# On the various mathematical connections with the Ramanujan's numbers $\mathbf{1 7 2 9}$, 728, the Ramanujan's class invariant, some sectors of Particle Physics and some formulae concerning the Supersymmetry 

Michele Nardelli ${ }^{1}$, Antonio Nardelli


#### Abstract

In the present research thesis, we have obtained various and interesting mathematical connections with the Ramanujan's numbers 1728, 1729, 728, 729 and some sectors of Particle Physics and Supersymmetry


[^0]

From:

## https://www.slideshare.net/SSridhar2/talk-on-ramanujan

From:
Ramanujan's Astonishing Knowledge of 1729 -Published May 12, 2016 -
https://thatsmaths.com/2016/05/12/ramanujans-astonishing-knowledge-of-1729/

Sf
(i) $\frac{1+53 x+9 x^{2}}{1-82 x-82 x^{2}+x^{3}}=a_{0}+a_{1} x+a_{2} x^{2}+a_{3} x^{3}+$ or $\frac{\alpha_{0}}{x^{2}}+\frac{\alpha_{1}}{x_{n}}+\frac{\alpha_{L}}{x_{0}}+$
(ii) $\frac{2-26 x-12 x^{2}}{1-82 x-84 x^{2}+x^{3}}=4_{0}+4, x+6_{2} x^{2}+L_{0} x+$ or $\frac{\beta_{0}}{x}+\frac{\beta_{1}}{x^{2}}+\frac{\beta_{L}}{x^{2}}+\cdots$.
(ii:) $\begin{aligned} \frac{2+8 x-10 x^{2}}{1-82 x-82 x^{2}+x^{3}}= & c_{0}+c_{1} x+c_{2} x^{2}+c_{5} x^{3}+ \\ & \text { or } \frac{x_{0}}{x}+\frac{x_{1}}{x_{2}}+\frac{x_{2}}{x_{3}}+.\end{aligned}$
then.

$$
\begin{aligned}
& \left.\quad \begin{array}{l}
a_{n}^{3}+c_{n}^{3}=c_{n}^{3}+(-1)^{n} \\
\text { and } \quad \alpha_{n}^{3}+\beta_{n}^{3}=\chi^{3}+(-1)^{n}
\end{array}\right\} \\
& \text { En_r.ples }
\end{aligned}
$$

$$
\begin{array}{ll}
135^{3}+138^{3}=172^{3}-1 & 9^{3}+10^{3}=12^{3}+1 \\
11161^{3}+11468^{3}=14258^{3}+1 & 6^{3}+8^{3}=9^{3}-1 \\
791^{3}+812^{3}=1010^{3}-1 & \\
65601^{3}+67402^{3}=83802^{3}+1 &
\end{array}
$$

Page from Ramanujan's Lost Notebook. Image credit: Trinity College Cambridge. Reproduced from Ono, 2015.]

We note the fundamental expressions:
$9^{3}+10^{3}=12^{3}+1 ; \quad 729+1000=1728+1$
$6^{3}+8^{3}=9^{3}-1 ; 216+512=729-1$
$135^{3}+138^{3}=172^{3}-1=5088447 ; \quad(5088447)^{1 / 32}=1,62024537 \ldots$. $(5088447)^{1 / 31}=1,645665103 \ldots ; \quad(5088447)^{1 / 30}=1,673219209 \ldots$
$5088447 / 1729=2943 ;$

```
838023}+1=588522607645609; 588522607645609 / 1729=340383231721
```



From: https://www.scienceandnonduality.com/article/the-secrets-of-ramanujansgarden

We have: $8 \mathrm{~J}+3$ and $64 \mathrm{~J}^{2}-24 \mathrm{~J}+9$
For $\mathrm{J}=1,3,30,165,20010$ we have:
$8 \mathrm{~J}+3=11 ; 8 \mathrm{~J}+3=27 ; 8 \mathrm{~J}+3=243 ; 8 \mathrm{~J}+3=1323 ; 8 \mathrm{~J}+3=160083 ;$
$11 ; 27 ; 27 * 3^{2}=243 ; 27 * 7^{2}=1323 ; 27 * 77^{2}=160083$;
$64 \mathrm{~J}^{2}-24 \mathrm{~J}+9=49 ; \quad 64 \mathrm{~J}^{2}-24 \mathrm{~J}+9=64 * 9-24 * 3+9=576-72+9=513 ;$
$64 \mathrm{~J}^{2}-24 \mathrm{~J}+9=64 * 900-24 * 30+9=57600-720+9=56889 ;$
$64 \mathrm{~J}^{2}-24 \mathrm{~J}+9=64 * 27225-24 * 165+9=1742400-3960+9=1738449$;
$64 \mathrm{~J}^{2}-24 \mathrm{~J}+9=64 * 400400100-24 * 20010+9=25625606400-480240+9=$ $=25625126169$;

Note that $64 \mathrm{~J}^{2}-24 \mathrm{~J}+9$ if set equal to zero, can be considered a quadratic equation. The quadratic formula for the roots of the general quadratic equation is:

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} .
$$

We have: $\quad 64 \mathrm{~J}^{2}-24 \mathrm{~J}+9=0$

$$
\begin{gathered}
\frac{24 \pm \sqrt{576-2304}}{128}=\frac{24 \pm \sqrt{-1728}}{128}=\frac{24}{128}+\frac{\sqrt{-1728}}{128} ; \quad \frac{24}{128}-\frac{\sqrt{-1728}}{128} \\
\frac{3}{16}+\frac{\sqrt{-1728}}{128} ; \quad \frac{3}{16}-\frac{\sqrt{-1728}}{128} ; \quad x_{1}=0,512259526 \quad x_{2}=-0,137259526
\end{gathered}
$$

We observe that the algebraic sum of the roots is: $x_{1}+x_{2}=0,375$ and that:

$$
\sqrt{\frac{1}{0,375}}=1,63299316185 \ldots
$$

Note that $(25625126169)^{1 / 3}=2948,1891086 \ldots . \quad$ value very near to the following charmonium particle:

| $\eta_{c}(1 S)$ | $2983.4 \pm 0.5$ |
| :--- | :--- |

$7^{2}=49 ; 27^{*} 19=513 ; 27 * 7^{2} * 43=27 * 49 * 43=56889 ;$
$27 * 31^{2} * 67=1738449 ; 27 * 2413^{2} * 163=25625126169 ;$
For $\mathrm{J}=1,3,30,165,20010$ we have also:

$$
\sqrt[6]{2 \sqrt{64 J^{2}-24 J+9}-(16 J-3)}=t
$$

$$
\begin{gathered}
\sqrt[6]{2 \sqrt{64-24+9}-(16-3)}=\sqrt[6]{2 \cdot 7-13}=1 \\
\sqrt[6]{2 \sqrt{64 \cdot 9-24 \cdot 3+9}-(16 \cdot 3-3)}=\sqrt[6]{2 \sqrt{513}-45}= \\
\sqrt[6]{45,299006611624498183420108841677-45}= \\
=0,81773665470181306492092503966592
\end{gathered}
$$

Or:

$$
\begin{aligned}
& \quad \sqrt[6]{2 \sqrt{64 \cdot 900-24 \cdot 30+9}-(16 \cdot 30-3)}=\sqrt[6]{2 \sqrt{56889}-477}= \\
& \sqrt[6]{477,02830104722298331495639065536-477}= \\
& =0,55203559829918124633667279829108 ;
\end{aligned}
$$

$$
\sqrt[6]{2 \sqrt{1738449}-2637}=0,41514887896093143232307651326475
$$

$$
\sqrt[6]{2 \sqrt{25625126169}-320157}=0,18656426483645848306470281669354
$$

The sum of the results is:
2.97148539679838422664537716791529

2,971485396
The difference is:
Result:
$-0.97148539679838422664537716791529$
$-0,971485396$
The algebraic sum between the two results is: 2
$10^{3}(1 / \pi * 2,971485396)=945,853178 \ldots$. very near to the mass of proton 938,27231(28)

And, for $\mathrm{J}=1,3,30,165,20010$

$$
\begin{gathered}
\frac{3 \sqrt{3}}{R^{6}}=\sqrt[2]{8 J+3}+\sqrt[2]{2 \sqrt{64 J^{2}-24 J+9}-8 J+6}= \\
\sqrt[2]{8 J+3}+\sqrt[2]{2 \sqrt{64 J^{2}-24 J+9}-8 J+6}=
\end{gathered}
$$

$3,3166247903553998491149327366707+3,4641016151377545870548926830117=$ $=6,7807264054931544361698254196824$

$$
\begin{gathered}
\sqrt[2]{27}+\sqrt[2]{2 \sqrt{64 \cdot 9-72+9}-24+6}= \\
\sqrt[2]{27}+\sqrt[2]{27,299006611624498183420108841677}=
\end{gathered}
$$

$=10,420997550710379532279250221154$

$$
\sqrt[2]{240+3}+\sqrt[2]{2 \sqrt{57600-720+9}-240+6}=
$$

$=31,177822266323734711257872601252$

$$
\sqrt[2]{1320+3}+\sqrt[2]{2 \sqrt{64 \cdot 165^{2}-24 \cdot 165+9}-1320+6}=
$$

$=72,746204291996970561556523525202$

$$
\sqrt[2]{160083}+\sqrt[2]{2 \sqrt{64 \cdot 20010^{2}-24 \cdot 20010+9}-8 \cdot 20010+6}=
$$

$$
=800,20747314951615853325696603331
$$

The sum of the results is:

Result:
921.3332236640403977745204378006004
$921,333223664 \ldots$ an approximation to the mass of the proton 938,27231(28)
and $(921,333223664)^{1 / 14}=1,628336104 \ldots$.
The difference is:

Result:
-907.7717708530540889021807869612356
$-907,771770853$ and $-(907,771770853)^{1 / 14}=-1,62661228 \ldots$
The difference between the two results is: 13,561452811 . This value is a good approximation of the energy spectrum of the hydrogen atom which is discrete, and the fundamental level is:

$$
E_{1}=-\frac{E_{h a}}{2}=-13.6 \mathrm{eV}
$$

Now, from:

$$
\frac{3 \sqrt{3}}{R^{6}}=\sqrt[2]{8 J+3}+\sqrt[2]{2 \sqrt{64 J^{2}-24 J+9}-8 J+6}
$$

We have:

$$
3 \sqrt{3}=5,1961524227066318805823390245176
$$

$\mathrm{R}^{6}=5,1961524227066318805823390245176 /$
$6,7807264054931544361698254196824=0,76631206$;
$\mathrm{R}^{6}=5,1961524227066318805823390245176 /$
$10,420997550710379532279250221154=0,498623323$;
$\mathrm{R}^{6}=5,1961524227066318805823390245176 /$
$31,177822266323734711257872601252=0,166661814$;
$\mathrm{R}^{6}=5,1961524227066318805823390245176 /$
$72,746204291996970561556523525202=0,0714285023$;
$\mathrm{R}^{6}=5,1961524227066318805823390245176 /$
$800,20747314951615853325696603331=0,00649350649$;
$\mathrm{R}=0,95660859082436004061727328369768$
$\mathrm{R}=0,89048942173962161448423500735182$
$\mathrm{R}=0,74183277566207698599349771995781$
$\mathrm{R}=0,64413751080965522991175648552217$
$\mathrm{R}=0,43192984468327433334089152205382$
$1 / R^{6}=1,3049514058280643527912114550305$
$1 / R^{6}=2,0055219117778812765242431309215$
$1 / R^{6}=6,0001747010865968373535163849831$
$1 / R^{6}=14,00001354921311292845069215458$
$1 / R^{6}=154,00000008316000004490640002425$
Note that from $64 \mathrm{~J}^{2}-24 \mathrm{~J}+9$ we have that $(64 * 24 * 9) / 8=13824 / 8=1728$
And $154-14+6-2+1,30=145,3 ;(145,3 * 12)-16=1727,6$
$154-14-6-2-1,30=130,7$
$154+14+6+2+1,30=177,3 ; \quad(177,3 * 10)-48=1725 ;$
And
0,95660859082436004061727328369768 + $0,89048942173962161448423500735182+$ $0,74183277566207698599349771995781+$ $0,64413751080965522991175648552217+$ 0,43192984468327433334089152205382

3,66499814

Result:
3.6649981437189882043476540185833
( $0.95660859082436004061727328369768+$
$0.89048942173962161448423500735182+$
$0.74183277566207698599349771995781+$
$0.64413751080965522991175648552217+$
$0.43192984468327433334089152205382) *(\mathrm{Pi} / 7)$

## Result:

1.6448473205325432233594072969452 ..
$130,7+3,66499814=134,36499814 ; \quad(134,36499814 * 13)-18=1728,74497582$
$(0,76631206+0,498623323+0,166661814+0,0714285023+0,00649350649)=$ $=1,50951920579$;
$1 / 1,50951920579=0,66246258 \ldots$.
$(0.76631206+0.498623323+0.166661814+0.0714285023+$ $\left.0.00649350649)^{*} \operatorname{sqrt}\left((1.085)^{\wedge} 18\right)\right)$

Input interpretation:
$(0.76631206+0.498623323+0.166661814+0.0714285023+0.00649350649)$
$\sqrt{1.085^{18}}$

## Result:

### 3.14562021156455784171215191839771484375

that is a good approximation to $\pi$

$$
\frac{3 \sqrt{3}}{R^{6}}=\sqrt[2]{8 J+3}+\sqrt[2]{2 \sqrt{64 J^{2}-24 J+9}-8 J+6}
$$

For $\mathrm{J}=3$

$$
\begin{gathered}
\frac{3 \sqrt{3}}{R^{6}}=\sqrt[2]{27}+\sqrt[2]{2 \sqrt{64 \cdot 9-72+9}-24+6}= \\
3 \sqrt{3} \cdot \frac{128}{64}=\sqrt[2]{27}+\sqrt[2]{27,299006611624498183420108841677}= \\
10,3923048454=10,4209975507 \\
\frac{1}{2 \pi} \cdot 3 \sqrt{3} \cdot \frac{128}{64}=\frac{10,392304845413263761164678049035}{2 \pi}=1,6539866862 \\
\frac{1}{2 \pi} \cdot \sqrt[2]{27}+\sqrt[2]{27,299006611624498183420108841677}=
\end{gathered}
$$

$$
=\frac{10,420997550710379532279250221154}{2 \pi}=1,658553272144
$$

$$
1,6539866862 \approx 1,658553272144
$$

The mean is: 1,656269979172
This result 1,656269 is very near to the fourteenth root of Ramanujan's class invariant 1164,2696 that is $1,65578 \ldots$, value very near to the mass of proton.

We have further, for $\mathrm{J}=1,3,30,165,20010$ :

$$
\begin{gathered}
\frac{1}{3} \sqrt{1+\frac{8}{3} J}=0,6382847 \ldots \\
\frac{1}{3} \sqrt{1+\frac{8}{3} \cdot 3}=1 \\
\frac{1}{3} \sqrt{1+\frac{8}{3} \cdot 30}=3 \\
\frac{1}{3} \sqrt{1+\frac{8}{3} \cdot 165}=7 \\
\frac{1}{3} \sqrt{1+\frac{8}{3} \cdot 20010}=77
\end{gathered}
$$

The sum of the results is:
$1+3+7+77=88 ;(88 * 16)^{1 / 15}=1,621462255 \ldots \quad(88 * 12)^{1 / 14}=1,6442808 \ldots$
$(1408)^{1 / 15}=1,621462255 \ldots \quad(1056)^{1 / 14}=1,6442808 \ldots$
We have also the following two equations:
$\mathrm{t}^{2}-14 \mathrm{t}-3=0 ;$ where $\mathrm{t}_{1}=14,2111025509 ; \mathrm{t}_{2}=-0,2111025509 ;$
$\mathrm{t}^{2}-26 \mathrm{t}-11=0 ;$ where $\mathrm{t}_{1}=26,4164078649 ; \mathrm{t}_{2}=-0,4164078649 ;$
we note that the algebraic sum of the two roots is: 14 and 26 , where $26-14=12$ and $(12)^{1 / 5}=1,64375182951 \ldots$

The various results highlighted in blue are good approximations to the electric charge of positron and to the mass of proton.

From: (http://www.sns.ias.edu/pitp2/2007files/Lecture\ Notes-
Problems/Witten Threedimgravity.pdf)
Let us give an example. If $k=1$, the partition function is simply the $J-$ function itself, so

$$
Z(q)=q^{-1}+196884 q+\ldots
$$

The number of black hole primaries of mass 2 is therefore 196883. The black hole entropy is therefore $\log (196883)=12.19$... The classical entropy of a black hole with $k=1$ and mass 2 is $4 \pi=12.57$... So we are off by just a few percent.

We note that the value that we have obtained 12 is a very good approximation of the value $12,19 \ldots$ that is the black hole entropy obtained from $\log (196883)$

In conclusion, we have the following equation:

$$
\frac{e^{\frac{\pi \sqrt{n}}{6}}+6 e^{-\frac{\pi \sqrt{n}}{6}}}{6 \sqrt{3}}
$$

$(2,4766322710964233011331665943154+2,4226446816602918287345554659281)$ / 10,392304845413263761164678049035

The result is:
0.471433144584769818249007360727972738250162080988804214746

0,47143314458476...
$\mathrm{e}^{0.471433144584769818249}=1,602288860133$

$$
e^{\left(e^{\frac{\pi \sqrt{n}}{6}}+6 e^{\left.-\frac{\pi \sqrt{n}}{6}\right) / 6 \sqrt{3}}\right.}=1,602288860133
$$

value 1,602288 very near to the electric charge of positron.
We have calculate the following integral:
$\mathrm{Pi}^{\wedge} 3 / 18$ * integrate $\operatorname{sqrt}(((1 /((2.4766322710964233011331665943154+$ $2.4226446816602918287345554659281) / 10.392304845413263761164678049035)))$ x

$$
\frac{\pi^{3}}{18} \int \sqrt{\frac{1}{\frac{2.4766322710964233011331665943154+2.4226446816602918287345554659281}{10.392304845413263761164678049035}} x} d x
$$

Result:
$1.6725372445167463037334360871954 x^{3 / 2}$

Indefinite integral assuming all variables are real:
$0.66901489780669852149337443487817 x^{5 / 2}+$ constant
The result 1,67253724 is very near to the value of the mass of proton.

## From: "SQUARE SERIES GENERATING FUNCTION TRANSFORMATIONS" MAXIE D. SCHMIDT - https://arxiv.org/abs/1609.02803v2

Corollary 4.7 (Special Values of Ramanujan's $\varphi$-Function). For any $k \in \mathbb{R}^{+}$, the variant of the Ramanujan $\varphi$-function, $\varphi\left(e^{-k \pi}\right) \equiv \vartheta_{3}\left(e^{-k \pi}\right)$, has the integral representation

$$
\begin{equation*}
\varphi\left(e^{-k \pi}\right)=1+\int_{0}^{\infty} \frac{e^{-t^{2} / 2}}{\sqrt{2 \pi}}\left[\frac{4 e^{k \pi}\left(e^{2 k \pi}-\cos (\sqrt{2 \pi k} t)\right)}{e^{4 k \pi}-2 e^{2 k \pi} \cos (\sqrt{2 \pi k} t)+1}\right] d t . \tag{33}
\end{equation*}
$$

Moreover, the special values of this function corresponding to the particular cases of $k \in$ $\{1,2,3,5\}$ in (33) have the respective integral representations

$$
\begin{align*}
\frac{\pi^{1 / 4}}{\Gamma\left(\frac{3}{4}\right)} & =1+\int_{0}^{\infty} \frac{e^{-t^{2} / 2}}{\sqrt{2 \pi}}\left[\frac{4 e^{\pi}\left(e^{2 \pi}-\cos (\sqrt{2 \pi} t)\right)}{e^{4 \pi}-2 e^{2 \pi} \cos (\sqrt{2 \pi} t)+1}\right] d t  \tag{34}\\
\frac{\pi^{1 / 4}}{\Gamma\left(\frac{3}{4}\right)} \cdot \frac{\sqrt{\sqrt{2}+2}}{2} & =1+\int_{0}^{\infty} \frac{e^{-t^{2} / 2}}{\sqrt{2 \pi}}\left[\frac{4 e^{2 \pi}\left(e^{4 \pi}-\cos (2 \sqrt{\pi} t)\right)}{e^{8 \pi}-2 e^{4 \pi} \cos (2 \sqrt{\pi} t)+1}\right] d t \\
\frac{\pi^{1 / 4}}{\Gamma\left(\frac{3}{4}\right)} \cdot \frac{\sqrt{\sqrt{3}+1}}{2^{1 / 4} 3^{3 / 8}} & =1+\int_{0}^{\infty} \frac{e^{-t^{2} / 2}}{\sqrt{2 \pi}}\left[\frac{4 e^{3 \pi}\left(e^{6 \pi}-\cos (\sqrt{6 \pi} t)\right)}{e^{12 \pi}-2 e^{6 \pi} \cos (\sqrt{6 \pi} t)+1}\right] d t \\
\frac{\pi^{1 / 4}}{\Gamma\left(\frac{3}{4}\right)} \cdot \frac{\sqrt{5+2 \sqrt{5}}}{5^{3 / 4}} & =1+\int_{0}^{\infty} \frac{e^{-t^{2} / 2}}{\sqrt{2 \pi}}\left[\frac{4 e^{5 \pi}\left(e^{10 \pi}-\cos (\sqrt{10 \pi} t)\right)}{e^{20 \pi}-2 e^{10 \pi} \cos (\sqrt{10 \pi} t)+1}\right] d t .
\end{align*}
$$

From the first of (34):

$$
\frac{\pi^{1 / 4}}{\Gamma\left(\frac{3}{4}\right)}=1+\int_{0}^{\infty} \frac{e^{-t^{2} / 2}}{\sqrt{2 \pi}}\left[\frac{4 e^{\pi}\left(e^{2 \pi}-\cos (\sqrt{2 \pi} t)\right)}{e^{4 \pi}-2 e^{2 \pi} \cos (\sqrt{2 \pi} t)+1}\right] d t
$$

we have:

$$
\begin{gathered}
\Gamma\left(\frac{3}{4}\right)=\frac{\pi \sqrt{2}}{\Gamma\left(\frac{1}{4}\right)}=\frac{4,44288293815}{3,625609908}=1,2254167025 \\
\frac{\pi^{1 / 4}}{\Gamma\left(\frac{3}{4}\right)}=\frac{1,3313353638}{1,2254167025}=1,08643481 \ldots
\end{gathered}
$$

For the integral, we have calculate as follows:
integrate $\left[\left(2.71828^{\wedge} 0.89\right) /(\right.$ sqrt6.283185307) $]\left[4 \mathrm{e}^{\wedge} 3.14159265 *\left(\mathrm{e}^{\wedge} 6.283185307-\right.\right.$ $\cos (($ sqrt 6.283185307$) 1.33416))] /\left[\mathrm{e}^{\wedge} 12.56637-2 \mathrm{e}^{\wedge} 6.283185307\right.$ $(\cos ($ sqrt6.283185307)1.33416) $)+1] \mathrm{x}$

```
Indefinite integral:
\(\int \frac{2.71828^{0.89}\left(4 e^{3.14159265}\left(e^{6.283185307}-\cos (\sqrt{6.283185307} 1.33416)\right)\right) x}{\sqrt{6.283185307}\left(e^{12.56637}-\left(2 e^{6.283185307}\right)(\cos (\sqrt{6.283185307}) 1.33416)+1\right)}\)
```

$d x=0.0837798 x^{2}+$ constant

Plot of the integral:


Alternate form assuming x is real:
$0.0837798 x^{2}+0+$ constant
Thence: $1+0.0837798=1.0837798$
and:
integrate $\left[\left(2.71828^{\wedge} 0.89\right) /(\right.$ sqrt6.283185307 $\left.)\right]\left[4 \mathrm{e}^{\wedge} 3.14159265 *\left(\mathrm{e}^{\wedge} 6.283185307-\right.\right.$ $\cos (($ sqrt 6.283185307$) 1.33416))] /\left[\mathrm{e}^{\wedge} 12.56637-2 \mathrm{e}^{\wedge} 6.283185307\right.$
$(\cos (\mathrm{sqrt6} .283185307) 1.33416))+1] \mathrm{x},[0,1]$

Definite integral:

$$
\int_{0}^{1} \frac{2.71828^{0.89}\left(4 e^{3.14159265}\left(e^{6.283185307}-\cos (\sqrt{6.283185307} 1.33416)\right)\right) x}{\sqrt{6.283185307}\left(e^{12.56637}-\left(2 e^{6.283185307}\right)(\cos (\sqrt{6.283185307}) 1.33416)+1\right)}
$$

$$
d x=0.0837798
$$

Visual representation of the integral:


Riemann sums:

$$
\text { left sum } \quad 0.0837798-\frac{0.0837798}{n}=0.0837798-\frac{0.0837798}{n}+O\left(\left(\frac{1}{n}\right)^{2}\right)
$$

(assuming subintervals of equal length)
Indefinite integral:

$$
\int \frac{2.71828^{0.89}\left(4 e^{3.14159265}\left(e^{6.283185307}-\cos (\sqrt{6.283185307} 1.33416)\right)\right) x}{\sqrt{6.283185307}\left(e^{12.56637}-\left(2 e^{6.283185307}\right)(\cos (\sqrt{6.283185307}) 1.33416)+1\right)}
$$

$$
d x=0.0837798 x^{2}+\text { constant }
$$

Thence: $1+0.0837798=1.0837798$
With regard the integral, from 0 to 0,58438 for $t=2$, where $\left(2.71828^{\wedge} 2\right) /(\operatorname{sqrt6} .283185307)=2,94780$ for $\mathrm{t}=2$, we have:
integrate $(2.94780)\left[4 \mathrm{e}^{\wedge} 3.14159265 *\left(\mathrm{e}^{\wedge} 6.283185307-\right.\right.$ $\cos (($ sqrt6.283185307)2) $)] /\left[\mathrm{e}^{\wedge} 12.56637-2 \mathrm{e}^{\wedge} 6.283185307\right.$ $(\cos (\operatorname{sqrt6.283185307)2}))+1] \mathrm{x},[0,0.58438]$
$\int_{0}^{0.58438} \frac{2.94780\left(4 e^{3.14159265}\left(e^{6.283185307}-\cos (\sqrt{6.283185307} 2)\right)\right) x}{e^{12.56637}-\left(2 e^{6.283185307}\right)(\cos (\sqrt{6.283185307}) 2)+1} d x=$
0.0864364
Thence, $1+0,0864364=1,0864364 ; \quad 1,08643481 \cong 1,0864364$.
In conclusion, the value of this, defined by us, "New Ramanujan's Constant" is 1.08643 .

In this and others our papers, we have used 1,08643 as a new "Ramanujan's constant" and we can see as this constant is fundamental for some results that we have obtained in various equations analyzed and developed.

## Search for pair production of higgsinos in final states with at least three $\boldsymbol{b}$-tagged jets in $\sqrt{s}=13 \mathrm{TeV} p p$ collisions using the ATLAS detector

The ATLAS Collaboration

A search for pair production of the supersymmetric partners of the Higgs boson (higgsinos $\tilde{H}$ ) in gauge-mediated scenarios is reported. Each higgsino is assumed to decay to a Higgs boson and a gravitino. Two complementary analyses, targeting high- and low-mass signals, are performed to maximize sensitivity. The two analyses utilize LHC $p p$ collision data at a center-of-mass energy $\sqrt{s}=13 \mathrm{TeV}$, the former with an integrated luminosity of $36.1 \mathrm{fb}^{-1}$ and the latter with $24.3 \mathrm{fb}^{-1}$, collected with the ATLAS detector in 2015 and 2016. The search is performed in events containing missing transverse momentum and several energetic jets, at least three of which must be identified as $b$-quark jets. No significant excess is found above the predicted background. Limits on the cross-section are set as a function of the mass of the $\tilde{H}$ in simplified models assuming production via mass-degenerate higgsinos decaying to a Higgs boson and a gravitino. Higgsinos with masses between 130 and 230 GeV and between 290 and 880 GeV are excluded at the $95 \%$ confidence level. Interpretations of the limits in terms of the branching ratio of the higgsino to a $Z$ boson or a Higgs boson are also presented, and a $45 \%$ branching ratio to a Higgs boson is excluded for $m_{\tilde{H}} \approx 400 \mathrm{GeV}$.

The signal region (SR) is defined by the requirement

$$
X_{h h}^{\mathrm{SR}}=\sqrt{\left(\frac{m_{2 j}^{\text {lead }}-120 \mathrm{GeV}}{0.1 \times m_{2 j}^{\text {lead }}}\right)^{2}+\left(\frac{m_{2 j}^{\text {subl }}-110 \mathrm{GeV}}{0.1 \times m_{2 j}^{\text {subl }}}\right)^{2}}<1.6,
$$

where $0.1 \times m_{2 j}^{\text {lead }}$ and $0.1 \times m_{2 j}^{\text {subl }}$ represent the approximate mass resolution of the leading and subleading Higgs boson candidates, respectively. The central values for the masses of the Higgs boson candidates of 120 GeV and 110 GeV , as well as the value of the $X_{h h}^{\mathrm{SR}}$ cut, were optimized using the data-driven background model described in Section 6.2.2 and simulated signal events.

For $m^{\text {lead }}=130$ and $m^{\text {subl }}=127$
$\sqrt{\left(\frac{130-120}{13}\right)^{2}+\left(\frac{127-110}{12.7}\right)^{2}}$
1.54387.

To derive the background model and estimate uncertainties in the background prediction, the following regions in the mass plane of the leading and subleading $p_{\mathrm{T}}$ Higgs boson candidates are defined: control region $(\mathrm{CR})$, validation region $1(\mathrm{VR} 1)$ and validation region 2 (VR2), using the variables

$$
\begin{aligned}
R_{h h}^{\mathrm{CR}} & \equiv \sqrt{\left(m_{2 j}^{\text {lead }}-126.0 \mathrm{GeV}\right)^{2}+\left(m_{2 j}^{\text {subl }}-115.5 \mathrm{GeV}\right)^{2}}, \\
X_{h h}^{\mathrm{VR} 1} & \equiv \sqrt{\left(\frac{m_{2 j}^{\text {lead }}-96 \mathrm{GeV}}{0.1 \times m_{2 j}^{\text {lead }}}\right)^{2}+\left(\frac{m_{2 j}^{\text {subl }}-88 \mathrm{GeV}}{0.1 \times m_{2 j}^{\text {subl }}}\right)^{2}} \\
X_{h h}^{\mathrm{VR} 2} & \equiv \sqrt{\left(\frac{m_{2 j}^{\text {lead }}-149 \mathrm{GeV}}{0.1 \times m_{2 j}^{\text {lead }}}\right)^{2}+\left(\frac{m_{2 j}^{\text {subl }}-137 \mathrm{GeV}}{0.1 \times m_{2 j}^{\text {subl }}}\right)^{2}} .
\end{aligned}
$$

All regions satisfy the same selection criteria as those for the SR , except for the requirement on $X_{h h}^{\mathrm{SR}}$. The control region is defined by $R_{h h}^{\mathrm{CR}}<55 \mathrm{GeV}$ and excludes the $\mathrm{SR}, X_{h h}^{\mathrm{SR}}>1.6$. The two validation regions are defined by functional forms similar to that of the SR but are displaced towards lower and higher Higgs boson candidate masses satisfying $X_{h h}^{\mathrm{VR} 1}<1.4$ and $X_{h h}^{\mathrm{VR} 2}<1.25$, respectively. The CR center $(126,115)$ was set so that the means of the Higgs candidates' mass distributions in the control region are equal to those in the signal region. The VR definitions were optimized to be similar to the SR while retaining sufficient statistical precision to test the background model. The CR and VRs are defined in both the 2-tag and 4-tag samples. Figure 5 shows the distributions of $m_{2 j}^{\text {lead }}$ versus $m_{2 j}^{\text {subl }}$ for the 2-tag and the 4-tag data after the event selection with the region definitions superimposed.

# Search for electroweak production of supersymmetric states in scenarios with compressed mass spectra at $\sqrt{s}=13 \mathrm{TeV}$ with the ATLAS detector 

M. Aaboud et al."<br>(ATLAS Collaboration)

(Received 21 December 2017; published 27 March 2018)
A search for electroweak production of supersymmetric particles in scenarios with compressed mass spectra in final states with two low-momentum leptons and missing transverse momentum is presented. This search uses proton-proton collision data recorded by the ATLAS detector at the Large Hadron Collider in 2015-2016, corresponding to $36.1 \mathrm{fb}^{-1}$ of integrated luminosity at $\sqrt{s}=13 \mathrm{TeV}$. Events with sameflavor pairs of electrons or muons with opposite electric charge are selected. The data are found to be consistent with the Standard Model prediction. Results are interpreted using simplified models of $R$-parityconserving supersymmetry in which there is a small mass difference between the masses of the produced supersymmetric particles and the lightest neutralino. Exclusion limits at $95 \%$ confidence level are set on next-to-lightest neutralino masses of up to 145 GeV for Higgsino production and 175 GeV for wino production, and slepton masses of up to 190 GeV for pair production of sleptons. In the compressed mass regime, the exclusion limits extend down to mass splittings of 2.5 GeV for Higgsino production, 2 GeV for wino production, and 1 GeV for slepton production. The results are also interpreted in the context of a radiatively-driven natural supersymmetry model with nonuniversal Higgs boson masses.

For $m^{\text {lead }}=160$ and $m^{\text {subl }}=157,45$
$\sqrt{(160-126)^{2}+(157.45-115.5)^{2}}$
53.9982...

For $m^{\text {lead }}=103$ and $m^{\text {subl }}=100$
$\sqrt{\left(\frac{103-96}{10.3}\right)^{2}+\left(\frac{100-88}{10}\right)^{2}}$
1.379084...

For $m^{\text {lead }}=157$ and $m^{\text {subl }}=154$
$\sqrt{\left(\frac{157-149}{15.7}\right)^{2}+\left(\frac{154-137}{15.4}\right)^{2}}$
1.21583..

Note that, for $m^{\text {lead }}=105$ and $m^{\text {subl }}=102$, we have:
$\sqrt{\left(\frac{105-96}{10.5}\right)^{2}+\left(\frac{102-88}{10.2}\right)^{2}}$
1.61820...

This result 1,61820 is practically the value of the golden ratio $1,61803398 \ldots$
149 Gev mass $=149 * 9 * 10^{16}=13410000000000000000 \mathrm{GeV}$;
and $1,65578 * 5 \varphi=13,395541517022 * 10^{18}=13395541517022000000 \mathrm{GeV}$

## From:

Search for pair production of supersymmetric particles with $R$-parity violating $L L \bar{E}$ couplings at $\sqrt{s}=192 \mathrm{GeV}$ to 202 GeV

C. Berat, E. Merle ISN Grenoble

Searches for $R_{p}$ effects in $\mathrm{e}^{+} \mathrm{e}^{-}$collisions at $\sqrt{s}-192 \mathrm{GeV}$ to 202 GeV have been performed with the DELPHI detector. The pair production of neutralinos, charginos and sleptons have been studied under the assumption that the LLE term is responsible for the supersymmetric particle decays into standard particles. No evidence for $R$-parity violation has been obscrved, allowing to update the limits previously obtained at $\sqrt{s}=189 \mathrm{CcV}$. A neutralino mass lower than $35.5 \mathrm{GeV} / c^{2}$ and a chargino mass lower than $99 \mathrm{GeV} / \mathrm{c}^{2}$ are excluded at $95 \%$ C.L
If the sneutrino is the LSP, the present limits are, with $\tan \beta-1.5$ :

- $m_{\tilde{\nu}_{\mathrm{e}}}>96 \mathrm{GeV} / c^{2}$ for $\mu=-150 \mathrm{GeV} / c^{2}$ and $\mathrm{M}_{2}=200 \mathrm{GeV} / c^{2}$;
- $m_{\tilde{\nu}_{\mu}}>81 \mathrm{GeV} / c^{2}$;
- $m_{\bar{\nu}_{-}}>86 \mathrm{GcV} / c^{2}$;

If $\tilde{\chi}_{1}^{0}$ is the ISP and the branching fraction $\tilde{\nu}(\tilde{\ell}) \rightarrow \nu(\ell) \tilde{\chi}_{1}^{0}$ is equal to 1 , taking into account the limit on the neutralino mass at $35.5 \mathrm{GeV} / c^{2}$, sneutrinos with mass lower than $83 \mathrm{GeV} / c^{2}$ and right-handed sleptons with mass lower than $87 \mathrm{GeV} / c^{2}$ were excluded at 95\% C.L.

We have that:
$m_{\tilde{\nu}_{\mathrm{e}}}>96 \mathrm{GeV} / c^{2}$ for $\mu=-150 \mathrm{GeV} / c^{2}$ and $\mathrm{M}_{2}=200 \mathrm{GeV} / c^{2}$;
For 108, we have: $108 * 9 * 10^{16}=9720000000000000000 \mathrm{GeV}$;
$(9720000000000000000)^{1 / 87}=1,65290935449971 \ldots$. or
$1164,2696 * \pi \sqrt{ } 7=9677,2609156539240463076732725537 * 10^{15}=$
$=9677260915653924046,3$
For 96, we have: $96 * 9 * 10^{16}=8640000000000000000 \mathrm{GeV}$;
$(8640000000000000000)^{1 / 87}=1,650673113624964$
$1164.2696 * \mathrm{e}^{2}=8602,84181522190464 * 10^{15}=8602841815221904640 \mathrm{GeV}$
where 1164.2696 is the Ramanujan's class invariant and $1,65291,65067$ are very near to the fourteenth root of 1164,2696 and to the mass of proton.

From:
Formulae for Supersymmetry | MSSM and more $\mid$
Toru Goto - KEK Theory Center, IPNS, KEK - Tsukuba, Ibaraki, 305-0801 JAPAN
Last Modified: March 31, 2019

We have that:

$$
\begin{align*}
E^{(3)}= & \left(\frac{1331}{2}-121 n_{l}+\frac{22}{3} n_{l}^{2}-\frac{4}{27} n_{l}^{3}\right) L_{\mu}^{3} \\
& +\left(\frac{4521}{2}-\frac{10955}{24} n_{l}+\frac{1027}{36} n_{l}^{2}-\frac{5}{9} n_{l}^{3}\right) L_{\mu}^{2} \\
& +\left[\frac{247675}{96}+\frac{32087}{108} \pi^{2}-\frac{99}{16} \pi^{4}+\frac{3025}{2} \zeta_{3}\right. \\
& -\left(\frac{166309}{288}+\frac{5095}{162} \pi^{2}-\frac{3}{8} \pi^{4}+\frac{902}{3} \zeta_{3}\right) n_{l} \\
& \left.+\left(\frac{10351}{288}+\frac{11}{9} \pi^{2}+\frac{158}{9} \zeta_{3}\right) n_{l}^{2}-\left(\frac{50}{81}+\frac{2}{81} \pi^{2}+\frac{8}{27} \zeta_{3}\right) n_{l}^{3}\right] L_{\mu} \\
- & \frac{865}{18} \pi^{2} L_{\alpha_{s}}+\frac{1267919}{1728}+\left(\frac{14286253}{38880}-\frac{7225}{162} \log 2\right) \pi^{2} \\
- & \frac{723119}{51840} \pi^{4}+\left(\frac{114917}{48}-\frac{1331}{8} \pi^{2}\right) \zeta_{3}+\frac{3993}{2} \zeta_{5}-\frac{128}{81} \pi^{2} L_{1}^{E}+\frac{a_{3}}{32} \\
- & {\left[\frac{52033}{288}+\frac{397591}{7776} \pi^{2}-\frac{59677}{77760} \pi^{4}+\left(\frac{8797}{18}-\frac{121}{4} \pi^{2}\right) \zeta_{3}+363 \zeta_{5}\right] n_{l} } \\
+ & {\left[\frac{3073}{288}+\frac{905}{432} \pi^{2}+\frac{11}{1080} \pi^{4}+\left(\frac{3239}{108}-\frac{11}{6} \pi^{2}\right) \zeta_{3}+22 \zeta_{5}\right] n_{l}^{2} } \\
& -\left[\frac{98}{729}+\frac{19}{486} \pi^{2}+\frac{1}{4860} \pi^{4}+\left(\frac{44}{81}-\frac{1}{27} \pi^{2}\right) \zeta_{3}+\frac{4}{9} \zeta_{5}\right] n_{l}^{3} . \tag{9.1.64c}
\end{align*}
$$

We have calculated and simplified the above expression. We have obtained:

Input interpretation:
$551.6851+1832.0138+4702.7364+878.1284+310.40515-36.52840+$
$53.4965+12.0628-0.669886+1208.0387+2208.6181376-1358.7647$
Open code

Enlarge Data Customize A Plaintext Interactive Result:
10361.2220016

Input interpretation:
$752.04873+1696.5-15.5964118+0.03125-$
$1163.7182131+66.23464189-1.1624358988$
Open code

Enlarge Data Customize A plaintext Interactive
Result:
1334.3375610912

Enlarge Data Customize A Plaintext Interactive
Result:
11695.5595626

11695,5595626.
Or, for $\alpha_{\mathrm{s}} \approx 5$, multiplying for 5, we obtain: 58477,797813
We have integrated the result 11695,5595626 :
$\mathrm{Pi}^{\wedge} 3$ * $1 /(1.6770424 \wedge 9)$ integrate $[10361.2220016+1334.337561] \mathrm{x}$
$\pi^{3} \times \frac{1}{1.6770424^{9}} \int(10361.2220016+1334.337561) x d x$
Result:
1728. $x^{2}$

The result is the Ramanujan's number 1729-1


Alternate form assuming x is real:
1728. $x^{2}+0$

Indefinite integral assuming all variables are real:
$576.001 x^{3}+$ constant
$\mathrm{Pi}^{\wedge} 3 /(27 * 4) * 1 /(1.6770424 \wedge 9)$ integrate $[10361.2220016+1334.337561] \mathrm{x}$
$\frac{\pi^{3}}{27 \times 4} \times \frac{1}{1.6770424^{9}} \int(10361.2220016+1334.337561) x d x$

Plot:


Alternate form assuming x is real:
16. $x^{2}+0$

Indefinite integral assuming all variables are real:
$5.33334 x^{3}+$ constant

Note that $(5,33334)^{1 / 21}=1,08297645043 \ldots .$. very near to the Ramanujan's new constant.

And
$\mathrm{Pi}^{\wedge} 3 /(64) * 1 /(1.6770424 \wedge 9)$ integrate $[10361.2220016+1334.337561] \mathrm{x}$
$\frac{\pi^{3}}{64} \times \frac{1}{1.6770424^{9}} \int(10361.2220016+1334.337561) x d x$

Result:
27. $x^{2}$

Plot:


Alternate form assuming x is real:
27. $x^{2}+0$

Indefinite integral assuming all variables are real:
$9.00001 x^{3}+$ constant
$\mathrm{Pi}^{\wedge} 3 /(8) * 1 /(2 * 9) * 1 /(1.6770424 \wedge 9)$ integrate $[10361.2220016+1334.337561] \mathrm{x}$
$\frac{\pi^{3}}{8} \times \frac{1}{2 \times 9} \times \frac{1}{1.6770424^{9}} \int(10361.2220016+1334.337561) x d x$
Result:
12. $x^{2}$

Plot:


Alternate form assuming $x$ is real:
12. $x^{2}+0$

Indefinite integral assuming all variables are real:
4. $x^{3}+$ constant

The result 12 is a good approximation to the value the black hole entropy $(12,19)$
$6.620 /(1728 * 4) * \mathrm{Pi}^{\wedge} 3 * 1 /(1.6770424 \wedge 9)$ integrate $[10361.2220016+1334.337561] \mathrm{x}$
$\frac{6.62}{1728 \times 4} \pi^{3} \times \frac{1}{1.6770424^{9}} \int(10361.2220016+1334.337561) x d x$
Result:
$1.655 x^{2}$

Plot:

$1.655 x^{2}+0$

Indefinite integral assuming all variables are real:
$0.551667 x^{3}+$ constant
$6.58 /(1723 * 4) * \mathrm{Pi}^{\wedge} 3 * 1 /(27 * 4)$ integrate $[10361.22+1334.337] \mathrm{x}$
$\frac{6.58}{1723 \times 4} \pi^{3} \times \frac{1}{27 \times 4} \int(10361.22+1334.337) x d x$

Result:
$1.60287 x^{2}$

Plot:


Alternate form assuming x is real:
$1.60287 x^{2}+0$

Indefinite integral assuming all variables are real:
$0.534289 x^{3}+$ constant
The results 1.655 and 1.602 are very near to the fourteenth root of Ramanujan's class invariant 1164.2696 and to the mass of proton and the electric charge of positron.

We have, for $\mathrm{n}_{1}=5$, that:

$$
E^{(1)}=\left(11-\frac{2}{3} n_{l}\right) L_{\mu}+\frac{97}{6}-\frac{11}{9} n_{l}
$$

And obtain: $17,72 \ldots$ or, for $n_{1}=1: 25,27$
And

$$
\begin{aligned}
E^{(2)}= & \left(\frac{363}{4}-11 n_{l}+\frac{1}{3} n_{l}^{2}\right) L_{\mu}^{2}+\left(\frac{927}{4}-\frac{193}{6} n_{l}+n_{l}^{2}\right) L_{\mu} \\
& +\frac{1793}{12}+\frac{2917}{216} \pi^{2}-\frac{9}{32} \pi^{4}+\frac{275}{4} \zeta_{3} \\
& -\left(\frac{1693}{72}+\frac{11}{18} \pi^{2}+\frac{19}{2} \zeta_{3}\right) n_{l}+\left(\frac{77}{108}+\frac{1}{54} \pi^{2}+\frac{2}{9} \zeta_{3}\right) n_{l}^{2},
\end{aligned}
$$

And obtain: $512833.4435 / 1728=296,778034$ or, for $n_{1}=1$ :
$979421.79734 / 1728=566,795$

We have that:

### 9.1.5 15 quarkonium mass

The mass of a 1 S quarkonium is decomposed into perturbative and nonperturbative contributions

$$
\begin{equation*}
M(1 \mathrm{~S})=2 m_{q, 0 \mathrm{OS}}\left|\Delta E^{\mathrm{p}}\right| \Delta E^{\mathrm{np}} \tag{9.1.57}
\end{equation*}
$$

The perturbative corrcetion $\Delta E^{\text {p }}$ is given in $N^{3} \mathrm{LO}$ as follows [1014] (scc also $[1015,1016,1017$, $1018,1019,1020,1021,1022,1023]$ ):

$$
\begin{equation*}
\Delta F_{s}^{\mathrm{P}}=-\frac{C_{F}^{2} \alpha_{s}^{2} \pi u_{q_{1} \mathrm{OS}}}{4}\left\{1+\frac{\alpha_{s}}{\pi} F^{(1)}+\left(\frac{\alpha_{s}}{\pi}\right)^{2} F^{(2)}+\left(\frac{\alpha_{s}}{\pi}\right)^{3} F^{(3)}+\cdots\right\} \tag{9.1.58а}
\end{equation*}
$$

For $\alpha_{\mathrm{s}}=5,13$ we have that:

$$
\begin{aligned}
& \left\{1+\frac{\alpha_{s}}{\pi} E^{(1)}+\left(\frac{\alpha_{s}}{\pi}\right)^{2} E^{(2)}+\left(\frac{\alpha_{s}}{\pi}\right)^{3} E^{(3)}+\cdots\right\} \\
= & 316959,3707073 \ldots
\end{aligned}
$$

And

$$
\Delta E^{\mathrm{p}}=-\frac{C_{F}^{2} \alpha_{s}^{2} m_{q, \mathrm{OS}}}{4}\left\{1+\frac{\alpha_{s}}{\pi} E^{(1)}+\left(\frac{\alpha_{s}}{\pi}\right)^{2} E^{(2)}+\left(\frac{\alpha_{s}}{\pi}\right)^{3} E^{(3)}+\cdots\right\}
$$

$$
=2085347,015742 \ldots
$$

For $\alpha_{\mathrm{s}}=1$, and $\mathrm{n}_{1}=1$, we have that:
$443,671771948 \ldots$ and $110,91794298 \ldots$ or, for $\alpha_{s}=5,13$ and the result of $\mathrm{E}^{(3)}=$ 11695,5595626:

52477,714003... and 345262,68791...
We have also that:

Nonperturbative contribution [1032, 1033] ${ }^{\text {|| }}$

$$
\Delta E^{\mathrm{np}}=\frac{\pi^{2} m_{q}}{\left(C_{F} \alpha_{s} m_{q}\right)^{4}} \frac{624}{425}\langle 0| \frac{\alpha_{s}}{\pi} G^{\mu \nu a} G_{\mu \nu}^{a}|0\rangle,
$$

where the gluon condensate is evaluated as [1034, 1035, 1036]

$$
\langle 0| \frac{\alpha_{s}}{\pi} G^{\mu \nu a} G_{\mu \nu}^{a}|0\rangle \approx 0.012 \mathrm{GeV}^{4}
$$

The mass of a 1 S quarkonium is:
$M(1 \mathrm{~S})=2 m_{q, \mathrm{OS}}+\Delta E^{\mathrm{P}}+\Delta E^{\mathrm{nP}}$
$\Delta \mathrm{E}^{\mathrm{np}}=2,5107715019136191675645344294841 \mathrm{e}-4$
Thence: $\mathrm{M}(1 \mathrm{~S})=2085349,0159$ or 345264,688161 or 113,0918338
We have that:
At this stage, all the factors, which are required for the evaluation of the quark masses at (QCD, $\overline{\mathrm{MS}}, n_{f}=6, \mu=\mu_{W}$ ), are determined, as well as some byproducts such as the quark masses in various schemes, and low energy values of $\alpha_{s}$. The quark masses $m_{q}\left(\mathrm{QCD}, \overline{\mathrm{MS}}, n_{f}=\right.$ $6, \mu=\mu_{W}$ ) are written as:

$$
\begin{align*}
m_{i}^{(6)}\left(\mu_{W}\right) & =\frac{m_{q}^{(6)}\left(\mu_{W}\right)}{m_{q}^{(6)}\left(\mu_{t}\right)} m_{i}^{(6)}\left(\mu_{t}\right),  \tag{3.2.2a}\\
m_{\dot{b}}^{(6)}\left(\mu_{W}\right) & -\frac{m_{q}^{(6)}\left(\mu_{W}\right)}{m_{q}^{(5)}\left(\mu_{W}\right)} \frac{m_{q}^{(5)}\left(\mu_{W}\right)}{m_{q}^{(5)}\left(\mu_{b}\right)} m_{b}^{(5)}\left(\mu_{b}\right),  \tag{3.2.2b}\\
m_{c}^{(6)}\left(\mu_{W}\right) & =\frac{m_{q}^{(6)}\left(\mu_{W}\right)}{m_{q}^{(5)}\left(\mu_{W}\right)} \frac{m_{q}^{(5)}\left(\mu_{W}\right)}{m_{q}^{(5)}\left(\mu_{b}\right)} \frac{m_{q}^{(5)}\left(\mu_{b}\right)}{m_{q}^{(q)}\left(\mu_{b}\right)} \frac{m_{q}^{(4)}\left(\mu_{b}\right)}{m_{q}^{(4)}\left(\mu_{c}\right)} m_{c}^{(4)}\left(\mu_{c}\right),  \tag{3.2.2c}\\
m_{d, u_{,},}^{(6)}\left(\mu_{W}\right) & =\frac{m_{q}^{(6)}\left(\mu_{W}\right)}{m_{q}^{(5)}\left(\mu_{W}\right)} \frac{m_{q}^{(5)}\left(\mu_{W}\right)}{m_{q}^{(5)}\left(\mu_{b}\right)} \frac{m_{q}^{(5)}\left(\mu_{b}\right)}{m_{q}^{(4)}\left(\mu_{b}\right)} \frac{m_{q}^{(4)}\left(\mu_{b}\right)}{m_{q}^{(4)}\left(\mu_{\mathrm{L}}\right)} m_{d, u, s}^{(4)}\left(\mu_{\mathrm{L}}\right) \quad \text { for } \mu_{\mathrm{L}}>\mu_{c} . \tag{3.2.2~d}
\end{align*}
$$

$m_{q}\left(\mathrm{QCD}, \overline{\mathrm{DR}}, n_{f}=6, \mu_{W}\right)$ arc obtaincd by (9.1.92a) with $n_{f}=6$.

