

# Newton's gravity depending on the topology of space.

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**Abstract:** Taking into account the fact that gravity is a curvature of 3D space, it can be represented as a conic section. And this leads to the formulation of the law of I. Newton's law in a general form, depending on the topology of space, which makes it possible to theoretically derive the speeds of the stars in galaxies. The theoretical derivation of the Hubble–Lemaître law is also given, and the relativistic form of the law of I. Newton is derived (in a generalized form). Explained relation between galaxies speeds with distance to it, on the basis of the A. Einstein's STR.

**Keywords:** I. Newton's law of gravity, the topology of space, the movement of stars in galaxies, the Hubble–Lemaître law, dark matter, dark energy.

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## INTRODUCTION.

The concept of "dark matter" go from the fact that the movement of stars in galaxies is different from the predicted. However, it can be strictly shown (based on Newton's gravity) that dark matter (and dark energy) is just a manifestation of gravity at large distances, if expressed more

accurately, it is a manifestation of gravity depending on the different topology of the three-dimensional curved space. For a rigorous proof, it is necessary to modify Newton's gravitational law, or rather, not to modify it, but to bring it into line with the different topology of space. Surprisingly, this will be quite enough, and this is quite logical. Next we give a rigorous proof.

It is worth noting that Newton's law of universal gravitation was experimentally confirmed with great precision (within the Earth — the Moon). For example, laser observations of the moon's orbit confirm the law of gravitation, at a distance from the earth to the moon, with an accuracy of  $3 \cdot 10^{-11}$  [1]. The fact of the great accuracy of the law reflects the Euclidean nature of the three-dimensional physical space. In a non-curved 3D space, the surface area of the sphere is precisely proportional to the square of its radius, and therefore the law of magnitude is exact. As will be shown later, in the curved 3D space, the law of gravitation must change, and take a more general form, on the basis of which you can easily explain both “dark matter” and “dark energy”.

## **RESULTS AND DISCUSSION.**

### **Newton's gravity depending on the topology of space.**

For further reasoning, let us recall the law of gravity by I. Newton.

Two bodies with masses  $m_1$  and  $m_2$  separated from each other by a distance  $r$  are attracted to each other with the force equal to:

$$m_1 \text{-----} r \text{-----} m_2$$

$$F = G (m_1 * m_2) / r^2$$

The forces of gravitational interaction between two point masses are directly proportional to the product of these masses, and inversely proportional to the square of the distance between them.

These forces always act and are directed along a straight line connecting these point masses.

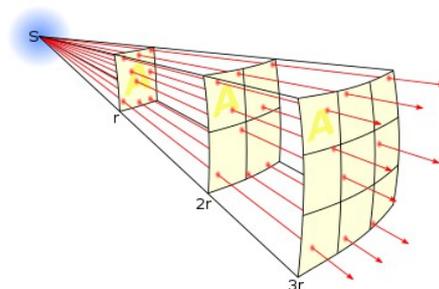
The gravitational force depends on the distance between the masses as  $1/r^2$ , that is,  $F = f(1/r^2)$ , this is the law of inverse squares (the value of some physical quantity at a given point of space is

inversely proportional to the square of the distance from the field source that characterizes this physical quantity).

The law of inverse squares was formulated in 1645 by Ismaël Bullialdus [2], the french astronomer. I. Bullialdus also propose the "conic hypothesis" [3, 2], which had a tremendous impact on the development of astronomy. Our representation of gravity through a conic section (hereinafter referred to as the text) is undoubtedly a development of the idea of the "conic hypothesis" of I. Bullialdus and its commitment to moving in a circle. And, it is this view that leads to the general formulation of the Newton's law of universal gravitation. We were just lucky that thanks to A. Einstein's GTR, we understand that gravitation it is a curvature of 3D space (or space-time continuum), then it was necessary to formulate purely technical details.

This law (inverse square law) is very clearly demonstrated as the decrease in intensity (that is, the energy per unit area per unit time) on the surface of the sphere from the distance of the source placed in the center of the sphere.

Let's look at the picture [4]:



«S represents the light source, while  $r$  represents the measured points. The lines represent the flux emanating from the source. The total number of flux lines depends on the strength of the source and is constant with increasing distance, where a greater density of flux lines (lines per unit area) means a stronger field. The density of flux lines is inversely proportional to the square of the distance from the source because the surface area of a sphere increases with the square of the radius. Thus the strength of the field is inversely proportional to the square of the distance from the source» [4].

It is important to note that we are considering the sphere. It is obvious that I. Newton, in formulating the law of gravity, proceeded from similar considerations. But, this is an ideal case that does not occur in reality. A confirmation of this is the movement of all bodies in elliptical orbits, and not in a circle (a ball, a circle is in fact an ideal curvature, and its sphere limits, remember conic hypothesis of I. Bullialdus).

Indeed, if we consider a sphere in the center of which some mass  $m_1$  (mathematical point) is located, then our three-dimensional space will be curved, and this curvature will be absolutely spherical (and ideal). The reason for this is similar to the reason why droplets of liquid in weightlessness have the shape of a ball. The water molecules inside the ball and the water molecules on the surface of the ball have different energy. Moreover, on the surface (of the ball) the molecules have more energy, and since the physical system tends to a minimum of energy, the number of such molecules should be minimal, that is, the area with a given mass (volume) should be minimal. This is possible only with the shape of a ball, which limits the sphere. In all other cases, with a given mass, the area will be much larger.

With curved space, everything is the same. Let, we have a curved space, and the curvature of space itself changes according to some law (with increasing distance from the source, the curvature decreases). In order for the system to have the lowest energy, the curvature must be strictly spherical (remember that the mass is located in the center of the sphere and represents a mathematical point).

Considering that the mass of the ball is evenly distributed, and taking into account the law of inverse squares, the transfer of mass to the center, to the mathematical point, is fully justified, and mathematically rigorous. But, this is an ideal case, and for such a case I. Newton formulated his famous law, and for this ideal case it is true. But, as will be shown below, it is still not strict for three-dimensional space, since strictly speaking, three-dimensional space is not additive: when

adding one space ball to another space ball, we always get a third space ball with some “straining” (see below proof).

