

**Abstract** There are many *ad hoc* expressions for the mass ratio of the proton to the electron. The models presented here are different in that they rely strictly on volumes and areas. One geometry is based on ellipsoids constructed with values taken from one of the two number sets:  $\{(4\pi), (4\pi - 1/\pi), (4\pi - 2/\pi)\}$  or  $\{(4\pi + 2), (4\pi - 2), (4\pi - 2/\pi)\}$ . The product of the three values of each number set approximates the value given by CODATA<sup>1</sup> for the mass ratio of the proton to the electron. Another approximate is formed from a solid ball of radius,  $r = (4\pi - 1/\pi)$ , with a conical sector, wedge, or internal ellipsoid removed. Each extracted solid has curved surface area of  $(4\pi - 1/\pi)\pi^{-2}$ . With the advent of the Higg's Boson, its value can be approximated by  $H^0 = (4\pi)(4\pi - 1/\pi)(4\pi - 2/\pi)(4\pi - 3/\pi)(4\pi - 4/\pi)$ .

**Formulas** Assume that the following statement<sup>2</sup> is true: “The most accurate measurement yet of the shape of the electron has shown it to be almost perfectly spherical.” This is not the case with the proton, however<sup>3</sup>. “Surprise To Physicists – Protons Aren’t Always Shaped Like A Basketball.”

Let  $m_e$ ,  $m_p$ ,  $m_n$ , and  $m_{H^0}$ , be the rest state mass of the electron, proton, neutron, and Higgs boson, respectively. Consider the following six approximations.

$$\frac{m_p}{m_e} \approx \prod_{k=0}^2 \left(4\pi - \frac{k}{\pi}\right) = 1836.15\dots \quad (1)$$

$$\frac{m_p}{m_e} \approx (4\pi + 2)(4\pi - 2) \left(4\pi - \frac{2}{\pi}\right) = 1836.15\dots \quad (2)$$

$$\frac{m_p}{m_e} \approx \left(4\pi - \frac{1}{\pi}\right)^3 - \left(4\pi - \frac{1}{\pi}\right) \left(\frac{1}{\pi^2}\right) = 1836.15\dots \quad (3)$$

$$\frac{m_n}{m_e} \approx \ln(4\pi) + (4\pi) \left(4\pi - \frac{1}{\pi}\right) \left(4\pi - \frac{2}{\pi}\right) = 1838.683\dots \quad (4)$$

$$\frac{m_{H^0}}{m_e} \approx \prod_{k=0}^4 \left(4\pi - \frac{k}{\pi}\right) = 240773.70\dots \quad (5)$$

$$\frac{m_{H^0}}{m_p} \approx \prod_{k=3}^4 \left(4\pi - \frac{k}{\pi}\right) = 131.1295\dots \quad (6)$$

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<sup>1</sup>[www.codata.org](http://www.codata.org)

<sup>2</sup><https://www.livescience.com/14322-electron-shape-standard-model-particle-physics.html>

<sup>3</sup><https://www.sciencedaily.com/releases/2003/04/030408085744.htm>

If the electron is a perfect sphere then its volume is given by the formula

$$V_e = \frac{4\pi}{3} \cdot r_e^3 \text{ and } m_e \propto V_e \quad (7)$$

Where  $V_e$  is the volume of the electron and  $r_e$  is the radius of the electron, determined by experiment to a high degree degree of precision. We assume that ( $m_{e+} = m_e$ ) and  $m_{e+}$  is the rest mass of the positron. Equation (1) approximates the mass ratio of the proton to the electron. The shape of the proton is consistent with it being an ellipsoid. Equations (2) and (3) are merely restatements of Equation (1). Equation (4) is an *empirical* approximation. Equations (5) and (6) are extensions of the product in Equation (1). The formulas presented here are different from others in that they rely strictly on volumes and areas. Equations (1), (2), and (3) have geometric persuasions as a proof.