### 9.1.6 $\overline{\mathrm{DR}}$ scheme

The leading order relation between the coupling constants in $\overline{\mathrm{MS}}$ and $\overline{\mathrm{DR}}$ schemes is given as [772, 1043]:

$$
\begin{equation*}
a_{\overline{\mathrm{DR}}}(\mu)^{-1}=a_{\overline{\mathrm{MS}}}(\mu)^{-1}-1 . \tag{9.1.90}
\end{equation*}
$$

The $O\left(\alpha_{s}^{2}\right)$ relation between the pole mass and the $\overline{\mathrm{DR}}$ running mass is given in Ref. [1001]:

$$
\begin{align*}
\frac{m_{\mathrm{pole}}}{m_{\overline{\mathrm{DR}}}(\mu)}=1+a_{\overline{\mathrm{DR}}}(\mu) & {\left[\frac{20}{3}-4 L_{\overline{\mathrm{DR}}}\right] } \\
& +a_{\overline{\mathrm{DR}}}^{2}(\mu) \\
& {\left[\frac{3043}{18}+\frac{32}{3}(2+\log 2) \zeta_{2}-\frac{8}{3} \zeta_{3}-\left(\frac{74}{9}+\frac{16}{3} \zeta_{2}\right) n_{f}+\frac{64}{3} \sum_{j=1}^{n_{f}} \Delta\left(\frac{m_{j}}{m}\right)\right.}  \tag{9.1.91}\\
& \left.-\left(\frac{350}{3}-\frac{52}{9} n_{f}\right) L_{\overline{\mathrm{DR}}}+\left(30-\frac{4}{3} n_{f}\right) L_{\overline{\mathrm{DR}}}^{2}\right],
\end{align*}
$$

where $L_{\overline{\mathrm{DR}}}=\log \left(m_{\overline{\mathrm{DR}}}(\mu) / \mu^{2}\right)=L_{\overline{\mathrm{MS}}}-\frac{8}{3} a(\mu)+O\left(a^{2}\right)$. The relation between the $\overline{\mathrm{MS}}$ and the $\overline{\mathrm{DR}}$ masses is derived from (9.1.21) and (9.1.91) as [1001, 1044]:

$$
\begin{align*}
\frac{m_{\overline{\mathrm{DR}}}(\mu)}{m_{\overline{\mathrm{MS}}}(\mu)} & =1-\frac{4}{3} a_{\overline{\mathrm{MS}}}(\mu)+\left(-\frac{73}{9}+\frac{n_{f}}{3}\right) a_{\overline{\mathrm{MS}}}^{2}(\mu)+O\left(a^{3}\right)  \tag{9.1.92a}\\
& =1-\frac{4}{3} a_{\overline{\mathrm{DR}}}(\mu)+\left(-\frac{61}{9}+\frac{n_{f}}{3}\right) a_{\overline{\mathrm{DR}}}^{2}(\mu)+O\left(a^{3}\right) . \tag{9.1.92b}
\end{align*}
$$

For $\mathrm{n}_{\mathrm{f}}=6$, we calculate $\mathrm{m}_{\mathrm{q}}$ :

$$
1-\frac{4}{3}+\left(-\frac{73}{9}+\frac{6}{3}\right)=-\frac{58}{9}=-6, \overline{4}=-6,444 \ldots
$$

## From:

$$
\begin{aligned}
& \Delta E^{\mathrm{p}}=-\frac{C_{F}^{2} \alpha_{s}^{2} m_{q, \mathrm{OS}}}{4}\left\{1+\frac{\alpha_{s}}{\pi} E^{(1)}+\left(\frac{\alpha_{s}}{\pi}\right)^{2} E^{(2)}+\left(\frac{\alpha_{s}}{\pi}\right)^{3} E^{(3)}+\cdots\right\} \\
& \Delta E^{\mathrm{np}}=\frac{\pi^{2} m_{q}}{\left(C_{F} \alpha_{s} m_{q}\right)^{4}} \frac{624}{425}\langle 0| \frac{\alpha_{s}}{\pi} G^{\mu \nu a} G_{\mu \nu}^{a}|0\rangle \\
& \langle 0| \frac{\alpha_{s}}{\pi} G^{\mu \nu a} G_{\mu \nu}^{a}|0\rangle \approx 0.012 \mathrm{GeV}^{4}
\end{aligned}
$$

We obtain:
$-(-6,4 / 4)$ * 443,671771948 = 709,8748351168;
$472,98302563428555096212907908526 / 713031,68=6,63340828 \mathrm{e}-4$
Thence:
$M(1 \mathrm{~S})=2 m_{q, \mathrm{OS}}+\Delta E^{\mathrm{p}}+\Delta E^{\mathrm{np}}$
$M(1 \mathrm{~S})=2 *-6.4+6.63340828 * 10^{-5}+709,8748351168=-722,67908$
For the value: 52477,714003 (for $\alpha_{s}=5,13$ ), we obtain:
$-\left(-6,4 * 5,13^{2} / 4\right) * 52477,714003=42,10704 * 52477,71403=2209681,203769 ;$
$M(1 \mathrm{~S})=2 *-6.4+6.63340828 * 10^{-5}+2209681,203769=-2209694,004 ;$
$-2209694 / 1278=-1729,025 ; \quad 1278=142 * 9 ;(142 * 12)+24=1728$

Note that $2209694,004 / 1728 * 100=12,7875810416666 \ldots$

From: (http://www.sns.ias.edu/pitp2/2007files/Lecture\ NotesProblems/Witten Threedimgravity.pdf)

Let us give an example. If $k=1$, the partition function is simply the $J$-function itself, so

$$
Z(q)=q^{-1}+196884 q+\ldots
$$

The number of black hole primaries of mass 2 is therefore 196883. The black hole entropy is therefore $\log (196883)=12.19 \ldots$. The classical entropy of a black hole with $k=1$ and mass 2 is $4 \pi=12.57$... So we are off by just a few percent.

We note that the value that we have obtained $12,7875 \ldots$ is a very good approximation of the value $12,57 \ldots$ that is the classical entropy of a black hole with $\mathrm{k}=1$ and mass 2

From:

## Breaking SU(3) Symmetry and Baryon Masses

Kai-Wai Wong, Gisela A. M. Dreschhoff, Högne J. N. Jungner

Department of Physics and Astronomy, University of Kansas, Lawrence, USA Radiocarbon Dating Lab, University of Helsinki, Helsinki, Finland Email: kww88ng@gmail.com - Received 16 July 2015; accepted 13 September 2015; published 16 September 2015

In the recent papers [3]-[6] we have shown how the meson masses are calculated, including the pion gluon potential pairs of intermediate quark currents, $u$ or $t$ and $d$. We explicitly obtained $U(\pi)$ as 121 MeV . Similarly, the proton gluon U can also be calculated with the gauge loop parameter $\mathrm{r}_{0}$ already determined [4] [6], and get $\mathrm{U}(\mathrm{p})=934.6 \mathrm{MeV}$ instead of the number fitting $\mathrm{U}(\mathrm{p})=928 \mathrm{MeV}$ we gave in Ref. [1]. It was also shown in ref. [4]-[6] that the inter quark interactions within hadrons are divided into 2 body for mesons and 3 body for baryons. The 3-body problem obeys the equilateral structure, meaning that all 3 pairs of relative distances are equal.

From this paper we observe that the proton gluon $U$ can be of value in a range of 928934,6 MeV

Note that:
From:
http://quantumpulse.com/page1.php - Physics Beyond the Standard Model

$$
\begin{array}{ll}
u=\text { mass of up quark } & u=\text { mass of up quark } \\
d=\text { mass of down quark } & d=\text { mass of down quark } \\
u=2.2431(46) \mathrm{MeV} & u=2.15(15) \mathrm{MeV} \\
d=4.8310(46) \mathrm{MeV} & d=4.70(20) \mathrm{MeV} \\
\frac{u}{d}=.4644(14) & \frac{u}{d}=.46(5) \\
& \\
d-u=2.5867(92) \mathrm{MeV} & d-u=2.55(35) \mathrm{MeV}
\end{array}
$$

${ }^{1} 2012$ Particle Data Group Update-Lawrence Berkeley Nation Laboratory A.V. Manohar (University of California, San Diego) and C.T. Sachrajda (University of Southampton)

We note that the mass of up quark is very near to the result of the expression, i.e. $2209694(2,209694)$

We calculate the following integral:
$\left(\mathrm{Pi}^{\wedge} 3 /(1.65578)^{\wedge} 6\right)$ * integrate [2209694]x where 1,65578 is the fourteenth root of the following Ramanujan's class invariant:

$$
\sqrt[14]{\left(\sqrt{\frac{113+5 \sqrt{505}}{8}}+\sqrt{\frac{105+5 \sqrt{505}}{8}}\right)^{3}}=1,65578 \ldots
$$

we obtain:
$\frac{\pi^{3}}{1.65578^{6}} \int 2209694 x d x$
Result:
$1.6624 \times 10^{6} x^{2}$

Plot:


The result 1,6624 is very near to the 1.65578 and to the mass of proton.

Now:

The gluon energy in the proton and neutron is shared between quarks; therefore the total energy in each quark is simply the gluon energy plus the quark mass. In the proton (see figure below), the total energy of the up quark equals its mass ( 2.243 MeV ) plus the proton gluon energy ( 928.956 MeV ) for a total of 931.199 MeV . The total energy of the down quark is 933.786 MeV (mass $4.830 \mathrm{MeV}+$ gluon 928.956 MeV ). A wide search range (0.1-35 MeV ) of possible quark mass values were tested in order to find the highest common factor between all quark total energies using this simple model

The highest common factor was 1.29333217 MeV , which is exactly equal to the mass difference between the neutron and the proton meaning this is also the highest possible common factor. The multiples of this factor in the proton are 720 in the up quark $(720 \times 1.29333217=931.199 \mathrm{MeV})$ and 722 in the down quark ( $722 \times 1.29333217$ $=933.786 \mathrm{MeV}$ ).


The quark mass search was concurrently run using the neutron model (see below). Again, the highest common factor was again 1.29333217 MeV , the difference in mass energy between the neutron and proton. The multiples of this factor in the neutron are 719 in the up quark ( $719 \times 1.29333217=929.906 \mathrm{MeV}$ ) and 721 in the down quark $(721 \times 1.29333217=932.492 \mathrm{MeV}$ ).


While concurrently searching for high common factors between the proton and neutron quark energy wavelengths, often times there was found a high common factor in one composite particle and not the other. Not only did the highest possible common factor ( 1.29333 MeV ) occur in one particle, it occurred in both at a up-down quark mass difference value within QCD predicted values ( 2.5867 MeV ).

We have that:

Proton mass: 938,27231 MeV/c²
Neutron mass: $939,56564217 \mathrm{MeV} / \mathrm{c}^{2}$
Difference: 939,56564217-938,27231 = 1,29333217;
Now:
$720 * 1,29333217=931,1991624=931,199$
$722 * 1,29333217=933,78582674=933,786$
In this computation the gluon value is 928,956 . Adding two time the value of quark up and the value of quark down, we have:
$928,956+4,830+2,243+2,243=938,272 \mathrm{MeV}$

And:

Proton mass: 938,27231 MeV/c²
Neutron mass: $939,56564217 \mathrm{MeV} / \mathrm{c}^{2}$
Difference: 939,56564217-938,27231 = 1,29333217;
Now:
$719 * 1,29333217=929,90583023=929,906$
$721 * 1,29333217=932,49249457=932,492$
In this computation the gluon value is 927,663 . Adding two time the value of quark down and the value of quark up, we have:
$927,663+4,830+4,830+2,243=939,566 \mathrm{MeV}$

To summarize, the proton and neutron are composite particles that form at the exact energy which creates the maximum possible common factor between their total quark energies (mass plus kinetic energy). These multiples range from 719 to 722 X the mass difference between the neutron and the proton ( 1.29333 MeV ). This occurred at a down-up mass difference of 2.5866 MeV , right where QCD predicted it should be.

We note that the values $719,720,721$ and 722 are very near to the value 728 . Indeed: $719+9=728 ; \quad 720+8=728 ; 721+7=728 ; \quad 722+6=728 ;$ note that:
$6+7+8+9=30=24+6$ where 24 and 6 are divisible for 1728.

From:

## http://quantumpulse.com/page1.php - Physics Beyond the Standard Model

With regard the usual Ramanujan invariant class:

$$
\left(\sqrt{\frac{113+5 \sqrt{505}}{8}}+\sqrt{\frac{105+5 \sqrt{505}}{8}}\right)^{3}=1164,2696
$$

and the numbers 728 and 1728 , it is possible to obtain some interesting mathematical connections with various values of particles' masses. We have the following gluon level:

$$
\mathrm{U}(\mathrm{~d})=\mathrm{U}_{0}+4 \mathrm{E}_{0}=934.6+178=1112.6 \mathrm{MeV} .
$$

$1112=1728-576-36-4 ;$
Should $\mathrm{U}(\mathrm{d})$ represent the gluon potential for $\Lambda^{0}$, then it's mass is given by (see Ch .8 of Ref. [1] for more details on the hadron mass formula)

$$
\begin{equation*}
\mathrm{M}\left(\Lambda^{0}\right)=\left\{1112.6^{2}+86^{2}\right\}^{0.5}=1115.9 \mathrm{MeV} \tag{2.4}
\end{equation*}
$$

$1115.9=1728-576-36=1116 ;$

$$
\begin{equation*}
\mathrm{U}(\mathrm{~s})=1183 \mathrm{MeV} \tag{2.7}
\end{equation*}
$$

Finally, we get

$$
\begin{equation*}
\mathrm{M}\left(\Sigma^{0}\right)=\left\{1183^{2}+146.9^{2}\right\}^{0.5}=1192.5 \mathrm{MeV} \tag{2.8}
\end{equation*}
$$

$$
\begin{aligned}
& 1183=1728-288-144-72-32-9 \\
& 1192=1728-288-144-72-32
\end{aligned}
$$

$$
\mathrm{M}\left(\Sigma^{+}\right)=\left\{1183^{2}+118^{2}\right\}^{0.5}=1189 \mathrm{MeV}
$$

$$
1189=1728-288-144-72-32-3
$$

$$
\begin{aligned}
& \mathrm{M}\left(\Sigma^{-}\right)=\left\{1188.9^{2}+98.2^{2}\right\}^{0.5}=1192.9 \mathrm{MeV} \\
& 1193=1728-288-144-64-36-3 \\
& \mathrm{U}\left(\Xi^{0}\right)=44.5\left\{16 \mathrm{f}+2 \times 4 \mathrm{f}^{2}+2 \mathrm{f}^{2}+1\right\}=1301 \mathrm{MeV} \\
& 1301=1728-288-108-27-4 \\
& 1301=1164+108+27+2 ; \\
& \mathrm{U}\left(\Xi^{-}\right)=44.5\left\{16 \mathrm{f}+2 \times 4 \mathrm{f}^{2}+2 \mathrm{f}^{2}+1+3 \times(\mathrm{f}-1)^{2}\right\}=1312.2 \mathrm{MeV} \\
& 1312=1728-288-64-36-16-12 \\
& 1312=1164+144+4 ; \\
& \mathrm{M}\left(\Xi^{-}\right)=\left\{1312.9^{2}+139.8^{2}\right\}^{0.5}=1320.4 \mathrm{MeV} \\
& 1320=1728-288-64-54-2 ; \\
& 1320=1164+144+12 ; \\
& \mathrm{M}\left(\Omega^{-}\right)=1672 \mathrm{MeV} . \\
& 1672=1728-54-2 ; \\
& 1672=1164+288+144+64+12 ; \\
& \mathrm{M}(\Lambda)=\left\{2248^{2}+111^{2}\right\}=2257 \mathrm{MeV} . \\
& \mathrm{U}(\mathrm{c})=2.0945 \times 1112.6+(1-2.0945) \times 44.5=2330.3-48.7=2281.6 \mathrm{MeV} . \\
& 2282=1164+728+288+64+36+2 \\
& 2257=1164+576+288+108+54+64+3 ; \\
& 2257=1164+728+288+54+16+4+3 ; \\
& 13
\end{aligned}
$$

Now, from: Formulae for Supersymmetry | MSSM and more | Toru Goto - KEK Theory Center, IPNS, KEK - Tsukuba, Ibaraki, 305-0801 JAPAN - Last Modified: March 31, 2019
we have:

$$
\begin{align*}
& \Delta_{\mathrm{MS} .4}^{(0)}=\frac{291716893}{6123600}-\frac{2362581983}{87091200} \zeta_{3}-\frac{76940219}{2177280} \zeta_{4}+\frac{1389}{256} \zeta_{5} \\
& +\frac{3031309}{.54432} \widetilde{a}_{4}+\frac{121}{36} \widetilde{a}_{5}-\frac{151369}{2177280} X_{0}  \tag{9.1.28a}\\
& =\frac{134805853579559}{43342154956800}-\frac{18233772727}{783820800} \zeta_{3}-\frac{254709337}{8709120} \zeta_{4}+\frac{4330717}{207360} \zeta_{5} \tag{9.1.28b}
\end{align*}
$$

$$
\begin{align*}
& \tilde{n}_{4}=\mathrm{T}_{i_{4}}\left(\frac{1}{2}\right)+\frac{1}{24}(\log 2)^{4}-\frac{1}{4} \zeta_{2}(\log 2)^{2},  \tag{9.1.29a}\\
& \widetilde{a}_{5}=\operatorname{Li}_{5}\left(\frac{1}{2}\right)-\frac{1}{120}(\log 2)^{5}+\frac{1}{12} \zeta_{2}(\log 2)^{2}+\frac{17}{16} \zeta_{4} \log 2,  \tag{9.1.29b}\\
& \bar{T}_{62,2}=T_{62,2}-\frac{64}{3} \zeta_{3}^{2}+360 \zeta_{6} . \tag{9.1.29c}
\end{align*}
$$

'The expressions (9.1.28a) and (9.1.28b) are found in Ref. [1010] and [1003], respectively. The constants $X_{0}, I_{62,2}^{\prime}$ and ' $I_{54,3}^{\prime}$ are numerically obtained [1011, 1012]:

$$
\begin{equation*}
X_{0}=1.80887954620833474 \tag{9.1.30a}
\end{equation*}
$$

$$
\begin{equation*}
T_{62,2}=-4553.4004372195263, \quad T_{54,3}=-8445.8046390310298 \tag{9.1.30b}
\end{equation*}
$$

Also a relation among $X_{0}\left(={ }_{9} '_{91,0}\right.$ in the notation in $\left.[1003,1012]\right), I_{62,2}$ and $'_{54,3}$ is available in Ref. [1012]:

$$
\begin{equation*}
X_{0}-\frac{5511907345}{7962624}-\frac{4103}{36} \zeta_{3}+\frac{89}{4} \zeta_{4}-\frac{273}{2} \zeta_{5}+176 \bar{a}_{4}-\frac{9}{256} T_{54,3}+\frac{3}{16} \widetilde{T}_{32,2} \tag{9.1.31}
\end{equation*}
$$

$T_{54,3}$ is solved with use of (9.1.28a), (9.1.28b) and (9.1.31) as

$$
\begin{equation*}
T_{51,3}=-\frac{4908181487}{279936}+\frac{1602496}{81} \zeta_{3}-\frac{335104}{9} \zeta_{1}-\frac{87296}{3} \zeta_{5}+\frac{315392}{9} \tilde{u}_{1}+32768 \tilde{u}_{5} \tag{9.1.32}
\end{equation*}
$$

Substituting (9.1.32), one obtains alternative expressions

$$
\begin{align*}
\Delta_{\frac{\mathrm{MS}, 4}{(0)}-} & -\frac{4852990063}{111974400}+\frac{2538746237}{87091200} \zeta_{3}-\frac{1113800801}{8709120} \zeta_{4}-\frac{27194483}{483840} \zeta_{5} \\
& +\frac{35137253}{272160} \widetilde{a}_{4}+\frac{315443}{3780} \widetilde{a}_{5}-\frac{151369}{11612160} \widetilde{T}_{62,2}  \tag{9.1.33}\\
X_{0}= & \frac{10469}{8}-\frac{1619}{2} \zeta_{3}+\frac{5325}{4} \zeta_{1}+\frac{1773}{2} \zeta_{5}-1056 \widetilde{a}_{4}-1152 \widetilde{a}_{5}+\frac{3}{16} \widetilde{T}_{62,2} . \tag{9.1.34}
\end{align*}
$$

## We have that:

$$
X_{0}=\frac{5511907345}{7962624}-\frac{4103}{36} \zeta_{3}+\frac{89}{4} \zeta_{4}-\frac{273}{2} \zeta_{5}+176 \widetilde{a}_{4}-\frac{9}{256} T_{54,3}+\frac{3}{16} \widetilde{T}_{62,2}
$$

is equal to $-47,232625$ for $\mathrm{a}=0,4082$;
$(-47,232625 * 37)-18=1747,607125-18=1729,607125$

$$
\begin{aligned}
\Delta \frac{(0)}{\mathrm{MS}, 4}= & -\frac{4852990063}{111974400}+\frac{2538746237}{87091200} \zeta_{3}-\frac{1113800801}{8709120} \zeta_{4}-\frac{27194483}{483840} \zeta_{5} \\
& +\frac{35137253}{272160} \widetilde{a}_{4}+\frac{315443}{3780} \widetilde{a}_{5}-\frac{151369}{11612160} \widetilde{T}_{62,2}
\end{aligned}
$$

is equal to $-58,8742714 ; \quad(-58,8742714 * 30)-36=1766,228-36=1730,228$;
Note that 1728 is divisible for 48 and 54.

$$
\begin{align*}
\Delta_{\mathrm{OS}, 4}^{(0)}= & \Delta_{\overline{\mathrm{MS}, 4}}^{(0)}-\frac{7478339}{139968}-\left[\frac{697121}{19440}-\frac{1027}{162} \log 2+\frac{11}{9}(\log 2)^{2}\right] \zeta_{2} \\
& -\left[\frac{341}{648}-\frac{1439}{216} \zeta_{2}\right] \zeta_{3}+\frac{3475}{1296} \zeta_{4}-\frac{1975}{648} \zeta_{5}+\frac{220}{81} \widetilde{a}_{4}  \tag{9.1.39a}\\
= & -\frac{141841753}{24494400}-\left[\frac{697121}{19440}-\frac{1027}{162} \log 2+\frac{11}{9}(\log 2)^{2}\right] \zeta_{2} \\
& -\left[\frac{2408412383}{8709120}-\frac{1439}{216} \zeta_{2}\right] \zeta_{3}-\frac{71102219}{2177280} \zeta_{4}+\frac{49309}{20736} \zeta_{5} \\
& +\frac{3179149}{54432} \widetilde{a}_{4}+\frac{121}{36} \widetilde{a}_{5}-\frac{151369}{2177280} X_{0} . \tag{9.1.39b}
\end{align*}
$$

That is equal to:
$-58,514996647845-319,36083504-4,38127424=-382,257106$
We have the following integral:
$1 / 48 *(728) / 1728$ integrate $[-382.257106] x$
$\frac{1}{48} \times \frac{728}{1728} \int-382.257106 x d x$

Result:
$-1.67754 x^{2}$


The result -1.67754 is very near to the value of the mass of the neutron with minus sign (antineutron)

We now note that:
8 integrate $\left[1 /\left(\left(\left(1+x^{\wedge}(1728)\right)\right)\left(1+x^{\wedge} 2\right)\right)\right][0,1]$
$8 \int_{0}^{\text {Input: }} \frac{1}{\left(1+x^{1728}\right)\left(1+x^{2}\right)} d x$
Computation result:
$8 \int_{0}^{1} \frac{1}{\left(1+x^{1728}\right)\left(1+x^{2}\right)} d x=6.28319$
Decimal approximation:
$6.281580800516077977125464725985681029862111334280455979192 \ldots$
and
$(64 * 3 \wedge 2) * 1 /(6 \mathrm{Pi})$ integrate $\left[1 /\left(\left(\left(1+\mathrm{x}^{\wedge}(1728)\right)\right)\left(1+\mathrm{x}^{\wedge} 2\right)\right)\right][0,1]$
$\left(64 \times 3^{2}\right) \times \frac{1}{6 \pi} \int_{0}^{1} \frac{1}{\left(1+x^{1728}\right)\left(1+x^{2}\right)} d x$
$\frac{64 \times 3^{2}}{6 \pi} \int_{0}^{1} \frac{1}{\left(1+x^{1728}\right)\left(1+x^{2}\right)} d x=24$.
The result $6,28158 \ldots$ is practically the length of a circle or radius equal to 1 , while 24 is connected with the dimension of bosonic string ( $D-2=26-2=24$ that are the physical degrees of freedom of the bosonic string).

From:

## Heterotic String

David J. Gross, Jeffrey A. Harvey, Emil Martinec, and Ryan Rohm Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544 (Received 21 November 1984)

A new type of superstring theory is constructed as a chiral combination of the closed $D=26$ bosonic and $D=10$ fermionic strings. The theory is supersymmetric, Lorentz invariant, and free of tachyons. Consistency requires the gauge group to be $\operatorname{Spin}(32) / Z_{2}$ or $E_{8} \times E_{8}$

The construction of the heterotic string is based on the observation that the states of the first quantized type-II closed strings, fermionic or bosonic, are essentially direct products of left- and right-moving modes. The physical degrees of freedom of the bosonic string are the 24 transverse coordinates $X^{i}(\tau-\sigma)$ and $\tilde{X}^{i}(\tau+\sigma)$ which describe right- (left-) moving twodimensional free fields, with periodic boundary conditions on the circle $0 \leqslant \sigma \leqslant \pi$. The fermionic string contains eight transverse coordinates as well as eight right- and left-moving two-dimensional real fermions, $S^{a}(\tau-\sigma)$ and $\tilde{S}^{a}(\tau+\sigma) \quad(a=1, \ldots, 8)$ which are Maiorana-Weyl ten-dimensional light-cone spinors. ${ }^{1}$ The right- and left-handed components of the string are tied together by the constraint that the total momentum and position of each component be identical. Thus the bosonic coordinates are given by the operators (we choose units in which the slope parameter is $\alpha^{\prime}=\frac{1}{2}$ )

Now, with 0,527 that is $1 / 5$ of 2,634547 (the Vertex solid angle of Icosahedron), thence 0,5269094 we calculate the following integrals:
$0.527 *$ integrate $4 *\left[1 /\left(\left(\left(1+x^{\wedge}(24494400 / 1728)\right)\right)\left(1+x^{\wedge} 2\right)\right)\right][0,1]$
$0.527 \int_{0}^{1} 4 \times \frac{1}{\left(1+x^{24494400 / 1728}\right)\left(1+x^{2}\right)} d x$
$0.527 \int_{0}^{1} \frac{4}{\left(1+x^{24494400 / 1728}\right)\left(1+x^{2}\right)} d x=1.65562$

Result:
$1.655567788609469639295445753091313466816342693599461489257 \ldots$
0.527 * integrate 4 * [1/(((1+x^(8709120/1728)))(1+x^2))][0,1]
$0.527 \int_{0}^{1} 4 \times \frac{1}{\left(1+x^{8709120 / 1728}\right)\left(1+x^{2}\right)} d x$
$0.527 \int_{0}^{1} \frac{4}{\left(1+x^{8709120 / 1728}\right)\left(1+x^{2}\right)} d x=1.65562$
$0.527 *$ integrate $4 *\left[1 /\left(\left(\left(1+x^{\wedge}(2177280 / 1728)\right)\right)\left(1+x^{\wedge} 2\right)\right)\right][0,1]$

$$
0.527 \int_{0}^{1} 4 \times \frac{1}{\left(1+x^{2177280 / 1728}\right)\left(1+x^{2}\right)} d x
$$

Computation result:
$0.527 \int_{0}^{1} \frac{4}{\left(1+x^{2177280 / 1728}\right)\left(1+x^{2}\right)} d x=1.65562$
Result:
1.655039505799136616108981534605007078142164652554115100193...

Where 24494400,8709120 and 2177280 are all divisible for 1728:
$24494400 / 1728=14175 ; \quad 8709120 / 1728=5040 ; 2177280 / 1728=1260 ;$

The three results 1.655 are practically equal to fourteenth root of Ramanujan's class invariant and very near to the mass of proton.

$$
\begin{align*}
d_{\frac{\mathrm{MS}, 4}{\prime(0)}}= & \frac{484}{27}-256 \Delta_{\overline{\mathrm{MS}, 4}}^{(0)}+\left[\frac{4770941}{8748}-\frac{3645913}{3888} \zeta_{3}+\frac{541549}{648} \zeta_{4}\right. \\
& \left.-\frac{460}{9} \zeta_{5}-\frac{2740}{81} \widetilde{a}_{4}\right] n_{l}+\left[\frac{271883}{17496}-\frac{668}{81} \zeta_{3}\right] n_{l}^{2} \tag{9.1.41e}
\end{align*}
$$

That is equal to

$$
\begin{aligned}
& \frac{484}{27}-256 \times(-58.8742714)+ \\
& \left(\frac{4770941}{8748}-\frac{3645913 \times 1.2025}{3888}+\frac{541549 \times 1.0823}{648}-\frac{460 \times 1.0362}{9}-\right. \\
& \left.\quad \frac{2740 \times 0.4082}{81}\right) \times 5+\left(\frac{271883}{17496}-\frac{668 \times 1.20205}{81}\right) \times 25
\end{aligned}
$$

Result:
16507.81833198081847279378143575674439871970736168267032464...

We have that:

$$
\begin{aligned}
& \operatorname{sqrt}\left(\left(\mathrm{Pi}^{\wedge} 2 /(492 * 288)\right) * 1 /(1728)\right) \text { integrate }\left[\left[484 / 27-256^{*}(-\right.\right. \\
& 58.8742714)+\left[4770941 / 8748-\left(3645913^{*} 1.2025\right) / 3888+(541549 * 1.0823) / 648-\right. \\
& \left.\left.(460 * 1.0362) / 9-(2740 * 0.4082) / 81] * 5+\left[271883 / 17496-\left(688^{*} 1.20205\right) / 81\right] * 25\right]\right] \mathrm{x} \\
& \sqrt{\frac{\pi^{2}}{492 \times 288} \times \frac{1}{1728}} \\
& \int\left(\frac{484}{27}-256 \times(-58.8742714)+\left(\frac{4770941}{8748}-\frac{3645913 \times 1.2025}{3888}+\right.\right. \\
& \left.\frac{541549 \times 1.0823}{460 \times 1.0362}-\frac{2740 \times 0.4082}{9}\right) \times 5+ \\
& \left.\left(\frac{271883}{17496}-\frac{688 \times 1.20205}{81}\right) \times 25\right) x d x
\end{aligned}
$$

Result:
$1.65639 x^{2}$


The result 1.65639 is practically equal to the fourteenth root of Ramanujan's class invariant and very near to the mass of proton.

$$
\begin{align*}
d_{\mathrm{OS}, 4}^{\prime(0)}= & \frac{196}{3}-256 \Delta_{\mathrm{OS}, 4}^{(0)}-\left[\frac{1773073}{2916}+\left(\frac{71296}{81}+\frac{5632}{81} \log 2-\frac{512}{27}(\log 2)^{2}\right) \zeta_{2}\right. \\
& \left.+\frac{4756441}{3888} \zeta_{3}-\frac{44653376}{4147} \zeta_{4}+\frac{460}{9} \zeta_{5}+\frac{692}{81} \widetilde{a}_{4}\right] n_{l} \\
& +\left[\frac{140825}{5832}+\frac{1664}{81} \zeta_{2}+\frac{76}{27} \zeta_{3}\right] n_{l}^{2}, \tag{9.1.43e}
\end{align*}
$$

$$
\begin{aligned}
& \frac{196}{3}-256 \times(-382.257106)- \\
& \left(\frac{1773073}{2916}+\left(\frac{71296}{81}+69.53 \log (2)-18.96 \times 0.48\right) \times 1.6449+\frac{4756441 \times 1.202}{3888}-\right. \\
& \left.\frac{44653376 \times 1.0823}{4147}+\frac{460 \times 1.03692}{9}+692 \times \frac{0.4082}{81}\right) \times 5+ \\
& \left(\frac{140825}{5832}+\frac{1664 \times 1.6449}{81}+2.8148 \times 1.202\right) \times 25
\end{aligned}
$$

## Result

$1.39489 \ldots \times 10^{5}$

139489

$$
\begin{align*}
\widetilde{d}_{\mathrm{OS}, 4}^{(0)}= & 256 \Delta_{\mathrm{OS}, 4}^{(0)}-\frac{392}{9}+\left[\frac{1773073}{2916}+\left(\frac{71296}{81}+\frac{5632}{81} \log 2-\frac{512}{27}(\log 2)^{2}\right) \zeta_{2}\right. \\
& \left.+\frac{4756441}{3888} \zeta_{3}-\frac{44653376}{4147} \zeta_{4}+\frac{460}{9} \zeta_{5}+\frac{692}{81} \widetilde{a}_{4}\right] n_{l} \\
& -\left[\frac{140825}{5832}+\frac{1664}{81} \zeta_{2}+\frac{76}{27} \zeta_{3}\right] n_{l}^{2}, \tag{9.1.47d}
\end{align*}
$$

$256 \times(-382.257106)-\frac{392}{9}+$

$$
\begin{aligned}
& \left(\frac{1773073}{2916}+\left(\frac{71296}{81}+69.53 \log (2)-18.96 \times 0.48\right) \times 1.6449+\frac{4756441 \times 1.202}{3888}-\right. \\
& \left.\frac{44653376 \times 1.0823}{4147}+\frac{460 \times 1.03692}{9}+692 \times \frac{0.4082}{81}\right) \times 5- \\
& \left(\frac{140825}{5832}+\frac{1664 \times 1.6449}{81}+2.8148 \times 1.202\right) \times 25
\end{aligned}
$$

Result:
$-1.39468 \ldots \times 10^{5}$
$-139468$

Now, we have that:
$\mathrm{Pi} /(1728)$ integrate $[[-97857.82-392 / 9+[608.0497+(71296 / 81+69.53 \ln 2-$
$18.96 * 0.480) * 1.645+(4756441 * 1.202) / 3888-$
$11653.8+(460 * 1.03692) / 9+692 * 0.4082 / 81] * 5-$
[24.1469+(1664*1.6449)/81+2.8148*1.202]*25]]x

$$
\begin{aligned}
& \frac{\pi}{1728} \\
& \int\left(-97857.82-\frac{392}{9}+\left(608.0497+\left(\frac{71296}{81}+69.53 \log (2)-18.96 \times 0.48\right) \times 1.645+\right.\right. \\
& \left.\frac{4756441 \times 1.202}{3888}-11653.8+\frac{460 \times 1.03692}{9}+692 \times \frac{0.4082}{81}\right) \times 5- \\
& \left.\left(24.1469+\frac{1664 \times 1.6449}{81}+2.8148 \times 1.202\right) \times 25\right) x d x
\end{aligned}
$$

Result:
$-126.779 x^{2}$


This result -126.779 is very near to the mass of Higgs boson $(125,09 \pm 0,24)$ with minus sign (Higgs antiboson)

Now, we take the sum of the various results that we have obtained:
-47.23265-58.8742714-382.257106+16507.8183+139489-139468-2209694+2085349
$-108304,546$; we note that $-108304.546 /(1728 * 10)=6,2676241898148 \ldots$ a value very near to $2 \pi$. Thence, we have a length of a circle $\mathrm{C}=2 \pi \mathrm{r}$ with $\mathrm{r}=1728^{*} 10$ or precisely $\mathrm{C}=2 \pi \mathrm{r}=108303,26513$ with $\mathrm{r}=1723,7^{*} 10$

And: $108304,546 /\left(5^{*} 1728\right)=12,535248379629$
From: (http://www.sns.ias.edu/pitp2/2007files/Lecture\ Notes-
Problems/Witten Threedimgravity.pdf)
Let us give an example. If $k=1$, the partition function is simply the $J$-function itself, so

$$
Z(q)=q^{-1}+196884 q+\ldots
$$

The number of black hole primaries of mass 2 is therefore 196883. The black hole entropy is therefore $\log (196883)=12.19 \ldots$ The classical entropy of a black hole with $k=1$ and mass 2 is $4 \pi=12.57 \ldots$... So we are off by just a few percent.

We note that the value that we have obtained $12,535 \ldots$ is a very good approximation of the value $12,57 \ldots$ that is the classical entropy of a black hole with $\mathrm{k}=1$ and mass 2.

Further: $(\ln 108304,546)^{1 / 5}=1,632438908 \ldots .$.
and
$\mathrm{Pi} / 178 * 1728$ integrate [108304.546]x where $178=144+34$ that are Fibonacci's numbers
$\frac{\pi}{178} \times 1728 \int 108304.546 x d x$
Result:
$1.65154 \times 10^{6} x^{2}$

Plot:


The results 1,6324 and 1.65154 are very near to the fourteenth root of Ramanujan's class invariant and to the mass of the proton

Furthermore:
$108304,546 \approx[1164,27 *(27 * 3+12)+27]=108304,11 \quad$ where $27=1728 / 64 ; 12=$ $1728 / 144$ and 1164.27 is equal to:

$$
\left(\sqrt{\frac{113+5 \sqrt{505}}{8}}+\sqrt{\frac{105+5 \sqrt{505}}{8}}\right)^{3}=1164,2696=1164,27
$$

## Now, we have:

### 9.1.3 Pole mass

The relation between the $\overline{\mathrm{MS}}$ running mass and the pole mass, assuming ( $n_{f}-1$ ) massless quarks and one massive quark, is written as a power series of $a^{\left[n_{f}\right]}(\mu)$ and $L_{\mathrm{OS}}=\log \left(m_{\mathrm{OS}}^{2} / \mu^{2}\right)$.

$$
\begin{align*}
\frac{m_{\frac{[n f}{\mathrm{MS}}}(\mu)}{m_{\mathrm{OS}}}= & 1+\sum_{k=1}^{\infty} a^{k}(\mu)\left[\sum_{m=0}^{k} C_{k}^{(m)} L_{\mathrm{OS}}^{m}\right]  \tag{9.1.19a}\\
C_{1}^{(0)}= & -\frac{16}{3},  \tag{9.1.19b}\\
C_{2}^{(0)}= & -\frac{3161}{18}-\frac{16}{3}(7+2 \log 2) \zeta_{2}+\frac{8}{3} \zeta_{3}+\left[\frac{71}{9}+\frac{16}{3} \zeta_{2}\right] n_{f},  \tag{9.1.19c}\\
C_{3}^{(0)}= & -\frac{1163813}{162}+\frac{608}{27}(\log 2)^{4}+\left(-\frac{2815124}{405}+\frac{36160}{27} \log 2+\frac{1024}{9}(\log 2)^{2}\right) \zeta_{2} \\
& +\left(-\frac{3304}{9}+\frac{11512}{9} \zeta_{2}\right) \zeta_{3}+\frac{6260}{9} \zeta_{4}-\frac{15800}{27} \zeta_{5}+\frac{4864}{9} \mathrm{Li}_{4}\left(\frac{1}{2}\right) \\
& +\left[\frac{167566}{243}-\frac{64}{81}(\log 2)^{4}+\left(\frac{16304}{27}+\frac{1408}{27} \log 2-\frac{256}{27}(\log 2)^{2}\right) \zeta_{2}\right. \\
& \left.+\frac{6232}{27} \zeta_{3}-\frac{4880}{27} \zeta_{4}-\frac{512}{27} \mathrm{Li}_{4}\left(\frac{1}{2}\right)\right] n_{f} \\
& +\left[-\frac{4706}{729}-\frac{416}{27} \zeta_{2}-\frac{224}{27} \zeta_{3}\right] n_{f}^{2}, \tag{9.1.19d}
\end{align*}
$$

where $\mathrm{Li}_{4}(x)$ is the polylogarithm (11.5.23) of the fourth order and $\mathrm{Li}_{4}(1 / 2)=0.51747906 \cdots$. $C_{k}^{(m)}$ for $m \geq 1$ are written in terms of $C_{k^{\prime}}^{(0)}\left(k^{\prime}<k\right)$ with the coefficients in the RGEs (9.1.1) and (9.1.11) as follows.

$$
\begin{align*}
C_{1}^{(1)} & =\frac{1}{2} \gamma_{m}^{(0)}=4,  \tag{9.1.20a}\\
C_{2}^{(1)} & =\frac{1}{2}\left[\gamma_{m}^{(1)}+\left(\gamma_{m}^{(0)}-2 \beta^{(0)}\right) C_{1}^{(0)}\right]=\frac{314}{3}-\frac{52}{9} n_{f},  \tag{9.1.20b}\\
C_{2}^{(2)} & =\frac{1}{8}\left(\gamma_{m}^{(0)}-2 \beta^{(0)}\right) \gamma_{m}^{(0)}=-14+\frac{4}{3} n_{f}, \tag{9.1.20c}
\end{align*}
$$

$$
\begin{align*}
C_{3}^{(1)}= & \frac{1}{2}\left[\gamma_{m}^{(2)}+\left(\gamma_{m}^{(1)}-2 \beta^{(1)}\right) C_{1}^{(0)}+\left(\gamma_{m}^{(0)}-4 \beta^{(0)}\right) C_{2}^{(0)}\right]  \tag{9.1.20d}\\
= & 41354,(672 । 192 \log 2) \zeta_{2} \\
& 48 \zeta_{3} \\
& -\left[\frac{13876}{27}+\left(\frac{1312}{9}+\frac{128}{9} \log 2\right) \zeta_{2}+\frac{448}{9} \zeta_{3}\right] n_{f}+\left[\frac{712}{81}+\frac{64}{9} \zeta_{2}\right] n_{f}^{2}(9.1 .20 \mathrm{e})  \tag{9.1.20f}\\
C_{3}^{(2)}= & \frac{1}{8}\left[-2 \beta^{(1)} \gamma_{m}^{(0)}+\left(\gamma_{m}^{(0)}-2 \beta^{(0)}\right)\left(2 \gamma_{m}^{(1)}+\left(\gamma_{m}^{(0)}-4 \beta^{(0)}\right) C_{1}^{(0)}\right)\right]  \tag{9.1.20g}\\
= & -\frac{3034}{3}+\frac{428}{3} n_{f}-\frac{104}{27} r_{f}^{2}, \\
C_{3}^{(3)}= & \frac{1}{48}\left(\gamma_{m}^{(0)}-4 \beta^{(0)}\right)\left(\gamma_{m}^{(0)}-2 \beta^{(0)}\right) \gamma_{m}^{(0)}=84-\frac{128}{9} n_{f}+\frac{16}{27} n_{f}^{2} .
\end{align*}
$$

$O\left(\alpha_{s}^{2}\right)$ terms are given in Ref. [996, 997, 990] and $O\left(\alpha_{s}^{3}\right)$ terms in Ref. [998, 999, 1000]. Inversion formulae of (9.1.19) is obtained as

## We obtain:

-5,333;
$-\frac{3161}{18}-\frac{16}{3}(7 \times 1.6449+2 \times 0.69314718 \times 1.6449)+$ $\frac{8}{3} \times 1.20205+\left(\frac{71}{9}+\frac{16}{3} \times 1.6449\right) \times 4$
$-179,33017 . .$.
$\left(-\frac{1163813}{162}+\frac{608}{27} \times 0.69314718^{4}+\right.$
$\left.\quad\left(-\frac{2815124}{405}+\frac{36160 \times 0.69314718}{27}+\frac{1}{9}\left(1024 \times 0.69314718^{2}\right)\right)\right) \times 1.6449$
Result:
More digits
-21625.1509257993171384286028538183763154299259259259259259...
-21625.1509...
$\left(-\frac{3304}{9}+\frac{11512 \times 1.6449}{9}\right) \times 1.20205+$
$\frac{6260 \times 1.0823}{9}-\frac{15800 \times 1.03692}{27}+\frac{4864 \times 0.517479062}{9}$
$2513.517388845 \overline{3}$ (period 1)
2513,51738...

$$
\begin{aligned}
& \left(\frac{167566}{243}-\frac{1}{81}\left(64 \times 0.69314718^{4}\right)+\right. \\
& \left.\quad\left(\frac{16304}{27}+\frac{1408 \times 0.69314718}{27}-\frac{1}{27}\left(256 \times 0.69314718^{2}\right)\right) \times 1.6449\right) \times 4
\end{aligned}
$$

6938.517876498652546258577447346465416954732510288065843621...
(period 27)
6938.51787
$\left(\frac{6232 \times 1.20205}{27}-\frac{4880 \times 1.0823}{27}-\frac{512 \times 0.517479062}{27}\right) \times 4$
288.089232630518 (period 3)
288.08923
$\left(-\frac{4706}{729}-\frac{416 \times 1.6449}{27}-\frac{224 \times 1.20205}{27}\right) \times 16$
-668.346012620027434842249657064471879286694101508916323731...
(period 81)
-668.3460126...

Result:
-12553.3724326
$-21625.1509+2513.51738+6938.51787+288.08923-668.3460126=-12553.3724326$
$-12553,3724326-179,33017-5,33333$
Result:
-12738.0359326
Final result
$-12738,0359326$ about $(728+21) * 17+5=12738 ; \quad 12738 / 1158=11$;
$1158=1164-6=1164-1728 / 288$
We have calculate the following integrals:
$32\left[\left(\mathrm{Pi}^{\wedge} 2 / 1164\right)\right]$ integrate $[-12738] \mathrm{x}$
$\frac{32 \pi^{2}}{1164} \int-12738 x d x \approx$ constant $+-1728.1 x^{2}$
Plot:


The result 1728 is the Ramanujan's number (1729-1)
integrate $\left(\left(\left(\left[2 *\left[\left((762 /(2 * \mathrm{Pi})) * \mathrm{Pi}^{\wedge} 2 / 1164.27\right) *[12738]\right)\right)\right)^{\wedge} 1 / 3 \mathrm{x}\right.\right.$
Indefinite integral:
$\int \frac{2\left(\left(762 \pi^{2}\right) \sqrt[3]{12738} x\right)}{(2 \pi) 1164.27} d x=24.0098 x^{2}+$ constant

Plot of the integral:


Note that $762 * 4 * 0.56706789=1728.42$ where 0.56706789 is about the Infinite power tower of $1 / \mathrm{e}$ (Omega constant) $0.567143 \ldots$

$0.5670678983907883690903972468241867067062310076575123455907248 \ldots$
and
$0.0680174 *$ integrate $\left(\left(\left(\left[2 *\left[\left((762 /(2 * \mathrm{Pi})) * \mathrm{Pi}^{\wedge} 2 /(1164.27) *[12738]\right)\right)\right)^{\wedge} 1 / 3 \mathrm{x}\right.\right.\right.$ 0.0680174761587831693972779 that is $1 / 10 *(\pi \sqrt{ } 3) / 8$ i.e. $1 / 10$ of "Body-centered cubic (bcc)"

Input interpretation:
$0.0680174 \int 2\left(\sqrt[3]{\frac{762}{2 \pi} \times \frac{\pi^{2}}{1164.27} \times 12738} x\right) d x$
Result:
$1.60322 x^{2}$


The result 1.60322 is very near to the value of the electric charge of the positron.
We have:
4;
$314 / 3-(52 * 4) / 9=81,5555 \ldots \ldots$
$-14+16 / 3=-8,6666 \ldots$.
$\frac{41354}{9}+(672+192 \times 0.69314718) \times 1.6449-48 \times 1.20205$
$5861.473585794232 \overline{8}$ (period 1)
5861,47358...
$-\left(\frac{13876}{27}+\left(\frac{1312}{9}+\frac{128 \times 0.69314718}{9}\right) \times 1.6449+\frac{448 \times 1.20205}{9}\right) \times 4+$
$\left(\frac{712}{81}+\frac{64 \times 1.6449}{9}\right) \times 16$
$-2991.271949700348 \overline{839506172}$ (period 9)
-2991.2719497
$5861,47358-2991,2719497=2870,2016303$
$-\frac{3034}{3}+\frac{428 \times 4}{3}-\frac{104 \times 16}{27}$

Exact result:
$-\frac{13562}{27}$
$-502 . \overline{296}($ period 3$)$
-502,296
$84-\frac{128 \times 4}{9}+\frac{16 \times 16}{27}$

Exact result:
$\frac{988}{27}$
$36 . \overline{592}$ (period 3)
36,592
2481,396
We have that: $-12738,0359326+2481,396=-10256,6399326$
$-10256,639 / 6=-1709,4398333 \ldots$
$-10256,639 / 14=732,61707$
$-12738,0359326-2481,396=-15219,4319326$
$-15219,4319326 / 8,8=-1729,4809014318$ where $8,8=0,55 * 16$;
$(-15219,4319326 / 108)-3,141592653 * 12=-1728,7471043471$
The mean of the two values obtained is: $-1729,114$ a value very near to the Ramanujan's number 1729

## From Wikipedia

In particle physics, quarkonium (from quark and -onium, pl. quarkonia) is a flavorless meson whose constituents are a heavy quark and its own antiquark, making it a neutral particle and the antiparticle of themselves. We take the following charmonium particle;

| Term symbol $n^{2 S+1}$ | $L_{J}$ | $\underline{\underline{I}}^{\mathrm{G}}\left(\underline{J}^{\mathrm{PC}}\right)$ | Particle |
| :--- | :--- | :--- | :--- |
| $1^{1} \mathrm{~S}_{0}$ | $0^{+}\left(0^{-}\right)$ | $\eta_{c}(1 S)$ | $2983.4 \pm 0.5$ |

Now, we take the mass equal to 2983,9 thence: $2983,9 * 9=2,68551 * 10^{19} \mathrm{MeV}$ is the correspondent energy and calculate the following integral:

1/(1728)* $1 /(2.68551)$ integrate [-15219.4319326]x
$\frac{1}{1728} \times \frac{1}{2.68551} \int-15219.4319326 x d x$

Result:
$-1.63983 x^{2}$


Alternate form assuming x is real:
$0-1.63983 x^{2}$

Indefinite integral assuming all variables are real:
$-0.546609 x^{3}+$ constant

$$
\left(\sqrt{\frac{113+5 \sqrt{505}}{8}}+\sqrt{\frac{105+5 \sqrt{505}}{8}}\right)^{3}=1164,2696=1164,27
$$

Note that:

$$
\sqrt[14.274]{\left(\sqrt{\frac{113+5 \sqrt{505}}{8}}+\sqrt{\frac{105+5 \sqrt{505}}{8}}\right)^{3}}=1,6398339 \ldots
$$

Note that $(1,0866246503513631746138436141496)^{32}=14,274$ where $1,08662465 \ldots$ is a very good approximation to the Ramanujan's new constant, i.e. 1,08643...

Furthermore:

$$
\begin{aligned}
& 1.08662465 \\
& \left(\sqrt{\frac{113+5 \sqrt{505}}{8}}+\sqrt{\frac{105+5 \sqrt{505}}{8}}\right)^{3} \\
& \\
& =6,6317902080152356725056810692356 \cdot 10^{-34}=
\end{aligned}
$$

The result 1,6393 and $6,631790208 \cdot 10^{-34}$ are very near to the value of the mass of proton and to the Planck's constant $6,626 * 10^{-34}$

With regard the $-15219,4319326 / 8,8=-1729,4809014318$ we observe that 8,8 is the $\Lambda(1520)$ branching ratios.

From Wikipedia, the free encyclopedia

In particle physics and nuclear physics, the branching fraction (or branching ratio) for a decay is the fraction of particles which decay by an individual decay mode with respect to the total number of particles which decay. It is equal to the ratio of the partial decay constant to the overall decay constant. Sometimes a partial half-life is given, but this term is misleading; due to competing modes it is not true that half of the particles will decay through a particular decay mode after its partial half-life.

$I\left(J^{P}\right)=0\left(\frac{3}{2}^{-}\right)$Status: ****

Discovered by FERRO-LUZZI 62; the elaboration in WATSON 63 is the classic paper on the Breit-Wigner analysis of a multichannel resonance.

The measurements of the mass, width, and elasticity published before 1975 are now obsolete and have been omitted. They were last listed in our 1982 edition Physics Letters 111B 1 (1982).

Production and formation experiments agree quite well, so they are listed together here.

## A(1520) MASS


$\Gamma(\boldsymbol{\Lambda} \gamma) / \Gamma_{\text {total }}$


VALUE (units $10^{-3}$ ) EVTS

## $8.5 \pm 1.5$ OUR ESTIMATE

## $8.8 \pm 1.1$ OUR FIT

$8.8 \pm 1.1$ OUR AVERAGE

| $10.7 \pm 2.9+1.5$ | 32 | TAYLOR | 05 | CLAS $\gamma p \rightarrow K+\Lambda \gamma$ |  |
| :--- | ---: | :--- | :--- | :--- | :--- |
| $10.2 \pm 2.1 \pm 1.5$ | 290 | ANTIPOV | 04 A | SPNX | $p N(C) \rightarrow \Lambda(1520) K+N(C)$ |
| $8.0 \pm 1.4$ | 238 | MAST | 683 HBC | Using $\Gamma(N K) / \Gamma_{\text {total }}=0.45$ |  |

We see as the value $8.8 \pm 1.1$ are present in the above table. We have that the value 15219,43 / 1729 that is the Ramanujan's taxicab number, is equal to 8,8024465 that is practically equal to the value of branching ratios

## From Wikipedia

The Lambda baryons are a family of subatomic hadron particles containing one up quark, one down quark, and a third quark from a higher flavour generation, in a combination where the quantum wave function changes sign upon the flavour of any two quarks being swapped (thus differing from a Sigma baryon). They are thus baryons, with total isospin of 0 , and have either neutral electric charge or the elementary charge +1 .

| Particle name | Symbol | Quark content | $\frac{\text { Rest mass }}{\left(\mathrm{MeV} / \underline{c}^{2}\right)}$ | 1 | ${ }^{\mathrm{J}} \mathrm{P}$ | $\frac{\mathrm{Q}}{(\text { e }}$ | S | C | B' | $\underline{T}$ | Mean lifetime (s) | Commonly decays to |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Lambda ${ }^{[6]}$ | $\Lambda^{0}$ | uds | $1115.683 \pm 0.006$ | 0 | $\frac{1}{2}+$ | 0 | -1 | 0 | 0 | 0 | $(2.631 \pm 0.020) \times 10^{-10}$ | $\mathrm{p}^{+}+\underline{\pi}^{-}$or $\underline{n}^{0}+\underline{\pi}^{0}$ |
| charmed <br> Lambda ${ }^{[15]}$ | $\Lambda_{c}^{+}$ | udc | $2286.46 \pm 0.14$ | 0 | $\frac{1}{2}$ + + | +1 | 0 | +1 | 0 | 0 | $(2.00 \pm 0.06) \times 10^{-13}$ | See $\wedge_{c}^{+}$ decay modes |
| bottom Lambda ${ }^{[16]}$ | $\Lambda_{b}^{0}$ | udb | $5620.2 \pm 1.6$ | 0 | $\frac{1}{2}$ + + | 0 | 0 | 0 | -1 | 0 | $1.409{ }_{-0.054}^{+0.055} \times 10^{-12}$ | See $\Lambda_{b}^{0}$ decay <br> modes |
| $\begin{aligned} & \text { top } \\ & \text { Lambda }^{\dagger} \end{aligned}$ | $\Lambda_{t}^{+}$ | udt | - | 0 | $\frac{1}{2}$ + + | +1 | 0 | 0 | 0 | +1 | - | - |

We remember that:
The relation between the $\overline{\mathrm{MS}}$ running mass and the pole mass, assuming $\left(n_{f}-1\right)$ massless quarks and one massive quark, is written as a power series of $a^{\left[n_{j}\right]}(\mu)$ and $L_{\mathrm{OS}}=\log \left(m_{\mathrm{OS}}^{2} / \mu^{2}\right)$.

