-metaarithmetics (tMA), because it states a potential infinite class of conjectures named VBGC(a,b) (with a and b being finite integers) stronger than BGC. [6,7,8]
7. **Discussion (3).** The [EMP](#)-generated IP-GP and [Zeno’s dichotomy paradox](#) also pose problems to [Quantum field theory](#) (QFT) and [Quantum mechanics](#) (QM) which both model (quantum) [elementary particles](#) (EPs) from the [Standard Model](#) (SM) as 0D point-like/zGP-like entities: this zGP-based approach strongly contradicts the 4D [spacetime continuum](#) conjectured by [Einstein’s General relativity theory](#) (GRT) (and modeled as a curved generalization of [Minkowski space](#)): it is clear that a zGP-like 0D-EP will have to accomplish an infinite number of steps (each step being defined as a zGP in such a spacetime continuum) thus taking an infinite amount of time (if the maximum speed in the universe is limited by GRT and QFT to the [speed of light in vacuum](#)) to reach from a zGP “A” to another distinct zGP “B” from the same spacetime continuum: this contra-argument obviously resembles [Zeno’s dichotomy paradox](#) (ZDP) and cannot be solved other than by IML or GIML; a step-by-step movement of a 0D-EP in a continuum spacetime cannot be accomplished in a finite time other than by a “teleportation”-like movement only, in which the zGP-like EP can surpass iGPs (made from an infinite number of zGPs) and not only zGPs. **In conclusion**, the IML-based tMM and TGP-based t-metageometry (tMG) can “absorb” both QFT and GRT by avoiding such paradoxes. The TGP-based tMM/tMG can also “absorb” all types n-dimensional [Hilbert spaces](#) and [Minkowski spaces](#) without generating paradoxes.
8. **Discussion (4).** As [superstring theories](#) (SSTs) and [M-Theory](#) (MT) also use 1D strings (aka “1-strings”) and 2D branes (aka “2-branes”) respectively, they cannot avoid IP-GP, BUT can be easily “absorbed” by the TGP/IML-based tMM/tMG. Additionally, SSTs and MT can be both unified in a “Volume”-Theory/V-Theory(VT) (which VT uses 3D branes [aka “3-branes”]/ “filled” 3D volumes: a special type of VT was also proposed by the author of this paper in an article called ” (Toy-model) A Simple “Digital” Vacuum Composed of Space Voxels with Quantized Energetic States” (aka “Digital Vacuum Toy-Model” [[DVTM](#)]) [9].

