Abhas Mitra and Eternally Collapsing Objects: A Review of 22 Years of Misconceptions

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Abstract

We critically review here the concepts which gave rise to the Eternally Collapsing Object (ECO) paradigm over almost 22 years. First we look into "non occurrence of trapped surfaces" then we analyze Mr. Mitra's claim regarding why $R = 0$ (inside black hole) should also be treated as another coordinate singularity! After that we review if indeed his another claim about "mass of Schwarzschild black hole being zero" is right or not. Do black holes really exist or they are just a fairy tale made by physicists to elude us into fiction? And are these so called "Black Holes", actually black hole or indeed as Mitra likes to call them "Eternally Collapsing Object" or ECO.

INTRODUCTION

> Most misunderstanding in the world could be avoided if people would simply take the time to ask, "What else could this mean?"
>
> – Shannon L. Alder

In this paper we first look into, Eternally Collapsing Object, which is an alternative solution to Black Hole Problem. Mr. Mitra believes that these so called Black Holes are actually Black Hole mimickers, since from outside the event horizon any black hole would behave like an object of mass M. But what does it even mean, when we say mass of the black hole? We will here look into Mr. Mitra's claim of "zero mass black hole", "non-occurrence of trapped surface", "Is singularity at R = 0 just another artifact of coordinate or are they real?" and then we go into studying the notion of ECO using Vaidya metric. ECOs are interesting objects since from outside they behave like Black Holes as they emit almost none to no radiation but because of some pressure fluctuation and such, often few photons do manage to escape which could explain hawking radiation! As they are so dense their collision can explain what was observed during the gravitational wave detection at LIGO. Existence of ECO and non existence of black hole can indeed solve a lot of issue and give us all a sense of relief that now we understand a lot more about the universe and there are few less mystery to unravel! But before we consider ECOs to be real and black holes as fairy tale we need to look into trapped surfaces, their occurrence or non-occurrence which begins by looking into his paper "Non occurrence of trapped surface", which is more of a base to his later work. After reviewing this we will move foreword in our analysis of Eternally Collapsing Object and see if they are based on undeniable mathematical rigour or birth of misconception! We will be using natural system where G = c = 1 unless these terms specifically appear.

I. Non occurrence of trapped surface

The real juice of this paper starts from section 5, where he begins by assuming few quantities [1] , which are actually used in few of the proofs he mentioned in the paper and I quote:

"we define a quantity, U, the rate of change of the circumference radius with the proper time following the radial worldline (at a constant r)",

$$U = \frac{\partial R}{\partial \tau}(r, \tau) \tag{a}$$

We further define another quantity, which expresses the rate of change of the circumference radius with the proper distance

$$\Gamma = \frac{\partial R}{\partial l}(r, \tau) \tag{b}$$

and since, there always exists a real proper time and proper distance, in a general fashion, we can define

$$\upsilon = \frac{\partial l}{\partial \tau} \tag{c}$$

after which he then assumes a static metric, and with that arrives at equation (75) $\left[\upsilon = \frac{dl}{\sqrt{g_{00}}dx^0}\right]$ in his paper, using

$$ds^2 = g_{00}(dx^0)^2 + g_{rr}dr^2$$

$$ds^2 = g_{00}(dx^0)^2 - dl^2$$

$$d\tau^2 = g_{00}(dx^0)^2 \tag{d}$$

Mr. Abhas Mitra is using confusing notation here! Originally he defined these quantities (a), (b) & (c) (with words) in terms of partial derivative (in reference to "*hydrodynamic Calculations of General-Relativistic Collapse*", which also defines them the same way!), but Mr. Mitra insisted on using derivative symbols throughout as can be spotted from his notations! In his notation they look something like this:

$$U = \frac{dR}{d\tau} = \frac{1}{\sqrt{g_{00}}}\frac{dR}{dt}; \tag{e}$$

$$\Gamma = \frac{dR}{dl};$$

$$\Gamma = \frac{1}{\sqrt{-g_{rr}}}\frac{dR}{dr} \qquad by\ definition\ dl^2 = -g_{rr}dr^2 \tag{f}$$

using the same methodology one also arrives at

$$\upsilon = \frac{dl}{d\tau} = \frac{1}{\sqrt{g_{00}}}\frac{dl}{dt} \tag{g}$$

Applying chain rule on (e), we get:

$$U = \frac{dR}{d\tau} = \frac{dR}{dl}\frac{dl}{d\tau} = \Gamma\upsilon \tag{h}$$

Now, Mr. Mitra used the result number (12) of "Hydrodynamic Calculations of General-Relativistic Collapse", which relates these above defined quantities for collapsing star in comoving frame, to his final proof on "non-occurrence of trapped surface", which is:

$$\Gamma^2 = 1 + U^2 - \frac{2GM}{R} \tag{i}$$

His proof goes something like this, and I quote

"For purely radial motion, one may ignore the angular part of the metric to write:

$$ds^2 = g_{00}dt^2 + g_{rr}dr^2$$

$$g_{00}\left(1 + \frac{g_{rr}dr^2}{g_{00}dt^2}\right) \geq 0 \qquad assuming, ds^2 \geq 0 \tag{j}$$

from section 7 and 8, there exists a positive definite quantity

$$\Gamma^2 = \frac{1}{-g_{rr}}\left(\frac{dR}{dr}\right)^2 = 1 + U^2 - \frac{2GM}{R} \tag{k}$$

From this he constructs an inequality and argues, since the whole inequality is positive, as they are the result of multiplication of two positive terms.

Mathematically he says,

$$since\ \Gamma \geq 0\ and\ \left(1+\frac{g_{rr}dr^2}{g_{00}dt^2}\right) \geq 0\ (as\ g_{00} \geq 0)$$

the resulting inequality should also be

$$\Gamma^2\left(1+\frac{g_{rr}dr^2}{g_{00}dt^2}\right) \geq 0 \tag{l}$$

on simplification, we get

$$\Gamma^2+\Gamma^2\left(\frac{g_{rr}dr^2}{g_{00}dt^2}\right) \geq 0$$

using his both (f) and (i) definition of "**Γ**", we get

$$\left(1+U^2-\frac{2GM}{R}\right)+\frac{1}{-g_{rr}}\left(\frac{dR}{dr}\right)^2\left(\frac{g_{rr}dr^2}{g_{00}dt^2}\right) \geq 0$$

$$\left(1+U^2-\frac{2GM}{R}\right)-\left(\frac{dR}{dr}\right)^2\left(\frac{dr^2}{g_{00}dt^2}\right) \geq 0$$

$$\left(1+U^2-\frac{2GM}{R}\right)-\left(\frac{1}{g_{00}}\right)\left(\frac{dR}{dr}\frac{dr}{dt}\right)^2 \geq 0 \tag{m}$$

Now If you look at the second part of the inequality, which is exactly U^2 but with "minus" sign so,

$$\left(1-\frac{2GM}{R}\right) \geq 0$$

or,

$$R \geq 2GM \tag{n}$$

and hence, no trapped surface will form!!!