From the equations concerning this argument, we have obtained the following result -15219,43

In quantum field theory, the pole mass of an elementary particle corresponds to the concept of rest mass in the special theory of relativity.
In particle physics, the invariant mass $m_{0}$ is equal to the mass in the rest frame of the particle, and can be calculated by the particle's energy $E$ and its momentum $\mathbf{p}$ as measured in any frame, by the energy-momentum relation:

$$
m_{0}^{2} c^{2}=\left(\frac{E}{c}\right)^{2}-\|\mathbf{p}\|^{2}
$$

or in natural units where $c=1$

$$
m_{0}^{2}=E^{2}-\|\mathbf{p}\|^{2} .
$$

This invariant mass is the same in all frames of reference (see also special relativity).
In quantum field theory, quantities like coupling constant and mass "run" with the energy scale of high energy physics. The running mass of a fermion or massive boson depends on the energy scale at which the observation occurs, in a way described by a renormalization group equation (RGE) and calculated by a renormalization scheme such as the on-shell scheme or the minimal subtraction scheme. The running mass refers to a Lagrangian parameter whose value changes with the energy scale at which the renormalization scheme is applied. A calculation, typically done by a computerized algorithm intractable by paper calculations, relates the running mass to the pole mass. The algorithm typically relies on a perturbative calculation of the self energy.

We note that dividing $15219,43 / 5620,2=2,70798726$; it is interesting observe that the square root of this value is: $\sqrt{ } 2,70798726=1,64559632 \ldots$ a good approximation to the mean of the two values obtained from the Ramanujan's class invariant. Indeed, we have:
$(1,6398339+1,6557845) / 2=1,6478092$
With regard 5620,2 we calculate the following integral:
$\mathrm{Pi}^{\wedge} 2 /\left(9^{*} 1729\right)^{*}$ integrate [5260.2]x

Input interpretation:

$$
\frac{\pi^{2}}{9 \times 1729} \int 5260.2 x d x
$$

## $1.66815 x^{2}$

Plot:


The mean between 1,64559 and 1,66815 is 1,65687 a value very near to the Ramanujan's class invariant and to the value of the mass of proton.

Now, we have:
$O\left(\alpha_{s}^{2}\right)$ terms are given in Ref. [996, 997, 990] and $O\left(\alpha_{s}^{3}\right)$ terms in Ref. [998, 999, 1000]. Inversion formulae of (9.1.19) is obtained as

$$
\begin{align*}
& \frac{m_{\mathrm{OS}}}{m_{\frac{\left[n_{f}\right]}{\mathrm{MS}}}(\mu)}=1+\sum_{k=1}^{\infty} a^{k}(\mu)\left[\sum_{m=0}^{k} C_{k}^{\prime(m)} L_{\overline{\mathrm{MS}}}^{m}\right], \quad L_{\overline{\mathrm{MS}}}=\log \frac{m_{\frac{\left[n_{f}\right] 2}{\mathrm{MS}}(\mu)}^{\mu^{2}},}{},  \tag{9.1.21a}\\
& C_{1}^{\prime(0)}=-C_{1}^{(0)}=\frac{16}{3}, \quad C_{1}^{r(1)}=-C_{1}^{(1)}=-4,  \tag{9.1.21b}\\
& C_{2}^{\prime(0)}=-C_{2}^{(0)}+2 C_{1}^{(1)} C_{1}^{(0)}+C_{1}^{(0) 2} \\
& =\frac{2905}{18}+\frac{16}{3}(7+2 \log 2) \zeta_{2}-\frac{8}{3} \zeta_{3}-\left[\frac{71}{9}+\frac{16}{3} \zeta_{2}\right] n_{f},  \tag{9.1.21c}\\
& C_{2}^{\prime(1)}=-C_{2}^{(1)}+2 C_{1}^{(1) 2}+2 C_{1}^{(1)} C_{1}^{(0)}=-\frac{346}{3}+\frac{52}{9} n_{f},  \tag{9.1.21d}\\
& C_{2}^{\prime(2)}=-C_{2}^{(2)}-C_{2}^{(1)} C_{1}^{(1) 2}+2 C_{1}^{(1) 2}+2 C_{1}^{(1)} C_{1}^{(0)}=30-\frac{4}{3} n_{f},  \tag{9.1.21e}\\
& C_{3}^{\prime(0)}=-C_{3}^{(0)}+2 C_{2}^{(1)} C_{1}^{(0)}+2 C_{2}^{(0)} C_{1}^{(1)}+2 C_{2}^{(0)} C_{1}^{(0)} \\
& -4 C_{1}^{(1) 2} C_{1}^{(0)}-5 C_{1}^{(1)} C_{1}^{(0) 2}-C_{1}^{(0) 3} \\
& =\frac{1046525}{162}-\frac{608}{27}(\log 2)^{4}+\left(\frac{2855444}{405}-\frac{35392}{27} \log 2-\frac{1024}{9}(\log 2)^{2}\right) \zeta_{2} \\
& +\left(360-\frac{11512}{9} \zeta_{2}\right) \zeta_{3}-\frac{6260}{9} \zeta_{4}+\frac{15800}{27} \zeta_{5}-\frac{4864}{9} \mathrm{Li}_{4}\left(\frac{1}{2}\right) \\
& +\left[-\frac{157702}{243}+\frac{64}{81}(\log 2)^{4}+\left(-\frac{16688}{27}-\frac{1408}{27} \log 2+\frac{256}{27}(\log 2)^{2}\right) \zeta_{2}\right. \\
& \left.-\frac{6232}{27} \zeta_{3}+\frac{4880}{27} \zeta_{4}+\frac{512}{27} \operatorname{Li}_{4}\left(\frac{1}{2}\right)\right] n_{f} \\
& +\left[\frac{4706}{729}+\frac{416}{27} \zeta_{2}+\frac{224}{27} \zeta_{3}\right] n_{f}^{2},  \tag{9.1.21f}\\
& C_{3}^{r(1)}=-C_{3}^{(1)}+4 C_{2}^{(2)} C_{1}^{(0)}+4 C_{2}^{(1)} C_{1}^{(1)}+2 C_{2}^{(1)} C_{1}^{(0)} \\
& +2 C_{2}^{(0)} C_{1}^{(1)}-4 C_{1}^{(1) 3}-10 C_{1}^{(1) 2} C_{1}^{(0)}-3 C_{1}^{(1)} C_{1}^{(0) 2} \\
& =-\frac{43982}{9}-\left(\frac{2912}{3}+\frac{832}{3} \log 2\right) \zeta_{2}+\frac{208}{3} \zeta_{3} \\
& +\left[\frac{4660}{9}+\left(\frac{1696}{9}+\frac{128}{9} \log 2\right) \zeta_{2}+\frac{448}{9} \zeta_{3}\right] n_{f}-\left[\frac{712}{81}+\frac{64}{9} \zeta_{2}\right] n_{f}^{2},(9.1 .21 \mathrm{~g}) \\
& C_{3}^{\prime(2)}=-C_{3}^{(2)}+6 C_{2}^{(2)} C_{1}^{(1)}+2 C_{2}^{(2)} C_{1}^{(0)}+2 C_{2}^{(1)} C_{1}^{(1)}-5 C_{1}^{(1) 3}-3 C_{1}^{(1) 2} C_{1}^{(0)} \\
& =1598-\frac{1540}{9} n_{f}+\frac{104}{27} n_{f}^{2} \text {, }  \tag{9.1.21~h}\\
& C_{3}^{\prime(3)}=-C_{3}^{(3)}+2 C_{2}^{(2)} C_{1}^{(1)}-C_{1}^{(1) 3}=-260+\frac{224}{9} n_{f}-\frac{16}{27} n_{f}^{2} . \tag{9.1.21i}
\end{align*}
$$

We have that:
$16 / 3=5,3333333333 ;-4 ;$

$$
\begin{aligned}
& \frac{2905}{18}+\frac{16}{3}(7 \times 1.6449+2 \times 0.69314718 \times 1.6449)- \\
& \frac{8 \times 1.20205}{3}-\frac{71 \times 4}{9}-\frac{1}{3}(16 \times 4 \times 1.64499)
\end{aligned}
$$

165.10602982807466

$$
-\frac{346}{3}+\frac{52 \times 4}{9}
$$

$-92.22222222222222$

$$
30-\frac{4 \times 4}{3}
$$

24.666666666666666

$$
\begin{aligned}
& \frac{1046525}{162}-\frac{1}{27}\left(608 \times 0.69314718^{4}\right)+ \\
& \left(\frac{2855444}{405}-\frac{35392 \times 0.69314718}{27}-\frac{1}{9}\left(1024 \times 0.69314718^{2}\right)\right) \times 1.6449
\end{aligned}
$$

16467.7117416566723149

$$
\begin{aligned}
& 360 \times 1.20205-\frac{1}{9}(11512 \times 1.6449 \times 1.20205)- \\
& \frac{6260 \times 1.0823}{9}+\frac{15800 \times 1.03692}{27}-\frac{4864 \times 0.517479062}{9}
\end{aligned}
$$

$-2522.0652999564444$
$4\left(-\frac{157702}{243}+\frac{1}{81}\left(64 \times 0.69314718^{4}\right)+\right.$

$$
\begin{aligned}
& \left(-\frac{16688}{27}-\frac{1408 \times 0.69314718}{27}+\frac{1}{27}\left(256 \times 0.69314718^{2}\right)\right) \times 1.6449- \\
& \left.\frac{6232 \times 1.20205}{27}+\frac{4880 \times 1.0823}{27}+\frac{512 \times 0.517479062}{27}\right)
\end{aligned}
$$

$-7157.81327209213402$
$4\left(\frac{4706}{729}+\frac{416 \times 1.6449}{27}+\frac{224 \times 1.20205}{27}\right)$
167.0865031550068587

```
5.3333333333-4 + 165.10602982807466-92.22222222222222+
    24.666666666666666 + 16467.7117416566723149 -
    2522.0652999564444-7157.81327209213402 + 167.0865031550068587
```

7053.8034803689198596 total partial

$$
\begin{aligned}
& \left(-\frac{43982}{9}-\frac{2912 \times 1.6449}{3}+\frac{1}{3}(832 \times 0.69314718 \times 1.6449)+\frac{208 \times 1.20205}{3}+\right. \\
& \left.\quad\left(\frac{4660}{9}+\frac{1696 \times 1.6449}{9}+\frac{1}{9}(128 \times 0.69314718 \times 1.6449)+\frac{448 \times 1.20205}{9}\right) \times 4\right)- \\
& \left(\frac{712}{81}+\frac{64 \times 1.6449}{9}\right) \times 16
\end{aligned}
$$

$-2796.5836362511913086$

$$
1598-\frac{1540 \times 4}{9}+\frac{104 \times 16}{27}
$$

975.185185185185185185

$$
-260+\frac{224 \times 4}{9}-\frac{16 \times 16}{27}
$$

$-169.925925925925925925$

```
7053.8034803689198596 - 2796.5836362511913086 -
    975.185185185185185185-169.925925925925925925
```


### 3112.10873300661743989 Final Result

$3112=389 * 8$
Note that $3112.108733 / 16^{2}=12,156674738307$..
From: (http://www.sns.ias.edu/pitp2/2007files/Lecture\ NotesProblems/Witten Threedimgravity.pdf)

Let us give an example. If $k=1$, the partition function is simply the J-function itself, so

$$
Z(q)=q^{-1}+196884 q+\ldots
$$

The number of black hole primaries of mass 2 is therefore 196883. The black hole entropy is therefore $\log (196883)=12.19 \ldots$ The classical entropy of a black hole with $k=1$ and mass 2 is $4 \pi=12.57 \ldots$ So we are off by just a few percent.

We note that the value that we have obtained $12,156 \ldots$ is a very good approximation of the value $12,19 \ldots$ that is the black hole entropy obtained from $\log (196883)$

We have that:

$$
\begin{gathered}
\sqrt[16]{3112.108733}=1,6531639364667 \ldots \\
\sqrt{\left(\frac{3112.108733}{1164.2696}\right)}=\sqrt{2,6730138217 \ldots}=1,6349354182 \ldots
\end{gathered}
$$

The result 1,63493 is very near to the fourteenth root of the Ramanujan's class invariant and to the mass of proton.

We note that: $3112,108733-1728=1384,108733$
With regard the baryon sigma, we have that (from: Citation: C. Amsler et al. (Particle Data Group), PL B667, 1 (2008) (URL: http://pdg.lbl.gov)

## $\Sigma(1385)^{0}$ MASS

| VALUE (MeV) | EVTS | DOCUMENT ID |  | TECN | COMMENT |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $1383.7 \pm 1.0$ OUR AVERAGE |  | Error includes scale factor of 1.4. See the ideogram below. |  |  |  |
| $1384.1+0.8$ | 5722 | AGIJII AR- | 81D | HBC | $K^{-} p \rightarrow \Lambda 3 \pi 4.2 \mathrm{GeV} / c$ |
| $1380 \pm 2$ | 3100 | ${ }^{5}$ BORENSTEIN | 74 | HBC | $\underset{\mathrm{GeV} / \mathrm{c}}{\mathrm{~K} \rightarrow} 13 \pi 2.18$ |
| $1385.1 \pm 2.5$ | 240 | 4 THOMAS | 73 | HBC | $\pi^{-} \rho \rightarrow \Lambda \pi^{0} K^{0}$ |

- . We do not use the following data for averages, fits, limits, etc.
$1389 \pm 3 \quad 500 \quad{ }^{6}$ BAUBILLIER 798 HBC $K^{-} p 8.25 \mathrm{GeV} / c$

We observe that:

$$
\text { 1383.7 } 1.0 \text { OUR AVERAGE }
$$

$$
1384.1 \pm 0.8 \quad 5722 \quad K p \rightarrow \Lambda 3 \pi 4.2 \quad \mathrm{GeV} / c
$$

$$
1380 \pm 2 \quad 3100 \quad K^{K} p \rightarrow \Lambda 3 \pi 2.18 \mathrm{GeV} / c
$$

Indeed: $1384.1 \pm 0.8$ is very near to the our result:
$3112,108733-1728=1384,108733$
From:

## A Study of Excited Charm-Strange Baryons with Evidence for new Baryons $\boldsymbol{\Xi}_{\mathrm{c}}$ $(3055)^{+}$and $\Xi_{\mathbf{c}}(3123)^{+}$- https://arxiv.org/pdf/0710.5763.pdf

Note that $\quad \Xi_{\mathrm{c}}(3123)(1.6 \pm 0.6 \pm 0.2) \mathrm{fb}<1.4 \mathrm{fb}$;
$\Xi_{\mathrm{c}}(3123)^{+}$Mass Resolution $\pm 0.3 \pm 1.5 \pm 5.0$ NA NA
Background Shape $\pm 0.2 \pm 0.6 \pm 6.9$ NA NA
Phase-Space Thresh. $\pm 0.1 \pm 0.5 \pm 3.0$ NA NA
Mass Scale $\pm 0.1$ NA NA NA NA
Total $\pm 0.3 \pm 1.7 \pm 8.9$ NA NA

In quoting upper limits for $\Xi_{\mathrm{c}}(3077)^{0}$ and $\Xi_{\mathrm{c}}(3077)^{+}$, we consider the integrated yield up to $3093 \mathrm{MeV} / \mathrm{c}^{2}$ and $3089 \mathrm{MeV} / \mathrm{c}^{2}$, respectively $\left(\approx 30 \mathrm{MeV} / \mathrm{c}^{2}\right.$ above threshold in each case).

We note that: $3123-0.3-1.7-8.9=3112,10$ that is perfectly the obtained result!

Furthermore, we have calculate the following integrals:
1728/(728+288) * integrate [3112.10873300661743989]x
Input interpretation:
$\frac{1728}{728+288} \int 3112.10873300661743989 x d x$
Result:
$2646.51766271428884652 x^{2}$


1728/(728+226) * integrate [3112.10873300661743989]x
Input interpretation:
$\frac{1728}{728+226} \int 3112.10873300661743989 x d x$
Result:
$2818.51356951542711537 x^{2}$
Plot:


Indefinite integral assuming all variables are real:
$939.504523171809038457 x^{3}+$ constant

1728/(728+236) * integrate [3112.10873300661743989]x
Input interpretation:
$\frac{1728}{728+236} \int 3112.10873300661743989 x d x$
Result:
$2789.27587688559903326 x^{2}$
Plot:


Indefinite integral assuming all variables are real:
$929.758625628533011087 x^{3}+$ constant

1728/(728+316) * integrate [3112.10873300661743989]x

Input interpretation:
$\frac{1728}{728+316} \int 3112.10873300661743989 x d x$
Result:
$2575.53826179857995025 x^{2}$

Plot:


1728/(728+360) * integrate [3112.10873300661743989]x

Input interpretation:
$\frac{1728}{728+360} \int 3112.10873300661743989 x d x$
Result:
$2471.38046444643149638 x^{2}$

Plot:

results that are very good approximations of the values of the mass of the charmed baryons.

We have other very significant connections between the number 1728 and its factors and all the mass of the charmed baryons:
$1728+576+144+96+36+16=2596$
$1728+576+288+144+64+16=2816$
$1728+576+288+144+48+6=2790$
$1728+576+288+48+6=2646$
$1728+576+144+64+48+16=2576$
$1728+576+144+24=2472$
$1728+576+144+108+64+8=2628$
and other...!
further, we have that $1728+728+288+144+64+16=2968$
Note that $3112.108733+2698=5810.108733$ value very near to the Sigma bottom that is $5807,8 \pm 2,7$

From the Ramanujan's equation above analyzed
$64 \mathrm{~J}^{2}-24 \mathrm{~J}+9=64 * 400400100-24 * 20010+9=25625606400-480240+9=$ $=25625126169$;
we have that $(25625126169)^{1 / 3}=2948,1891086 \ldots$.
and $3112,10-728=2384,108733 ; 2384+144+108+24=2660 ;$
$3112-144-27=2941 ; 3112-36=3076 ;$
$3112-108-24=2980 \quad 3112-144-64-24=2880$
$2384+64+72=2520 ; 2384+288+96=2768 ;$
$2384+576+16=2976 \quad 2384+288+144+64=2880$
$2384+288+144+96+27=2939$

TABLE II: Mass spectra and decay widths (in units of MeV ) of charmed baryons. Experimental values are taken from the Particle Data Group [3] except $\Lambda_{c}(2880), \Lambda_{c}(2940), \Xi_{c}(2980)^{+, 0}, \Xi_{c}(3077)^{+, 0}$ and $\Omega_{c}(2768)$ for which we use the most recent available BaBar and Belle measurements.

| State | quark content | $J^{P}$ | Mass | Width |
| :---: | :---: | :---: | :---: | :---: |
| $\Lambda_{c}^{+}$ | $u d c$ | $\frac{1}{2}^{+}$ | $2286.46 \pm 0.14$ |  |
| $\Lambda_{c}(2593)^{+}$ | $u d c$ | $\frac{1}{2}^{-}$ | $2595.4 \pm 0.6$ | $3.6{ }_{-1.3}^{+2.0}$ |
| $\Lambda_{c}(2625)^{+}$ | $u d c$ | $\frac{3}{2}^{-}$ | $2628.1 \pm 0.6$ | $<1.9$ |
| $\Lambda_{c}(2765)^{+}$ | $u d c$ | ? ${ }^{\text {? }}$ | $2766.6 \pm 2.4$ | 50 |
| $\Lambda_{c}(2880)^{+}$ | $u \mathrm{dc}$ | $\frac{5}{2}^{+}$ | $2881.5 \pm 0.3$ | $5.5 \pm 0.6$ |
| $\Lambda_{c}(2940)^{+}$ | $u d c$ | ? | $2938.8 \pm 1.1$ | $13.0 \pm 5.0$ |
| $\Sigma_{c}(2455)^{++}$ | uиe | $\frac{1}{2}^{+}$ | $2454.02 \pm 0.18$ | $2.23 \pm 0.30$ |
| $\Sigma_{c}(2455)^{+}$ | $u d c$ | $\frac{1}{2}^{+}$ | $2452.9 \pm 0.4$ | $<4.6$ |
| $\Sigma_{c}(2455)^{0}$ | $d d c$ | $\frac{1}{2}^{+}$ | $2453.76 \pm 0.18$ | $2.2 \pm 0.4$ |
| $\Sigma_{c}(2520)^{++}$ | uиe | $\frac{3}{2}^{+}$ | $2518.4 \pm 0.6$ | $14.9 \pm 1.9$ |
| $\Sigma_{c}(2520)^{+}$ | $u d c$ | $\frac{3}{2}^{+}$ | $2517.5 \pm 2.3$ | $<17$ |
| $\Sigma_{c}(2520)^{0}$ | $d d c$ | $\frac{3}{2}^{+}$ | $2518.0 \pm 0.5$ | $16.1 \pm 2.1$ |
| $\Sigma_{c}(2800)^{++}$ | uис | ?? | $28011_{-6}^{+4}$ | $75_{-17}^{+22}$ |
| $\Sigma_{c}(2800)^{+}$ | $u d c$ | ?? | $2792-5$ | $62_{-40}^{+60}$ |
| $\Sigma_{c}(2800)^{0}$ | $d d c$ | ? ${ }^{\text {? }}$ | $2802_{-7}^{+4}$ | $61_{-18}^{+28}$ |
| $\Xi_{c}^{+}$ | $u s c$ | $\frac{1}{2}^{+}$ | $2467.9 \pm 0.4$ |  |
| $\Xi_{c}^{0}$ | $d s c$ | $\frac{1}{2}^{+}$ | $2471.0 \pm 0.4$ |  |
| $\Xi_{c}^{\prime+}$ | usc | $\frac{1}{2}^{+}$ | $2575.7 \pm 3.1$ |  |
| $\Xi_{c}^{\prime 0}$ | $d s c$ | $\frac{1}{2}^{+}$ | $2578.0 \pm 2.9$ |  |
| $\Xi_{c}(2645)^{+}$ | usc | $\frac{3}{2}^{+}$ | $2646.6 \pm 1.4$ | $<3.1$ |
| $\Xi_{c}(2645)^{0}$ | $d s c$ | $\frac{3}{2}^{+}$ | $2646.1 \pm 1.2$ | $<5.5$ |
| $\Xi_{c}(2790)^{+}$ | usc | $\frac{1}{2}$ | $2789.2 \pm 3.2$ | <15 |
| $\Xi_{c}(2790)^{0}$ | $d s c$ | $\frac{1}{2}$ | $2791.9 \pm 3.3$ | $<12$ |
| $\Xi_{c}(2815)^{+}$ | usc | $\frac{3}{2}$ | $2816.5 \pm 1.2$ | $<3.5$ |
| $\Xi_{c}(2815)^{0}$ | $d s c$ | $\frac{3}{2}^{-}$ | $2818.2 \pm 2.1$ | $<6.5$ |
| $\Xi_{c}(2980)^{+}$ | usc | ?? | $2971.1 \pm 1.7$ | $25.2 \pm 3.0$ |
| $\Xi_{c}(2980)^{0}$ | $d s c$ | ?? | $2977.1 \pm 9.5$ | 43.5 |
| $\Xi_{c}(3077)^{+}$ | usc | ?? | $3076.5 \pm 0.6$ | $6.2 \pm 1.1$ |
| $\Xi_{c}(3077)^{0}$ | $d s c$ | ?? | $3082.8 \pm 2.3$ | $5.2 \pm 3.6$ |
| $\Omega_{c}^{0}$ | SSC | $\frac{1}{2}^{+}$ | $2697.5 \pm 2.6$ |  |
| $\Omega_{c}(2768)^{0}$ | ssc | $\frac{3}{2}^{+}$ | $2768.3 \pm 3.0$ |  |

We take the precedent values: $10256,639 \quad 15219,4319 \quad 108304,5462209694$ and we have that:
$108304,546 / 48=2256,344708333 \quad 108304,546 / 36=3008,4596111$
$2209694 / 864=2557,5162037$
Values of some charmed baryons
$\begin{array}{llllll}2575.7 & 2578 & 2517.5 & 2518.4 & 2286.46 & 2977.1\end{array}$
From the following calculations:
$2557,5+24=2581.25 \quad 2564,159+12=2576.159 \quad 2536,57-18=2518.57$
$2256,34+32=2288.34 \quad 3008,459-32=2976.459$
we obtain the following very good approximations:
$2576.159 \approx 2575.7 \quad 2581.25 \approx 2578 \quad 2518.57 \approx 2517.5-2518.4$
$2288.34 \approx 2286.46 \quad 2976.459 \approx 2977.1$

Now, we have:
$\delta_{c}^{(k)}$ are coefficients in the $\overline{\text { MS }}$-pole conversion formula for the charm quark mass.

$$
\begin{align*}
\frac{m_{c, \cap S}}{m_{c, \overline{\mathrm{MS}}}\left(\mu_{c}\right)} & =1+\epsilon \delta_{c}^{(1)}+\epsilon^{2} \delta_{c}^{[2)}+\epsilon^{3} \delta_{c}^{(3)}+\cdots,  \tag{9.1.78a}\\
\delta_{c}^{(1)} & =\frac{\alpha_{c}^{[4]}(\mu)}{\pi}\left[\frac{4}{3}-I_{c, \overline{\mathrm{NS}}}\right], \tag{9.1.78h}
\end{align*}
$$

$$
\begin{align*}
\delta_{c}^{(2)}= & \frac{\alpha_{s}^{[4] 2}(\mu)}{\pi^{2}}\left[\frac{779}{96}+\frac{1}{6} \pi^{2}+\frac{1}{9} \pi^{2} \log 2-\frac{1}{6} \zeta_{3}-\frac{25}{9} L_{c, \overline{\mathrm{MS}}}^{[\mu]}\right. \\
& \left.-\left(\frac{215}{72}-\frac{25}{12} L_{c, \overline{\mathrm{MS}}}^{[\mu]}\right) L_{c, \overline{\mathrm{MS}}}-\frac{13}{24} L_{c, \overline{\mathrm{MS}}}^{2}\right]  \tag{9.1.78c}\\
\delta_{c}^{(3)}= & \frac{\alpha_{s}^{[4] 3}(\mu)}{\pi^{3}}\left[\frac{5784469}{93312}+\frac{488501}{38880} \pi^{2}+\frac{37}{7776} \pi^{4}-\frac{49}{162}(\log 2)^{4}-\frac{641}{162} \pi^{2} \log 2\right. \\
& -\frac{16}{81} \pi^{2}(\log 2)^{2}-\frac{1453}{216} \zeta_{3}-\frac{1439}{432} \pi^{2} \zeta_{3}+\frac{1975}{216} \zeta_{5}-\frac{196}{27} L_{4}\left(\frac{1}{2}\right) \\
& +\left(-\frac{7313}{192}-\frac{25}{36} \pi^{2}-\frac{25}{54} \pi^{2} \log 2+\frac{25}{36} \zeta_{3}\right) L_{c, \overline{\mathrm{MS}}}^{[\mu]}+\frac{625}{108} L_{c, \overline{\mathrm{MS}}}^{[\mu] 2} \\
& +\left(-\frac{42019}{5184}-\frac{1}{6} \pi^{2}-\frac{1}{9} \pi^{2} \log 2+\frac{7}{2} \zeta_{3}+\frac{6761}{432} L_{c, \overline{\mathrm{MS}}}^{[\mu]}-\frac{625}{144} L_{c, \overline{\mathrm{MS}}}^{[\mu] 2}\right) L_{c, \overline{\mathrm{MS}}} \\
& \left.+\left(-\frac{5357}{864}+\frac{325}{288} L_{c, \mathrm{MS}}^{[\mu]}\right) L_{c, \overline{\mathrm{MS}}}^{2}-\frac{247}{432} L_{c, \overline{\mathrm{MS}}}^{3}\right] \tag{9.1.78d}
\end{align*}
$$

where

$$
\begin{equation*}
L_{c, \overline{\mathrm{MS}}}=\log \frac{m_{c, \overline{\mathrm{MS}}}^{2}\left(\mu_{c}\right)}{\mu_{c}^{2}}, \quad L_{c, \overline{\mathrm{MS}}}^{[\mu]}=\log \frac{m_{c, \overline{\mathrm{MS}}}^{2}\left(\mu_{c}\right)}{\mu^{2}} . \tag{9.1.79}
\end{equation*}
$$

Eq. (9.1.78) is obtained by setting $n_{f}=4$ in Eq. (9.1.22). $\Delta_{2}^{\prime}\left(L_{\mu}, a\right)$ and $\Delta_{3}^{\prime}\left(L_{\mu}, a\right)$ are expanded
$\frac{5.13 \times 2}{\pi}\left(\frac{4}{3}-4.852\right)$
$-11.4915 \ldots$
-11.4915

$$
\begin{aligned}
& \frac{5.13 \times 2}{\pi^{2}}\left(\frac{779}{96}+\frac{\pi^{2}}{6}+\frac{1}{9}\left(\pi^{2} \times 0.69314718\right)-\frac{1.20205}{6}-\frac{25 \times 5.7693668}{9}-\right. \\
& \left.\frac{215 \times 4.852}{72}+\frac{1}{12}(25 \times 5.7693668 \times 4.852)-\frac{1}{24}\left(13 \times 5.7693668^{2}\right)\right)
\end{aligned}
$$

20.8885...
20.8885
0.330901 *

$$
\begin{aligned}
& \frac{5784469}{93312}+\frac{488501 \pi^{2}}{38880}+\frac{37 \pi^{4}}{7776}- \\
& \frac{1}{162}\left(49 \times 0.69314718^{4}\right)-\frac{1}{162}\left(641 \pi^{2} \times 0.69314718\right)
\end{aligned}
$$

159.320475...
159.320475

Integral representations:

$$
\begin{aligned}
& \frac{5784469}{93312}+\frac{488501 \pi^{2}}{38880}+\frac{37 \pi^{4}}{7776}-\frac{49 \times 0.693147^{4}}{162}-\frac{641 \pi^{2} 0.693147}{162}= \\
& 0.0761317\left(1.58097+\left(\int_{0}^{\infty} \frac{1}{1+t^{2}} d t\right)^{2}\right)\left(514.456+\left(\int_{0}^{\infty} \frac{1}{1+t^{2}} d t\right)^{2}\right) \\
& \frac{5784469}{93312}+\frac{488501 \pi^{2}}{38880}+\frac{37 \pi^{4}}{7776}-\frac{49 \times 0.693147^{4}}{162}-\frac{641 \pi^{2} 0.693147}{162}= \\
& 0.0761317\left(1.58097+\left(\int_{0}^{\infty} \frac{\sin (t)}{t} d t\right)^{2}\right)\left(514.456+\left(\int_{0}^{\infty} \frac{\sin (t)}{t} d t\right)^{2}\right)
\end{aligned}
$$

$$
\begin{aligned}
& \frac{5784469}{93312}+\frac{488501 \pi^{2}}{38880}+\frac{37 \pi^{4}}{7776}-\frac{49 \times 0.693147^{4}}{162}-\frac{641 \pi^{2} 0.693147}{162}= \\
& 1.21811\left(0.395242+\left(\int_{0}^{1} \sqrt{1-t^{2}} d t\right)^{2}\right)\left(128.614+\left(\int_{0}^{1} \sqrt{1-t^{2}} d t\right)^{2}\right)
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{1}{81}\left(16 \pi^{2} \times 0.69314718^{2}\right)-\frac{1453 \times 1.20205}{216}- \\
& \frac{1}{432}\left(1439 \pi^{2} \times 1.20205\right)+\frac{1975 \times 1.036929}{216}-\frac{196 \times 0.517479062}{27}
\end{aligned}
$$

-42.8164...
-42.8164

Integral representations:

$$
\begin{aligned}
& -\frac{1}{81}\left(16 \pi^{2} 0.693147^{2}\right)-\frac{1453 \times 1.20205}{216}-\frac{1439 \pi^{2} 1.20205}{432}+ \\
& \frac{1975 \times 1.03693}{216}-\frac{196 \times 0.517479}{27}=-2.36135-16.3958\left(\int_{0}^{\infty} \frac{1}{1+t^{2}} d t\right)^{2} \\
& -\frac{1}{81}\left(16 \pi^{2} 0.693147^{2}\right)-\frac{1453 \times 1.20205}{216}-\frac{1439 \pi^{2} 1.20205}{432}+ \\
& \quad \frac{1975 \times 1.03693}{216}-\frac{196 \times 0.517479}{27}=-2.36135-65.5833\left(\int_{0}^{1} \sqrt{1-t^{2}} d t\right)^{2}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{1}{81}\left(16 \pi^{2} 0.693147^{2}\right)-\frac{1453 \times 1.20205}{216}-\frac{1439 \pi^{2} 1.20205}{432}+ \\
& \frac{1975 \times 1.03693}{216}-\frac{196 \times 0.517479}{27}=-2.36135-16.3958\left(\int_{0}^{\infty} \frac{\sin (t)}{t} d t\right)^{2} \\
& \left(-\frac{7313}{192}-\frac{1}{36}\left(25 \pi^{2}\right)-\frac{1}{54}\left(25 \pi^{2} \times 0.69314718\right)+\frac{25 \times 1.20205}{36}\right) \times 5.76936+ \\
& \frac{1}{108}\left(625 \times 5.76936^{2}\right)
\end{aligned}
$$

-80.1211...
$-80.1211$
Integral representation

$$
\begin{aligned}
& \left(-\frac{7313}{192}-\frac{25 \pi^{2}}{36}-\frac{25 \pi^{2} 0.693147}{54}+\frac{25 \times 1.20205}{36}\right) 5.76936+\frac{625 \times 5.76936^{2}}{108}= \\
& \quad-22.306-23.4316\left(\int_{0}^{\infty} \frac{1}{1+t^{2}} d t\right)^{2} \\
& \left(-\frac{7313}{192}-\frac{25 \pi^{2}}{36}-\frac{25 \pi^{2} 0.693147}{54}+\frac{25 \times 1.20205}{36}\right) 5.76936+\frac{625 \times 5.76936^{2}}{108}= \\
& \quad-22.306-93.7263\left(\int_{0}^{1} \sqrt{1-t^{2}} d t\right)^{2}
\end{aligned}
$$

$$
\begin{aligned}
& \left(-\frac{7313}{192}-\frac{25 \pi^{2}}{36}-\frac{25 \pi^{2} 0.693147}{54}+\frac{25 \times 1.20205}{36}\right) 5.76936+\frac{625 \times 5.76936^{2}}{108}= \\
& \quad-22.306-23.4316\left(\int_{0}^{\infty} \frac{\sin (t)}{t} d t\right)^{2}
\end{aligned}
$$

$$
\left(-\frac{42019}{5184}-\frac{\pi^{2}}{6}-\frac{1}{9}\left(\pi^{2} \times 0.69314718\right)+\right.
$$

$$
\left.\frac{7 \times 1.20205}{2}+\frac{6761 \times 5.76936}{432}-\frac{1}{144}\left(625 \times 5.76936^{2}\right)\right) \times 4.852
$$

-293.442...
-293.442

$$
\begin{aligned}
& \left(\begin{array}{l}
\left.-\frac{42019}{5184}-\frac{\pi^{2}}{6}-\frac{\pi^{2} 0.693147}{9}+\frac{7 \times 1.20205}{2}+\frac{6761 \times 5.76936}{432}-\frac{625 \times 5.76936^{2}}{144}\right) \\
4.852=-281.773-4.7294\left(\int_{0}^{\infty} \frac{1}{1+t^{2}} d t\right)^{2} \\
\left(-\frac{42019}{5184}-\frac{\pi^{2}}{6}-\frac{\pi^{2} 0.693147}{9}+\frac{7 \times 1.20205}{2}+\frac{6761 \times 5.76936}{432}-\frac{625 \times 5.76936^{2}}{144}\right) \\
4.852=-281.773-18.9176\left(\int_{0}^{1} \sqrt{1-t^{2}} d t\right)^{2} \\
\left(-\frac{42019}{5184}-\frac{\pi^{2}}{6}-\frac{\pi^{2} 0.693147}{9}+\frac{7 \times 1.20205}{2}+\frac{6761 \times 5.76936}{432}-\frac{625 \times 5.76936^{2}}{144}\right) \\
4.852=-281.773-4.7294\left(\int_{0}^{\infty} \frac{\sin (t)}{t} d t\right)^{2} \\
\left(-\frac{5357}{864}+\frac{325 \times 5.76936}{288}\right) \times 4.852^{2}-\frac{1}{432}\left(247 \times 4.852^{3}\right) \\
-58.003600281 \overline{1074}(\text { period } 3) \\
-58.003600281074
\end{array}\right.
\end{aligned}
$$

```
1-11.4915+20.8885 +
    0.330901 (159.320475-42.8164-80.1211-293.442-58.0036)
```

$1-11.4915+20.8885+0.330901(159.320475-42.8164-80.1211-293.442-58.0036)$
Result:
-93.857537675125

Final result: - 93,8575
We calculate the following integral:
$1 /(9 \mathrm{Pi})$ integrate $[1-11.4915+20.8885+0.330901(159.320475-42.8164-80.1211-$ 293.442-58.0036)]x

$$
\begin{aligned}
& \frac{1}{9 \pi} \int(1-11.4915+20.8885+ \\
& \quad 0.330901(159.320475-42.8164-80.1211-293.442-58.0036)) x d x
\end{aligned}
$$

$$
-1.65977 x^{2}
$$

Plot:

$1 / 29$ integrate $[1-11.4915+20.8885+0.330901(159.320475-42.8164-80.1211-$ 293.442-58.0036)]x

Input interpretation:

$$
\begin{aligned}
& \frac{1}{29} \int(1-11.4915+20.8885+ \\
& 0.330901(159.320475-42.8164-80.1211-293.442-58.0036)) x d x
\end{aligned}
$$

Result:
$-1.61823 x^{2}$
Plot:


The results -1.6597 and -1.61823 are very near to fourteenth root of Ramanujan's class invariant and to the mass of proton with minus sign and to the electric charge of the electron ( $1.61823 \approx$ golden ratio).

We note that:
From:

Citation: J. Beringer ot al. (Particlc Date Croup), PR D86, 010001 (2012) and 2013 partial update for the 2014 cdition (URL: http://pdg.lbl.gov)
The parity of the $\Lambda_{c}^{+}$is defined to be positive (as are the parities of the proton, neutron, and $\Lambda$ ). The quark content is $u d c$. Results of an analysis of $p K^{-} \pi^{+}$decays (JEZABEK 92) are consistent with $J$ $=1 / 2$. Nobody doubts that the spin is indeed $1 / 2$.

## $\Lambda_{c}^{+}$BRANCHING RATIOS

Hadronic modes with a $p: S=-1$ final states

## $\Gamma\left(\Lambda \rho^{+}\right) / \Gamma\left(p K^{-} \pi^{+}\right)$


$\Gamma\left(\Sigma^{0} \pi^{+}\right) / \Gamma\left(\Lambda \pi^{+}\right)$
DOCUMENT ID TECN COMMENT $\quad \Gamma_{\mathbf{3 9}} / \Gamma_{\mathbf{2 3}}$
VALUE
$0.98 \pm 0.05$ OUR FIT
$0.98 \pm 0.05$ OUR AVERAGE

| $0.977 \pm 0.015 \pm 0.051$ | $33 k$ | AUBERT | 07U BABR $e^{+} e^{-} \approx \gamma(4 S)$ |
| :--- | :--- | :--- | :--- | :--- |
| $1.09 \pm 0.11 \pm 0.19$ | 750 | LINK | O5F FOCS $\gamma$ nucleus, $\bar{E}_{\gamma} \approx 180 \mathrm{GeV}$ |

$0.98-0.05=0.93 ; ~ 0.977-0.015-0.051=0.911 ;<0.95$
Between 0.93 and 0.95 there is the value 0.94 very near to the value 0.938575
Now from 93,8575 we obtain:
$\sqrt[9]{93,8575}=1,65639236 \ldots$ value that is also very near to the fourteenth root of Ramanujan's class invariant

$$
\sqrt[14]{\left(\sqrt{\frac{113+5 \sqrt{505}}{8}}+\sqrt{\frac{105+5 \sqrt{505}}{8}}\right)^{3}}=1,65578454 \ldots
$$

Further:
$(94 * 8)-24=728 ; \quad(728+24) / 8=94 ;$

Now, we have:

The relation between the 1 S and the $\overline{\mathrm{MS}}$ masses of the bottom quark is derived by combining 1S-pole and $\overline{\mathrm{MS}}$ pole mass relations. The latter is written as

$$
\begin{align*}
& \frac{m_{b, \mathrm{OS}}}{m_{b, \overline{\mathrm{MS}}}\left(\mu_{b}\right)}=1+\epsilon \delta_{b}^{(1)}+\epsilon^{2} \delta_{b}^{(2)}+\epsilon^{3} \delta_{b}^{(3)}+\cdots, \quad \delta_{b}^{(k)}=\delta_{b, 0}^{(k)}+\delta_{b, m}^{(k)}, \\
& \delta_{b}^{(1)}=\frac{\alpha_{s}^{[4]}(\mu)}{\pi}\left[\frac{4}{3}-L_{b, \overline{\mathrm{MS}}}\right],  \tag{9.1.84a}\\
& \delta_{b, 0}^{(2)}=\frac{\alpha_{s}^{[4] 2}(\mu)}{\pi^{2}}\left[\frac{2195}{288}+\frac{1}{9} \pi^{2}+\frac{1}{9} \pi^{2} \log 2-\frac{1}{6} \zeta_{3}-\frac{25}{9} L_{b, \mathrm{MS}}^{[\mu]}\right. \\
& \left.-\left(\frac{205}{72}-\frac{25}{12} L_{b, \overline{\mathrm{MS}}}^{[\mu]}\right) L_{b, \overline{\mathrm{MS}}}-\frac{11}{24} L_{b, \overline{\mathrm{MS}}}^{2}\right],  \tag{9.1.84~b}\\
& \delta_{b, 0}^{(3)}=\frac{\alpha_{s}^{[4] 3}(\mu)}{\pi^{3}}\left[\frac{4903957}{93312}+\frac{439961}{38880} \pi^{2}+\frac{281}{7776} \pi^{4}-\frac{47}{162}(\log 2)^{4}-\frac{221}{54} \pi^{2} \log 2\right. \\
& -\frac{14}{81} \pi^{2}(\log 2)^{2}-\frac{55}{6} \zeta_{3}-\frac{1439}{432} \pi^{2} \zeta_{3}+\frac{1975}{216} \zeta_{5}-\frac{188}{27} \operatorname{Li}_{4}\left(\frac{1}{2}\right) \\
& +\left(-\frac{62267}{1728}-\frac{25}{54} \pi^{2}-\frac{25}{54} \pi^{2} \log 2+\frac{25}{36} \zeta_{3}\right) L_{b, \overline{\mathrm{MS}}}^{[\mu]}+\frac{625}{108} L_{b, \mathrm{MS}}^{[\mu] 2} \\
& -\left(-\frac{54859}{5184}-\frac{1}{9} \pi^{2}-\frac{1}{9} \pi^{2} \log 2+\frac{13}{3} \zeta_{3}+\frac{6511}{132} L_{b, \mathrm{MS}}^{\mu]}-\frac{625}{141} L_{b, \mathrm{MS}}^{[\mu] 2}\right) L_{b, \overline{\mathrm{MS}}} \\
& \left.-\left(\frac{841}{864}+\frac{275}{144} L_{b, \overline{\mathrm{MS}}}^{[\mu]}\right) L_{b, \overline{\mathrm{MS}}}^{2}-\frac{1231}{432} L_{b, \overline{\mathrm{MS}}}^{3}\right],  \tag{9.1.84c}\\
& \delta_{b, m}^{(1)}=0, \quad \delta_{b, m}^{(2)}=\frac{\alpha_{s}^{[4] 2}(\mu)}{\pi^{2}} \frac{4}{3} \Delta\left(\rho_{\overline{\mathrm{MS}}}\right), \\
& \delta_{b, m}^{(3)} \approx \frac{\alpha_{s}^{[4] 3}(\mu)}{\pi^{3}} \frac{\pi^{2}}{12} \rho_{\overline{\mathrm{MS}}}\left[\beta^{(0)[4]}\left(-L_{c, \overline{\mathrm{MS}}}^{[\mu]}-4 \log 2+\frac{14}{3}\right)\right. \\
& \left.-\frac{4}{3}\left(\frac{29}{15}+2 \log 2\right)+\frac{76}{3 \pi}\left(c_{1} c_{2}+d_{1} d_{2}\right)+2\left(L_{b, \overline{\mathrm{MS}}}-L_{c_{,} \overline{\mathrm{MS}}}\right)\right],  \tag{9.1.84e}\\
& L_{b, \overline{\mathrm{MS}}}=\log \frac{m_{b, \overline{\mathrm{MS}}}^{2}\left(\mu_{b}\right)}{\mu_{b}^{2}}, \quad L_{b, \overline{\mathrm{MS}}}^{[\mu]}=\log \frac{m_{b, \mathrm{MS}}^{2}\left(\mu_{b}\right)}{\mu^{2}}, \quad \rho_{\overline{\mathrm{MS}}}=\frac{m_{r, \overline{\mathrm{MS}}}\left(\mu_{c}\right)}{m_{b, \overline{\mathrm{MS}}}\left(\mu_{b}\right)} . \tag{9.1.84f}
\end{align*}
$$

$\delta_{b}^{(1)}, \delta_{b, 0}^{(2)}$ and $\delta_{b, 0}^{(3)}$ are obtained by setting $n_{f}=5$ in Eq. (9.1.22) and re-expanding $\alpha_{s}^{[5]}$ in terms of $\alpha_{s}^{[4]}$ with use of Eqs. (9.1.40) and (9.1.41). $\delta_{l_{, z n}^{(2)}}^{(2)}$ is the term corresponding to Eq. (9.1.23).

$$
\frac{\alpha_{s}^{[4]}(\mu)}{\pi}\left[\frac{4}{3}-L_{b, \overline{\mathrm{MS}}}\right]
$$

$-11,49147072$
5.33291891559407 * [[[(2195/288+((Pi^2)/9)+((Pi^2)/9)*0.693147)-(1.20205/6)-((25*5.769)/9)-((205*4.852)/72-((25*5.769*4.852)/12)-((11*4.852^2)/24)]]]
$5.33291891559407\left(\left(\frac{2195}{288}+\frac{\pi^{2}}{9}+\frac{\pi^{2}}{9} \times 0.693147\right)-\frac{1.20205}{6}-\right.$

$$
\left.\frac{25 \times 5.769}{9}-\left(\frac{205 \times 4.852}{72}-\frac{1}{12}(25 \times 5.769 \times 4.852)-\frac{1}{24}\left(11 \times 4.852^{2}\right)\right)\right)
$$

Result:
258.877...

Integral representations:
$5.332918915594070000\left(\left(\frac{2195}{288}+\frac{\pi^{2}}{9}+\frac{0.693147 \pi^{2}}{9}\right)-\frac{1.20205}{6}-\right.$

$$
\left.\frac{25 \times 5.769}{9}-\left(\frac{205 \times 4.852}{72}-\frac{25 \times 5.769 \times 4.852}{12}-\frac{11 \times 4.852^{2}}{24}\right)\right)=
$$

$248.975+4.01307\left(\int_{0}^{\infty} \frac{1}{1+t^{2}} d t\right)^{2}$
$5.332918915594070000\left(\left(\frac{2195}{288}+\frac{\pi^{2}}{9}+\frac{0.693147 \pi^{2}}{9}\right)-\frac{1.20205}{6}-\right.$

$$
\left.\frac{25 \times 5.769}{9}-\left(\frac{205 \times 4.852}{72}-\frac{25 \times 5.769 \times 4.852}{12}-\frac{11 \times 4.852^{2}}{24}\right)\right)=
$$

$248.975+16.0523\left(\int_{0}^{1} \sqrt{1-t^{2}} d t\right)^{2}$
$5.332918915594070000\left(\left(\frac{2195}{288}+\frac{\pi^{2}}{9}+\frac{0.693147 \pi^{2}}{9}\right)-\frac{1.20205}{6}-\right.$

$$
\left.\frac{25 \times 5.769}{9}-\left(\frac{205 \times 4.852}{72}-\frac{25 \times 5.769 \times 4.852}{12}-\frac{11 \times 4.852^{2}}{24}\right)\right)=
$$

$248.975+4.01307\left(\int_{0}^{\infty} \frac{\sin (t)}{t} d t\right)^{2}$
8.7082817709 (49.2811243840744-55.0963-357.286)
49.2811243840744
-55.0963...
-357.286..
8.7082817709 (49.2811243840744-55.0963-357.286)

Input interpretation:
8.7082817709 (49.2811243840744-55.0963-357.286)

Result:
-3161.98734860852448221064504
$-3161.9873486$
This value is a good approximation to the value of rest mass of vector meson $\mathrm{J} / \mathrm{Psi}$ that is $3096.916 \pm 0.011$

Note that:
$(864+1728)-8.7082817709(49.2811243840744-55.0963-357.286)$
Input interpretation:
$(864+1728)+(49.2811243840744-55.0963-357.286) \times(-8.7082817709)$
Result:
5753.98734860852448221064504
5753.987

This result is a very good approximation to the value of the rest mass of bottom Xi baryon, that is $5787.8 \pm 5.0 \pm 1.3 ; \quad 5791.1 \pm 2.2$

Now we calculate the following integrals:
integrate $(1728+729+64) /\left(\mathrm{Pi}^{\wedge} 7\right) \quad[-8.7082817709$ (49.2811243840744-55.0963357.286)]x

Indefinite integral:
$\int \frac{(1728+729+64)(-8.7082817709(49.2811243840744-55.0963-357.286)) x}{\pi^{7}}$ $d x=1319.64 x^{2}+$ constant


Alternate form assuming x is real:
$1319.64 x^{2}+0+$ constant

The result 1319.64 is a good approximation to the value of rest mass of baryon Xi $1314.86 \pm 0.20 \quad 1321.71 \pm 0.07$

Now:
integrate sqrt [[1/(1164.2696) [-8.7082817709 (49.2811243840744-55.0963357.286)]]]

$$
\begin{aligned}
& \int \sqrt{-\frac{8.7082817709(49.2811243840744-55.0963-357.286)}{1164.2696}} d x= \\
& 1.64799 x+\text { constant }
\end{aligned}
$$

The result 1,64799 is very near to the fourteenth root of Ramanujan's class invariant and to the mass of proton
and
integrate $[(1729 /(1.08643 \wedge 2)-24) \ln [-8.7082817709$ (49.2811243840744-55.0963357.286)]x

$$
\begin{aligned}
& \text { Indefinite integral: } \\
& \int\left(\frac{1729}{1.08643^{2}}-24\right) \log (-8.7082817709(49.2811243840744-55.0963-357.286)) x \\
& \quad d x=5805.85 x^{2}+\text { constant }
\end{aligned}
$$



The result 5805.85 is practically equal to the rest mass of the baryon bottom Sigma, that is $5811.3 \pm 1.7+(0.9-0.8) \quad 5815.5 \pm 1.7+(0.6-0.5)$

In conclusion, we remember that:

Nonperturbative contribution $[1032,1033]^{\mid}$

$$
\begin{equation*}
\Delta E^{\mathrm{np}}=\frac{\pi^{2} m_{q}}{\left(C_{F} \alpha_{s} m_{q}\right)^{4}} \frac{624}{425}\langle 0| \frac{\alpha_{s}}{\pi} G^{\mu \nu a} G_{\mu \nu}^{a}|0\rangle, \tag{9.1.65}
\end{equation*}
$$

where the gluon condensate is evaluated as $[1034,1035,1036]$

$$
\begin{equation*}
\langle 0| \frac{\alpha_{s}}{\pi} G^{\mu \nu c} G_{\mu \nu}^{a}|0\rangle \approx 0.012 \mathrm{GeV}^{4} . \tag{9.1.66}
\end{equation*}
$$

and

These quantities are evaluated at a hadronic energy scale $\mu_{\mathrm{H}} \sim 1 \mathrm{GeV}$. In Ref. [582, 584, 576], $\mu_{\mathrm{H}}$ is chosen such that the strong coupling constant satisfies $g_{3}\left(\mu_{\mathrm{H}}\right)=4 \pi / \sqrt{6}$. See Sec. 9.3.6 for the QCD correction factors due to the running effect between the EW scale and $\mu_{\mathrm{H}}$.
$\beta^{(k)}$ are evaluated with $n_{f}=n_{l}$ in (9.1.1). The $\mathrm{SU}(3)$ color factors are

$$
\begin{equation*}
C_{A}=3, \quad C_{F}=\frac{4}{3}, \quad T_{F}=\frac{1}{2} . \tag{9.1.60}
\end{equation*}
$$

$\alpha_{\mathrm{s}}=4 \pi / \sqrt{ } 6=5.130199=5.13$
For $C_{F}=4 / 3 \alpha_{\mathrm{s}}=5.13$ and $\mathrm{m}_{\mathrm{q}}=4.776483 \mathrm{MeV} / \mathrm{c}^{2}=0.004776483 \mathrm{GeV} / \mathrm{c}^{2}$ (the mass of quark down is $4.8 \pm 0.5 \pm 0.3=4.776483 \mathrm{MeV} / \mathrm{c}^{2}$ ), we obtain:
$\left.\left[\mathrm{Pi}^{\wedge} 2 *(0.004776483) * 624 * 0.012\right] /\left[425 *\left(((4 / 3) * 5.13 *(0.004776483))^{\wedge} 4\right)\right)\right]$

Input interpretation:
$\frac{\pi^{2} \times 0.004776483 \times 624 \times 0.012}{425\left(\frac{4}{3} \times 5.13 \times 0.004776483\right)^{4}}$
Result:
729.000...

