Proof of Twin Prime Conjecture that can be obtained by using Contradiction method in Mathematics

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Author’s Biography

The author of this research paper is K.H.K. Geerasee Wijesuriya . And this proof of twin prime conjecture is completely K.H.K. Geerasee Wijesuriya's proof.

Geerasee she studied before at Faculty of Science, University of Colombo Sri Lanka. And she graduated with BSc (Hons) in Physics and Mathematics from the University of Colombo, Sri Lanka in 2014. And in March 2018, she completed her first Doctorate Degree in Physics with first class recognition. Now she is following her second PhD in Astrophysics with Belarusian National Technical University.

Geerasee has been invited by several Astronomy/Physics institutions and organizations world-wide, asking to get involve with them. Also, She has received several invitations from some private researchers around the world asking to contribute to their researches. She worked as Mathematics tutor/Instructor at Mathematics department, Faculty of Engineering, University of Moratuwa, Sri Lanka. Now she is a research scientist in Physics as her career. Furthermore she has achieved several other scientific achievements already.

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Abstract

Twin prime numbers are two prime numbers which have the difference of 2 exactly. In other words, twin primes is a pair of prime that has a prime gap of two. Sometimes the term twin prime is used for a pair of twin primes; an alternative name for this is prime twin or prime pair. Up to date there is no any valid proof/disproof for twin prime conjecture. Through this research paper, my attempt is to provide a valid proof for twin prime conjecture.

Literature Review

The question of whether there exist infinitely many twin primes has been one of the great open questions in number theory for many years. This is the content of the twin prime conjecture, which states that there are infinitely many primes p such that p + 2 is also prime. In 1849, de Polignac made the more general conjecture that for every natural number k, there are infinitely many primes p such that p + 2k is also prime. The case k = 1 of de Polignac's conjecture is the twin prime conjecture.

A stronger form of the twin prime conjecture, the Hardy–Littlewood conjecture, postulates a distribution law for twin primes akin to the prime number theorem. On April 17, 2013, Yitang Zhang announced a proof that for some integer N that is less than 70 million, there are infinitely many pairs of primes that differ by N. Zhang's paper was accepted by Annals of Mathematics in early May 2013. Terence Tao subsequently proposed a Polymath Project collaborative effort to optimize Zhang's bound. As of April 14, 2014, one year after Zhang's announcement, the bound has been reduced to 246. Further, assuming the Elliott–Halberstam conjecture and its generalized form, the Polymath project wiki states that the bound has been reduced to 12 and 6, respectively. These improved bounds were discovered using a different approach that was simpler than Zhang's and was discovered independently by James Maynard and Terence Tao.

Assumption

Let's assume that there are finitely many twin prime numbers……………(1.0)

Therefore we proceed by considering that there are finitely many twin prime numbers. Then let the highest twin prime numbers are P_{n-1} and $(P_{n-1} + 2)$. Then for all prime numbers P_N greater than $(P_{n-1} + 2)$, $(P_N + 2)$ is not a prime number.

Methodology

With this mathematical proof, I use the contradiction method to prove that there are infinitely many twin prime numbers.

Let P_n is an odd number greater than 1. But let P_3 is divisible by x_3. But x^2_3 does not divide P_3. And let P_n is not divisible by x_3. We choose P_n such that $P_n = (M + 4) – (D_0.P_3 / x_3)$; for some integer $D_0 \neq 0$. Where D_0 is not divisible by x_3 . But D_0 is divisible by B_2.

To see the meaning of P_3 , x_3, B_2 and M, please refer the below content.

Let P_N is an arbitrary prime number greater than $(P_{n-1} + 2)$. Because there are infinitely many prime numbers. And here $(P_N – 2) > (P_{n-1} + 2)$. Thus $(P_N – 2)$ is not a prime number.