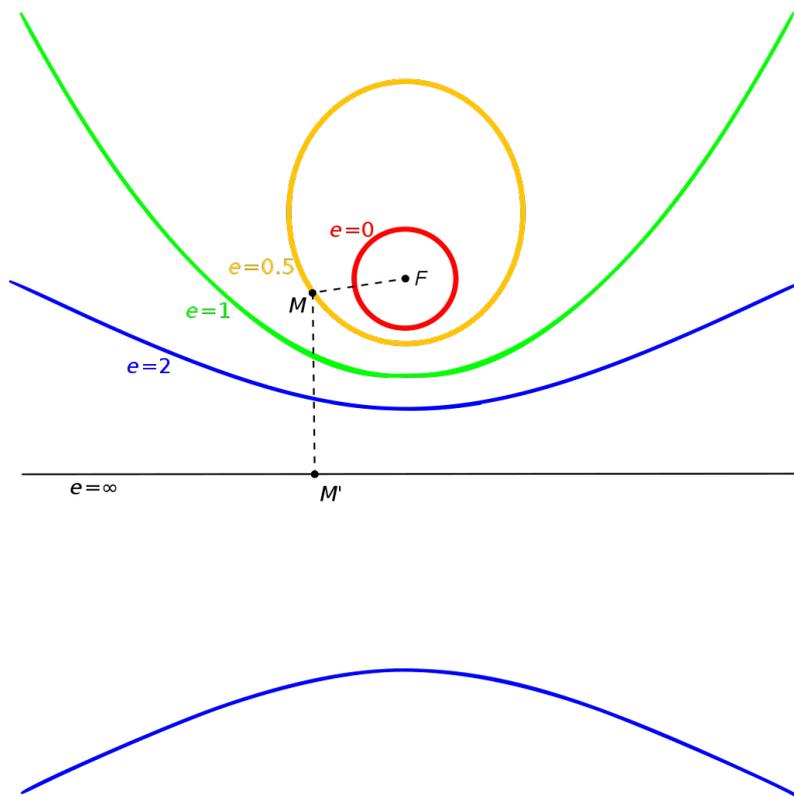
Therefore, if we place a real, material object, for example, the Sun (or a planet, etc.) in the center of the ball, then the distribution of matter in the object will not be strictly uniform (due to gravity there will always be a certain density gradient, convection, etc.), then the curvature of our ideally curved spatial sphere (or ball) inevitably arises. The rotation of the Sun (planets, etc.) around its axis also inevitably leads to the curvature of the three-dimensional ball (and hence the sphere, which limits it). The curvature of the ideal spatial ball (sphere) will also be caused by the introduction of another material object with mass  $m_2$  in the gravity field.

But, once again, we note that since three-dimensional space is not additive by definition, the curvature of the three-dimensional ball in the material world will always be, even when the distribution of matter in the ball is strictly uniform. That is, the space will be bent even without mass, although it does not exist without mass. Let's prove this.

Taking into account Fermat's Last Theorem. let us single out the case for  $n = 3$  ( $r_1^3 + r_2^3 = r_3^3$ ), since the ball with dimension 3 represents our three-dimensional space or rather its properties. We already know that with the addition of two balls with rational radii, we will never get a third ball with a rational radius. The ball with  $n = 3$  is actually our space, and the fact that there can be no addition of two balls with rational radii means that our space does not have the property of additivity. Since the addition of two balls always there is some kind of "tension", which in turn means that our space can not be three-dimensional by definition. Since then, with an arbitrarily large number of additions, an arbitrarily large “tension” of space can be achieved. And the “stretching” can be so big that the three-dimensional space will collapse. This means only one thing: our world at the fundamental level has the dimension of space less than three, and what we perceive as three-dimensional space is a kind of illusion of the fundamental quantum world. Imagine a rotating circle around its center in all directions - so we get the illusion that the object is

three-dimensional (and at a high oscillation speed it will manifest itself as a ball: for example, the ball will bounce off of it), but in fact (on fundamental level) the circle is two-dimensional. Similarly, the quantum world has a dimension less than three (maybe 2, then quantization is clear (according to the Pythagorean theorem), maybe 1, or even dimension 0, or dimension tending to zero), but we perceive the world around us as three-dimensional space. It is clear from the above that approximations when spaces with dimensions greater than 3 are investigated cannot be true by definition, since all spaces with dimensions greater than 2 are not additive.

Now, let us ask ourselves the question: how will our ideal three-dimensional ball be curved (and therefore our ideal sphere, which limits it)? Moreover, the answer will be to look for a common, for all cases of curvature. To answer this question, let us recall the definition of conic sections, more precisely, we will define conic sections from a physical point of view, we will introduce a physical interpretation into the definition. To do this, consider the figure that defines the conic sections [5].



So, the definition of a conic section (mathematical):

Choose a point  $F$  and a line  $d$  on the plane and set a real number  $e$  ( $e \geq 0$ ). Then the locus of the points for which the distance to the point  $F$  and to the straight line  $d$  differs by a factor of  $e$  is a conic section. The point  $F$  is called the focus of the conic section, the straight line  $d$  - is the directrix, the number  $e$  - is the eccentricity.

$$e = MF/MM'$$

Depending on the eccentricity, you get:

when  $e = 2$  — hyperbole,

when  $e = 1$  — parabola,

when  $e = 1/2$  — ellipse.

Now let's give this definition of physical meaning.

Let the mass  $m$  be placed at the point  $F$ , which will cause some curvature of the two-dimensional space, which varies according to a certain law. And let the straight  $d$  is a ray of light, which, passing at a certain distance from the mass  $m$ , no longer bends, but moves straight, that is, this distance at which the curvature of space no longer exists.

Then, by definition, a conic section will determine the locus of points with the same curvature of space (see the mathematical definition), that is, points with the same gravitational potential. Or in other words, it will be the geometric location of points at which the same gravitational force will act on the test body, due to the gravitational force to the point  $F$ . That is, in a two-dimensional world we define an equipotential line, and in a three-dimensional world we define an equipotential surface of the gravitational field of mass  $m$  located at point  $F$ . This is the physical definition of conic sections, and it describes both closed trajectories (ellipse) and unclosed ones.

It is also necessary to understand that at the point  $F$  we can also place small masses, literally even elementary particles. Imagine how complex and interesting their interference will be, or rather the interference of their equipotential lines and equipotential surfaces ... It's impossible not to recall the interfering Universe [6].

For this reason (as the conic section) in a gravitational field, such as the Sun, another material body will always move along an ellipse (if it is a closed trajectory), and never will circle around (as we know, this is an ideal case that does not occur in the real world, more precisely in three-dimensional space). It is wonderful that all conic sections are mathematically written as a single equation. So, in the Cartesian coordinate system, this equation has the form:

$$A*x^2 + B*x*y + C*y^2 + D*x + E*y + F = 0$$

Therefore, we can clarify I. Newton's law of the world with a more accurate and general formulation:

$$F = (K1*M*m) / (r^2) + (K2*M*m) / r + K3*M*m$$

This formula strictly follows from the fact that with the gravitational curvature of three-dimensional space (general case), we always get one or another conic section, in the plane of which the test body will move (according to the law of conservation of angular momentum).

This can be clearly explained for the closed trajectories of motion of the test body. For closed trajectories, we always get some kind of ellipsoid (curved space, three-dimensional body), which will be limited to some equipotential surface (curved sphere, ellipsoid)!

Let us now recall the change in the intensity of the lines of force limited by an ideal sphere (or illumination intensity), through a unit of area. But, since in reality we have an ellipsoid, our surface will no longer be a sphere (this is obvious), but a curved sphere (ellipsoid). The conic section of such an ellipsoid (for closed trajectories) will always be ellipse. But, what is great, the intensity of the lines of force that will pass through a unit of area of such a curved sphere (ellipsoid as a surface) will be as follows:

$$F = f(1/r^2) + f(1/r) + C$$

Moreover, the change in the intensity of the lines of force across a unit of area, for a bounding surface, generally for all movements (closed, unclosed) will also obey the above formula.

Therefore, the reduced formula

$$F = (K1 * M * m) / (r^2) + (K2 * M * m) / r + K3 * M * m$$

is a general formulation of the law of gravitation, and in fact it expresses what is reflected in A.Einstein's GR: that attraction between bodies depends on the topology of a curved three-dimensional space. As an approximation for an ideal sphere, this formula implies the formula of I. Newton:

$$F = (G * M * m) / (r^2)$$

Now once again turn to the figure defining conic sections.

$$e = MF/MM'$$

when  $e = 2$  — hyperbole,

when  $e = 1$  — parabola,

when  $e = 1/2$  — ellipse.