729
And for $\mathrm{m}_{\mathrm{q}}=4.77867 \mathrm{MeV} / \mathrm{c}^{2}=0.00477867 \mathrm{GeV} / \mathrm{c}^{2}$, we obtain
$\left.\left[\mathrm{Pi}^{\wedge} 2 *(0.00477867) * 624 * 0.012\right] /\left[425^{*}\left(((4 / 3) * 5.13 *(0.00477867))^{\wedge} 4\right)\right)\right]$

Input interpretation:
$\frac{\pi^{2} \times 0.00477867 \times 624 \times 0.012}{425\left(\frac{4}{3} \times 5.13 \times 0.00477867\right)^{4}}$
Result:
728.000...

728

We know that:

$9^{3}+10^{3}=12^{3}+1=1729 ; \quad 6^{3}+8^{3}=9^{3}-1=728 ;$

Practically, the result of the above expression concerning the nonperturbative contributions of the mass of a 1S quarkonium, is equal to a fundamental Ramanujan's numbers: 728 and 729. Furthermore, from the same formula, we obtain the number 1729. Indeed:

$$
\left.10^{\wedge} 3+\left[\mathrm{Pi}^{\wedge} 2^{*}(0.004776483) * 624 * 0.012\right] /\left[425^{*}\left(((4 / 3) * 5.13 *(0.004776483))^{\wedge} 4\right)\right)\right]
$$

Input interpretation:
$10^{3}+\frac{\pi^{2} \times 0.004776483 \times 624 \times 0.012}{425\left(\frac{4}{3} \times 5.13 \times 0.004776483\right)^{4}}$

Result:
1729.00...

1729
Further, 1729 is also the fundamental number that is in the range of the mass of the candidate "glueball" $\mathrm{f}_{0}(1710)$ :

## $f_{0}(1710)$ MASS

| VALUE ( MeV ) | EVTS | DOCUMENT ID | TECN | COMMENT |
| :---: | :---: | :---: | :---: | :---: |
| $1723+5$ | VERAGE | Error includes s | tor of | 6. See the |


| $\begin{aligned} & 1720 \pm 10 \quad \pm 10 \\ & 1742 \pm 15 \end{aligned}$ |  | ${ }^{9}$ BALTRUSAIT... 87 |  | MRK3 $J / \psi \rightarrow \gamma K^{+} K^{-}$ <br> MPSF $200 \pi^{-} N \rightarrow 2 K_{S}^{0} \mathrm{X}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1726 \pm 7$ | 74 | 13 CHEKANOV | 04 |  | ep $\rightarrow$ | $K_{S}^{0}$ |
| $1732+15$ |  | 14 ANISOVICH | 03 | RVUE |  |  |
| $1726 \pm 7$ | 74 | 13 CHEKANOV | 04 | ZEUS | $e p \rightarrow K$ | $K_{S}^{0} K_{S}^{0}$ |
| 1732+15 |  | 14 ANISOVICH | 03 | RVUE |  |  |
| $1744 \pm 15$ |  | 22 ALDE 9 | 92D | GAM2 3 | $38 \pi^{-} p \rightarrow$ | $\rightarrow \eta \eta n$ |
| $1730{ }_{-10}^{+}$ |  | ${ }^{7}$ LONGACRE 86 | RV | UUE 22 | $\pi^{-} p \rightarrow n$ | $n 2 K_{S}^{0}$ |

Indeed, in we take the various masses, we have the following means: 1729,1731 , $1729,1729,1744-15=1729 ; 1730+2=1732$, with a partial mean of 1729,83 . The mean adding the number with the minus sign is 1726,22 , while with the sign positive is 1740,77 that less the algebraic sum of the difference $-69+77=8$ is equal to 1732.77 . The final mean is 1729,6

The complete develop of the two above expressions is:
$\left.\left[\mathrm{Pi}^{\wedge} 2 *(0.004776483) * 624 * 0.012\right] /\left[425 *\left(((4 / 3) * 5.13 *(0.004776483))^{\wedge} 4\right)\right)\right]$
$\frac{\pi^{2} \times 0.004776483 \times 624 \times 0.012}{425\left(\frac{4}{3} \times 5.13 \times 0.004776483\right)^{4}}$

Result:
729.000...

Alternative representations:
$\frac{\pi^{2}(0.00477648 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}}=\frac{0.0357663\left(180^{\circ}\right)^{2}}{425 \times 0.0326711^{4}}$
$\frac{\pi^{2}(0.00477648 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}}=\frac{0.0357663(-i \log (-1))^{2}}{425 \times 0.0326711^{4}}$
$\frac{\pi^{2}(0.00477648 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}}=\frac{0.214598 \zeta(2)}{425 \times 0.0326711^{4}}$

Series representations:

$$
\begin{aligned}
& \frac{\pi^{2}(0.00477648 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}}=1181.81\left(\sum_{k=0}^{\infty} \frac{(-1)^{k}}{1+2 k}\right)^{2} \\
& \frac{\pi^{2}(0.00477648 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}}=295.453\left(-1+\sum_{k=1}^{\infty} \frac{2^{k}}{\binom{2 k}{k}}\right)^{2}
\end{aligned}
$$

$$
\frac{\pi^{2}(0.00477648 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}}=73.8631\left(\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50 k)}{\binom{3 k}{k}}\right)^{2}
$$

$$
\begin{aligned}
& \frac{\pi^{2}(0.00477648 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}}=295.453\left(\int_{0}^{\infty} \frac{1}{1+t^{2}} d t\right)^{2} \\
& \frac{\pi^{2}(0.00477648 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}}=1181.81\left(\int_{0}^{1} \sqrt{1-t^{2}} d t\right)^{2} \\
& \frac{\pi^{2}(0.00477648 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}}=295.453\left(\int_{0}^{\infty} \frac{\sin (t)}{t} d t\right)^{2}
\end{aligned}
$$

and
$\left.10^{\wedge} 3+\left[\operatorname{Pi}^{\wedge} 2 *(0.004776483) * 624 * 0.012\right] /\left[425 *\left(((4 / 3) * 5.13 *(0.004776483))^{\wedge} 4\right)\right)\right]$
$10^{3}+\frac{\pi^{2} \times 0.004776483 \times 624 \times 0.012}{425\left(\frac{4}{3} \times 5.13 \times 0.004776483\right)^{4}}$
Result:
1729.00...

Alternative representations:
$10^{3}+\frac{\pi^{2}(0.00477648 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}}=10^{3}+\frac{0.0357663\left(180^{\circ}\right)^{2}}{425 \times 0.0326711^{4}}$
$10^{3}+\frac{\pi^{2}(0.00477648 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}}=10^{3}+\frac{0.0357663(-i \log (-1))^{2}}{425 \times 0.0326711^{4}}$
$10^{3}+\frac{\pi^{2}(0.00477648 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}}=10^{3}+\frac{0.214598 \zeta(2)}{425 \times 0.0326711^{4}}$

Series representations:
$10^{3}+\frac{\pi^{2}(0.00477648 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}}=1000+1181.81\left(\sum_{k=0}^{\infty} \frac{(-1)^{k}}{1+2 k}\right)^{2}$

$$
\begin{aligned}
& 10^{3}+\frac{\pi^{2}(0.00477648 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}}=1000+295.453\left(-1+\sum_{k=1}^{\infty} \frac{2^{k}}{\binom{2 k}{k}}\right)^{2} \\
& 10^{3}+\frac{\pi^{2}(0.00477648 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}}=1000+73.8631\left(\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50 k)}{\binom{3 k}{k}}\right)^{2}
\end{aligned}
$$

Integral representations:

$$
10^{3}+\frac{\pi^{2}(0.00477648 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}}=1000+295.453\left(\int_{0}^{\infty} \frac{1}{1+t^{2}} d t\right)^{2}
$$

$$
10^{3}+\frac{\pi^{2}(0.00477648 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}}=1000+1181.81\left(\int_{0}^{1} \sqrt{1-t^{2}} d t\right)^{2}
$$

$$
10^{3}+\frac{\pi^{2}(0.00477648 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}}=1000+295.453\left(\int_{0}^{\infty} \frac{\sin (t)}{t} d t\right)^{2}
$$

Now, we have:

$$
\begin{aligned}
& \frac{\pi^{2}(0.00477648 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}}=1181.81\left(\int_{0}^{1} \sqrt{1-t^{2}} d t\right)^{2} \\
& 10^{3}+\frac{\pi^{2}(0.00477648 \times 624 \times 0.012)}{425\left(\frac{4 \times 5.13 \times 0.00477648}{3}\right)^{4}}=1000+1181.81\left(\int_{0}^{1} \sqrt{1-t^{2}} d t\right)^{2}
\end{aligned}
$$

From the following integral representations of 729 and 1729 , we can to obtain the value 1181,81 a number very near to 1164.2696 that is the following Ramanujan's class invariant $Q=\left(G_{505} / G_{101 / 5}\right)^{3}=1164,2696$

$$
\left(\sqrt{\frac{113+5 \sqrt{505}}{8}}+\sqrt{\frac{105+5 \sqrt{505}}{8}}\right)^{3}=1164,269601267364
$$

For the value 1181,81 we have that:

$$
\sqrt[14]{1181,81}=1,657554016
$$

and

$$
\sqrt[14]{\left(\sqrt{\frac{113+5 \sqrt{505}}{8}}+\sqrt{\frac{105+5 \sqrt{505}}{8}}\right)^{3}}=1,65578 \ldots
$$

We note that $1,65755 \approx 1,65578$. The values are also very near to the mass of the proton.

## From Ramanujan's Notebook part II:

Integrals and asymptotic expansions:
Entry 9. If

$$
\varphi(m)=\int_{0}^{\infty} \frac{e^{-m^{2} x^{2}}}{1+x^{2}} d x
$$

and if $|m| \geq|n|$, where $m$ and $n$ are real, then

$$
\begin{equation*}
\int_{0}^{\infty} \frac{e^{-m^{2} x^{2}}}{1+x^{2}} \cos (2 m n x) d x=\frac{e^{-n^{2}}}{2}\{\varphi(m+n)+\varphi(m-n)\} \tag{9.1}
\end{equation*}
$$

We have that, for $\mathrm{m}=32$ and $\mathrm{n}=27$ :
integrate $\left[\left(\mathrm{e}^{\wedge}\left(-32 \mathrm{x}^{\wedge} 2\right) /\left(1+\mathrm{x}^{\wedge} 2\right)\right) \cos 1728\right] \mathrm{x}$,[0..infinity]

Definite integral:
$\int_{0}^{\infty} \frac{\left(e^{-32 x^{2}} \cos \left(1728^{\circ}\right)\right) x}{1+x^{2}} d x=-\frac{1}{8}(\sqrt{5}-1) e^{32} \operatorname{Ei}(-32) \approx 0.00468615$

- $\mathrm{Ei}(x)$ is the exponential integral Ei

Indefinite integral:
$\int \frac{\left(e^{-32 x^{2}} \cos \left(1728^{\circ}\right) x\right.}{1+x^{2}} d x=\frac{1}{8}(\sqrt{5}-1) e^{32} \operatorname{Ei}\left(-32\left(x^{2}+1\right)\right)+$ constant
Integral representations

$$
\begin{aligned}
& \frac{1}{8}(-1+\sqrt{5})(-1) e^{32} \mathrm{Ei}(-32)=-\frac{1}{8}(-1+\sqrt{5}) e^{32}\left(\gamma+\int_{0}^{-32} \frac{-1+e^{t}}{t} d t+\log (32)\right) \\
& \frac{1}{8}(-1+\sqrt{5})(-1) e^{32} \mathrm{Ei}(-32)=-\frac{1}{8}(-1+\sqrt{5}) e^{32} \mathcal{P} \int_{-\infty}^{-32} \frac{e^{t}}{t} d t \\
& \frac{1}{8}(-1+\sqrt{5})(-1) e^{32} \mathrm{Ei}(-32)=\frac{1}{8}(-1+\sqrt{5}) e^{32} \mathcal{\rho} \int_{32}^{\infty} \frac{e^{-t}}{t} d t
\end{aligned}
$$

And $1 / 0,00468615=213,394791$

We have, with regard the meson particles:

## $\Gamma\left(\phi f_{1}(1285)\right) / \Gamma_{\text {total }}$ <br> 「52/Г

VALUE (units $10^{-4}$ ) EVTS DOCUMENT ID TECN COMMENT
2.6 $\pm \mathbf{0 . 5}$ OUR AVERAGE Error includes scale factor of 1.1 .

| $3.2 \pm 0.6 \pm 0.4$ |  | JOUSSET | 90 | DM2 | $J / \psi \rightarrow \phi 2\left(\pi^{+} \pi^{-}\right)$ |
| :--- | ---: | ---: | ---: | :--- | :--- |
| $2.1 \pm 0.5 \pm 0.4$ | 74 JOUSSET | 90 | DM2 | $J / \psi \rightarrow \phi \eta \pi^{+} \pi^{-}$ |  |

$\Gamma(\rho \eta) / \Gamma_{\text {total }}$
$\Gamma_{54 / \Gamma}$
$\frac{V A L U E \text { (units } 10^{-3} \text { ) }}{0.193 \pm 0.023 \text { OUR AVE }} \frac{\text { EVTS }}{\text { RAGE }}$
$0.193 \pm 0.023$ OUR AVERAGE
$0.194 \pm 0.017 \pm 0.029 \quad 299$
JOUSSET $\quad 90$ DM2 $J / \psi \rightarrow$ hadrons
$0.193 \pm 0.013 \pm 0.029$
COFFMAN 88 MRK3 $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \eta$

Note that $2.6-0.5=2.1$ or $3.2-0.6-0.4=2.2$ and $0.193+0.023=0.216$ are value very near to 213,394791 that is the result of the integral.

From:
In Ramanujan's famous last letter to Hardy in 1920, he gives 17 examples of mock theta functions, without giving any complete definition of this term. A typical example (Ramanujan's second mock theta function of "order 7" - a notion that he also does not define) is

$$
\begin{aligned}
\mathcal{F}_{7}(\tau) & =-q^{-25 / 168} \sum_{n=1}^{\infty} \frac{q^{n^{2}}}{\left(1-q^{n}\right) \cdots\left(1-q^{2 n-1}\right)} \\
& =-q^{143 / 168}\left(1+q+q^{2}+2 q^{3}+\cdots\right)
\end{aligned}
$$

For $\mathrm{q}=\mathrm{e}^{2 \pi \mathrm{i} \tau}$ for $\mathrm{i} \tau>0$ (we take $\mathrm{i} \tau=1$ ), we obtain:
$\left(-\mathrm{e}^{2 \pi}\right)^{143 / 168}\left(1+\left(\mathrm{e}^{2 \pi}\right)+\left(\mathrm{e}^{2 \pi}\right)^{2}+2\left(\mathrm{e}^{2 \pi}\right)^{3}+\ldots\right)=$
$=-210,2269147(1+535,49165+286751,313+307105870,79+\ldots)=$
$=-64622315330,61$;
We have: $64622315330,61 / 1728^{3}=12,5242376$
From: (http://www.sns.ias.edu/pitp2/2007files/Lecture\ Notes-
Problems/Witten_Threedimgravity.pdf)
Let us give an example. If $k=1$, the partition function is simply the J-function itself, so

$$
Z(q)=q^{-1}+196884 q+\ldots
$$

The number of black hole primaries of mass 2 is therefore 196883. The black hole entropy is therefore $\log (196883)=12.19 \ldots$. The classical entropy of a black hole with $k=1$ and mass 2 is $4 \pi=12.57 \ldots$ So we are off by just a few percent.

We note that the value that we have obtained $12,524 \ldots$ is a very good approximation of the value $12,57 \ldots$ that is the classical entropy of a black hole with $\mathrm{k}=1$ and mass 2.

Note that $-\sqrt[49]{64622315330,61}=1,661958 \ldots$

$$
-\sqrt[50]{64622315330,61}=1,645158 \ldots
$$

The results 1,661958 and 1,645158 are very near to the fourteenth root of Ramanujan class invariant and to the mass of proton.

From $\quad-\ln (64622315330,61)=24,8918256 \ldots$
We have, with regard the meson particle:

$$
\begin{aligned}
& \boldsymbol{\Gamma}\left(\mathbf{2}\left(\boldsymbol{\pi}^{+} \boldsymbol{\pi}^{-}\right) \boldsymbol{K}^{+} \boldsymbol{K}^{-}\right) \times \boldsymbol{\Gamma}\left(\boldsymbol{e}^{+} \boldsymbol{e}^{-}\right) / \boldsymbol{\Gamma}_{\text {total }} \\
& \frac{\text { VALUE }\left(10^{-2} \mathrm{keV}\right)}{\mathbf{2 . 7 5} \pm \mathbf{0 . 2 3} \pm \mathbf{0 . 1 7}} \frac{\text { EVTS }}{205} \quad \frac{\text { DOCUMENT ID }}{\text { AUBERT }} \frac{\text { TECN }}{\text { BABR }} \frac{\Gamma_{\mathbf{1 0 2}} \boldsymbol{\Gamma}_{\mathbf{3}} / \boldsymbol{\Gamma}}{\substack{\text { COMMENT }}} \begin{array}{l}
10.6 e^{+} e^{-} \overrightarrow{K^{+}} K^{-} 2\left(\pi^{+} \pi^{-}\right) \gamma
\end{array}
\end{aligned}
$$

VALUE (units $10^{-4}$ ) EVTS
$21.8 \pm 2.3$ OUR AVERAGE
$20.8 \pm 2.7 \pm 3.9 \quad 195 \pm 25$
$29.6 \pm 3.7 \pm 4.7 \quad 238 \pm 30$
$20.7 \pm 2.4 \pm 3.0$
$20 \pm 3 \pm 3$
$155 \pm 20$
$\Gamma(p \bar{\rho} \phi) / \Gamma_{\text {total }}$
V/ALUE (unils $10^{-4}$ )
$0.45 \pm 0.13 \pm 0.07$
DOCUMENT ID TECN COMMENT
ABLIKIM U8E BES2 J/ $\psi \rightarrow \phi K_{S}^{0} K^{ \pm} \pi^{\mp}$
ABLIKIM 08E BES2 J/ $\psi \rightarrow \phi K^{+} K^{-} \pi^{0}$
FALVARD 88 DM2 $J / \psi \rightarrow$ hadrons
BECKER 87 MRK3 $e^{+} e^{-} \rightarrow$ hadrons
$\frac{\text { DOCUMENT ID }}{\text { FALVARD } 88} \frac{\text { TECN }}{\text { DM2 }} \frac{\Gamma_{\mathbf{9 8}} / \boldsymbol{\Gamma}}{J / \psi, \text { hadrons }}$

## $\Gamma\left(2\left(\pi^{+} \pi^{-}\right) \eta\right) / \Gamma_{\text {total }}$

$\Gamma_{88} / \Gamma$
VALUE (units $10^{-3}$ ) EVTS
DOCUMENT ID TECN COMMENT

## $2.29 \pm 0.24$ OUR AVERAGE

| $2.35 \pm 0.39 \pm 0.20$ | 85 | 100 AUBERT | 07AU BABR $10.6 e^{+} e^{-} \rightarrow 2\left(\pi^{+} \pi^{-}\right) \eta \gamma$ |
| :--- | ---: | ---: | :--- |
| $2.26 \pm 0.08 \pm 0.27$ | 4839 | ABLIKIM | 05C BES2 |
| $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow 2\left(\pi^{+} \pi^{-}\right) \eta$ |  |  |  |

100 AUBERT 07AU quotes $\Gamma_{e e}^{J / \psi} \cdot \mathrm{B}\left(J / \psi \rightarrow 2\left(\pi^{+} \pi^{-}\right) \eta\right) \cdot \mathrm{B}(\eta \rightarrow \gamma \gamma)=5.16 \pm 0.85 \pm$ 0.39 eV .

We have: $2.75-0.23-0.17=\mathbf{2 . 3 5} \quad 21.8+2.3=\mathbf{2 4 . 1} 0.45-0.13-0.07=\mathbf{0 . 2 5}$
$2.29+2.24=\mathbf{2 . 5 3}$ values very near to the result of the $\ln$ of Ramanujan's second mock theta function of "order 7 " that is $\mathbf{2 4 , 8 9 1 8 2 5 6}$
and for the following values of lambda charmed baryon:


VALUL
$0.21 \pm 0.03 \pm 0.02$

DOCUMLNT ID TLCN TLCN COMMLNT
LINK 05 FOCS $\gamma$ nucleus, $\bar{E}_{\gamma} \approx 180 \mathrm{GeV}$
$\frac{\text { VALUE }}{\mathbf{0 . 1 7} \pm \mathbf{0 . 0 4} \pm \mathbf{0 . 0 3}} \frac{\text { EVTS }}{54} \quad \frac{\text { DOCUMENT ID }}{\text { AMMAR }} \frac{\text { TECN }}{\text { CLE2 }} \frac{\text { COMMENT }}{e^{+} e^{-} \approx r(4 S)}$

## $\Gamma\left(\Sigma^{0} \pi^{+}\right) / \Gamma\left(p K^{=} \pi^{+}\right)$

$0.210 \pm 0.018$ OUR FIT
$0.20 \pm 0.04$ OUR AVERAGE

| $0.21 \pm 0.02$ | $\pm 0.04$ | 196 | AVERY | 94 | CLE2 | $e^{+} e^{-} \approx r(3 S), r(4 S)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $0.17 \pm 0.06$ | $\pm 0.04$ |  | ALBRECHT | 92 | ARG | $e^{+} e^{-} \approx 10.4 \mathrm{GeV}$ |

$\Gamma\left(\Sigma^{+} \pi^{0}\right) / \Gamma\left(p K^{-} \pi^{+}\right)$
$\frac{\text { VAI IIF }}{\mathbf{0 . 2 0} \pm \mathbf{0 . 0 3} \pm \mathbf{0 . 0 3}} \frac{\text { FVTS }}{93}$
$\frac{\text { DOCIMMFNT in }}{\text { KUBOTA } 93} \frac{\text { TFCN }}{\text { CLE2 }} \frac{\text { COMMFNT }}{\mathrm{e}^{+} \mathrm{e}^{-} \approx \Upsilon(4 S)}$
we have: $0.21+0.03+0.02=\mathbf{0 . 2 6} ; \quad 0.17+0.04+0.03=\mathbf{0 . 2 4} ; \quad 0.20+0.04=\mathbf{0 . 2 4}$;
$0.20+0.03+0.03=\mathbf{0 . 2 6}$ all values very near to the value $\mathbf{2 4 , 8 9} 18 \ldots$

We have the following formulae:

$$
\begin{aligned}
F(\tau) & =\frac{1}{\eta^{24}(\tau)}=\sum_{n=-1}^{\infty} c(n) q^{n} \\
d(n) & :=c(n)=p_{24}(n+1)
\end{aligned}
$$

where $\eta(\tau)$ is the familiar Dedekind eta function

$$
\eta(\tau)=q^{\frac{1}{24}} \prod_{n=1}^{\infty}\left(1-q^{n}\right) \quad \text { with } \quad q:=e^{2 \pi i \tau}
$$

$$
\psi_{m}^{\mathrm{P}}:=\frac{p_{24}(m+1)}{\eta^{24}(\tau)} \sum_{s \in \mathbb{Z}} \frac{q^{m s^{2}+s} y^{2 m s+1}}{\left(1-q^{s} y\right)^{2}}
$$

$$
\tau_{2}^{3 / 2} \frac{\partial}{\partial \bar{\tau}} \widehat{\psi_{m}^{F}}(\tau, z)=\sqrt{\frac{m}{8 \pi i}} \frac{p_{24}(m+1)}{\eta^{24}(\tau)} \sum_{\ell \bmod 2 m} \overline{\vartheta_{m, \ell}(\tau)} \vartheta_{m, \ell}(\tau, z)
$$

We have that for $\mathrm{i} \tau=1, \mathrm{n}=6, \mathrm{c}(\mathrm{n})=\mathrm{d}(\mathrm{n})=-12 ; \quad \eta(\tau)=30634746108626862,17$
$\mathrm{ms}=4 ; \mathrm{y}=3$
$\frac{p_{24}(m+1)}{\eta^{24}(\tau)}$ is equal to $-1,678766 * 10^{-16}$
$\frac{q^{m s^{2}+s} y^{2 m s+1}}{\left(1-q^{s} y\right)^{2}}$ is equal to $=3,4864841910330477 * 10^{41}$
$\psi_{m}^{\mathrm{P}}:=\frac{p_{24}(m+1)}{\eta^{24}(\tau)} \sum_{s \in \mathbb{Z}} \frac{q^{m s^{2}+s} y^{2 m s+1}}{\left(1-q^{s} y\right)^{2}}$
$=-5,8529911194437853551382000 * 10^{25}$
We have that:
$\psi_{m}^{\mathrm{P}}$ is the counting function of multi-centered black holes
And 5852,9 is very near to the value of $5832.1 \pm 0.7$ and $5835.1 \pm 0.6$ that is the mass of bottom Sigma baryons ${\underline{\Sigma x^{+}}}_{\underline{b}}$ and $\underline{\Sigma *}_{\underline{b}}^{-}$.

## From Ramanujan's Notebook part II:

Corollary (ii). If $n$ is a positive integer, as $x$ tends to $\infty$,

$$
\begin{equation*}
\sum_{k=0}^{\infty}\left(\frac{x^{k}}{k!}\right)^{n} \sim \frac{\exp \left\{n x+\frac{n^{2}-1}{24}\left(\frac{1}{n x}+\frac{1}{2 n^{2} x^{2}}+\cdots\right)\right\}}{\sqrt{n}(2 \pi x)^{(n-1) / 2}} \tag{10.23}
\end{equation*}
$$

For $\mathrm{x}=3, \mathrm{n}=\sqrt{ } 1729$
$\exp \left(\left(\left((\operatorname{sqrt}(1729) * 3)+\left(\left(\left(\operatorname{sqrt}(1729)^{\wedge} 2\right)-\right.\right.\right.\right.\right.$
1))/24)) $)^{*}\left(\left(1 /\left(\operatorname{sqrt}(1729)^{*} 3\right)+\left(1 /\left(\left(2 *\left(\operatorname{sqrt}((1729))^{\wedge} 2\right) 3^{\wedge} 2\right)\right)\right)\right)\right)$

$$
\exp \left(\sqrt{1729} \times 3+\frac{1}{24}\left(\sqrt{1729}^{2}-1\right)\right)\left(\frac{1}{\sqrt{1729} \times 3}+\frac{1}{2 \sqrt{1729}^{2} \times 3^{2}}\right)
$$

$\left(\frac{1}{31122}+\frac{1}{3 \sqrt{1729}}\right) e^{72+3 \sqrt{1729}}$

Decimal approximation:

$$
\left(\frac{1}{31122}+\frac{1}{3 \sqrt{1729}}\right) e^{72+3 \sqrt{1729}} \text { is a transcendental number }
$$

Alternate forms:
$\frac{(1+6 \sqrt{1729}) e^{72+3 \sqrt{1729}}}{31122}$
$\underline{(10374+\sqrt{1729}) e^{72+3 \sqrt{1729}}}$

$$
31122 \sqrt{1729}
$$

$\frac{e^{72+3 \sqrt{1729}}}{31122}+\frac{e^{72+3 \sqrt{1729}}}{3 \sqrt{1729}}$
Comparison:
$\approx 2200 \times$ the number of atoms in the visible universe $\left(\approx 10^{80}\right)$
Series representations:
$\exp \left(\sqrt{1729} 3+\frac{1}{24}\left(\sqrt{1729}^{2}-1\right)\right)\left(\frac{1}{\sqrt{1729} 3}+\frac{1}{2{\sqrt{1729} 3^{2}}^{2}}\right)=$ $\left(\exp \left(\frac{1}{24}\left(-1+72 \sqrt{1728} \sum_{k=0}^{\infty} 1728^{-k}\binom{\frac{1}{2}}{k}+\sqrt{1728}^{2}\left(\sum_{k=0}^{\infty} 1728^{-k}\binom{\frac{1}{2}}{k}\right)\right)^{2}\right)\right)$
$\left(1+6 \sqrt{1728} \sum_{k=0}^{\infty} 1728^{-k}\binom{\frac{1}{2}}{k}\right) /\left(18 \sqrt{1728}^{2}\left(\sum_{k=0}^{\infty} 1728^{-k}\binom{\frac{1}{2}}{k}\right)^{2}\right)$
$\exp \left(\sqrt{1729} 3+\frac{1}{24}\left(\sqrt{1729}^{2}-1\right)\right)\left(\frac{1}{\sqrt{1729} 3}+\frac{1}{2{\sqrt{1729} 3^{2}}^{2}}\right)=$ $\left(\exp \left(\frac{1}{24}\left(-1+72 \sqrt{1728} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{1728}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}+\sqrt{1728}^{2}\left(\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{1728}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)^{2}\right)\right)\right.$ $\left.\left(1+6 \sqrt{1728} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{1728}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)\right) /\left(18 \sqrt{1728}^{2}\left(\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{1728}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)^{2}\right)$
Open code

$$
\begin{aligned}
& \exp \left(\sqrt{1729} 3+\frac{1}{24}\left(\sqrt{1729}^{2}-1\right)\right)\left(\frac{1}{\sqrt{1729} 3}+\frac{1}{2 \sqrt{1729}^{2} 3^{2}}\right)= \\
& \left(2 \operatorname { e x p } \left(\frac { 1 } { 9 6 \sqrt { \pi } ^ { 2 } } \left(-4 \sqrt{\pi}^{2}+144 \sqrt{\pi} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 1728^{-5} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)+\right.\right.\right. \\
& \left.\left.\left.\quad\left(\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 1728^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)\right)^{2}\right)\right)\right) \\
& \left.\sqrt{\pi}\left(\sqrt{\pi}+3 \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 1728^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)\right)\right) / \\
& \left(9\left(\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 1728^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)\right)^{2}\right)
\end{aligned}
$$

$\left.1 /\left(\left(\operatorname{sqrt}(\operatorname{sqrt}(1729))^{*}\left(2 \mathrm{Pi}^{*} 3\right)^{\wedge} 20.2906225\right)\right)\right)$
$\frac{1}{\sqrt{\sqrt{1729}}(2 \pi \times 3)^{20.2906225}}$

Result:
$2.060144 \ldots \times 10^{-27}$
Series representations:

$$
\begin{aligned}
& \frac{1}{\sqrt{\sqrt{1729}}(2 \pi 3)^{20.2906}}=\frac{1.6249 \times 10^{-16}}{\pi^{20.2906} \sqrt{-1+\sqrt{1729}} \sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}(-1+\sqrt{1729})^{-k}} \\
& \frac{1}{\sqrt{\sqrt{1729}}(2 \pi 3)^{20.2906}}=\frac{1.6249 \times 10^{-16}}{\pi^{20.2906} \sqrt{-1+\sqrt{1729}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(-1+\sqrt{1729})^{-k}}{k!}} \\
& 1 \\
& \frac{1}{\sqrt{\sqrt{1729}}(2 \pi 3)^{20.2906}}=\frac{\pi^{20.2906} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)(-1+\sqrt{1729})^{-s}}{3.2498 \times 10^{-16} \sqrt{\pi}}
\end{aligned}
$$

$\exp ((((\operatorname{sqrt}(1729) * 3)+(((\operatorname{sqrt}(1729) \wedge 2)-$
$1)) / 24)))^{*}\left(\left(1 /(\operatorname{sqrt}(1729) * 3)+\left(1 /\left(\left(2 *\left(\operatorname{sqrt}((1729))^{\wedge} 2\right) 3^{\wedge} 2\right)\right)\right)\right)\right) *\left(2.060144^{*} 10^{\wedge}-27\right)$

$$
\begin{aligned}
& \exp \left(\sqrt{1729} \times 3+\frac{1}{24}\left(\sqrt{1729}^{2}-1\right)\right) \\
& \left(\frac{1}{\sqrt{1729} \times 3}+\frac{1}{2 \sqrt{1729}^{2} \times 3^{2}}\right) \times 2.060144 \times 10^{-27}
\end{aligned}
$$

## Result:

$4.616742 \ldots \times 10^{56}$

Comparisons:
$\approx 570 \times$ the size of the Monster group $\left(\approx 8.1 \times 10^{53}\right)$
$\approx 8.8 \times 10^{6} \times$ the number of chess positions $\left(\approx 5.2 \times 10^{49}\right)$
$1 /\left(1728000^{\wedge} 8\right) * 1 /(1728+576) \exp ((((\operatorname{sqrt}(1729) * 3)+(((\operatorname{sqrt}(1729) \wedge 2)-$
$1)) / 24)))^{*}\left(\left(1 /(\operatorname{sqrt}(1729) * 3)+\left(1 /\left(\left(2 *\left(\operatorname{sqrt}((1729))^{\wedge} 2\right) 3^{\wedge} 2\right)\right)\right)\right)\right) *\left(2.060144^{*} 10^{\wedge}-27\right)$

$$
\begin{aligned}
& \frac{1}{1728000^{8}} \times \frac{1}{1728+576}\left(\exp \left(\sqrt{1729} \times 3+\frac{1}{24}\left(\sqrt{1729}^{2}-1\right)\right)\right. \\
& \quad\left(\frac{1}{\sqrt{1729} \times 3}+\frac{1}{\left.\left.2{\sqrt{1729}^{2} \times 3^{2}}^{\text {In }}\right) \times 2.060144 \times 10^{-27}\right)}\right.
\end{aligned}
$$

Result:
2520.596.

## Note that:

Baryons are composite particles made of three quarks, as opposed to mesons, which are composite particles made of one quark and one antiquark. Baryons and mesons are both hadrons, which are particles composed solely of quarks or both quarks and antiquarks.

| $f=3 / 2^{+}$baryons |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Particle name | Symbol | Quark content | $\frac{\text { Rest mass }}{\left(\mathrm{MeV} / \underline{c}^{2}\right)}$ | 1 | $\underline{J}^{\text {P }}$ | $\underline{Q}(\mathrm{e})$ | $\underline{s}$ | $\underline{C}$ | $\underline{B}$ | Mean lifetime(s) | Commonly decays to |


| charmed Sigma ${ }^{311]}$ | $\Sigma_{c}^{*++}(2520)$ | Uuc | $2517.9 \pm 0.6$ | 1 | 3/2+ | +2 | 0 | +1 | 0 | $(4.42 \pm 0.44) \times 10^{-233^{[n]}}$ | $\underline{\Lambda s}_{\underline{+}}+\underline{\Pi^{+}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| charmed <br> Sigma ${ }^{[31]}$ | $\sum_{c}^{*+}(2520)$ | udc | $2517.5 \pm 2.3$ | 1 | $3 / 2+$ | +1 | 0 | +1 | 0 | >3.87 $\times 10^{-233^{[m]}}$ | $\underline{\Lambda L}_{+}^{+}+\underline{\pi}^{0}$ |
| charmed Sigma ${ }^{[31]}$ | $\Sigma_{C}^{* 0}(2520)$ | ddc | $2518.8 \pm 0.6$ | 1 | $3 / 2^{+}$ | 0 | 0 | +1 | 0 | $(4.54 \pm 0.47) \times 10^{-233^{[\mathrm{M}}}$ | $\underline{\Delta}_{+}^{+}+\underline{\pi^{-}}$ |

## CHARMED BARYONS

 ( $C=+1$ )$\Lambda_{c}^{+}=u d c, \quad \Sigma_{c}^{++}=u u c, \quad \Sigma_{c}^{+}=u d c, \quad \Sigma_{c}^{0}=d d c$,
$\bar{E}_{c}^{+}=u s c, \quad \Xi_{c}^{0}=d s c, \quad \Omega_{c}^{0}=s s c$
$\Sigma_{c}(2520)$

$$
I\left(J^{P}\right)=1\left(\frac{3}{2}^{+}\right)
$$

$J^{P}$ has not been measured; $\frac{3}{2}^{+}$is the quark-model prediction.

$$
\begin{aligned}
& \Sigma_{c}(2520)^{++} \text {mass } m=2518.4 \pm 0.6 \mathrm{MeV} \quad(\mathrm{~S}=1.4) \\
& \Sigma_{c}(2520)^{+} \text {mass } m=2517.5 \pm 2.3 \mathrm{MeV} \\
& \Sigma_{c}(2520)^{0} \quad \text { mass } m=2518.0 \pm 0.5 \mathrm{MeV} \\
& m_{\Sigma_{c}(2520)^{+}}-m_{\Lambda_{c}^{\prime}}=231.9+0.6 \mathrm{MeV} \quad(\mathrm{~S}=1.5) \\
& m_{\Sigma_{c}(2520)^{+}}-m_{\Lambda_{c}^{+}}=231.0 \pm 2.3 \mathrm{MeV} \\
& m_{\Sigma_{c}(2520)^{0}}-m_{\Lambda_{c}^{+}}=231.6 \pm 0.5 \mathrm{MeV} \quad(\mathrm{~S}=1.1) \\
& \left.m_{\Sigma_{c}(2520}\right)^{++} \quad m_{\Sigma_{c}(2520)^{0}}=0.3 \pm 0.6 \mathrm{MeV} \quad(\mathrm{~S}=1.2) \\
& \Sigma_{c}(2520)^{++} \\
& \text {full width } \Gamma=14.9 \pm 1.9 \mathrm{MeV} \\
& \Sigma_{c}(2520)^{+} \\
& \Sigma_{c}(2520)^{0} \\
& \text { full width } \Gamma<17 \mathrm{MeV}, \mathrm{CL}=90 \% \\
& \text { full width } \Gamma=16.1 \pm 2.1 \mathrm{MeV}
\end{aligned}
$$

$\Lambda_{C}^{+} \pi$ is the only strong decay allowed to a $\Sigma_{C}$ having this mass.

| $\boldsymbol{\Sigma}_{\boldsymbol{c}} \mathbf{( 2 5 2 0 )}$ DECAY MODES | Fraction $\left(\Gamma_{i} / \Gamma\right)$ | $p(\mathrm{MeV} / \mathrm{c})$ |
| :--- | :--- | ---: |
| $\Lambda_{c}^{+} \pi$ | $\approx 100 \%$ | 180 |

The values of the mass of the charmed baryons, precisely the charmed Sigma $\Sigma_{\mathrm{c}}(2520)$, that are: $2517.9 \pm 0.6 \quad 2517.5 \pm 2.3 \quad 2518.8 \pm 0.6$, are all very good approximations to the result obtained from the Ramanujan expression analyzed above: 2520.596

If we calculate the following simple integral:
1728/(2*10^56) integrate $\exp \left(\left(\left((\operatorname{sqrt}(1729) * 3)+\left(\left(\left(\operatorname{sqrt}(1729)^{\wedge} 2\right)-\right.\right.\right.\right.\right.$
$1)) / 24)))^{*}\left(\left(1 /\left(\operatorname{sqrt}(1729)^{*} 3\right)+\left(1 /\left(\left(2 *\left(\operatorname{sqt}((1729))^{\wedge} 2\right) 3^{\wedge} 2\right)\right)\right)\right)\right){ }^{*}\left(2.060144^{*} 10^{\wedge}-27\right) x$

$$
\begin{aligned}
& \frac{1728}{2 \times 10^{56}} \int \exp \left(\sqrt{1729} \times 3+\frac{1}{24}\left(\sqrt{1729}^{2}-1\right)\right) \\
& \quad\left(\frac{1}{\sqrt{1729} \times 3}+\frac{1}{2 \sqrt{1729}^{2} \times 3^{2}}\right) \times 2.060144 \times 10^{-27} x d x
\end{aligned}
$$

Result:
$1994.43 x^{2}$
Plot:


Alternate form assuming x is real:
$1994.43 x^{2}+0$

Indefinite integral assuming all variables are real:
$664.811 x^{3}+$ constant
the result 1994.43 is very near to the pseudoscalar meson strange D mass and to the vector meson $D$ mass $1968.49 \pm 0.34 \quad 2006.97 \pm 0.19$ with a difference of -26 and 12 (also 26 and 12 are significant numbers).

Now:

Entry 11 (i). As $x$ tends to $\infty$,

$$
\sum_{k=0}^{\infty}\left(\frac{e x}{k}\right)^{k} \sim \sqrt{2 \pi x} \exp \left(x-\frac{1}{24 x}-\frac{1}{48 x^{2}}-\left(\frac{1}{36}+\frac{1}{5760}\right) \frac{1}{x^{3}}+\cdots\right)
$$

For $x=6$, we obtain:
$\sqrt{12 \pi} \exp \left(6-\frac{1}{144}-\frac{1}{48 \times 36}-\left(\frac{1}{36}+\frac{1}{5760}\right) \times \frac{1}{216}\right)$

Exact result:
$2 e^{7455439 / 1244160} \sqrt{3 \pi}$
Decimal approximation:
2458.153445356729373894365991193740075188431305325431030927...

Continued fraction:
$[2458 ; 6,1,1,14,4,2,3,1,35,3,2,2,11,3,1,5,1,5,1,17,2,32,1,7,3,1,2, \ldots]$

$$
\begin{aligned}
& \sqrt{12 \pi} \exp \left(6-\frac{1}{144}-\frac{1}{48 \times 36}-\frac{1}{216}\left(\frac{1}{36}+\frac{1}{5760}\right)\right)= \\
& \exp \left(\frac{7455439}{1244160}\right) \sqrt{-1+12 \pi} \sum_{k=0}^{\infty}(-1+12 \pi)^{-k}\binom{\frac{1}{2}}{k} \\
& \sqrt{12 \pi} \exp \left(6-\frac{1}{144}-\frac{1}{48 \times 36}-\frac{1}{216}\left(\frac{1}{36}+\frac{1}{5760}\right)\right)= \\
& \exp \left(\frac{7455439}{1244160}\right) \sqrt{-1+12 \pi} \sum_{k=0}^{\infty} \frac{(-1)^{k}(-1+12 \pi)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}
\end{aligned}
$$

$$
\begin{aligned}
& \sqrt{12 \pi} \exp \left(6-\frac{1}{144}-\frac{1}{48 \times 36}-\frac{1}{216}\left(\frac{1}{36}+\frac{1}{5760}\right)\right)= \\
& \quad \exp \left(\frac{7455439}{1244160}\right) \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(12 \pi-z_{0}\right)^{k} z_{0}^{-k}}{k!}
\end{aligned}
$$

$$
\text { for } \operatorname{not}\left(\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right)\right)
$$

The result 2458,1534 is very near to the value of the mass of charmed Sigma: $2453.98 \pm 0.16 \quad 2452.9 \pm 0.4 \quad 2453.74 \pm 0.16$ of charmed Xi 2470.88(+0.34-0.80) and to decay width ${ }^{2}$ of boson Z that is $2.4952 \pm 0.0023 \mathrm{GeV} / c^{2}=2495.2 \pm 2.3 \mathrm{MeV} / c^{2}$

[^1]
## $\boldsymbol{\Sigma}_{\boldsymbol{c}}(\mathbf{2 4 5 5}) \mathrm{MASSES}$

The masses are obtained from the mass-difference measurements that follow.

## $\boldsymbol{\Sigma}_{c}(2455)^{++}$MASS

$\frac{\text { VALUE (MeV) }}{2453.98 \pm 0.16 \text { OUR FIT }}$ $\qquad$
$\boldsymbol{\Sigma}_{c}(2455)^{+}$MASS
$\frac{\text { VALUE (MeV) }}{2452.9 \pm 0.4 \text { OUR FIT }}$ $\qquad$
$\boldsymbol{\Sigma}_{c}(\mathbf{2 4 5 5})^{0}$ MASS
VALUE (MeV) $\qquad$
$2453.74 \pm 0.16$ OUR FIT

From Wikipedia:
The W and Z bosons are together known as the weak or more generally as the intermediate vector bosons. These elementary particles mediate the weak interaction; the respective symbols are $\mathrm{W}^{+}, \mathrm{W}^{-}$, and Z . The W bosons have either a positive or negative electric charge of 1 elementary charge and are each other's antiparticles. The Z boson is electrically neutral and is its own antiparticle. The three particles have a spin of 1 . The W bosons have a magnetic moment, but the Z has none.
Z bosons decay into a fermion and its antiparticle. As the Z boson is a mixture of the pre-symmetry-breaking $\mathrm{W}^{0}$ and $\mathrm{B}^{0}$ bosons (see weak mixing angle), each vertex factor includes a factor $T_{3}-Q \sin ^{2} \theta_{\mathrm{W} \text {; where } T_{3} \text { is the third component of the weak }}$ isospin of the fermion, $Q$ is the electric charge of the fermion (in units of the elementary charge), and $\theta_{\mathrm{W}}$ is the weak mixing angle. Because the weak isospin is different for fermions of different chirality, either left-handed or right-handed, the coupling is different as well.

If we calculate the following simple integral:
integrate $(\operatorname{sqrt}(12 \mathrm{Pi})) *[(\exp (6-1 / 144-1 /(48 * 36)-(1 / 36+1 / 5760) * 1 / 216))] \mathrm{x}$
Indefinite integral:
$\int \sqrt{12 \pi} \exp \left(6-\frac{1}{144}-\frac{1}{48 \times 36}-\frac{1}{216}\left(\frac{1}{36}+\frac{1}{5760}\right)\right) x d x \approx$ constant $+1229.08 x^{2}$

[^2]
we obtain the value very near to the delta baryons rest mass: $1232 \pm 2$

## $\Delta(1232)$ BREIT-WIGNER MASSES

| MIXED CHARGES <br> Value (MeV) | DOCUMENT ID |  | TECN | COMMENT |
| :---: | :---: | :---: | :---: | :---: |
| 1230 to 1234 ( $\approx 1232$ ) OUR ESTIMATE |  |  |  |  |
| $1228 \pm 2$ | ANISOVICH | 12A | DPWA | Multichannel |
| $1233.4 \pm 0.4$ | ARNDT | 06 | DPWA | $\pi N \rightarrow \pi N, \eta N$ |
| $1232 \pm 3$ | CUTKOSKY | 80 | IPWA | $\pi N \rightarrow \pi N$ |
| $1233 \pm 2$ | HOEHLER | 79 | IPWA | $\pi N \rightarrow \pi N$ |
| - - We do not use the following data for averages, fits, limits, etc. - . |  |  |  |  |
| $1231.1 \pm 0.2$ | SHRESTHA | 12A | DPWA | Multichannel |
| $1230 \pm 2$ | ANISOVICH | 10 | DPWA | Multichannel |
| $1232.9 \pm 1.2$ | ARNDT | 04 | DPWA | $\pi N \rightarrow \pi N, \eta N$ |
| $1228 \pm 1$ | PENNER | 02c | DPWA | Multichannel |
| $1234 \pm 5$ | VRANA | 00 | DPWA | Multichannel |
| 1233 | ARNDT | 95 | DPWA | $\pi N \rightarrow N \pi$ |
| $1231 \pm 1$ | MANLEY | 92 | IPWA | $\pi N \rightarrow \pi N \& N \pi \pi$ |

$\Delta(1232)^{++}$MASS
VALUE (MeV) DOCUMENT ID TECN COMMENT

-     - We do not use the following data for averages, fits, limits, etc. - .
$1230.55+0.20 \quad$ GRIDNEV 06 DPWA $\pi N \rightarrow \pi N$
$1231.88 \pm 0.29 \quad$ BERNICHA $96 \quad$ Fit to PEDRONI 78
$1230.5 \pm 0.2 \quad$ ABAEV 95 IPWA $\pi N \rightarrow \pi N$
$1230.9 \pm 0.3 \quad$ KOCH 80 B IPWA $\pi N \rightarrow \pi N$
$1231.1 \pm 0.2 \quad$ PEDRONI $78 \quad \pi N \rightarrow \pi N 70-370 \mathrm{MeV}$


## $\Delta(1232)^{+}$MASS

VALUE (MeV) DOCUMENT ID COMMENT

-     - We do not use the following data for averages, fits, limits, etc. - -
$1234.9 \pm 1.4$ MIROSHNIC... 79 Fit photoproduction


## $\Delta(1232)^{0}$ MASS

VALUE (MeV) DOCUMENT ID TECN COMMENT

-     - We do not use the following data for averages, fits, limits, e

| $1231.3 \pm 0.6$ | BREITSCHOP..06 | CNTR Using new CHEX data |  |  |
| :--- | :--- | :--- | :--- | :--- |
| $1233.40 \pm 0.22$ | GRIDNEV | 06 | DPWA $\pi N \rightarrow \pi N$ |  |
| $1234.35 \pm 0.75$ | BERNICHA | 96 |  | Fit to PEDRONI 78 |
| $1233.1 \pm 0.3$ | ABAEV | 95 | IPWA $\pi N \rightarrow \pi N$ |  |
| $1233.6 \pm 0.5$ | KOCH | $80 B$ | IPWA $\pi N \rightarrow \pi N$ |  |
| $1233.8 \pm 0.2$ | PEDRONI | 78 |  | $\pi N \rightarrow \pi N 70-370 \mathrm{MeV}$ |

Entry 11(ii). As $n$ tends to $\infty$,

$$
I_{n}:=\int_{0}^{\infty} \frac{x^{n-1} d x}{\sum_{k=0}^{\infty}(x / k)^{k}} \sim n^{n}\left(\frac{1}{n}+\frac{1}{2 n^{2}}+\frac{1}{3 n^{3}}+\frac{3}{8 n^{4}}+\cdots\right) .
$$

For $\mathrm{n}=6$, we have:
$6^{6}\left(\frac{1}{6}+\frac{1}{2 \times 6^{2}}+\frac{1}{3 \times 6^{3}}+\frac{3}{8 \times 6^{4}}\right)$
8509.5

Possible closed forms:

$$
\begin{aligned}
& \frac{27087 \pi}{10} \approx 8509.63202 \\
& \csc ^{2}\left(\frac{1}{40}(13-4 \pi)\right) \approx 8509.41266 \\
& \frac{17019}{2}=8509.5
\end{aligned}
$$

For $\mathrm{n}=8$, we have

Decimal approximation:

## $2.24068266666666666666666666666666666666666666666666666 \ldots \times 10^{6}$

2240682,666666

We calculate the following simple integrals 1 ):
$728 *\left(1 / 10^{\wedge} 3\right)$ integrate $([8509.5]) \mathrm{x}$
Input interpretation:
$728 \times \frac{1}{10^{3}} \int 8509.5 x d x$
Result:
$3097.46 x^{2}$
Plot:


We observe that the result is practically equal to the mass of $J / \psi(1 S)$ MASS value that is $3096.900 \pm 0.006 \mathrm{MeV}$.

The $\mathbf{J} / \mathbf{\psi}(\mathbf{J} / \mathbf{p s i})$ meson or psion is a subatomic particle, a flavor-neutral meson consisting of a charm quark and a charm antiquark. Mesons formed by a bound state of a charm quark and a charm anti-quark are generally known as "charmonium". The
$\mathrm{J} / \psi$ is the most common form of charmonium, due to its low rest mass. The $\mathrm{J} / \psi$ has a rest mass of $3.0969 \mathrm{GeV} / c^{2}$ (or, equivalently, $3096.9 \mathrm{MeV} / \mathrm{c}^{2}$ ).
and 2):
Pi/1728 integral [2240682.666666]x

Input interpretation:
$\frac{\pi}{1728} \int 2.240682666666 \times 10^{6} x d x$
Result:
$2036.838022171 x^{2}$
Plot:


Indefinite integral assuming all variables are real:
$678.9460073904 x^{3}+$ constant

The value 2036.838 is very near to the mass of charmed meson $D(2010)$, i.e. $D^{*}(2010)^{ \pm}$mass, with a difference of a -26 .