Where each arbitrary P_N prime number obey (that means we choose a set of P_N (> $(P_{n-1} + 2)$) arbitrarily such that each arbitrarily chosen P_N give us :

1. P_3 is divisible by x_3. But x^2_3 does not divide P_3 ; whenever $(P_N – 2) = P_3.x_3$
2. We choose x_3 as it gives $P_L \mid (x_3 – 1)$ for some P_L an odd integer greater than 0 ; whenever $(P_N – 2) = P_3.x_3$

And here $(P_N + 2) > (P_{n-1} + 2)$. Then according to our assumption, $(P_N + 2)$ is also not a prime number. Here P_N is a prime number such that $(P_N + 2)$ is dividing by prime number P_2. …………………(1)

Thus $(P_N + 2) = P_2 * x_2$ for some x_2 natural number. Since P_N is a prime number, for some r_2 (rational number which is not a natural number): $P_N / r_2 = P_2$. Thus $(P_N + 2) = P_2 * x_2$ ………………(02) and $P_N = r_2 * P_2$ ……………….(03)

x_2 is a natural number and P_2 is a prime number. Since P_N is a prime number , $(P_N – 2)$ is also not a prime number (Since $P_N – 2 > P_{n-1} + 2$). Then for some integer P_3 greater than 1 such that $(P_N – 2) / P_3 = x_3$; where x_3 is an integer greater than 1. But here we considered that $x_3 | P_3$.

But we must have chosen x_3 and P_3 such that as they give $P_L | (x_3 – 1)$ for some P_L an odd integer (not equals to 0). Then $P_L | [P_L - (x_3 - 1)]$ ……….(3.1)

$(P_N – 2) = P_3 * x_3$ ……………………..(04)

But $(P_N + 2)$, P_n both are odd numbers. Thus $(P_N + 2) = P_n + 2.l$; for some l integer (where $l \neq 0$) ……………………..(05)

Then $(P_N - 2) = P_n + 2.l - 4 = P_n + 2 . (l - 2)$ ....(6.1)’

And we know that $(P_N + 2) = P_n + 2.l \rightarrow P_N = P_n + 2.l - 2$ ……………………(*)

Thus by (*): $P_n + 2.l - 2 = P_N$. Thus by (04) and (*): $P_3 * x_3 + 2 = P_n + 2.l - 2$

Thus $P_3 * x_3 - 2.l + 4 = P_n$ ....(6.1.0)

Thus $P_3 * x_3 + 2. (l - 2) = P_n + 4. (l - 2) = P_n + 2.P_N - 4 - 2.P_n = 2.P_N – 4 – P_n$ (by (6.1)’)

Thus $P_3 * x_3 + 2. (l - 2) = 2.P_N – 4 – P_n = P_{n''}$

Thus $P_3 * x_3 + 2. (l - 2) = P_{n''} = 2. P_3 * x_3 - P_n$ ....(7)

Thus $P_3 * x_3 +2.l = 4 + 2. P_3 * x_3 - P_n$

$P_3 * x_3 + (2.l + M) = (4 + M - P_n) + 2. P_3 * x_3$; Where M is an integer ($M \neq 0$)

$(2.l + M) = (4 + M - P_n) + P_3 * x_3$; Where M is an integer $\neq 0$………(8)

But we chose M such that (M + 4) is divisible by x_3. But let (M + 4) is not divisible by P_3.

But we know that P_3 is divisible by x_3. But x^2_3 does not divide P_3. And we know that $(P_3 * x_3)$ is divisible by x_3. And we know that P_n is not divisible by x_3................(8.1) .

Thus by (8): x_3 does not divide ($2.l + M$). Since P_3 is divisible by x_3, P_3 does not divide ($2.l + M$) …………….(i)

But P_N is an arbitrary prime greater than $(P_{n-1} + 2)$. Then let $\{ (P_N + A_1) , P_N \}$ are two arbitrary consecutive primes set such that each primes are greater than $(P_{n-1} + 2)$. Where each arbitrary P_N prime number obey:

1. P_3 is divisible by x_3. But x^2_3 does not divide P_3 ; whenever $(P_N – 2) = P_3.x_3$
2. We choose x_3 as it gives $P_L \mid (x_3 – 1)$ for some P_L an odd integer greater than 0 ; whenever $(P_N – 2) = P_3.x_3$

Here since $P_N > (P_{n-1} + 2)$ and since $(P_N + A_1) > (P_{n-1} + 2)$, $A_1 \neq (+/-) 2$. Because for any two arbitrary consecutive primes greater than $(P_{n-1} + 2)$, the difference between those consecutive primes is greater than 2 (since the greatest twin primes are P_{n-1} and $[P_{n-1} + 2]$).