It is interesting to note that if  $MM' \rightarrow 0$ , then the eccentricity will tend to infinity  $e \rightarrow \infty$ , that is, the conic section will turn into a straight line. If  $MF \rightarrow 0$ , then the eccentricity will also tend to zero  $e \rightarrow 0$ , that is, the conic section will turn into a circle (but never reach it).

As we have already determined, in the physical interpretation a conic section (for example, an ellipse) is an equipotential line, that is, a line where the force what act on test body has the same value at each point. From here it is easy to understand why the body speed when moving along an ellipse will be different: the force of the gravitation is constant at every point of the ellipse, but according to Newton's second law

$$F = m * a$$

and acceleration during curvilinear motion is

$$a = v^2/R$$

where  $R$  is the radius of curvature (i.e., the radius of the contacting circle).

Naturally, since the gravitational force is constant, the acceleration for the test body will also be constant  $a = \text{const}$  (since  $m = \text{const}$ ). That is, as the radius of curvature of the directory increases,

the speed of the test body should also increase (for  $a = \text{const}$ ). And with a decrease in the radius of curvature of the directory, the velocity of the test body will decrease accordingly. Therefore, when moving along an ellipse, the speed of the body will periodically change (depending on the place on the line).

It is interesting to note that the movement along a straight line can also be considered as a limiting case of curvilinear motion: when the radius of curvature (the radius of the contacting circle) is infinitely large. Then the body having a certain speed will have an acceleration tending to zero (that is, equal to zero).

Now, consider the motion of a test body along a hyperbola or parabola (that is, along open paths). As we know, these lines are also equipotential, that is, at any point in space on this line, even infinitely distant, the force of gravitational attraction will remain constant (this follows from the fact that these are equipotential lines). How is this possible? Perhaps, if we consider that this is a curved space. Especially, given the formula

$$F = m*a \quad \text{and} \quad a = v^2/R$$

and the fact that with open motion the test body will move along a curve whose radius of curvature will increase all the time (as the curvature will decrease). And since the force is constant (equipotential line!), Then the speed will also increase (relative to point F), according to the formula  $a = v^2/R$ , with  $a = \text{const}$ . If the curvature of the curve is zero, then the adjacent circle degenerates into a straight line (the radius of such a curvature tends to infinity).

This is one of the reasons why galaxies increase their speed all the time (they move along unclosed trajectories, move to infinity with respect to us).

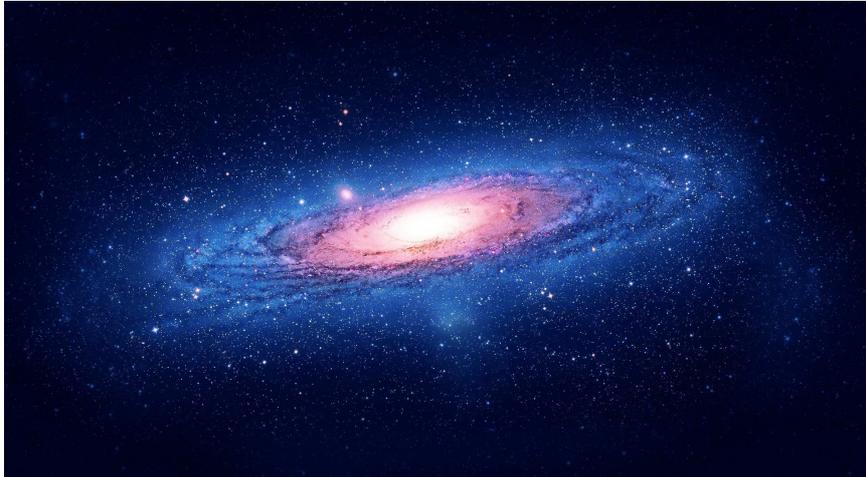
Follow from formule

$$F = (K1*M*m) / (r^2) + (K2*M*m) / r + K3*M*m$$

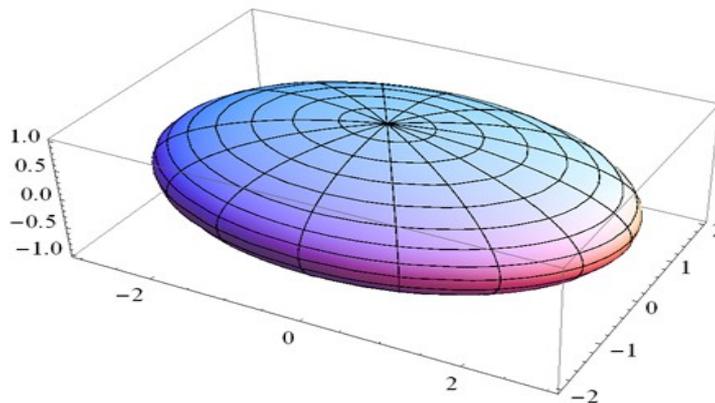
and given the topology of space, it is easy to explain the anomaly in the motion of stars in galaxies.

Obviously, here the second term of the equation ( $(K^2 * M * m) / r$ ) comes into play, but if you take into account the topology of the curved three-dimensional space, then everything is much more interesting.

So, consider any flat galaxy, see the picture (Andromeda Galaxy) [7].



In the center of each galaxy there is a massive object, or rather a black hole, which rotates around its axis with great speed. Given this, let us ask ourselves the question: what will be the topology of the curved space of a flat galaxy? The answer is obvious - it is oblate ellipsoid. See picture [8].



It is clear that the more massive the black hole in the center of the galaxy, and the faster it rotates, the more compressed the ellipsoid will be compressed, and the more it will approximate the shape of a circle. In the extreme case, it will twist so much that it will turn into a flat circle. But, it must be remembered that an ellipsoid (like a surface) is a surface obtained by rotating an ellipse

(conic section), that is, obtained by rotating an equipotential line. And this means that an ellipsoid is an equipotential surface, that is, a surface, at which point, the same force acting on the test body ( $F = \text{const}$ ). We especially note that it is constant force! On this basis, it is easy to explain the speed of stars in galaxies. Again, remember the movement in a circle:

$$F = m \cdot a \quad \text{and} \quad a = v^2/R$$

if  $F = \text{const}$ , then for the same mass, the acceleration is also constant, that is  $a = \text{const}$ .

