A revisit of the problems of Eternally Collapsing Object by Abhas Mitra

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ABSTRACT

We critically review here the concepts which gave rise to the Eternally Collapsing Object (ECO) paradigm over almost 22 years. All the mathematical analysis will be dealt here to conclude the proofs required for ECO paradigm to begin with is ad-hoc. First part in rejecting black holes always considers the possibility of formation of trapped surfaces so we will begin our work by looking into "non occurrence of trapped surfaces" then we analyze Dr. Mitra's indirect claim regarding how $R = 0$ (inside black hole) could also be treated as another coordinate singularity! After that we review if indeed his another claim about "mass of Schwarzschild black hole being zero" is right or not. Do black holes really exist or they are just a fairy tale made by physicists to elude us into fiction? And are these so called "Black Holes", actually black hole or indeed as Mitra likes to call them "Eternally Collapsing Object" or ECO. Are ECOs really a true alternative to Black Holes? The analysis presented here will show us, why ECOs are baseless can not really a solution to black hole problem.

INTRODUCTION

> Most misunderstanding in the world could be avoided if people would simply take the time to ask, "What else could this mean?"
>
> – Shannon L. Alder

In this paper we first look into, Eternally Collapsing Object, which is an alternative solution to Black Hole Problem. Dr. Mitra believes that these so called Black Holes are actually Black Hole mimickers, since from outside the event horizon any black hole would behave like an object of mass M. But what does it even mean, when we say mass of the black hole? We will here look into Dr. Mitra's claim of "zero mass black hole", "non-occurrence of trapped surface", "Is singularity at R = 0 just another artifact of coordinate or are they real?"(section 4.1 of Black Holes or Eternally Collapsing Objects A Review of 90 Years of Misconceptions) and then we go into studying the notion of ECO using Vaidya metric. ECOs are interesting objects since from outside they behave like Black Holes as they emit almost none to no radiation but because of some pressure fluctuation and such, often few photons do manage to escape which could explain hawking radiation! As they are so dense their collision can explain what was observed during the gravitational wave detection at LIGO. Existence of ECO and non existence of black hole can indeed solve a lot of issue and give us all a sense of relief that now we understand a lot more about the universe and there are few less mystery to unravel! But before we consider ECOs to be real and black holes as fairy tale we need to look into trapped surfaces, their occurrence or non-occurrence which begins by looking into his paper "Non occurrence of trapped surface", which is more of a base to his later work. After reviewing this we will move foreword in our analysis of Eternally Collapsing Object and see if they are based on undeniable mathematical rigour or birth of misconception! We will be using natural system where $G = c = 1$ unless these terms specifically appear. All the misproofs and their calculation shown here are exactly what Dr. Mitra used in his papers.

I. Non occurrence of trapped surface

The analysis of the non occurrence of trapped surfaces by dr. Abhas Mitra begins with section 5 of the paper[1], where he starts off by assuming few quantities. These variables are actually used in few of the proofs he mentioned in the paper:

First "we define a quantity U, the rate of change of the circumference radius with the proper time following the radial worldline (at a constant r)",

$$U = \frac{\partial R}{\partial \tau}(r, \tau) \tag{a}$$

"We further define another quantity, which expresses the rate of change of the circumference radius with the proper distance"

$$\Gamma = \frac{\partial R}{\partial l}(r, \tau) \tag{b}$$

and since, there always exists a real proper time and proper distance, in a general fashion, we can define

$$\upsilon = \frac{\partial l}{\partial \tau} \tag{c}$$

after which he then assumes a static metric, and with that arrives at equation (75) $\left[\upsilon = \frac{dl}{\sqrt{g_{00}} dx^0}\right]$ in his paper, using

$$ds^2 = g_{00}(dx^0)^2 + g_{rr}dr^2$$

$$ds^2 = g_{00}(dx^0)^2 - dl^2$$

$$d\tau^2 = g_{00}(dx^0)^2 \tag{d}$$

I shall remind you that Dr. Abhas Mitra insisted on using derivative notation instead of partial derivative throughout the calculation! In his notation the above defined quantities look something like this:

$$U = \frac{dR}{d\tau}$$

$$= \frac{1}{\sqrt{g_{00}}}\frac{dR}{dt}; \tag{e}$$

since by definition $dl^2 = -g_{rr}dr^2$

$$\Gamma = \frac{dR}{dl}$$

$$= \frac{1}{\sqrt{-g_{rr}}}\frac{dR}{dr} \tag{f}$$

using the same methodology:

$$\upsilon = \frac{dl}{d\tau} = \frac{1}{\sqrt{g_{00}}}\frac{dl}{dt} \tag{g}$$

Applying chain rule on (e), we get:

$$U = \frac{dR}{d\tau} = \frac{dR}{dl}\frac{dl}{d\tau} = \Gamma\upsilon \tag{h}$$

These three physical variables obeys one particular equation describing a collapsing fluid. This equation is the result (12) of *"Hydrodynamic Calculations of General-Relativistic Collapse"*, which is a solution of Einstein Field Equation for collapsing fluid having non-zero pressure in comoving frame (i.e. the fluid is at rest in this frame):

$$\Gamma^2 = 1 + U^2 - \frac{2GM}{R} \tag{i}$$

For purely radial motion, one may ignore the angular part of the metric to write:

$$ds^2 = g_{00}dt^2 + g_{rr}dr^2$$

assuming, $ds^2 \geq 0$

$$g_{00}\left(1 + \frac{g_{rr}dr^2}{g_{00}dt^2}\right) \geq 0 \tag{j}$$

from equation (f), there exists a positive definite quantity

$$\Gamma^2 = \frac{1}{-g_{rr}}\left(\frac{dR}{dr}\right)^2 = 1 + U^2 - \frac{2GM}{R} \tag{k}$$

From this we can construct an inequality and which has L.H.S. as positive number.

Mathematically,

$$\text{since}\ \ \Gamma \geq 0 \ \ \text{and} \ \ \left(1 + \frac{g_{rr}dr^2}{g_{00}dt^2}\right) \geq 0 \qquad \text{(as } g_{00} \geq 0\text{)}$$

the resulting inequality should also be

$$\Gamma^2\left(1 + \frac{g_{rr}dr^2}{g_{00}dt^2}\right) \geq 0 \tag{l}$$

on simplification, we get

$$\Gamma^2 + \Gamma^2\left(\frac{g_{rr}dr^2}{g_{00}dt^2}\right) \geq 0$$

using his both (f) and (i) definition of "Γ", we get

$$\left(1 + U^2 - \frac{2GM}{R}\right) + \frac{1}{-g_{rr}}\left(\frac{dR}{dr}\right)^2\left(\frac{g_{rr}dr^2}{g_{00}dt^2}\right) \geq 0$$

$$\left(1 + U^2 - \frac{2GM}{R}\right) - \left(\frac{dR}{dr}\right)^2\left(\frac{dr^2}{g_{00}dt^2}\right) \geq 0$$

$$\left(1 + U^2 - \frac{2GM}{R}\right) - \left(\frac{1}{g_{00}}\right)\left(\frac{dR}{dr}\frac{dr}{dt}\right)^2 \geq 0 \tag{m}$$

The term in the second part of the inequality is exactly U^2 but with "$-$" sign so,

$$\left(1 - \frac{2GM}{R}\right) \geq 0$$

or,

$$R \geq 2GM \tag{n}$$

and hence, no trapped surface will form!!!

Calculation seems to suggest that event horizon will never form but this result is just a consequence of treating partial derivative as derivative. If we notice, both the term "U" & "Γ" were originally defined in terms of partial derivative, even the other paper "*Hydrodynamic Calculations of General-Relativistic Collapse*" by Dr. Micheal May and Richard White[2], defined them like partial derivatives and arrived at the result (i), which Dr. Mitra used in his proof shown above.

During the proof if we used equation **(f)** and applied chain rule on **(m)** to arrive at **(n)** which is a violation of partial derivatives. If he treated them like partial derivatives, he would have gotten an uglier equation, like this:

$$\left(1 + U^2 - \frac{2GM}{R}\right) + \left(\frac{\partial R}{\partial r}\frac{dr}{dl} + \frac{\partial R}{\partial \tau}\frac{d\tau}{dl}\right)^2 \left(\frac{g_{rr}dr^2}{g_{00}dt^2}\right) \geq 0$$

Also the fact that "U^2" got canceled was because he treated ∂ as $\mathbf{d}$. It was also clear from the fact that Einstein Field Equation is a system of non-linear partial differential equations, and the result (i), it's solution (both papers derive them from EFE)! But since he insisted on using derivative notation, he ended up treating them like one!

Another somewhat similar analysis in favor of non-occurrence of trapped surface was presented as an "ultimate proof" was provided by Dr. Abhas Mitra. In this section author proceeds by using equation (i) and (h) to get,

$$\Gamma^2 = 1 + (\Gamma\upsilon)^2 - \frac{2GM}{R} \qquad \text{(substituting } U = \Gamma\upsilon\text{)}$$

$$\Gamma^2 - (\Gamma\upsilon)^2 = 1 - \frac{2GM}{R}$$

$$\Gamma^2(1 - \upsilon^2) = 1 - \frac{2GM}{R}$$

$$\frac{\Gamma^2}{\gamma^2} = 1 - \frac{2GM}{R} \tag{o}$$

since, $\frac{\Gamma}{\gamma} \geq 0$ the R.H.S. of the equation **(o)** should be ≥ 0,

hence,

$$1 - \frac{2GM}{R} \geq 0$$

or,

$$R \geq 2GM$$

The above equation we arrived at was only possible because the simplification involved $U = \Gamma\upsilon$, which is completely wrong. If we treat them like partial derivatives it will never lead to $R > 2GM$.

$$U = \frac{dR}{d\tau} = \frac{dR}{dl}\frac{dl}{d\tau} = \Gamma\upsilon \tag{p}$$

$$U = \frac{\partial R}{\partial \tau}(r, \tau) \tag{q}$$

We can actually defend the misproofs by assuming a path having $dR = 0$ and from this we can get (p) and the last equation (o), but we should be very careful before taking it for granted, because the equation $dR = R'dr + \dot{R}dt = 0$ does not represent the surface of the star!