But $A_1 \neq 2.(x_3\text{-}1)$. But now choose two particular two consecutive primes (greater than $(P_{n-1} + 2)$) from the arbitrary prime number set $\{ (P_N + A_1) , P_N \}$ such that those chosen two particular consecutive primes obey $[P_3 \mid (A_1 – 2)]$. i.e. where particularly we choose A_1 such that $P_L – x_3 = B_2$. Where $(A_1 - 2) / P_3 = B_2$. But by (3.1), $P_L \mid [P_L – (x_3 - 1)]$. But $P_L – x_3 = B_2$. Thus $P_L \mid (B_2 + 1)$. Where $P_L = (P - 4) / P_3$. Here P = chosen particular prime $(P_N + A_1)$. **Since $A_1 \neq - 2$, there exists an odd number P_3 greater than 1 such that $[P_3 \mid (A_1 – 2)]$. Refer the 'Proof' below to see the existence of two consecutive primes $(P_N + A_1)$ and P_N such that $[P_3 \mid (A_1 – 2)]$. And refer 'Proof 1' to see the existence of an integer $(P_N - 2)$ such that $(P_N - 2) = P_3.x_3$ such that P_3 is divisible by x_3. But x^2_3 does not divide P_3.**

But we know that $(P_N + A_1) > (P_{n-1} + 2)$. Thus here $A_1 \neq (+/-) 2$, since there are finite number of twin primes according to our assumption. BUT REMEMBER THAT P_N AND $(P_N + A_1)$ ARE CONSECUTIVE PRIMES greater than $(P_{n-1} + 2)$.

{ Here $(P_N – 2) = P_3.x_3$ and $(P_N + A_1) = P$ = Prime. That means $P_3.x_3 + (A_1 + 2) = P$

But $(A_1 - 2)$ is divisible by P_3 . Thus $(A_1 + 2)$ is not divisible by P_3. Because P_3 does not divide 4.

But since $P_3 * x_3$ is divisible by P_3 , P is not divisible by P_3.

But $(A_1 - 2)$ is divisible by P_3 and since $(x_3 \mid P_3)$, $x_3 \mid (A_1 - 2)$. Thus $(A_1 + 2)$ is not divisible by x_3. Because x_3 does not divide 4 since x_3 is an odd number (since $(P_N – 2) = P_3.x_3$).

But since $P_3 * x_3$ is divisible by x_3 , P is not divisible by x_3.

But $P = P_3.x_3 + A_1 + 2 \neq P_3.x_3 + 2.(x_3 - 1) + 2 = P_3.x_3 + 2.x_3 = x_3.(P_3 + 2)$. Thus $P \neq x_3.(P_3 + 2)$.

Therefore according to above steps, we can write $P_3.x_3 + (A_1 + 2) = P$ as a prime }

But $(2.l + M) = P_N - P_n + 2 + M = (P_N + A_1) + (M + 2 - A_1 - P_n)$…………..(9)

By (8.1): $x_3 \mid (M + 4)$. But $[P_3 \mid (A_1 – 2)]$. …….(10)

But since $[P_3 \mid (P_N - 2)]$ and since P_3 does not divide $(A_1 + 2)$, $\{ (A_1 + 2) + (P_N - 2) \}$ does not divide by P_3 . i.e. P (= $(P_N + A_1)$) does not divide by P_3. Thus our choice of A_1 such that $[P_3 \mid (A_1 – 2)]$ is okay.