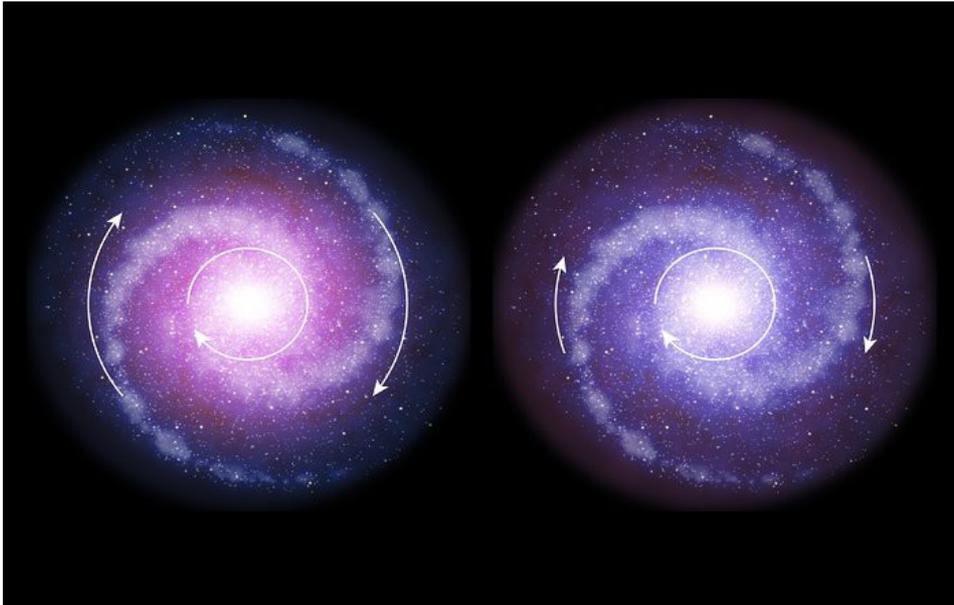
$$a = \text{const} = v^2/R$$

And in order for the acceleration to be constant with an increase in the radius, an increase in speed is necessary, which is observed during the rotation of stars in galaxies. Simply put, space in galaxies curves to a two-dimensional circle, and behaves like a holistic (solid) surface. If a complete circle is rotated around its axis, then for the same reason, as the distance from the center increases, the linear velocity of the circle points increases (since, for the same time, the point passes an arc of greater length).

Similarly, in galaxies: the equipotential surface of galaxies is actually an analogue of the solid circle, therefore, the speeds increase with increasing distance from the center, which is observed in reality.

## The theoretical derivation of the speeds of stars in galaxies.

Surprisingly, the maximum weight of a galaxy in the universe is limited by the speed of light in a vacuum. The mass restriction for a galaxy strictly follows from the law of gravity by I. Newton (the general formulation, depending on the topology of space). Consider the standard case. See the picture [9].



We have a galaxy with a black hole in the center. Around the circle, around the center of the galaxy, the stars move, and their speed is almost the same (then we strictly derive this dependence).

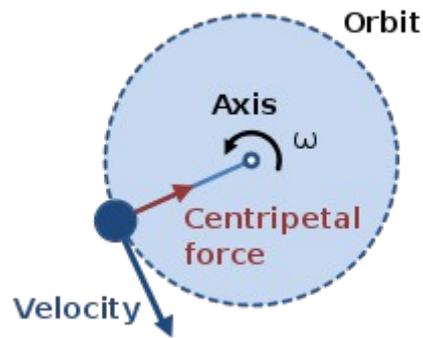
Now apply Newton's law of gravity:

$$F = (K1 * M * m) / (r^2) + (K2 * M * m) / r + K3 * M * m$$

In galaxies, only the 2nd member will “work” (this depends on the values of the coefficients K1, K2, K3, which depend on the topology of the space)  $(K2 * M * m) / r$ .

$$F(2) = (K2 * M * m) / r$$

Now consider the movement of stars in the galaxy on the model. See picture [10].



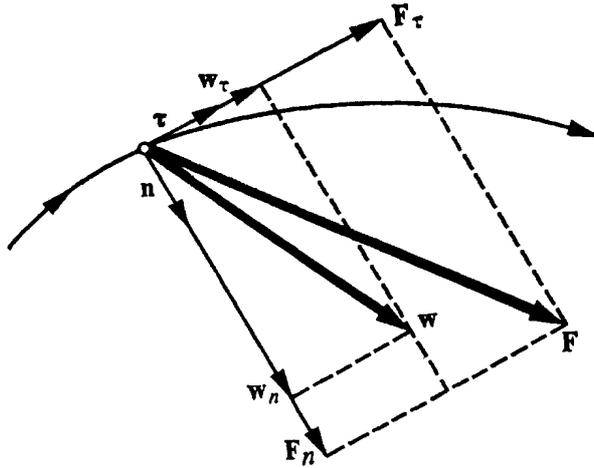
Suppose the mass  $M_0$  is concentrated in the center of the galaxy (in fact, this point mass  $M_0$  will be the mass of the whole circle from the star to the center, and this is mathematically strict). A star of mass  $m_0$  moves in a circle around the center at a speed of  $v$ . The star is affected by the force of gravitation to the center of the galaxy  $F(2)$  ( $r_0$  is the radius of the circle along which the star moves around the center of the galaxy):

$$F(2) = (K^2 * M_0 * m_0) / r_0$$

The star will have a centripetal acceleration directed toward the center (and perpendicular to the speed)  $a$ :

$$a = v^2 / r_0$$

Further calculations will be carried out taking into account relativistic effects. For this, the interaction of a galaxy (mass  $M_0$ ) with a star of mass  $m_0$  will be viewed from an inertial reference frame associated with the center of the galaxy (the reference frame  $K$ ). An observer in the frame of reference  $K$  (at a certain moment in time) “sees” that the star has a mass  $m$ , a velocity  $v$ , and a distance of  $r_0$ . The force of gravitational attraction of the star is directed to the center of the galaxy (as well as acceleration). Note that the star's speed and acceleration are perpendicular ( $v \rightarrow$ ,  $a \uparrow$ ), therefore, to take into account relativistic effects, we will take into account the “transverse mass” (for more information about the transverse and longitudinal mass, see the book, see pictures) [11].



$$\begin{aligned} \frac{F_n}{w_n} &= \frac{m_0}{\sqrt{1 - v^2/c^2}}, \\ \frac{F_\tau}{w_\tau} &= \frac{m_0}{(1 - v^2/c^2)^{3/2}}. \end{aligned}$$

$$\tau \frac{m_0}{(1 - v^2/c^2)^{3/2}} \frac{dv}{dt} = F_\tau, \quad n \frac{m_0}{\sqrt{1 - v^2/c^2}} \frac{v^2}{R} = F_n. \quad (21.6)$$

Therefore, when relativistically taking into account the increase in the mass of the star (which the observer sees in K), we will take into account precisely the “transverse mass” ( $v \rightarrow, a \uparrow$ ):

$$m = m_0 / (1 - v^2/c^2)^{0.5}$$

Then law of gravitation will be written like this (observer in K):

$$F(2) = (K_2 * M_0 * m) / r_0 = (K_2 * M_0 * m_0) / (r_0 * (1 - v^2/c^2)^{0.5})$$

And the second law of Newton ( $v \rightarrow, a \uparrow$ ),

$$F = m * a$$

written like this

$$F = (m_0 * a) / (1 - v^2 / c^2)^{0.5}$$

Taking into account  $F = F(2)$ ,

and  $a = v^2 / r_0$ , write:

$$(K_2 * M_0 * m_0) / (r_0 * (1 - v^2 / c^2)^{0.5}) = (m_0 * a) / (1 - v^2 / c^2)^{0.5}$$

Did little arithmetic reduction we will get:

$$v^2 = K_2 * M_0$$

Since the speed of light in vacuum is maximal, then from here we get the limiting (maximum) mass of the galaxy in the Universe:

$$M_0 < c^2 / K_2$$

substituting constants it is easy to get the maximum mass of the galaxy, also knowing the speed of stars in the galaxy and the mass of the galaxy it is easy to get the numerical value of  $K_2$ .