## $D^{*}(2010)^{ \pm}$MASS

The fit includes $D^{ \pm}, D^{0}, D_{s}^{ \pm}, D^{* \pm}, D^{* 0}, D_{s}^{* \pm}, D_{1}(2420)^{0}, D_{2}^{*}(2460)^{0}$, and $D_{s 1}(2536)^{ \pm}$mass and mass difference measurements.

```
VALUE (MeV)
DOCUMENT ID TECN CHG COMMENT
```


## $2010.26 \pm 0.05$ OUR FIT

```
- - We do not use the following data for averages, fits, limits, etc.
\begin{tabular}{llllll}
2008 & \(\pm 3\) & \(1^{2}\) GOLDHABER & 77 & MRK1 & \(\pm\) \\
\(e^{+} e^{-}\) \\
2008.6 & \(\pm 1.0\) & 2 PERUZZI & 77 & LGW & \(\pm\) \\
\(e^{+} e^{-}\)
\end{tabular}
\({ }^{1}\) From simultaneous fit to \(D^{*}(2010)^{+}, D^{*}(2007)^{0}, D^{+}\), and \(D^{0}\); not independent of FELDMAN 77B mass difference below.
\({ }^{2}\) PERUZZI 77 mass not independent of FELDMAN 77B mass difference below and PERUZZI \(77 D^{0}\) mass value.
```

We have:

Example. For $n, a>0$,

$$
\int_{0}^{\infty} \frac{\cos (n x) d x}{a^{2}+x^{2}}=\frac{\pi}{2 a} e^{-n a}
$$

For $\mathrm{n}=1$, $\mathrm{a}=2$, we obtain:

Input:
$\frac{\pi}{4} \exp (-2)$
Exact result:
$\frac{\pi}{4 e^{2}}$
Decimal approximation:
$0.106292082896909082109780590302250510262385101997650436041 \ldots$
Value very near to the branching ratio of $D^{+}$:

## $D^{+}$BRANCHING RATIOS

Some now-obsolete measurements have been omitted from these Listings.

## $c$-quark decays

$\Gamma\left(c \rightarrow e^{+}\right.$anything $) / \Gamma(c \rightarrow$ anything $)$
For the Summary Table, we only use the average of $e^{+}$and $\mu^{+}$measurements from $Z^{0} \rightarrow c \bar{C}$ decays; see the second data block below.

| VALUE | EVTS | DOCUMENT ID | TECN | COMMENT |
| :---: | :---: | :---: | :---: | :---: |
| $0.103 \pm 0.009{ }_{-0.008}^{+0.009}$ | 378 | ${ }^{1}$ ABBIENDI | OPAL | $z^{0} \rightarrow c \bar{c}$ |

${ }^{1}$ ABBIENDI 99 K uses the excess of right-sign over wrong-sign leptons opposite reconstructed $D^{*}(2010)^{+} \rightarrow D^{0} \pi^{+}$decays in $Z^{0} \rightarrow c \overline{\boldsymbol{c}}$.

| $\Gamma\left(K^{+} K^{-} \pi^{+}\right) / \Gamma\left(K^{-2} 2 \pi^{+}\right)$ |  | DOCUMENT ID |  | $\Gamma_{97} / \Gamma_{43}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| VALUE | EVTS |  |  | TECN | COMMENT |
| $0.1059 \pm 0.0018$ OUR FIT |  |  |  |  |  |
| $\mathbf{0 . 1 0 5 9} \pm \mathbf{0 . 0 0 1 8}$ OUR AVERAGE |  |  |  |  |  |
| $0.106 \pm 0.002 \pm 0.003$ |  | BONVICINI | 14 | CLEO | All CLEO-c runs |
| $0.117 \pm 0.013 \pm 0.007$ | $181 \pm 20$ | ABLIKIM | 05F | BES | $e^{+} e^{-} \approx \psi(3770)$ |
| $0.107 \pm 0.001 \pm 0.002$ | 43k | AUBERT | 05s | BABR | $e^{+} e^{-} \approx r(4 S)$ |
| $0.093 \pm 0.010+0.008$ |  | JUN | 00 | SELX | $\Sigma^{-}$nucleus, 600 GeV |
| $0.0976 \pm 0.0042 \pm 0.0046$ |  | FRABETTI | 95B | E687 | $\gamma \mathrm{Be}, \bar{E}_{\gamma} \approx 200 \mathrm{GeV}$ |

From the inverse of the expression, for $\mathrm{n}=4, \quad \mathrm{a}=3$, we have:
Input:
$\frac{1}{\frac{\pi}{6}} \exp (-12)$
$\frac{6 e^{12}}{\pi}$
Decimal approximation:
310838.7547946983720610230743772732246045552446944186835683.
$310838.75479=3108.3875 * 10^{2}$ and 3108.38 is a value very near to the vector meson $\mathrm{J} / \mathrm{Psi}=3096.916 \pm 0.011$

If we calculate the following integral, we have:
$\left(\mathrm{Pi}^{\wedge} 2\right) / 1728$ integrate $\mathrm{x} /\left(\mathrm{Pi} / 6^{*} \exp (-12)\right.$
$\frac{\pi^{2}}{1728} \int \frac{x}{\frac{1}{6} \pi \exp (-12)} d x \approx$ constant $+887.69 x^{2}$
Where 887,69 is a very good approximation to the masses of vector meson Kaon $=$ $891.66 \pm 0.026 \quad 896.00 \pm 0.025$

Now:

Entry 16. For $a$ and $n$ both real, and $n$ integral in (iv),
(i) $\int_{0}^{\infty} \frac{\sinh (a x)}{\sinh (\pi x)} \cos (n x) d x=\frac{1}{2} \frac{\sin a}{\cosh n+\cos a}, \quad|a|<\pi$,
for $\mathrm{a}=3$ and $\mathrm{n}=1 / 1728$, we have:
$\frac{1}{2} \times \frac{\sin (3)}{\cosh \left(\frac{1}{1728}\right)+\cos (3)}$
Exact result:
$\frac{\sin (3)}{2\left(\cos (3)+\cosh \left(\frac{1}{1728}\right)\right)}$

Decimal approximation:
7.050592000652599429873182695638317820716138104149441462737...

If we calculate the following integral, we obtain:
$\mathrm{Pi}^{\wedge} 6 / 3$ integrate $1 / 2((\sin (3) /(((\cosh (1 /(1728))+\cos (3))) \mathrm{x}$
$\frac{\pi^{6}}{3} \int \frac{\sin (3)}{2\left(\left(\cosh \left(\frac{1}{1728}\right)+\cos (3)\right) x\right)} d x \approx$ constant $+2259.45 \log (0.0200153 x)$
(assuming a complex-valued logarithm)

Alternate forms

$$
\frac{\sqrt[1728]{e} \pi^{6} \sin (3) \log \left(2 x\left(\cos (3)+\cosh \left(\frac{1}{1728}\right)\right)\right)}{3(1+\sqrt[864]{e}+2 \sqrt[1728]{e} \cos (3))}
$$

$$
\frac{\pi^{6} \sin (3) \log \left(2 x\left(\cos (3)+\cosh \left(\frac{1}{1728}\right)\right)\right)}{6\left(\frac{1}{2 \sqrt[1728]{e}}+\frac{1728}{2}+\cos (3)\right)}
$$

```
\(\sqrt[1728]{e} \pi^{6} \sin (3)(1728 \log (x(1+\sqrt[864]{e}+2 \sqrt[1728]{e} \cos (3)))-1)\)
\[
5184(1+\sqrt[864]{e}+2 \sqrt[1728]{e} \cos (3))
\]
```

Alternate form assuming $\mathrm{x}>0$ :
$\frac{\pi^{6} \sin (3) \log (x)}{6\left(\cos (3)+\cosh \left(\frac{1}{1728}\right)\right)}+\frac{\pi^{6} \sin (3) \log \left(\cos (3)+\cosh \left(\frac{1}{1728}\right)\right)}{6\left(\cos (3)+\cosh \left(\frac{1}{1728}\right)\right)}+\frac{\pi^{6} \log (2) \sin (3)}{6\left(\cos (3)+\cosh \left(\frac{1}{1728}\right)\right)}$

Series expansion of the integral at $\mathrm{x}=0$ :
$\frac{\pi^{6} \sin (3)\left(\log (x)+\log (2)+\log \left(\cos (3)+\cosh \left(\frac{1}{1728}\right)\right)\right)}{6\left(\cos (3)+\cosh \left(\frac{1}{1728}\right)\right)}+O\left(x^{6}\right)$
(generalized Puiseux series)

Series expansion of the integral at $\mathrm{x}=\infty$ :
$\frac{\pi^{6} \sin (3)\left(\log (x)+\log (2)+\log \left(\cos (3)+\cosh \left(\frac{1}{1728}\right)\right)\right)}{6\left(\cos (3)+\cosh \left(\frac{1}{1728}\right)\right)}+O\left(\left(\frac{1}{x}\right)^{6}\right)$
(generalized Puiseux series)
where $(2259.45 * 3.91) / 4=2209.32$ or $(2259.45 * 3.91) / 3.91=2259.45$ that is very near to the mass of meson $\mathrm{f}_{2}(2300)$ :

## $f_{2}(2300)$ MASS


$2297 \pm 28$
$\frac{\text { DOCUMENT ID }}{1} \frac{\text { TECN }}{\text { ETKIN }} \frac{88}{\text { MPS }} \quad \frac{\text { COMMENT }}{22 \pi^{-} p \rightarrow \phi \phi n}$

-     - We do not use the following data for averages, fits, limits, etc.

| $2243_{-}^{+}+7+39$ | UEHARA | 13 | BELL $\gamma \gamma \rightarrow K_{S}^{0} K_{S}^{0}$ |
| :--- | :--- | ---: | :--- | :--- |
| $2270 \pm 12$ | VLADIMIRSK...06 | SPEC $40 \pi^{-} p \rightarrow K_{S}^{0} K_{S}^{0} n$ |  |
| $2327 \pm 9 \pm 6$ | ABE | 04 | BELL $10.6 e^{+} e^{-} \rightarrow e^{+} e^{-} K^{+} K^{-}$ |
| $2231 \pm 10$ | BOOTH | 86 | OMEG $85 \pi^{-} \mathrm{Be} \rightarrow 2 \phi \mathrm{Be}$ |
| $2220_{-20}^{+90}$ | LINDENBAUM 84 | RVUE |  |
| $2320 \pm 40$ | ETKIN | 82 | MPS $22 \pi^{-} p \rightarrow 2 \phi n$ |

${ }^{1}$ Includes data of ETKIN 85. The percentage of the resonance going into $\phi \phi 2++S_{2}$, $D_{2}$, and $D_{0}$ is $6_{-5}^{+15}, 25_{-14}^{+18}$, and $69_{-27}^{+16}$, respectively.

Indeed, if we take 2297, we have the minimum value $2297-28=2269$. While 2209.32 is practically equal to the mass of meson $\mathrm{f}_{0}(2200)$ :

## $f_{0}(2200)$ MASS

VALUE (MeV) EVTS

DOCUMENT ID TECN COMMENT

## $2189 \pm 13$ OUR AVERAGE

$2170 \pm 20+10$
$2210 \pm 50$
$2197 \pm 17$

| ABLIKIM | $05 Q$ | BES2 | $\psi(2 S) \rightarrow$ |
| :--- | :--- | :--- | :--- |
|  |  |  | $\gamma \pi^{+} \pi^{-} K^{+} K^{-}$ |
| 1 BINON | 05 | GAMS | $33 \pi^{-} p, \eta \eta n$ |
| 2 AUGUSTIN | 88 | DM2 | $J / \psi \rightarrow \gamma K_{S}^{0} K_{S}^{0}$ |

- We do not use the following data for averages, fits, limits, etc.

| $2206 \pm 12 \pm 8$ | 381 | 3,4 DOBBS | 15 | $J / \psi \rightarrow \gamma K^{+} K^{-}$ |
| :--- | ---: | ---: | ---: | :--- |
| $2188 \pm 17 \pm 16$ | 203 | 3,4 DOBBS | 15 | $\psi(2 S) \rightarrow \gamma K^{+} K^{-}$ |
| $\sim$ |  | HASAN | 94 | RVUE $\bar{p} p \rightarrow \pi \pi$ |
| $\sim$ | 2122 |  | HASAN | 94 |
|  |  | RVUE $\bar{p} p \rightarrow \pi \pi$ |  |  |

${ }^{1}$ First solution, PWA is ambiguous.
${ }^{2}$ Cannot determine spin to be 0 .
${ }^{3}$ Using CLEO-c data but not authored by the CLEO Collaboration.
${ }^{4}$ From a fit to a Breit-Wigncr linc shape with fixed $\Gamma=238 \mathrm{McV}$.

Indeed: $2189+13=2202 ; 2197+17=2214$;
If we calculate the following integral:
125/2 integrate $1 / 2((\sin (3) /(((\cosh (1 /(1728))+\cos (3))) x$
$\frac{125}{2} \int \frac{\sin (3)}{2\left(\left(\cosh \left(\frac{1}{1728}\right)+\cos (3)\right) x\right)} d x \approx$ constant $+440.662 \log (0.0200153 x)$
(assuming a complex-valued logarithm)

$$
\frac{125 \sqrt[1728]{e} \sin (3) \log \left(2 x\left(\cos (3)+\cosh \left(\frac{1}{1728}\right)\right)\right)}{2(1+\sqrt[864]{e}+2 \sqrt[1728]{e} \cos (3))}
$$

$$
\begin{aligned}
& \frac{125 \sin (3) \log \left(2 x\left(\cos (3)+\cosh \left(\frac{1}{1728}\right)\right)\right)}{4\left(\frac{1}{2 \sqrt[1728]{e}}+\frac{172 \sqrt[8]{e}}{2}+\cos (3)\right)} \\
& \frac{125 \sqrt[1728]{e} \sin (3)(1728 \log (x(1+\sqrt[864]{e}+2 \sqrt[1728]{e} \cos (3)))-1)}{3456(1+\sqrt[864]{e}+2 \sqrt[1728]{e} \cos (3))}
\end{aligned}
$$

$\frac{125 \sin (3) \log (x)}{4\left(\cos (3)+\cosh \left(\frac{1}{1728}\right)\right)}+\frac{125 \sin (3) \log \left(\cos (3)+\cosh \left(\frac{1}{1728}\right)\right)}{4\left(\cos (3)+\cosh \left(\frac{1}{1728}\right)\right)}+\frac{125 \log (2) \sin (3)}{4\left(\cos (3)+\cosh \left(\frac{1}{1728}\right)\right)}$

Series expansion of the integral at $x=0$ :
$\frac{125 \sin (3)\left(\log (x)+\log (2)+\log \left(\cos (3)+\cosh \left(\frac{1}{1728}\right)\right)\right)}{4\left(\cos (3)+\cosh \left(\frac{1}{1728}\right)\right)}+O\left(x^{6}\right)$
(generalized Puiseux series)

Series expansion of the integral at $x=\infty$ :
$\frac{125 \sin (3)\left(\log (x)+\log (2)+\log \left(\cos (3)+\cosh \left(\frac{1}{1728}\right)\right)\right)}{4\left(\cos (3)+\cosh \left(\frac{1}{1728}\right)\right)}+O\left(\left(\frac{1}{x}\right)^{6}\right)$
(generalized Puiseux series)

Now:
$440.662 \log (0.0200153)$
-1723.54...
The result $-1723,54$ is exactly the mass with sign minus of $f_{0}(1710)$ that has been identified as possible particle named "glueball". Indeed:

## $f_{0}(1710)$ MASS

VALUE (MeV) EVTS DOCUMENTID TECN COMMENT
$1723_{-}^{+} \mathbf{5}$ OUR AVERAGE Error includes scale factor of 1.6. See the ideogram below.

| $1759 \pm 6$ | $\begin{aligned} & +14 \\ & -25 \end{aligned}$ | 5.5k | ${ }^{1}$ ABLIKIM | 13N | BES3 | $\epsilon^{+} \mathrm{e}^{-} \rightarrow J / \psi \rightarrow \gamma \eta \eta$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1750 \pm 7$ | $\begin{aligned} & +29 \\ & -18 \end{aligned}$ |  | UEHARA | 13 | BELL | $\gamma \gamma \rightarrow K_{S}^{0} K_{S}^{0}$ |
| $1701 \pm 5$ | + $+\quad 2$ $-\quad$ | 4 k | 2 CHEKANOV | 08 | ZEUS | $\epsilon p \rightarrow K_{S}^{0} K_{S}^{0} X$ |
| $1765-4$ | $\pm 13$ |  | ABLIKIM | 06V | BES2 | $\epsilon^{+} e^{-} \rightarrow J / \psi \rightarrow \gamma \pi^{+} \pi^{-}$ |
| $1760 \pm 15$ | $\begin{array}{r} +15 \\ -10 \end{array}$ |  | ${ }^{3}$ ABLIKIM | 05Q | BES2 | $\psi(2 S) \rightarrow \gamma \pi^{+} \pi^{-} K^{+} K^{-}$ |
| $1738 \pm 30$ |  |  | ABLIKIM | 04E | BES2 | $J / \psi \rightarrow \omega K^{\prime} K$ |
| $1740 \pm 4$ | +10 -25 |  | ${ }^{4} \mathrm{BAI}$ | 03G | BES | $J / \psi \rightarrow \gamma K \bar{K}$ |
| $1740+30$ |  |  | $4^{4} \mathrm{BAI}$ | 00A | BES | $J / \psi \rightarrow \gamma\left(\pi^{+} \pi^{-} \pi^{+} \pi^{-}\right.$ |
| $1698 \pm 18$ |  |  | ${ }^{5}$ BARBERIS | 00E |  | $450 p p \rightarrow p_{f} \eta \eta \rho_{s}$ |
| $1710 \pm 12$ | $\pm 11$ |  | ${ }^{6}$ BARBERIS | 99D | OMEG | $450 p p \rightarrow K^{+} K^{-} . \pi^{+} \pi^{-}$ |
| $1710 \pm 25$ |  |  | ${ }^{7}$ FRENCH | 99 |  | $300 p p \rightarrow p_{f}\left(K^{+} K^{-}\right) p_{S}$ |
| $1707 \pm 10$ |  |  | ${ }^{8}$ AUGUSTIN | 88 | DM2 | $J / \psi \rightarrow \gamma K^{+} K^{-}, K_{S}^{0} K_{S}^{0}$ |
| $1698 \pm 15$ |  |  | ${ }^{8}$ AUGUSTIN | 87 | DM2 | $J / \psi \rightarrow \gamma \pi^{+} \pi^{-}$ |
| $1720 \pm 10$ | $\pm 10$ |  | ${ }^{9}$ BALTRUSAIT. | . 87 | MRK3 | $J / \psi \rightarrow \gamma \mathrm{K}^{+} \mathrm{K}^{-}$ |
| $1742+15$ |  |  | 8 WILLIAMS | 84 | MPSF | $200 \pi-N \rightarrow 2 K_{S}^{0} x$ |
| $1670 \pm 50$ |  |  | BLOOM | 83 | CBAL | $J / \psi \rightarrow \gamma 2 \eta$ |

Note that the value $-1723,54$ is practically equal to value $1723(+6,-5)$ in MeV with sign minus

From:

## Strong Effective Coupling, Meson Ground States, and Glueball within Analytic Confinement

Gurjav Ganbold 1,2
Bogoliubov Laboratory of Theoretical Physics, JINR, Joliot-Curie 6, 141980 Dubna, Russia; ganbold@theor.jinr.ru - Institute of Physics and Technology, Mongolian Academy of Sciences, Enkh Taivan 54b,13330 Ulaanbaatar, Mongolia - Received: 13 February 2019; Accepted: 18 March 2019; Published: 1 April 2019

Our model has a minimal set of free parameters: $\left\{\hat{\alpha}, \Lambda, m_{u d}, m_{s}, m_{c}, m_{b}\right\}$. The glueball mass depends on $\{\hat{\alpha}, \Lambda\}$. We fix $\Lambda$ by fitting the expected glueball mass. Particularly, for $\Lambda=236 \mathrm{MeV}$ and $\hat{\alpha}\left(M_{G}\right)$ defined in Equation (28) we obtain new estimates:

$$
\begin{equation*}
M_{0++}=1739 \mathrm{MeV}, \quad \hat{\alpha}\left(M_{0++}\right)=0.451 \tag{33}
\end{equation*}
$$

The new value of $M_{0++}$ in (33) agrees not only with our previous estimate [27], but also with other predictions expecting the lightest glueball located in the scalar channel in the mass range $\sim 1500 \div$ $1800 \mathrm{MeV}[12,16,46,51]$. The often referred quenched QCD calculations predict $1750 \pm 50 \pm 80 \mathrm{MeV}$ for the mass of the lightest glueball [17]. The recent quenched lattice estimate with improved lattice spacing favors a scalar glueball mass $M_{G}=1710 \pm 50 \pm 58 \mathrm{MeV}$ [49].

Another important property of the scalar glueball is its size, the 'radius' which should depend somehow on the glueball mass. We estimate the glueball radius roughly as follows:

$$
\begin{equation*}
r_{0^{++}} \sim \frac{1}{2 \Lambda} \sqrt{\frac{\int d^{4} x x^{2} W_{\Lambda}(x) U^{2}(x)}{\int d^{4} x W_{\Lambda}(x) U^{2}(x)}} \approx \frac{1}{394.3 \mathrm{MeV}} \approx 0.51 \mathrm{fm} \tag{34}
\end{equation*}
$$

This may indicate that the dominant forces binding gluons are provided by vacuum fluctuations of correlation length $\sim 0.5 \mathrm{fm}$. On the other side, typical energy-momentum transfers inside a scalar glueball should occur in the confinement domain $\sim 236 \mathrm{MeV} \sim 0.85 \mathrm{fm}$, rather than at the chiral symmetry breaking scale $\Lambda_{\chi} \sim 1 \mathrm{GeV} \sim 0.2 \mathrm{fm}$.

The gluon condensate is a non-perturbative property of the QCD vacuum and may be partly responsible for giving masses to certain hadrons. The correlation function in QCD dictates the value of corresponding condensate. Particularly, with $\Lambda=236 \mathrm{MeV}$ and $\hat{\alpha}_{s}=0.451$ we calculate the lowest non-vanishing gluon condensate in the leading-order (ladder) approximation:

$$
\frac{\hat{\alpha}_{s}}{\pi}\left\langle F_{\mu \nu}^{A} F_{A}^{\mu \nu}\right\rangle=\frac{16 N_{c}}{\pi} \Lambda^{4} \approx 0.0214 \mathrm{GeV}^{4}
$$

which is in accordance with a refereed value [52]

$$
\alpha_{s}\left\langle G^{2}\right\rangle=(7.0 \pm 1.3) \cdot 10^{-2} \mathrm{GeV}^{4}, \quad \text { or, } \quad \frac{\alpha_{s}}{\pi}\left\langle G^{2}\right\rangle=(2.2 \pm 0.4) \cdot 10^{-2} \mathrm{GeV}^{4}
$$

## 7. Conclusions

In conclusion, we demonstrate that many properties of the low-energy phenomena such as strong running coupling, hadronization processes, mass generation for quark-antiquark and di-gluon bound states may be explained reasonably within a QCD-inspired model with infrared-confined propagators. We derived a meson mass equation and by exploiting it revealed a specific new behavior of the strong coupling $\alpha_{S}(M)$ in dependence of mass scale. An infrared freezing point $\alpha_{S}(0)=1.03198$ at origin $M=0$ has been found and it did not depend on the particular choice of the confinement scale $\Lambda>0$. A new estimate of the lowest (scalar) glueball mass has been performed and it was found at $\approx 1739 \mathrm{MeV}$. The scalar glueball 'size' has also been calculated: $r_{G} \approx 0.51 \mathrm{fm}$. A nontrivial value of the gluon condensate has also been obtained. We have estimated the spectrum of conventional mesons by introducing a minimal set of parameters: four masses of constituent quarks ( $u=d, s$,
c, b) and 1 . The obtained values fit the latest experimental data with relative errors less than 1.8 percent. Accurate estimates of the leptonic decay constants of pseudoscalar and vector mesons have also been performed ${ }^{3}$.

Now we take the integral of pg. 76
integrate $(\operatorname{sqrt}(12 \mathrm{Pi})) *[(\exp (6-1 / 144-1 /(48 * 36)-(1 / 36+1 / 5760) * 1 / 216))] \mathrm{x}$ multiplied for $(1.08643)^{4}$. We obtain:

$$
1.08643^{4} \int \sqrt{12 \pi} \exp \left(6-\frac{1}{144}-\frac{1}{48 \times 36}-\left(\frac{1}{36}+\frac{1}{5760}\right) \times \frac{1}{216}\right) x d x
$$

## Result:

$1712.32 x^{2}$

Plot:


Alternate form assuming x is real:
$1712.32 x^{2}+0$

Indefinite integral assuming all variables are real:

## $570.775 x^{3}+$ constant

Also here the value 1712,32 is a good approximation to the mass of $f_{0}(1710)$.
We calculate this other integral (see pg.73):
$(729+9+16) *\left(1 / 10^{\wedge} 56\right)$ integrate $\exp \left(\left(\left((\operatorname{sqrt}(1729) * 3)+\left(\left((\operatorname{sqrt}(1729))^{2}\right)-\right.\right.\right.\right.$
$1)) / 24)))^{*}\left(\left(1 /(\operatorname{sqrt}(1729) * 3)+\left(1 /\left(\left(2^{*}\left(\operatorname{sqrt}((1729))^{\wedge} 2\right) 3^{\wedge} 2\right)\right)\right)\right)\right) *\left(2.060144 * 10^{\wedge}-27\right) \mathrm{x}$

[^3]\[

$$
\begin{aligned}
& (729+9+16) \times \frac{1}{10^{56}} \int \exp \left(\sqrt{1729} \times 3+\frac{1}{24}\left(\sqrt{1729}^{2}-1\right)\right) \\
& \left(\frac{1}{\sqrt{1729} \times 3}+\frac{1}{2 \sqrt{1729}^{2} \times 3^{2}}\right) \times 2.060144 \times 10^{-27} x d x
\end{aligned}
$$
\]

Result:
$1740.51 x^{2}$

Plot:


Alternate form assuming x is real:
$1740.51 x^{2}+0$

Indefinite integral assuming all variables are real:
$580.171 x^{3}+$ constant

We note that this value i.e. 1740,51 correspond exactly to the new estimate of the lowest (scalar) glueball mass, that is $\approx 1739 \mathrm{MeV}$.

Furthermore $\mathrm{e}^{1739}=1,7302307644949 * 10^{754}=1730,2307644949 * 10^{751}$ that is about a multiple of 1730 .

Further, from pg.40, we can calculate the following integral:
$-47.23265-58.8742714-382.257106+16507.8183+139489-139468-2209694+2085349$
$1 / 31 *$ integrate $[-47.23265-58.8742714-382.257106+16507.8183+139489-139468-$ $2209694+2085349$ ]x

$$
\begin{aligned}
& \frac{1}{31} \int(-47.23265-58.8742714-382.257106+ \\
& 16507.8183+139489-139468-2209694+2085349) x d x
\end{aligned}
$$

```
Result:
\(-1746.85 x^{2}\)
```

Plot:


Alternate form assuming x is real:
$0-1746.85 x^{2}$

Indefinite integral assuming all variables are real:
$-582.283 x^{3}+$ constant

The result -1746.85 is a good approximation of the new estimate of the lowest (scalar) glueball mass, that is $\approx 1739 \mathrm{MeV}$.

Note that:
$(1 / 52) *(1 / 1728) *$ integrate [-2209694]x
Indefinite integral

$$
\frac{1}{52 \times 1728} \int-2209694 x d x=-\frac{1104847 x^{2}}{89856}+\text { constant }
$$

$-12,29575097 \mathrm{x}^{2}$
and
$\left((1728+216)^{*} 1164.2696\right) / 10^{\wedge} 10$ * integrate $[-47.23265-58.8742714-$ $382.257106+16507.8183+139489-139468-2209694+2085349$ ]x

[^4]results 12.29 and 12.25 , that are very near to the value of the black hole entropy (12.19) with minus sign.

We have also:
$\left(\mathrm{Pi}^{\wedge} 2\right) /(2 \mathrm{e}) *\left(1 /\left(10^{\wedge} 5\right)\right) *$ integrate $(\operatorname{sqrt}(12 \mathrm{Pi})) *[(\exp (6-1 / 144-1 /(48 * 36)-$ $\left.\left.\left.(1 / 36+1 / 5760)^{*} 1 / 216\right)\right)\right] \mathrm{x}$

Indefinite integral:

$$
\begin{aligned}
& \frac{\pi^{2}}{(2 e) 10^{5}} \int \sqrt{12 \pi} \exp \left(6-\frac{1}{144}-\frac{1}{48 \times 36}-\frac{1}{216}\left(\frac{1}{36}+\frac{1}{5760}\right)\right) x d x \approx \\
& \text { constant }+0.0223128 x^{2}
\end{aligned}
$$

Plot:


The value 0,0223128 is a good approximation to the value of the lowest nonvanishing gluon condensate, that is $\approx 0.0214 \mathrm{GeV}^{4}$.
and:
$\left(\mathrm{Pi}^{\wedge} 2\right) /(\operatorname{sqrt}(9 \mathrm{Pi} / 5)) *\left(1 /\left(10^{\wedge} 4\right)\right) *$ integrate $(\operatorname{sqrt}(12 \mathrm{Pi})) *[(\exp (6-1 / 144-1 /(48 * 36)-$ $\left.\left.\left.(1 / 36+1 / 5760)^{*} 1 / 216\right)\right)\right] \mathrm{x}$

Indefinite integral
$\frac{\pi^{2}}{\sqrt{\frac{9 \pi}{5}} 10^{4}} \int \sqrt{12 \pi} \exp \left(6-\frac{1}{144}-\frac{1}{48 \times 36}-\frac{1}{216}\left(\frac{1}{36}+\frac{1}{5760}\right)\right) x d x \approx$
constant $+0.510114 x^{2}$


We observe that the value 0.510114 is exactly the value of the scalar glueball 'size' that is $\mathrm{r}_{\mathrm{G}} \approx 0.51 \mathrm{fm}$

From the integral of pg.80, we have:
$0.577211 /\left(10^{\wedge} 3\right)\left(\mathrm{Pi}^{\wedge} 2\right) / 1728$ integrate $\mathrm{x} /\left(\mathrm{Pi} / 6^{*} \exp (-12)\right)$
(where 0.57721 is the Eulero-Mascheroni constant)

Input:
$0.57721 \times \frac{1}{10^{3}} \times \frac{\pi^{2}}{1728} \int \frac{x}{\frac{\pi}{6} \exp (-12)} d x$
Result:
$0.512383 x^{2}$
Plot:


The value 0.512383 is very near to the value of the scalar glueball 'size' that is $\mathrm{r}_{\mathrm{G}} \approx$ 0.51 fm
and
$0.57721 /(\operatorname{sqrt}(2 \mathrm{e})) \quad 1 /\left(10^{\wedge} 4\right)\left(\mathrm{Pi}^{\wedge} 2\right) / 1728$ integrate $\mathrm{x} /(\mathrm{Pi} / 6 * \exp (-12))$
Input:
$\frac{0.57721}{\sqrt{2 e}} \times \frac{1}{10^{4}} \times \frac{\pi^{2}}{1728} \int \frac{x}{\frac{\pi}{6} \exp (-12)} d x$

Result:
$0.0219752 x^{2}$

Plot:


The value 0.0219752 is very near to the value of the lowest non-vanishing gluon condensate, that is $\approx 0.0214 \mathrm{GeV}^{4}$.

Now:
Our version of Example ( $i$ ) is different from that of Ramanujan, who writes that the maximum value of $a^{x} / \Gamma(x+1)$ is

$$
\begin{gathered}
a^{a-1 / 2} \\
\Gamma\left(a+\frac{1}{2}\right)
\end{gathered} \exp \left(\frac{1}{1152 a^{3}+323.2 a}\right)
$$

For $\mathrm{a}=16$
$\frac{a^{a-1 / 2}}{\Gamma\left(a+\frac{1}{2}\right)} \exp \left(\frac{1}{1152 a^{3}+323.2 a}\right)$
4611686994701242309,804652561838 / (gamma 33/2)
$4.611686994701242309804652561838 \times 10^{18}$

$$
\Gamma\left(\frac{33}{2}\right)
$$

$888571.9401322504183973732124617 .$.
Gamma 3/2 $=\frac{191898783962510625 \sqrt{\pi}}{65536}$
$5.1899984530401250830724817743776669334491731891236353 \ldots \times 10^{12}$
$16^{3}=4096 ; 4096 * 1152=4718592 ; 4718592+323.2 * 16=4723763,2$ and
$e^{1 / 4723763.2}=1,0000002116956467775140917669629$
$\left(16^{\wedge} 15.5\right) * 1.0000002116956467775140917669629 /\left(5.18999845304012508 * 10^{12}\right)$
$1 /(32 * 8)$ integrate ((16^15.5)*
$1.0000002116956467775140917669629)) /\left(5.18999845304012508^{*} 10^{\wedge} 12\right) \mathrm{x}$

Input interpretation:

$$
\frac{1}{32 \times 8} \int \frac{16^{15.5} \times 1.0000002116956467775140917669629}{5.18999845304012508 \times 10^{12}} x d x
$$

Result:
$1735.49 x^{2}$
Plot:


Alternate form assuming x is real:
$1735.49 x^{2}+0$

Indefinite integral assuming all variables are real:
$578.497 x^{3}+$ constant

Also this result 1735.49 is a good approximation of the new estimate of the lowest (scalar) glueball mass, that is $\approx 1739 \mathrm{MeV}$.

Note that:
1/(144*32*8) integrate ((16^15.5)*
$1.0000002116956467775140917669629)) /\left(5.18999845304012508^{*} 10^{\wedge} 12\right) \mathrm{x}$
$\frac{1}{144 \times 32 \times 8} \int \frac{16^{15.5} \times 1.0000002116956467775140917669629}{5.18999845304012508 \times 10^{12}} x d x$

The result 12.052 is very near to the value of black hole entropy 12.19
Furthermore, we have:
$1 /\left(144^{\wedge} 2\right) 1 /\left(10^{\wedge} 3\right)$ integrate $\left(\left(16^{\wedge} 15.5\right)^{*}\right.$
$1.0000002116956467775140917669629)) /\left(5.18999845304012508 * 10^{\wedge} 12\right) \mathrm{x}$
$\frac{1}{144^{2}} \times \frac{1}{10^{3}} \int \frac{16^{15.5} \times 1.0000002116956467775140917669629}{5.18999845304012508 \times 10^{12}} x d x$
Result:
$0.0214258 x^{2}$

and
$1 /\left(\mathrm{Pi}^{\wedge} 2-3^{\wedge} 2\right) 1 /\left(10^{\wedge} 6\right)$ integrate $\left(\left(16^{\wedge} 15.5\right)^{*}\right.$ $1.0000002116956467775140917669629)) /\left(5.18999845304012508^{*} 10^{\wedge} 12\right) \mathrm{x}$

$$
\frac{1}{\pi^{2}-3^{2}} \times \frac{1}{10^{6}} \int \frac{16^{15.5} \times 1.0000002116956467775140917669629}{5.18999845304012508 \times 10^{12}} x d x
$$

Result:
$0.510906 x^{2}$


The results 0.0214258 and 0.510906 are exactly the value of the lowest nonvanishing gluon condensate, that is $\approx 0.0214 \mathrm{GeV}^{4}$ and the value of the scalar glueball 'size' that is $\mathrm{r}_{\mathrm{G}} \approx 0.51 \mathrm{fm}$.

Now:

Entry 30(i). If $n$ is a nonnegative integer, then

$$
\int_{0}^{\infty} \frac{\sin ^{2 n+1} x}{x} d x=\int_{0}^{\infty} \frac{\sin ^{2 n+2} x}{x^{2}} d x=\frac{\sqrt{\pi} \Gamma\left(n+\frac{1}{2}\right)}{2 n!} .
$$

We have, for $\mathrm{n}=3$ :
$\frac{\sqrt{\pi} r\left(\frac{7}{2}\right)}{6!} \quad \frac{\pi}{384}$
$0.008181230868723419891829800477290372094263461977539338075 \ldots$
And
$\frac{1}{\frac{\sqrt{\pi} r\left(\frac{Z}{2}\right)}{6!}}$
$122.2309962945756178705027302700910300424650079286705526382 \ldots$
This result 122,23 is very near to the value of the mass of the Higgs boson ( $125,09 \pm 0,24$ ).

Multiplying the expression for the square of the golden ratio, we obtain:
$(((\operatorname{sqrt}(5)+1)) / 2)^{\wedge} 2\left(\left(\left(\operatorname{sqrt}(\operatorname{Pi})^{*} \operatorname{gamma}(7 / 2)\right)\right) /(6!)\right.$
$\left(\frac{1}{2}(\sqrt{5}+1)\right)^{2} \times \frac{\sqrt{\pi} \Gamma\left(\frac{7}{2}\right)}{6!}$

Exact result:
$\frac{(1+\sqrt{5})^{2} \pi}{1536}$

Decimal approximation:
0.021418740484127742328833730199275264911533876803636093597 ..

Property:
$\frac{(1+\sqrt{5})^{2} \pi}{1536}$ is a transcendental number

Alternate forms:
$\frac{1}{768}(3+\sqrt{5}) \pi$
$\frac{\pi}{256}+\frac{\sqrt{5} \pi}{768}$

Continued fraction:
$[0 ; 46,1,2,4,1,5,1,5,1,3,3,6,6,1,1,1,3,1,10,1,1,25,1,13,1,1,2,20,1, \ldots]$
Alternative representations:
$\frac{\left(\frac{1}{2}(\sqrt{5}+1)\right)^{2}\left(\sqrt{\pi} \Gamma\left(\frac{7}{2}\right)\right)}{6!}=\frac{e^{-\log \mathrm{G}(7 / 2)+\log \mathrm{G}(9 / 2)}\left(\frac{1}{2}(1+\sqrt{5})\right)^{2} \sqrt{\pi}}{(1)_{6}}$
$\frac{\left(\frac{1}{2}(\sqrt{5}+1)\right)^{2}\left(\sqrt{\pi} \Gamma\left(\frac{7}{2}\right)\right)}{6!}=\frac{e^{-\log G(7 / 2)+\log G(9 / 2)}\left(\frac{1}{2}(1+\sqrt{5})\right)^{2} \sqrt{\pi}}{5!!\times 6!!}$
$\frac{\left(\frac{1}{2}(\sqrt{5}+1)\right)^{2}\left(\sqrt{\pi} \Gamma\left(\frac{7}{2}\right)\right)}{6!}=\frac{e^{-\log \mathrm{G}(7 / 2)+\log \mathrm{G}(9 / 2)}\left(\frac{1}{2}(1+\sqrt{5})\right)^{2} \sqrt{\pi}}{e^{\log (7)}}$
Series representations:

$$
\begin{aligned}
& \frac{\left(\frac{1}{2}(\sqrt{5}+1)\right)^{2}\left(\sqrt{\pi} \Gamma\left(\frac{7}{2}\right)\right)}{6!}=\left(\exp \left(i \pi \left\lvert\, \frac{\arg (\pi-x)}{2 \pi}\right.\right]\right) \sqrt{x} \\
& \left(1+\exp \left(i \pi\left[\frac{\arg (5-x)}{2 \pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^{k}(5-x)^{k} x^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)^{2} \\
& \left.\quad \sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}}(\pi-x)^{k_{1}} x^{-k_{1}}\left(-\frac{1}{2}\right)_{k_{1}}\left(\frac{7}{2}-z_{0}\right)^{k_{2}} \Gamma^{\left(k_{2}\right)}\left(z_{0}\right)}{k_{1}!k_{2}!}\right) / \\
& \left(4 \sum_{k=0}^{\infty} \frac{\left(6-n_{0}\right)^{k} \Gamma^{(k)}\left(1+n_{0}\right)}{k!}\right)
\end{aligned}
$$

for $\left(x \in \mathbb{R}\right.$ and ( $n_{0} \notin \mathbb{Z}$ or $\left.n_{0} \geq 0\right)$ and ( $z_{0} \notin \mathbb{Z}$ or $z_{0}>0$ ) and $x<0$ and $n_{0} \rightarrow 6$ )

$$
\begin{aligned}
& \frac{\left(\frac{1}{2}(\sqrt{5}+1)\right)^{2}\left(\sqrt{\pi} \Gamma\left(\frac{7}{2}\right)\right)}{6!}=\left(\left(\frac{1}{z_{0}}\right)^{1 / 2\left\lfloor\arg \left(\pi-z_{0}\right) /(2 \pi)\right\rfloor} z_{0}^{\left.1 / 2+1 / 2 \arg \left(\pi-z_{0}\right) /(2 \pi)\right\rfloor}\right. \\
& \left(1+2\left(\frac{1}{z_{0}}\right)^{\left.1 / 2 \arg \left(5-z_{0}\right) /(2 \pi)\right\rfloor} z_{0}^{1 / 2+1 / 2\left\lfloor\arg \left(5-z_{0}\right) /(2 \pi)\right\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k!}+\right. \\
& \left.\quad\left(\frac{1}{z_{0}}\right)^{\left.\arg \left(5-z_{0}\right) /(2 \pi)\right\rfloor} z_{0}^{\left.\left.1+\arg \left(5-z_{0}\right)\right)(2 \pi)\right\rfloor}\left(\sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5-z_{0}\right)^{k} z_{0}^{k}}{k!}\right)^{2}\right) \\
& \left.\quad \sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}}\left(-\frac{1}{2}\right)_{k_{1}}\left(\frac{7}{2}-z_{0}\right)^{k_{2}}\left(\pi-z_{0}\right)^{k_{1}} z_{0}^{-k_{1}} \Gamma^{\left.\left.\left(k_{2}\right)\right)_{0}\right)}}{k_{1}!k_{2}!}\right) / \\
& \left(4 \sum_{k=0}^{\infty} \frac{\left(6-n_{0}\right)^{k} \Gamma^{(k)}\left(1+n_{0}\right)}{k!}\right)
\end{aligned}
$$

for $\left(\left(n_{0} \notin \mathbb{Z}\right.\right.$ or $\left.n_{0} \geq 0\right)$ and ( $z_{0} \notin \mathbb{Z}$ or $z_{0}>0$ ) and $\left.n_{0} \rightarrow 6\right)$
Integral representations:
$\frac{\left(\frac{1}{2}(\sqrt{5}+1)\right)^{2}\left(\sqrt{\pi} \Gamma\left(\frac{7}{2}\right)\right)}{6!}=\frac{\Gamma\left(\frac{7}{2}\right)(1+\sqrt{5})^{2} \sqrt{\pi}}{4 \int_{0}^{\infty} e^{-t} t^{6} d t}$
$\frac{\left(\frac{1}{2}(\sqrt{5}+1)\right)^{2}\left(\sqrt{\pi} \Gamma\left(\frac{7}{2}\right)\right)}{6!}=$
$\frac{i \pi(1+\sqrt{5})^{2} \sqrt{\pi}}{2\left(720+e^{-\infty}(-(\infty+6 \infty+30 \infty+120 \infty+360 \infty+720) \infty+-720)\right) \oint_{L} \frac{t^{t}}{t^{7 / 2}} d t}$
$\frac{\left(\frac{1}{2}(\sqrt{5}+1)\right)^{2}\left(\sqrt{\pi} \Gamma\left(\frac{7}{2}\right)\right)}{6!}=\frac{i \pi(1+\sqrt{5})^{2} \sqrt{\pi}}{1440 \oint_{L} \frac{t}{t^{7 / 2}} d t}$

And
$24(((\operatorname{sqrt}(5)+1)) / 2)^{\wedge} 2(((\operatorname{sqrt}(\mathrm{Pi}) * \operatorname{gamma}(7 / 2))) /(6!)$
$24\left(\frac{1}{2}(\sqrt{5}+1)\right)^{2} \times \frac{\sqrt{\pi} \Gamma\left(\frac{7}{2}\right)}{6!}$
$\frac{1}{64}(1+\sqrt{5})^{2} \pi$
Decimal approximation:
$0.514049771619065815892009524782606357876813043287266246341 \ldots$
Property:
$\frac{1}{64}(1+\sqrt{5})^{2} \pi$ is a transcendental number

$$
\begin{aligned}
& \frac{\text { Altermate forms: }}{\frac{1}{32}(3+\sqrt{5}) \pi} \\
& \frac{3 \pi}{32}+\frac{\sqrt{5} \pi}{32}
\end{aligned}
$$

Continued fraction:
$[0 ; 1,1,17,3,2,2,14,2,1,2,2,5,2,2,1,2,1,2,1,11,1,1,3,2,2,1,1,7,1,1, \ldots]$
Alternative representations:
$\frac{\left(24\left(\frac{1}{2}(\sqrt{5}+1)\right)^{2}\right)\left(\sqrt{\pi} \Gamma\left(\frac{7}{2}\right)\right)}{6!}=\frac{24 e^{-\log G(7 / 2)+\log G(9 / 2)}\left(\frac{1}{2}(1+\sqrt{5})\right)^{2} \sqrt{\pi}}{(1)_{6}}$
$\frac{\left(24\left(\frac{1}{2}(\sqrt{5}+1)\right)^{2}\right)\left(\sqrt{\pi} \Gamma\left(\frac{7}{2}\right)\right)}{6!}=\frac{24 e^{-\log G(7 / 2)+\log G(9 / 2)}\left(\frac{1}{2}(1+\sqrt{5})\right)^{2} \sqrt{\pi}}{5!!\times 6!!}$
$\frac{\left(24\left(\frac{1}{2}(\sqrt{5}+1)\right)^{2}\right)\left(\sqrt{\pi} \Gamma\left(\frac{7}{2}\right)\right)}{6!}=\frac{24 e^{-\log \mathrm{G}(7 / 2)+\log \mathrm{G}(9 / 2)}\left(\frac{1}{2}(1+\sqrt{5})\right)^{2} \sqrt{\pi}}{e^{\log \Gamma(7)}}$

Series representations:

$$
\begin{aligned}
& \frac{\left(24\left(\frac{1}{2}(\sqrt{5}+1)\right)^{2}\right)\left(\sqrt{\pi} \Gamma\left(\frac{7}{2}\right)\right)}{6!}=\left(6 \exp \left(i \pi\left\lfloor\frac{\arg (\pi-x)}{2 \pi}\right]\right) \sqrt{x}\right. \\
& \left(1+\exp \left(i \pi\left\lfloor\frac{\arg (5-x)}{2 \pi}\right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^{k}(5-x)^{k} x^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)^{2} \\
& \left.\sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}}(\pi-x)^{k_{1}} x^{-k_{1}}\left(-\frac{1}{2}\right)_{k_{1}}\left(\frac{7}{2}-z_{0}\right)^{k_{2}} \Gamma^{\left(k_{2}\right)}\left(z_{0}\right)}{k_{1}!k_{2}!}\right) / \\
& \left(\sum_{k=0}^{\infty} \frac{\left(6-n_{0}\right)^{k} \Gamma^{(k)}\left(1+n_{0}\right)}{k!}\right)
\end{aligned}
$$

for ( $x \in \mathbb{R}$ and ( $n_{0} \notin \mathbb{Z}$ or $n_{0} \geq 0$ ) and ( $z_{0} \notin \mathbb{Z}$ or $z_{0}>0$ ) and $x<0$ and $n_{0} \rightarrow 6$ )

$$
\begin{aligned}
& \frac{\left(24\left(\frac{1}{2}(\sqrt{5}+1)\right)^{2}\right)\left(\sqrt{\pi} \Gamma\left(\frac{7}{2}\right)\right)}{6!}=\left(6\left(\frac{1}{z_{0}}\right)^{1 / 2\left\lfloor\arg \left(\pi-z_{0}\right) /(2 \pi)\right\rfloor} z_{0}^{\left.1 / 2+1 / 2\left\lfloor\arg \left(\pi-z_{0}\right)\right)(2 \pi)\right\rfloor}\right. \\
& \left(1+2\left(\frac{1}{z_{0}}\right)^{1 / 2\left\lfloor\arg \left(5-z_{0}\right) /(2 \pi)\right\rfloor} z_{0}^{1 / 2+1 / 2\left\lfloor\arg \left(5-z_{0}\right) /(2 \pi)\right\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5-z_{0}\right)^{k} z_{0}^{k}}{k!}+\right. \\
& \left.\left(\frac{1}{z_{0}}\right)^{\left.\arg \left(5-z_{0}\right) /(2 \pi)\right\rfloor} z_{0}^{1+\left\lfloor\arg \left(5-z_{0}\right) /(2 \pi)\right\rfloor}\left(\sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right)^{2}\right) \\
& \left.\quad \sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}( }\left(-\frac{1}{2}\right)_{k_{1}}\left(\frac{7}{2}-z_{0}\right)^{k_{2}\left(\pi-z_{0}\right)^{k_{1}} z_{0}^{-k_{1}} \Gamma^{\left(k_{2}\right)}\left(z_{0}\right)}}{k_{1}!k_{2}!}\right) / \\
& \left(\sum_{k=0}^{\infty} \frac{\left(6-n_{0}\right)^{k} \Gamma^{(k)}\left(1+n_{0}\right)}{k!}\right)
\end{aligned}
$$

$$
\text { for }\left(\left(n_{0} \triangleq \mathbb{Z} \text { or } n_{0} \geq 0\right) \text { and }\left(z_{0} \notin \mathbb{Z} \text { or } z_{0}>0\right) \text { and } n_{0} \rightarrow 6\right)
$$

Integral representations:

$$
\begin{aligned}
& \frac{\left(24\left(\frac{1}{2}(\sqrt{5}+1)\right)^{2}\right)\left(\sqrt{\pi} \Gamma\left(\frac{7}{2}\right)\right)}{6!}=\frac{6 \Gamma\left(\frac{7}{2}\right)(1+\sqrt{5})^{2} \sqrt{\pi}}{\int_{0}^{\infty} e^{-t} t^{6} d t} \\
& \frac{\left(24\left(\frac{1}{2}(\sqrt{5}+1)\right)^{2}\right)\left(\sqrt{\pi} \Gamma\left(\frac{7}{2}\right)\right)}{6!}= \\
& \frac{12 i \pi(1+\sqrt{5})^{2} \sqrt{\pi}}{720+e^{-\infty}(-(\infty+6 \infty+30 \infty+120 \infty+360 \infty+720) \infty+-720) \oint_{L} t^{t} \frac{t}{7 / 2} d t} \\
& \frac{\left(24\left(\frac{1}{2}(\sqrt{5}+1)\right)^{2}\right)\left(\sqrt{\pi} \Gamma\left(\frac{7}{2}\right)\right)}{6!}=\frac{12 i \pi(1+\sqrt{5})^{2} \sqrt{\pi}}{720 \oint_{L} \frac{t}{t^{7 / 2}} d t}
\end{aligned}
$$

where the results 0.02141874 and 0.51404977 are exactly the value of the lowest non-vanishing gluon condensate, that is $\approx 0.0214 \mathrm{GeV}^{4}$ and the value of the scalar glueball 'size' that is $\mathrm{r}_{\mathrm{G}} \approx 0.51 \mathrm{fm}$.

Now, we calculate the following integral:
$(9 * \mathrm{Pi})$ integrate $\mathrm{x} /[(((\operatorname{sqrt}(\mathrm{Pi}) * \operatorname{gamma}(7 / 2))) /(6!)]$
(9 $\int^{\text {Indefinite e integral: }} \frac{x}{\frac{\sqrt{\pi} \Gamma\left(\frac{z}{2}\right)}{6!}} d x=1728 x^{2}+$ constant

Plot:


1728
$(\operatorname{sqrt}(1.08643)+55 / 2)$ integrate $\mathrm{x} /\left[\left(\left(\left(\operatorname{sqrt}(\mathrm{Pi})^{*} \operatorname{gamma}(7 / 2)\right)\right) /(6!)\right]\right.$
$\sqrt{1.08643}+\frac{55}{2} \int \frac{x}{\frac{\sqrt{\pi} \Gamma\left(\frac{7}{2}\right)}{6!}} d x$

Result:
$1744.38 x^{2}$
Plot:

The result 1744.38 is very near the new estimate of the lowest (scalar) glueball mass, that is $\approx 1739 \mathrm{MeV}$.