But $[P_3 \mid (P_N - 2)]$ and $[P_3 \mid (A_1 – 2)]$. Thus $(P_N - 2) = P_3.x_3$ and $(A_1 – 2) = P_3.B_2$; where x_3 and B_2 are integers and each of them not equals to 0.

Thus $(P_N + A_1 – 4) = P_3.x_3 + P_3.B_2 = (P – 4)$

i.e $P_3 \mid (P - 4)$………………(11)

Let's consider M integer such that $M = P – C$; for some integer 'C' $\neq 0$ ……….……….(12).

But $x_3 \mid (M + 4)$ and $P_3 \mid (P - 4)$ by (8.1) and (11).

By (12): $P = (M + C)$. Thus $[(M + C)] – 4 = P_3 . P_L$ …………(13)

Where $P_L = [(P - 4) / P_3]$ = integer, but not equals to 0.

Then $(P_L / x_3) = [(P - 4) / (x_3 P_3)] = (P – 4) / (P_N - 2)$

$= [(P_N - 2) + (A_1 - 2)] / (P_N - 2) = 1 + [(A_1 - 2) / (P_N - 2)]$

If $A_1 < 0$, then either $1 + [(A_1 - 2) / (P_N - 2)] < 0$ or $0 < 1 + [(A_1 - 2) / (P_N - 2)] < 1$.

But $P_L = [(P - 4) / P_3]$. Since $(P - 4) > 0$ and since $x_3 > 1$, $P_3 > 1$; $P_L > 0$. Thus it is impossible to have $1 + [(A_1 - 2) / (P_N - 2)] < 0$. Thus if $A_1 < 0$, we have $0 < 1 + [(A_1 - 2) / (P_N - 2)] < 1$. Then (P_L / x_3) is not an integer. If $A_1 > 0$, then $A_1 > 2$. Then $(P_N – A_1) < (P_N – 2)$……….(13)'

Then $(P_L / x_3) = 1 + [(A_1 - 2) / (P_N - 2)] = 1 + [(A_1 - P_N + P_N - 2) / (P_N - 2)]$

$= 2 + [(A_1 - P_N) / (P_N - 2)] = 2 - [(P_N - A_1) / (P_N - 2)]$.

Since $A_1 \neq (+ / -) 2$ and by (13)' : $[(P_N – A_1) / (P_N - 2)]$ is not an integer.

Then $2 - [(P_N - A_1) / (P_N - 2)] = (P_L / x_3)$ is not an integer.

Then for $A_1 > 0$ and for $A_1 < 0$: (P_L / x_3) is not an integer.

Thus here x_3 does not divide P_L…………………(13.1)

But $[(M + 4) / x_3] = P_Q$ = integer, but not equals to 0.

Thus by (13): $[(x_3 .P_Q – 4 + C)] – 4 = P_3. P_L$

Thus $C - 4 = [(P_3 .P_L + 4) – x_3 .P_Q]$ ……………………….(14)

By (09): $(2.l + M) = (P_N + A_1) + (M + 2 - A_1 - P_n) = P + (M + 2 - A_1 - P_n)$

$= P + P – C + 2 - A_1 - P_n = 2.P – C – P_3.B_2 - P_n$ ………..(15)

By (14): $C = [(P_3 .P_L + 8) – x_3 .P_Q]$. Then $2.P – C – P_n = 2.P + x_3 .P_Q - (P_3 .P_L + 8) - P_n$

$= 2.(P - 4) + x_3 .P_Q - P_n - (P_3 .P_L) = 2. P_3 .P_L + x_3 .P_Q - P_n - (P_3 .P_L) = P_3.P_L + [x_3 .P_Q - P_n]$

$= P_3.P_L + P_3. [x_3 .(P_Q / P_3) - (P_n / P_3)]$……………………..(16)

But we chose P_n such that $(x_3. P_n) = x_3. (M + 4) – D_0.P_3$; for some integer D_0.