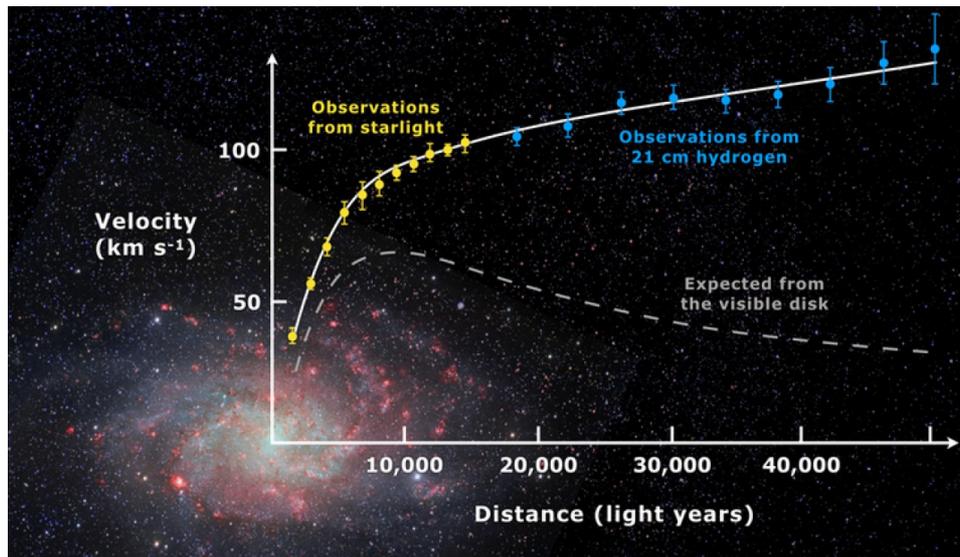
You can also get the speed of the stars, which depends only on the mass of the galaxy (or more precisely, on the mass of the circle of the galaxy, from the center to the star):

$$v = (K_2 * M_0)^{0.5}$$

where  $K_2$  is a constant, and  $M_0$  is the mass of the galaxy (the mass of the circle of the galaxy, from the center to the star). Note that this formula is a relativistic formula, and it is correct up to the speed of light. In fact, this formula strictly implies that stars in large galaxies (on the periphery) can speed up to the speed of light, this limits the size of galaxies.

As we see, the obtained dependence of the speed of the stars in galaxies (depending on the distance from the center of the galaxy) corresponds exactly to that obtained experimentally. Here it is only necessary to take into account that the “mass of the galaxy” (the mass of the circle to the star) will increase (from the increase in the distance to the star). And therefore, we obtain the dependence of the speed on the distance to the center, which will actually correspond to the

dependence  $y = k * x^{0.5}$ , which corresponds to the experimental observations [12, 13, 14, 15]. See picture [16].



That is, the rotation speed will not depend on the radius of rotation, but will be constant. But, if we take into account that with an increase in the radius, the mass that will attract ( $M_0$ ) will increase (as the amount of substance increases), then the speed will still increase slightly (due to the increase in mass). In the end, due to the fact that the addition of mass will be insignificant (by the end of the galaxy), the speed must become constant.

For the same reason (due to an increase in mass), the anomaly of Pioneer can be explained [17]. That is, the slowdown of the Pioneer satellite (or other objects) on the outskirts of our solar system occurs due to the fact that the mass that attracts the Pioneer increases. Moreover, it should be borne in mind that the solar system is an almost ideal sphere (slightly curved), and therefore, the whole mass that lies within this sphere, and not only within the solar system (as a circle), must be taken into account. We assume that if we take into account this effect, then we will manage to explain the anomaly of Pioneer.

## The theoretical derivation of the Hubble–Lemaître law.

Our Universe is forced by gravity to expand and accelerate, more precisely, gravity at large distances. To confirm our words, we theoretically, strictly derive the Hubble-Lemaître law [18, 19], which, as it turns out, has a non-linear relationship.

Let, we have a galaxy with a mass  $m_0$  (this is a rest mass), which moves away from the center  $M_0$ , with a speed  $v$ , and acceleration  $a$  ( $v \rightarrow$ ,  $a \rightarrow$ ).

$$M_0 \text{-----} h_0 \text{-----} m_0, \quad v \rightarrow, a \rightarrow$$

The reason for the deletion is the law of Newton's gravity, more precisely, its third member (at these distances only he “works”, the coefficients  $K_1$  and  $K_2$  “turn off” other members).

$$F = (K_1 * M * m) / (r^2) + (K_2 * M * m) / r + K_3 * M * m$$

$$F (3) = K_3 * M * m$$

For taking into account for relativistic effects, we place the inertial report system  $K$ , at the point  $M_0$ . Then, at a certain point in time, the observer (in frame  $K$ ) will see a galaxy of mass  $m$ , at a distance  $h_0$ , which moves at a speed  $v$ , and an acceleration  $a$  ( $v \rightarrow$ ,  $a \rightarrow$ ). We especially note that the speed and acceleration are directed along the same line, that is, in relativistic effects, we will take into account the “longitudinal mass” [11].

Also note that the mass  $M_0$  is the mass of all matter that is within the radius ball  $h_0$ , that is, it is the mass of the Universe of radius  $h_0$ . Let us assume that at  $h_0 = 0$ , the galaxy  $m_0$  had a speed  $v = 0$ .

Galaxy on distance  $h_0$  have speed  $v$ , and acceleration  $a = v^2 / (2 * h_0)$ .

As we know, the longitudinal mass will be equal to:

$$m = m_0 / (1 - v^2 / c^2)^{1.5}$$

Therefore, Newton’s law of gravity (his third term) is written in the form:

$$F(3) = K_3 * M_0 * m = (K_3 * M_0 * m_0) / (1 - v^2/c^2)^{1.5}$$

And second law of Newton ( $v \rightarrow, a \rightarrow$ ),

$$F = m * a$$

written like

$$F = (m_0 * a) / (1 - v^2 / c^2)^{1.5}$$

Taking into account  $F = F(3)$ ,

and  $a = v^2 / (2 * h_0)$ ,

write:

$$(K_3 * M_0 * m_0) / (1 - v^2 / c^2)^{1.5} = (m_0 * a) / (1 - v^2 / c^2)^{1.5}$$

Making the necessary arithmetic abbreviations we get:

$$K_3 * M_0 = a$$

$$v^2 / (2 * h_0) = K_3 * M_0$$

Then, get

$$v^2 = 2 * K_3 * M_0 * h_0$$

And for speed, get:

$$v = (2 * K_3 * M_0)^{0.5} * h_0^{0.5}$$

The value  $(2 * K_3 * M_0)^{0.5}$  is the Hubble's constant, which depends only on the mass of the Universe (of course, radius  $h_0$ ), and therefore as the distance increases (and increases in mass), the Hubble's constant will increase slightly, and therefore the galaxies will accelerate .

From the resulting formula:

$$v = (2 * K_3 * M_0)^{0.5} * h_0^{0.5}$$

it follows that the rate of removal of galaxies nonlinearly depends on distance.

This formula expresses the Hubble-Lemaître's Law. Also note that this is a relativistic formula. And since, the speed of light is maximum, this formula practically defines the event horizon, which an observer can see from the report system K.

$$h_0 < c^2 / (2 * K_3 * M_0)$$

The removal rate of galaxies (from a distance) will correspond to the formula:  $y = k * x^{0.5}$

Knowing the galaxy removal rate and distance to it, it is easy to calculate the coefficient  $K_3$ .

$$K_3 = v^2 / (2 * M_0 * h_0)$$

The foregoing also explains why the adjustments to the Hubble's constant occurred.