Note that:
$1 / 144$ * (sqrt(1.08643)+55/2) integrate $\mathrm{x} /\left[\left(\left(\left(\operatorname{sqrt(Pi)}{ }^{*} \operatorname{gamma}(7 / 2)\right)\right) /(6!)\right]\right.$
Input interpretation:
$\frac{1}{144}\left(\sqrt{1.08643}+\frac{55}{2}\right) \int \frac{x}{\frac{\sqrt{\pi} \Gamma\left(\frac{7}{2}\right)}{6!}} d x$
Result:
$12.1137 x^{2}$

The result 12.11 is very near to the value of black hole entropy 12.19
Now:

Entry 35. Let $n$ denote a nonnegative integer, and let $\alpha, \beta>0$ with $\alpha \beta=\pi$. Then

$$
\sqrt{\alpha}\left\{1+2 \sum_{k=1}^{\infty} \frac{1}{\left(1+\alpha^{2} k^{2}\right)^{n+1}}\right\}=\sqrt{\beta} \frac{\Gamma\left(n+\frac{1}{2}\right)}{\Gamma(n+1)}\left\{1+2 \sum_{k=1}^{\infty} e^{-2 \beta k} \varphi(4 \beta k)\right\}
$$

where

$$
\begin{array}{r}
\varphi(t)=\frac{n!}{(2 n)!} \sum_{k=0}^{n} \frac{(n+k)!t^{n-k}}{(n-k)!k!} . \\
4 \int_{0}^{\infty} \frac{\cos (2 \pi k x)}{\left(1+\alpha^{2} x^{2}\right)^{n+1}} d x=\sqrt{\frac{\beta}{\alpha}} \frac{\Gamma\left(n+\frac{1}{2}\right)}{\Gamma(n+1)} 2 e^{-2 \beta k} \varphi(4 \beta k)
\end{array}
$$

For $\mathrm{n}=3, \mathrm{k}=2, \alpha \beta=\pi$
$[(3!) /(6!)] \operatorname{sum}[5!t /(1!2!)]$

$$
\frac{\sum 60 t}{120}
$$

0.5
$\operatorname{sqrt}(\mathrm{Pi}) *(3.32335097) /(6) *(2(\exp (-4 \mathrm{Pi}) * 0.5(8 \mathrm{Pi})$
$\operatorname{gamma}(7 / 2)^{3.323350970447842551184064031264647217745405230229475865400 \ldots}$
$\operatorname{gamma}(4)=6$
$\left.\left.((\operatorname{sqrt}(\mathrm{Pi}) *(3.32335097) /(6))) *\left(2 * \mathrm{e}^{\wedge}(-4 \mathrm{Pi})\right) * 0.5(8 \mathrm{Pi})\right)\right)$

Input interpretation:
$\left(\sqrt{\pi} \times \frac{3.32335097}{6}\right)\left(2 e^{-4 \pi}\right) \times 0.5(8 \pi)$
Result:
$0.0000860467 \ldots$
$\frac{1}{\left(\sqrt{\pi} \times \frac{3.32335097}{6}\right)\left(2 e^{-4 \pi}\right) \times 0.5(8 \pi)}$

Result:
11621.6...

The result 11621,6 is very near to the value of the Ramanujan's class invariant 1164,2696 multiplied by 10.

Now:
$\left.\left.\left.\left(27^{*} 2\right)-(((\operatorname{sqrt}(5)-1) / 8))\right)^{*} 1 /\left[\left(\left(\operatorname{sqrt}(\mathrm{Pi})^{*}(3.32335097) /(6)\right)\right)^{*}\left(2^{*} \mathrm{e}^{\wedge}(-4 \mathrm{Pi})\right)^{*} 0.5(8 \mathrm{Pi})\right)\right)\right]$

Input interpretation:
$27 \times 2-\left(\frac{1}{8}(\sqrt{5}-1)\right) \times \frac{1}{\left(\sqrt{\pi} \times \frac{3.32335097}{6}\right)\left(2 e^{-4 \pi}\right) \times 0.5(8 \pi)}$
Result:
-1741.63...
Series representations:

The result $-1741,63$ is very near the new estimate of the lowest (scalar) glueball mass, that is $\approx 1739 \mathrm{MeV}$.

$$
\begin{aligned}
& 27 \times 2-\frac{\sqrt{5}-1}{\frac{1}{6}\left(\left(2 e^{-4 \pi}\right) \sqrt{\pi} 3.32335 \times 0.5(8 \pi)\right) 8}= \\
& -\left(\left(0 . 0 2 8 2 0 9 5 \left(-e^{4 \pi}+e^{4 \pi} \sqrt{4} \sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}-1914.25 \pi \sqrt{-1+\pi}\right.\right.\right. \\
& \left.\left.\left.\quad \sum_{k=0}^{\infty}(-1+\pi)^{-k}\binom{\frac{1}{2}}{k}\right)\right) /\left(\pi \sqrt{-1+\pi} \sum_{k=0}^{\infty}(-1+\pi)^{-k}\left(\begin{array}{c}
\frac{1}{2} \\
2 \\
k
\end{array}\right)\right)\right)
\end{aligned}
$$

$27 \times 2-\frac{\sqrt{5}-1}{\frac{1}{6}\left(\left(2 e^{-4 \pi}\right) \sqrt{\pi} 3.32335 \times 0.5(8 \pi)\right) 8}=$

$$
\begin{aligned}
&-\left(\left(0 . 0 2 8 2 0 9 5 \left(-e^{4 \pi}+e^{4 \pi} \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}-1914.25 \pi \sqrt{-1+\pi}\right.\right.\right. \\
&\left.\left.\left.\sum_{k=0}^{\infty} \frac{(-1)^{k}(-1+\pi)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)\right) /\left(\pi \sqrt{-1+\pi} \sum_{k=0}^{\infty} \frac{(-1)^{k}(-1+\pi)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)\right)
\end{aligned}
$$

$27 \times 2-\frac{\sqrt{5}-1}{\frac{1}{6}\left(\left(2 e^{-4 \pi}\right) \sqrt{\pi} 3.32335 \times 0.5(8 \pi)\right) 8}=$

$$
\begin{aligned}
& -\left(\left(0 . 0 2 8 2 0 9 5 \left(-e^{4 \pi}+e^{4 \pi} \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k!}-\right.\right.\right. \\
& \left.\left.1914.25 \pi \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(\pi-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right)\right) / \\
& \left.\left(\pi \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(\pi-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right)\right) \text { for } \operatorname{not}\left(\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right)\right)
\end{aligned}
$$

$1 /(192+9)^{\wedge} 2$ integrate $[-1741.63 x]$
Input interpretation:
$\frac{1}{(192+9)^{2}} \int-1741.63 x d x$

Result:
$-0.0215543 x^{2}$

$8^{*} 1 /(144-27)^{\wedge} 2$ integrate $[-1741.63 x]$
$8 \times \frac{1}{(144-27)^{2}} \int-1741.63 x d x$
Result:
$-0.508914 x^{2}$
Plot:

$0-0.508914 x^{2}$

Indefinite integral assuming all variables are real:
$-0.169638 x^{3}+$ constant
where the results -0.0215543 and -0.508914 are exactly the value of the lowest nonvanishing gluon condensate, that is $\approx 0.0214 \mathrm{GeV}^{4}$ and the value of the scalar glueball 'size' that is $\mathrm{r}_{\mathrm{G}} \approx 0.51 \mathrm{fm}$.

Now, we calculate the following integral already previously analyzed:
integrate $\left.\left.1 /\left[\left(\left(\operatorname{sqrt}(\mathrm{Pi})^{*}(3.32335097) /(6)\right)\right)^{*}\left(2^{*} \mathrm{e}^{\wedge}(-4 \mathrm{Pi})\right)^{*} 0.5(8 \mathrm{Pi})\right)\right)\right] \mathrm{x}$

$$
\int \frac{x}{\frac{1}{6}(\sqrt{\pi} 3.32335097)\left(2 e^{-4 \pi}\right) 0.5(8 \pi)} d x=5810.8 x^{2}+\text { constant }
$$

Plot of the integral:


Note that:
$1729 /(26 \mathrm{Pi}) * 1 / 10^{\wedge} 4$ integrate $1 /\left[((\operatorname{sqrt}(\mathrm{Pi}) *(3.32335097) /(6))) *\left(2 * \mathrm{e}^{\wedge}(-\right.\right.$ $\left.4 \mathrm{Pi}))^{*} 0.5(8 \mathrm{Pi})\right)$ )]x
$\frac{1729}{26 \pi} \times \frac{1}{10^{4}} \int \frac{1}{\left(\sqrt{\pi} \times \frac{3.32335097}{6}\right)\left(2 e^{-4 \pi}\right) \times 0.5(8 \pi)} x d x$

Result:
$12.3001 x^{2}$

The result 12.30 is very near to the value of the black hole entropy 12.19-12.57
We have also (see pg. 19) this other integral and we can to obtain:
integrate $[10361.2220016+1334.337561] x$

$$
\int(10361.2220016+1334.337561) x d x=5847.78 x^{2}+\text { constant }
$$

Plot of the integral:


We note that:
$1729 /(26 \mathrm{Pi}) * 1 / 10^{\wedge} 4$ integrate $[10361.2220016+1334.337561] \mathrm{x}$
$\frac{\begin{array}{c}\text { Input interpretation: } \\ \text { Open code }\end{array}}{\frac{1729}{10^{4}}} \times \frac{1}{10361.2220016+1334.337561) x d x}$

Result:
$12.3784 x^{2}$

The result 12.378 is very near to the value of black hole entropy $12.19-12.57$
integrate $1 / 2$ [10361.2220016+1334.337561]+96
$\int\left(\frac{10361.2220016+1334.337561}{2}+96\right) d x=5943.78 x+$ constant
Plot of the integral:

integrate $-(288+48)+2 \mathrm{Pi} /(\ln 1729)) 1 /\left[((\operatorname{sqrt}(\mathrm{Pi}) *(3.32335097) /(6))) *\left(2 * \mathrm{e}^{\wedge}(-\right.\right.$ $4 \mathrm{Pi})) * 0.5(8 \mathrm{Pi})))]$

$$
\begin{aligned}
& \int\left(-(288+48)+\frac{2 \pi}{\frac{1}{6} \log (1729)\left((\sqrt{\pi} 3.32335097)\left(2 e^{-4 \pi}\right) 0.5(8 \pi)\right)}\right) d x= \\
& 9458.46 x+\text { constant }
\end{aligned}
$$

Plot of the integral:

integrate $(96+48+9)+\mathrm{Pi} / 6$ [11695.5595626]
$\left.\int(96+48+9)+\frac{\pi 11695.5595626}{6}\right) d x=6276.78066691 x+$ constant
Plot of the integral:


We note that the results of the calculated integrals are very good approximations of the bottom Xi, bottom Sigma, charmed B meson and Upsilon meson.

Now, from:
A holographic description of heavy-flavoured baryonic matter decay involving glueball - Si-wen Li - arXiv:1812.03482v2 [hep-th]

$$
\begin{gather*}
y=r \cos \Theta, \quad z=r \sin \Theta, \\
U^{3}=U_{K K}^{3}+U_{K K} r^{2}, \quad \Theta=\frac{2 \pi}{\beta} X^{4}=\frac{3}{2} \frac{U_{K K}^{1 / 2}}{R^{3 / 2}} . \tag{B-8}
\end{gather*}
$$

In the standard WSS model, the probe $\mathrm{D} 8 / \overline{\mathrm{D} 8}$-branes are embedded at $\Theta=+\frac{1}{2} \pi$ respectively i.e. the position of $y=0$, which exactly corresponds to the antipodal $\mathrm{D} 8 / \overline{\mathrm{D} 8}$-branes (blue) in Figure 1. In this case, the solution for the embedding function is $X^{4}(U)=\frac{1}{4} \beta$ and $U_{0}=U_{K K}$. In aldition, the (B-7) also allows the non-antipodal solution if we chouse $\Theta= \pm \Theta_{H} \neq \pm \frac{1}{2} \pi, U_{0}=$ $U_{H} \nleftarrow U_{K K}$ which corresponds to the non-antipodal D8/D8-branes (red) in Figure 1. On the other hand, while each endpoints of the IIL string could move along the flavoured branes, in our setup it is stretched between the heavy- (non-antipodal) and light-flavoured (antipodal) D8/D8brancs. So it conncets the positions respectively on the heavy- and light-flavoured D8/D8branes which are most close to each other and in the $U \quad X^{4}$ planc, they are the positions of $\left(U_{K K}, 0\right)$ on the light flavoured branes and $\left(U_{H}, 0\right)$ on the heavy flavoured branes. And this is the configuration of the HL string with minimal length i.e. the VEV.

The eigenvalue equations for $H_{E, D, T}$ are given in (A-9) and (A-14). In the rescaled coordinate $Z \rightarrow \lambda^{-1 / 2} Z$, the equations are written as,
$Z^{2}=1 / \lambda$
We have that:

| Excitation of glueball ( $n$ ) | $n=0$ |
| :---: | :---: |
| Glueball mass $M_{E}^{(n)}$ | 0.901 |
| Glueball mass $M_{D, T}^{(n)}$ | 1.567 |
| The coefficients | $n=0$ |
| $\mathcal{C}_{E}$ | 144.545 |
| $\mathcal{C}_{D}$ | 29.772 |
| $\mathcal{C}_{T}$ | 72.927 |

and:

$$
\begin{aligned}
& \lambda=g_{\mathrm{YM}}^{2} N_{c}, g_{\mathrm{YM}}^{2}=2 \pi g_{s} l_{s} M_{K K}, \\
= & -2,31830159 * 10^{-34}
\end{aligned}
$$

$$
X^{4} \sim X^{4}+2 \pi \delta X^{4}, \delta X^{4}=\frac{1}{M_{K K}}
$$

For $\beta=1, \frac{1}{4}+\frac{2 \pi}{M_{K K}}=0 ; \quad \frac{1}{4}=-\frac{2 \pi}{M_{K K}} ; \quad M_{K K}=-8 \pi$

$$
\begin{gathered}
l=0, N_{Q}=1, N_{c}=3, N_{f}=2 . \\
Z^{2}=1 / \lambda=-4,313502611 * 10^{31}
\end{gathered}
$$

Now, from the eq. (3.2):

$$
\begin{aligned}
H_{E}(z) & =\mathcal{C}_{E}\left(1-\frac{3 M_{E}^{2}+16 M_{K K}^{2}}{12 M_{K K}^{2} \lambda} Z^{2}\right) \lambda^{-1 / 2} N_{c}^{-1} M_{K K}^{-1}+\mathcal{O}\left(\lambda^{-3 / 2}\right), \\
H_{D, T}(z) & =\mathcal{C}_{D, T}\left(1-\frac{M_{D, T}^{2}}{4 M_{K K}^{2} \lambda} Z^{2}\right) \lambda^{-1 / 2} N_{c}^{-1} M_{K K}^{-1}+\mathcal{O}\left(\lambda^{-3 / 2}\right) .
\end{aligned}
$$

we obtain:
$144.545\left(\left(\left(\left(1-\left(3^{*} 0.901^{\wedge} 2+16^{*}\left(64 \mathrm{Pi}^{\wedge} 2\right)^{*}-4.313502611^{*} 10^{\wedge} 31\right) /\left(12^{*}\left(64 \mathrm{Pi}^{\wedge} 2\right)^{*}-\right.\right.\right.\right.\right.$ $\left.\left.2.31830^{*} 10^{\wedge}-34\right)\right)^{*}\left(\left(-2.0768980^{*} 10^{\wedge} 16\right)^{*}(1 / 3)^{*}(-0.03978873)\right)$
$144.545\left(\left(1-\frac{3 \times 0.901^{2}+16\left(64 \pi^{2}\right)\left(-4.313502611 \times 10^{31}\right)}{12\left(64 \pi^{2}\right)\left(-2.31830 \times 10^{-34}\right)}\right)\right.$

$$
\left.\left(-2.0768980 \times 10^{16} \times \frac{1}{3} \times(-0.03978873)\right)\right)
$$

Result:
$-9.87771 \ldots \times 10^{81}$
$\left(-9.87771^{*} 10^{\wedge} 81\right)+\left(8.958711419 * 10^{\wedge} 47\right)$
$-9.87771 \times 10^{81}+8.958711419 \times 10^{47}$

## Result:

-9877709999999999999999999999999999104128858100000000000000:
000000000000000000000000
$-9.877709 * 10^{81}$
Now:
$3.152565 \ldots \times 10^{20}$

And
$\left(\ln \left[-\left(-9.87771^{*} 10^{\wedge} 81\right)+\left(8.958711419^{*} 10^{\wedge} 47\right)\right]^{\wedge} 1 / 11\right.$
$\sqrt[11]{\log \left(-\left(-9.87771 \times 10^{81}\right)+8.958711419 \times 10^{47}\right)}$

Result:
1.61030894...

The result 1,61030894 is very near to the electric charge of the positron.
$\mathrm{Pi}^{\wedge} 2 / 11\left(\ln \left[-\left(-9.87771^{*} 10^{\wedge} 81\right)+\left(8.958711419^{*} 10^{\wedge} 47\right)\right]^{\wedge} 1 / 2\right.$
$\frac{\pi^{2}}{11} \sqrt{\log \left(-\left(-9.87771 \times 10^{81}\right)+8.958711419 \times 10^{47}\right)}$

Result:
12.3284273..

This result is very near to the value of Black Hole Entropy 12,57
Now, we calculate the following integral:
integrate $1 /(27 * 1728) * 1 /\left(10^{\wedge} 37\right)^{\wedge} 2 *\left[\left(-9.87771 * 10^{\wedge} 81\right)+\left(8.958711419 * 10^{\wedge} 47\right)\right]$
$\int \frac{-9.87771 \times 10^{81}+8.958711419 \times 10^{47}}{(27 \times 1728)\left(10^{37}\right)^{2}} d x=-2117.14 x+$ constant

Plot of the integral:


The result $-2117,14$ is very near to the rest mass of strange $D$ meson $2112.3 \pm 0.5$

Now, for the second expression of (3.2):
$8.958711419^{*} 10^{\wedge} 47+29.772((()-(1.567 \wedge 2$ * $(-$
$\left.\left.4.313502611^{*} 10^{\wedge} 31\right)\right) /\left(4^{*}\left(64 \mathrm{Pi}^{\wedge} 2\right)\right)^{*}\left(\left(-2.0768980^{*} 10^{\wedge} 16\right)^{*}(1 / 3) *(-0.03978873)\right)$

Input interpretation:
$8.958711419 \times 10^{47}+29.772$

$$
\left(1-\frac{1.567^{2}\left(-4.313502611 \times 10^{31}\right)}{4\left(64 \pi^{2}\right)}\left(-2.0768980 \times 10^{16} \times \frac{1}{3} \times(-0.03978873)\right)\right)
$$

Result:
$8.9621493 \ldots \times 10^{47}$
$8.9621493 * 10^{47}$

Percent increase:
$8.958711419 \times 10^{47}+29.772$

$$
\left(1-\frac{\left(1.567^{2}\left(-4.313502611 \times 10^{31}\right)\right)\left(-2.0768980 \times 10^{16}(-0.03978873)\right)}{\left(4\left(64 \pi^{2}\right)\right) 3}\right)=
$$

$$
8.96215 \times 10^{47} \text { is } 0.0383747 \% \text { larger than } 8.958711419 \times 10^{47}=8.95871 \times 10^{47} .
$$

## Comparisons

$\approx 1.1 \times 10^{-6} \times$ the size of the Monster group $\left(\approx 8.1 \times 10^{53}\right)$
$\approx 0.017 \times$ the number of chess positions $\left(\approx 5.2 \times 10^{49}\right.$ )
$\left[\ln \left(8.9621493 * 10^{\wedge} 47\right)^{\wedge} 1 / 10\right]$
$\sqrt[10]{\log \left(8.9621493 \times 10^{47}\right)}$

Result:
1.6006729829...

The result 1,6006729 is very near to the value of the electric charge of the positron.
$\left[\ln \left(8.9621493 * 10^{\wedge} 47\right) \mathrm{Pi}^{\wedge} 2 / 89\right]$

Input interpretation:
$\log \left(8.9621493 \times 10^{47}\right) \times \frac{\pi^{2}}{89}$

Result:
12.24435425...

Note that the result 12.24 is very near to the value of black hole entropy 12.19

We calculate the following integral:
integrate $-16+1 /\left((1728+1728) * 10^{\wedge} 41\right)$ [8.958711419*10^47 + 29.772((() $1-$ $\left(1.567 \wedge 2 *\left(-4.313502611^{*} 10^{\wedge} 31\right)\right) /\left(4 *\left(64 \mathrm{Pi}^{\wedge} 2\right)\right)^{*}\left(\left(-2.0768980^{*} 10^{\wedge} 16\right) *(1 / 3) *(-\right.$ $0.03978873)$ )]

$$
\begin{aligned}
& \int\left(-16+\left(8.958711419 \times 10^{47}+\right.\right. \\
& 29.772\left(1-\frac{1}{\left(4\left(64 \pi^{2}\right)\right) 3}\left(1.567^{2}\left(-4.313502611 \times 10^{31}\right)\right)\right. \\
& \left.\left.\left(-2.0768980 \times 10^{16}(-0.03978873)\right)\right)\right) / \\
& \left.\left((1728+1728) 10^{41}\right)\right) d x=2577.21 x+\text { constant }
\end{aligned}
$$

Plot of the integral:

The result 2577.21 is practically equal to $2575.6 \pm 3.1$ and $2577.9 \pm 2.9$ that are the values of the baryons charmed Xi prime.

We note that:
integrate $1 /(144+64) *\left(-16+1 /\left((1728+1728) * 10^{\wedge} 41\right)\left[8.958711419 * 10^{\wedge} 47+\right.\right.$ $29.772\left(\left(\left(\left(1-\left(1.567 \wedge 2 *\left(-4.313502611^{*} 10^{\wedge} 31\right)\right) /\left(4^{*}\left(64 \mathrm{Pi}^{\wedge} 2\right)\right)^{*}((-\right.\right.\right.\right.$ $\left.\left.\left.2.0768980 * 10^{\wedge} 16\right) *(1 / 3) *(-0.03978873)\right)\right]$

$$
\begin{aligned}
& \int \frac{1}{144+64} \\
& \left.\begin{array}{r}
\left(-16+\left(8.958711419 \times 10^{47}+29.772\left(1-\frac{1}{\left(4\left(64 \pi^{2}\right)\right) 3}\left(1.567^{2}(-4.313502611\right.\right.\right.\right. \\
\left.\left.\left.\left.10^{31}\right)\right)\left(-2.0768980 \times 10^{16}(-0.03978873)\right)\right)\right) / \\
\left.\left((1728+1728) 10^{41}\right)\right) d x
\end{array}\right)=12.3905 x+\text { constant }
\end{aligned}
$$

The result 12.39 is very near to the value of black hole entropy 12.57
From the ratio between the two obtained results we have the following expression:
$\left.\left.\left.\left(\left(\left(-9.877709 * 10^{\wedge} 81\right) /\left(8.9621493 * 10^{\wedge} 47\right)\right)^{\wedge} 2\right)\right)^{\wedge} 1 / 23\right)\right)-16$
$\sqrt[23]{\left(\frac{-9.877709 \times 10^{81}}{8.9621493 \times 10^{47}}\right)^{2}}-16$
896.42071...

Percent decrease:

$$
\begin{aligned}
& \sqrt[23]{\left(-\frac{9.877709 \times 10^{81}}{8.9621493 \times 10^{47}}\right)^{2}}-16=896.421 \text { is } 1.75358 \\
& \text { \% smaller than } \sqrt[23]{\left(-\frac{9.877709 \times 10^{81}}{8.9621493 \times 10^{47}}\right)^{2}}=912.421
\end{aligned}
$$

The result 896,42 is equal to the value of rest mass of the Kaon $896.00 \pm 0.025$ and:
$\left(\ln \left(-\left(-9.877709 * 10^{\wedge} 81\right) *\left(8.9621493 * 10^{\wedge} 47\right)\right)\right)^{\wedge} 1 / 3$
$\sqrt[3]{\log \left(-\left(-9.877709 \times 10^{81}\right)\left(8.9621493 \times 10^{47}\right)\right)}$
Result:

### 6.688479367...

(we observe that 6.68847 is very near to the value of $\mathrm{G}_{\mathrm{N}}=6.70872$ that is the gravitational constant 4 d of string theory)
and:
$\left(\ln \left(-\left(-9.877709 * 10^{\wedge} 81\right) *(8.9621493 * 10 \wedge 47)\right)\right)^{\wedge} 1 / 12$
$\sqrt[12]{\log \left(-\left(-9.877709 \times 10^{81}\right)\left(8.9621493 \times 10^{47}\right)\right)}$

Result:
1.6081695990...

The result 1,6081695 is very near to the value of the electric charge of the positron. Also:

2 * (-(-9.877709 * $\left.\left.10^{\wedge} 81\right)+\left(8.9621493 * 10^{\wedge} 47\right)\right)^{\wedge} 1 /(139 * 3) \quad$ where $139 * 3=417$, is the difference between the value of "glueball" and the baryon Xi 1321.71 $\pm 0.07$. Indeed: $1739-1321.71=417.29 \approx 417$

Input interpretation:
$2^{139 \times 3} \sqrt{-\left(-9.877709 \times 10^{81}\right)+8.9621493 \times 10^{47}}$
Result:
3.145284007...

This result is a good approximation to $\pi$
With the difference between the value of the strange D meson $2112.3 \pm 0.5$ and the value of "glueball" $1738 \pm 30$, we have that: $2112.3-1738=374.3$ that is about the value of the root: 375 . Thence, we have:

$$
\left(-\left(-9.877709 * 10^{\wedge} 81\right)+\left(8.9621493 * 10^{\wedge} 47\right)\right)^{\wedge} 1 /(375)
$$

Input interpretation:
$\sqrt[375]{-\left(-9.877709 \times 10^{81}\right)+8.9621493 \times 10^{47}}$

## Result:

1.654445314...

With the value of the Omega meson multiplied for $1 / 2,(782.65 \pm 0.12)^{*} 1 / 2$, we obtain:
$\left(-\left(-9.877709 * 10^{\wedge} 81\right)+\left(8.9621493 * 10^{\wedge} 47\right)\right)^{\wedge} 1 /(782.65 / 2)$

Input interpretation:

$$
\sqrt[\frac{782.65}{2}]{-\left(-9.877709 \times 10^{81}\right)+8.9621493 \times 10^{47}}
$$

Result:
1.62006...

The results 1,65444 and 1,62006 are very near to fourteenth root of the Ramanujan's class invariant and to the mass of proton and the electric charge of the positron.

In conclusion, multiplying the two results and calculating the following integral, we obtain:

10 integrate $\ln \left(-\left(-9.877709 * 10^{\wedge} 81\right) *\left(8.9621493 * 10^{\wedge} 47\right)\right) \mathrm{x}$
$10 \int^{\text {Input interpretation: }} \log \left(-\left(-9.877709 \times 10^{81}\right)\left(8.9621493 \times 10^{47}\right)\right) x d x$
Result:
$1496.07 x^{2}$


Alternate form assuming x is real:
$1496.07 x^{2}+0$
Indefinite integral assuming all variables are real:
$498.69 x^{3}+$ constant
where 1496 is a value very near to the $f_{0}(1500)$ mass:

## $f_{0}(1500)$ MASS

VALUE (MeV) EVTS DOCUMENT ID TECN COMMENT

1504\# 6 OUR AVERAGE Error includes scale factor of 1.3. See the ideogram below.

| $1468{ }_{-15}^{+14}-73$ | 5.5 k | 1 ABLIKIM | 13N | BES3 | $e^{+} e^{-} \rightarrow J / \psi \rightarrow \gamma \eta \eta$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $1466 \pm 6 \pm 20$ |  | ABLIKIM | 06V | BES2 | $e^{+} e^{-} \rightarrow J / \psi \rightarrow \gamma \pi^{+} \pi^{-}$ |
| $1515 \pm 12$ |  | 2 BARBERIS | 00A |  | $450 p p \rightarrow p_{f} \eta \eta p_{s}$ |
| $1511 \pm 9$ |  | 2,3 BARBERIS | 00C |  | $450 p p \rightarrow p_{f} 4 \pi p_{s}$ |
| $1510 \pm 8$ |  | 2 BARBERIS | 00E |  | $450 p p \rightarrow p_{f} \eta \eta p_{s}$ |
| $1522 \pm 25$ |  | BERTIN | 98 | OBLX | 0.05-0.405 $\bar{n} p \rightarrow \pi^{+} \pi^{+} \pi^{-}$ |
| $1449 \pm 20$ |  | 2 BERTIN | 97C | OBLX | $0.0 \bar{p} p \rightarrow \pi^{+} \pi^{-} \pi^{0}$ |
| $1515 \pm 20$ |  | ABELE | 96B | CBAR | $0.0 \bar{p} p \rightarrow \pi^{0} K_{L}^{0} K_{L}^{0}$ |
| $1500 \pm 15$ |  | ${ }^{4}$ AMSLER | 95B | CBAR | $0.0 \bar{p} p \rightarrow 3 \pi^{0}$ |
| $1505 \pm 15$ |  | ${ }^{5}$ AMSLER | 95C | CBAR | $0.0 \bar{p} p \rightarrow \eta \eta \pi^{0}$ |

We have that:

$$
\left(\left(10^{\wedge} 3 *\left(\left(\left(-\left(-9.877709 * 10^{\wedge} 81\right) *\left(8.9621493 * 10^{\wedge} 47\right)\right)^{\wedge} 1 /(240)\right) \mathrm{x}\right)\right)\right)
$$

[^5]$10^{3}\left(\sqrt[240]{-\left(-9.877709 \times 10^{81}\right)\left(8.9621493 \times 10^{47}\right)} x\right)$
Result:
$3478.93 x$

Plot:


Geometric figure:
line
Alternate form assuming x is real:
$3478.93 x+0$

Property as a function:
Parity:
odd
Properties as a real function:
Domain:
$\mathbb{R}$ (all real numbers)

Range:
$\mathbb{R}$ (all real numbers)

Bijectivity:
bijective from its domain to $\mathbb{R}$
Parity:
odd
Derivative:
$\frac{d}{d x}(3478.93 x)=3478.93$
Indefinite integral:
$\int 10^{3}\left(\sqrt[240]{-\left(-9.877709 \times 10^{81}\right)\left(8.9621493 \times 10^{47}\right)} x\right) d x=1739.47 x^{2}+$ constant
1739.47

Definite integral after subtraction of diverging parts:
$\int_{0}^{\infty}(3478.93 x-3478.93 x) d x=0$

The result of indefinite integral is: 1739,47

And
$\left.\left(\left(10^{\wedge} 3 *\left(\left(\left(-\left(-9.877709 * 10^{\wedge} 81\right)+\left(8.9621493 * 10^{\wedge} 47\right)\right)\right)^{\wedge} 1 /(152)\right) \mathrm{x}\right)\right)\right)$
Input interpretation:
$10^{3}\left(\sqrt[152]{-\left(-9.877709 \times 10^{81}\right)+8.9621493 \times 10^{47}} x\right)$

Result:
$3462.89 x$

Plot:


Geometric figure:
line

Alternate form assuming x is real:
$3462.89 x+0$
Properties as a real function:
Domain:
$R$ (all real numbers)
Range:
R (all real numbers)
Bijectivity:
bijective from its domain to $\mathbb{R}$
Parity:
odd

Derivative:
$\frac{d}{d x}(3462.89 x)=3462.89$

Indefinite integral:
$\int 10^{3}\left(\sqrt[152]{-\left(-9.877709 \times 10^{81}\right)+8.9621493 \times 10^{47}} x\right) d x=1731.44 x^{2}+$ constant
1731.44

Definite integral after subtraction of diverging parts:
$\int_{0}^{\infty}(3462.89 x-3462.89 x) d x=0$

The result of indefinite integral is: 1731,44
We note that:
integrate $2 \mathrm{Pi} / 1728 \quad[-16+10 \wedge 3 *(((-(-9.877709 * 10 \wedge 81)+(8.9621493 *$ $\left.\left.\left.\left.10^{\wedge} 47\right)\right)^{\wedge} 1 / 152\right)\right)$ )]

$$
\int \frac{2 \pi\left(-16+10^{3} \sqrt[152]{\left.-\left(-9.877709 \times 10^{81}\right)+8.9621493 \times 10^{47}\right)}\right.}{1728} d x=
$$

The result 12.53 is practically equal to the value of black hole entropy 12.57
We calculate also the following double integral:
((integrate integrate $-16+10^{\wedge} 3 *\left(\left(\left(-\left(-9.877709 * 10^{\wedge} 81\right)+(8.9621493 *\right.\right.\right.$ $\left.\left.\left.\left.10^{\wedge} 47\right)\right)^{\wedge} 1 / 152\right)\right)$ ))

Input interpretation:
$\int\left(\int\left(-16+10^{3} \sqrt[152]{-\left(-9.877709 \times 10^{81}\right)+8.9621493 \times 10^{47}}\right) d x\right) d x$

Result:
$1723.44 x^{2}$

Indefinite integral assuming all variables are real:
$574.481 x^{3}+$ constant

$$
\begin{aligned}
& \iint 3446.89 d x d x=10828.7 R^{2} \\
& 2 x^{2}<R^{2} \\
& \text { Definite integral over a square of edge length } 2 \mathrm{~L} \text { : } \\
& \int_{-L}^{L} \int_{-L}^{L} 3446.89 d x d x=13787.6 L^{2}
\end{aligned}
$$

Result: 1723,44

Note that $1723.44 \quad 1739.47$ and 1731.44 are results that are practically in the range of the mass of the candidate "glueball" $\mathrm{f}_{0}(1710)$. Indeed, as we can see from the following Table, all the values highlighted in yellow and the average are very near, or equal, to the results of the various integrations.

## $f_{0}(1710)$ MASS

VALUE $(\mathrm{MeV})$ EVTS DOCUMENT ID TECN COMMENT

1723 $\pm 5$ OUR AVERAGE Error includes scale factor of 1.6. See the ideogram below.

| $1759 \pm 6$ | $\begin{aligned} & +14 \\ & -25 \end{aligned}$ | 5.5k | 1 ABLIKIM | 13 N | BES3 | $e^{+} e^{-} \rightarrow J / \psi \rightarrow \gamma \eta \eta$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1750 \pm 7$ | $\begin{array}{r} +29 \\ +18 \end{array}$ |  | UEHARA | 13 | BELL | $\gamma \gamma \rightarrow K_{S}^{0} K_{S}^{0}$ |
| $1701 \pm 5$ | + +9 $-\quad 2$ | 4k | ${ }^{2}$ CHEKANOV | 08 | ZEUS | $e p \rightarrow K_{S}^{0} K_{S}^{0} X$ |
| $1765 \pm 4$ | $\pm 13$ |  | ABLIKIM | 06 V | BES2 | $e^{+} e^{-} \rightarrow J / \psi \rightarrow \gamma \pi^{+} \pi^{-}$ |
| $1760 \pm 15$ | $\begin{aligned} & +15 \\ & -10 \end{aligned}$ |  | ${ }^{3}$ ABLIKIM | 05Q | BES2 | $\psi(2 S) \rightarrow \gamma \pi^{+} \pi^{-} K^{+} K^{-}$ |
| $1738 \pm 30$ |  |  | ABLIKIM | 04E | BES2 | $J / \psi \rightarrow \omega K^{+} K^{-}$ |
| $1740 \pm 4$ | +10 -25 |  | ${ }^{4} \mathrm{BAI}$ | 03G | BES | $J / \psi \rightarrow \gamma K \bar{K}$ |
| ${ }_{1740}+30$ |  |  | ${ }^{4} \mathrm{BAI}$ | 00A | BES | $J / \psi \rightarrow \gamma\left(\pi^{+} \pi^{-} \pi^{+} \pi^{-}\right)$ |
| $1698 \pm 18$ |  |  | ${ }^{5}$ BARBERIS | 00E |  | $450 \mathrm{pp} \rightarrow p_{f} \eta \eta p_{s}$ |
| $1710 \pm 12$ | $\pm 11$ |  | ${ }^{6}$ BARBERIS | 99D | OMEG | $450 p p \rightarrow K^{+} K^{-}, \pi^{+} \pi^{-}$ |
| $1710 \pm 25$ |  |  | ${ }^{7}$ FRENCH | 99 |  | $300 p p \rightarrow p_{f}\left(K^{+} K^{-}\right) p_{s}$ |
| $1707 \pm 10$ |  |  | ${ }^{8}$ AUGUSTIN | 88 | DM2 | $J / \psi \rightarrow \gamma K^{+} K^{-}, K_{S}^{0} K_{S}^{\sigma}$ |
| $1698 \pm 15$ |  |  | ${ }^{8}$ AUGUSTIN | 87 | DM2 | $J / \psi \rightarrow \gamma \pi^{+} \pi^{-}$ |
| $1720 \pm 10$ | $\pm 10$ |  | ${ }^{9}$ BALTRUSAIT.. | 87 | MRK3 | $J / \psi \rightarrow \gamma K^{+} K^{-}$ |
| $1742 \pm 15$ |  |  | ${ }^{8}$ WILLIAMS | 84 | MPSF | $200 \pi^{-} N \rightarrow 2 K_{S}^{0} X$ |
| $1670 \pm 50$ |  |  | BLOOM | 83 | CBAL | $\boldsymbol{J} / \psi \rightarrow \gamma 2 \eta$ |

-     - We do not use the following data for averages, fits, limits, etc.

| $1744 \pm 7 \pm 5$ | 381 | 10,11 DOBBS | 15 |  | $\boldsymbol{J} / \psi \rightarrow \gamma \pi^{+} \pi$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $1705 \pm 11 \pm 5$ | 237 | 10,11 DOBBS | 15 |  | $\psi(2 S) \rightarrow \gamma \pi^{+} \pi^{-}$ |
| $1706 \pm 4 \pm 5$ | 1.0 k | 10,11 DOBBS | 15 |  | $J / \psi \rightarrow \gamma K^{+} K^{-}$ |
| $1690 \pm 8 \pm 3$ | 349 | 10,11 DOBBS | 15 |  | $\psi(2 S) \rightarrow \gamma K^{+} K^{-}$ |
| $1750 \pm 13$ |  | AMSLER | 06 | CBAR | $1.64 \bar{p} p \rightarrow K^{+} K^{-} \pi^{0}$ |
| $1747 \pm 5$ | 80k | 12,13 UMAN | 06 | E835 | $5.2 \bar{p} p \rightarrow \eta \eta \pi^{0}$ |
| $1776 \pm 15$ |  | VLADIMIRSK... |  | SPEC | $40 \pi^{-} p \rightarrow K_{S}^{0} K_{S}^{0}{ }^{n}$ |
| $1790 \pm 40$ |  | ${ }^{3}$ ABLIKIM | 05 | BES2 | $J / \psi \rightarrow \phi \pi^{+} \pi^{-}$ |
| $1670 \pm 20$ |  | 12 BINON | 05 | GAMS | $33 \pi^{-} p \rightarrow \eta \eta n$ |
| $1726 \pm 7$ | 74 | 13 CHEKANOV | 04 | ZEUS | $e p \rightarrow K_{S}^{0} K_{S}^{0} X$ |
| $1732+15$ |  | 14 ANISOVICH | 03 | RVUE |  |
| $1682 \pm 16$ |  | TIKHOMIROV |  | SPEC | $40.0 \pi^{-} \mathrm{C} \rightarrow K_{S}^{0} K_{S}^{0} K_{L}^{0} \mathrm{X}$ |
| $1670 \pm 26$ | 3.6k | 4,15 NICHITIU | 02 | OBLX |  |
| $1770 \pm 12$ |  | 16,17 ANISOVICH | 99 B | SPEC | 0.6-1.2 $p \bar{P} \rightarrow \eta \eta \pi^{0}$ |


| $1730 \pm 15$ |  | ${ }^{4}$ BARBERIS | 99 | OMEG | $450 p p \rightarrow p_{s} p_{f} K^{+} K^{-}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $1750 \pm 20$ |  | ${ }^{4}$ BARBERIS | 99B | OMEG | $450 p p \rightarrow p_{s} p_{f} \pi^{+} \pi^{-}$ |
| $1750 \pm 30$ |  | 18 ANISOVICH | 98B | RVUE | Compilation |
| $1720 \pm 39$ |  | BAI | 98H | BES | $J / \psi \rightarrow \gamma \pi^{0} \pi^{0}$ |
| $1775 \pm 1.5$ | 57 | 19 BARKOV | 98 |  | $\pi^{-} p \rightarrow K_{S}^{0} K_{S}^{0} n$ |
| $1690 \pm 11$ |  | 20 ABREU | 96C | DLPH | $Z^{0} \rightarrow K^{+} K^{-}+X$ |
| $1696 \pm 5 \quad \pm \begin{array}{r}\text { - }\end{array}$ |  | ${ }^{9} \mathrm{BAI}$ | 96C | BES | $J / \psi \rightarrow \gamma K^{+} K^{-}$ |
| $1781 \pm 8 \quad+10$ |  | ${ }^{4} \mathrm{BAI}$ | 96C | BES | $J / \psi \rightarrow \gamma K^{+} K^{-}$ |
| $1768 \pm 14$ |  | BALOSHIN | 95 | SPEC | $40 \pi^{-} \mathrm{C} \rightarrow K_{S}^{0} K_{S}^{0} \mathrm{X}$ |
| $1750 \pm 15$ |  | 21 BUGG | 95 | MRK3 | $J / \psi \rightarrow \gamma \pi^{+} \pi^{-} \pi^{+} \pi^{-}$ |
| $1620 \pm 16$ |  | ${ }^{9}$ BUGG | 95 | MRK3 | $J / \psi \rightarrow \gamma \pi^{+} \pi^{-} \pi^{+} \pi^{-}$ |
| $1748 \pm 10$ |  | ${ }^{8}$ ARMSTRONG | 93C | E760 | $\bar{p} p \rightarrow \pi^{0} \eta \eta \rightarrow 6 \gamma$ |
| $\sim 1750$ |  | BREAKSTONE | E93 | SFM | $p p \rightarrow p p \pi^{+} \pi^{-} \pi^{+} \pi^{-}$ |
| $1744 \pm 15$ |  | 22 ALDE | 92D | GAM2 | $38 \pi^{-} p \rightarrow \eta \eta n$ |
| $1713 \pm 10$ |  | 23 ARMSTRONG | 89D | OMEG | $300 \mathrm{pp} \rightarrow \mathrm{ppK} \mathrm{K}^{+} \mathrm{K}^{-}$ |
| $1706 \pm 10$ |  | 23 ARMSTRONG | 89D | OMEG | $300 p p \rightarrow p p K_{S}^{0} K_{S}^{0}$ |
| $1700 \pm 15$ |  | ${ }^{9}$ BOLONKIN | 88 | SPEC | $40 \pi^{-} p \rightarrow K_{S}^{0} K_{S}^{0} n$ |
| $1720 \pm 60$ |  | 4 BOLONKIN | 88 | SPEC | $40 \pi^{-} p \rightarrow K_{S}^{0} K_{S}^{0} n$ |
| $1638 \pm 10$ |  | 24 FALVARD | 88 | DM2 | $J / \psi \rightarrow \phi K^{+} K^{-}, K_{S}^{0} K_{S}^{0}$ |
| $1690 \pm 4$ |  | 25 FALVARD | 88 | DM2 | $J / \psi \rightarrow \phi K^{+} K^{-}, K_{S}^{0} K_{S}^{O}$ |
| $1755 \pm 8$ |  | 26 ALDE | 86C | GAM2 | $38 \pi^{-} p \rightarrow n 2 \eta$ |
| $1730 \pm 2$ |  | 27 LONGACRE | 86 | RVUE | $22 \pi^{-} p \rightarrow n 2 K_{S}^{0}$ |
| $1650 \pm 50$ |  | BURKE | 82 | MRK2 | $J / \psi \rightarrow \gamma 2 \rho$ |
| $1640 \pm 50$ |  | 28,29 EDWARDS | 82D | CBAL | $J / \psi \rightarrow \gamma 2 \eta$ |
| $1730 \pm 10 \pm 20$ |  | 30 ETKIN | 82C | MPS | $23 \pi^{-} p \rightarrow n 2 K_{S}^{0}$ |

Now:

| Excitation of glueball $(n)$ | $n=0$ | $n=1$ | $n=2$ | $n=3$ | $n=4$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Glueball mass $M_{E}^{(n)}$ | 0.901 | 2.285 | 3.240 | 4.149 | 5.041 |
| Glueball mass $M_{D, T}^{(n)}$ | 1.567 | 2.485 | 3.373 | 4.252 | 5.124 |
| The coefficients | $n=0$ | $n=1$ | $n=2$ | $n=3$ | $n=4$ |
| $\mathcal{C}_{E}$ | 144.545 | 114.871 | 131.283 | 146.259 | 157.832 |
| $\mathcal{C}_{D}$ | 29.772 | 36.583 | 42.237 | 47.220 | 51.724 |
| $\mathcal{C}_{T}$ | 72.927 | 89.609 | 103.46 | 115.664 | 126.696 |

We note that dividing the two numbers, 3.240 for $\mathrm{M}_{\mathrm{E}} \mathrm{n}=2$ and 2.485 for $\mathrm{M}_{\mathrm{D}} \mathrm{n}=1$, that are the mass of the dilatonic and exotic scalar glueball, we obtain $3.240 / 2.485=$

1,303822937 which is very near to the mass ratio of the glueball candidates. Indeed, we have: $1.723 / 1.504=1,1456117$ that with the recent value of $f_{0}(1710)$ is:
$1.739 / 1.504=1,15625$. If we take 5.041 for $M_{E} n=4$ and 4.252 for $M_{D} n=3$, we obtain: $5.041 / 4.252=1,18555973$ very near to 1,15625 . We observe, also that 1,18555 is practically equal to $(1,08643)^{2}=1,18033$ where 1,08643 is the "new Ramanujan's constant".

Now:

|  | I | II | III | IV |
| :---: | :---: | :---: | :---: | :---: |
| $\Gamma$ | $\left(0.013922 \lambda^{-1}\right.$ | $0.06628 \lambda^{-1}$ | $0.01785 \lambda^{-1}$ | $0.1046 \lambda^{-1}$ |
|  | V | VI | VII | VIII |
| $\Gamma$ | $0.1671 \lambda^{-1}$ | $0.2093 \lambda^{-1}$ | $0.6316 \lambda^{-1}$ | $1.0527 \lambda^{-1}$ |

Table 3: The corresponding decay rates in the units of $m_{H}$ to the transitions in (3.7) by setting $l=0, N_{Q}=1, N_{c}=3, N_{f}=2$.
and this result would be in agreement with the previous discussion in [19]. Therefore we could conclude that only the decay process VIII in (3.7) might be realistic. This transition describes the decay of the baryonic meson consisted of one heavy- and one light- flavoured quark. So while the identification of the other transitions might be less clear, the transition VIII could be interpreted as the decay of the baryonic B-meson involving the glueball candidate $f_{0}(1710)$ as discussed e.g. in $[8,9,10]$ since the corresponding quantum numbers of the states could be identified.

From Table 3 we have the decay process VIII that is:

$$
1.0527 \lambda^{-1}=1.0527 *-4,313502611 * 10^{31}=-4,5408241985997 * 10^{31}
$$

Now, we have that:
The rest mass of baryon Xi is $1314.86 \pm 0.20$. We have that:
integrate $\left.1 /\left((1729)^{*} 10^{\wedge} 25\right)\right)\left[\left(-4.5408241985997 * 10^{\wedge} 31\right)\right] \mathrm{x}$
Indefinite integral:
$\int-\frac{\left(4.5408241985997 \times 10^{31}\right) x}{1729 \times 10^{25}} d x=-1313.1359741468 x^{2}+$ constant

[^6]

Or:
integrate $1 /\left(10^{\wedge} 33\right)\left[\left(-4.5408241985997 * 10^{\wedge} 31\right)\right] x,\left[1729 /(1.63721868)^{\wedge} 4,0\right]$
Definite integral:
$\int_{\frac{1729}{1.63721868^{4}}}^{0}-\frac{\left(4.5408241985997 \times 10^{31}\right) x}{10^{33}} d x=1314.74$
The result 1314.74 is equal to the rest mass of baryon Xi that is $1314.86 \pm 0.20$


Riemann sums

$$
\text { left sum } \quad 1314.74+\frac{1314.74}{n}=1314.74+\frac{1314.74}{n}+O\left(\left(\frac{1}{n}\right)^{2}\right)
$$

(assuming subintervals of equal length)
Indefinite integral:
$\int-\frac{\left(4.5408241985997 \times 10^{31}\right) x}{10^{33}} d x=-0.022704120992999 x^{2}+$ constant

The value 0,02270412 is a good approximation to the value of the lowest nonvanishing gluon condensate, that is $\approx 0.0214 \mathrm{GeV}^{4}$

And:
integrate $1 /\left(10^{\wedge} 33\right)\left[\left(-4.5408241985997 * 10^{\wedge} 31\right)\right] \mathrm{x},[1728 /(\ln 36), 0]$

$$
\int_{\frac{1728}{\log (36)}}^{0}-\frac{\left(4.5408241985997 \times 10^{31}\right) x}{10^{33}} d x=5279.2564694966
$$

The result 5279.256 is practically equal to the rest mass of B meson, that is $5279.15 \pm 0.31 \quad 5279.53 \pm 33$

Visual representation of the integral:


Riemann sums:

$$
\begin{array}{l|l}
\text { left sum } & \frac{5279.256469497}{n}+5279.2564694966= \\
& 5279.2564694966+\frac{5279.256469497}{n}+O\left(\left(\frac{1}{n}\right)^{2}\right)
\end{array}
$$

## (assuming subintervals of equal length)

Indefinite integral:
$\int-\frac{\left(4.5408241985997 \times 10^{31}\right) x}{10^{33}} d x=-0.022704120992999 x^{2}+$ constant

Also here, the value 0,02270412 is a good approximation to the value of the lowest non-vanishing gluon condensate, that is $\approx 0.0214 \mathrm{GeV}^{4}$

We have that:
integrate $1 /\left(\left(1729 * 3 *(\operatorname{sqrt}(5)+5) / 2 * 10^{\wedge} 26\right)\right)\left[\left(-4.5408241985997 * 10^{\wedge} 31\right)\right] x$

Indefinite integral:
$\int-\frac{\left(4.5408241985997 \times 10^{31}\right) x}{\frac{1729}{2} \times 3(\sqrt{5}+5) 10^{26}} d x=-12.098061896138 x^{2}+$ constant
Open code


The result -12.098 is very near to the value of black hole entropy 12.19 , but with the sign minus

Now:

The value of $\mathcal{Q}$ corresponds to the situation of a baryonic bound state consisting of $N_{Q}$ heavy flavoured quarks. The eigenfunctions and mass spectrum of (C-7) can be evaluated by solving its Schrodinger equation, respectively they are obtained as ${ }^{8}$,

$$
\begin{align*}
\psi\left(y_{I}\right) & =R(\rho) T^{(l)}\left(a_{I}\right), R(\rho)=e^{-\frac{m_{y} \cup_{\rho}}{2} \rho^{z}} \rho^{i} \text { Hypergeometric } F_{1}\left(-n_{\rho}, \tilde{l}+2 ; m_{y} \omega_{\rho} \rho^{2}\right), \\
F\left(l, n_{\rho}, n_{z}\right) & =\omega_{\rho}\left(\tilde{l}+2 n_{\rho}+2\right)=\sqrt{\frac{(l+1)^{2}}{6}+\frac{640}{3} a^{2} \pi^{4} Q^{2}}+\frac{2\left(n_{\rho}+n_{z}\right)+2}{\sqrt{6}} . \tag{C-9}
\end{align*}
$$

Notice that $T^{(l)}\left(a_{I}\right)$ satisfies $\nabla_{S^{3}}^{2} T^{(l)}=-l(l+2) T^{(l)}$ which is the function of the spherical part because $H_{y}$ can be written with the radial coordinate $\rho$ as,

$$
\begin{equation*}
H_{y}=-\frac{1}{2 m_{y}}\left\lceil\frac{1}{\rho^{3}} \partial_{\rho}\left(\rho^{3} \partial_{\rho}\right)+\frac{1}{\rho^{2}}\left(\nabla_{S^{3}}^{2}-2 m_{y} \mathcal{Q}\right)\right\rceil+\frac{1}{2} m_{y} \omega_{r}^{2} \rho^{2} \tag{C-10}
\end{equation*}
$$

For $\mathrm{l}=0$ and $\mathrm{Q}=-3,29867$ we obtain:
$\left(\left(\operatorname{sqrt}\left(640 / 3 *\left(1 /\left(216 \mathrm{Pi}^{\wedge} 3\right)\right)^{\wedge} 2^{*} \mathrm{Pi}^{\wedge} 4^{*}(-3.29867)^{\wedge} 2\right)\right)+((2 *(3+2)+2)) /(\operatorname{sqrt}(6))\right.$
Input interpretation:
$\sqrt{\frac{640}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} \pi^{4}(-3.29867)^{2}+\frac{2(3+2)+2}{\sqrt{6}}}$
4.9699805...

$$
\begin{aligned}
& \sqrt{\frac{1}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} 640 \pi^{4}(-3.29867)^{2}}+\frac{2(3+2)+2}{\sqrt{6}}= \\
& \frac{12+{\sqrt{z_{0}}}^{2} \sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}\left(6-z_{0} k^{k_{1}}\left(\frac{0.0497541}{\pi^{2}}-z_{0}\right)^{k_{2}} z_{0}^{-k_{1}-k_{2}}\right.}{k_{1}!k_{2}!}}{\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(6-z_{0}\right)^{k} z_{0}^{-k}}{k!}}
\end{aligned}
$$

$$
\text { for } \operatorname{not}\left(\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right)\right)
$$

$$
\begin{aligned}
& \sqrt{\frac{1}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} 640 \pi^{4}(-3.29867)^{2}+\frac{2(3+2)+2}{\sqrt{6}}=} \\
& \left(12+\exp \left(i \pi\left\lfloor\left.\frac{\arg (6-x)}{2 \pi} \right\rvert\,\right) \exp \left(i \pi\left[\frac{\arg \left(\frac{0.0497541}{\pi^{2}}-x\right)}{2 \pi}\right]\right) \sqrt{x}^{2}\right.\right. \\
& \left.\sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}}(6-x)^{k_{1}}\left(\frac{0.0497541}{\pi^{2}}-x\right)^{k_{2}} x^{-k_{1}-k_{2}}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}}{k_{1}!k_{2}!}\right) / \\
& \left(\exp \left(i \pi\left\lfloor\frac{\arg (6-x)}{2 \pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^{k}(6-x)^{k} x^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right) \text { for }(x \in \mathbb{R} \text { and } x<0)
\end{aligned}
$$

$$
\sqrt{\frac{1}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} 640 \pi^{4}(-3.29867)^{2}}+\frac{2(3+2)+2}{\sqrt{6}}=
$$

$$
\left(\left(\frac{1}{z_{0}}\right)^{-1 / 2\left\lfloor\arg \left(6-z_{0}\right) /(2 \pi)\right\rfloor} z_{0}^{-1 / 2-1 / 2\left\lfloor\arg \left(6-z_{0}\right) /(2 \pi)\right\rfloor}\right.
$$

$$
\left(12+\left(\frac{1}{z_{0}}\right)^{1 / 2\left\lfloor\arg \left(6-z_{0}\right) /(2 \pi)\right\rfloor+1 / 2}\left[\arg \left(\frac{0.0497541}{\pi^{2}}-z_{0}\right) /(2 \pi)\right]\right.
$$

$$
z_{0}^{1+1 / 2\left\lfloor\arg \left(6-z_{0}\right) /(2 \pi)\right\rfloor+1 / 2}\left\lfloor\arg \left(\frac{0.0497541}{\pi^{2}}-z_{0}\right) /(2 \pi)\right]
$$

$$
\left.\sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}\left(6-z_{0}\right)^{k_{1}}\left(\frac{0.0497541}{\pi^{2}}-z_{0}\right)^{k_{2}} z_{0}^{-k_{1}-k_{2}}}{k_{1}!k_{2}!}\right) / /
$$

$$
\left(\sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(6-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right)
$$

For $\mathrm{l}=1,616 * 10^{-35}$ and $\mathrm{Q}=-3,29867$ we obtain:
$\left(\left(\operatorname{sqrt}\left(1 / 6+640 / 3 *\left(1 /\left(216 \mathrm{Pi}^{\wedge} 3\right)\right)^{\wedge} 2 * \mathrm{Pi}^{\wedge} 4^{*}(-3.29867)^{\wedge} 2\right)\right)+\left(\left(2^{*}(3+2)+2\right)\right) /(\operatorname{sqrt}(6))\right.$

Input interpretation:
$\sqrt{\frac{1}{6}+\frac{640}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} \pi^{4}(-3.29867)^{2}}+\frac{2(3+2)+2}{\sqrt{6}}$

Result:
5.31335590...