Since we are in comoving frame of the fluid, surface is represented by $dr_s = 0$ (also only outside $r = r_s$, the equality $R(r, t) = r$ will hold because of birkhoff theorem) which makes

$$dR(r_s, t) = R'dr + \dot{R}dt = 0$$
$$dR(r_s.t) = R' \times 0 + \dot{R}dt = \dot{R}dt = 0$$

We should realise here and now that using Birchoff Theorem we related r from comoving frame to R from non-comoving frame which also represents *circumference variable* in comoving frame. But leaving that aside for a moment and looking at them in our comoving frame, birkhoff theorem implies $R(r_s, \tau) = r_s$ that makes $\dot{R} = \dot{r_s} = 0$ for the surface of collapsing fluid. Taking a closer look for $r = r_s$.

$$dR(r_s, t) = R'(r_s, t)dr_s + \dot{R}(r_s, t)dt$$

$$= {R'}_s \frac{dr_s}{dt}dt + \dot{R}_s dt$$

$$= \left({R'}_s \frac{dr_s}{dt} + \dot{R}_s\right) dt = 0$$

from we find, as $\dot{r}_s = 0$

$$\dot{R}_s = 0$$

This result is indeed very crucial which describes the collapsing surface using that with the original definition of U and Γ [2]:

$$U = \frac{\dot{R}_s}{\sqrt{g_{00}}}$$

$$= \frac{0}{\sqrt{g_{00}}} = 0 \qquad \text{for } r = r_s$$

and

$$\Gamma = \frac{{R'}_s}{\sqrt{-g_{rr}}}$$

$$= \frac{1}{\sqrt{-g_{rr}}} \qquad \text{for } r = r_s$$

Where $\dot{}$ and $'$ represent $\frac{\partial}{\partial t}$ and $\frac{\partial}{\partial r}$ respectively, and both of Γ and U are constraint to obey this equation at $r = r_s$[2]

$$\Gamma^2 = 1 + U^2 - \frac{2GM}{r_s}$$

$$\frac{1}{-g_{rr}} = 1 + 0 - \frac{2GM}{r_s}$$

$$g_{rr} = -\frac{1}{1 - 2GM/r_s}$$

This is in agreement with Birchoff theorem and does not describe the condition for *non-occurrence of trapped surface*, where we used these results for simplification:

$$\Gamma = \frac{R'_s}{\sqrt{-g_{rr}}} = \frac{1}{\sqrt{-g_{rr}}}$$

and

$$R'_s = \frac{\partial R_s}{\partial r} = \frac{\partial r_s}{\partial r} = 1$$

We can quickly perform a domain check to arrive at $R \geq 2GM$, since $\Gamma = \frac{R'_s}{\sqrt{-g_{rr}}}$ and the term inside $\sqrt{-g_{rr}}$ should be positive, which makes $-g_{rr} > 0$ and from that

$$\infty \geq -g_{rr} > 0$$

$$0 \leq \frac{1}{-g_{rr}} < \infty$$

$$0 \leq 1 - \frac{2GM}{r_s} < \infty$$

$$1 \geq \frac{2GM}{r_s} > -\infty$$

Removing the possibility of negative mass and it becomes

$$0 \leq \frac{2GM}{r_s} \leq 1$$

The above argument does open up a possibility for $r_s = 2GM$. Since for $g_{rr} = \infty$, $\frac{1}{g_{rr}} = 0$ and $r_s = 2GM$.But the shown proof describing the collapsing surface does not describe the condition for *non-occurrence of trapped surface*. To conclude this we should go back to our calculations and observe closely.

1. The collapse of the star obeying the relation (i) began when $r >> 2GM$.
2. We assumed an observer having $dR = 0$.
3. But using Birchoff Theorem implies R is radial variable only at/outside the star.
4. Meaning the observer obeying $dR = 0$ is well outside $r = 2GM$.
5. Relating it to radial variable using Birchoff Theorem implies $dR = 0$ in non-comoving frame.

6. Which means the observer obeying $dR = 0$ is not moving along with the collapsing surface.
7. As $dR = 0$ describes a stationary observer but the fluid is collapsing soon it will become an outside observer
8. and for him collapse never completes.

So, the result is already in agreement with what was proposed by Oppenheimer and Snyder[3]. In summary ***if R is inside the star, then we can use equation (i) but R would not represent the radial variable, if R is outside the star, we can interpret it as radial variable but collapse never completes in his frame of reference and if R represents the surface then dR=0 represents a non-comoving observer which will become an outside observer very soon.***

There are few more results which seems to prove the *non-existence of trapped surface* but are based on treating partial derivative as derivative due to bad notation used in the paper. This next calculation can be found in the papers "nonoccurence of trapped surface".

Let's look at equation 201 from his paper "Non-occurrence of trapped surface" for the speed of particle

$$\upsilon^2 = \left(\frac{dl}{d\tau}\right)^2 = \frac{\left(\frac{dR}{d\tau}\right)^2}{\left(\frac{dR}{dl}\right)^2} = \frac{U^2}{\Gamma^2} = \frac{\Gamma^2 - 1 + \frac{2GM}{R}}{\Gamma^2} \tag{r}$$

since particles are not allowed to move faster than the speed of light, i.e. $\upsilon^2 \leq 1$

$$\upsilon^2 = \frac{\Gamma^2 - 1 + \frac{2GM}{R}}{\Gamma^2} \leq 1$$

or,

$$\frac{2GM}{R} \leq 1$$

As was stated earlier this result is also a consequence of bad notation, which resulted when we treated, partial derivatives as derivatives and in general

$$\left(\frac{dl}{d\tau}\right)^2 \neq \frac{\left(\frac{\partial R}{\partial \tau}\right)^2}{\left(\frac{\partial R}{\partial l}\right)^2} \neq \left(\frac{\partial l}{\partial \tau}\right)^2$$

rendering this equation (r) or (201) in his paper, useless.

Now we will study the domain check where one more misconception regarding "non-occurence of trapped surface" emerges. The equations used in the calculation to show that are equation (32) and (36) of the paper *On Continued Gravitational Contraction*. This is equation (36) which is the function of r and τ [2]:

$$t(r,\tau) = \frac{2}{3}\sqrt{R_{gb}}\left(\sqrt{r_b^3} - \sqrt{R_{gb}^3 y^3}\right) - 2R_{gb}\sqrt{y} + R_{gb}\ln\frac{\sqrt{y}+1}{\sqrt{y}-1} \tag{s}$$

and this is equation (32) :

$$y = \frac{1}{2}\left[\left(\frac{r}{r_b}\right)^2 - 1\right] + \frac{r_b R}{R_{gb} r} \tag{t}$$

where,

$$R = \left(\frac{-3\sqrt{R_{gb}}}{2}\left(\frac{r}{r_b}\right)^{3/2}\tau + \sqrt{r^3}\right)^{2/3} \qquad \text{for } r \leq r_b \tag{u}$$

$$R_{gb} = 2GM \tag{v}$$

From the above equations it can easily spotted $t(r,\tau)$ contains a logarithmic term, which is only defined if the term inside is >0.

Mathematically,

$$\frac{\sqrt{y}+1}{\sqrt{y}-1} > 0$$

It asserts this is possible only if, $y > 1$. At $r = r_b$ first term in the expression of y collapses and all we have is

$$y_{r=r_b} = \frac{R}{R_{gb}} > 1$$

substituting back the variable, $R_{gb} = 2GM$ we get,

$$R > 2GM \tag{w}$$

This might tempt us to say, it the proof for "non-occurrence of trapped surface" but before we jump to any conclusion, we should recall that R is not the radial distance. It's just a placeholder which is there to make calculation easier and had been defined in terms of r and τ by Oppenheimer and Snyder in equation (27) of their paper[3].

$$R = \left(\frac{-3\sqrt{R_{gb}}}{2} \left(\frac{r}{r_b}\right)^{3/2} \tau + \sqrt{r^3} \right)^{2/3} \qquad \text{for } r \leq r_b$$

Putting it back in equation **(w)**, we get something really different in terms of *radial variable*!

$$\left(\frac{-3\sqrt{2GM}}{2} \left(\frac{r}{r_b}\right)^{3/2} \tau + \sqrt{r^3} \right)^{2/3} > 2GM \tag{x}$$

This equation has dependence on τ and r, and can not be represented as a proof for *non-occurence of trapped surface.*

Few more proofs can be presented using Tolman-Oppenheimer-Volkoff equation on pressureless dust to argued that if $p = 0$ then, so must $\rho = 0$. Where we should look back at the internal assumption which says that the **Tolman-Oppenheimer-Volkoff equation is only valid for hydrodynamic equilibrium**[4] and hence can't be used to conclude anything about a collapsing fluid. Yes, this is indeed not very realistic, yes real fluids do have pressure and the solution provided by Oppenheimer-Snyder is very much ideal, but there had been some other studies on introducing pressure gradient and studying the collapse, one such work is "Gravitational collapse of an imperfect non-adiabatic fluid" by R. Chan with some success, as his result introduced the emergence of exotic energy during the end of collapse [5].