But we choose D_0 integer such that $x^2_3 \mid (P_L.x_3 + D_0))$. Where $D_0 \neq 0$ and D_0 is not divisible by x_3.

Since P_L is not divisible by x_3 , there exists such an integer number D_0 (other than zero), such that $x^2_3 \mid (P_L.x_3 + D_0)$. Whenever D_0 is not divisible by x_3.

To see the proof that proves that there exists an integer D_0 ($\neq 0$) such that $x^2_3 \mid (P_L.x_3 + D_0)$, please refer 'Proof 2' below.

Then $P_n = (M + 4) - (D_0.P_3 / x_3)$. Then $[x_3 .(P_Q / P_3) - (P_n / P_3)] = (D_0 / x_3)$

Then by (16): $2.P – C – P_n = P_3.(P_L + (D_0/ x_3)) = P_3. x_3 [(P_L / x_3) + (D_0 / x^2_3)]$; where (P_L / x_3) and (D_0 / x^2_3) are not integers (by 13.1). But we choose D_0 such that $x^2_3 \mid (P_L.x_3 + D_0)$.

Then $2.P – C – P_n = P_3. x_3 [[(P_L . x_3) + D_0] / x^2_3] = P_3. x_3 . D''$;

where $D'' = (P_L.x_3 + D_0) / x^2_3$ = integer , but not equals to 0.

Then $P_3 \mid (2.P – C – P_n)$………………………..(17)

by (15), (17): $P_3 \mid (2.l + M)$………………(18)

Thus by (i): P_3 does not divide $(2.l + M)$…………………….(19). Thus by (18) and (19): We have a contradiction………………..(20)

Therefore the only possibility is: our assumption (1.0) is false. Therefore there are infinitely many Twin Prime Numbers.

Proof

Let's prove that there exists consecutive primes P_N and $(P_N + A_1)$ such that $[P_3 \mid (A_1 – 2)]$ for some odd integer P_3 which is not equal to 1 (when there exist consecutive prime numbers P_N and $(P_N + A_1)$ which both are greater than $[P_{n-1} + 2]$) through this 'Proof' as below.

By 2nd reference: $P_{N-1} = (P_N + A_1) = 2 + \sum_{j=1}^{N-2} hj$,where $h_j = P_{j+1} – P_j$ for all $j \in \{1 , 2,.., (N-2) \}$ or $P_{N+1} = (P_N + A_1) = 2 + \sum_{j=1}^{N} hj$ when $j \in \{1 , 2,…., N\}$. Here $(P_N + A_1) = P_{N+1}$ or P_{N-1} , depends on the sign of A_1.

$P_{N-1} = P_N + A_1 = 2 + \sum_{j=1}^{N-2} hj$ (when $A_1 < 0$). If $A_1 > 0$, $P_{N+1} = P_N + A_1 = 2 + \sum_{j=1}^{N} hj$

Consider the case that $A_1 < 0$.

Then $(A_1 – 2) = - P_N + \sum_{j=1}^{N-2} hj$. Then $(2 – A_1) = P_N – \sum_{j=1}^{N-2} hj$

Then $(2 – A_1) = P_N - k' – \sum_{j=1}^{N-2} hj$ ……………………(21) Because here inside the term $\sum_{j=1}^{N-2} hj$, I have included (+ k') term.

But by 2nd reference: for all $\epsilon > 0$, there is a natural number 'm' such that for all (N- 2) > m;

$h_{N-2} < P_{N-2} . \epsilon$

Let ϵ_s is a positive real number $\epsilon_s = [- B + C_s + k' + P_N + B_2. P_3] / P_s > 0$, such that $h_s < P_s{*} \epsilon_s$ for all s > (N - 3) . But here $P_L \mid (B_2 + 1)$. Let here the chosen ϵ_s implies that m = (N - 3) (Here s is going from 1 to (N - 2). Then " for all s > (N - 3)" means s = (N – 2). Where k' is an integer number which not equals to 0 and we choose k' such that k' / (N - 2) is an integer. Here the chosen k' integer number is responsible for $h_s < P_s * Є_s$ for all s > (N - 3) (i.e. s = N- 2) and $Є_s > 0$. That means here the value of k' is responsible to say " $Є_s$ is existing such that $h_s < P_s * Є_s$,