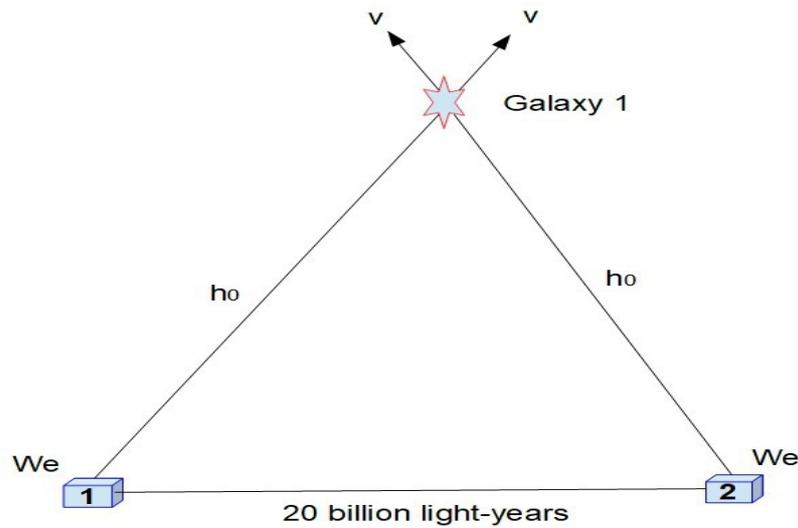
The stationary, statistical Universe is actually restored by the presence in the Universe of the maximum speed of energy transfer, since this inevitably leads us to the "horizon of visibility" of our Universe:

$$h_0 = c^2 / (2 * K_3 * M_0)$$

where  $h_0$  — is the radius of the visible Universe.

The speed of light limits the visible Universe, which the observer will see, because in connection with effects of relativity, at the edge of the visible Universe, objects will move at speeds close to the speed of light in a vacuum. But, this does not mean that outside our visible sphere of radius  $h_0$ , there are no galaxies, stars and other matter. On the contrary, outside our sphere, the Universe is unlimited in size. And if we could move in any direction by 20 - 30 - 40 billion light years, then we would see completely different galaxies, stars, nebulae. Note that for this you do not need to go to another Universe, or to a different dimension of the world. That is, we would see another part of our Universe, invisible from our location, since  $h_0$  is just the radius of our visible Universe. This is similar to how we see a different starry sky in the southern and northern hemisphere of our planet.

Considering Einstein's STR, this is not surprising, since we will see a slightly different picture of events from different reference systems. And our visible Universe, with a radius of about 14 billion light years, is part of the cosmos that we see from our reference system (our location), given the finite speed of light. We explain this in the figures, see figure.



If we move from position 1 to position 2, take a distance of 20 billion light-years between them, then we will still see galaxy 1, but for the observer from position 2, it will move in a completely different direction (see figure). That is, the movement of galaxies, which we observe is a consequence of Einstein's STR on a cosmological scale. And if you think about it, you can explain why with increasing distance, we see an increase in speed, which is limited by the speed of light.

To do this, we will try to answer a simple question: what is the difference between the two inertial reference systems? The answer is simple: they move relative to each other at a certain speed. And what is the reason for this, how are they fundamentally different? Considering Einstein's STR, the answer is obvious: they differ in the speed of time flow. That is, if time in two inertial reference systems proceeds at different speeds, then the observer will see (both external and internal) that they move relative to each other at a certain speed. If the time flow rate (in different reference systems) is the same, then the observer will see that they do not move relative to each other. The extent to which time in different reference systems proceeds differently reflect the well-known formulas of A. Einstein.

Given the above, it is easy to explain the speed of the movement of galaxies. For this, let the galaxy be one inertial reference system, and the rest of the Universe in the center of which is the observer, let it be the second inertial reference system. Then it is clear that the greater the  $h_0$  will be (the greater the mass will be), the more will be the speed of time flow different in the galaxy and in our Universe. This means that the observer "from the center of the Universe" will see the greater speed of the galaxy movement, the greater the distance  $h_0$  to the galaxy (which is reflected by the Hubble–Lemaître Law). It is clear that this will be the effect of gravitational deceleration (acceleration) of time on cosmological scales: the greater the mass, the slower the time passes, and the more  $h_0$  the greater the mass of "our visible" Universe.

Then, Newton's stationary Universe is quite logical and consistent (without expansion and collapse). And the various observable effects at the cosmological level are simply a consequence of Einstein's STR (at the cosmological level).

It is also possible to define the concept of "movement". Movement is the flow of time at different speeds in different objects (! this is why movement is relative).

Given the apparent radius of the Universe  $h_0$ , one can easily explain the cosmological paradoxes.

Photometric cosmological paradox: since the visible part of the Universe is limited by the speed of light in a vacuum, the number of stars that we will see will also be limited and will not be infinite.

Gravitational cosmological paradox: with an infinite, unlocked Universe, the gravitational potential at a point will not be infinitely large (at any point), since the gravitational interaction between the masses will occur only within the "visible Universe" radius  $h_0$  (you need to clarify that for an observer who located in the center of such a visible Universe).

## Relativistic form of the law of gravity of I. Newton.

We showed how Newton's law of gravity can be written depending on the topology of space, in general terms:

$$F = (K1 * M * m) / (r^2) + (K2 * M * m) / r + K3 * M * m$$

But it is possible to strictly show that speed must be included in I. Newton's law of gravity. This follows logically from the considerations cited for the Coulomb's law in accordance with the STR of A. Einstein [20]. We will need (literally) to replace the charge on the mass and bring the proof given by reference.

Further in the text, we logically derive the relativistic form of the law of gravity I. Newton in general (there will be a restriction on the speed of bodies, and the dependence of the force of gravity on the speed, the dependence of the force of gravity in various inertial reference systems).

But first, we will show why speed must be included in I. Newton's law of gravity. Let us give the proof.

The interaction of fixed masses (points) is completely described by the law of I. Newton's Gravity (general form):

$$F = (K1 * M * m) / (r^2) + (K2 * M * m) / r + K3 * M * m$$

$$M \text{ ----- } r \text{ ----- } m$$

Consider the interaction of two point masses, which are at rest in the frame of reference K1.

However, in another frame of reference K2, moving relative to K1, these masses move with the same speed, and their interaction becomes more complex.

I. Newton's law of gravity (in a static form) is insufficient for analyzing the interaction of moving masses, and this is related to the relativistic properties of space and time, and the relativistic equation of motion (Newton's law of gravity has nothing to do with this). This follows from the following considerations.

Relativistic equations of motion:

$$dp/dt = F \quad (1)$$

Is invariant and has the same form in all inertial frame of reference. So in the coordinate system K2, which moves rectilinearly and uniformly with respect to K1:

$$dp^2/dt^2 = F^2 \quad (2)$$

The left-hand sides of equations (1) and (2) include purely mechanical quantities (the behavior of which is known when passing from one coordinate system to another). Consequently, the left-hand sides of equations (1) and (2) can be related by some formula. But then the right parts of these equations (the equations of force) are related. Such a bond is conditioned the requirement of relativistic invariance of the equation of motion. Since speed enter the left-hand sides of equations (1) and (2), we conclude that the interaction of moving charges depends on the speed of motion and does not reduce to the Newton's law of gravitation force.

Thus, it is proved that the interaction of moving masses should be described by Newton's relativistic gravity equation, that is, the equation of gravity which should include the velocity of the masses. This strictly follows from the fact that the speed of the gravitational interaction is equal to the speed of light in a vacuum, that is, it is finite [21].