The above misproofs of non-occurrences were all provided by Dr. Mitra which ware based on misinterpretation. Right now, we can neither conclude that the true mathematical black hole as predicted by GR is real, nor it can be rejected unless shown with mathematical rigor. But we all hope that the singularity at the heart of the black hole isn't real and just a limitation of General Relativity being a classical theory! Until we have an experimentally tested model of quantum gravity we can't be sure of what's really inside the event horizon! Here I am assuming that the event horizon is indeed formed, as several papers have shown this to some extent. So, I remain optimistic! Before we move onto the next part we shall see one more independent work in favor of *non-occurrence of trapped surface* by Robertson and Leiter[27]:

$$ds^2 = c^2 d\tau^2 = c^2 d\tau_{syn}^2 - dl^2 > 0$$

where $d\tau_{syn}$ represents the element of proper time synchronized along the particle trajectory and is given by (Landau & Lifshitz vol.2 pg. 270):

$$d\tau_{syn} = \left(1 + \frac{g_{0k}}{g_{00}}\frac{dx^k}{dx^0}\right)\frac{\sqrt{g_{00}}dx^0}{c} \tag{1}$$

and dl represents the element of spatial distance (Landau & Lifshitz vol.2 pg. 253):

$$dl^2 = \left(\frac{g_{ok}g_{oj}}{g_{00}} - g_{kj}\right)dx^k dx^j = \gamma_{kj}dx^k dx^j$$

Defining a synchronized velocity by measuring the synchronized proper time along the worldline.

$$V^k = \frac{dx^k}{d\tau_{syn}}$$

corresponding synchronized 3-speed becomes:

$$V = \frac{dl}{d\tau_{syn}} = \frac{(\gamma_{kj}dx^k dx^j)^{1/2}}{d\tau_{syn}} = \left(\frac{\gamma_{kj}dx^k dx^j}{d\tau_{syn}^2}\right)^{1/2} = |\gamma_{kj}V^k V^j|^{1/2}$$

Redefining the spacetime interval as in terms of V:

$$ds^2 = c^2 d\tau_{syn}^2\left(1 - \frac{dl^2}{c^2 d\tau_{syn}^2}\right) = c^2 d\tau_{syn}^2\left(1 - \frac{V^2}{c^2}\right)$$

since $d\tau = ds/c$, we can express $d\tau$ in terms of τ_{syn}:

$$d\tau = d\tau_{syn}\left(1 - \frac{V^2}{c^2}\right)^{1/2} \tag{2}$$

Proper 3-velocity u^k can be expressed in terms of synchronized velocity V^k

$$u^k = \frac{dx^k}{d\tau_{syn}\left(1 - \frac{V^2}{c^2}\right)^{1/2}} = V^k\left(1 - \frac{V^2}{c^2}\right)^{-1/2}$$

and

$$u^0 = \frac{dx^0}{d\tau} = \frac{dx^0}{d\tau_{syn}}\left(1 - \frac{V^2}{c^2}\right)^{-1/2} = \left(1 + \frac{g_{0k}}{g_{00}}\frac{dx^k}{dx^0}\right)^{-1}\frac{c}{\sqrt{g_{00}}}\left(1 - \frac{V^2}{c^2}\right)^{-1/2}$$

Calculating the proper 3 speed:

$$u = |\gamma_{kj}u^k u^j|^{1/2}$$

$$= \left|\gamma_{kj}V^k V^j\left(1 - \frac{V^2}{c^2}\right)^{-1}\right|^{1/2}$$

$$= \left|V^2\left(1 - \frac{V^2}{c^2}\right)^{-1}\right|^{1/2}$$

$$u^2 = \frac{V^2}{1 - V^2/c^2} \tag{3}$$

$$u^2 - \frac{u^2 V^2}{c^2} = V^2$$

$$u^2 = V^2 \left(1 + \frac{u^2}{c^2}\right) \tag{4}$$

Since equation (2) and (3) are essentially same we have:

$$\frac{V^2}{1 - V^2/c^2} = V^2 \left(1 + \frac{u^2}{c^2}\right)$$

$$\implies 1 - \frac{V^2}{c^2} = \left(1 + \frac{u^2}{c^2}\right)^{-1} \tag{5}$$

Substituting equation (1) in equation (2) we get:

$$d\tau = d\tau_{syn} \left(1 - \frac{V^2}{c^2}\right)^{1/2}$$

$$= \frac{\sqrt{g_{00}}dx^0}{c} \left(1 + \frac{g_{0k}}{g_{00}} \frac{dx^k}{dx^0}\right) \left(1 - \frac{V^2}{c^2}\right)^{1/2} \qquad \text{(using equation 1)}$$

$$= \frac{\sqrt{g_{00}}dx^0}{c} \left(1 + \frac{g_{0k}}{g_{00}} \frac{dx^k}{dx^0}\right) \left(1 + \frac{u^2}{c^2}\right)^{-1/2} \qquad \text{(using equation 5)}$$

Up until now everything Leiter and Robertson did was correct but the equation (2j) which they arrived at in their calculation, because of bad typo was mistyped as:

$$d\tau = dt \left\{ (g_{00})^{1/2} \left[1 + \frac{g_{ok}}{g_{00}} \frac{v^k}{c^2}\right] \left(1 + \frac{u^2}{c^2}\right)^{-1/2} \right\}$$

$$= \frac{dt}{1+z}$$

Which is obviously incorrect, the first equation is typo and the latter is misinterpreted consequence of it. In general redshift factor for zero angular momentum is:

$$1 + z = \sqrt{\frac{c + dr/dt}{c - dr/dt}}$$

and using schwarzschild coordinate in flat spacetime i.e. $M = 0$, we would get something very different:

$$d\tau = \frac{dt}{\sqrt{1 + u^2/c^2}}$$

Authors then proceed with a transformation $u = t - R_s/c$. For ease in calculation we will switch back to natural system having $G = c = 1$.

From the above transformation we will get:

$$du = dt - dR$$

$$= dt - \frac{\partial R}{\partial l} dl - \frac{\partial R}{\partial \tau} d\tau$$

$$= d\tau_{syn}\sqrt{g_{00}}\left(1 + \frac{g_{0k}}{g_{00}}\frac{dx^k}{dx^0}\right) - \Gamma dl - U d\tau_{syn}\left(1 - \frac{V^2}{c^2}\right)^{1/2}$$

$$du = d\tau_{syn}\left[\sqrt{g_{00}} + \frac{g_{0k}}{\sqrt{g_{00}}}\frac{dx^k}{dt} - \Gamma dl - U\left(1 - \frac{V^2}{c^2}\right)^{1/2}\right]$$

Authors use another equation here as well, to justify *non-occurrence of trapped surfaces.* Their ad-hoc version of the equation is:

$$d\tau_{syn} = \frac{du}{1+z} = du(\Gamma + U)$$

The above equation has been used over and over again by Leiter and Robertson in their paper to justify multiple things like *non-occurrence of trapped surfaces* but ultimately it's incorrect. We can indeed arrive at the relation

$$\frac{1}{1+z} = \Gamma + U$$

By comparing equation 37 of [32] and equation 6 of [33]

$$L_\infty = L_s(\Gamma_s + U_s)^2$$

$$= \frac{L_s}{(1+z)^2}$$

But we should realize these equations are for outside observer and if we them then we are assuming an outside observer and it means collapse never completes for him which is a well established result [3].

By now, we have established the claims regarding "nonoccurence of trapped surface" is based on ad hoc maths, confusing notation and erroneous calculation or using wrong equations. Let's proceed ahead and see some of his other claims like how "Black Hole is a misconception"!

II. Is Black Hole a misconception?

The paper titled "Black Holes or Eternally Collapsing Objects A Review of 90 Years of Misconceptions" by Dr. Mitra can be categorically divided into four parts, where he discussed the possibility of non existence of black holes and claimed these so called objects are not really true black holes but are Black Hole mimickers!

- Denial of singularity at r = 0
- Convincing trapped surface isn't formed (already covered)
- Mass of Schwarzschild black hole is **zero**
- Arguing in favor of ECOs.

Denial of Singularity at $\mathbf{R} = 0$

Principle of Equivalence states, *"in small enough regions of spacetime, the laws of physics reduces to those of special relativity; it is impossible to detect the existence of gravitational field by means of* ***local experiments****."* (Carroll pg 50). The indirect claim of denial comes from studying infinitesimal neighborhood of singularity at $R = 0$ using local coordinate frame and assuming $g_{ij} \approx \eta_{ij}$ via POE.

If we are to study the singularity at $R = 0$ using LIC, we must be very careful because the assumption of LIC (Locally Inertial Frame) is based on $g_{ij}(p) = n_{ij}$ and $\Gamma^k_{ij}(p) = 0$ with,

$$g_{ij}(p + \delta p) \approx n_{ij}$$

$$\Gamma^k_{ij}(p + \delta p) \approx 0$$

During the analysis if we extend δp to include $R = 0$, then $\Gamma^k_{ij}(p + \delta p)$ can no longer be approximated to "0" and in fact it would blow up. For the moment if we ignore that and study $R = 0$ in LIC which because of curvature singularity can't be removed in the context of GR seems to assert that $R = 0$ in flat spacetime is not just a coordinate singularity, it must be real but using different coordinate to descibe $R = 0$ in LIC says otherwise that there is no singularity!. This is exactly, what Dr. Mitra exactly used to argue that singularity at $R = 0$ also behaves like a coordinate artifact just as $R = 2M$. This is his exact words where it can be seen he asserts, singularity at $R = 0$ in LIC is real:

> The metric $ds^2 = dT^2 - dR^2 - R^2\left(d\theta^2 + sin\theta d\phi^2\right)$ could also represent the spacetime in the infinitesimal neighborhood of a source of mass-energy (at $R = 0$) in a locally free falling frame. In this case, neither the choice of $R = 0$ is geometrically arbitrary nor are the curvature components identically zero. In such a case, the singularity at $R = 0$ might be a genuine physical singularity since the singularity persists at the location of the massenpunkt and actually cannot be physically removed by any coordinate transformation.

This quotation is from his paper "Black Holes or Eternally Collapsing Objects A Review of 90 Years of Misconceptions". Here one could as well have used another metric in LIC, like $ds^2 = dT^2 - dx^2 - dy^2 - dz^2$ which doesn't have singularity at $(x, y, z) = (0, 0, 0)$, but even this can't be extended to include $(x, y, z) = (0, 0, 0)$ when there is singularity at $R = 0$, as that would lead to the violation of $g_{ij}(p + \delta p) \approx n_{ij}$ and $\Gamma^k_{ij}(p + \delta p) \approx 0$.