**History** It was on the 28th of June, 1989, that I observed that  $64\pi^3 - 48\pi + 8/\pi = 1836.15\dots$ . This is a very good approximation to the dimensionless mass ratio of the proton to the electron. The “gold standard” for approximations is  $22/7$  for  $\pi$ . There their relative error is  $(22/7 - \pi)/\pi \approx 0.0004$ . For the mass ratio of the experimental value versus the algebraic approximation, we have:

$$\text{relative error of ratio} \leq 0.00000051 \quad (8)$$

Relative error is a measure of the uncertainty of measurement compared to the size of the measurement. Relative error is also known as relative uncertainty or approximation error.<sup>4</sup> We see that our approximation is remarkable. Now there *are* many good approximations. We need to see if there are other consistent approximations as well.

**Semi-Axes** Consider the following equality:

$$\prod_{k=0}^1 \left( 4\pi - \frac{k}{\pi} \right) = (4\pi)^2 - 4 = (4\pi + 2)(4\pi - 2) \quad (9)$$

It is clear that Equations (1), (2), and (3) are behaving as if the three factors in each one is a semi-axis of a tri-axial ellipsoid. There are three points in

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<sup>4</sup><https://www.thoughtco.com/definition-of-relative-error-605609> by Anne Marie Helmenstine, Ph.D.

each equation whose product is  $1836.15\dots$ . Equation (1) extends the notion of a proton by continuing the products *empirical approximation*.

**Higgs Boson** The Higgs boson ( $H^0$ ) is an elementary particle in the Standard Model. An approximation to the empirical value of the mass ratio of the Higgs Boson to the proton is approximately 131. This is 240773.70 times the mass of the electron. This value is found in Equation (4). It decays to a sphere of radius  $r = (4\pi)$ . The sphere of radius  $r = 4\pi - \pi^{-1}$  is created by the application of the *Inversion of the Spheres*. Also unstable, the second sphere ejects either an interior sector, an interior wedge, or an interior ellipsoid with semi-axes of  $\{(4\pi - \pi^{-1}), (\pi^{-1}), (\pi^{-1})\}$ .

An alternative is an ellipsoid, as in Equation (1)  $\{(4\pi), (4\pi - \pi^{-1}), (4\pi - 2\pi^{-1})\}$ . These geometric models help to explain the deflections from an electron beam. The surface areas for the shapes suggested by Equation (1) and by Equation (3) are approximately 1884.97431 and 1910.20448, respectively. This suggests difference from the other shapes.<sup>5</sup>

**New Model** The volume of the ellipsoid, whose semi-axes are  $\{(4\pi + 2), (4\pi - 2), (4\pi - 2\pi^{-1})\}$ , is  $1836.15\dots$ . This tri-axial ellipsoid surface deviates significantly from the sphere. Moreover, using a ball of radius  $r = (2)$ , place an “attachment point” on the ball surface and add to the radius to get the major semi-axis of  $4\pi + 2$  and on the opposing side of the ball, obtain a semi-axis of  $4\pi - 2$ . Their product being the same as the value from an “area” of  $(4\pi)(4\pi - 1/\pi)$ .

**A Function** Define the function  $f$  as follows: Let the initial set be the positive integers  $Z^+$ , the final set be the real numbers,  $\mathfrak{R}$ , and the rule assigning each member of the initial set to one member of the final set:

$$f(m) = 4\pi - (m - 1)/\pi \tag{10}$$

Define the function  $F$  as follows: Let the initial set be the positive integers  $Z^+$ , the final set be the real numbers,  $\mathfrak{R}$ , and the rule assigning each member of the initial set to one member of the final set:

$$F(m) = (4\pi) \cdots (4\pi - (m - 1)/\pi) \tag{11}$$

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<sup>5</sup><https://planetcalc.com/149/>

**Conclusion** The function  $F(2) = 1836.15\dots$  approximates the experimental value of the mass ratio of the proton to the electron and  $F(4)$  approximates the mass ratio of the Higg's Boson to the electron. The neutron-to-electron ratio is approximated with  $\ln(4\pi) + F(2)$ .