«From the point of view of the theory of relativity, the addition of the law of gravitation of Newton by the mere principle of a finite velocity of gravitation is unacceptable, since the field of the metric tensor is due not only to geometry, but also to the kinematics of masses. The solution (II, 4.2) already shows that the effects caused by the kinematics of masses and the finite velocity of gravity have the same orders, so that the refinement of Newton's theory by taking into account only one of these effects is erroneous» [22].

The real body movement during gravitational interaction will always occur on the conical section, that is, the body will always have acceleration (curvilinear movement), which means the body will always radiate gravitational waves (but very weak).

In principle, any accelerated mass movement generates gravitational waves. Gravitational waves will not be radiated by centrally symmetric compression and expansion of a spherical body. Also, gravitational waves will not be emitted when a perfectly symmetric body rotates around the symmetry axis.

«Albert Einstein originally predicted the existence of gravitational waves in 1916,[24][25] on the basis of his theory of general relativity.[26] General relativity interprets gravity as a consequence of distortions in space-time, caused by mass. Therefore, Einstein also predicted that events in the cosmos would cause "ripples" in space-time – distortions of space-time itself – which would spread outward, although they would be so minuscule that they would be nearly impossible to detect by any technology foreseen at that time.[13] It was also predicted that objects moving in an orbit would lose energy for this reason (a consequence of the law of conservation of energy), as some energy would be given off as gravitational waves, although this would be insignificantly small in all but the most extreme cases.[27]» [23].

Due to the accelerated motion of the masses (along conic sections), the gravitational field at each point in space is variable, and therefore gravitational waves will form, but very weak.

Now we give evidence of how using A. Einstein's STR we can write down Newton's law of gravity in a general relativistic form, taking into account the velocity of the masses.

If the velocity is zero, then the law of gravity is written in the form:

$$F = (K1 * M * m) / (r ^ 2) + (K2 * M * m) / r + K3 * M * m$$

As is known from A. Einstein's STR, if we have a K1 reference system in which a body of mass  $m_0$  has a speed  $v = 0$ , then in another reference system K2, which relative to K1 moves rectilinearly and evenly with speed  $v$ , the body will have a mass  $m$ :

$$m = m_0 / (1 - v^2 / c^2)^{0.5}$$

Naturally, the time interval  $\Delta t$  in K2 will be equal ( $\Delta t_0$  is the time interval in K1):

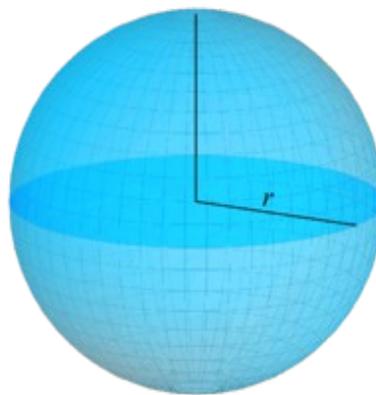
$$\Delta t = \Delta t_0 / (1 - v^2 / c^2)^{0.5}$$

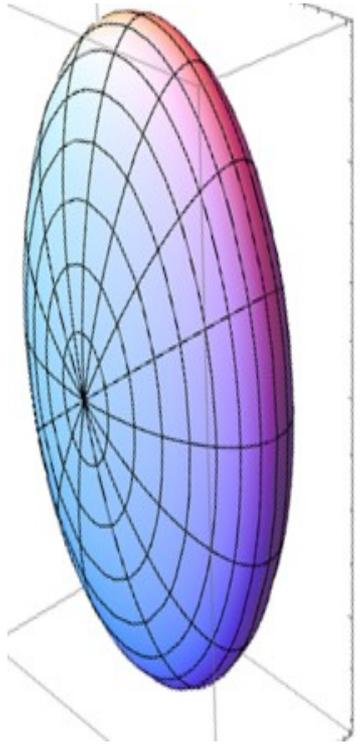
There will also be a decrease in body length in the K2 reference system (in the direction of the K2 reference system):

$$\Delta L = \Delta L_0 * (1 - v^2 / c^2)^{0.5}$$

These are the well-known equations from the A. Einstein's STR. But, here we pay special attention to reducing the length of bodies (in the direction of movement of the reference systems). What does it mean? And how can this be interpreted from the point of view of movement along conic sections?

Note that our space, ideally curved by some mathematical mass  $m$ , can be considered as an ideal sphere. Then the answer is obvious: when the reference frame moves at a speed  $v$  (for example, K2 moves relative to K1), our ideal spatial sphere is curved into an oblate ellipsoid (in the direction of travel). Therefore, the observer in K2 sees objects in K1 through his “flattened ellipsoid” of space (there is a surface), and therefore, in the direction of travel, the length of physical bodies decreases (which the observer from K2 sees, according to the well-known formula). See the drawings that are clearly demonstrated by the above ( sphere, and flattened ellipsoid) [24, 8].





Naturally, at the speed of light ( $v = c$ ), the 3D-space will turn into a 2D-space (the length in the direction of travel will be zero).

Therefore, reducing the length of objects in K1 (in the direction of travel), which the observer sees in the K2 reference system, is the result of the curvature of the ideal spatial sphere into a flattened ellipsoid (that is, in fact, into a flattened ellipsoid, the K2 reference system goes). Since the “flattening of the ellipsoid” of the K2 reference system is an increase in the curvature of space (or space-time continuum), and therefore the mass increases, time slows down, and linear dimensions decrease.

Given the above, we need to write the general formula of Newton's gravity

$$F = (K1 * M * m) / (r^2) + (K2 * M * m) / r + K3 * M * m$$

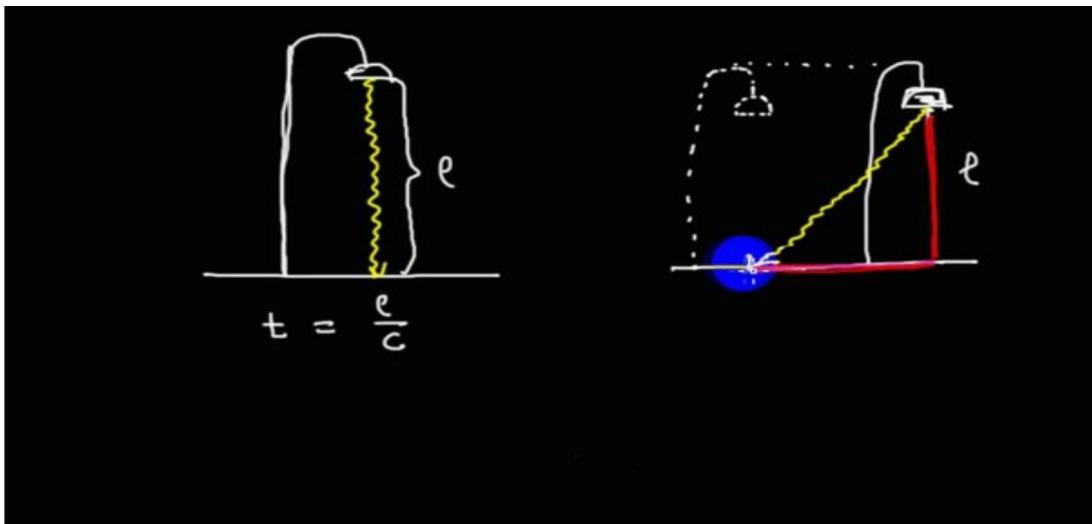
in a relativistic form.