Mass of Schwarzschild black hole is zero

There have been many proofs describing why the mass of Black Hole should actually be zero. Here are some of them, first by considering pressureless dust, which was used in Oppenheimer-Snyder solution for collapse, speed of sound in dust and few more, let's visit them one by one.

1. Using Tolman-Oppenheimer-Volkoff equation

$$\frac{dp}{dR} = -G\frac{M(r) + 4\pi pR^3}{R^2\left(1 - \frac{2GM}{R^2}\right)}[p + \rho(0)]$$

This is equation (70) of his paper, and he mentioned if $p = 0$, it would mean $\dfrac{dp}{dR} = 0$. Which means,

$$0 = \left[-G\frac{M(r) + 4\pi pR^3}{R^2\left(1 - \frac{2GM}{R^2}\right)}\right]\rho(0)$$

Leading to $\rho(0) = 0$. But by now, know that the Tolman-Oppenheimer-Volkoff equation can only be used in case of hydrodynamic equilibrium! And as Oppenheimer-Snyder paper never assumed pressure to be zero before the collapse, this is not a violation!

2. Speed of sound in dust! The moment Oppenheimer-Snyder assumed pressure-less dust, they also assumed no sound waves (indirectly). As one knows sound is caused due to oscillation of particles and that means change in pressure. But as already assumed, no pressure means no sound and not using $c_s = \sqrt{\frac{dp}{d\rho}}$. But in this section that's exactly what Dr. Mitra did and I quote[6]:

> "In order that, $c_s \neq 0$ at the boundary of the dust ball (considered as a fluid) where $dp = 0$, one must again have $d\rho = 0$."

Here he implemented $c_s = \sqrt{\frac{dp}{d\rho}}$ for $dp = 0$ to evaluate $c_s = 0$ and only other way out was to make the equation indeterminate or $\frac{0}{0}$ form! Again as Oppenheimer and Snyder had already assumed, no pressure, it means no sound and not using $c_s = \sqrt{\frac{dp}{d\rho}}$.

3. Invariance of 4-Volume This section is my personal favorite, as this proof had me questioning black hole. Dr. Mitra begins by stating the fact that

$$\int \sqrt{-g} d^4x = invariant \tag{y}$$

Applying this on Eddington-Finkelstein coordinate and Hilbert (we call it Schwarzschild) coordinate:

$$\int \sqrt{-g_*} dT_* \; dR \; d\theta \; d\phi = \int \sqrt{-g} \; dT \; dR \; d\theta \; d\phi$$

also from (which he defined)

$$T_* = T \pm \alpha_0 \; ln \left(\frac{R - \alpha_0}{\alpha_0} \right)$$

which leads to

$$-g_* = R^4 sin^2\theta = -g$$

and substituting them simplifies the equations further.

$$\int R^2 sin\theta \; dT_* \; dR \; d\theta \; d\phi = \int R^2 sin\theta \; dT \; dR \; d\theta \; d\phi \tag{z}$$

carrying out integral on both sides on $d\theta$ and $d\phi$, we get:

$$\int R^2 \; dT_* \; dR = \int R^2 \; dT \; dR$$

we have,

$$\int R^2 \left(dT + \frac{\alpha_0}{R - \alpha_0} dR \right) \; dR = \int R^2 \; dT \; dR$$

$$\int R^2 \; dT \; dR + \int R^2 \frac{\alpha_0}{R - \alpha_0} dR \; dR = \int R^2 \; dT \; dR$$

$$\alpha_0 \int \frac{R^2}{R - \alpha_0} dR \; dR = 0$$

This yields, $\alpha_0 = 0$ and I quote [6]:

The integration constant $\alpha_0 = 0$ which arose in the solution for the spacetime around a "massenpunkt", turned out, after 90 years, to be actually zero.

Now, that would mean there is no curvature and spacetime in-fact is *flat!* Thank goodness, there already was a paper proving this result erroneous, titled *Comment on 'Comment on The Euclidean gravitational action as black hole entropy, singularities, and spacetime voids'* by Prasun K. Kundu. I'm going to use that paper and show via calculation to point out where this went wrong. Let's go back to equation (z):

$$\int R^2 sin\theta \; dT_* \; dR \; d\theta \; d\phi = \int R^2 sin\theta \; dT \; dR \; d\theta \; d\phi$$

which doesn't actually look like this, but:

$$\int R^2 sin\theta \; dT_* \wedge dR \wedge d\theta \wedge d\phi = \int R^2 sin\theta \; dT \wedge dR \wedge d\theta \wedge d\phi \tag{A}$$

Here after doing everything he did, the last equation will be:

$$\int R^2 \; dT \wedge dR + \int R^2 \frac{\alpha_0}{R - \alpha_0} dR \wedge dR = \int R^2 \; dT \wedge dR$$

Now from differential geometry we know $dR \wedge dR = 0$. Which actually yields,

$$\int R^2 \; dT \wedge dR = \int R^2 \; dT \wedge dR$$

Well, there's nothing much you can draw from this. But it does teach us one important lesson i.e. treating 1-form as differential is very much misleading! This is what led Dr. Mitra to the erroneous conclusion that Mass of a Schwarzschild Black Hole is actually "Zero". Also we can also cross check this and I quote:

> "on an n-dimensional manifold M, the integrand is properly understood as an n-form."
>
> – Carroll, Spacetime and Geometry, pg 88

Since spacetime is a 4 dimensional manifold, there is no denying in equation , and if we check equation 2.94 of textbook "Spacetime and Geometry" by Carroll, is

$$\sqrt{|g'|}dx^{0'} \wedge \cdots \wedge dx^{(n-1)'} = \sqrt{|g|}dx^0 \wedge \cdots \wedge dx^{(n-1)} \tag{B}$$

and Carroll also mentions this, "In the interest of simplicity we will usually write the volume element as $\sqrt{|g|}d^4x$, rather than as the explicit wedge product:

$$\sqrt{|g|}d^n x = \sqrt{|g|}dx^0 \wedge \cdots \wedge dx^{(n-1)} \tag{C}$$

This calculation showing the exact error in Dr. Mitra's claim was missing in Dr. Kundu's paper, which is why I mentioned it here along with citation to one of the standard textbook in this field.! Dr. Mitra did try to defend his result by arguing that, does this mean 170 years Jacobi formula is wrong?? Well, not quite but that one is limited. See Jacobian Determinant equation was defined in the context of linear algebra and vector space, and didn't involve any wedge product. But here these dx's are not differentials but one form or (0,1) anti-symmetric tensor. And this equation which we used to show the actual calculation has been defined in the context of differential geometry over manifolds, but since vector space too form a manifold, this result can be viewed as a generalization to Jacobi equation.

4. Why oppenheimer snyder black holes doesn't correspond to M=0. This part looks back at why exactly the Oppenheimer Snyder BHs don't correspond to $M = 0$. In the first paper titled "The fallacy of Oppenheimer Snyder Collapse" looking at equation (22) of the paper, which is

$$\int_0^{R_b} 4\pi R^2 \frac{\rho(t)v^2}{1-v^2} dR = 0$$

This quite clearly suggests, $\rho(t)v^2 = 0$ but one must be very careful here and why, we shall just see here. The equation (3) and equation (14) represent the mass of black hole in different coordinate systems which where used in the calculations are[24],

Mass in comoving coordinate/frame:

$$M(r,t) = \int_0^{r} 4\pi\rho(r,t)R^2R'dr$$

Mass in non-comoving coordinate/frame:

$$M(R,T) = \int_0^{R} 4\pi\frac{\rho(R,T)}{1-v^2}R^2 dR$$

via Birchoff theorem, he asserts that both of these should be equal which also seems obvious and then, as $dR(r,t) = R'dr$ at a particular moment

$$\int_0^{R_b} 4\pi\rho R^2 dR = \int_0^{R_b} 4\pi\frac{\rho}{1-v^2}R^2 dR$$

$$\int_0^{R_b} 4\pi\rho R^2 dR - \int_0^{R_b} 4\pi\frac{\rho}{1-v^2}R^2 dR = 0$$

$$\int_0^{R_b} 4\pi\left(\rho - \frac{\rho}{1-v^2}\right) R^2 dR = 0$$

$$\int_0^{R_b} 4\pi\left(\frac{\rho(t)v^2}{1-v^2}\right) R^2 dR = 0$$

From above we can conclude that $\rho(t)v^2 = 0$, but during the calculation we hid the part where both of these ρ in L.H.S. and R.H.S. of the equation were originally written in different coordinate system and hence had different form, if we redo all the calculation keeping that in mind we will arrive at:

$$\int_0^{R_b} 4\pi\left(\rho(r,t) - \frac{\rho(R,T)}{1-v^2}\right) R^2 dR = 0$$

This will again never lead to the same form, which Dr. Mitra used in his proof. Here "t" represents the time in comoving clock, and "T" represents time in non-comoving frame and in general both are different! Birchoff theorem could only be used to put a constraint on metric not on density and every other physical variable to suit one's need.

In the same paper concerning *the fallacies of Oppenheimer-Snyder collapse*, author asserts another point which had been used quite a lot in his other papers as well and that is using equation (4) of "Finite Self-Energy of Classical Point Particles". The equation goes something like this:

$$m = \lim_{\epsilon \to 0} 2m_0 \left[1 + \left(1 + \frac{m_0}{8\pi\epsilon} \right)^{1/2} \right]^{-1}$$

Dr. Mitra asserts here that ϵ is radius of the particle and as $\epsilon \to 0$, $m \to 0$. Calculation-wise it makes sense, but let's take a look at what the original authors had to say about this! In their paper, they considered the total energy of the gravitational field as[25]

$$E = \int (g_{ij,j} - g_{jj,i}) dS_i$$

Then, they consider a neutral static point particle and isotropic coordinate ($g_{ij} = \chi^4(r)\delta_{ij}$) and then the Einstein field equation becomes:

$$\begin{aligned} -2\left(R_0^0 - \frac{1}{2}R\right)({}^3g)^{1/2} &\equiv -8\chi \bigtriangledown^2 \chi \\ &= -T_0^0({}^3g)^{1/2} = m_0\delta(r) \end{aligned}$$

Solving and doing all the calculation, they arrive at:

$$m = \lim_{\epsilon \to 0} 2m_0 \left[1 + \left(1 + \frac{m_0}{8\pi\epsilon} \right)^{1/2} \right]^{-1}$$

And they mentioned, ϵ to be "the radius of the δ^3 function" in exact same word! The whole paper was about one particle system, which Dr. Mitra is misinterpreting and using in his calculation for dust and everywhere else it seems fit to justify his point! While arriving at this result they asserted the "energy to be correctly interpreted as total mass of the particle", which was "the total energy of the combined system". The statement under quotation are their exact words[25].

