## Relationship between Napier number e and Pi without i

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The length  $\sqrt{x}$ , that x is a prime number, can be set in a geometric pattern composed of circles with a diameter 1. And in many lines or curves through the center of circles of prime, some limited conditions, which is on the center of the circles of x=3 and 7(Fig.1), shows some graphs. Especially, graph of Gauss' prime number theorem. which meets the limited condition, shows relationship between Napier number e and  $\pi$  without imaginary number i.

Key Words: Napier number e,  $\pi$ , imaginary unit i

## 1. Introduction

There is unknown area in prime numbers. Therefore, this research aimed to deepen knowledge on unknown areas through basic research on prime numbers.

## 2. Geometric pattern and limited condition

The center of a circle with a close-packed structure of a circle with a radius of 1/2 is set as the O, is focused on x at a distance  $\sqrt{x}$  from O.

The limited the condition is go through the centers of two circles of x=3&7.



Fig.1 two curves which one meets the limited conditions and another doesn't meet it.

## 3. Curve of Gauss' prime number theorem graph

First, I thought of some quadratic curves. However, it did not give a desired result.

Second, I considered a curve of Gauss' prime number theorem graph.

x=1.78131217...on the curve which meets the limited condition  $x:\frac{x}{1-\sqrt{3}}=1:\sqrt{3}$ 

$$\log(x) = \frac{1}{\sqrt{3}}$$
$$x = e^{\frac{1}{\sqrt{3}}} = 1.78131217 \cdots$$

Next, I considered more deeply about the curve of the Gauss prime theorem graph.

4. Formula about Napier number e and pi without i. I thought about b of the equation below.

$$e = \frac{a}{\log(a)} \qquad (\because a = e^{\frac{1}{\sqrt{3}}} + b)$$
$$a = e^{\frac{1}{\sqrt{3}}} + b\left(\because b = \frac{14}{15}\right)$$

And as a result,

$$\frac{a}{\log(a)} \cong e$$

And I thought two things below.

(1) : 
$$\left(e^{\frac{1}{\sqrt{3}}}\right)^2 = 3.17307306 \cdots \cong \frac{10}{\pi} - \frac{1}{100}$$
  
(2) :  $\frac{\left(e^{\frac{1}{\sqrt{3}}}\right)^2}{\frac{1}{\pi}} = 9.96850302 \cdots \cong \pi^2 \qquad \sqrt{\pi} \times \left(e^{\frac{1}{\sqrt{3}}}\right) \cong \pi,$   
 $\log(\sqrt{\pi}) \cong \frac{1}{\sqrt{3}}$ 

Here, I noticed below.

$$\frac{1}{\log(\sqrt{\pi})} = 1.74713705 \dots \cong \sqrt{14} - 2$$
$$\sqrt{14} \times \log(\sqrt{\pi}) = 2.14159352 \dots \cong \pi - 1$$

And after trial & error, I could finally get the following equation.



 $\left(: \sqrt{14} \cdot \ln\left(\sqrt{\pi}\right) - \pi = A, \sqrt{14} \cdot \ln\left(\sqrt{\pi}\right) + 1 = B\right)$  $\left(: a_n = p \pm 1, n = 2^p \cdot (2 \cdot q + 1), (: p, q = \text{int eger})\right)$ 

5. Conclusion

The equation (1) that I found shows the relationship between Napier number e and  $\pi$ . Furthermore, the equation (1) doesn't include imaginary unit i of e^(i  $\pi$ )=-1.<sup>[1]</sup>

6. References

[1].新装版 オイラーの贈物一人類の至宝 e<sup>A</sup>i π =-1 を学ぶ 吉田武(著) 2010 年 1 月 1 日

(New edition Euler's gift-The treasure of humanity. Learn e ^ i  $\pi = 1$ . Takeshi Yoshida. 2010 Jan 1)(by google translation)