We must take into account the fact (stated above) that the space in the K2 reference system is bent into a “flattened ellipsoid” in the direction of travel. Therefore, the masses in the K1 reference system (in which the masses do not move) must be placed perpendicular to the movement

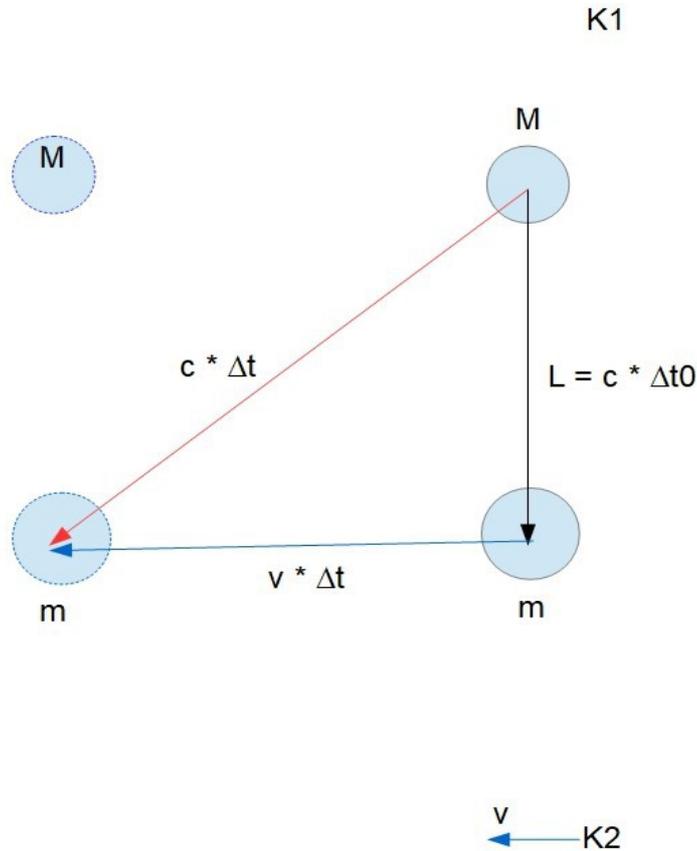
of the K2 reference system. That is, if K2 moves at a speed  $v$ , relative to K1, along the X axis, then the masses  $M$  and  $m$  must be placed either along the Y axis or along the Z axis. Then, the curvature of the ellipsoid along the X axis (its flattening along the X axis) does not will “interfere” with the derivation of the relativistic formula of gravity.

And then the derivation of the relativistic formula of gravity will be similar to the conclusion of the basic laws of the special theory of A. Einstein:

when light is viewed from the lamp to the platform, on the railroad platform (K1 reference system), and in a uniformly moving train (K2 reference system), see the figure [25].



We will have a complete analogy: instead of a flashlight, we will consider the mass  $M$ , and on the perone we will consider another interacting mass  $m$ . The distance between them in the reference frame K1 associated with the iron road peron is  $L$ . Instead of a ray of light, we will have a ray of gravitational interaction (which is transmitted from the mass  $M$  to the mass  $m$ ). Another frame of reference, K2, will be connected with the train, which moves relative to the train at speeds of  $v$ . We look at the picture.



That is, in the reference frame K1 (in which the masses M and m are at rest) the beam of gravitational interaction “released” from the mass M passes the distance L, and it will take time  $\Delta t_0$  from it. Naturally, the gravitational interaction is transmitted at the speed of light in a vacuum, therefore

$$L = c * \Delta t_0$$

In the K2 reference system (which moves relative to K1 at a speed  $v$ ), the observer will see that the ray of gravitational interaction passes a path (from mass M to mass m)  $c * \Delta t$ , that is, it will be the hypotenuse of the triangle. Details, such as the fact that the beam of gravitational interaction was released from M, at that moment when K2 was near m, are clear from the standard consideration at Einstein's STR, and we will not dwell on this here, either, or on the elementary conclusions of the formulas (all the same). Naturally, the reference system K2 will pass the way equal to  $v * \Delta t$ .

Using the Pythagorean theorem, we write:

$$c^2 * \Delta t^2 = v^2 * \Delta t^2 + c^2 * \Delta t_0^2$$

Using elementary arithmetic transformations, we get that the distance between the masses M and m, which the observer sees in K2, has increased in comparison with the distance in the reference frame K1 (we take it in K1 equal to L0):

$$L = L_0 / (1 - v^2 / c^2)^{0.5}$$

Let us pay special attention that the distance between the masses has increased (in K2), compared with the distance in K1 (where the masses do not move).

Time and mass in the K2 reference system changes in accordance with the Einstein's STR:

$$\Delta t = \Delta t_0 / (1 - v^2 / c^2)^{0.5}$$

$$m = m_0 / (1 - v^2 / c^2)^{0.5}$$

Where  $\Delta t$  is the time interval in K2, and  $\Delta t_0$  is the time interval in K1, M, m are the masses in K2, and M0, m0 are the masses in K1, that is, we have the standard consideration in SRT.

So, in the reference frame K1, we have two masses M0 and m0, the distance between which is equal to r0. The force of gravitational interaction between them, in the frame of reference K1, will be equal to:

$$F = (K_1 * M_0 * m_0) / (r_0^2) + (K_2 * M_0 * m_0) / r_0 + K_3 * M_0 * m_0$$

An observer in the K2 reference system, which moves relative to K1, uniformly and straightforwardly at speed v, “sees” the masses M and m, and the distance between them r:

$$M = M_0 / (1 - v^2/c^2)^{0.5}$$

$$m = m_0 / (1 - v^2/c^2)^{0.5}$$

$$r = r_0 / (1 - v^2/c^2)^{0.5}$$

If we put it into formule:

$$F = (K_1 * M * m) / (r^2) + (K_2 * M * m) / r + K_3 * M * m$$

to we get the relativistic formula of gravitational interaction:

$$F = (K_1 * M_0 * m_0) / (r_0^2) + (K_2 * M_0 * m_0) / (r_0 * (1 - v^2 / c^2)^{0.5}) + K_3 * M_0 * m_0 / (1 - v^2 / c^2)$$

This formula is suitable for calculating the gravitational interaction in any inertial frame of reference. Note that the first term (Newton's gravity) does not depend on the speed of the reference system. But, the second and especially the third term of the equation, strongly depend on relativistic effects. Obviously, the third term is a cosmological constant, and it will be the greater, the speed of the reference system will be closer to the speed of light, that is, the galaxies will be accelerated more and more.

We also note that knowing that the inertial reference system is a compressed ellipsoid, and taking into account the concept of transverse and longitudinal masses, it is possible to calculate the mass of a body of any shape, practically for all movements, since it is clear the change of any point of the ellipsoid, and hence the change in body mass ( $m = f(\alpha)$ ,  $\alpha$  is the angle of rotation relative to the transverse mass (or longitudinal)).

## **CONCLUSION.**

Thus, introducing gravity through conic sections (since gravity is a curvature of 3D space) it was possible to formulate the general form of the law of I. Newton, as well as to derive its relativistic form. Using the generalized law of I. Newton, one can easily explain the anomaly of the speeds of stars in galaxies, without using the concept of "dark matter". A theoretical conclusion for the velocities of stars in galaxies is also given. Moreover, the generalized law gives the possibility of theoretically deriving the Hubble-Lemaître law, which, as it turns out, has a non-linear relationship. It is worth noting that the concept of "dark energy" also not used.

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