for s = (N - 2) ". Here $h_j = b_j$ - [k'/ (N- 2)] for all j < (N – 2) = s. And where $\Sigma\, b_j = B$ for j < (N – 2) = s. Then for some C_s , $h_s = P_s * Є_s - C_s$; here s ≡ (N – 2). *** the meaning of 'j' is the order number and h_j is the prime gap between P_{j+1} and P_j . Please refer the below content and the 2nd reference. But here we chose C_{N-2} such that $h_{N-2} = P_{N-2} * Є_{N-2} - C_{N-2}$

But $h_{N-2} = P_{N-2} * Є_{N-2} - C_{N-2} = (- B + k' + P_N + B_2 . P_3)$. Where k' is a natural number. Now let's use the 2nd reference to proceed further. By (21):

$(A_1 - 2) = - P_N + \sum_{j=1}^{N-2} hj = - P_N + (- B + k' + P_N + B_2 . P_3) + B - (N - 2).[k'/ (N- 2)] = B_2 . P_3$ …………………………..(22)

Thus by (22): $(A_1 - 2) = P_3 . B_2$. Thus there exist consecutive prime numbers P_N and $(P_N + A_1)$ both greater than $(P_{n-1} + 2)$ such that $(A_1 - 2) = B_2 . P_3$; for integer B_2 (≠ 0).

And here we chose integer P_L such that $P_L \mid (B_2 + 1)$.

Similar to above, if $A_1 > 0$, we can proceed with the similar steps to prove that $(A_1 - 2) = B_2 . P_3$; for integer B_2 (≠ 0) when $A_1 > 0$.

Proof 1

Let's prove the existence of an integer $(P_N - 2)$ (> $P_{n-1} + 2$) such that $(P_N - 2) = P_3 . x_3$ such that P_3 is divisible by x_3. But x^2_3 does not divide P_3 as below.

By 2nd reference: $P_N = 2 + \sum_{j=1}^{N-1} gj$, where $g_j = P_{j+1} - P_j$ for all j ϵ {1 , 2, ……., N- 1 }

Then $(P_N - 2) = \sum_{j=1}^{N-1} gj$ ……………………….(23)

But by 2nd reference: for all Є > 0, there is a natural number 'm_0' such that for all N > m_0; $g_N < P_N . Є$.

Let $Є_s$ is a positive real number $Є_s = [- A + C_s + x^2_3 . k_1] / P_s > 0$, such that $h_s < P_s * Є_s$ for all s > (N - 2) . Let here the chosen $Є_s$ implies that m_0 = (N - 2) (Here s is going from 1 to N-1. Then

" for all s > (N - 2)" means s = (N – 1)). Where k_1 is an integer number which is not divisible by x_3. Here the chosen k_1 integer number ($\neq 0$) is responsible for $g_s < P_s * \epsilon_s$ for all s > (N - 2) (i.e. s = N- 1) and $\epsilon_s > 0$. That means here the value of k_1 is responsible to say " ϵ_s is existing such that $g_s < P_s * \epsilon_s$, for s = N - 1 ". Here $g_j = a_j$ for all j < (N – 1) = s. And where $\Sigma a_j = A$ for j < (N – 1) = s. Then for some C_s , $g_s = P_s * \epsilon_s - C_s$; here s ≡ (N – 1). *** the meaning of 'j' is the order number and g_j is the prime gap between P_{j+1} and P_j . Please refer the below content and the 2nd reference. But here we chose C_{N-1} such that $g_{N-1} = P_{N-1} * \epsilon_{N-1} - C_{N-1}$.