Dr. Mitra has also considered the equation of sort $M = \int_0^{R_0} 4\pi R^2 \rho dR$ in equation (25) and (26) of his other paper titled "Black Holes or Eternally Collapsing Objects A Review of 90 Years of Misconceptions", to assert that for $R_0 \to 0, M \to 0$ but while doing so he contradicted himself as he considered an extended body and for extended body $R_0 \neq 0$. The same proof also appears in his "The mass of the Oppenheimer-Snyder black hole no general relativistic collapse at all, no black hole, no physical singularity" and "Kruskal Coordinates and Mass of Schwarzschild Black Holes".

Arguing in favor of ECOs

From the above discussions Dr. Mitra concluded since, trapped surface isn't formed, singularity at $R = 0$ isn't real and the mass of a black hole is "0" (as he tried to prove), mixing that with proper time of collapse as "∞" , he went onto looking for a solution, which he did find! Eternally Collapsing Object, which takes infinite proper time to collapse and by the time it does collapse, it has already radiated away all of it's mass, leading to $M = 0$, which via equation (56) of the paper, which is in agreement with his idea, he concludes black holes are not formed and in-fact they are ever collapsing object, radiating bit by bit before it ever collapses to form a true Schwarzschild black hole! Just for the sake of clarity let's list them one by one!

1. Mass of a Schwarzschild Black Hole is **zero**
2. Proper Time of collapse is inversely proportional to M and for $M = 0$, he concluded $\tau = \infty$
3. Since Trapped Surface isn't formed
4. It must be radiating via H-K process (which used non-relativistic equations in his derivation and indirectly assumed weak gravitational field by using Newtonian equations!) and always be losing it's mass!

He concluded, if a we take these points seriously then a collapsing star will lost mass in the form of radiation, ever decreasing it's Schwarzschild Radius, and with that loosing some mass. Mix it up with proper time of collapse and voila you get infinite proper time.

Since we already saw his proof of "$M = 0$" and "non-occurrence of trapped surface" is based on erroneous calculations, he hasn't really proven that the Black Holes are not formed and also reviewing H-K process/mechanism in his paper, one will easily see his calculations over there is non-relativistic, which raises the doubt to even believe in it! Since his claims rely that stars will radiate out before the trapped surface is even formed! But as can be seen in "An analytical solution for gravitational collapse with radiation" by P. C. Vaidya, where he did consider pressure and still arrived at equation similar to Oppenheimer-Snyder paper titled "On Continued Gravitational Contraction", and I quote [7].

> "The rate of contraction $D_t R_0$ of the boundary of the sphere is given by an equation similar to the corresponding equation of Oppenheimer-Snyder."
>
> "which is of the same form as the corresponding equation for the rate of contraction of Oppenheimer-Snyder spheres"

From this, we conclude even the contribution of pressure, can not, for sure ever truly stop contraction. And now I would like to mention the assumption that mixes $M = 0$ with "radiating away" by Dr. Mitra which means, everything that made up the star will go away before it ever collapses. Which can happen in one of two ways, either because of some pressure fluctuation, trapped baryonic matter will escape, or these trapped matter gets converted to photon and then escape (highly unlikely as it leads to the violation of conservation of baryon number)! If the latter case happens, then we would also have to consider "exchange force", which in case of baryon was the source of degeneracy pressure but in case of bosons it acts like an attractive force and perhaps will support the collapse (because as the energy is there and this lead to decrease in degeneracy pressure but increment in radiation pressure, also decrement in degeneracy pressure wouldn't be completely fulfilled by radiation pressure as one depends on the density while latter on the energy/intensity of individual photons.)!

In first case, if baryonic particles escapes because of pressure fluctuation, then all that pressure can/will do, is provide an initial kick leading to escapes. But since that pressure isn't working outside the star, and it's not photon, the baryon will come back falling following it's geodesic back into the collapsing star!(considering isolated neutron star for simplification). Since this too doesn't lead to $M = 0$, we conclude that ECO are perhaps a birth of misconception, erroneous calculation and ad hoc mathematics.

III. Why event horizon is not a Physical Singularity!

Event Horizon is often defined as null hypersurface which had often been related to physical variables like speed or acceleration of a particle to assert that it's not just a coordinate singularity but also physical singularity and hence must not really exist.
If we define the **3-speed** of the particle something like this (equation (137) of [16]).

$$\upsilon^2 = \upsilon^\alpha \upsilon_\alpha = g_{\alpha\beta}\upsilon^\alpha \upsilon^\beta = g_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = \frac{dx^\alpha}{d\tau}\frac{g_{\alpha\beta}dx^\beta}{d\tau} = \frac{g_{\alpha\beta}dx^\alpha dx^\beta}{d\tau^2} = \left(\frac{dl}{d\tau}\right)^2$$

using schwarzschild coordinate for simplification:

$$\upsilon^2 = \left(\frac{dl}{d\tau}\right)^2 = \frac{-g_{RR}dR^2}{g_{TT}dT^2} = \left(1 - \frac{2GM}{R}\right)^{-2}\frac{dR^2}{dT^2}$$

For radially moving particle schwarzschild coordinate takes this form:

$$ds^2 = \left(1 - \frac{2GM}{R}\right)dT^2 - \frac{dR^2}{\left(1 - \frac{2GM}{R}\right)}$$

From using timelike killing vector we arrive at the total energy of particle in schwarzschild coordinate as[28]:

$$E_* = g_{00}\frac{dT}{ds} = \left(1 - \frac{2GM}{R}\right)\frac{dT}{ds}$$

$$\implies \frac{dT}{ds} = \frac{E_*}{1 - 2GM/R}$$

Dividing both side of Schwarzschild metric by ds^2 and simplifying we get:

$$1 = \left(1 - \frac{2GM}{R}\right)\frac{dT^2}{ds^2} - \left(1 - \frac{2GM}{R}\right)^{-1}\frac{dR^2}{ds^2}$$

$$1 = \left(1 - \frac{2GM}{R}\right)\left(\frac{dT}{ds}\right)^2 - \left(1 - \frac{2GM}{R}\right)^{-1}\left(\frac{dR}{dT}\frac{dT}{ds}\right)^2$$

$$1 = \left[\left(1 - \frac{2GM}{R}\right) - \left(1 - \frac{2GM}{R}\right)^{-1}\left(\frac{dR}{dT}\right)^2\right]\left(\frac{dT}{ds}\right)^2$$

$$1 = \left[\left(1 - \frac{2GM}{R}\right) - \left(1 - \frac{2GM}{R}\right)^{-1}\left(\frac{dR}{dT}\right)^2\right]\frac{E_*^2}{(1 - 2GM/R)^2}$$

$$\frac{(1 - 2GM/R)^2}{E_*^2} = \left(1 - \frac{2GM}{R}\right) - \left(1 - \frac{2GM}{R}\right)^{-1}\left(\frac{dR}{dT}\right)^2$$

$$\left(1 - \frac{2GM}{R}\right)^{-1}\left(\frac{dR}{dT}\right)^2 = \left(1 - \frac{2GM}{R}\right) - \frac{(1 - 2GM/R)^2}{E_*^2}$$

$$\left(\frac{dR}{dT}\right)^2 = \left(1 - \frac{2GM}{R}\right)^2 - \frac{(1 - 2GM/R)^3}{E_*^2}$$

$$\frac{dR}{dT} = \frac{1 - 2GM/R}{E_*}\left(E_*^2 - \left(1 - \frac{2GM}{R}\right)\right)^{1/2}$$

Using this expression to simplify υ we get:

$$\upsilon^2 = \frac{E_* - \left(1 - \frac{2GM}{R}\right)}{E_*}$$

$$\lim_{R \to 2GM} \upsilon^2 = 1$$

The very definition of υ^2 involves a term like $(1-2GM/R)^{-2}$ in the beginning which is not defined at $R = 2GM$ at the same time $dR/dT = 0$ making the resulting equation a limit at $R = 2GM$ in schwarzschild coordinate which is not applicable at $R = 2GM$. There is one more related claim, so let's discuss that and then move onto explaining what could it mean. This claim is discussed in section 11.2 of this paper. Here his equation (186), which can also be found in The Classical Theory of Fields by Landau and Lifshitz is

$$\upsilon^2 = \frac{\left(g_{01}^2 - g_{11}g_{00}\right)\left(\frac{dx^1}{dx^0}\right)^2}{\left(g_{00} + g_{01}\frac{dx^1}{dx^0}\right)^2} \tag{D}$$

This result also assumes $d\theta = d\phi = 0$. At $R = \alpha_0 = 2GM$, Eddington-Finkelstein metric becomes

$$g_{00} = \left(1 - \frac{2GM}{R}\right)$$

$$g_{01} = g_{10} = \pm 1$$

It results in:

$$\lim_{R\to 2M} \upsilon = 1$$

$$\lim_{R\to 0} \upsilon = \infty$$

Here author claimed that "if there would be any spacetime below the EH, one would have $\upsilon^2 > 1$ for a material particle in direct violation of relativity" and 'Obviously this *Physical Invariant representing a local intrinsic property of the spacetime* ***does blow up*** *at the EH,* $R = \alpha_0 = 2M_0$. *Hence the EH must be a region of true physical singularity*'[16].