III. ADDENDUM SECTION

1. Cites from Henri Poincaré: [extracts from pages XX-XXI, 58-59, 82, 102, 152, 153-154, 155](#) of the English translation variant of his book “[Science and hypothesis](#)” (first published in 1905). In this quotes, Poincaré argues that geometry is mainly a convenient complex convention.
- a. [pages XX-XXI from Author’s preface] “Space is another framework which we impose on the world. Whence are the first principles of geometry derived? Are they imposed on us by logic? Lobatschewsky, by inventing non-Euclidean geometries, has shown that this is not the case. Is space revealed to us by our senses? No; for the space revealed to us by our senses is absolutely different from the space of geometry. Is geometry derived from experience? Careful discussion will give the answer—no! We therefore conclude that the principles of geometry are only conventions; but these conventions are not arbitrary, and if transported into another world (which I shall call the non-Euclidean world, and which I shall endeavour to describe), we shall find ourselves compelled to adopt more of them. In mechanics we shall be led to analogous conclusions, and we shall see that the principles of this science, although more directly based on experience, still share the conventional character of the geometrical postulates. So far, nominalism triumphs; but we now come to the physical sciences, properly so called, and here the scene changes. We meet with hypotheses of another kind, and we fully grasp how fruitful they are. No doubt at the outset theories seem unsound, and the history of science shows us how ephemeral they are; but they do not entirely perish, and of each of them some traces still remain. It is these traces which we must try to discover, because in them and in them alone is the true reality.”
 - b. [pages 58-59] “The properties of light and its propagation in a straight line have also given rise to some of the propositions of geometry, and in particular to those of projective geometry, so that from that point of view one would be tempted to say that metrical geometry is the study of solids, and projective geometry that of light. But a difficulty remains, and is unsurmountable. If geometry were an experimental science, it would not be an exact science. It would be subjected to continual revision. Nay, it would from that day forth be proved to be erroneous, for we know that no rigorously invariable solid exists. The geometrical axioms are therefore neither synthetic à priori intuitions nor experimental facts. They are conventions. Our choice among all possible conventions is guided by experimental facts; but it remains free, non-Euclidean geometries. and is only limited by the necessity of avoiding every contradiction, and thus it is that postulates may remain rigorously true even when the experimental laws which have determined their adoption are only approximate. In other words, the axioms of geometry (I do not speak of those of arithmetic) are only definitions in disguise. What, then, are we to think of the question: Is Euclidean geometry true? It has no meaning. We might as well ask if the metric system is true, and if the old weights and measures are false; if Cartesian co-ordinates are true and polar co-ordinates false. One geometry cannot be more true than another; it can only be more convenient. Now, Euclidean geometry is, and will remain, the most convenient: 1st, because it is the simplest, and it is not so only because of our mental habits or because of the kind of direct intuition that we have of Euclidean space; it is the simplest in itself, just as a polynomial of the first degree is simpler than a polynomial of the second degree; 2nd, because it sufficiently agrees with the properties of natural solids, those bodies which we can compare and measure by means of our senses. [...] [page 82] Experiment guides us in this choice, which it does not impose on us. It tells us not what is the truest, but what is the most convenient geometry.”
 - c. [page 102] “Finally, is not our Euclidean geometry in itself only a kind of convention of language? Mechanical facts might be enunciated with reference to a non-Euclidean space which would be less convenient but quite as legitimate as our ordinary space; the enunciation would become more complicated, but it still would be possible.”
 - d. [page 124] “Are the laws of acceleration and of the composition of forces only arbitrary conventions? Conventions, yes; arbitrary, no—they would be so if we lost sight of the experiments

which led the founders of the science to adopt them, and which, imperfect as they were, were sufficient to justify their adoption. It is well from time to time to let our attention dwell on the experimental origin of these conventions. “

- e. [pages 152-153] “The fundamental propositions of geometry, for instance, Euclid’s postulate, are only conventions, and it is quite as unreasonable to ask if they are true or false as to ask if the metric system is true or false. Only, these conventions are convenient, and there are certain experiments which prove it to us. [...] The experiments which have led us to adopt as more convenient the fundamental conventions of geometry refer to bodies which have nothing in common with those that are studied by geometry. They refer to the properties of solid bodies and to the propagation of light in a straight line. These are mechanical, optical experiments. In no way can they be regarded as geometrical experiments. And even the probable reason why our geometry seems convenient to us is, that our bodies, our hands, and our limbs enjoy the properties of solid bodies. Our fundamental experiments are pre-eminently physiological experiments which refer, not to the space which is the object that geometry must study, but to our body—that is to say, to the instrument which we use for that study. On the other hand, the fundamental conventions of mechanics and the experiments which prove to us that they are convenient, certainly refer to the same objects or to analogous objects. Conventional and general principles are the natural and direct generalisations of experimental and particular principles. Let it not be said that I am thus tracing artificial frontiers between the sciences; that I am separating by a barrier geometry properly so called from the study of solid bodies. I might just as well raise a barrier between experimental mechanics and the conventional mechanics of general principles. Who does not see, in fact, that by separating these two sciences we mutilate both, and that what will remain of the conventional mechanics when it is isolated will be but very little, and can in no way be compared with that grand body of doctrine which is called geometry.”
- f. [page 155] “Principles are conventions and definitions in disguise. They are, however, deduced from experimental laws, and these laws have, so to speak, been erected into principles to which our mind attributes an absolute value. Some philosophers have generalised far too much. They have thought that the principles were the whole of science, and therefore that the whole of science was conventional. This paradoxical doctrine, which is called Nominalism, cannot stand examination. How can a law become a principle? It expressed a relation between two real terms, A and B; but it was not rigorously true, it was only approximate. We introduce arbitrarily an intermediate term, C, more or less imaginary, and C is by definition that which has with A exactly the relation expressed by the law. So our law is decomposed into an absolute and rigorous principle which expresses the relation of A to C, and an approximate experimental and revisable law which expresses the relation of C to B. But it is clear that however far this decomposition may be carried, laws will always remain.”

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