Well, you can check the calculation as many times as you want, it's indeed correct. Which is why I have mentioned it, but by now you maybe wondering, so perhaps Mr. Mitra is right and the collapse doesn't lead to formation of event horizon!!!

Calculation is indeed correct but the conclusion drawn from it, is very much wrong. If you notice, both the term "U" & "Γ" were originally defined in terms of partial derivative, even the other paper "*Hydrodynamic Calculations of General-Relativistic Collapse*" by Mr. Micheal May and Richard White[2], defined them like partial derivatives and arrived at the result (i), which Mr. Mitra used in his proof shown above.

During the proof he used equation **(f)** and applied chain rule on **(m)** to arrive at **(n)** which is a violation of partial derivatives. If he treated them like partial derivatives, he would have gotten an uglier equation, like this:

$$\left(1 + U^2 - \frac{2GM}{R}\right) + \left(\frac{\partial R}{\partial r}\frac{dr}{dl} + \frac{\partial R}{\partial \tau}\frac{d\tau}{dl}\right)^2 \left(\frac{g_{rr}dr^2}{g_{00}dt^2}\right) \geq 0$$

Also the fact that "U^2" got canceled was because he treated ∂ as **d.** It was also clear from the fact that Einstein Field Equation is a system of non-linear partial differential equations, and the result (I), it's solution (both papers derive them from EFE)! But since he insisted on using derivative notation, he ended up treating them like one! Anyways, let's proceed ahead and look into his second proof, as he said the "ultimate proof".

Here he proceeds by using equation (i) and (h) to get,

$$\Gamma^2 = 1 + (\Gamma v)^2 - \frac{2GM}{R} \qquad (substituting\ U = \Gamma v)$$

$$\Gamma^2 - (\Gamma v)^2 = 1 - \frac{2GM}{R}$$

$$\Gamma^2(1 - v^2) = 1 - \frac{2GM}{R}$$

$$\frac{\Gamma^2}{\gamma^2} = 1 - \frac{2GM}{R} \tag{o}$$

since, $\frac{\Gamma}{\gamma} \geq 0$ the R.H.S. of the equation **(o)** should be ≥ 0,

hence,

$$1 - \frac{2GM}{R} \geq 0$$

or,

$$R \geq 2GM$$

As you can see from this, the drawn conclusion from this scenario is same as earlier equation (n) but the part where he used $U = \Gamma \upsilon$ is completely wrong, as they are partial derivatives it will never take that form!

$$U = \frac{dR}{d\tau} = \frac{dR}{dl}\frac{dl}{d\tau} = \Gamma \upsilon \quad \text{(p)}$$

$$U = \frac{\partial R}{\partial \tau}(r, \tau) \quad \text{(q)}$$

As the result (p) is based on adhoc maths, applying chain rule on **(q)** never would have gotten him the same result., from which he drew/based his conclusions.! Actually Mr. Mitra did indeed try to justify equation (p) by assuming a path $dR = 0$ and from this he did get (p) and the last equation (o), but be careful before taking it for granted $dR = R'dr + \dot{R}d\tau = 0$ does not represent the surface of the star! As we are in comoving frame of the fluid, surface is represented by $dr = 0$ (also only at $r = r_s$, the equality $R(r_s, \tau) = r_s$ will hold because of birkhoff theorem) which makes $dR = \dot{R}d\tau = 0$, makes sense as we're in comoving frame and as birkhoff theorem implies $R(r_s, \tau) = r_s$, it makes $\dot{R} = \dot{r_s} = 0$ as it's comoving frame, so using proper mathematics even this can't help Mr. Mitra derive that result!

There are a lot of mistakes one could find in his papers like "nonoccurence of trapped surface", but to stay on point let's first discuss his another attempt at disproving black hole. This time he drew his conclusions from, Oppenheimer-Snyder paper, titled "On Continued Gravitational Contraction" but just before we do that, let's discuss one more fallacy in his arguments

Let's look at equation 201 from his paper "Non-occurrence of trapped surface" for the speed of particle

$$\upsilon^2 = \left(\frac{dl}{d\tau}\right)^2 = \frac{\left(\frac{dR}{d\tau}\right)^2}{\left(\frac{dR}{dl}\right)^2} = \frac{U^2}{\Gamma^2} = \frac{\Gamma^2 - 1 + \frac{2GM}{R}}{\Gamma^2} \quad \text{(r)}$$

since we all know particles are not allowed to move faster than the speed of light, i.e. $\upsilon^2 \leq 1$

$$\upsilon^2 = \frac{\Gamma^2 - 1 + \frac{2GM}{R}}{\Gamma^2} \leq 1$$

or,

$$\frac{2GM}{R} \leq 1$$

Again, same conclusion which resulted from treating, partial derivatives as derivatives. Since, originally he introduced υ as partial derivative but even if denies that, this result is still wrong as "**U**" and "**Γ**" were both defined and worked on in terms of partial derivatives and in general

$$\left(\frac{dl}{d\tau}\right)^2 \neq \frac{\left(\frac{\partial R}{\partial \tau}\right)^2}{\left(\frac{\partial R}{\partial l}\right)^2} \neq \left(\frac{\partial l}{\partial \tau}\right)^2$$

rendering his equation (r) or (201) in his paper, useless.

Let's come back to Oppenheimer-Snyder paper, where Mr. Mitra used equation (32) and (36) of this paper. This is equation (36) which is the function of r and τ [2]:

$$t(r,\tau) = \frac{2}{3}\sqrt{R}_{gb}\left(\sqrt{r_b^3} - \sqrt{R_{gb}^3 y^3}\right) - 2R_{gb}\sqrt{y} + R_{gb} ln\frac{\sqrt{y}+1}{\sqrt{y}-1} \qquad \text{(s)}$$

and this is equation (32) :

$$y = \frac{1}{2}\left[\left(\frac{r}{r_b}\right)^2 - 1\right] + \frac{r_b R}{R_{gb} r} \qquad \text{(t)}$$

where,

$$R = \left(\frac{-3\sqrt{R_{gb}}}{2}\left(\frac{r}{r_b}\right)^{3/2}\tau + \sqrt{r^3}\right)^{2/3} \qquad for\ r \leq r_b \qquad \text{(u)}$$

$$R_{gb} = 2GM \qquad \text{(v)}$$

Before you get afraid just looking at these equations, let me assure you we're not gonna do anything complicated, just a simple domain search, as you might have spotted that $t(r,\tau)$ contains a logarithmic term which is only defined if the term inside is > 0.