But I beg to differ, The above used equation (D) uses the proper time **as measured by the clock synchronized along the trajectory**. During the synchronization, Landau and Lifshitz[17] used light and a mirror to reflect and didn't consider redshift of any sort. But at event horizon where the gravitational redshift is so huge that any photon emitted at the surface would be undetectable by the observer outside! This changes things as the specified synchronization of the clock which depends on emission of photon from the infalling particle and getting reflected by a mirror can't happen. Since that is out of the question the resulting equation (D) can't as well be used to describe anything about the falling observer at/inside $R = 2GM$. The entire notion of physical variable blowing up at event horizon is based on use of some equation which isn't even allowed to be used at or inside the event horizon.

The same claim about event horizon being a physical singularity by Dr. Mitra uses the analysis of "norm of 4-acceleration". This is equation (45) from Dr. Mitra's paper[6] and equation (20) from Doughty [9], both of them are same:

$$a = \frac{M_0}{R^2}\frac{1}{\sqrt{1 - \frac{2M_0}{R}}} \tag{E}$$

Obviously, it diverges at $R = 2M_0$ which raises the question that *is our equation correct and applicable at event horizon?* The first point that need to remind ourselves is that this equation is for an outside observer at rest and for any outside observer (as seen from the geodesic equation), nothing never crosses the horizon(it takes infinite amount of coordinate time)! I would also like to add a comment on the derivation of equation (E) which will provide some clarification. We will do this calculation in schwarzschild metric:

$$ds^2 = g_{TT}dT^2 + g_{RR}dR^2 + R^2 d\Omega$$

A static particle will have:

$$u^\alpha = \frac{dx^\alpha}{d\tau} = \left(\frac{1}{\sqrt{g_{TT}}}, 0, 0, 0\right)$$

the proper acceleration of that particle:

$$a^\alpha = \frac{Du^\alpha}{d\tau} = \frac{dx^v}{d\tau}\frac{Du^\alpha}{dx^v} = \left(\frac{du^\alpha}{dx^v} + \Gamma^\alpha_{\pi v} u^\pi\right) u^v$$

Since, $a^\alpha \propto u^v$ and $u^r = u^\theta = u^\phi = 0$ that makes:

$$a^\alpha = \left(\frac{du^\alpha}{d\tau} + \Gamma^\alpha_{TT} u^T\right) u^T$$

But since $u^j = 0$ and for our static (metric is time independent) coordinate system

$$\frac{du^T}{d\tau} = \frac{d}{d\tau}\left(\frac{dT}{d\tau}\right) = \frac{d}{d\tau}\left(\frac{1}{\sqrt{g_{TT}}}\right) = 0$$

$$a^\alpha = \frac{du^T}{d\tau} + \Gamma^\alpha_{TT} u^T u^T = \Gamma^\alpha_{TT} u^T u^T = \Gamma^\alpha_{TT}\left(\frac{1}{g_{TT}}\right)$$

$$\Gamma^R_{TT} = -\frac{1}{2g_{RR}}\frac{\partial g_{TT}}{\partial R} \quad \text{and} \quad \Gamma^T_{TT} = \Gamma^\theta_{TT} = \Gamma^\phi_{TT} = 0$$

using which the proper acceleration then becomes:

$$\begin{aligned} a^R &= \left[-\frac{1}{2g_{RR}}\frac{\partial g_{TT}}{\partial R}\right]\frac{1}{g_{tt}} \\ &= \frac{1}{2}\frac{\partial g_{TT}}{\partial R} && \text{as } g_{RR}g_{TT} = -1 \\ &= \frac{1}{2}\frac{\partial}{\partial R}\left(1 - \frac{2GM}{R}\right) \\ &= \frac{GM}{R^2} \end{aligned}$$

Norm of the acceleration is:

$$a = \sqrt{a^\mu a_\mu} = \sqrt{g_{\mu\nu} a^\mu a^\nu} = \sqrt{g_{RR} a^R a^R}$$

Since, $a^t = a^\theta = a^\phi = 0$,

$$a^R = \frac{GM}{R^2}\left(1 - \frac{2GM}{R}\right)^{-1}$$

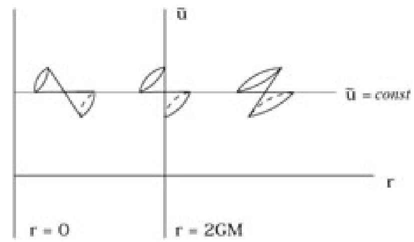

Figure 1:

Here authors[9] assume that the particle is **not moving**, but if the particle is at rest, then it has to be inside it's light cone, and hence as the timelike observer remains timelike and lightlike observers remain lightlike. The only thing that can stay at $R = 2GM$ is photon (as seen from Fig.1).

Assuming stationary observer at $R = 2GM$ was automatically converting a timelike observer to lightlike! ($ds^2 = g_{TT}dT^2 + g_{RR} \times 0 = 0 \times dT^2 = 0$ in Eddington-Finkelstein coordinate). This suggests why we couldn't use equation (E) at event horizon. But Dr. Mitra interpreted it as a symbol of singularity, norm of 4 acceleration is indeed a physical invariant but it's isn't the property of spacetime, it describes the particle not spacetime..! Which is why we use kretschmann scalar to deduce if the singularity at event horizon is real or not. Kretschmann scalar depends on Ricci Tensor, which does describe the spacetime geometry not the 4 acceleration which is the byproduct of curvature (because there is Γ^k_{ij} term which depends on curvature!).

From the above discussion we can conclude that, one must be careful about which equation to use and when to use or else it can lead to disastrous conclusion. From most of these analysis on properties of event horizon, Dr. Mitra looking at the weird result concluded since this can not be true and we can get rid of it by assuming black holes don't exist or $\alpha_0 = 2GM = 0$, which is like saying "a non existent problem doesn't need a solution" and with that we're back in flat spacetime, where life is simple. But now, if since Schwarzschild Solution doesn't describe what happens at/inside the event horizon, what then? Well, we perform coordinate transformation, but why? Dr. Mitra did say "why should R and T be bad coordinates particularly when, in GTR, there are really no "bad" or "good" coordinates"

Spacetime is a 4 dimensional manifold (Sean Carroll, Spacetime and Geometry), and from the very definition of manifolds we can't use one map to cover it all, and since the absurd results we get by using Schwarzschild metric, it means this map can not cover (event horizon + everything inside) it and we need a way out, which is "Eddington-Finkelstein coordinate". Like if you use that equation (D) for speed, using Schwarzschild metric, you will get ∞ as an answer which is also really absurd! But before we move on, I would also like to mention that in General Relativity all observers are equivalent, coordinate system and observers are related but not same, a particular observer can use/have multiple coordinates to study and analyze but before he/she jumps to any conclusion he would need to check if his coordinates do "cover" the parts of spacetime he/she is studying!

Before we proceed ahead I would like to address Dr. Mitra's claim about the emergence of modulus in tortoise coordinate[16], where he said that "many authors quietly put a modular sign in the argument of logarithmic term of R_* without even mentioning so:"

$$R_* = R + 2GM \ln \left| \frac{R - 2GM}{2GM} \right| \qquad \textbf{(F)}$$

This transformation is used to arrive at Eddington-Finkelstein coordinate. If we use this equation **(F)** without modular sign in region $R < 2GM$, it would lead to R_* being imaginary. And since, even this coordinate transformation couldn't fix the issue, we added modular sign just so the notion of Black Holes and spacetime inside event horizon survives!

Let me try to derive the same result and see for ourselves if he is indeed right and we just come up with modular sign!

Let's begin by considering a null radial geodesic for which, $ds^2 = 0$.

$$ds^2 = 0 = \left(1 - \frac{2GM}{R}\right) dT^2 - \left(1 - \frac{2GM}{R}\right)^{-1} dR^2$$

$$\left(1 - \frac{2GM}{R}\right) dT^2 = \left(1 - \frac{2GM}{R}\right)^{-1} dR^2$$

$$\frac{dT^2}{dR^2} = \left(\frac{dT}{dR}\right)^2 = \left(1 - \frac{2GM}{R}\right)^{-2}$$

$$\left(\frac{dT}{dR}\right) = \pm\left(1 - \frac{2GM}{R}\right)^{-1} \qquad (\text{as } \sqrt{x^2} = |x|) \text{ (G)}$$

$$dT = \pm\left(1 - \frac{2GM}{R}\right)^{-1} dR$$

$$dT = \pm\frac{dR}{\left(1 - \frac{2GM}{R}\right)} \qquad \text{(H)}$$

$$dT = \pm\frac{\left(1 - \frac{2GM}{R} + \frac{2GM}{R}\right)}{\left(1 - \frac{2GM}{R}\right)} dR$$

$$\int dT = \pm\int dR \pm \int \frac{\frac{2GM}{R}}{\left(1 - \frac{2GM}{R}\right)} dR$$

Integrating both sides and assuming $\int dT = T_*$ we get,

$$T_* = \pm R \pm \frac{\frac{2GM}{R}}{\left(1 - \frac{2GM}{R}\right)} dR + constant$$

Let's simplify it further by multiplying both numerator and denominator by $\dfrac{R}{2GM}$

$$T_* = \pm R \pm \frac{\frac{2GM}{R} \times \frac{R}{2GM}}{\left(1 - \frac{2GM}{R}\right) \times \frac{R}{2GM}} dR + constant$$

$$= \pm R \pm \frac{1}{\left(\frac{R}{2GM} - 1\right)} dR + constant$$

Substituting $\dfrac{R}{2GM}$ as β we get, $d\beta = \dfrac{dR}{2GM}$

$$T_* = \pm R \pm 2GM \int \frac{d\beta}{(\beta - 1)} + constant$$

$$T_* = \pm R \pm 2GM \ln|\beta - 1| + constant \qquad \text{as } \int \frac{1}{x}dx = \ln|x|$$

substituting β back into the equation we get,

$$T_* = \pm R \pm 2GM \ln\left|\frac{R}{2GM} - 1\right| + constant$$

Here we assume the rather complicated term to be R_* and then get the solution in new coordinate which looks rather simple! i.e.

$$T_* = \pm R_* + constant$$

From this mathematical analysis, one can see that the modulus was, actually the consequence of performing integration the right way! Not some ad hoc mathematics, and yes indeed many textbook miss this part, which is why I had to derive it from the scratch. If one looks at equation (G) of mine, there is used $\pm$ sign, which if you ask me, is there because both of these terms upon squaring lead to same conclusion, i.e. $(+x)^2 = (-x)^2 = x^2$ and we didn't want to leave any possibility.