But $g_{N-1} = (- A + x^2_3 . k_1)$. Now let's use the 2nd reference to proceed further. By (23):

$(P_N - 2) = \sum_{j=1}^{N-1} gj$

But $\sum_{j=1}^{N-1} gj = A + (- A + x^2_3 . k_1) = x^2_3 . k_1$ ………………….(24)

Thus by (23) and (24): $(P_N - 2) = x^2_3 . k_1$; where k_1 is not divisible by x_3.

Then $(P_N - 2) = x_3.(x_3.k_1) = x_3 . P_3$; where P_3 is divisible by x_3. But since k_1 is not divisible by x_3, P_3 is not divisible by x^2_3 .

Thus $(P_N - 2) = P_3 . x_3$; where P_3 is divisible by x_3. But P_3 is not divisible by x^2_3 . Thus there exists an integer set { $P_3 . x_3$; where P_3 is divisible by x_3 but P_3 is not divisible by x^2_3 }.

Proof 2

Now let's prove that there exists an integer D_0 ($\neq 0$) such that $x^2_3 \mid (P_L.x_3 + D_0)$.

Consider D is an integer ($\neq$0) such that $B_2 \mid D$. And let D is not divisible by x_3.

Let choose D' = (x_3 / G), D = G where D' ≠ 1 and G is an integer (≠ 0).

Then (D'.D) = x_3. Then [(1/D').x_3 - D] = 0.

Then [$(D')^2.x_3$ + 1]. [(1/D').x_3 - D] = 0 = $D'.x_3^2$ - D + [(x_3 /D') – D.$(D')^2.x_3$]

Then $D'.x_3^2$ - D + [(x_3 /D') – D.$(D')^2.x_3$] = 0.

Then $D'.x_3^2$ - D = D.$(D')^2.x_3$ - (x_3 /D') …….(24.1)

Let's consider D.$(D')^2$ – (1/D').

Then D.$(D')^2$ – (1/D') = G. $(x_3 / G)^2$ – (G / x_3) = G.[$(x_3 / G)^2$ – (1/ x_3)]

$= (G / (G^2. x_3)). [x^3{}_3 – G^2] = (1/ G.x_3). [x^3{}_3 – G^2]$…………(25)

By (25): $D.(D')^2 – (1/D') = (1/ G.x_3). [x^3{}_3 – G^2] = (1/ D.x_3). [x^3{}_3 – D^2]$

$= [(x^2{}_3 / D) – (D/ x_3)]$. But $(P_L – B_2) = x_3$. Because $(P_N – 2) = P_3.x_3 = P_3.P_L – P_3.B_2$

Thus $D.(D')^2 – (1/D') = (x^2{}_3 / D) – (D/ x_3) = (P_L – B_2)^2 / D – [D / (P_L – B_2)]$

$= [(P_L – B_2)^3 – D^2] / [D.(P_L – B_2)]$

But we chose D such that $(P_L – B_2)^3 = k^3.D^3$; k is a real number ($\neq 1$), but $k \neq (1 / T)$ for all T integers.

Then $D.(D')^2 – (1/D') = [k^3.D^3 - D^2] / (D.k.D) = k^2.D – (1/ k) = [k^3.D – 1] / k$…….(25.1)

But we chose D such that $k = (- D / B_2)$. Then $(P_L – B_2) = - D^2 / B_2$; where $B_2 | D$.

Then $k \neq (1 / T)$ for all T integer. Then $[k^3.D – 1] / k = k^2.D + B_2 / D = (P_L – B_2)^2 / D + B_2 / D$.

Then $[k^3.D – 1] / k = [(P_L – B_2)^2 + B_2] / D$ ………………..(25.2)

We chose M' such that $(P_L – B_2)^2 = M' – B_2$; where M' is an integer number.

Then $[(P_L – B_2)^2 + B_2] = M'$ ………….(26)

But $(P_L – B_2)^2 = P_L – B_2 + K'$; where K' is an integer number.