Series representations:

$$
\begin{aligned}
& \sqrt{\frac{1}{6}+\frac{1}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} 640 \pi^{4}(-3.29867)^{2}+\frac{2(3+2)+2}{\sqrt{6}}=} \\
& \frac{12+{\sqrt{z_{0}}}^{2} \sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}\left(6-z_{0} k^{k_{1}}\left(\frac{1}{6}+\frac{0.0497541}{\pi^{2}}-z_{0}\right)^{k_{2}} z_{0}^{-k_{1}-k_{2}}\right.}}{k_{1}!k_{2}!}}{\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(6-z_{0}\right)^{k} z_{0}^{-k}}{k!}}
\end{aligned}
$$

$$
\text { for } \operatorname{not}\left(\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right)\right)
$$

$$
\begin{aligned}
& \sqrt{\frac{1}{6}+\frac{1}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} 640 \pi^{4}(-3.29867)^{2}}+\frac{2(3+2)+2}{\sqrt{6}}= \\
& \left.12+\exp \left(i \pi\left[\frac{\arg (6-x)}{2 \pi}\right]\right) \exp \left(i \pi \left\lvert\, \frac{\arg \left(\frac{1}{6}+\frac{0.0497541}{\pi^{2}}-x\right)}{2 \pi}\right.\right)\right) \sqrt{x}^{2} \\
& \left.\sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}}(6-x)^{k_{1}}\left(\frac{1}{6}+\frac{0.0497541}{\pi^{2}}-x\right)^{k_{2}} x^{-k_{1}-k_{2}}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}}{k_{1}!k_{2}!}\right) / \\
& \left(\exp \left(i \pi\left[\frac{\arg (6-x)}{2 \pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^{k}(6-x)^{k} x^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right) \text { for }(x \in \mathbb{R} \text { and } x<0)
\end{aligned}
$$

$$
\begin{aligned}
& \sqrt{\frac{1}{6}+\frac{1}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} 640 \pi^{4}(-3.29867)^{2}}+\frac{2(3+2)+2}{\sqrt{6}}= \\
& \left(\left(\frac{1}{z_{0}}\right)^{-1 / 2\left\lfloor\arg \left(6-z_{0}\right) /(2 \pi)\right\rfloor} z_{0}^{\left.-1 / 2-1 / 2\left\lfloor\arg \left(6-z_{0}\right)\right)(2 \pi)\right\rfloor}\right. \\
& \left(12+\left(\frac{1}{z_{0}}\right)^{1 / 2\left\lfloor\arg \left(6-z_{0}\right) /(2 \pi)+1 / 2\right.}\left\lfloor\arg \left(\frac{1}{6}+\frac{0.0497541}{\pi^{2}}-z_{0}\right) /(2 \pi)\right]\right. \\
& \left.z_{0}^{\left.1+1 / 2\left\lfloor\arg \left(6-z_{0}\right)\right)(2 \pi)\right\rfloor+1 / 2} \operatorname{ang}^{1}\left(\frac{1}{6}+\frac{0.0497541}{\pi^{2}}-z_{0}\right) /(2 \pi)\right] \\
& \sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}\left(6-z_{0}\right)^{k_{1}}\left(\frac{1}{6}+\frac{0.0497541}{\pi^{2}}-z_{0}\right)^{k_{2}} z_{0}^{-k_{1}-k_{2}}}{k_{1}!k_{2}!} \\
& ) / / \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(6-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right)
\end{aligned}
$$

For $1=1,616 * 10^{-35}$ and $\mathrm{Q}=-54,192473$ we obtain:
$\left(\left(\operatorname{sqrt}\left(1 / 6+640 / 3^{*}\left(1 /\left(216 \mathrm{Pi}^{\wedge} 3\right)\right)^{\wedge} 2^{*} \mathrm{Pi}^{\wedge} 4^{*}(-54.192473)^{\wedge} 2\right)\right)+\left(\left(2^{*}(3+2)+2\right)\right) /(\operatorname{sqrt}(6))\right.$
Input interpretation:
$\sqrt{\frac{1}{6}+\frac{640}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} \pi^{4}(-54.192473)^{2}+\frac{2(3+2)+2}{\sqrt{6}}}$
Result:
$6.13480446 \ldots$
Series representations:

$$
\begin{aligned}
& \sqrt{\frac{1}{6}+\frac{1}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} 640 \pi^{4}(-54.1925)^{2}+\frac{2(3+2)+2}{\sqrt{6}}}= \\
& \frac{12+{\sqrt{z_{0}}}^{2} \sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}\left(6-z_{0}\right)^{k_{1}}\left(\frac{1}{6}+\frac{13.4286}{\pi^{2}}-z_{0}\right)^{k_{2}} z_{0}^{-k_{1}-k_{2}}}{k_{1}!k_{2}!}}{\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(6-z_{0}\right)^{k} z_{0}^{-k}}{k!}}
\end{aligned}
$$

$$
\text { for } \operatorname{not}\left(\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right)\right)
$$

$$
\begin{aligned}
& \sqrt{\frac{1}{6}+\frac{1}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} 640 \pi^{4}(-54.1925)^{2}}+\frac{2(3+2)+2}{\sqrt{6}}= \\
& \left(12+\exp \left(i \pi\left\lfloor\frac{\arg (6-x)}{2 \pi}\right]\right) \exp \left(i \pi\left[\frac{\arg \left(\frac{1}{6}+\frac{13.4286}{\pi^{2}}-x\right)}{2 \pi}\right]\right) \sqrt{x}^{2}\right. \\
& \left.\sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}}(6-x)^{k_{1}}\left(\frac{1}{6}+\frac{13.4286}{\pi^{2}}-x\right)^{k_{2}} x^{-k_{1}-k_{2}}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}}{k_{1}!k_{2}!}\right) / \\
& \left(\exp \left(i \pi\left\lfloor\frac{\arg (6-x)}{2 \pi}\right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^{k}(6-x)^{k} x^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right) \text { for }(x \in \mathbb{R} \text { and } x<0) \\
& \sqrt{\frac{1}{6}+\frac{1}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} 640 \pi^{4}(-54.1925)^{2}}+\frac{2(3+2)+2}{\sqrt{6}}= \\
& \left(\left(\frac{1}{z_{0}}\right)^{-1 / 2\left\lfloor\arg \left(6-z_{0}\right) /(2 \pi)\right\rfloor} z_{0}^{-1 / 2-1 / 2\left\lfloor\operatorname{agg}\left(6-z_{0}\right) /(2 \pi)\right\rfloor}\right. \\
& \left(\left.12+\left(\frac{1}{z_{0}}\right)^{1 / 2\left\lfloor\arg \left(6-z_{0}\right) /(2 \pi)\right)+1 / 2} \right\rvert\, \arg \left(\frac{1}{6}+\frac{13.4286}{\pi^{2}}-z_{0}\right) /(2 \pi)\right] \\
& \left.z_{0}^{1+1 / 2\left\lfloor\arg \left(6-z_{0}\right) /(2 \pi)+1 / 2\right.} \arg \left(\frac{1}{6}+\frac{13.4286}{\pi^{2}}-z_{0}\right) /(2 \pi)\right] \\
& \left.\left.\sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}\left(6-z_{0}\right)^{k_{1}}\left(\frac{1}{6}+\frac{13.4286}{\pi^{2}}-z_{0}\right)^{k_{2}} z_{0}^{-k_{1}-k_{2}}}{k_{1}!k_{2}!}\right)\right) \\
& /\left(\sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(6-z_{0}\right)^{k} z_{0}^{k}}{k!}\right)
\end{aligned}
$$

For $1=1,616 * 10^{-35}$ and $\mathrm{Q}=50,893800$ we obtain:
$\left(\left(\operatorname{sqrt}\left(1 / 6+640 / 3 *\left(1 /\left(216 \mathrm{Pi}^{\wedge} 3\right)\right)^{\wedge} 2 * \mathrm{Pi}^{\wedge} 4^{*}(50.893800)^{\wedge} 2\right)\right)+((2 *(3+2)+2)) /(\operatorname{sqrt}(6))\right.$

$$
\sqrt{\frac{1}{6}+\frac{640}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} \pi^{4} \times 50.893800^{2}}+\frac{2(3+2)+2}{\sqrt{6}}
$$

Series representations

$$
\begin{aligned}
& \sqrt{\frac{1}{6}+\frac{1}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} 640 \pi^{4} 50.8938^{2}+\frac{2(3+2)+2}{\sqrt{6}}=} \\
& \frac{12+{\sqrt{z_{0}}}^{2} \sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}\left(6-z_{0}\right)^{k_{1}}\left(\frac{1}{6}+\frac{11.8435}{\pi^{2}}-z_{0}\right)^{k_{2}} z_{0}^{-k_{1}-k_{2}}}{k_{1}!k_{2}!}}{\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(6-z_{0}\right)^{k} z_{0}^{-k}}{k!}}
\end{aligned}
$$

$$
\text { for } \operatorname{not}\left(\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right)\right)
$$

$$
\begin{aligned}
& \sqrt{\frac{1}{6}+\frac{1}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} 640 \pi^{4} 50.8938^{2}}+\frac{2(3+2)+2}{\sqrt{6}}= \\
& \left(12+\exp \left(i \pi \left\lvert\, \frac{\arg (6-x)}{2 \pi}\right.\right]\right) \exp \left(i \pi\left[\frac{\arg \left(\frac{1}{6}+\frac{11.8435}{\pi^{2}}-x\right)}{2 \pi}\right]\right) \sqrt{x}^{2} \\
& \left(\sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}}(6-x)^{k_{1}}\left(\frac{1}{6}+\frac{11.8435}{\pi^{2}}-x\right)^{k_{2}} x^{-k_{1}-k_{2}}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}}{k_{1}!k_{2}!}\right) / \\
& \left(\exp \left(i \pi\left\lfloor\frac{\arg (6-x)}{2 \pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^{k}(6-x)^{k} x^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right) \text { for }(x \in \mathbb{R} \text { and } x<0)
\end{aligned}
$$

$$
\sqrt{\frac{1}{6}+\frac{1}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} 640 \pi^{4} 50.8938^{2}}+\frac{2(3+2)+2}{\sqrt{6}}=
$$

$$
\left(\left(\frac{1}{z_{0}}\right)^{-1 / 2\left\lfloor\arg \left(6-z_{0}\right) /(2 \pi)\right\rfloor} z_{0}^{-1 / 2-1 / 2\left\lfloor\arg \left(6-z_{0}\right) /(2 \pi)\right\rfloor}\right.
$$

$$
\left(12+\left(\frac{1}{z_{0}}\right)^{1 / 2\left\lfloor\arg \left(6-z_{0}\right) /(2 \pi)\right\rfloor+1 / 2\left[\arg \left(\frac{1}{6}+\frac{11.8435}{\pi^{2}}-z_{0}\right) /(2 \pi)\right]}\right.
$$

$$
\begin{aligned}
& 1+1 / 2\left\lfloor\arg \left(6-z_{0}\right) /(2 \pi)\right\rfloor+1 / 2\left\lfloor\arg \left(\frac{1}{6}+\frac{11.8435}{\pi^{2}}-z_{0}\right) /(2 \pi)\right] \\
& z_{0}
\end{aligned}
$$

$$
\left.\left.\sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}\left(6-z_{0}\right)^{k_{1}}\left(\frac{1}{6}+\frac{11.8435}{\pi^{2}}-z_{0}\right)^{k_{2}} z_{0}^{-k_{1}-k_{2}}}{k_{1}!k_{2}!}\right)\right)
$$

$$
/\left(\sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(6-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right)
$$

Now, we calculate the exp of the above expression for $1=1,616 * 10^{-35}$ and $\mathrm{Q}=-$ 54,192473 and multiplied it for $\pi$. We obtain:
$\mathrm{Pi} * \exp \left(\left(\operatorname{sqrt}\left(1 / 6+640 / 3 *\left(1 /\left(216 \mathrm{Pi}^{\wedge} 3\right)\right)^{\wedge} 2 * \mathrm{Pi}^{\wedge} 4^{*}(-54.192473)^{\wedge} 2\right)\right)+\right.$ $\left(\left(2^{*}(3+2)+2\right)\right) /(\operatorname{sqrt}(6))$
$\pi \exp \left(\sqrt{\frac{1}{6}+\frac{640}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} \pi^{4}(-54.192473)^{2}}+\frac{2(3+2)+2}{\sqrt{6}}\right)$
Result:
1450.3125...

Series representations:

$$
\begin{aligned}
& \pi \exp \left(\sqrt{\frac{1}{6}+\frac{1}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} 640 \pi^{4}(-54.1925)^{2}}+\frac{2(3+2)+2}{\sqrt{6}}\right)= \\
& \quad \pi \exp \left(\frac{12}{\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(6-z_{0}\right)^{k} z_{0}^{k}}{k!}}+\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(\frac{1}{6}+\frac{13.4286}{\pi^{2}}-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right) \\
& \text { for } \operatorname{not}\left(\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
& \pi \exp \left(\sqrt{\frac{1}{6}+\frac{1}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} 640 \pi^{4}(-54.1925)^{2}}+\frac{2(3+2)+2}{\sqrt{6}}\right)= \\
& \pi \exp \left(\frac{12}{\exp \left(i \pi\left\lfloor\frac{\arg (6-x)}{2 \pi}\right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^{k}(6-x)^{k} x^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}}+\right. \\
& \quad \exp \left(i \pi\left[\frac{\arg \left(\frac{1}{6}+\frac{13.4286}{\pi^{2}}-x\right)}{2 \pi}\right)\right] \sqrt{x} \\
& \left.\sum_{k=0}^{\infty} \frac{(-1)^{k}\left(\frac{1}{6}+\frac{13.4286}{\pi^{2}}-x\right)^{k} x^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right) \text { for }(x \in \mathbb{R} \text { and } x<0)
\end{aligned}
$$

$$
\begin{aligned}
& \pi \exp \left(\sqrt{\frac{1}{6}+\frac{1}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} 640 \pi^{4}(-54.1925)^{2}}+\frac{2(3+2)+2}{\sqrt{6}}\right)= \\
& \pi \exp \left(\frac{12\left(\frac{1}{z_{0}}\right)^{-1 / 2\left\lfloor\arg \left(6-z_{0}\right) /(2 \pi)\right\rfloor} z_{0}^{-1 / 2-1 / 2\left\lfloor\arg \left(6-z_{0}\right) /(2 \pi)\right\rfloor}}{\sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(6-z_{0}\right)^{k} z_{0}^{-k}}{k!}}+\left(\frac{1}{z_{0}}\right)^{1 / 2\left\lfloor\arg \left(\frac{1}{6}+\frac{13.4286}{\pi^{2}}-z_{0}\right) /(2 \pi)\right]}\right. \\
& \left.z_{0}^{1 / 2\left(1+\left[\arg \left(\frac{1}{6}+\frac{13.4286}{\pi^{2}}-z_{0}\right) /(2 \pi)\right]\right)} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(\frac{1}{6}+\frac{13.4286}{\pi^{2}}-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right) \\
& \pi \exp \left(\sqrt{\frac{1}{6}+\frac{1}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} 640 \pi^{4}(-54.1925)^{2}}+\frac{2(3+2)+2}{\sqrt{6}}\right)= \\
& \pi \exp \left(\frac{12\left(\frac{1}{z_{0}}\right)^{-1 / 2\left\lfloor\arg \left(6-z_{0}\right) /(2 \pi)\right\rfloor} z_{0}^{1 / 2\left(-1-\left\lfloor\arg \left(6-z_{0}\right) /(2 \pi)\right\rfloor\right)}}{\sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(6-z_{0}\right)^{k} z_{0}^{-k}}{k!}}+\left(\frac{1}{z_{0}}\right)^{1 / 2\left[\arg \left(\frac{1}{6}+\frac{13.4286}{\pi^{2}}-z_{0}\right) /(2 \pi)\right]}\right. \\
& \left.\left.z_{0}^{1 / 2}\left(1+\arg \left(\frac{1}{6}+\frac{13.4286}{\pi^{2}}-z_{0}\right) /(2 \pi)\right]\right) \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(\frac{1}{6}+\frac{13.4286}{\pi^{2}}-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right)
\end{aligned}
$$

From:
72. $\rho(1450)$ and $\rho(1700)$

Updated November 2015 by S. Eidelman (Novosibirsk), C. Hanhart (Juelich) and G. Venanzoni (Frascati).

This scenario with two overlapping resonances is supported by other data. Bisello [9] measured the pion form factor in the interval $1.35-2.4 \mathrm{GeV}$, and observed a deep minimum around 1.6 GeV . The best fit was obtained with the hypothesis of $\rho$-like resonances at 1420 and 1770 MeV , with widths of about 250 MeV . Antonelli [10] found that the $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \eta \pi+\pi-$ cross section is better fitted with two fully interfering Breit-Wigners, with parameters in fair agreement with those of [2] and [9]. These results can be considered as a confirmation of the $\rho(1450)$.

The result of the above expression is 1450,3125 that is practically equal to the mass of meson $\rho(1450)$. Indeed, as we can see from the next Table, all the values highlighted in yellow and the average are very near, or equal, to the results of the analyzed expression.

## $\rho(\mathbf{1 4 5 0})$ MASS

VALUE (MeV)

DOCUMENT ID

$1465 \pm 25$ OUR ESTIMATE This is only an educated guess; the error given is larger than the error on the average of the published values.

## $\eta \rho^{0}$ MODE

VALUE $(\mathrm{MeV})$ EVTS DOCUMENT ID TECN COMMENT

-     - We do not use the following data for averages, fits, limits, etc.

| $1500 \pm 10$ | 7.4k | ${ }^{1}$ ACHASOV | 18 | SND | $1.22-2.00 e^{+} e^{-} \rightarrow \eta \pi^{+} \pi^{-}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $1497+14$ |  | 2 AKHMETSHIN | 018 | CMD2 | $e^{+} e^{-} \rightarrow \eta \gamma$ |
| $1421 \pm 15$ |  | ${ }^{3}$ AKHMETSHIN | 00D | CMD2 | $e^{+} e^{-} \rightarrow \eta \pi^{+} \pi^{-}$ |
| $1470 \pm 20$ |  | ANTONELLI | 88 | DM2 | $e^{+} e^{-} \rightarrow \eta \pi^{+} \pi^{-}$ |
| $1446 \pm 10$ |  | FUKUI | 88 | SPEC | $8.95 \pi^{-} p \rightarrow \eta \pi^{+} \pi^{-} n$ |

${ }^{1}$ From the combined fit of AULCHENKO 15 and ACHASOV 18 in the model with the interfering $\rho(1450), \rho(1700)$ and $\rho(2150)$ with the parameters of the $\rho(1450)$ and $\rho(1700)$ floating and the mass and width of the $\rho(2150)$ fixed at 2155 MeV and 320 MeV , respectively. The phases of the resonances are $\pi, 0$ and $\pi$, respectively.
${ }^{2}$ Using the data of AKHMETSHIN 01B on $e^{+} e^{-} \rightarrow \eta \gamma$. AKHMETSHIN OOD and ANTONELLI 88 on $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \eta \pi^{+} \pi^{-}$.
${ }^{3}$ Using the data of ANTONELLI 88, DOLINSKY 91, and AKHMETSHIN 00D. The energyindependent width of the $\rho(1450)$ and $\rho(1700)$ mesons assumed.

## $\omega \pi$ MODE

VALUE (MeV) $\qquad$ DOCUMENT ID
TECN $\qquad$

-     - We do not use the following data for averages, fits, limits, etc. - -

| $1510 \pm 7$ | 10.2 k | ${ }^{1}$ ACHASOV | 16D | SND | $1.05-2.00 e^{+} e^{-}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $1544 \pm 22 \pm 41$ | 821 | 2 MATVIENKO | 15 | BELL | $\bar{B}^{0} \rightarrow D^{*+} \omega \pi^{-}$ |
| $1491 \pm 19$ | 7815 | ${ }^{3}$ ACHASOV | 13 | SND | 1.05-2.00 $e^{+} e^{-}$ |
| $1582 \pm 17 \pm 25$ | 2382 | ${ }^{4}$ AKHMETSHIN | 03B | CMD2 | $e^{+} e \rightarrow \pi^{0} \pi^{0} \gamma$ |
| $1349 \pm 25 \pm 5$ | 341 | ${ }^{5}$ ALEXANDER | 01B | CLE2 | $B \rightarrow D^{(*)} \omega \pi^{-}$ |
| $1523 \pm 10$ |  | ${ }^{6}$ EDWARDS | 00A | CLE2 | $\tau^{-} \rightarrow \omega \pi^{-} \nu_{\tau}$ |
| $1463 \pm 25$ |  | 7 CLEGG | 94 | RVUE |  |
| 1250 |  | ${ }^{8}$ ASTON | 80c | OMEG | 20-70 $\gamma p \rightarrow \omega \pi^{0} p$ |
| $1290 \pm 40$ |  | ${ }^{8}$ BARBER | 80C | SPEC | $3-5 \gamma p \rightarrow \omega \pi^{0} p$ |

${ }^{1}$ From a phenomenological model based on vector meson dominance with interfering $\rho(770), \rho(1450)$, and $\rho(1700)$. The $\rho(1700)$ mass and width are fixed at 1720 MeV and 250 MeV , respectively. Systematic uncertainties not estimated. Supersedes ACHASOV 13.
${ }^{2}$ Using Breit-Wigner parameterization of the $\rho(1450)$ and assuming equal probabilities of the $\rho(1450) \rightarrow \pi \pi$ and $\rho(1450) \rightarrow \omega \pi$ decays.
${ }^{3}$ From a phenomenological model based on vector meson dominance with the interfering $\rho(1450)$ and $\rho(1700)$ and their widths fixed at 400 and 250 MeV , respectively. Systematic uncertainty not estimated.
${ }^{4}$ Using the data of AKHMETSHIN 03B and BISELLO 91B assuming the $\omega \pi^{0}$ and $\pi^{+} \pi^{-}$ mass dependence of the total width. $\rho(1700)$ mass and width fixed at 1700 MeV and 240 MeV , respectively.
${ }^{5}$ Using Breit-Wigner parameterization of the $\rho(\mathbf{1 4 5 0})$ and assuming the $\omega \pi^{-}$mass dependence for the total width.
${ }^{6}$ Mass-independent width parameterization. $\rho(1700)$ mass and width fixed at 1700 MeV and 235 MeV respectively.
${ }^{7}$ Using data from BISELLO 91B, DOLINSKY 86 and ALBRECHT 87 L.
${ }^{8}$ Not separated from $b_{1}(1235)$, not pure $J^{P}=1^{-}$effect.
$4 \pi$ MODE
VALUE (MeV) DOCUMENT ID TECN COMMENT

-     - We do not use the following data for averages, fits, limits, etc.

| $1435 \pm 40$ | ABELE | 01 B CBAR $0.0 \bar{p} n \rightarrow 2 \pi^{-} 2 \pi^{0} \pi^{+}$ |
| :--- | :---: | :--- |
| $1350 \pm 50$ | ACHASOV 97 RVUE $e^{+} e^{-} \rightarrow 2\left(\pi^{+} \pi^{-}\right)$ |  |
| $1449 \pm 4$ | $\mathbf{1}$ ARMSTRONG 89 E OMEG $300 p p \rightarrow p p^{2} 2\left(\pi^{+} \pi^{-}\right)$ |  |

${ }^{1}$ Not clear whether this observation has $I=1$ or 0 .

## $\pi \pi$ MODE

VALUE (MeV) EVTS DOCUMENT ID TECN COMMENT

-     - We do not use the following data for averages, fits, limits, etc.

| 1326.35 | $\pm 3.46$ |  |  | ${ }^{1}$ BARTOS | 17 | RVUE | $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1342.3 | $\pm 46.62$ |  |  | ${ }^{2}$ BARTOS | 17A | RVUE | $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-}$ |
| 1373.83 | +11.37 |  |  | ${ }^{3}$ BARTOS | 17A | RVUE | $\tau^{-} \rightarrow \pi^{-} \pi^{0} \nu_{\tau}$ |
| 1429 | $\pm 41$ |  | 20K | ${ }^{4}$ LEES | 17C | BABR | $J / \psi \rightarrow \pi^{+} \pi^{-} \pi^{0}$ |
| 1350 | $\pm 20$ | $\begin{aligned} & +20 \\ & \mathbf{3} 0 \end{aligned}$ | 63.5k | 5 ABRAMOWI | 12 | ZEUS | $e p \rightarrow e \pi^{+} \pi^{-} p$ |
| 1493 | $\pm 15$ |  |  | ${ }^{6}$ LEES | 12G | BABR | $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \gamma$ |
| 1446 | $\pm 7$ | $\pm 28$ | 5.4 M | 7,8 FUJIKAWA | 08 | BELL | $\tau^{-} \rightarrow \pi^{-} \pi^{0} \nu_{\tau}$ |
| 1328 | $\pm 15$ |  |  | $9{ }^{9}$ SCHAEL | 05C | ALEP | $\tau^{-} \rightarrow \pi^{-} \pi^{0} \nu_{\tau}$ |
| 1406 | $\pm 15$ |  | 87k | 7,10 ANDERSON | 00A | CLE2 | $\tau^{-} \rightarrow \pi^{-} \pi^{0} \nu_{\tau}$ |
| $\sim 1368$ |  |  |  | 11 ABELE | 99C | CBAR | $0.0 \bar{p} d \rightarrow \pi^{+} \pi^{-} \pi^{-} \rho$ |
| 1348 | $\pm 33$ |  |  | BERTIN | 98 | OBLX | $0.05-0.405 \bar{n} p \rightarrow$ |
| 1411 | $\pm 14$ |  |  | 12 ABELE | 97 | CBAR | $\bar{p} n \rightarrow \pi^{-} \pi^{0} \pi^{0}$ |
| 1370 | $\begin{aligned} & +90 \\ & -70 \end{aligned}$ |  |  | ACHASOV | 97 | RVUE | $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-}$ |
| 1359 | $\pm 40$ |  |  | 10 BERTIN | 97 C | OBLX | $0.0 \bar{p} p \rightarrow \pi^{+} \pi^{-} \pi^{0}$ |
| 1282 | $\pm 37$ |  |  | BERTIN | 97D | OBLX | $0.05 \bar{p} p \rightarrow 2 \pi^{+} 2 \pi^{-}$ |
| 1424 | $\pm 25$ |  |  | BISELLO | 89 | DM2 | $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-}$ |
| 1265.5 | $\pm 75.3$ |  |  | DUBNICKA | 89 | RVUE | $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-}$ |
| 1292 | $\pm 17$ |  |  | 13 KURDADZE | 83 | OLYA | $0.64-1.4 e^{+} e^{-} \rightarrow$ |

${ }^{1}$ Applies the Unitary \& Analytic Model of the pion electromagnetic form factor of DUBNICKA 10 to analyze the data of LEES 12G and ABLIKIM 16C.
${ }^{2}$ Applies the Unitary \& Analytic Model of the pion electromagnetic form factor of DUBNICKA 10 to analyze the data of ACHASOV 06, AKHMETSHIN 07, AUBERT 09AS, and AMBROSINO 11A.
${ }^{3}$ Applies the Unitary \& Analytic Model of the pion electromagnetic form factor of DUBNICKA 10 to analyze the data of FUJIKAWA 08.
${ }^{4}$ From a Dalitz plot analysis in an isobar model with $\rho(1450)$ and $\rho(1700)$ masses and widths floating.

Now, we calculate the following integral:
integrate $(((1728 * 9+(728+1164.2696)+\mathrm{Pi} * \exp (($ sqrt $(1 / 6+$
$\left.\left.\left.\left.\left.640 / 3 *\left(1 /\left(216 \mathrm{Pi}^{\wedge} 3\right)\right)^{\wedge} 2 * \mathrm{Pi}^{\wedge} 4 *(-54.192473)^{\wedge} 2\right)\right)+((2 *(3+2)+2)) /(\operatorname{sqrt}(6))\right)\right)\right) \mathrm{x}$

$$
\left.\begin{array}{l}
\int(1728 \times 9+(728+1164.2696)+ \\
\left.\pi \exp \left(\sqrt{\frac{1}{6}+\frac{640}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} \pi^{4}(-54.192473)^{2}}+\frac{2(3+2)+2}{\sqrt{6}}\right)\right) \\
x d x
\end{array}\right)
$$



The result 9447,29 is a good approximation to the rest mass of Upsilon meson, that is $9460.30 \pm 0.26$
Note that 1164,2696 is the following Ramanujan's class invariant $Q=\left(G_{505} /\right.$ $\left.G_{101 / 5}\right)^{3}=1164,2696$

$$
\left(\sqrt{\frac{113+5 \sqrt{505}}{8}}+\sqrt{\frac{105+5 \sqrt{505}}{8}}\right)^{3}=1164,269601267364
$$

Note that, we have also:
integrate $1 /(744+6.582119))\left(\left(\left(1728^{*} 9+(728+1164.2696)+\mathrm{Pi} * \exp ((\right.\right.\right.$ sqrt $(1 / 6+$ $\left.\left.\left.\left.\left.640 / 3 *\left(1 /\left(216 \mathrm{Pi}^{\wedge} 3\right)\right)^{\wedge} 2 * \mathrm{Pi}^{\wedge} 4 *(-54.192473)^{\wedge} 2\right)\right)+((2 *(3+2)+2)) /(\operatorname{sqrt}(6))\right)\right)\right) \mathrm{x}$

$$
\begin{aligned}
& \int \frac{1}{744+6.582119}(1728 \times 9+(728+1164.2696)+ \\
& \left.\pi \exp \left(\sqrt{\frac{1}{6}+\frac{640}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} \pi^{4}(-54.192473)^{2}}+\frac{2(3+2)+2}{\sqrt{6}}\right)\right) \\
& x d x=12.5866 x^{2}+\text { constant }
\end{aligned}
$$

The result 12,5866 is practically equal to the value of black hole entropy 12,57

Now we calculate the $\exp$ for $1=0$ and $Q=-3,29867$
$\left(\left(\operatorname{sqrt}\left(640 / 3 *\left(1 /\left(216 \mathrm{Pi}^{\wedge} 3\right)\right)^{\wedge} 2 * \operatorname{Pi}^{\wedge} 4 *(-3.29867)^{\wedge} 2\right)\right)+((2 *(3+2)+2)) /(\operatorname{sqrt}(6))\right.$

5- $\exp \left(\left(\left(\left(\operatorname{sqrt}\left(640 / 3 *\left(1 /\left(216 \mathrm{Pi}^{\wedge} 3\right)\right)^{\wedge} 2 * \operatorname{Pi}^{\wedge} 4 *(-3.29867)^{\wedge} 2\right)\right)+((2 *(3+2)+2)) /(\operatorname{sqrt}(6))\right.\right.\right.$
$5-\exp \left(\sqrt{\frac{640}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} \pi^{4}(-3.29867)^{2}}+\frac{2(3+2)+2}{\sqrt{6}}\right)$
Result:
-139.0241...

Series representations:

$$
\begin{aligned}
& 5-\exp \left(\sqrt{\frac{1}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} 640 \pi^{4}(-3.29867)^{2}}+\frac{2(3+2)+2}{\sqrt{6}}\right)= \\
& 5-\exp \left(\frac{12}{\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(6-z_{0} k^{k} z_{0}^{-k}\right.}{k!}}+\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{\left.\left.(-1)^{k}\left(-\frac{1}{2}\right)_{k}\right) \frac{0.0497541}{\pi^{2}}-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right)
\end{aligned}
$$

for $\operatorname{not}\left(\left(z_{0} \in \mathbb{R}\right.\right.$ and $\left.\left.-\infty<z_{0} \leq 0\right)\right)$

$$
\begin{aligned}
& 5-\exp \left(\sqrt{\frac{1}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} 640 \pi^{4}(-3.29867)^{2}}+\frac{2(3+2)+2}{\sqrt{6}}\right)=5- \\
& \exp \left(\frac{12}{\exp \left(i \pi\left\lfloor\frac{\arg (6-x)}{2 \pi}\right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^{k}(6-x)^{k} x^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}}+\exp \left(i \pi\left[\frac{\arg \left(\frac{0.0497541}{\pi^{2}}-x\right)}{2 \pi}\right]\right)\right. \\
& \left.\sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(\frac{0.0497541}{\pi^{2}}-x\right)^{k} x^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right) \text { for }(x \in \mathbb{R} \text { and } x<0)
\end{aligned}
$$

$$
\begin{aligned}
& 5-\exp \left(\sqrt{\frac{1}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} 640 \pi^{4}(-3.29867)^{2}}+\frac{2(3+2)+2}{\sqrt{6}}\right)=5- \\
& \exp \left(\frac{12\left(\frac{1}{z_{0}}\right)^{\left.-1 / 2 \operatorname{agg}\left(6-z_{0}\right) /(2 \pi)\right\rfloor} z_{0}^{\left.-1 / 2-1 / 2 \arg \left(6-z_{0}\right) /(2 \pi)\right\rfloor}}{\sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(6-z_{0}\right)^{k} z_{0}^{-k}}{k!}}+\left(\frac{1}{z_{0}}\right)^{1 / 2\left\lfloor\arg \left(\frac{0.0497541}{\pi^{2}}-z_{0}\right) /(2 \pi)\right]}\right. \\
& \left.z_{0}^{1 / 2}\left(1+\left[\arg \left(\frac{0.0497541}{\pi^{2}}-z_{0}\right) /(2 \pi)\right]\right) \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(\frac{0.0497541}{\pi^{2}}-z_{0}\right)^{k} z_{0}^{k}}{k!}\right) \\
& 5-\exp \left(\sqrt{\frac{1}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} 640 \pi^{4}(-3.29867)^{2}}+\frac{2(3+2)+2}{\sqrt{6}}\right)= \\
& 5-\exp \left(\frac{12\left(\frac{1}{z_{0}}\right)^{\left.-1 / 2 \arg \left(6-z_{0}\right) /(2 \pi)\right\rfloor} z_{0}^{\left.1 / 2\left(-1-\arg \left(6-z_{0}\right) /(2 \pi)\right)\right)}}{\sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(6-z_{0}\right)^{k} z_{0}^{k}}{k!}}+\left(\frac{1}{z_{0}}\right)^{1 / 2\left|\arg \left(\frac{0.0497541}{\pi^{2}}-z_{0}\right) /(2 \pi)\right|}\right. \\
& \left.z_{0}^{1 / 2\left(1+\left\lfloor\arg \left(\frac{0.0497541}{\pi^{2}}-z_{0}\right) /(2 \pi)\right]\right)} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(\frac{0.0497541}{\pi^{2}}-z_{0}\right)^{k} z_{0}^{k}}{k!}\right)
\end{aligned}
$$

The result $-139,0241$ is practically equal, but with sign minus, to the rest mass of Pion meson, that is $139.57018 \pm 0.00035$
integrate $\left(\left(\left(1728+1164.2696+\mathrm{Pi}^{*} \exp \left(\left(\operatorname{sqrt}\left(640 / 3 *\left(1 /\left(216 \mathrm{Pi}^{\wedge} 3\right)\right)^{\wedge} 2^{*} \mathrm{Pi}^{\wedge} 4^{*}(-\right.\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.3.29867)^{\wedge} 2\right)\right)+((2 *(3+2)+2)) /(\operatorname{sqrt}(6))\right)\right)\right) \mathrm{x}$

$$
\begin{aligned}
& \int\left(1728+1164.2696+\pi \exp \left(\sqrt{\frac{640}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} \pi^{4}(-3.29867)^{2}}+\frac{2(3+2)+2}{\sqrt{6}}\right)\right) x d x= \\
& \\
& \quad 1672.37 x^{2}+\text { constant }
\end{aligned}
$$



This value 1672,37 is practically equal to the rest mass of Omega baryon $1672.45 \pm 0.29$

For $\mathrm{l}=1,616 * 10^{-35}$ and $\mathrm{Q}=-3,29867$ and the previously expression
$\left(\left(\operatorname{sqrt}\left(1 / 6+640 / 3 *\left(1 /\left(216 \mathrm{Pi}^{\wedge} 3\right)\right)^{\wedge} 2 * \mathrm{Pi}^{\wedge} 4^{*}(-3.29867)^{\wedge} 2\right)\right)+((2 *(3+2)+2)) /(\operatorname{sqrt}(6))\right.$
we calculate the $\exp$ together to the following integral:
integrate $\left(\left(\left(1728+1164.2696+\mathrm{Pi}^{*} \exp \left(\left(\operatorname{sqrt}\left(1 / 6+640 / 3 *\left(1 /\left(216 \mathrm{Pi}^{\wedge} 3\right)\right)^{\wedge} 2 * \mathrm{Pi}^{\wedge} 4^{*}(-\right.\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.3.29867)^{\wedge} 2\right)\right)+\left(\left(2^{*}(3+2)+2\right)\right) /(\operatorname{sqrt}(6))\right)\right)\right) \mathrm{x}$

$$
\begin{aligned}
& \int\left(1728+1164.2696+\pi \exp \left(\sqrt{\frac{1}{6}+\frac{640}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} \pi^{4}(-3.29867)^{2}}+\frac{2(3+2)+2}{\sqrt{6}}\right)\right) x \\
& \quad d x=1765.05 x^{2}+\text { constant }
\end{aligned}
$$



We note that the result 1765,05 is a good approximation to the mass of strange meson $\mathrm{K}_{2}(1770)$. Indeed, as we can see from the next Table, all the values highlighted in yellow and the average are very near, or equal, to the results of the analyzed expression.

## $K_{2}(1770)$ MASS

| VALUE (MeV) | EVTS | DOCUMENT ID |  | TECN | CHG | COMMENT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1773童 8 OUR | VERAG |  |  |  |  |  |
| $1777 \pm 35 \pm 122$ | 4289 | ${ }^{1}$ AAIJ | 17 c | LHCB |  | $\mathrm{B}^{+} \rightarrow \mathrm{J} / \psi \phi K^{+}$ |
| $1773 \pm 8$ |  | ${ }^{2}$ ASTO | 93 | LASS |  | $11 K^{-} p \rightarrow K^{-} \omega p$ |

-     - We do not use the following data for averages, fits, limits, etc.

| $1743 \pm 15$ |  | TIKHOMIROV |  | SPEC |  | $\begin{aligned} & 40.0 \pi-c \mid \\ & K_{S}^{0} K_{S}^{0} \vec{K}_{L}^{0} X \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1810 \pm 20$ |  | FRAME | 86 | OMEG | $+$ | $13 K^{+} p \rightarrow \phi K^{+} p$ |
| $\sim 1730$ |  | ARMSTRONG | 83 | OMEG | - | $18.5 K^{-} p \rightarrow 3 K p$ |
| $\sim 1780$ |  | 3 DAUM | 81C | CNTR | - | $63 K^{-} p \rightarrow K^{-} 2 \pi p$ |
| $1710 \pm 15$ | 60 | CHUNG | 74 | HBC | - | $7.3 K^{-} p \rightarrow K^{-} \omega p$ |
| $1767 \pm 6$ |  | BLIEDEN | 72 | MMS | - | 11-16 $K^{-}{ }^{-}$ |
| $1730 \pm 20$ | 306 | 4 FIRESTONE | 72B | DBC | $+$ | $12 K^{+}{ }_{d}$ |
| $1765 \pm 40$ |  | $5^{5}$ COLLEY | 71 | HBC | $+$ | $10 K^{+} p \rightarrow K 2 \pi N$ |
| 1740 |  | DENEGRI | 71 | DBC | - | $12.6 K^{-} d \rightarrow \bar{K} 2 \pi d$ |
| $1745 \pm 20$ |  | AGUILAR-... | 70C | HBC | - | $4.6 K^{-} p$ |
| $1780 \pm 15$ |  | BARTSCH | 70C | HBC | - | $10.1 K^{-} p$ |
| $1760 \pm 15$ |  | LUDLAM | 70 | HBC | - | $12.6 K^{-}{ }_{p}$ |

For $\mathrm{l}=1,616 * 10^{-35}$ and $\mathrm{Q}=50,893800$ and the previously expression $\left(\left(\operatorname{sqrt}\left(1 / 6+640 / 3^{*}\left(1 /\left(216 \mathrm{Pi}^{\wedge} 3\right)\right)^{\wedge} 2^{*} \mathrm{Pi}^{\wedge} 4^{*}(50.893800)^{\wedge} 2\right)\right)+\left(\left(2^{*}(3+2)+2\right)\right) /(\operatorname{sqrt}(6))\right.$ we calculate the exp together to the following integral:
integrate ( ( $(1728+1164.2696-$
$24+\mathrm{Pi}^{*} \exp \left(\left(\operatorname{sqrt}\left(1 / 6+640 / 3^{*}\left(1 /\left(216 \mathrm{Pi}^{\wedge} 3\right)\right)^{\wedge} 2^{*} \mathrm{Pi}^{\wedge} 4^{*}(50.893800)^{\wedge} 2\right)\right)+\left(\left(2^{*}(3+2)+2\right)\right) /(\mathrm{s}\right.$ $\operatorname{qrt(6))))}) \mathrm{x}$

$$
\begin{aligned}
& \int(1728+1164.2696-24+ \\
& \begin{aligned}
\pi & \left.\exp \left(\sqrt{\frac{1}{6}+\frac{640}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} \pi^{4} 50.893800^{2}}+\frac{2(3+2)+2}{\sqrt{6}}\right)\right) \\
x d x & =2112.45 x^{2}+\text { constant }
\end{aligned}
\end{aligned}
$$

Plot of the integral:


This value 2112,45 is practically equal to the rest mass of strange D meson $2112.3 \pm 0.5$

Indeed, as we can see from the next Table, the value of mass is equal to the result of the analyzed expression.

## $D_{s}^{* \pm}$ MASS

The fit includes $D^{ \pm}, D^{0}, D_{s}^{ \pm}, D^{* \pm}, D^{* 0}$, and $D_{s}^{* \pm}$ mass and mass difference measurements.


Now, from the precedent integrals, we can to obtain also:
integrate $[(((1728 * 9+(728+1164.2696)+\mathrm{Pi} * \exp ((\operatorname{sqrt}(1 / 6+$
$\left.\left.\left.\left.\left.\left.640 / 3^{*}\left(1 /\left(216 \mathrm{Pi}^{\wedge} 3\right)\right)^{\wedge} 2 * \mathrm{Pi}^{\wedge} 4 *(-54.192473)^{\wedge} 2\right)\right)+((2 *(3+2)+2)) /(\operatorname{sqrt}(6))\right)\right)\right)\right]^{\wedge} 0.048$

$$
\begin{aligned}
& \int(1728 \times 9+(728+1164.2696)+ \\
& \left.\quad \pi \exp \left(\sqrt{\frac{1}{6}+\frac{640}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} \pi^{4}(-54.192473)^{2}}+\frac{2(3+2)+2}{\sqrt{6}}\right)\right)^{0.048} \\
& d x=
\end{aligned}
$$

integrate $[(((1728 * 9+(728+1164.2696)+\mathrm{Pi} * \exp ((\operatorname{sqrt}(1 / 6+$ $\left.\left.\left.\left.\left.\left.640 / 3^{*}\left(1 /\left(216 \mathrm{Pi}^{\wedge} 3\right)\right)^{\wedge} 2 * \mathrm{Pi}^{\wedge} 4^{*}(50.893800)^{\wedge} 2\right)\right)+((2 *(3+2)+2)) /(\operatorname{sqrt}(6))\right)\right)\right)\right]^{\wedge} 0.048$

$$
\begin{aligned}
& \int(1728 \times 9+(728+1164.2696)+ \\
& \left.\quad \pi \exp \left(\sqrt{\frac{1}{6}+\frac{640}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} \pi^{4} 50.893800^{2}}+\frac{2(3+2)+2}{\sqrt{6}}\right)\right)^{0.048}
\end{aligned}
$$

$$
d x=1.60384 x+\text { constant }
$$

Results that practically are equals to the value of the electric charge of the positron.
Also:
integrate $\left(\left(\left(1728+1164.2696+\mathrm{Pi}^{*} \exp \left(\left(\operatorname{sqrt}\left(640 / 3 *\left(1 /\left(216 \mathrm{Pi}^{\wedge} 3\right)\right)^{\wedge} 2 * \mathrm{Pi}^{\wedge} 4^{*}(-\right.\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.3.29867)^{\wedge} 2\right)\right)+\left(\left(2^{*}(3+2)+2\right)\right) /(\operatorname{sqrt}(6))\right)\right)\right)^{\wedge}(1 / 17)$

$$
\begin{aligned}
& \left.\int \sqrt[171]{1728+1164.2696+\pi \exp \left(\sqrt{\frac{640}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} \pi^{4}(-3.29867)^{2}}\right.}+\frac{2(3+2)+2}{\sqrt{6}}\right) \\
& d x=1.61182 x+\text { constant }
\end{aligned}
$$

and
integrate $\left(\left(\left(1728+1164.2696+\mathrm{Pi}^{*} \exp \left(\left(\operatorname{sqrt}\left(1 / 6+640 / 3 *\left(1 /\left(216 \mathrm{Pi}^{\wedge} 3\right)\right)^{\wedge} 2 * \mathrm{Pi}^{\wedge} 4^{*}(-\right.\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.3.29867)^{\wedge} 2\right)\right)+((2 *(3+2)+2)) /(\operatorname{sqrt}(6))\right)\right)\right)^{\wedge}(1 / 17)$

$$
\begin{aligned}
& \int \sqrt[17]{1728+1164.2696+\pi \exp \left(\sqrt{\frac{1}{6}+\frac{640}{3}\left(\frac{1}{216 \pi^{3}}\right)^{2} \pi^{4}(-3.29867)^{2}}+\frac{2(3+2)+2}{\sqrt{6}}\right)} \\
& d x=1.61694 x+\text { constant }
\end{aligned}
$$

are results that practically are very near to the value of the electric charge of the positron.

Now, we have:

$$
\begin{align*}
D_{M} D_{M} \Phi_{N}-D_{N} D_{M} \Phi_{M}+2 \mathcal{F}_{N M} \Phi_{M}+\mathcal{O}\left(\lambda^{-1}\right) & =0 \\
D_{M}\left(D_{0} \Phi_{M}-D_{M} \Phi_{0}\right)-\mathcal{F}^{0 M} \Phi_{M}-\frac{1}{64 \pi^{2} a} \epsilon_{M N P Q} K_{M N P Q}+\mathcal{O}\left(\lambda^{-1}\right) & =0 \tag{2.8}
\end{align*}
$$

where $x^{M}=\left\{x^{i}, Z\right\}, i=1,2,3$ and the 4 -form $K_{M N P Q}$ is given as,

$$
\begin{equation*}
K_{M N P Q}=\partial_{M} \mathcal{A}_{N} \partial_{P} \Phi_{Q}\left|\mathcal{A}_{M} \mathcal{A}_{N} \partial_{P} \Phi_{Q}\right| \partial_{M} \mathcal{A}_{N} \mathcal{A}_{P} \Phi_{Q} \mid{ }_{6}^{5} \mathcal{A}_{M} \mathcal{A}_{N} \mathcal{A}_{P} \Phi_{Q} \tag{2.9}
\end{equation*}
$$

Since the holographic approach is valid in the strongly compling limit. $\lambda \rightarrow \infty$, the contributions from $\mathcal{O}\left(\lambda^{-1}\right)$ have been dropped off. Note that the light flavoured gauge field $\mathcal{A}_{a}$ satisfies the equations of motion obtained by varying the action ( $\mathrm{C}-1$ ), so their solution remains to be (C-2) in the large $\lambda$ limit. And we could further define $\Phi_{a}=\phi_{a} e^{ \pm i m_{H} x^{j}}$ in the heavy quark limit i.e. $m_{H} \rightarrow \infty$ as in $[25,26]$ so that $D_{0} \Phi_{M}=\left(D_{0}+i m_{H}\right) \phi_{M}$ where " + " corresponds to quark and anti-quark respectively. By keeping these in mind, altogether we find the full solution for $(2.8)$ as,

$$
\begin{align*}
& \phi_{0}=-\frac{1}{1024 a \pi^{2}}\left[\frac{25 \rho}{2\left(x^{2}+\rho^{2}\right)^{5 / 2}}+\frac{7}{\rho\left(x^{2}+\rho^{2}\right)^{3 / 2}}\right] \chi, \\
& \phi_{M}=\frac{\rho}{\left(x^{2}+\rho^{2}\right)^{3 / 2}} o_{M \chi}, \tag{2.10}
\end{align*}
$$

where $\chi$ is a spinor independent on $x^{M}$. Then in the double limit i.e. $\lambda \rightarrow \infty$ followed by $m_{H} \rightarrow$ $\infty$, the Hamiltonion for the collective modes involving the heavy flavour could be calculated as in (C-7) by following the procedures in Appendix C.