There is one more aspect of Event Horizon which Dr. Mitra considered, "using Karlhede Invariant as a event horizon detector". Karlhede Invariant incase of a "non-rotating" black hole takes the form of:

$$\mathcal{I} = -\frac{720M^2(R - 2M)}{R^9}$$

from which it is clearly evident that $\mathcal{I}$ changes the sign at $R = 2GM$, and using that we could in theory detect event horizon.! But in reality Karlhede invariant is not a trustable event horizon detector as it changes it's sign for the simple case of *rotating black hole* not at event horizon but ergosphere. This can be seen in equation (18) of the paper titled "Karlhede's invariant and the black hole firewall proposal", which changes things drastically! For neutral rotating black hole, we have[29]

$$\mathcal{I} = \frac{-720M^2(a^2cos^2\theta + R^2 - 2MR)Q_1Q_2}{(a^2cos^2\theta + R^2)^9}$$

with

$$Q_1 = (acos\theta - R)^4 - 4aR^2cos\theta(3acos\theta - 2R),$$

,

$$Q_2 = (acos\theta - R)^4 - 4a^2Rcos^2\theta(3R - 2acos\theta).$$

From here it can be seen quite easily that $\mathcal{I}$ changes sign not at $R = 2M$ but at $R = M \pm \sqrt{M^2 - a^2cos^2\theta}$, i.e. at ergosphere! Which implies it can not be used as a reliable event horizon detector. But if we are to continue treating Karlhede invariant as event horizon detector, then the "fact that the vanishing of Karlhede's invariant $\mathcal{I}$ at the Schwarzschild event horizon $R = 2M$ can be physically measured, based on classical general relativity, as an observer freely falls into a black hole implies the reality of the event horizon as a physical membrane"[29], which changes things as earlier we were considering event horizon as a horizon in space not an object/entity!

IV. Non Occurrence of Trapped Surfaces again!

We have already debunked many of such claims but there is one more in section 14.2.2 of his paper [6] which needs some special care. He argues that the equation $M \leq 2L$ is in accordance with his theory and Abhay Ashtekar had rejected it without reading. Dr. Mitra mentioned that this paper [10] was all about self energy in static gravitational field and no collapse, but Dr. Mitra perhaps didn't go through the whole paper carefully, perhaps he just skimmed over and saw the equation $M \leq 2L$ and concluded that even this paper [10] supports his claim!

Up until now, in all the results R meant the proper radius of object and M the total mass (which includes the negative binding energy [11]) , but in this result $M \leq 2L$, which he again is misinterpreting, M is the mass contained inside the coordinate sphere (without binding energy) of radius "R", for which the condition of formation of trapped surface is very different, as here Bizon didn't consider the self energy! In his other paper, Bizon also proved for , $M \geq L$ trapped surface would form! [10] ($L \leq M \leq 2L$ event horizon will form and until $M \leq L$ there won't be any trapped surface!)

In equation (1) of his paper "*Trapped Surfaces in spherical stars*" he argued if $M \geq L$ then, Ω must contain a trapped surface, here again by "M" he meant the mass inside the surface Ω with proper radius L and in the last paragraph of his work he concluded,[11]

"The key results, (1) and (3), have only been derived in the spherically symmetric case, but, of course, they are obviously valid (with some minor adjustments of the constants) for any data which are close to spherical symmetry."

V. Vaidya Metric and ECO

In this section, Dr. Mitra had really strong argument since stars are radiating body and they are always emitting electromagnetic wave and such so perhaps we should study them from Vaidya Metric and I quote!:

The exterior spacetime of a collapsing and radiating body is described by the Vaidya metric

$$ds^2 = \left(1 - \frac{2M(v)}{R_0}\right) dv^2 - 2dvdR_0 - R_0(d\theta^2 + sin^2\theta d\phi)$$

where v is retarded time. The determinant of this (external) metric is

$$g = -R_0^4 \; sin^2\theta$$

the expression for $g_{ij}^{interior}$ is not known but it is expected that both the metric and g must be continuous everywhere including at the boundary of the body. Also, since there is no vacuum spacetime, the question of supposed "Schwarzschild Metric Singularity" also should not arise. Thus it is expected that, the boundary of the collapsing fluid smoothly approaches $R_0 = 0$ as $g \to 0$. This however requires that, $g_{vv}(R) \geq 0$ and no event horizon forms:

$$\frac{2M(v)}{R_0} \leq 1$$

As $R \to 0$, one must have $M \to 0$ demanding the entire mass energy to be radiated out. However in case, one would have, $2M(v)/R_0 < 1$, in this limit, i.e. if an event horizon would not form, there would be further emission of radiation and, M can travel to $-\infty$! To avoid this unphysical occurrence, one must have

$$\lim_{R_0 \to 0} \frac{2M(v)}{R_0} = 1$$

But, if so the world line of the particle on the boundary will become non-timelike. Therefore this state of $R = 0$ must not ever be reached, in other words, the comoving proper time for the formation of the eventual zero mass black hole must be infinite!

Well at first it does indeed looks like there is no coordinate singularity at $R = 2M$ so no event horizon and collapse? Perhaps there is hope for, us all? Perhaps radiating objects like the sun, can't form an event horizon?

But looking back at Equation (4.11) in Vaidya's original paper of 1951 titled "The gravitational field of a radiating star", where he actually mentioned his metric using Schwarzschild coordinate is! [12]

$$ds^2 = \frac{\dot{M}^2}{f^2}\left(1 - \frac{2M}{R}\right)dT^2 - \frac{1}{\left(1 - \frac{2M}{R}\right)}dR^2 - R^2 d\Omega^2$$

Which at $R = 2M$ does have the coordinate singularity! Also in the paper where Dr. Vaidya derived this result he assumed at equation (3.3) that the radiation would follow null geodesic i.e. it would have zero rest mass! The same metric can also be found as equation (9.31) of the textbook "Exact Space-Times in Einstein's General Relativity" by Jerry B. Griffiths and Jiri Podosky[14]:

> "In fact, the metric (9.31) can be expressed in a much more useful form (for outgoing radiation) by the introduction of a null coordinate "υ", *such that* $d\upsilon = -\frac{1}{f(M)}dM$. *With this, the line element becomes* $ds^2 = \left(1 - \frac{2M(\upsilon)}{R_0}\right)d\upsilon^2 - 2d\upsilon dR_0 - R_0(d\theta^2 + sin^2\theta d\phi)$"

Now it is important to note that, this metric is only applicable to radiating null dusts. But stars won't always radiate and the time when, this emission of radiation via nuclear fusion will come to halt and we won't be able to apply this metric! Do note that $R = 2GM$ in Schwarzschild form of Vaidya metric isn't about event horizon and I quote:

Apart from the presence here of pure radiation, the main difference between this and the Schwarzschild space-time is that the coordinate singularity at $R = 2M(\upsilon)$ is not a null hypersurface and therefore cannot be an event horizon. In fact, it is an example of an *apparent horizon.* In the text Dr. Podolsky and Griffiths also mentioned the possibility of collapse and the likely outcome of it in vaidya spacetime and they said:

> In fact, two possible scenarios arise. If the surface of the body passes through its Schwarzschild horizon, then a black hole is formed and the structure of the space-time will be that illustrated in Figure 9.17. On the other hand, if the outer surface of the body remains outside its Schwarzschild horizon while its volume reduces to zero, then the structure of the space-time will be that illustrated in Figure 9.18 in which a Vaidya region is matched to a subsequent Minkowski region.

But looking at the first possibility that it will **radiate all of it's mass** and studying that under Vaidya metric means, we're talking about the emission of particles with zero rest mass, but since that can not happen! As the matter inside the star is baryons and leptons, which is interacting with higgs field.

That's not the only point, these baryons can't also convert to photonic matter or anything which moves at the speed of light as that would be the violation of conservation of baryon/lepton number! But yes, Dr. Mitra's study of radiating matter using Vaidya metric does have a use, a star or object made purely of trapped photonic matter or anything with zero rest mass like graviton, which is also radiating. But that is unlikely yet possible under some circumstances! Almost every work in favor of such ever collapsing object[18,23] which looses mass bit by bit via radiation uses in one way or the other, "Vaidya Metric" and rely on this assumption that the body will never cross it's Schwarzschild radius because of that. An impractical result which doesn't consider the fact that the stars are actually made up of baryons and the condition of $M = 0$ can't ever be achieved this way. Soon the time will arrive when the emission of radiation which moves along the null geodesic will stop and then, the "vaidya metric" will resemble schwarzschild and the same work, done by every other physicist in the field will prevail.

