## Relationship between Napier number e and Pi without i

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I am Yuji Masuda from Japan．I have a job of lending rooms to companies，which called＂share office or share w ork place＂．And I am working there as an hourly－employee．

The length $\sqrt{ } \mathrm{x}$ ，that x is a prime number，can be set in a geometric pattern composed of circles with a diameter 1 ．And in many lines or curves through the center of circles of prime，some limited conditions，which is on the center of the circles of $\mathrm{x}=3$ and 7（Fig．1），shows some graphs．Especially，graph of Gauss＇prime number theorem．which meets the limited condition， shows relationship between Napier number e and $\pi$ without imaginary number i．

Key Words：Napier number e，$\pi$ ，imaginary unit i

## 1．Introduction

There is unknown area in prime numbers．Therefore，this research aimed to deepen knowledge on unknown areas through basic research on prime numbers．

## 2．Geometric pattern and limited condition

The center of a circle with a close－packed structure of a circle with a radius of $1 / 2$ is set as the O ，is focused on x at a distance $\sqrt{x}$ from O ．

The limited the condition is go through the centers of two circles of $x=3 \& 7$ ．


Fig． 1 two curves which one meets the limited conditions and another doesn＇t meet it．

3．Curve of Gauss＇prime number theorem graph
First，I thought of some quadratic curves．However，it did not give a desired result．

Second，I considered a curve of Gauss＇prime number theorem graph．
$\mathrm{x}=1.78131217 \ldots$ on the curve which meets the limited condition

$$
\begin{aligned}
& x: \frac{x}{\log (x)}=1: \sqrt{3} \\
& \log (x)=\frac{1}{\sqrt{3}} \\
& x=e^{\frac{1}{\sqrt{3}}}=1.78131217 \ldots
\end{aligned}
$$

Next，I considered more deeply about the curve of the Gauss prime theorem graph．

4．Formula about Napier number e and pi without i．
I thought about $b$ of the equation below．

$$
e=\frac{a}{\log (a)} \quad\left(\because a=e^{\frac{1}{\sqrt{3}}}+b\right)
$$

And as a result，

$$
\begin{aligned}
& a=e^{\frac{1}{\sqrt{3}}}+b\left(\because b=\frac{14}{15}\right) \\
& \frac{a}{\log (a)} \cong e
\end{aligned}
$$

And I thought two things below．

$$
\begin{aligned}
& \text { (1) }:\left(e^{\frac{1}{\sqrt{3}}}\right)^{2}=3.17307306 \cdots \cong \frac{10}{\pi}-\frac{1}{100} \\
& \text { (2): } \frac{\left(e^{\frac{1}{\sqrt{3}}}\right)^{2}}{\frac{1}{\pi}}=9.96850302 \cdots \cong \pi^{2} \quad \sqrt{\pi} \times\left(e^{\frac{1}{\sqrt{3}}}\right) \cong \pi, \\
& \\
& \text { Here, I noticed below. }
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{\log (\sqrt{\pi})}=1.74713705 \cdots \cong \sqrt{14}-2 \\
& \sqrt{14} \times \log (\sqrt{\pi})=2.14159352 \cdots \cong \pi-1
\end{aligned}
$$

And after trial \＆error，I could finally get the following equation．

$=-1 \quad(1)$
$(\because \sqrt{14} \cdot \ln (\sqrt{\pi})-\pi=A, \sqrt{14} \cdot \ln (\sqrt{\pi})+1=B)$
$\left(\because a_{n}=p \pm 1, n=2^{p} \cdot(2 \cdot q+1),(\because p, q=\right.$ int eger $\left.)\right)$

## 5．Conclusion

The equation（1）that I found shows the relationship between
Napier number e and $\pi$ ．Furthermore，the equation（1） doesn＇t include imaginary unit i of $\mathrm{e}^{\wedge}(\mathrm{i} \pi)=-1$ ．${ }^{[1]}$

## 6．References

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