Mathematically,

$$\frac{\sqrt{y}+1}{\sqrt{y}-1} > 0$$

which is possible only if, $y > 1$ but at $r = r_b$ first term collapses and all we have is

$$y_{r=r_b} = \frac{R}{R_{gb}} > 1$$

substituting, $R_{gb} = 2GM$ we get,

$$R > 2GM \quad \text{(w)}$$

Again similar looking equation but before you jump to any conclusion, remember R is not the radial distance, it's just a placeholder which is there to make calculation easier and has been defined in terms of r and τ by Oppenheimer and Snyder in equation (27) of their paper [3].

$$R = \left(\frac{-3\sqrt{R_{gb}}}{2}\left(\frac{r}{r_b}\right)^{3/2}\tau + \sqrt{r^3}\right)^{2/3} \qquad for\ r \leq r_b$$

which makes Mr. Mitra's equation **(w)**, really look something like this!

$$\left(\frac{-3\sqrt{2GM}}{2}\left(\frac{r}{r_b}\right)^{3/2}\tau + \sqrt{r^3}\right)^{2/3} > 2GM \quad \text{(x)}$$

with dependence on τ and r, this isn't something Mr. Mitra wanted to show as a result in his favor. Which is why with change in notation, he really tried to make it as confusing as possible to crosscheck, and only mentioned equation (t) as equation (139) in his paper, but hid equation (u) [1].

In the same paper at around equation (195) of his, he also tried to relate pressureless dust to Tolman-Oppenheimer-Volkoff equation and argued that if $p = 0$ then, so must $\rho = 0$. But perhaps he didn't considered that the **Tolman-Oppenheimer-Volkoff equation is only valid for hydrodynamic equilibrium**[4] and hence can't be used to conclude anything about a collapsing fluid. Yes, this is indeed not very realistic, yes real fluids do have pressure and the solution provided by Oppenheimer-Snyder is very much ideal, but there had been some other studies on introducing pressure gradient and studying the collapse, one such work is "Gravitational collapse of an imperfect non-adiabatic fluid" by R. Chan with some success, as his result introduced the emergence of exotic energy during the end of collapse[5]

From these above analysis, we can conclude that Mr. Mitra started out good, but got confused his own mind, based and drew conclusions from misconceptions or erroneous calculations. Right now, we can neither conclude that the true mathematical black hole as predicted by GR is real, nor it can be rejected unless shown with mathematical rigor. But we all hope that the singularity at the heart of the black hole isn't real and just a limitation of General Relativity being a classical theory! Until we have an experimentally tested model of quantum gravity we can't be sure of what's really inside the event horizon! Here I am assuming that the event horizon is indeed formed, as several papers have shown this to some extent. So, I remain optimistic!

Since by now, we have established Mr. Mitra's claim of "nonoccurence of trapped surface" is based on ad hoc maths, confusing notation and erroneous calculation. Let's proceed ahead and see some of his other claims like how "Black Hole is a misconception"!

II. Is Black Hole a misconception?

This paper titled "Black Holes or Eternally Collapsing Objects A Review of 90 Years of Misconceptions" by Mr. Mitra can be categorically divided into four parts, where he discussed the possibility of non existence of black holes and claimed these so called objects are not really true black holes but are Black Hole mimickers!

- Denial of singularity at r = 0
- Convincing trapped surface isn't formed (already covered)
- Mass of Schwarzschild black hole is **zero**
- Arguing in favor of ECOs.

Denial of Singularity at R = 0

This can be seen from his multiple arguments using Principle of Equivalence which states, *"in small enough regions of spacetime, the laws of physics reduces to those of special relativity; it is impossible to detect the existence of gravitational field by means of* ***local experiments****."* (Carroll pg 50). Here Mr. Mitra's claim comes from studying infinitesimal neighborhood of singularity at $R = 0$ using local coordinate frame and assuming $g_{ij} \approx \eta_{ij}$ via POE.

But one must be careful when doing something like that, because the assumption of LIC (Locally Inertial Frame) is based on $g_{ij}(p) = n_{ij}$ and $\Gamma^k_{ij}(p) = 0$ with,

$$g_{ij}(p + \delta p) \approx n_{ij}$$

$$\Gamma^k_{ij}(p + \delta p) \approx 0$$

This is all correct but how big our δp can get, depends on the curvature, i.e. till how far we can study using this methodology, depends on how strong the curvature is! Which Principle of Equivalence also mentions in the line *"in small enough regions of spacetime"*. But what Mr. Mitra did was extend δp so big as to include $R = 0, \ \Gamma^k_{ij}(p + \delta p)$ can no longer be approximated to "0" and in fact it would blow up, rendering his conclusions drawn using this, useless! This is exactly, what Mr. Mitra exactly used to argue that singularity at $R = 0$ also behaves like a coordinate artifact just as $R = 2M$. And I quote:

> "The metric $ds^2 = dT^2 - dR^2 - R^2\left(d\theta^2 + sin\theta d\phi^2\right)$ could also represent the spacetime in the infinitesimal neighborhood of a source of mass-energy (at $R = 0$) in a locally free falling frame. In this case, neither the choice of $R = 0$ is geometrically arbitrary nor are the curvature components identically zero. In such a case, the singularity at $R = 0$ might be a genuine physical singularity since the singularity persists at the location of the massenpunkt and actually cannot be physically removed by any coordinate transformation."

This quotation from his paper "Black Holes or Eternally Collapsing Objects A Review of 90 Years of Misconceptions" clearly shows how much he had actually misinterpreted in order to disprove black holes. Here one could as well have used another metric, like for LIC as $ds^2 = dT^2 - dx^2 - dy^2 - dz^2$ which doesn't have singularity at $(x, y, z) = (0, 0, 0)$, but even this can't be extended to include $(x, y, z) = (0, 0, 0)$ when there is singularity at $R = 0$, as that would lead to the violation of $g_{ij}(p + \delta p) \approx n_{ij}$ and $\Gamma^k_{ij}(p + \delta p) \approx 0$.

Mass of Schwarzschild black hole is zero

Mr. Mitra has tried to show this in multiple way, first by considering pressureless dust, which was used in Oppenheimer-Snyder solution for collapse, speed of sound in dust and few more, let's visit them one by one.

1. Using Tolman-Oppenheimer-Volkoff equation

$$\frac{dp}{dR} = -G\frac{M(r) + 4\pi pR^3}{R^2\left(1 - \frac{2GM}{R^2}\right)}[p + \rho(0)]$$

This is equation (70) of his paper, and he mentioned if $p = 0$, it would mean $\dfrac{dp}{dR} = 0$. Which means,

$$0 = \left[-G \frac{M(r) + 4\pi p R^3}{R^2 \left(1 - \frac{2GM}{R^2}\right)} \right] \rho(0)$$

Leading to $\rho(0) = 0$. But by now, know that the Tolman-Oppenheimer-Volkoff equation can only be used in case of hydrodynamic equilibrium! And as Oppenheimer-Snyder paper never assumed pressure to be zero before the collapse, this is not a violation!

2. Speed of sound in dust!

The moment Oppenheimer-Snyder assumed pressureless dust, he also assumed no sound waves (indirectly). As one knows sound is caused due to oscillation of particles and that means change in pressure. But as already assumed, no pressure means no sound and not using $c_s = \sqrt{\frac{dp}{d\rho}}$. But in this section that's exactly what Mr. Mitra did and I quote[6]:

> "In order that, $c_s \neq 0$ at the boundary of the dust ball (considered as a fluid) where $dp = 0$, one must again have $d\rho = 0$."

Here he implemented $c_s = \sqrt{\frac{dp}{d\rho}}$ for $dp = 0$ to evaluate $c_s = 0$ and only other way out was to make the equation indeterminate or $\frac{0}{0}$ form! Again as Oppenheimer and Snyder had already assumed, no pressure, it means no sound and not using $c_s = \sqrt{\frac{dp}{d\rho}}$.

3. Invariance of 4-Volume

This section is my personal favorite, as this proof had me questioning black hole.
Mr. Mitra begins by stating the fact that

$$\int \sqrt{-g} d^4x = invariant \qquad \text{(y)}$$

Applying this on Eddington-Finkelstein coordinate and Hilbert (we call it Schwarzschild) coordinate:

$$\int \sqrt{-g_*} dT_* \; dR \; d\theta \; d\phi = \int \sqrt{-g} \; dT \; dR \; d\theta \; d\phi$$

also from (which he defined)

$$T_* = T \pm \alpha_0 \; ln \left(\frac{R - \alpha_0}{\alpha_0} \right)$$

which leads to

$$-g_* = R^4 sin^2\theta = -g$$

and substituting them simplifies the equations further.

$$\int R^2 sin\theta \; dT_* \; dR \; d\theta \; d\phi = \int R^2 sin\theta \; dT \; dR \; d\theta \; d\phi \qquad \text{(z)}$$

carrying out integral on both sides on $d\theta$ and $d\phi$, we get:

$$\int R^2 \; dT_* \; dR = \int R^2 \; dT \; dR$$

we have,

$$\int R^2 \; \left(dT + \frac{\alpha_0}{R - \alpha_0} dR \right) \; dR = \int R^2 \; dT \; dR$$

$$\int R^2 \; dT \; dR + \int R^2 \frac{\alpha_0}{R - \alpha_0} dR \; dR = \int R^2 \; dT \; dR$$

$$\alpha_0 \int \frac{R^2}{R - \alpha_0} dR \; dR = 0$$

This yields, $\alpha_0 = 0$ and I quote [6]:

> the integration constant $\alpha_0 = 0$ which arose in the solution for the spacetime around a "massenpunkt", turned out, after 90 years, to be actually zero

Now, that would mean there is no curvature and spacetime in-fact is *flat!* Thank goodness, there already was a paper proving this result erroneous, titled *Comment on 'Comment on The Euclidean gravitational action as black hole entropy, singularities, and spacetime voids'* by Prasun K. Kundu. I'm going to use that paper and show via calculation to point out where this went wrong. Let's go back to equation (z):

$$\int R^2 sin\theta \ dT_* \ dR \ d\theta \ d\phi = \int R^2 sin\theta \ dT \ dR \ d\theta \ d\phi$$

which doesn't actually look like this, but:

$$\int R^2 sin\theta \ dT_* \wedge dR \wedge d\theta \wedge d\phi = \int R^2 sin\theta \ dT \wedge dR \wedge d\theta \wedge d\phi \qquad \text{(A)}$$

Here after doing everything he did, the last equation will be:

$$\int R^2 \ dT \wedge dR + \int R^2 \frac{\alpha_0}{R - \alpha_0} dR \wedge dR = \int R^2 \ dT \wedge dR$$

Now from differential geometry we know $dR \wedge dR = 0$. Which actually yields,

$$\int R^2 \ dT \wedge dR = \int R^2 \ dT \wedge dR$$

Well, there's nothing much you can draw from this. But it does teach us one important lesson i.e. treating 1-form as differential is very much misleading! This is what led Mr. Mitra to the erroneous conclusion that Mass of a Schwarzschild Black Hole is actually "Zero". Also we can also cross check this and I quote:

> "on an n-dimensional manifold M, the integrand is properly understood as an n-form."
>
> ---(Carroll, Spacetime and Geometry, pg 88)

Since spacetime is a 4 dimensional manifold, there is no denying in equation , and if we check equation 2.94 of textbook "Spacetime and Geometry" by Carroll, is

$$\sqrt{|g'|} dx^{0'} \wedge \cdots \wedge dx^{(n-1)'} = \sqrt{|g|} dx^0 \wedge \cdots \wedge dx^{(n-1)} \qquad \text{(B)}$$

and Carroll also mentions this, "In the interest of simplicity we will usually write the volume element as $\sqrt{|g|} d^4x$, rather than as the explicit wedge product:

$$\sqrt{|g|}d^4x = \sqrt{|g|}dx^0 \wedge \cdots \wedge dx^{(n-1)} \tag{C}$$

This calculation showing the exact error in Mr. Mitra's claim was missing in Mr. Kundu's paper, which is why I mentioned it here along with citation to one of the standard textbook in this field.! Mr. Mitra did try to defend his result by arguing that, does this mean 170 years Jacobi formula is wrong?? Well, not quite but that one is limited. See Jacobian Determinant equation was defined in the context of linear algebra and vector space, and didn't involve any wedge product. But here these dx's are not differentials but one form or (0,1) anti-symmetric tensor. And this equation which we used to show the actual calculation has been defined in the context of differential geometry over manifolds, but since vector space too form a manifold, this result can be viewed as a generalization to Jacobi equation.

Arguing in favor of ECOs

From the above discussions Mr. Mitra concluded since, trapped surface isn't formed, singularity at $R = 0$ isn't real and the mass of a black hole is "0" (as he tried to prove), mixing that with proper time of collapse as "∞" , he went onto looking for a solution, which he did find! Eternally Collapsing Object, which takes infinite proper time to collapse and by the time it does collapse, it has already radiated away all of it's mass, leading to $M = 0$, which via equation (56) of the paper, which is in agreement with his idea, he concludes black holes are not formed and in-fact they are ever collapsing object, radiating bit by bit before it ever collapses to form a true Schwarzschild black hole! Just for the sake of clarity let's list them one by one!

1. Mass of a Schwarzschild Black Hole is **zero**
2. Proper Time of collapse is inversely proportional to M and for $M = 0$, he concluded $\tau = \infty$
3. Since Trapped Surface isn't formed
4. It must be radiating via H-K process (which used non-relativistic equations in his derivation and indirectly assumed weak gravitational field by using Newtonian equations!) and always be losing it's mass!

He concluded, if a we take these points seriously then a collapsing star will lost mass in the form of radiation, ever decreasing it's Schwarzschild Radius, and with that loosing some mass. Mix it up with proper time of collapse and voila you get infinite proper time.

Since we already saw his proof of "$M = 0$" and "non-occurrence of trapped surface" is based on erroneous calculations, he hasn't really proven that the Black Holes are not formed and also reviewing H-K process/mechanism in his paper, one will easily see his calculations over there is non-relativistic, which raises the doubt to even believe in it! Since his claims rely that stars will radiate out before the trapped surface is even formed! But as can be seen in "An analytical solution for gravitational collapse with radiation" by P. C. Vaidya, where he did consider pressure and still arrived at equation similar to Oppenheimer-Snyder paper titled "On Continued Gravitational Contraction", and I quote [7].

> "The rate of contraction $D_t R_0$ of the boundary of the sphere is given by an equation similar to the corresponding equation of Oppenheimer-Snyder."

> "which is of the same form as the corresponding equation for the rate of contraction of Oppenheimer-Snyder spheres"

From this, we conclude even the contribution of pressure, can not, for sure ever truly stop contraction. And now I would like to mention the assumption that mixes $M = 0$ with "radiating away" by Mr. Mitra which means, everything that made up the star will go away before it ever collapses. Which can happen in one of two ways, either because of some pressure fluctuation, trapped baryonic matter will escape, or these trapped matter gets converted to photon and then escape (highly unlikely as it leads to the violation of conservation of baryon number)! If the latter case happens, then we would also have to consider "exchange force", which in case of baryon was the source of degeneracy pressure but in case of bosons it acts like an attractive force and perhaps will support the collapse (because as the energy is there and this lead to decrease in degeneracy pressure but increment in radiation pressure, also decrement in degeneracy pressure wouldn't be completely fulfilled by radiation pressure as one depends on the density while latter on the energy/intensity of individual photons.)!

In first case, if baryonic particles escapes because of pressure fluctuation, then all that pressure can/will do, is provide an initial kick leading to escapes. But since that pressure isn't working outside the star, and it's not photon, the baryon will come back falling following it's geodesic back into the collapsing star!(considering isolated neutron star for simplification). Since this too doesn't lead to $M = 0$, we conclude that ECO are perhaps a birth of misconception, erroneous calculation and ad hoc mathematics.

III. Some Bonus Analysis of why event horizon isn't real!

If you look equation (137) of this paper, you'd find he defines **3-speed** something like this (in context of GR). Which he used along with Schwarzschild radius to claim why speed of particle is 1 at event horizon!

$$v^2 = v^\alpha v_\alpha = \left(\frac{dl}{d\tau}\right)^2$$

Which comes from

$$v^2 = v^\alpha v_\alpha = g_{\alpha\beta} v^\alpha v^\beta = \left(\frac{dl}{d\tau}\right)^2$$

where, his equation (138) tells us what υ^α are

$$\upsilon^\alpha = \frac{dx^\alpha}{d\tau}$$

But

$$\upsilon^2 = \upsilon^\alpha \upsilon_\alpha = g_{\alpha\beta}\upsilon^\alpha \upsilon^\beta = g_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = \frac{dx^\alpha}{d\tau}\frac{g_{\alpha\beta}dx^\beta}{d\tau} = \frac{g_{\alpha\beta}dx^\alpha dx^\beta}{d\tau^2}$$

as

$$d\tau^2 = g_{00}dx^0dx^0 + g_{\alpha\beta}dx^\alpha dx^\beta$$

here you can clearly spot the error, he used $dl^2 = g_{\alpha\beta}dx^\alpha dx^\beta$ (because the definition assumes $d\tau = 0$, which means you're already assuming $\upsilon^2 = \infty$)! But perhaps it was just a placeholder not proper distance, so let's forgive him for this.

Now that we have established, his definition of υ, let's look at his conclusion at equation 155, via substituting Schwarzschild metric, which is based on it,
he gets:

$$\upsilon^2 = \left(\frac{dl}{d\tau}\right)^2 = \frac{-g_{RR}dR^2}{g_{TT}dT^2} = (1-\alpha_0)^2\frac{dR^2}{dT^2}$$

he substitutes the value of

$$\frac{dR}{dT} = \frac{1-\alpha_0}{E_*}\left(E_*^2 - \left(1 - \frac{\alpha_0}{R}\right)\right)^{1/2}$$

which he derived earlier and gets:

$$\upsilon^2 = \frac{E_* - \left(1 - \frac{\alpha_0}{R}\right)}{E_*}$$

$$\lim_{R\to\alpha_0} \upsilon^2 = 1$$

Not just the very definition of υ^2 was troublesome but also the above calculation has typo error. The real calculation would have been:

$$\left(\frac{dl}{d\tau}\right)^2 = \frac{-g_{RR}dR^2}{g_{TT}dT^2} = \left(1-\frac{\alpha_0}{R}\right)^{-2}\frac{dR^2}{dT^2}$$

$$\frac{dR}{dT} = \frac{1-\frac{\alpha_0}{R}}{E_*}\left(E_*^2-\left(1-\frac{\alpha_0}{R}\right)\right)^{1/2}$$

Which still result in same thing! So, much calculation/typo errors (these errors repeat!) can make one doubt the entire paper to recheck all of his calculation! There is one more related claim, so let's discuss that and then move onto explaining what could it mean. This claim is discussed in section 11.2 of this paper. Here his equation (186), which can also be found as equation 22 in "Observer and Velocity measurement in General Relativity" by Paulo Crawford and Ismael Tereno is

$$\upsilon^2 = \frac{\left(g_{01}^2-g_{11}g_{00}\right)\left(\frac{dx^1}{dx^0}\right)^2}{\left(g_{00}+g_{01}\frac{dx^1}{dx^0}\right)^2} \quad \text{(D)}$$

Keep in mind, this result (D) assumes $d\theta = d\phi = 0$, also Crawford and Tereno defined the velocity of the particle with respect to a static observer (r = constant) as

$$\upsilon^2 = \frac{1}{-g_{00}^2}\left(\frac{dR}{dt}\right)^2$$

Both papers [6], [8] claims for at $R = \alpha_0 = 2GM$, Eddington-Finkelstein metric becomes

$$g_{00} = \left(1-\frac{2GM}{R}\right)$$

$$g_{01} = g_{10} = \pm 1$$

It results in 25 of [8] and conclusion of equation 187 in [6]

$$\lim_{R\to 2M}\upsilon = 1$$

$$\lim_{R\to 0} \upsilon = \infty$$

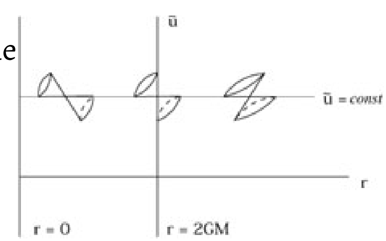
Mr. Mitra and Crawford both claimed that this result will be found in all other coordinate system, but Mr. Mitra took it a step further and I quote

> "And hence if there would be any spacetime below the EH, one would have $\upsilon^2 > 1$ for a material particle in direct violation of relativity"
>
> "Obviously this *Physical Invariant representing a local intrinsic property of the spacetime* **does blow up** at the EH, $R = \alpha_0 = 2M_0$. Hence the *EH must be a region of true physical singularity*."

This is equation (45) from Mr. Mitra's paper[6] and (29) from Crawford[8], they both are same

$$a = \frac{M_0}{R^2}\frac{1}{\sqrt{1-\frac{2M_0}{R}}} \tag{E}$$

Which diverges at $r = 2M_0$ and is also indeed correct! But as can be seen from the geodesic equation, for outside observer, anything never crosses the horizon(it takes infinite amount of coordinate time)! And this acceleration as calculated in equation (20) of "Acceleration of a static observer near the event horizon of a static isolated black hole"[9] , says something similar, when it does touch the event horizon, acceleration due to gravity is so huge that he **wouldn't be able to hover over and freeze in time**!! And I would also like to add a comment on the derivation of equation (E) which will provide some clarification. Here authors assume that the particle isn't moving, but if the particle is moving, then it has to be inside it's light cone, and hence as the timelike observer remains timelike and lightlike observers remain lightlike only thing that can stay at $r = 2GM$ is photon(seen from the diagram). Assuming stationary observer at $r = 2GM$ was automatically converting a timelike observer to lightlike! ($ds^2 = g_{tt}dt^2 + g_{rr} \times 0 = 0 \times dt^2 = 0$ in Eddington-Finkelstein coordinate). This suggests why we couldn't use equation (E) at event horizon. But Mr. Mitra interpreted it as a symbol of singularity, norm of 4 acceleration is indeed a physical invariant but it's isn't the property of spacetime, it describes the particle not spacetime..! Which is why we use kretschmann scalar to deduce if the singularity at event horizon is real or not. Kretschmann scalar depends on Ricci Tensor, which does describe the spacetime geometry not the 4 acceleration which is the byproduct of curvature (because there is Γ^k_{ij} term which depends



on curvature!). Well that was what Mr. Mitra concluded, but the Mr. Crawford tried to took it different way and resolved the claim by going over the earlier definitions and I quote:

> "Notice also that an observer cannot stay at rest in a Schwarzschild field at $r = 2m$, where $g_{ab}u^a u^b = 0$, for he (or she) cannot have there a timelike four-velocity field tangent to its worldline. This means that only a photon can stay at rest at $r = 2m$, and with respect to this "photon-frame" all particles have $v^2 = 1$, as it should be expected"
>
> "But on the surface of a neutron star $(\sqrt{g_{11}})$ may exceed unity by a very large factor, and for a black hole $a \to \infty$ (acceleration) as $r \to 2m$. It follows that a 'particle' at rest in the space at $r = 2m$ would have to be a photon"
>
> "This emphasizes the point that one cannot use expressions like (17) or (22) at the surface $r = 2m$. In other words, there is no observer at rest on that surface"

Where Taylor and Wheeler mentioned in their textbook *Exploring Black Holes* (p. 13-15) said:

> "Shell–and shell observers–cannot exist inside the horizon or even *at* the horizon, where the spherical shells experiences infinite stresses"

These explanations are perfectly fine (in a way) but we should realize that 3-speed and ordinary speed both are very different things, one is for ease in calculation(as it's invariant under Lorentz transformation) whereas other truly represents what the outside observer sees. For the sake of clarification and difference between the two, I'm going to quote Griffith from his book "Introduction to Elementary Particles"

> "Proper velocity is a hybrid quantity, in the sense that distance is measured in the *lab* frame, whereas time is measured in the particle frame. Some people object to the adjective "proper" in this context holding that this should be reserved for quantities measured entirely in the particle frame. Of course, in it's own frame the particle never moves at all – it's velocity is zero. If my terminology disturbs you, call η the 'four-velocity'. I should add that although **the proper velocity is the more convenient quantity to *calculate* with, ordinary velocity is still the more *natural* quantity** from the point of view of an observer watching the particle fly."
>
> --- (page 97, footnote)

These defined 3-speed is **not** coordinate speed! It's obvious how fast an object is moving is defined by

$$speed = \frac{distance\ covered}{time\ taken} = \frac{dx}{dt}$$

In this definition, both the measurement of dx and dt are done in the same frame of reference. But do note, at slow speed and in weak gravitation both the equations are same, as the elements of metric essentially becomes $+1$!

From this we can conclude that, one must be careful about which equation to use and when to use or else it can lead to disastrous conclusion. From most of these analysis on properties of event horizon, Mr. Mitra looking at the weird result concluded since this can not be true and we can get rid of it by assuming black holes don't exist or $\alpha_0 = 2GM = 0$, which is like saying "a non existent problem doesn't need a solution" and with that we're back in flat spacetime, where life is simple. But now, if since Schwarzschild Solution doesn't describe what happens at/inside the event horizon, what then? Well, we perform coordinate transformation, but why? And I quote Mr. Mitra.

> Then why should *R* and *T* be bad coordinates particularly when, in GTR, there are really no "bad" or "good" coordinates

Well, no remember spacetime is a 4 dimensional manifold (Sean Carroll, Spacetime and Geometry), and like any manifold which has some curvature you can't use one map to cover it all, and since the absurd results we get by using Schwarzschild metric, it means this map can not cover (event horizon + everything inside) it and we need a way out, "Eddington-Finkelstein" coordinate. Like if you use that equation (D) for speed, using Schwarzschild metric, you will get ∞ as an answer which is also really absurd! But before we move on, I would also like to mention that in General Relativity all observers are equivalent, coordinate system and observers are related but not same, a particular observer can use/have multiple coordinates to study and analyze but before he/she jumps to any conclusion he would need to check if his coordinates do "cover" the parts of spacetime he/she is studying!

Before we proceed ahead I would like to clear Mr. Mitra's doubt in equation (89) of his, where he said and I quote:

> many authors quietly put a modular sign in the argument of logarithmic term of R_* without even mentioning so:

$$R_* = R + \alpha_0\ ln\left|\frac{R - \alpha_0}{\alpha_0}\right| \quad \textbf{(F)}$$

This is in Eddington-Finkelstein, and if you check this is indeed what you will find. Here Mr. Mitra's claim is based on the assumption that, since (R, T) of Schwarzschild metric is not well behaved inside Event Horizon, we perform a coordinate transformation but even after that, this behavior still persists.

If we use this equation (**F**) without modular sign in region $R < \alpha_0$, it would lead to R_* being imaginary. And since, even this coordinate transformation couldn't fix the issue, we added modular sign just so the notion of Black Holes and spacetime inside event horizon survives!

Let me try to derive the same result and see for ourselves if he is indeed right or we just come up with modular sign!

Let's begin by considering a null radial geodesic for which, $ds^2 = 0$

$$ds^2 = 0 = \left(1 - \frac{2GM}{R}\right) dT^2 - \left(1 - \frac{2GM}{R}\right)^{-1} dR^2$$

$$\left(1 - \frac{2GM}{R}\right) dT^2 = \left(1 - \frac{2GM}{R}\right)^{-1} dR^2$$

$$\frac{dT^2}{dR^2} = \left(\frac{dT}{dR}\right)^2 = \left(1 - \frac{2GM}{R}\right)^{-2}$$

$$\left(\frac{dT}{dR}\right) = \pm \left(1 - \frac{2GM}{R}\right)^{-1} \qquad (as\ \sqrt{x^2} = |x|) \qquad \text{(G)}$$

$$dT = \pm \left(1 - \frac{2GM}{R}\right)^{-1} dR$$

$$dT = \pm \frac{dR}{\left(1 - \frac{2GM}{R}\right)} \qquad \text{(H)}$$

$$dT = \pm \frac{\left(1 - \frac{2GM}{R} + \frac{2GM}{R}\right)}{\left(1 - \frac{2GM}{R}\right)} dR$$

$$\int dT = \pm \int dR \pm \int \frac{\frac{2GM}{R}}{\left(1 - \frac{2GM}{R}\right)} dR$$

Integrating both sides and assuming $\int dT = T_*$ we get,

$$T_* = \pm R \pm \frac{\frac{2GM}{R}}{\left(1 - \frac{2GM}{R}\right)} dR + constant$$

Let's simplify it further by multiplying both numerator and denominator by $\frac{R}{2GM}$

$$T_* = \pm R \pm \frac{\frac{2GM}{R} \times \frac{R}{2GM}}{\left(1 - \frac{2GM}{R}\right) \times \frac{R}{2GM}} dR + constant$$

$$T_* = \pm R \pm \frac{1}{\left(\frac{R}{2GM} - 1\right)} dR + constant$$

Substituting $\frac{R}{2GM}$ as β we get, $d\beta = \frac{dR}{2GM}$

$$T_* = \pm R \pm 2GM \int \frac{d\beta}{(\beta - 1)} + constant$$

$$T_* = \pm R \pm 2GM\ ln|\beta - 1| + constant \qquad \left(as \int \frac{1}{x} dx = ln|x|\right)$$

substituting β back into the equation we get,

$$T_* = \pm R \pm 2GM\ ln\left|\frac{R}{2GM} - 1\right| + constant$$

Here we assume the rather complicated term to be R_* and then get the solution in new coordinate which looks rather simple! i.e.

$$T_* = \pm R_* + constant$$

From this, one can see that the modulus was, actually the consequence of performing integration the right way! Not some ad hoc mathematics, and yes indeed many textbook miss this part, which is why I had to derive it from the scratch. I hope this will give Mr. Mitra some answers he wanted from these textbooks but couldn't find. If one looks at equation (G) of mine, there is used $\pm$ sign, which if you ask me, is there because both of these terms upon squaring lead to same conclusion, i.e. $(+x)^2 = (-x)^2 = x^2$ and we didn't want to leave any possibility.

IV. Non Occurrence of Trapped Surfaces again!

We have already debunked many of such claims but there is one more in section 14.2.2 of his paper [6] which needs some special care. He argues that the equation $M \leq 2L$ is in accordance with his theory and Abhay Ashtekar had rejected it without reading. Mr. Mitra mentioned that this paper [10] was all about self energy in static gravitational field and no collapse, but Mr. Mitra perhaps didn't go through the whole paper carefully, perhaps he just skimmed over and saw the equation $M \leq 2L$ and concluded that even this paper [10] supports his claim!

Up until now, in all the results R meant the proper radius of object and M the total mass (which includes the negative binding energy [11]) , but in this result $M \leq 2L$, which he again is misinterpreting, M is the mass contained inside the coordinate sphere (without binding energy) of radius "R", for which the condition of formation of trapped surface is very different, as here Bizon didn't consider the self energy! In his other paper, Bizon also proved for , $M \geq L$ trapped surface would form! [10] ($L \leq M \leq 2L$ event horizon will form and until $M \leq L$ there won't be any trapped surface!)

In equation (1) of his paper "*Trapped Surfaces in spherical stars*" he argued if $M \geq L$ then, Ω must contain a trapped surface, here again by "M" he meant the mass inside the surface Ω with proper radius L and in the last paragraph of his work he concluded, [11]

> "The key results, (1) and (3), have only been derived in the spherically symmetric case, but, of course, they are obviously valid (with some minor adjustments of the constants) for any data which are close to spherical symmetry."

V. Vaidya Metric and ECO

In this section, Mr. Mitra had really strong argument since stars are radiating body and they are always emitting electromagnetic wave and such so perhaps we should study them from Vaidya Metric and I quote!:

> The exterior spacetime of a collapsing and radiating body is described by the Vaidya metric
>
> $$ds^2 = \left(1 - \frac{2M(\upsilon)}{R_0}\right) d\upsilon^2 - 2\upsilon dR_0 - R_0(d\theta^2 + sin^2\theta d\phi)$$
>
> where υ is retarded time. The determinant of this (external) metric is
>
> $$g = -R_0^4 \ sin^2\theta$$
>
> the expression for $g_{ij}^{interior}$ is not known but it is expected that both the metric and g must be continuous everywhere including at the boundary of the body. Also, since there is no vacuum spacetime, the question of supposed "Schwarzschild Metric Singularity" also should not arise. Thus it is expected that, the boundary of the collapsing fluid smoothly approaches $R_0 = 0$ as $g \to 0$. This however requires that, $g_{\upsilon\upsilon}(R) \geq 0$ and no event horizon forms:
>
> $$\frac{2M(\upsilon)}{R_0} \leq 1$$
>
> As $R \to 0$, one must have $M \to 0$ demanding the entire mass energy to be radiated out. However in case, one would have, $\frac{2M(\upsilon)}{R_0} < 1$, in this limit, i.e. if an event horizon would not form, there would be further emission of radiation and, M can travel to $-\infty$! To avoid this unphysical occurrence, one must have
>
> $$\lim_{R_0 \to 0} \frac{2M(\upsilon)}{R_0} = 1$$

> But, if so the world line of the particle on the boundary will become non-timelike. Therefore this state of R = 0 must not ever be reached, in other words, the comoving proper time for the formation of the eventual zero mass black hole must be infinite!

Well at first it does indeed looks like there is no coordinate singularity at $R = 2M$ so no event horizon and collapse? Perhaps there is hope for, us all? Perhaps radiating objects like the sun, can't form an event horizon? But looking back at Equation (4.11) in Vaidya's original paper of 1951 titled "The gravitational field of a radiating star", where he actually mentioned his metric using Schwarzschild coordinate! is [12]

$$ds^2 = \frac{\dot{M}^2}{f^2}\left(1 - \frac{2M}{R}\right)dT^2 - \frac{1}{\left(1 - \frac{2M}{R}\right)}dR^2 - R^2 d\Omega^2$$

which at $R = 2M$ does have the coordinate singularity! Also in the paper where Mr. Vaidya derived this result he mentioned/assumed the radiation was electromagnetic and will follow null geodesic i.e. it would have zero rest mass! The same metric can also be found as equation (9.31) of the textbook "Exact Space-Times in Einstein's General Relativity" by Jerry B. Griffiths and Jiri Podosky. The exact form used by Mr. Mitra here, is also mentioned in Exact Space-Times in Einstein's General Relativity and I quote [14]:

> "In fact, the metric (9.31) can be expressed in a much more useful form (for outgoing radiation) by the introduction of a null coordinate υ such that $d\upsilon = -\frac{1}{f(M)}dM$. With this, the line element becomes
>
> $$ds^2 = \left(1 - \frac{2M(\upsilon)}{R_0}\right)d\upsilon^2 - 2\upsilon dR_0 - R_0(d\theta^2 + sin^2\theta d\phi)"$$

Now it is important to note that, this metric is only applicable to null dusts. But stars won't always radiate and the time when, this emission of radiation via nuclear fusion will come to halt and we won't be able to apply this metric! Do note that R = 2GM in Schwarzschild form of Vaidya metric isn't about event horizon and I quote:

> Apart from the presence here of pure radiation, the main difference between this and the Schwarzschild space-time is that the coordinate singularity at $r = 2M(\upsilon)$ is not a null hypersurface and therefore cannot be an event horizon. In fact, it is an example of an *apparent horizon.*

In the text Mr. Podolsky and Griffiths also mentioned the possibility of collapse and the likely outcome of it in vaidya spacetime, which I quote:

> In fact, two possible scenarios arise. If the surface of the body passes through its Schwarzschild horizon, then a black hole is formed and the structure of the space-time will be that illustrated in Figure 9.17. On the other hand, if the outer surface of the body remains outside its Schwarzschild horizon while its volume reduces to zero, then the structure of the space-time will be that illustrated in Figure 9.18 in which a Vaidya region is matched to a subsequent Minkowski region.

But looking at the first possibility that it will **radiate all of it's mass** and studying that under Vaidya metric means, we're talking about the emission of particles with zero rest mass, but since that can not happen! As the matter inside the star is baryons and leptons, which is interacting with higgs field.

That's not the only point, these baryons can't also convert to photonic matter or anything which moves at the speed of light as that would be the violation of conservation of baryon/lepton number! But yes, Mr. Mitra's study of radiating matter using Vaidya metric does have a use, a star or object made purely of trapped photonic matter or anything with zero rest mass like graviton, which is also radiating. But that is unlikely yet possible under some circumstances!

VI. Conclusion

In presented paper, we looked into "non-occurrence of trapped surface" which was based on misinterpretation, using confusing symbols, erroneous calculation and hiding results. Later we analyzed Mr. Mitra's logic about, if indeed the Mass of Schwarzschild Black Hole is zero or not and concluded his proof of zero mass black hole to be wrong and just another erroneous result! Later we dived into the result and learned that treating 1-forms as differentials can be dangerous specially when it comes to Relativity. Today every standard textbook has a section of n-forms and their use in defining integrals on manifolds, but perhaps Mr. Mitra missed that part or skipped it or maybe his textbook didn't include the chapter or he didn't study it from textbooks instead learned from research papers!. We also studied the basis for eternally collapsing object, really a simple logic, one based on another and concluded indeed perhaps ECOs are nothing more than a misconception and denial of our lack of understanding.

Acknowledgment

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