Eddington Luminosity in General Relativistic Case

The possibility of existence of relativistic radiation pressure supported stars seem quite fascinating at first, but till now there have been no such work proving the existence of such stars, until 2006 when Dr. Mitra published his work claiming ECOs as *Radiation pressure supported stars*[26]. This work is based on using one very crucial equation which he published in 1998 in his other paper entitled "Maximum Accretion Efficiency in General Theory of Relativity".

$$L_{ed} = \frac{4\pi G M_x c}{\kappa}(1+z)$$

This paper doesn't have any real derivation for this equation to hold other than it fits the boundary condition perfectly. There is indeed a derivation but was proved later by Leiter and Robertson[27]. They attempted to derive it from equation A9a of their paper, which is.

$$\frac{dU_s}{d\tau} = \left[\frac{\Gamma}{\rho + P/c^2}\right]\left[-\frac{\partial P}{\partial R}_s - \frac{G\left\{M + 4\pi R^3 \frac{(P+q)}{c^2}\right\}}{R^2}\right]$$

Somewhat similar equation to the one mentioned above also appears in 1.12-U of Relativistic Equations for Adiabatic, Spherically Symmetric Gravitational Collapse and equation A-2 of Observer Time as a Coordinate in Relativistic Spherical Hydrodynamics, equation 3.37 of Relativistic Transport Theory by Lindquist. His proof of L_{ed} comes from Appendix B of the paper [27] where he assumes $\frac{dU}{d\tau} = 0$ and then, the equation become (for obvious reasons)

$$-\left(\frac{\partial P}{\partial R}\right)_s = \frac{G\left\{M + 4\pi R^3 \frac{(P+q)}{c^2}\right\}}{R^2}$$

But what Leiter and Robertson used was something else entirely!

$$\frac{\Gamma}{\rho + P/c^2}\left(-\frac{\partial P}{\partial R}\right)_s = \frac{GM}{R^2}$$

There is no explanation other than "in the Eddington limit at the surface S", and he accepted that:

$$\frac{\Gamma}{\rho + P/c^2} \neq 0$$

with,

$$q = E_0^0 c^2 \quad \text{and} \quad L = 4\pi R^2 q c$$

Implementing these will in no way lead to,

$$L_{ed} = \frac{4\pi G M_x c}{\kappa}(1+z)$$

Which had been used by him many of his papers to prove the existence of relativistic radiation pressure supported star! He could come up with alot of ludicrous explanation of why these equations should hold but he can't really prove them, also he just happens to forget that proving equilibrium condition is not enough, one also has to prove the stability of such radiation pressure supported star as was shown and done by Weinberg in his book "Gravitation and Cosmology". This does raise a doubt about how come his peer reviewed paper used such questionable equations for which there exists no real proof! There can be other form of the same equation, which satisfies the relation

$$L_\infty = \frac{L}{(1+z)^2}$$

This relation seems indeed to be true as a clear and concise derivation of it can be found in some paper but the one used by Dr. Mitra in his peer reviewed papers are clearly wrong. Perhaps his peers just checked the calculation and took that particular equation for granted as it looks similar to non relativistic eddington limit!

There is indeed somewhat similar equation in Relativistic Transport Theory, where author describes the equation 3.37

$$D_t U = \Gamma D_r \phi - \frac{m + 4\pi R^3(p + K)}{R^2}$$

But it can't be used to conclude the result Robertson was looking for, as the work by author explicitly uses radiation flux(H) and as can be seen there L_s near equation 3.42a.

$$L_s = 4\pi R_s^2 H(r_s, u)$$

But that's not the end, equation 3.40c of the very same paper author asserts $\phi(r_s, t) = 0$ as a boundary condition. Other paper, entitled "Observer Time as a Coordinate in Relativistic Spherical Hydrodynamics" using equation A2 and A8 we can arrive at:

$$\frac{\partial U}{\partial \tau} = \Gamma \left(-\frac{\frac{\partial P}{\partial l} + nC}{\epsilon + P} \right) - \frac{M + 4\pi R^3 P}{R^2} - \frac{L}{R}$$

If we assume that there is no transfer of energy from fluid to radiation then $C = 0$: [32]

$$\frac{\partial U}{\partial \tau} = \frac{\Gamma}{\epsilon + P} \left(-\frac{\partial P}{\partial l} \right) - \frac{M + 4\pi R^3 P}{R^2} - \frac{L}{R}$$

Looking over the original equations we can't conclude the equation dr. Robertson and Leiter concluded.

VI. Radiation Pressure Supported Star and ECO

From thermodynamics it is quite evident that at a finite temperature, every object has certain amount of radiation being emitted from it due thermal radiation. Considering that with the non-adiabatic collapse of a radiating star, can that lead to the possibility of radiation pressure supported star? This question was also tackled by Dr. Abhas Mitra in his work in 2006[26]. The analysis begins with defining few quantities regarding radiation following a null geodesic. If the radiation fluid is moving with the bulk speed of υ_{eff} then the comoving energy density of radiation is:

$$\rho_r = \frac{L}{4\pi R^2 \upsilon_{eff}} \tag{I}$$

Defining a critical parameter relating the Luminosity of the star to it's Eddington Luminosity.

$$L = \alpha L_{ed} \tag{J}$$

Using this we can redefine equation (I) as:

$$\rho_r = \frac{\alpha L_{ed}}{4\pi R^2 \upsilon_{eff}} \tag{K}$$

In relativistic limit, baryonic energy density is $\rho_0 = m_0 n c^2$ and [30]

$$\begin{aligned} \upsilon_{eff} &= \frac{1}{g_{TT} g_{RR}} \frac{c}{Rn\sigma} \\ &= \frac{1}{g_{TT} g_{RR}} \frac{c(m_0 c^2)}{R(\rho_0)\sigma} \\ &= \frac{1}{g_{TT} g_{RR}} \frac{m_0 c^3}{R\rho_0 \sigma} \end{aligned}$$

This makes the equation (K):

$$\rho_r = \frac{\alpha L_{ed}}{4\pi R^2}\frac{g_{TT}g_{RR}R\rho_0\sigma}{m_0c^3}$$

$$= g_{TT}g_{RR}\frac{\alpha L_{ed}}{4\pi R}\frac{\rho_0\sigma}{m_0c^3} \tag{L}$$

and if we use $L_{ed} = \frac{4\pi GMm_0c}{\sigma}(1+z)$ we will get,

$$\rho_r = g_{TT}g_{RR}\frac{\alpha\frac{4\pi GMm_0c}{\sigma}(1+z)}{4\pi R}\frac{\rho_0\sigma}{m_0c^3}$$

$$= g_{TT}g_{RR}\frac{\alpha GM(1+z)}{R}\frac{\rho_0}{c^2}$$

$$= g_{TT}g_{RR}\frac{\alpha GM\rho_0}{Rc^2}(1+z)$$

From this we can conclude ρ_r/ρ_0

$$\frac{\rho_r}{\rho_0} = g_{TT}g_{RR}\frac{\alpha GM}{Rc^2}(1+z) \tag{M}$$

From equation (12) and (13) we can relate the ratio of energy density to pressure for a uniform density star in the equation of the form:

$$\frac{p_r}{p_0} = \frac{m_0c^2}{3kT}\frac{\rho_r}{\rho_0} = \frac{m_0c^2}{3kT}\frac{\alpha GM}{2Rc^2}(1+z) \tag{N}$$

There is a big if in equality of equation (N), because it was derived using non-relativistic equations and we are assuming the same relation to hold in relativistic case as well! From this it is quite obvious to conclude that as the star collapses z will begin to diverge and at $R = 2GM$ it will blow up, but with that from equation (M) the radiation pressure density will also blow up. But before all that happens, there will be a particular time when, p_r will perfectly counter the gravitational force of attraction and we can have a radiation pressure supported star even in relativistic case! Because whatever the case the above equation (N) implies at $R \approxeq 2GM$, $p_r/p_0 \approxeq \alpha z$.

The above discussion is based on one assumption, i.e. a radiation pressure supported star exist at the eddington limit of L_{ed} and has the form of $L_{ed} = \frac{4\pi GMm_0c}{\sigma}(1+z)$. We discussed this in earlier section that this result is not necessarily true and derivation regarding it was ad-hoc. We could just as well have considered

$$L_{ed} = \frac{4\pi GMm_0c}{\sigma}$$

This too satisfies the relation that:

$$L_\infty = \frac{L}{(1+z)^2} \implies L_\infty = \frac{4\pi GMm_0c}{(1+z)^2\sigma}$$

The above discussion seems to suggest that the arguments regarding why such a radiation pressure supported star in relativistic limit should exist is not very rigor and is based on many impractical and strong assumptions. Even though by now we have not fully solved the Einstein field equation for radiating star having non-zero pressure but all other approximate work seems to suggest that the collapse will happen in one way or the other.

VII. Conclusion

From the above discussions we can conclude all the proofs regarding why trapped surfaces shouldn't form or why black holes should be massless are either wrong or misinterpretation. This changes things completely, because the notion of ECO rests completely either on mass loss due to radiation, non existence of trapped surfaces and results like $L_{ed} = \frac{4\pi G M_x c}{\kappa}(1+z)$ in analysis to prove the existence of radiation pressure supported star in relativistic limit.

Non-Existence of Trapped Surface could have opened up a door for eternally collapsing radiating star which attains the condition for $M = 0$ via radiation. But the possibility of losing all of stellar mass due to radiation is in violation with conservation of quantum numbers. If a radiating baryonic star has mass M_b due to existence of baryons. Then the metric describing a radiating star like Vaidya metric will take the same form as Schwarzschild or Eddington-Finkelstein metric when the star looses all of kinetic energy in the form of radiation attaining the case of $E = M_b c^2$. At this point no more photons can be emitted and hence no massloss. This will convert the vaidya metric back to Eddington-Finkelstein metric and with that all the work concerning Black Hole will come back into the picture.

But before trapped surface really forms there is a possibility that the star could somehow attain equilibrium but the analysis showing the possibility for such an event to occur seems to be using an equation which is purely coincidental that it just meets the boundary condition and there is no clear reason for us to believe in such equation. Using other equation that meets the conditions never leads to the same conclusion. All of proof is based on somehow getting the z factor in numerator and using non-relativistic thermodynamics to argue the existence of diverging radiation and then justifying it with ad-hoc calculation. In non-relativistic weak gravitation case, gas pressure dominates the overall radiation pressure and keeps the star in equilibrium but that can't happen in strong gravitational case. The radiation pressure may seem to dominate but it doesn't actually counteract the gravity just like degeneracy pressure. The above discussion makes ECO baseless and their existence very much unlikely as there are no clear mathematical analysis showing why such stars should even exist in the first place. The work of eternally collapsing object could have one place which doesn't voilate any conservation laws and that is a star made entirely of photons or massless particles.

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