Then $M' = P_L – B_2 + K' + B_2$. Then $M' = P_L + K'$

But $M' = P_L + P_L. (E-1) = P_L.E$; **whenever $K' = P_L. (E-1)$.**

{

Because $K' = (P_L – B_2). [P_L – B_2 - 1] = P_L. (E-1)$. Because we can consider that

$P_L.N' = [P_L – B_2 - 1]$ and $(P_L – B_2) = (E - 1) / N'$ for N' integer number not equals to 0.

Because: $(x_3 – 1) = \{ [(P – 4) – P_3. B_2] / P_3 \} - 1$. Then $[(P - 4) / P_3] – B_2 - 1 = (x_3 – 1)$

Then $P_L – (B_2 + 1) = (x_3 – 1)$. But $P_L – (B_2 + 1) = P_L. (1 – [B_2 + 1] / P_L) = P_L.N'$

Thus $P_L.N' = (x_3 – 1)$. Where $N' = (P_L – B_2 - 1) / P_L = 1 – (B_2 + 1) / P_L$. But as in 'Proof ' , we chose $(P_N + A_1)$ and P_N such that $(B_2 + 1)$ is divisible by P_L. Thus N' is an integer.

But we choose integer D such that $(B_2. x_3. N') + B_2 = D$ = integer, for the integer number N'. Where $D \neq 0$.

Thus there exists N' an integer number ($\neq 0$) such that $P_L.N' = [P_L– B_2 - 1]$ and

$(P_L – B_2) = ((D/B_2) - 1) / N'$. Where $(D / B_2) = E$ is an integer.

Then $K' = (P_L – B_2). [P_L – B_2 - 1] = P_L. (E-1)$

}

Then $(M' / E) = P_L$. Then by (26):

$B_2. [(P_L – B_2)^2 + B_2] / D = P_L = [k^3.D – 1] .(B_2 / k) = B_2. [D.(D')^2 – (1/D')]$ (by 25.1 and 25.2)

Then by (24.1): $B_2. [D'.x_3^2 - D] = P_L.x_3$. Then $[D''. x^2{}_3 – D_0] = P_L.x_3$; where $B_2. D' = D''$ and $(D.B_2) = D_0$ (Since D_0 is divisible by B_2, we have the capability to say: D is still an integer, but which is not equal to zero). And here we know that both D and D_0 are not divisible by x_3.

Then $x^2{}_3 \mid (x_3.P_L + D_0)$ since D'' is an integer.

Discussion

We assumed initially that there are finitely many twin primes. After proceeding with that, I ended up with a contradiction. But to get the contradiction, I used that P_N as a prime number greater than $(P_{n-1} + 2)$. And we chose P_n odd integer (> 1) and also we chose an integer A_1 such that $P_3 \mid (A_1 - 2)$. Also to get the contradiction, I used the facts that $(P_N + 2)$ and $(P_N - 2)$ as non-primes since $P_N - 2 > (P_{n-1} + 2)$. And also I have used that x_2 and x_3 as natural numbers (since, $(P_N + 2)$ and $(P_N – 2)$ are not prime numbers). And also I have used the fact (to get the contradiction as in (20)): The difference between any two consecutive prime numbers (which are greater than $(P_{n-1} + 2)$) is greater than 2. Therefore to get the contradiction, I have used the facts got from our assumption (1.0). Then the only possibility is our assumption (1.0) is false.

Results

Therefore I have used our assumption (1.0) to get the contradiction finally, as showed in (20). Therefore it is possible to conclude that our assumption (1.0) is false. Thus the negation of the assumption (1.0) is true.

Thus there are infinitely many twin prime numbers.

Appendix

Prime number: A natural number which divides by 1 and itself only.

Twin Prime Numbers: Two prime numbers which have the difference exactly 2.

We denote 'i' th prime gap $g_i = P_{i+1} - P_i$

Then according to the 2nd reference; Prime number $P_N = 2 + \sum_{j=1}^{N-1} gj$

Also by 2nd reference: for all $\epsilon > 0$, there is a natural number 'n' such that for all $N - 1 > n$;

$g_{N-1} < P_{N-1} . \epsilon$

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