We have that:
$\left.-\left(\left(\left(216^{*} \mathrm{Pi}^{\wedge} 3\right)\right) /\left(1024^{*} \mathrm{Pi}^{\wedge} 2\right)\right)\right) *\left[\left(25^{*}-6.5677261^{*} 10^{\wedge} 16\right) /\left(\left(2^{*}((1+((-\right.\right.\right.$
$\left.\left.\left.6.5677261 * 10^{\wedge} 16\right)^{\wedge} 2\right)\right)^{\wedge} 2.5$
Input interpretation:
$-\frac{216 \pi^{3}}{1024 \pi^{2}} \times \frac{25\left(-6.5677261 \times 10^{16}\right)}{2\left(1+\left(-6.5677261 \times 10^{16}\right)^{2}\right)^{2.5}}$
Result:
$4.45198 \ldots \times 10^{-67}$
$\left.-\left(\left(\left(216 * \mathrm{Pi}^{\wedge} 3\right)\right) /\left(1024 * \mathrm{Pi}^{\wedge} 2\right)\right)\right) *\left[7 /\left(\left(\left(-6.5677261^{*} 10^{\wedge} 16\right)^{*}(1+(-\right.\right.\right.$
$\left.\left.\left.\left.6.5677261 * 10^{\wedge} 16\right)^{\wedge} 2\right)^{\wedge} 1.5\right)\right]$
Input interpretation:
$-\frac{216 \pi^{3}}{1024 \pi^{2}}\left(-\frac{7}{6.5677261 \times 10^{16}\left(1+\left(-6.5677261 \times 10^{16}\right)^{2}\right)^{1.5}}\right)$

Result:
$2.49311 \ldots \times 10^{-67}$
$\left(4.45198^{*} 10^{\wedge}-67\right)+\left(2.49311^{*} 10^{\wedge}-67\right)$

Input interpretation:
$4.45198 \times 10^{-67}+2.49311 \times 10^{-67}$
Result:
$6.94509 \times 10^{-67}$

And
$\left.\left.\left.\left.\left.\left(\left(\left(-6.5677261 * 10^{\wedge} 16\right) * 585\right)\right) /\left(1+\left(6.5677261 * 10^{\wedge} 16\right)^{\wedge} 2\right)\right)^{\wedge} 1.5\right)\right)\right)\right)$
$\frac{-6.5677261 \times 10^{16} \times 585}{\left(1+\left(6.5677261 \times 10^{16}\right)^{2}\right)^{1.5}}$
Result:
$-1.35621 \ldots \times 10^{-31}$

We calculate the following integrals:
integrate $1 /\left(\left((1.65578)^{\wedge} 1.08643 \wedge(2 \mathrm{Pi})\right)\right) *\left(1164.2696^{*} 10^{\wedge} 67\right) *\left[\left(4.45198^{*} 10^{\wedge}-\right.\right.$ $\left.67)+\left(2.49311^{*} 10^{\wedge}-67\right)\right] \mathrm{x}$

$$
\begin{aligned}
& \text { Indefinite integral: } \\
& \int \frac{\left(1164.2696 \times 10^{67}\right)\left(4.45198 \times 10^{-67}+2.49311 \times 10^{-67}\right) x}{1.65578^{1.08643^{2 \pi}}} d x= \\
& 1729.88 x^{2}+\text { constant }
\end{aligned}
$$



We have that:
$1 /(142)$ integrate $1 /\left(\left((1.65578)^{\wedge} 1.08643^{\wedge}(2 \mathrm{Pi})\right)\right)^{*}\left(1164.2696^{*} 10^{\wedge} 67\right)$ * $\left[\left(4.45198^{*} 10^{\wedge}-67\right)+\left(2.49311^{*} 10^{\wedge}-67\right)\right] x$
$\frac{1}{142} \int \frac{1}{1.65578^{1.08643^{2 \pi}}}\left(1164.2696 \times 10^{67}\right)\left(4.45198 \times 10^{-67}+2.49311 \times 10^{-67}\right) x d x$
Result:
$12.1823 x^{2}$

The result 12,182 is practically equal to the value of black hole entropy 12,19
$\left.\left.\left.\left.\left.\left.\left.\left.6.5677261^{*} 10^{\wedge} 16\right)^{*} 585\right)\right) /\left(1+\left(6.5677261^{*} 10^{\wedge} 16\right)^{\wedge} 2\right)\right)^{\wedge} 1.5\right)\right)\right)\right) \mathrm{x}$


$$
-1710.24 x^{2}+\text { constant }
$$

Plot of the integral:


Now:
$\frac{6.94509}{10^{67}}-\frac{1.35621}{10^{31}}$

Result:
$-1.35620999999999999999999999999999999305491 \times 10^{-31}$

We calculate the following integral of the algebraic sum of the two results:
integrate (1164.2696-9)* 10^31 [(6.94509/10^67) - (1.35621/10^31)]x
$\int(1164.2696-9) 10^{31}\left(\left(\frac{6.94509}{10^{67}}-\frac{1.35621}{10^{31}}\right) x\right) d x=-783.394 x^{2}+$ constant

Plot of the integral:


This result -783.394 is practically equal, with sign minus, to the rest mass of Omega meson $782.65 \pm 0.12$. Indeed

| $\omega$ (782) MASS |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\operatorname{VALUE}(\mathrm{MeV})$ | EVTS | DOCUMENT ID |  | TECN | COMMENT |
| $782.65 \pm 0.12$ OUR AVERAGE |  | Error includes scale factor |  | of 1.9. | See the ideogram below. |
| $783.20 \pm 0.13 \pm 0.16$ | 18680 | AKHMETSHIN 05 |  | CMD2 | $\underset{\pi^{0}}{0.60-1.38} e^{+} e^{-} \rightarrow$ |
| $782.68 \pm 0.09 \pm 0.04$ | 11200 | ${ }^{1}$ AKHMETSHIN 04 |  | CMD2 | $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ |
| $782.79 \pm 0.08 \pm 0.09$ | 1.2M | ${ }^{2}$ ACHASOV | 03D | RVUE | $\begin{gathered} 0.44-2.00 \\ \pi^{+} \pi^{-} \\ \pi_{0}^{+} \end{gathered}$ |
| $782.7 \pm 0.1 \pm 1.5$ | 19500 | WURZINGER | 95 | SPEC | $1.33 \mathrm{pd} \rightarrow{ }^{3} \mathrm{Hew}$ |
| $781.96 \pm 0.17 \pm 0.80$ | 11k | ${ }^{3}$ AMSLER | 94C | CBAR | $0.0 \bar{p} p \rightarrow \omega \eta \pi^{0}$ |
| $782.08 \pm 0.36 \pm 0.82$ | 3463 | ${ }^{4}$ AMSLER | 94C | CBAR | $0.0 \bar{p} p \rightarrow \omega \eta \pi^{0}$ |
| $781.96 \pm 0.13 \pm 0.17$ | 15k | AMSLER | 93B | CBAR | $0.0 \bar{p} p \rightarrow \omega \pi^{0} \pi^{0}$ |
| $782.4 \pm 0.2$ | 270k | WEIDENAUER |  | ASTE | $\bar{p} p \rightarrow 2 \pi^{+} 2 \pi^{-} \pi^{0}$ |
| $782.2 \pm 0.4$ | 1488 | KURDADZE | 83B | OLYA | $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ |
| $782.4 \pm 0.5$ | 7000 | ${ }^{5}$ KEYNE | 76 | CNTR | $\pi^{-} p \rightarrow \omega n$ |
| - - We do not use the following data for averages, fits, limits, etc. - - |  |  |  |  |  |
| $781.78 \pm 0.10$ |  | ${ }^{6}$ BARKOV | 87 | CMD | $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ |
| $783.3 \pm 0.4$ | 433 | CORDIER | 80 | DM1 | $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ |
| $782.5 \pm 0.8$ | 33260 | ROOS | 80 | RVUE | 0.0-3.6 $\bar{p} p$ |
| $782.6 \pm 0.8$ | 3000 | BENKHEIRI | 79 | OMEG | $9-12 \pi^{ \pm} p$ |
| $781.8 \pm 0.6$ | 1430 | COOPER | 78B | HBC | $0.7-0.8 \bar{p} p \rightarrow 5 \pi$ |
| $782.7 \pm 0.9$ | 535 | VANAPEL... | 78 | HBC | $7.2 \bar{p} p \rightarrow \bar{p} p \omega$ |
| $783.5 \pm 0.8$ | 2100 | GESSAROLI | 77 | HBC | $11 \pi^{-} p \rightarrow \omega n$ |
| $782.5 \pm 0.8$ | 418 | AGUILAR-... | 72B | HBC | 3.9,4.6 $K^{-} p$ |
| $783.4 \pm 1.0$ | 248 | BIZZARRI | 71 | HBC | $0.0 p \bar{p} \rightarrow K^{+} K^{-} \omega$ |
| $781.0 \pm 0.6$ | 510 | BIZZARRI | 71 | HBC | $0.0 p \bar{p} \rightarrow K_{1} K_{1} \omega$ |
| $783.7 \pm 1.0$ | 3583 | ${ }^{7}$ COYNE | 71 | HBC | $\begin{aligned} & 3.7 \pi^{+} p \rightarrow \\ & p \pi^{+} \pi^{+} \pi^{-} \pi^{0} \end{aligned}$ |
| $784.1 \pm 1.2$ | 750 | ABRAMOVI... | 70 | HBC | $3.9 \pi^{-} p$ |
| $783.2 \pm 1.6$ |  | ${ }^{8}$ BIGGS | 70 B | CNTR | $<4.1 \gamma \mathrm{C} \rightarrow \pi^{+} \pi^{-} \mathrm{C}$ |
| $782.4 \pm 0.5$ | 2400 | BIZZARRI | 69 | HBC | $0.0 \bar{p} p$ |

Furthermore, from the above integral we have also:
$(26+1) /(1728)$ integrate (1164.2696-9)* $10^{\wedge} 31\left[\left(6.94509 / 10^{\wedge} 67\right)-\right.$ (1.35621/10^31)]x

$$
\frac{26+1}{1728} \int(1164.2696-9) \times 10^{31}\left(\left(\frac{6.94509}{10^{67}}-\frac{1.35621}{10^{31}}\right) x\right) d x
$$

Result:
$-12.2405 x^{2}$

The result $-12,24$ is very near to the value of black hole entropy 12,19 with sign minus

Now, we have:

For the dilatonic scalar glueball, the following formula:

$$
\begin{aligned}
\frac{\mathcal{L}_{\Psi}^{D}}{a \mathcal{C}_{D}}= & v^{2} \frac{\left(N_{f}+1\right)^{2}}{N_{f}^{2}}\left[-\frac{\partial^{i} \partial^{j} G_{D}}{3 M_{D}^{2} M_{K K}} \Phi_{i}^{\dagger} \Phi_{j}+\frac{2 G_{D}}{3 M_{K K}} \eta^{i j} \Phi_{i}^{\dagger} \Phi_{j}\right. \\
& \left.+\frac{\partial^{2} G_{D}}{6 M_{D}^{2} M_{K K}} \eta^{i j} \Phi_{i}^{\dagger} \Phi_{j}+\frac{G_{D}}{3 M_{K K}} \Phi_{Z}^{\dagger} \Phi_{Z}+\frac{\partial^{2} G_{D}}{6 M_{D}^{2} M_{K K}} \Phi_{Z}^{\dagger} \Phi_{Z}\right] .
\end{aligned}
$$

(1.2371318784*10^63)[-(0.249996/185.1395)-
$(0.999992 \mathrm{i} /(24 \mathrm{Pi}))+(0.249996 /(48 \mathrm{Pi} 2.455489))-$
$(0.499996 \mathrm{i} /(24 \mathrm{Pi}))+(0.249996 /(48 \mathrm{Pi} * 2.455489)]$
$1.2371318784 \times 10^{63}\left(-\frac{0.249996}{185.1395}-0.999992 \times \frac{i}{24 \pi}+\right.$

$$
\left.\frac{0.249996}{48 \pi \times 2.455489}-0.499996 \times \frac{i}{24 \pi}+\frac{0.249996}{48 \pi \times 2.455489}\right)
$$

Result:
$-8.01323 \ldots \times 10^{52}-$
$2.46118 \ldots 10^{61}$,

Polar coordinates
$r=2.46118 \times 10^{61}$ (radius), $\quad \theta=-90 .^{\circ}$ (angle)

And:
$[2.46118 * 10 \wedge 61]^{*}\left(\left(1 /\left(216 * \mathrm{Pi}^{\wedge} 3\right)\right) * 29.772\right.$
$2.46118 \times 10^{61}\left(\frac{1}{216 \pi^{3}} \times 29.772\right)$

Result:
$1.09408 \ldots \times 10^{59}$

Comparison:
$\approx 1.4 \times 10^{5} \times$ the size of the Monster group $\left(\approx 8.1 \times 10^{53}\right)$

Now, we have the following integral:
$(1164.2696+1729-144) *\left(1 /\left(10^{\wedge} 59\right)\right)$ integrate
$\left[\left(2.46118^{*} 10^{\wedge} 61\right)^{*}\left(\left(1 /\left(216^{*} \mathrm{Pi}^{\wedge} 3\right)\right)^{*} 29.772\right] \mathrm{x}\right.$

Input interpretation
$(1164.2696+1729-144) \times \frac{1}{10^{59}} \int 2.46118 \times 10^{61}\left(\frac{1}{216 \pi^{3}} \times 29.772 x\right) d x$
Result:
$1503.96 x^{2}$


Alternate form assuming x is real:
$1503.96 x^{2}+0$

Indefinite integral assuming all variables are real
$501.319 x^{3}+$ constant

The result 1503.96 is practically equal to the following value of meson $f_{0}(1500)$ mass:

## $f_{0}(1500)$ MASS

VALUE (MeV) EVTS DOCUMENT ID TECN COMMENT

1504志 6 OUR AVERAGE Error includes scale factor of 1.3 . See the ideogram below.

| $1468{ }_{-15}^{+14}+74$ | 5.5 k | ${ }^{1}$ ABLIKIM | 13 N | BES3 | $e^{+} e^{-} \rightarrow$ | $J / \psi \rightarrow \gamma \eta \eta$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1466 \pm 6 \pm 20$ |  | ABLIKIM | 06 v | BES2 | $e^{+} e^{-} \rightarrow$ | $J / \psi \rightarrow \gamma \pi^{+} \pi^{-}$ |
| $1515 \pm 12$ |  | 2 BARBERIS | 00A |  | 450 pp $\rightarrow$ | $p_{f} \eta \eta p_{s}$ |
| $1511 \pm 9$ |  | 2,3 BARBERIS | O0C |  | 450 pp $\rightarrow$ | $p_{f} 4 \pi p_{s}$ |
| $1510 \pm 8$ |  | 2 BARBERIS | O0E |  | $450 p p \rightarrow$ | $p_{f} \eta \eta p_{s}$ |
| $1522 \pm 25$ |  | BERTIN | 98 | OBLX | 0.05-0.405 | $\bar{n} p \rightarrow \pi^{+} \pi^{+} \pi^{-}$ |
| $1449 \pm 20$ |  | 2 BERTIN | 97c | OBLX | $0.0 \bar{p} p \rightarrow$ | $\pi^{+} \pi^{-} \pi^{0}$ |
| $1515 \pm 20$ |  | ABELE | 96B | CBAR | $0.0 \bar{p} p \rightarrow$ | $\pi^{0} K_{L}^{0} K_{L}^{0}$ |
| $1500 \pm 15$ |  | ${ }^{4}$ AMSLER | 95B | CBAR | $0.0 \bar{p} p \rightarrow$ |  |
| $1505 \pm 15$ |  | ${ }^{5}$ AMSLER | 95 C | CBAR | $0.0 \bar{p} p \rightarrow$ | $\eta \eta \pi^{0}$ |

Indeed, we have also that, from:

# Production of $\mathrm{f} 0(1710), \mathrm{f} 0(1500)$, and $\mathrm{f} 0(1370)$ in $\mathrm{J} / \psi$ hadronic decays 

Frank E. Close and Qiang Zhao


#### Abstract

The importance of glueball $Q \bar{Q}$ mixing is also highlighted by the indispensible contributions from the doubly disconnected processes, which turn out to be nomperturbative and violate the OZI rule. Since the coupling $g g \rightarrow Q Q$ in the dunbly discomected processes is essentially the same as the glueball- $Q \bar{Q}$ mixing, the nonperturbative feature of the doubly disconnected processes is self-consistent with the proposed confguration mixing scheme for these three $f_{0}$ states. In this sense, our results not only provide an understanding of the recent "puzzling" experimental cata from BES $\left[13,14,15\right.$, but also highlight the strong possibility of the existence of glueball contents in the $f_{0}(1500)$, and its sizeable interferences in $f_{0}(1710)$. Furthermore, due to the configuration mixing, the $|n n\rangle$ dominant $f_{0}(1370)$ tends to have a lower mass lower than 1370 MeV , which also agrees with a recent more refined analysis [13, 22].

With Eqs. (9) and (10), and applying the method of Ref. [6], the the relative decay widths (excluding phase space) for $f_{0}^{i}>\gamma \gamma$ arc found to be $f_{0}(1370): f_{0}(1500): f_{\mathrm{v}}(1710) \sim 12: 2: 1$. The results are consistent with those of Ret. [6], which should not be surprizing since the mixing matrices are similar to each other. In this factorization scheme, a quantitative normalization of the scalar glueball production rate in the $J / \psi \rightarrow V G$ is also accessible. With the pure gluehall mass in a range of $1.46-1.52 \mathrm{GeV}$, we ohtain the branching ratios $b r_{J / \psi \rightarrow \phi G} \simeq \frac{1}{2} b r_{J / \psi \rightarrow \omega G} \simeq(1 \sim 2) \times 10^{-4}$. Although a direct measurement of the glueball production seems


In this sense, our results not only provide an understanding of the recent "puzzling" experimental data from BES $[13,14,15]$, but also highlight the strong possibility of the existence of glueball contents in the $\mathrm{f} 0(1500)$, and its sizeable interferences in $\mathrm{f} 0(1710)$. Furthermore, due to the configuration mixing, the $\mid \mathrm{n}^{-}$ni dominant $\mathrm{f} 0(1370)$ tends to have a lower mass lower than 1370 MeV , which also agrees with a recent more refined analysis.

From the above integral, we have also that:

$$
\begin{aligned}
& 1 /\left(192-64-e^{*} 1.65578\right)(1164.2696+1729-144) *\left(1 /\left(10^{\wedge} 59\right)\right) \text { integrate } \\
& {\left[\left(2.46118^{*} 10^{\wedge} 61\right) *\left(\left(1 /\left(216^{*} \mathrm{Pi}^{\wedge} 3\right)\right) * 29.772\right] \mathrm{x}\right.}
\end{aligned}
$$

Input interpretation:

$$
\begin{aligned}
& \frac{1}{192-64+e \times(-1.65578)}\left((1164.2696+1729-144) \times \frac{1}{10^{59}}\right) \\
& \int 2.46118 \times 10^{61}\left(\frac{1}{216 \pi^{3}} \times 29.772 x\right) d x
\end{aligned}
$$

Result:
$12.1779 x^{2}$

Furthermore:
$\operatorname{sqrt}\left(\ln \left[1729 *\left(2.46118^{*} 10^{\wedge} 61\right)\right)\right.$

Input interpretation:
$\sqrt{\log \left(1729 \times 2.46118 \times 10^{61}\right)}$

## Result:

12.198919...

And
$\operatorname{sqrt}\left(\ln \left[64 \mathrm{Pi}^{*} 1729^{*}\left(1.09408^{*} 10^{\wedge} 59\right)\right)\right.$

Input interpretation:
$\sqrt{\log \left(64 \pi \times 1729 \times 1.09408 \times 10^{59}\right)}$

Result:
12.194316..

All the results $12,177912,1989$ and 12,1943 are very near to the value of black hole entropy 12,19

From the following formula of the the exotic scalar glueball:

$$
\begin{aligned}
\mathcal{L}_{\Psi}^{E}= & -v^{2} \frac{\left(N_{f}+1\right)^{2}}{N_{f}^{2}}\left[-\frac{5}{12 M_{E}^{2} M_{K K}} \partial^{i} \partial^{j} G_{E} \Phi_{i}^{\dagger} \Phi_{j}+\frac{5}{24 M_{E}^{2} M_{K K}} \partial^{2} G_{E} \delta^{i j} \Phi_{i}^{\dagger} \Phi_{j}\right. \\
& \left.-\frac{5}{12 M_{K K}} G_{E} \Phi_{Z}^{\dagger} \Phi_{Z}+\frac{5}{24 M_{E}^{2} M_{K K}} \partial^{2} G_{E} \Phi_{Z}^{\dagger} \Phi_{Z}\right] .
\end{aligned}
$$

(1.2371318784*10^63)[-5*-0.153644/(12*0.901^2*(-8Pi))+(5*(-
$0.153644) /(24 * 0.901 \wedge 2 *(-8 \mathrm{Pi}))-(5 * 0.391974 \mathrm{i}) /\left(12^{*}(-8 \mathrm{Pi})\right)+(5 *-$
$0.153644 /(24 * 0.901 \wedge 2 *(-8 \mathrm{Pi})]$
$1.2371318784 \times 10^{63}\left(-5\left(-\frac{0.153644}{12 \times 0.901^{2}(-8 \pi)}\right)+\right.$

$$
\left.\left(5\left(-\frac{0.153644}{24 \times 0.901^{2}(-8 \pi)}\right)-\frac{5 \times 0.391974 i}{12(-8 \pi)}+5\left(-\frac{0.153644}{24 \times 0.901^{2}(-8 \pi)}\right)\right)\right)
$$

Result:
$8.03937 \ldots \times 10^{60}{ }_{i}$

Polar coordinates:
$r=8.03937 \times 10^{60}$ (radius), $\quad \theta=90^{\circ}$ (angle)

We calculate the following integral:
$\left.\left.\operatorname{sqrt}[(((\operatorname{sqrt}(5)+1) / 2)))^{\wedge} 2+\operatorname{sqrt}(5) / 2\right]^{*}(1729 * 1164.2696+729 * 4) / 1.65578\right) *(1 /(10 \wedge 63))$ integrate $\left[8.03937 * 10^{\wedge} 60\right] \mathrm{x}$
$\sqrt{\left(\frac{1}{2}(\sqrt{5}+1)\right)^{2}+\frac{\sqrt{5}}{2}} \times \frac{1729 \times 1164.2696+729 \times 4}{1.65578} \times \frac{1}{10^{63}} \int 8.03937 \times 10^{60} x d x$

Result:
$9459.63 x^{2}$
or:
(1/(10^63))
$\left.\left.\operatorname{sqrt}[(((\operatorname{sqrt}(5)+1) / 2)))^{\wedge} 2+\operatorname{sqrt}(5) / 2\right]^{*}(1729 * 1164.2696+(1729+729))^{*} 1 / 1.65578\right)$
integrate [8.03937* $\left.10^{\wedge} 60\right] \mathrm{x}$

$$
\begin{aligned}
& \frac{1}{10^{63}}\left(\sqrt{\left(\frac{1}{2}(\sqrt{5}+1)\right)^{2}+\frac{\sqrt{5}}{2}}(1729 \times 1164.2696+(1729+729)) \times \frac{1}{1.65578}\right) \\
& \quad \int 8.03937 \times 10^{60} x d x \\
& \begin{array}{l}
\text { Result: } \\
9457.48 x^{2}
\end{array}
\end{aligned}
$$

The two results 9459.63 and 9457.48 are practically equals to the rest mass of Upsilon meson $9460.30 \pm 0.26$

Now:
$\mathrm{Pi}+\ln \left[\left[\left[\operatorname{sqrt}[(((\operatorname{sqrt}(5)+1) / 2)))^{\wedge} 2+\operatorname{sqrt}(5) / 2\right]^{*}(1729 * 1164.2696+729 * 4) / 1.65578\right) *\right.$ (1/(10^63)) integrate [8.03937* $\left.\left.\left.\left.10^{\wedge} 60\right] x\right]\right]\right]$

$$
\begin{aligned}
& \pi+\log \left(\sqrt{\left(\frac{1}{2}(\sqrt{5}+1)\right)^{2}+\frac{\sqrt{5}}{2}} \times \frac{1729 \times 1164.2696+729 \times 4}{1.65578} \times \frac{1}{10^{63}}\right. \\
& \left.\int 8.03937 \times 10^{60} x d x\right)
\end{aligned}
$$

Result:
$\log \left(9459.63 x^{2}\right)+\pi$

Input interpretation:
$\pi+\log (9459.63)$
Result:
12.29638...
$\mathrm{Pi}+\ln \left[\left[\left(1 /\left(10^{\wedge} 63\right)\right)\right.\right.$
$\left.\left.\operatorname{sqrt}[(((\operatorname{sqrt}(5)+1) / 2)))^{\wedge} 2+\operatorname{sqrt}(5) / 2\right]^{*}(1729 * 1164.2696+(1729+729)) * 1 / 1.65578\right)$
integrate [8.03937*10^60]x]]]

$$
\begin{aligned}
& \pi+\log \left(\frac{1}{10^{63}}\left(\sqrt{\left(\frac{1}{2}(\sqrt{5}+1)\right)^{2}+\frac{\sqrt{5}}{2}}(1729 \times 1164.2696+(1729+729)) \times \frac{1}{1.65578}\right)\right. \\
& \left.\quad \int 8.03937 \times 10^{60} x d x\right) \\
& \text { Result: }
\end{aligned}
$$

$\log \left(9457.48 x^{2}\right)+\pi$

Input interpretation:
$\pi+\log (9457.48)$
Result:
12.29615..

The results 12.29638 and 12.29615 are very near to the value of black hole entropy 12.19

We have also that:
$\left.\operatorname{sqrt}[(((\operatorname{sqrt}(5)+1) / 2)))^{\wedge} 2+\operatorname{sqrt}(5) / 2\right]^{*}(((1729 * 1164.2696)+729)) /(288+\mathrm{Pi}) *$ (1/(10^61)) integrate [8.03937* $\left.10^{\wedge} 60\right] x$

Input interpretation:
$\sqrt{\left(\frac{1}{2}(\sqrt{5}+1)\right)^{2}+\frac{\sqrt{5}}{2}} \times \frac{1729 \times 1164.2696+729}{288+\pi} \times \frac{1}{10^{61}} \int 8.03937 \times 10^{60} x d x$

Result:
$5374.04 x^{2}$

This result 5374.04 is very near to the rest mass of Strange B meson $5366.3 \pm 0.6$
We have also that:
$1 / 75 *(\operatorname{sqrt}(2.618+\operatorname{sqrt}(5) / 2) * 1729 * 1164.2696) / 1.65578) *\left(1 /\left(10^{\wedge} 63\right)\right)$ integrate [8.03937* $\left.10^{\wedge} 60\right] \mathrm{x}$

Input interpretation:
$\frac{1}{75} \times \frac{\sqrt{2.618+\frac{\sqrt{5}}{2}} \times 1729 \times 1164.2696}{1.65578} \times \frac{1}{10^{63}} \int 8.03937 \times 10^{60} x d x$

Result:
$125.945 x^{2}$
(xfrom-1.2 to 1.2)

The result 125.945 is very near to the value of the mass of Higgs boson that is $125,09 \pm 0,24$

And
$\operatorname{sqrt}(13)+\ln \left[\operatorname{sqrt}[(((\operatorname{sqrt}(5)+1) / 2)))^{\wedge} 2+\operatorname{sqrt}(5) / 2\right]^{*}\left(\left(\left(1729^{*} 1164.2696\right)+729\right)\right) /(288+\mathrm{Pi})$ * $\left(1 /\left(10^{\wedge} 61\right)\right)$ integrate $\left.\left[8.03937 * 10^{\wedge} 60\right] x\right]$
$\sqrt{13}+\log ($
$\left.\sqrt{\left(\frac{1}{2}(\sqrt{5}+1)\right)^{2}+\frac{\sqrt{5}}{2}} \times \frac{1729 \times 1164.2696+729}{288+\pi} \times \frac{1}{10^{61}} \int 8.03937 \times 10^{60} x d x\right)$

Result:
$\log \left(5374.04 x^{2}\right)+\sqrt{13}$

Input interpretation:
$\sqrt{13}+\log (5374.04)$

Result:
12.19489..

The result 12.19489 is practically equal to the value of the black hole entropy 12.19 From:

Monstrous Moonshine and the Entropy of the Smallest Black Hole
Last Update: 14th September 2008

The reason for the $j$-function being of interest is too lengthy to discuss, so suffice it to say that it has played a role in mathematics since Gauss and features strongly in number theory. It also features in the theory of elliptic curves, which provides an alternative, purely algebraic, definition. However, what we are interested in is the Laurent expansion of $j$ in powers of $q$,
$j(\tau)=\frac{1}{q}+744+196884 q+21493760 q^{2}+864299970 q^{3}+20245856256 q^{4}+\cdots$,
Finally we see the moonshine. The coefficient of ' $q$ ' is none other than the dimension of the first non-trivial representation of the Monster group (plus 1). Coincidence? It could have been, but for the fact that the subsequent coefficients of the j-function also have a simple numerical relationship with the dimensions of the Monster group's representations, as follows,
$196,884=1+196,883$
$21,493,760=1+196,883+21,296,876$
$864,299,970=1+1+196,883+196,883+21,296,876+842,609,326$
Relationships along these lines have been proved to continue for all the expansion coefficients and dimensions. Moreover, this is not all. It turns out that there are further numerical coincidences connecting the j -function and the Monster group.

The conformal theory considered has the interesting property that the cosmological constant is quantised by an integer $\mathrm{k}=1,2,3 \ldots$ The total vacuum energy of the spacetime is also quantised by this integer. The magnitude of the cosmological constant in our universe is notoriously tiny when expressed in Planck units, of order $10^{-123}$. In Witten's 3 d spacetime it is $-1 /(16 \mathrm{k})^{2}$, and hence, as well as being of different sign, is comparatively enormous in magnitude for modest vales of $k\left(\right.$ i.e. $\sim 10^{-3}$ ).

However, for any given k , there is a minimum size of black hole which can exist in this spacetime. What Witten does is to find the number of quantum states of a black hole of minimum size, and how it depends upon k . He does this by arguing that the partition function of the theory should differ from that of the corresponding classical theory only by linear terms, and that it should also be expressible as a power series in the j -function. This leads to a set of functions,

$$
\begin{aligned}
Z_{1}(q) & =j(q)=q^{-1}+196884 q+21493760 q^{2}+864299970 q^{3}+20245856256 q^{4}+\ldots \\
Z_{2}(q) & =j(q)^{2}-393767=q^{-2}+1+42987520 q+40491909396 q 2+\ldots \\
Z_{3}(q) & =j(q)^{3}-590651 j(q)-64481279 \\
& =q^{-3}+q^{-1}+1+2593096794 q+12756091394048 q^{2}+\ldots \\
Z_{4}(q) & =j(q)^{4}-787535 j(q)^{2}-8597555039 j(q)-644481279 \\
& =q^{-4}+q^{-2}+q^{-1}+2+81026609428 q+1604671292452452276 q^{2}+\ldots
\end{aligned}
$$

(where j has been redefined for convenience by omitting the constant term, 744).
The coefficient of $q$ in each of the functions $Z_{k}$ is the number of quantum states of the minimal black hole for that value of k (to an accuracy of within one or two states, at least). Thus, for $\mathrm{k}=1$, the entropy of the minimal black hole is $\ln (196884)=12.190$, whereas for $\mathrm{k}=4$ the entropy is $\ln (81026609428)=25.118$.

Now the point here is that the entropy of a black hole is also known from semiclassical arguments (i.e. neglecting the quantisation of gravity) by a formula known as the Bekenstein-Hawking formula, which in this case becomes $S=4 \pi \sqrt{k}$. So for $\mathrm{k}=1$ we expect the result $4 \pi=12.566$, which compares with Witten's 12.190 , and for $\mathrm{k}=4$ we expect $8 \pi=25.133$ which compares with Wittens' 25.118 . The comparison for the first 4 values of k is,

| k | Bekenstein- <br> Hawking | Witten | Difference <br> $\mathbf{( \% )}$ |
| :---: | :---: | :---: | :---: |
| 1 | 12.566 | 12.190 | $3.0 \%$ |
| 2 | 17.772 | 17.576 | $1.1 \%$ |
| 3 | 21.766 | 21.676 | $0.4 \%$ |
| 4 | 25.133 | 25.118 | $0.06 \%$ |

We note that the results 12.194316 and 12.198919 are very near to the value obtained from Witten for $\mathrm{k}=1$, for the entropy of a black hole, considering the $\ln$ (196884) that is 12.190

We note also that:
$1729 / 142=12,17605$
$728 / 7=104 ; \quad \ln (1729 * 104)=12.09968$
$\ln (729 / 6 * 1729)=12.255$
all results that are very near to the value of black hole entropy 12,19
Note that from the sum of the Ramanujan's numbers
$\left(14258^{3}+1+1010^{3}-1+172^{3}-1+12^{3}+1+9^{3}-1\right)$ we calculate the following expressions:
$\left.\ln \left(14258^{\wedge} 3+1-1010^{\wedge} 3-1-172^{\wedge} 3-1-12^{\wedge} 3+1-9^{\wedge} 3-1\right)-((\operatorname{sqrt}(5)+5) / 2)\right)$
$\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)$
$\frac{1}{2}(-5-\sqrt{5})+\log (2897481469606)$

Decimal approximation:
25.07682902808574525405759734333809832819946081084836122198...

Property:
$\frac{1}{2}(-5-\sqrt{5})+\log (2897481469606)$ is a transcendental number

Alternate forms:
$-\frac{5}{2}-\frac{\sqrt{5}}{2}+\log (2897481469606)$
$\frac{1}{2}(-5-\sqrt{5}+2 \log (2897481469606))$

$$
\frac{1}{2}(-5-\sqrt{5}+2 \log (2)+2 \log (1448740734803))
$$

## Continued fraction:

$[25 ; 13,62,1,5,4,1,4,1,2,1,4,4,4,1,1,2,13,5,6,2,2,1,14,2,12,1,3,3, \ldots]$
Alternative representations:
$\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)=$ $\log _{e}\left(-1-9^{3}-12^{3}-172^{3}-1010^{3}+14258^{3}\right)+\frac{1}{2}(-5-\sqrt{5})$
$\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)=$ $-\operatorname{Li}_{1}\left(2+9^{3}+12^{3}+172^{3}+1010^{3}-14258^{3}\right)+\frac{1}{2}(-5-\sqrt{5})$
$\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)=$ $\log (a) \log _{a}\left(-1-9^{3}-12^{3}-172^{3}-1010^{3}+14258^{3}\right)+\frac{1}{2}(-5-\sqrt{5})$

Series representations

$$
\begin{aligned}
& \log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)= \\
& \quad-\frac{5}{2}-\frac{\sqrt{5}}{2}+\log (2897481469605)-\sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2897481469605}\right)^{k}}{k}
\end{aligned}
$$

$\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)=$

$$
\begin{aligned}
& -\frac{5}{2}-\frac{\sqrt{5}}{2}+2 i \pi\left|\frac{\arg (2897481469606-x)}{2 \pi}\right|+ \\
& \log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k}(2897481469606-x)^{k} x^{-k}}{k} \text { for } x<0
\end{aligned}
$$

$\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)=$

$$
-\frac{5}{2}-\frac{\sqrt{5}}{2}+\left\lfloor\frac{\arg \left(2897481469606-z_{0}\right)}{2 \pi}\right\rfloor \log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)+
$$

$$
\left\lfloor\frac{\arg \left(2897481469606-z_{0}\right)}{2 \pi}\right\rfloor \log \left(z_{0}\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(2897481469606-z_{0}\right)^{k} z_{0}^{-k}}{k}
$$

Integral representations:
$\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)=$

$$
-\frac{5}{2}-\frac{\sqrt{5}}{2}+\int_{1}^{2897481469606} \frac{1}{t} d t
$$

$\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)=$

$$
-\frac{5}{2}-\frac{\sqrt{5}}{2}-\frac{i}{2 \pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{2897481469605^{-s} \Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s \text { for }-1<\gamma<0
$$

$1 / 2\left(\left(\ln \left(14258^{\wedge} 3+1-1010^{\wedge} 3-1-172^{\wedge} 3-1-12^{\wedge} 3+1-9^{\wedge} 3-1\right)-((\operatorname{sqrt}(5)+5) / 2)\right)\right)$
$\frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)$
$\frac{1}{2}\left(\frac{1}{2}(-5-\sqrt{5})+\log (2897481469606)\right)$
Decimal approximation:
$12.53841451404287262702879867166904916409973040542418061099 \ldots$
$\frac{1}{2}\left(\frac{1}{2}(-5-\sqrt{5})+\log (2897481469606)\right)$ is a transcendental number

Alternate forms:
$\frac{1}{4}(-5-\sqrt{5}+2 \log (2897481469606))$
$-\frac{5}{4}-\frac{\sqrt{5}}{4}+\frac{\log (2897481469606)}{2}$
$\frac{1}{4}(-5-\sqrt{5})+\frac{\log (2897481469606)}{2}$
Continued fraction:
$[12 ; 1,1,6,125,1,2,9,1,1,1,6,1,1,1,1,1,1,1,1,1,3,1,26,2,1,1,2,1,2, \ldots]$

$$
\begin{aligned}
& \frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)= \\
& \frac{1}{2}\left(\log _{e}\left(-1-9^{3}-12^{3}-172^{3}-1010^{3}+14258^{3}\right)+\frac{1}{2}(-5-\sqrt{5})\right)
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)= \\
& \frac{1}{2}\left(-\operatorname{Li}_{1}\left(2+9^{3}+12^{3}+172^{3}+1010^{3}-14258^{3}\right)+\frac{1}{2}(-5-\sqrt{5})\right) \\
& \frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)= \\
& \frac{1}{2}\left(\log (a) \log _{a}\left(-1-9^{3}-12^{3}-172^{3}-1010^{3}+14258^{3}\right)+\frac{1}{2}(-5-\sqrt{5})\right)
\end{aligned}
$$

Series representations:

$$
\begin{aligned}
& \frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)= \\
& -\frac{5}{4}-\frac{\sqrt{5}}{4}+\frac{\log (2897481469605)}{2}-\frac{1}{2} \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2897481469605}\right)^{k}}{k}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)= \\
& -\frac{5}{4}-\frac{\sqrt{5}}{4}+i \pi\left[\left.\frac{\arg (2897481469606-x)}{2 \pi} \right\rvert\,+\frac{\log (x)}{2}-\right. \\
& \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^{k}(2897481469606-x)^{k} x^{-k}}{k} \text { for } x<0
\end{aligned}
$$

$$
\frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)=
$$

$$
-\frac{5}{4}-\frac{\sqrt{5}}{4}+i \pi\left[\frac{\pi-\arg \left(\frac{1}{z_{0}}\right)-\arg \left(z_{0}\right)}{2 \pi}\right]+
$$

$$
\frac{\log \left(z_{0}\right)}{2}-\frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(2897481469606-z_{0}\right)^{k} z_{0}^{-k}}{k}
$$

$$
\begin{aligned}
& \frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)= \\
& -\frac{5}{4}-\frac{\sqrt{5}}{4}+\frac{1}{2} \int_{1}^{2897481469606} \frac{1}{t} d t
\end{aligned}
$$

$$
\frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)=
$$

$$
-\frac{5}{4}-\frac{\sqrt{5}}{4}-\frac{i}{4 \pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{2897481469605^{-s} \Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s \text { for }-1<\gamma<0
$$

$5+1 / 2\left(\left(\ln \left(14258^{\wedge} 3+1-1010^{\wedge} 3-1-172^{\wedge} 3-1-12^{\wedge} 3+1-9^{\wedge} 3-1\right)-\right.\right.$ $((\operatorname{sqrt}(5)+5) / 2)))$
$5+\frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)$
Exact result:
$5+\frac{1}{2}\left(\frac{1}{2}(-5-\sqrt{5})+\log (2897481469606)\right)$

Decimal approximation:
$17.53841451404287262702879867166904916409973040542418061099 \ldots$
$5+\frac{1}{2}\left(\frac{1}{2}(-5-\sqrt{5})+\log (2897481469606)\right)$ is a transcendental number
$\frac{1}{4}(15-\sqrt{5}+2 \log (2897481469606))$
$\frac{15}{4}-\frac{\sqrt{5}}{4}+\frac{\log (2897481469606)}{2}$
$\frac{1}{4}(15-\sqrt{5})+\frac{\log (2897481469606)}{2}$

Continued fraction:
$[17 ; 1,1,6,125,1,2,9,1,1,1,6,1,1,1,1,1,1,1,1,1,3,1,26,2,1,1,2,1,2, \ldots]$

Alternative representations:

$$
\begin{aligned}
5 & +\frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)= \\
5 & +\frac{1}{2}\left(\log _{e}\left(-1-9^{3}-12^{3}-172^{3}-1010^{3}+14258^{3}\right)+\frac{1}{2}(-5-\sqrt{5})\right)
\end{aligned}
$$

$$
\begin{aligned}
& 5+\frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)= \\
& 5+\frac{1}{2}\left(-\mathrm{Li}_{1}\left(2+9^{3}+12^{3}+172^{3}+1010^{3}-14258^{3}\right)+\frac{1}{2}(-5-\sqrt{5})\right) \\
& 5+\frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)= \\
& 5+\frac{1}{2}\left(\log (a) \log _{a}\left(-1-9^{3}-12^{3}-172^{3}-1010^{3}+14258^{3}\right)+\frac{1}{2}(-5-\sqrt{5})\right)
\end{aligned}
$$

Series representations:

$$
\begin{aligned}
& 5+\frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)= \\
& \frac{15}{4}-\frac{\sqrt{5}}{4}+\frac{\log (2897481469605)}{2}-\frac{1}{2} \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2897481469605}\right)^{k}}{k} \\
& 5+\frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)= \\
& \frac{15}{4}-\frac{\sqrt{5}}{4}+i \pi\left|\frac{\arg (2897481469606-x)}{2 \pi}\right|+\frac{\log (x)}{2}- \\
& \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^{k}(2897481469606-x)^{k} x^{-k}}{k} \text { for } x<0
\end{aligned}
$$

$$
5+\frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)=
$$

$$
\frac{15}{4}-\frac{\sqrt{5}}{4}+i \pi\left[\left.\frac{\pi-\arg \left(\frac{1}{z_{0}}\right)-\arg \left(z_{0}\right)}{2 \pi} \right\rvert\,+\right.
$$

$$
\frac{\log \left(z_{0}\right)}{2}-\frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(2897481469606-z_{0}\right)^{k} z_{0}^{-k}}{k}
$$

Integral representations:
$5+\frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)=$ $\frac{15}{4}-\frac{\sqrt{5}}{4}+\frac{1}{2} \int_{1}^{2897481469606} \frac{1}{t} d t$

$$
5+\frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)=
$$

$$
\frac{15}{4}-\frac{\sqrt{5}}{4}-\frac{i}{4 \pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{2897481469605^{-s} \Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s \text { for }-1<\gamma<0
$$

$9+1 / 2\left(\left(\ln \left(14258^{\wedge} 3+1-1010^{\wedge} 3-1-172^{\wedge} 3-1-12^{\wedge} 3+1-9^{\wedge} 3-1\right)-\right.\right.$ $((\operatorname{sqrt}(5)+5) / 2)))$
$9+\frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)$
$9+\frac{1}{2}\left(\frac{1}{2}(-5-\sqrt{5})+\log (2897481469606)\right)$
Decimal approximation:
$21.53841451404287262702879867166904916409973040542418061099 \ldots$
$9+\frac{1}{2}\left(\frac{1}{2}(-5-\sqrt{5})+\log (2897481469606)\right)$ is a transcendental number

$$
\frac{1}{4}(31-\sqrt{5}+2 \log (2897481469606))
$$

Enlarge Data Customize A Plaintext Interactive
$\frac{31}{4}-\frac{\sqrt{5}}{4}+\frac{\log (2897481469606)}{2}$
$\frac{1}{4}(31-\sqrt{5})+\frac{\log (2897481469606)}{2}$

Continued fraction:
$[21 ; 1,1,6,125,1,2,9,1,1,1,6,1,1,1,1,1,1,1,1,1,3,1,26,2,1,1,2,1,2, \ldots]$

Alternative representations

$$
\begin{aligned}
9+ & \frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)= \\
& 9+\frac{1}{2}\left(\log _{e}\left(-1-9^{3}-12^{3}-172^{3}-1010^{3}+14258^{3}\right)+\frac{1}{2}(-5-\sqrt{5})\right)
\end{aligned}
$$

$$
\begin{aligned}
& 9+\frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)= \\
& 9+\frac{1}{2}\left(-\operatorname{Li}_{1}\left(2+9^{3}+12^{3}+172^{3}+1010^{3}-14258^{3}\right)+\frac{1}{2}(-5-\sqrt{5})\right) \\
& 9+\frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)= \\
& 9+\frac{1}{2}\left(\log (a) \log _{a}\left(-1-9^{3}-12^{3}-172^{3}-1010^{3}+14258^{3}\right)+\frac{1}{2}(-5-\sqrt{5})\right)
\end{aligned}
$$

Series representations:

$$
\begin{aligned}
& 9+\frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)= \\
& \frac{31}{4}-\frac{\sqrt{5}}{4}+\frac{\log (2897481469605)}{2}-\frac{1}{2} \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2897481469605}\right)^{k}}{k} \\
& 9+\frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)= \\
& \frac{31}{4}-\frac{\sqrt{5}}{4}+i \pi\left|\frac{\arg (2897481469606-x)}{2 \pi}\right|+\frac{\log (x)}{2}- \\
& \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^{k}(2897481469606-x)^{k} x^{-k}}{k} \text { for } x<0
\end{aligned}
$$

$$
9+\frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)=
$$

$$
\frac{31}{4}-\frac{\sqrt{5}}{4}+i \pi\left[\frac{\pi-\arg \left(\frac{1}{z_{0}}\right)-\arg \left(z_{0}\right)}{2 \pi}\right]+
$$

$$
\frac{\log \left(z_{0}\right)}{2}-\frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(2897481469606-z_{0}\right)^{k} z_{0}^{-k}}{k}
$$

Integral representations:
$9+\frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)=$ $\frac{31}{4}-\frac{\sqrt{5}}{4}+\frac{1}{2} \int_{1}^{2897481460606} \frac{1}{t} d t$

$$
\begin{array}{r}
9+\frac{1}{2}\left(\log \left(14258^{3}+1-1010^{3}-1-172^{3}-1-12^{3}+1-9^{3}-1\right)-\frac{1}{2}(\sqrt{5}+5)\right)= \\
\frac{31}{4}-\frac{\sqrt{5}}{4}-\frac{i}{4 \pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{2897481469605^{-s} \Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s \text { for }-1<\gamma<0
\end{array}
$$

The results obtained $25,07612,53817,538$ and 21,538 are very near to the various values of the black hole entropy as showed in the following table:

| k | Bekenstein- <br> Hawking | Witten | Difference <br> $\mathbf{( \% )}$ |
| :---: | :---: | :---: | :---: |
| 1 | 12.566 | 12.190 | $3.0 \%$ |
| 2 | 17.772 | 17.576 | $1.1 \%$ |
| 3 | 21.766 | 21.676 | $0.4 \%$ |
| 4 | 25.133 | 25.118 | $0.06 \%$ |

$$
\begin{aligned}
& \text { sf } \\
& \text { (i) } \frac{1+53 x+9 x^{2}}{1-82 x-82 x^{2}+x^{3}}=a_{0}+a_{1} x+a_{2} x^{2}+a_{3} x^{3}+ \\
& \text { or } \frac{\alpha_{0}}{x}+\frac{\alpha_{1}}{x^{2}}+\frac{\alpha_{L}}{x^{0}}+\text {. } \\
& \text { (ii) } \frac{2-26 x-12 x^{2}}{1-82 x-84 x^{2}+x^{3}}=L_{0}+4, x+4 x_{2} x^{2}+4 x+ \\
& \text { or } \frac{\beta_{0}}{x}+\frac{\beta_{1}}{x^{L}}+\frac{\beta_{L}}{x^{0}}+ \\
& \text { (iii) } \frac{2+8 x-10 x^{2}}{1-82 x-82 x^{2}+x^{3}}=c_{0}+c_{1} x+c_{2} x^{2}+c_{0} x^{3}+ \\
& \text { or } \frac{x_{0}}{x}+\frac{x_{1}}{x_{2}}+\frac{x_{2}}{x_{0}}+ \\
& \text { then } \\
& \left.a_{n}^{3}+a_{n}^{3}=c_{n}^{3}+(-1)^{n}\right\} \\
& \text { and } \left.\quad \alpha_{n}^{3}+\beta_{n}^{3}=\gamma_{n}^{3}+(-1)^{n}\right\} \\
& \text { Examples } \\
& 135^{3}+138^{3}=172^{3}-1 \\
& 11161^{3}+11468^{3}=1425-8^{3}+1 \\
& 791^{3}+812^{3}=1010^{3}-1 \\
& 9^{3}+10^{3}=12^{3}+1 \\
& 6^{3}+8^{3}=9^{3}-1
\end{aligned}
$$

## Conclusion

From the different results highlighted during this research, it is possible to propose the number 1729 (and also the 728), defined by the mathematical genius S . Ramanujan as "very interesting", which plays a fundamental role in number theory, as a new physical constant from which emerge various properties of the Standard Model particles, including the masses, and also the mass values of the "glueballs" and also in many cases, the value of the entropy of black holes. Since the entropy of black holes also takes negative values, we tend to propose that they be white holes. For supersymmetry, as for each particle there is a superpartner, with each black hole there is a white hole. As from a black hole nothing can come out, from a white hole the reverse happens. It is therefore easy to think that all white holes are big bang singularities. In reality, not all the black holes that evaporate pass the information to the corresponding white holes from which possible bubble universes will emerge, but only a well-defined number that will form the universes subsets of the multiverse. For all others, information will pass directly into the infinite-dimensional Hilbert space. This further strengthens the proposal of a multiverse composed of a very high but finite set of bubbles (perhaps $8.08 * 10 \wedge 53$ ). Once the expansion-acceleration phase is complete, every bubble-universe of the multiverse becomes the final phase, when each galaxy, star, etc. ends its cycle, an immense black hole. The final giant n-black holes, connected to each other in a sort of entanglement, as happens for the particles, will evaporate simultaneously in an incalculable but finite time, passing once they become infinitely small, more than an atomic nucleus, (symmetry with the initial singularity) the n -information in the infinite-dimensional space. The evident similar behavior of the physics of black holes and particles, even in the entanglement effect, could explain the evident connection that is obtained from the equations of the physics of subatomic particles inherent to the Standard Model, whose solutions are very close and often even equal to the entropy value of a black hole. This with the appropriate use of Ramanujan's mathematics which can then be applied to both black holes and particle physics. This could also be a further indication that elementary particles, such as electrons, mediators of fundamental forces, massive and scalar bosons and glueballs, are in fact a sort of quantum black holes. All these connections obtained by integrating and / or using different equations from various expressions of Ramanujan in different ways, reinforce our belief that this mathematics can be the way to go to reach a sort of "mathematical TOE"

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[^0]:    ${ }^{11}$ M.Nardelli have studied by Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10-80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" Università degli Studi di Napoli "Federico II" - Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

[^1]:    ${ }^{2}$ Note in Italian "La distribuzione Breit-Wigner relativistica (chiamata così dai nomi di Gregory Breit e Eugene Wigner) è una distribuzione di probabilità continua con la seguente funzione di densità di probabilità

    $$
    f(E) \sim \frac{1}{\left(E^{2}-M^{2}\right)^{2}+M^{2} \Gamma^{2}}
    $$

    (Questa equazione è scritta usando unità naturali, $\hbar=c=1$.) Viene molto più spesso usata per modellare le risonanze (particelle instabili) nella fisica ad alta energia. In questo caso $E$ è l'energia del centro di massa che produce la risonanza, $M$ è la massa della risonanza, e $\Gamma$ è la larghezza di risonanza (o larghezza di decadimento "decay width"), relativa alla sua vita media secondo la formula $\tau=\hbar / \Gamma$. La probabilità di produrre risonanza a una data energia $E$ è proporzionale a $\mathrm{f}(E)$, in modo che il grafico del tasso di produzione della particella instabile in funzione dell'energia tracci la forma della distribuzione Breit-Wigner relativistica.

    In generale, $\Gamma$ può essere anche una funzione di $E$; questa dipendenza è in genere importante solo quando $\Gamma$ non è piccola in confronto a $M$ e la dipendenza spazio-fase della larghezza va presa in considerazione. (Per esempio, nel decadimento del mesone rho in una coppia di pioni.) Il fattore $M^{2}$ che moltiplica $\Gamma^{2}$ andrebbe sostituito con $E^{2}$ (o $E^{4} / M^{2}$, ecc.) quando la risonanza è ampia. ${ }^{[2]}$

    La forma della distribuzione Breit-Wigner relativistica sorge dal propagatore di una particella instabile, che ha un denominatore della forma $p^{2}-M^{2}$ $+\mathrm{i} \Gamma$. Qui $\mathrm{p}^{2}$ è il quadrato del quadrimpulso portato dalla particella. Il propagatore appare nella ampiezza della meccanica quantistica per il processo che produce la risonanza; la distribuzione della probabilità risultante è proporzionale al quadrato assoluto dell'ampiezza, producendo la distribuzione Breit-Wigner relativistica per la funzione di densità della probabilità come descritta precedentemente.

    La forma di questa distribuzione è simile alla soluzione dell'equazione classica del moto per una oscillatore armonico smorzato (dumped) condotto da una forza esterna sinusoidale"

[^2]:    Plot of the integral:

[^3]:    ${ }^{3}$ Note in Italian "In conclusione, dimostriamo che molte proprietà dei fenomeni di bassa energia, come il forte accoppiamento in movimento, i processi di adronizzazione, la generazione di massa per stati legati a quark-antiquark e di di-gluon possono essere spiegati ragionevolmente all'interno di un modello ispirato alla QCD con propagatori confinati con infrarossi. Abbiamo derivato un'equazione di massa del mesone e sfruttandola ha rivelato un nuovo comportamento specifico dell'accoppiamento forte $\alpha S(M)$ in dipendenza della scala di massa. È stato trovato un punto di congelamento a infrarossi $\alpha S(0)=1.03198$ all'origine $M=0$ non dipendente dalla particolare scelta della scala di confinamento $\Lambda>0$. È stata eseguita una nuova stima della massa di glueball più bassa (scalare) e è stato trovato $\mathrm{a} \approx 1739 \mathrm{MeV}$. È stata calcolata anche la "dimensione" del glueball scalare: $\mathrm{rG} \approx 0,51 \mathrm{fm}$. E stato ottenuto anche un valore non banale del condensato di gluone. Abbiamo stimato lo spettro dei mesoni convenzionali introducendo un set minimo di parametri: quattro masse di quark costituenti ( $u=d, s, c, b$ ) e $\Lambda$. I valori ottenuti si adattano agli ultimi dati sperimentali con errori relativi inferiori all' $1,8 \%$. Sono state inoltre eseguite stime accurate delle costanti di decadimento dei mesoni pseudoscalari e vettoriali"

[^4]:    Input interpretation.
    $\frac{(1728+216) \times 1164.2696}{10^{10}}$
    $\int(-47.23265-58.8742714-382.257106+16507.8183+139489-$ $139468-2209694+2085$ 349) $x d x$

[^5]:    Input interpretation:

[^6]:    Plot of the integral:

