On the Ramanujan's mathematics applied to some parameters of Extended Gauged Supergravity, Inflaton Potentials and some sectors of String Theory: New possible mathematical connections.

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#### Abstract

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In this research thesis, we have described some Ramanujan expressions applied to several parameters of Extended Gauged Supergravity, Inflaton Potentials and some sectors of String Theory, obtaining new possible mathematical connections.

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https://www.britannica.com/biography/Srinivasa-Ramanujan



https://futurism.com/brane-science-complex-notions-of-superstring-theory

From:

**Axial Symmetric Kahler manifolds, the D-map of Inflaton Potentials and the Picard-Fuchs Equation** - *Pietro Fre, Alexander S. Sorin* – arXiv:1310.5278v2 [hep-th] 26 Oct 2013

#### We remember that $U(\phi)$ is the potential of the inflaton field, $\phi$

#### We have that:

As an illustration of reconstruction of the Kähler potential in the series (5.1), we utilize the best fit model  $\gamma = -\frac{7}{6}$  proposed by Sagnotti. Inserting the value  $\gamma = -\frac{7}{6}$  in eq.(8.1) and furthermore redefining the parameters as in equations (8.9), (8.10), (8.11), we obtain:

$$R_{-\frac{7}{6}}(\phi) = -\frac{2744 + 5996e^{\frac{13\phi}{2\sqrt{3}}} + 9844e^{\frac{13\phi}{\sqrt{3}}} + e^{\frac{13\sqrt{3}\phi}{2}}}{12\left(1 + e^{\frac{13\phi}{2\sqrt{3}}}\right)^2 \left(14 + e^{\frac{13\phi}{2\sqrt{3}}}\right)}$$
(8.24)

where the overall scale a and the parameter  $\lambda$  cancel. The function  $R_{-\frac{\gamma}{a}}(\phi)$  has the property:

$$R_{-\frac{7}{6}}(-\infty) = \frac{49}{12} \quad ; \quad R_{-\frac{7}{6}}(\infty) = \frac{1}{48}$$
(8.25)

$$R_{-\frac{7}{6}}(\phi) = -\frac{2744 + 5996e^{\frac{13\phi}{2\sqrt{3}}} + 9844e^{\frac{13\phi}{\sqrt{3}}} + e^{\frac{13\sqrt{3}\phi}{2}}}{12\left(1 + e^{\frac{13\phi}{2\sqrt{3}}}\right)^2 \left(14 + e^{\frac{13\phi}{2\sqrt{3}}}\right)}$$
(8.24)

 $e^{(13*x/(2sqrt3))} = 42.63931648 = 40.915$  for x = 1 or x = 0.989 (i.e.  $\phi$ )  $e^{(13*x/(sqrt3))} = 1818.1113 = 1674.04$  as above  $e^{(x(13sqrt3)/2)} = 77523.023543 = 68493.1$  as above

 $-(2744+5996*(e^{(13*x/(2sqrt3))+9844*(e^{(13*x/(sqrt3))})+e^{(x(13sqrt3)/2)}) / 12((((1+(e^{(13*x/(2sqrt3))})^2 (14+e^{(13*x/(2sqrt3))}))) = 49/12)))) = 49/12$ 

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-(2744+5996*42.63931648+9844*1818.1113+77523.023543) /
(((12(1+42.63931648)^2 (14+42.63931648))))
```

#### **Input interpretation:**

 $2744 + 5996 \times 42.63931648 + 9844 \times 1818.1113 + 77523.023543$ 

12 (1+42.63931648)<sup>2</sup> (14+42.63931648)

#### **Result:**

 $-14.0868213521128690592315012838303183739419265007086254459\ldots$ 

-14.086821352...

#### From which:

-3(((-(2744+5996\*42.63931648+9844\*1818.1113+77523.023543)/ (((12(1+42.63931648)^2 (14+42.63931648))))))))

#### **Input interpretation:**

 $-3\left(-\frac{2744+5996\times 42.63931648+9844\times 1818.1113+77523.023543}{12\left(1+42.63931648\right)^2\left(14+42.63931648\right)}\right)$ 

#### **Result:**

42.26046405633860717769450385149095512182577950212587633787... 42.260464...

 $-(2744+5996*40.915+9844*1674.04+68493.1) / (((12(1+40.915)^{2}(14+40.915)))))$ 

#### **Input interpretation:**

 $2744 + 5996 \times 40.915 + 9844 \times 1674.04 + 68\,493.1$ 

12 (1 + 40.915)<sup>2</sup> (14 + 40.915)

#### **Result:**

-14.5074091940429788271474544775280445229326038202670939833... -14.507409194...

With regard the eqs. (8.25)

$$R_{-\frac{7}{6}}(-\infty) = \frac{49}{12}$$
;  $R_{-\frac{7}{6}}(\infty) = \frac{1}{48}$ 

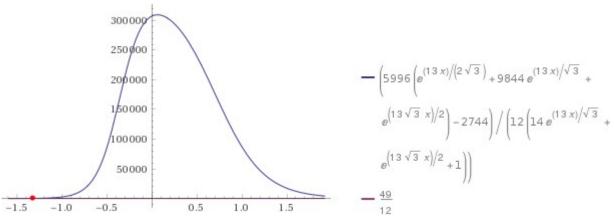
where 49/12 = 4.08333... and 1/48 = 0.0208333... we have the following calculations:

#### Input:

$$\frac{-2744 + 5996 \left(e^{13 \times x / \left(2 \sqrt{3}\right)} + 9844 e^{13 \times x / \sqrt{3}} + e^{x \left(1/2 \left(13 \sqrt{3}\right)\right)}\right)}{12 \left(1 + \left(e^{13 \times x / \left(2 \sqrt{3}\right)}\right)^2 \left(14 + e^{13 \times x / \left(2 \sqrt{3}\right)}\right)\right)} = \frac{49}{12}$$

## Exact result: $\frac{5996 \left( e^{(13 x)/(2 \sqrt{3})} + 9844 e^{(13 x)/\sqrt{3}} + e^{(13 \sqrt{3} x)/2} \right) - 2744}{12 \left( e^{(13 x)/\sqrt{3}} \left( e^{(13 x)/(2 \sqrt{3})} + 14 \right) + 1 \right)} = \frac{49}{12}$

**Plot:** 



Solutions:  

$$\begin{aligned} x &= \frac{2}{13} \sqrt{3} \left[ 2i\pi c_1 + \log\left(\frac{1}{5947} \left(-19\,674\,646 + \frac{1}{3^{2/3}} \left(\left(\frac{1}{2} \left(-137\,086\,051\,000\,810\,387\,372\,047 + 5947\,i\right) \sqrt{6\,892\,239\,879\,645\,850\,388\,988\,114\,807}\right)\right) ^{(1/3)} + 1161\,275\,050\,017\,736 / \left(\left(\frac{3}{2} \left(-137\,086\,051\,000\,810\,387\,372\,047 + 5947\,i\right) \sqrt{6\,892\,239\,879\,645\,850\,388\,988\,114\,807}\right)\right) ^{(1/3)} \right) \right\| = 0.266469 \\ ((6.28319\,i)\,c_1 - (4.98668 + 2.2017 \times 10^{-9}\,i)) \\ for \\ c_1 \in \mathbb{Z} \end{aligned}$$

$$x = \frac{2}{13} \sqrt{3} \left(2\,i\pi c_1 + \log\left(-\frac{19\,674\,646}{5947} - \frac{1}{11\,894 \times 3^{2/3}} + \left(1+i\sqrt{3}\right)\left(\frac{1}{2} \left(-137\,086\,051\,000\,810\,387\,372\,047 + 5947\,i\sqrt{6\,892\,239\,879\,645\,850\,388\,988\,114\,807}\right)\right) ^{(1/3)} - \left(5947\left(\frac{3}{2} \left(-137\,086\,051\,000\,810\,387\,372\,047 + 5947\,i\sqrt{6\,892\,239\,879\,645\,850\,388\,988\,114\,807}\right)\right) ^{(1/3)} - \left(5947\left(\frac{3}{2} \left(-137\,086\,051\,000\,810\,387\,372\,047 + 5947\,i\sqrt{6\,892\,239\,879\,645\,850\,388\,988\,114\,807}\right)\right) ^{(1/3)} \right) \right) = 0.266469 \\ ((6.28319\,i)\,c_1 - (4.97191 - 3.14159\,i)) \text{ for } c_1 \in \mathbb{Z} \end{aligned}$$

$$x = \frac{2}{13} \sqrt{3} \left(2\,i\pi c_1 + \log\left(-\frac{19\,674\,646}{5947} - \frac{1}{11\,894 \times 3^{2/3}} + \left(1-i\sqrt{3}\right)\left(\frac{1}{2} \left(-137\,086\,051\,000\,810\,387\,372\,047 + 5947\,i\sqrt{6\,892\,239\,879\,645\,850\,388\,988\,114\,807}\right)\right) ^{(1/3)} - \left(580\,637\,525\,008\,868\,\left(1+i\sqrt{3}\right)\right) \right/ (5947\,\left(\frac{3}{2} \left(-137\,086\,051\,000\,810\,387\,372\,047 + 5947\,i\sqrt{6\,892\,239\,879\,645\,850\,388\,988\,114\,807}\right)\right) ^{(1/3)} - \left(580\,637\,525\,008\,868\,\left(1+i\sqrt{3}\right)\right) \right) / (5947\,\left(\frac{3}{2} \left(-137\,086\,051\,000\,810\,387\,372\,047 + 5947\,i\sqrt{6\,892\,239\,879\,645\,850\,388\,988\,114\,807}\right)\right) ^{(1/3)} - \left(580\,637\,525\,008\,868\,\left(1+i\sqrt{3}\right)\right) \right) / (5947\,\left(\frac{3}{2} \left(-137\,086\,051\,000\,810\,387\,372\,047 + 5947\,i\sqrt{6\,892\,239\,879\,645\,850\,388\,988\,114\,807}\right)\right) ^{(1/3)} - \left(580\,637\,525\,008\,868\,\left(1+i\sqrt{3}\right)\right) \right) / (5947\,\left(\frac{3}{2} \left(-137\,086\,051\,000\,810\,387\,372\,047 + 5947\,i\sqrt{6\,892\,239\,879\,645\,850\,388\,988\,114\,807}\right)\right) ^{(1/3)} \right) \right\| = 0.266469 \\ ((9.20281 - 3.14159\,9\,) + (6.28319\,9\,0\,1)\,for\,c_1 \in \mathbb{Z}$$

 $\log(x)$  is the natural logarithm

 $\mathbb Z$  is the set of integers

#### **Real solution:**

 $x \approx -1.3288$ 

#### $-1.3288 = \phi$

#### Solutions:

 $x \approx 0.266469 \left( (6.28319 \, i) \, n + (9.20281 + 3.14159 \, i) \right), \quad n \in \mathbb{Z}$ 

 $x\approx 0.266469\,(-\,(4.97191\,-\,3.14159\,i)\,+\,(6.28319\,i)\,n)\,,\quad n\in\mathbb{Z}$ 

 $x \approx 0.266469 \, ((6.28319 \, i) \, n - 4.98668) \,, \quad n \in \mathbb{Z}$ 

ℤ is the set of integers

#### Note that:

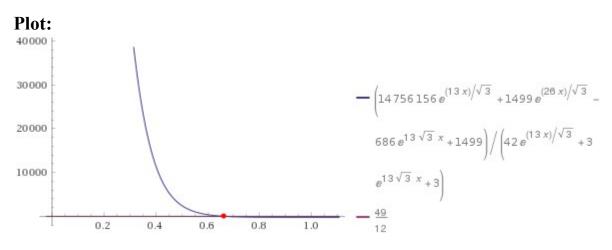
 $(((-2744+5996*(e^{(13*-2x/(2sqrt3))+9844*(e^{(13*-2x/(sqrt3)))+e^{(-2x(13sqrt3)/2))})) / (((12((1+(e^{(13*-2x/(2sqrt3)))^2 (14+e^{(13*-2x/(2sqrt3)))))))))) = 49/12$ 

#### **Input:**

$$\frac{-2744 + 5996 \left(e^{13 \times (-2) \times x / \left(2\sqrt{3}\right)} + 9844 e^{13 \times (-2) \times x / \sqrt{3}} + e^{-2x \left(1/2 \left(13\sqrt{3}\right)\right)}\right)}{12 \left(1 + \left(e^{13 \times (-2) \times x / \left(2\sqrt{3}\right)}\right)^2 \left(14 + e^{13 \times (-2) \times x / \left(2\sqrt{3}\right)}\right)\right)} = \frac{49}{12}$$

#### **Exact result:**

$$\frac{5996 \left(9844 e^{-(26 x)/\sqrt{3}} + e^{-(13 x)/\sqrt{3}} + e^{-13 \sqrt{3} x}\right) - 2744}{12 \left(e^{-(26 x)/\sqrt{3}} \left(e^{-(13 x)/\sqrt{3}} + 14\right) + 1\right)} = \frac{49}{12}$$



Solutions:

$$\begin{split} x &= \frac{1}{13} \sqrt{3} \\ & \left(2 i \pi c_1 + \log \left(\frac{1}{8379} \left(5996 + 494 597528 518 \right) / \left(\left(\frac{1}{2} \left(8897857349216929 + 8379 i \sqrt{6892239879645850388988114807}\right)\right)^{\wedge} (1/3)\right) + \left(\frac{1}{2} \left(8897857349216929 + 8379 i \sqrt{6892239879645850388988114807}\right)\right)^{\wedge} (1/3)\right) \right) \approx \\ & \left(1/3\right) + \left(\frac{1}{2} \left(8897857349216929 + 8379 i \sqrt{6892239879645850388988114807}\right)\right)^{\wedge} (1/3)\right) \right) \approx \\ & 0.133235 (4.98668 + (6.28319 i) c_1) \text{ for } \\ c_1 \in \mathbb{Z} \\ x &= \frac{1}{13} \sqrt{3} \left(2 i \pi c_1 + \log \left(\frac{5996}{8379} - \left(247298764259\left(1 + i\sqrt{3}\right)\right) / \left(8379\left(\frac{1}{2} \left(8897857349216929 + 8379 i \sqrt{6892239879645850388988114807}\right)\right)^{\wedge} (1/3)\right) - \\ & \frac{1}{16758} \left(1 - i\sqrt{3}\right) \left(\frac{1}{2} \left(8897857349216929 + 8379 i \sqrt{6892239879645850388988114807}\right)\right)^{\wedge} (1/3)\right) \\ & 0.133235 ((4.97191 + 3.14159i) + (6.28319i) c_1) \\ \text{ for } \\ c_1 \in \mathbb{Z} \\ x &= \frac{1}{13} \sqrt{3} \left(2 i \pi c_1 + \log \left(\frac{5996}{8379} - \left(247298764259\left(1 - i\sqrt{3}\right)\right) / \left(8379\left(\frac{1}{2} \left(8897857349216929 + 8379i i \sqrt{6892239879}645850388988114807\right)\right)^{\wedge} (1/3)\right) - \\ & \frac{1}{16758} \left(1 + i\sqrt{3}\right) \left(\frac{1}{2} \left(8897857349216929 + 8379i i \sqrt{6892239879}645850388988114807\right)\right)^{\wedge} (1/3)\right) - \\ & \frac{1}{16758} \left(1 + i\sqrt{3}\right) \left(\frac{1}{2} \left(8897857349216929 + 8379i i \sqrt{6892239879}645850388988114807\right)\right)^{\wedge} (1/3)\right) \\ & 0.133235 ((6.28319i) i c_1 - (9.20281 - 3.14159i)) \\ \text{ for } \\ c_1 \in \mathbb{Z} \\ z \end{array}$$

 $\log(x)$  is the natural logarithm

ℤ is the set of integers

#### **Real solution:**

 $x \approx 0.66440$ 

 $0.66440 = -\phi/2$ 

#### Solutions:

 $x \approx 0.133235 \left( (6.28319 \, i) \, n + (4.97191 + 3.14159 \, i) \right), \quad n \in \mathbb{Z}$ 

 $x \approx 0.133235 (-(9.20281 - 3.14159 i) + (6.28319 i) n), \quad n \in \mathbb{Z}$ 

 $x \approx 0.133235$  ((6.28319 i) n + 4.98668),  $n \in \mathbb{Z}$ 

 $(((-2744+5996*(e^{(13*(-1.3288)/(2sqrt3))+9844*(e^{(13*(-1.3288)/(sqrt3)))+e^{((-1.3288)(13sqrt3)/2)))}) / (((12((1+(e^{(13*(-1.3288)/(2sqrt3)))^2 (14+e^{(13*(-1.3288)/(2sqrt3)))}))))))))$ 

$$\frac{-2744 + 5996 \left(e^{13 \left(-1.3288 / \left(2 \sqrt{3}\right)\right)} + 9844 e^{13 \left(-1.3288 / \sqrt{3}\right)} + e^{-1.3288 \left(1/2 \left(13 \sqrt{3}\right)\right)}\right)}{12 \left(1 + \left(e^{13 \left(-1.3288 / \left(2 \sqrt{3}\right)\right)}\right)^2 \left(14 + e^{13 \left(-1.3288 / \left(2 \sqrt{3}\right)\right)}\right)\right)}$$

#### **Result:**

4.07659...

 $4.07659\ldots {\approx}\,49/12$ 

## Series representations:

$$\frac{-2744 + 5996 \left(e^{(13(-1.3288))/(2\sqrt{3})} + 9844 e^{(13(-1.3288))/\sqrt{3}} + e^{1/2(13\sqrt{3})(-1)1.3288)}\right)}{12 \left(1 + \left(e^{(13(-1.3288))/(2\sqrt{3})}\right)^2 \left(14 + e^{(13(-1.3288))/(2\sqrt{3})}\right)\right)} = -\left[\left(e^{-8.6372\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1/2}{k}}\right) - \frac{1499 e^{25.9116/\left(\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1}{2}\right)}}{\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1}{2}} + 8.6372\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1}{2}}\right) - \frac{1499 \exp\left(\frac{17.2744}{\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1}{2}}{k} + 8.6372\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1}{2}}\right) + \frac{686 \exp\left(\frac{25.9116}{\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1}{2}}{k} + 8.6372\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1}{2}}\right) + \frac{686 \exp\left(\frac{25.9116}{\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1}{2}}{k} + 8.6372\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1}{2}}\right) + \frac{3\left(1 + 14e^{8.6372/\left(\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1}{2}}\right) + e^{25.9116/\left(\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1}{2}}{k}\right)}\right)\right)}{25.9116/\left(\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1}{2}}\right)}\right)$$

$$\frac{-2744 + 5996 \left(e^{(13(-1.3288))/(2\sqrt{3})} + 9844 e^{(13(-1.3288))/\sqrt{3}} + e^{1/2 \left(13\sqrt{3}\right)(-1)(1.3288)}\right)}{12 \left(1 + \left(e^{(13(-1.3288))/(2\sqrt{3})}\right)^2 \left(14 + e^{(13(-1.3288))/(2\sqrt{3})}\right)\right)} = -\left[\left(\exp\left(-8.6372\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right) \left(-1499 e^{\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} - 1499 e^{\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} - 1499 e^{\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} + 8.6372\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}\right) - 1499 e^{\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} + 8.6372\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} + e^{\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}\right) - 686 e^{\left(\frac{25.9116}{\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} + 8.6372\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}\right)}{\left(3\left(1 + 14e^{\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + e^{\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}\right)\right)\right)$$

$$\begin{split} & \frac{-2744 + 5996 \left(e^{(13(-1.3288))/(2\sqrt{3})} + 9844 e^{(13(-1.3288))/\sqrt{3}} + e^{1/2 \left(13\sqrt{3}\right)(-1)1.3288}\right)}{12 \left(1 + \left(e^{(13(-1.3288))/(2\sqrt{3})}\right)^2 \left(14 + e^{(13(-1.3288))/(2\sqrt{3})}\right)\right)} \\ & - \left(\left(\exp\left(-\frac{4.3186 \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right)\Gamma(s\right)}{\sqrt{\pi}}\right)\right) \\ & \left(-1499 \exp\left(\frac{51.8232 \sqrt{\pi}}{\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right) - \\ & 14756 156 \exp\left(\frac{17.2744 \sqrt{\pi}}{\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right) - \\ & 14756 156 \exp\left(\frac{34.5488 \sqrt{\pi}}{\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right) - \\ & 1499 \exp\left(\frac{34.5488 \sqrt{\pi}}{\sqrt{\pi}}\right) - \\ & 1499 \exp\left(\frac{34.5488 \sqrt{\pi}}{\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right) + \\ & \frac{4.3186 \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\sqrt{\pi}}\right) + \\ & 686 \exp\left(\frac{51.8232 \sqrt{\pi}}{\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right) + \\ & \frac{4.3186 \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\sqrt{\pi}}\right) + \\ & \left(3\left(1 + 14 \exp\left(\frac{17.2744 \sqrt{\pi}}{\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right) + \\ & \exp\left(\frac{51.8232 \sqrt{\pi}}{\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right)\right) \right) \right) \right) \right) \right) \\ \end{array}$$

We have also:

#### Input interpretation:

$$\sqrt[e]{\frac{-2744 + 5996 \left(e^{13 \left(-1.3288 / \left(2 \sqrt{3}\right)\right)} + 9844 e^{13 \left(-1.3288 / \sqrt{3}\right)} + e^{-1.3288 \left(1/2 \left(13 \sqrt{3}\right)\right)}\right)}{12 \left(1 + \left(e^{13 \left(-1.3288 / \left(2 \sqrt{3}\right)\right)}\right)^2 \left(14 + e^{13 \left(-1.3288 / \left(2 \sqrt{3}\right)\right)}\right)\right)}}$$

#### **Result:**

 $1.676933774582334581657001861376930679192936661895708451250\ldots$ 

1.67693377458233458....

#### Series representations:

$$\begin{split} \sqrt{\frac{-2744 + 5996 \left(e^{(13(-1.3288))/(2\sqrt{3})} + 9844 e^{(13(-1.3288))/\sqrt{3}} + e^{1/2(13\sqrt{3})(-1)(1.3288)}\right)}{12 \left(1 + \left(e^{(13(-1.3288))/(2\sqrt{3})}\right)^2 \left(14 + e^{(13(-1.3288))/(2\sqrt{3})}\right)\right)} = \\ 12^{-1/e} \left( \left[ -2744 + 5996 \left(9844 e^{-17.2744/\left(\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\left(\frac{1}{k}\right)\right)\right) + e^{-8.6372\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\left(\frac{1}{k}\right)}\right) + e^{-8.6372\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\left(\frac{1}{k}\right)}\right) \right) \\ \left( 1 + e^{-17.2744/\left(\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\left(\frac{1}{k}\right)\right) + e^{-8.6372/\left(\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\left(\frac{1}{k}\right)\right)}\right) \right) \cap (1/e)} \\ \sqrt{\frac{-2744 + 5996 \left(e^{(13(-1.3288))/(2\sqrt{3})} + 9844 e^{(13(-1.3288))/\sqrt{3}} + e^{1/2(13\sqrt{3})(-1)(1.3288)}\right)}{12 \left(1 + \left(e^{(13(-1.3288))/(2\sqrt{3})}\right)^2 \left(14 + e^{(13(-1.3288))/(2\sqrt{3})}\right)\right)} = \\ 12^{-1/e} \\ \left( \left[ -2744 + 5996 \left(9844 \exp\left(-\frac{17.2744}{\sqrt{2}\sum_{k=0}^{\infty}\left(\frac{-1}{2}\frac{k}{k}\left(-\frac{1}{2}k}\right)\right) + \exp\left(-\frac{8.6372}{\sqrt{2}\sum_{k=0}^{\infty}\left(\frac{-1}{2}\frac{k}{k}\left(-\frac{1}{2}k}\right)}{k!}\right) + \exp\left(-\frac{8.6372}{\sqrt{2}\sum_{k=0}^{\infty}\left(\frac{-1}{2}\frac{k}{k}\left(-\frac{1}{2}k}\right)}{k!}\right) + \exp\left(-\frac{17.2744}{\sqrt{2}\sum_{k=0}^{\infty}\left(\frac{-1}{2}\frac{k}{k}\left(-\frac{1}{2}k}\right)}{k!}\right) \right) \right) \right) \cap (1/e) \\ \sqrt{2} \sum_{k=0}^{\infty}\left(\frac{-17.2744}{\sqrt{2}\sum_{k=0}^{\infty}\left(\frac{-1}{2}\frac{k}{k}\left(-\frac{1}{2}k}\right)}{k!}\right)} \right) \left(14 + \exp\left(-\frac{8.6372}{\sqrt{2}\sum_{k=0}^{\infty}\left(\frac{-1}{2}\frac{k}{k}\left(-\frac{1}{2}k}\right)}{\sqrt{2}\sum_{k=0}^{\infty}\left(\frac{-1}{2}\frac{k}{k}\left(-\frac{1}{2}k}\right)}{k!}\right)} \right) \right) \right) \cap (1/e) \\ \sqrt{2} \sum_{k=0}^{\infty}\left(\frac{-17.2744}{k!}\right) \left(14 + \exp\left(-\frac{8.6372}{\sqrt{2}\sum_{k=0}^{\infty}\left(\frac{-1}{2}\frac{k}{k}\left(-\frac{1}{2}k}\right)}{\sqrt{2}\sum_{k=0}^{\infty}\left(\frac{-1}{2}\frac{k}{k!}\right)}\right) \right) \right) \right) \right) \right) \right)$$

$$\left\{ \begin{array}{l} \displaystyle \frac{-2744 + 5996 \left( e^{(13(-1.3288))/(2\sqrt{3})} + 9844 \ e^{(13(-1.3288))/\sqrt{3}} + e^{1/2 \left(13\sqrt{3}\right)(-1) 1.3288\right)} \right)}{12 \left(1 + \left( e^{(13(-1.3288))/(2\sqrt{3})} \right)^2 \left(14 + e^{(13(-1.3288))/(2\sqrt{3})} \right) \right)} \\ 12^{-1/e} \left( \left( -2744 + 5996 \left( 9844 \exp \left( -\frac{34.5488\sqrt{\pi}}{\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma \left(-\frac{1}{2} - s\right) \Gamma (s)} \right) \right) + \\ & \exp \left( -\frac{17.2744\sqrt{\pi}}{\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma \left(-\frac{1}{2} - s\right) \Gamma (s)} \right) \right) + \\ & \exp \left( -\frac{4.3186 \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma \left(-\frac{1}{2} - s\right) \Gamma (s)}{\sqrt{\pi}} \right) \right) \right) \right) \\ & \left( 1 + \exp \left( -\frac{34.5488\sqrt{\pi}}{\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma \left(-\frac{1}{2} - s\right) \Gamma (s)} \right) \right) \\ & \left( 14 + \exp \left( -\frac{17.2744\sqrt{\pi}}{\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma \left(-\frac{1}{2} - s\right) \Gamma (s)} \right) \right) \right) \right) \land (1/e)$$

Integral representation:  $(1+z)^{a} = \frac{\int_{-i \ \infty + \gamma}^{i \ \infty + \gamma} \frac{\Gamma(s) \Gamma(-a-s)}{z^{s}} ds}{(2 \pi i) \Gamma(-a)} \text{ for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$ 

 $\operatorname{Re}(z)$  is the real part of z

arg(z) is the complex argument

|z| is the absolute value of z

i is the imaginary unit

and:

#### Input interpretation:

$$= \frac{-(55+4) \times \frac{1}{10^{3}} + \frac{-2744 + 5996 \left(e^{13 \left(-1.3288 / \left(2 \sqrt{3}\right)\right)} + 9844 e^{13 \left(-1.3288 / \sqrt{3}\right)} + e^{-1.3288 \left(1/2 \left(13 \sqrt{3}\right)\right)}\right)}{12 \left(1 + \left(e^{13 \left(-1.3288 / \left(2 \sqrt{3}\right)\right)}\right)^{2} \left(14 + e^{13 \left(-1.3288 / \left(2 \sqrt{3}\right)\right)}\right)\right)}$$

#### **Result:**

 $1.617933774582334581657001861376930679192936661895708451250\ldots$ 

1.61793377458233.... result that is a very good approximation to the value of the golden ratio 1,618033988749...

#### Series representations:

$$\begin{split} & -\frac{55+4}{10^3} + \\ & \sqrt{\frac{-2744+5996\left(e^{(13(-1.3288))/(2\sqrt{3})}+9844\,e^{(13(-1.3288))/\sqrt{3}}+e^{1/2\left(13\sqrt{3}\right)(-1)\,1.3288\right)}\right)}{12\left(1+\left(e^{(13(-1.3288))/(2\sqrt{3})}\right)^2\left(14+e^{(13(-1.3288))/(2\sqrt{3})}\right)\right)} \\ & = -\frac{1}{125}\times2^{-3-2/e}\times3^{-1/e} \\ & \left(59\sqrt[e]{12}-1000\left(\left[-2744+5996\left(9844\,e^{-17.2744}/\left(\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\left(\frac{1}{2}\right)\right)\right)+e^{-8.6372\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\left(\frac{1}{2}\right)}\right)+e^{-8.6372\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\left(\frac{1}{2}\right)}\right)\right) \\ & -\frac{e^{-8.6372}/\left(\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\left(\frac{1}{2}\right)\right)}{14+e}\left(14+e^{-8.6372}/\left(\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\left(\frac{1}{2}\right)\right)\right)\right)} \land (1/e) \end{split}$$

$$\begin{split} -\frac{55+4}{10^3} + \\ \sqrt{\frac{2744+5996\left(e^{(13(-1.3288))/(2\sqrt{3})}+9844e^{(13(-1.3288))/\sqrt{3}}+e^{1/2\left(13\sqrt{3}\right)(-1)1.3288\right)}\right)}{12\left(1+\left(e^{(13(-1.3288))/(2\sqrt{3})}\right)^2\left(14+e^{(13(-1.3288))/(2\sqrt{3})}\right)\right)} \\ = -\frac{1}{125} \times 2^{-3-2/e} \times 3^{-1/e} \\ \left(59\sqrt[e]{12}-1000\left(\left[-2744+5996\left(9844\exp\left[-\frac{17.2744}{\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right]}{\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)\right] + \\ \exp\left[-\frac{8.6372}{\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}{\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{17.2744}{\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}{\sqrt{1}\left(1+\exp\left[-\frac{17.2744}{\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}\right]} \\ \left(14+\exp\left[-\frac{8.6372}{\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right]\right) \wedge (1/e) \end{split}$$

$$\begin{split} & -\frac{55+4}{10^3} + \\ & \sqrt{\frac{-2744+5996\left(e^{(13(-1.3288))/(2\sqrt{3})}+9844\,e^{(13(-1.3288))/\sqrt{3}}+e^{1/2\left(13\sqrt{3}\right)(-1)\,1.3288\right)}\right)}{12\left(1+\left(e^{(13(-1.3288))/(2\sqrt{3})}\right)^2\left(14+e^{(13(-1.3288))/(2\sqrt{3})}\right)\right)} \\ & = -\frac{1}{125} \times 2^{-3-2/e} \times 3^{-1/e} \\ & \left(59\,\frac{e}{\sqrt{12}}-1000\left(\left[-2744+5996\left(9844\exp\left(-\frac{34.5488\sqrt{\pi}}{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}}\,2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right]\right) + \\ & \quad \exp\left(-\frac{17.2744\sqrt{\pi}}{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}}\,2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right)\right) + \\ & \quad \left(1+\exp\left(-\frac{34.5488\sqrt{\pi}}{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}}\,2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right)\right) \right) \\ & \quad \left(14+\exp\left(-\frac{17.2744\sqrt{\pi}}{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}}\,2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right)\right)\right) \land (1/e) \end{split}$$

 $\binom{n}{m}$  is the binomial coefficient

n! is the factorial function

 $(a)_n$  is the Pochhammer symbol (rising factorial)

 $\Gamma(x)$  is the gamma function

Resf is a complex residue  $s=z_0$ 

$$\left( \left( (-2744 + 5996*(e^{(13*x/(2sqrt3))} + 9844*(e^{(13*x/(sqrt3))}) + e^{(x(13sqrt3)/2)}) \right) \right) \\ \left( \left( (12((1+(e^{(13*x/(2sqrt3))})^2 (14+e^{(13*x/(2sqrt3))}))) \right) = 1/48 \right) \\ \right) \\ \left( (12((1+(e^{(13*x/(2sqrt3))})^2 (14+e^{(13*x/(2sqrt3))}))) + e^{(x(13sqrt3)/2)}) \right) \\ \left( (12((1+(e^{(13*x/(2sqrt3))})^2 (14+e^{(13*x/(2sqrt3))}))) + e^{(x(13sqrt3)/2)}) \right) \\ \left( (12((1+(e^{(13*x/(2sqrt3))})^2 (14+e^{(13*x/(2sqrt3))})) + e^{(x(13sqrt3)/2)}) \right) \right) \\ \left( (12((1+(e^{(13*x/(2sqrt3))})^2 (14+e^{(13*x/(2sqrt3))}))) + e^{(x(13sqrt3)/2)}) \right) \\ \left( (12((1+(e^{(13*x/(2sqrt3))})^2 (14+e^{(13*x/(2sqrt3))})) + e^{(x(13sqrt3)/2)}) \right) \\ \left( (12((1+(e^{(13*x/(2sqrt3))}))^2 (14+e^{(13*x/(2sqrt3))})) + e^{(x(13sqrt3)/2)}) \right) \\ \left( (12((1+(e^{(13*x/(2sqrt3))}))^2 (14+e^{(13*x/(2sqrt3))}) + e^{(x(13sqrt3))}) \right) \\ \left( (12((1+(e^{(13*x/(2sqrt3))}))^2 (14+e^{(13*x/(2sqrt3))})) + e^{(x(13xqrt3))}) \right) \\ \left( (12((1+(e^{(13*x/(2sqrt3))}))^2 (14+e^{(13*x/(2sqrt3))}) + e^{(x(13xqrt3))}) \right) \\ \left( (12((1+(e^{(13*x/(2sqrt3))}))^2 (14+e^{(13*x/(2sqrt3))}) + e^{(x(13xqrt3))}) \right) \\ \left( (12((1+(e^{(13*x/(2sqrt3))})) + e^{(x(13xqrt3))}) + e^{(x(13xqrt3))}) \right) \right) \\ \left( (12((1+(e^{(13*x/(2sqrt3))})) + e^{(x(13xqrt3))}) + e^{(x(13xqrt3))}) \right) \\ \left( (12((1+(e^{(13*xqrt3)})) + e^{(x(13xqrt3))}) + e^{(x(13xqrt3))}) \right) \right) \\ \left( (12((1+(e^{(13xqrt3)})) + e^{(x(13xqrt3))}) + e^{(x(13xqrt3))}) \right) \\ \left( (12((1+(e^{(13xqrt3)})) + e^{(x(13xqrt3))}) + e^{(x(13xqrt3))}) \right) \right) \\ \left( (12((1+(e^{(13xqrt3)})) + e^{(x(13xqrt3))}) + e^{(x(13xqrt3))}) \right) \\ \left( (12((1+(e^{(13xqrt3)})) + e^{(x(13xqrt3))}) \right) \right) \\ \left( (12((1+(e^{(13xqrt3)})) + e^{(x(13xqrt3)}) + e^{(x(13xqrt3)}) \right) \right) \\ \left( (12((1+(e^{(13xqrt3)})) + e^{(x(13xqrt3)}) + e^{(x(13xqrt3)}) + e^{(x(13xqrt3)}) \right) \right) \\ \left( (12((1+(e^{(13xqrt3)})) + e^{(x(13xqrt3)}) + e^{(x(13xqrt3)}) \right) \right) \\ \left( (12((1+(e^{(13xqrt3)})) + e^{(x(13xqrt3)}) + e^{(x(13xqrt3)}) \right) \right) \\ \left( (12((1+(e^{(13xqrt3)})) + e^{(x(13xqrt3)}) + e^{(x(13xqrt3)}) \right) \right) \\ \left( (12((1+(e^{(13xqrt3)})) + e^{(x(13xqrt3)}) + e^{(x(13xqrt3)}) \right) \right) \\ \left( (12((1+(e^{(13xqrt3)}$$

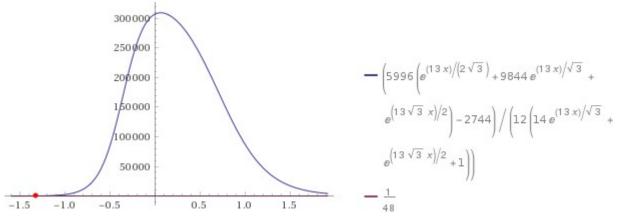
**Input:** 

$$\frac{-2744 + 5996 \left(e^{13 \times x / \left(2 \sqrt{3}\right)} + 9844 e^{13 \times x / \sqrt{3}} + e^{x \left(1/2 \left(13 \sqrt{3}\right)\right)}\right)}{12 \left(1 + \left(e^{13 \times x / \left(2 \sqrt{3}\right)}\right)^2 \left(14 + e^{13 \times x / \left(2 \sqrt{3}\right)}\right)\right)} = \frac{1}{48}$$

**Exact result:** 

$$\frac{5996 \left( e^{(13 x)/(2 \sqrt{3})} + 9844 e^{(13 x)/\sqrt{3}} + e^{(13 \sqrt{3} x)/2} \right) - 2744}{12 \left( e^{(13 x)/\sqrt{3}} \left( e^{(13 x)/(2 \sqrt{3})} + 14 \right) + 1 \right)} = \frac{1}{48}$$

**Plot:** 



Solutions:  

$$x = \frac{2}{13} \sqrt{3} \left( 2 i \pi c_1 + \log \left( \frac{1}{23983} \right) \left( -78 \, 699 \, 494 + \frac{1}{3^{2/3}} \left( \left( \frac{1}{2} \left( -8 \, 773 \, 811 \, 611 \, 228 \, 315 \, 720 \, 231 \, 623 + 23 \, 983 \, i \right) \sqrt{1733 \, 678 \, 535 \, 658 \, 161 \, 449 \, 854 \, 094 \, 472 \, 687} \right) \right)^{\wedge}$$

$$(1/3) + 18 \, 580 \, 830 \, 492 \, 359 \, 836 \, /$$

$$\left( \left( \frac{3}{2} \left( -8 \, 773 \, 811 \, 611 \, 228 \, 315 \, 720 \, 231 \, 623 + 23 \, 983 \, i \right) \sqrt{1733 \, 678 \, 535 \, 658 \, 161 \, 449 \, 854 \, 094 \, 472 \, 687} \right) \right)^{\wedge}$$

$$(1/3) \right) \right) \approx 0.266469 \, (-4.99555 + (6.28319 \, i) \, c_1) \text{ for } c_1 \in \mathbb{Z}$$

$$\begin{split} x &= \frac{2}{13} \sqrt{3} \left( 2 i \pi c_1 + \log \left( -\frac{78699494}{23983} - \frac{1}{47966 \times 3^{2/3}} \right) \right)^{-1} \\ &= \left( 1 + i \sqrt{3} \right) \left( \frac{1}{2} \left( -8773811611228315720231623 + 23983i \sqrt{1733678535658161449854094472687} \right) \right)^{-1} \\ &= \left( 1/3 \right) - \left( 9290415246179918 \left( 1 - i \sqrt{3} \right) \right) \right)^{-1} \\ &= \left( 23983 \left( \frac{3}{2} \left( -8773811611228315720231623 + 23983i \sqrt{1733678535658161449854094472687} \right) \right)^{-1} \\ &= \left( 3 \right)^{-1} \\ &= \left( 3 \right)^{-1} \right) \left( 2 i \pi c_1 + \log \left( -\frac{78699494}{23983} - \frac{1}{47966 \times 3^{2/3}} \right) \\ &= \left( 1 - i \sqrt{3} \right) \left( \frac{1}{2} \left( -8773811611228315720231623 + 23983i \sqrt{1733678535658161449854094472687} \right) \right)^{-1} \\ &= \left( 1 - i \sqrt{3} \right) \left( \frac{1}{2} \left( -8773811611228315720231623 + 23983i \sqrt{1733678535658161449854094472687} \right) \right)^{-1} \\ &= \left( 1 - i \sqrt{3} \right) \left( \frac{1}{2} \left( -8773811611228315720231623 + 23983i \sqrt{1733678535658161449854094472687} \right) \right)^{-1} \\ &= \left( 1/3 \right) - \left( 9290415246179918 \left( 1 + i \sqrt{3} \right) \right) \right)^{-1} \\ &= \left( 23983 \left( \frac{3}{2} \left( -8773811611228315720231623 + 23983i \sqrt{1733678535658161449854094472687} \right) \right)^{-1} \\ &= \left( 1/3 \right) - \left( 9290415246179918 \left( 1 + i \sqrt{3} \right) \right) \right)^{-1} \\ &= \left( 2 (9.19466 + 3.14159i) + (6.28319i)c_1 \right) \text{ for } c_1 \in \mathbb{Z} \right)$$

 $\log(x)$  is the natural logarithm

 $\mathbb Z$  is the set of integers

#### **Real solution:**

 $x \approx -1.3312$ 

 $-1.3312 = \phi$ 

#### Solutions:

 $x\approx 0.266469\,((6.28319\,i)\,n+(9.19466+3.14159\,i)\,)\,,\quad n\in\mathbb{Z}$ 

 $x\approx 0.266469\,(-\,(4.98065-3.14159\,i)\,+(6.28319\,i)\,n)\,,\quad n\in\mathbb{Z}$ 

 $x \approx 0.266469 \ ((6.28319 \ i) \ n - 4.99555), \quad n \in \mathbb{Z}$ 

Note that:

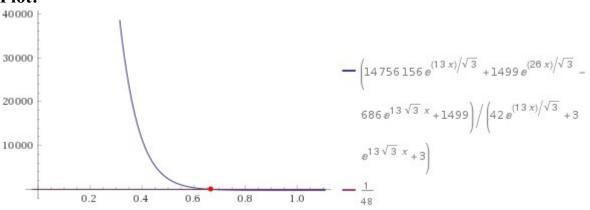
#### Input:

$$\frac{-2744 + 5996 \left(e^{13 \times (-2) \times x / \left(2\sqrt{3}\right)} + 9844 e^{13 \times (-2) \times x / \sqrt{3}} + e^{-2x \left(1/2 \left(13\sqrt{3}\right)\right)}\right)}{12 \left(1 + \left(e^{13 \times (-2) \times x / \left(2\sqrt{3}\right)}\right)^2 \left(14 + e^{13 \times (-2) \times x / \left(2\sqrt{3}\right)}\right)\right)} = \frac{1}{48}$$

#### **Exact result:**

$$\frac{5996 \left(9844 e^{-(26x)/\sqrt{3}} + e^{-(13x)/\sqrt{3}} + e^{-13\sqrt{3}x}\right) - 2744}{12 \left(e^{-(26x)/\sqrt{3}} \left(e^{-(13x)/\sqrt{3}} + 14\right) + 1\right)} = \frac{1}{48}$$

**Plot:** 



#### Solutions:

$$\begin{aligned} x &= \frac{1}{13} \sqrt{3} \left( 2 i \pi c_1 + \log \left( \frac{1}{32\,931} \right) \\ &\left( 23\,984 + 7\,775\,534\,342\,998 \right) \left( \left( \frac{1}{2} \left( 559\,529\,475\,824\,767\,381 + 32\,931\,i \right) \sqrt{1\,733\,678\,535\,658\,161\,449\,854\,094\,472\,687} \right) \right) \\ &\left( 1/3 \right) + \left( \frac{1}{2} \left( 559\,529\,475\,824\,767\,381 + 32\,931\,i \sqrt{1\,733\,678\,535\,658\,161\,449\,854\,094\,472\,687} \right) \right) \\ &\left( 1/3 \right) \right) \\ &\left( 1/3 \right) \right) \\ &\approx 0.133235\,(4.99555 + (6.28319\,i)\,c_1) \text{ for } c_1 \in \mathbb{Z} \end{aligned}$$

$$\begin{split} x &= \frac{1}{13} \sqrt{3} \left( 2 i \pi c_1 + \log \left( \frac{23\,984}{32\,931} - \left( 3\,887\,767\,171\,499\,\left( 1 + i\,\sqrt{3} \right) \right) \right) \right) \\ & \left( 32\,931 \left( \frac{1}{2} \left( 559\,529\,475\,824\,767\,381 + 32\,931\,i \right) \right) \\ & \left( 1/3 \right) \right) - \frac{1}{65\,862} \left( 1 - i\,\sqrt{3} \right) \\ & \left( \frac{1}{2} \left( 559\,529\,475\,824\,767\,381 + 32\,931\,i \right) \\ & \sqrt{1733\,678\,535\,658\,161\,449\,854\,094\,472\,687} \right) \right) \\ & \left( 1/3 \right) \right) = \frac{1}{65\,862} \left( 1 - i\,\sqrt{3} \right) \\ & \left( 1,33235\,((4.98065 - 3.14159\,i) + (6.28319\,i)\,c_1) \right) \\ & \text{for} \\ & c_1 \in \\ \mathbb{Z} \\ x &= \frac{1}{13} \sqrt{3} \left( 2\,i \pi\,c_1 + \log \left( \frac{23\,984}{32\,931} - \left( 3\,887\,767\,171\,499\,\left( 1 - i\,\sqrt{3} \right) \right) \right) \right) \\ & \left( 32\,931 \left( \frac{1}{2} \left( 559\,529\,475\,824\,767\,381 + 32\,931\,i \right) \\ & \sqrt{1733\,678\,535\,658\,161\,449\,854\,094\,472\,687} \right) \right) \\ & \left( 1/3 \right) - \frac{1}{65\,862} \left( 1 + i\,\sqrt{3} \right) \\ & \left( \frac{1}{2} \left( 559\,529\,475\,824\,767\,381 + 32\,931\,i \right) \\ & \sqrt{1733\,678\,535\,658\,161\,449\,854\,094\,472\,687} \right) \right) \\ & \left( 1/3 \right) - \frac{1}{65\,862} \left( 1 + i\,\sqrt{3} \right) \\ & \left( \frac{1}{2} \left( 559\,529\,475\,824\,767\,381 + 32\,931\,i \right) \\ & \sqrt{1733\,678\,535\,658\,161\,449\,854\,094\,472\,687} \right) \right) \\ & \left( 1/3 \right) = \frac{1}{65\,862} \left( 1 + i\,\sqrt{3} \right) \\ & \left( \frac{1}{2} \left( 559\,529\,475\,824\,767\,381 + 32\,931\,i \right) \\ & \sqrt{1733\,678\,535\,658\,161\,449\,854\,094\,472\,687} \right) \right) \\ & \left( 1.332235\,((6.28319\,i)\,c_1 - (9.19466\,- 3.14159\,i)) \right) \\ & \text{for} \\ & c_1 \in \\ \mathbb{Z} \\ \end{split}$$

log(x) is the natural logarithm

 $\ensuremath{\mathbb{Z}}$  is the set of integers

#### **Real solution:**

 $x \approx 0.66558$ 

 $0.66558 = -\phi/2$ 

#### Solutions:

 $x\approx 0.133235\,((6.28319\,i)\,n+(4.98065+3.14159\,i)\,)\,,\quad n\in\mathbb{Z}$ 

 $x\approx 0.133235\,(-\,(9.19466-3.14159\,i)\,+(6.28319\,i)\,n)\,,\quad n\in\mathbb{Z}$ 

 $x\approx 0.133235\,((6.28319\,i)\,n\!+\!4.99555)\,,\quad n\in\mathbb{Z}$ 

## Input interpretation:

$$\begin{pmatrix} -2744 + 5996 \\ \left( e^{13\left(-1.33116109/(2\sqrt{3})\right)} + 9844 e^{13\left(-1.33116109/\sqrt{3}\right)} + e^{-1.33116109\left(1/2\left(13\sqrt{3}\right)\right)} \end{pmatrix} \right) \\ \left( 12\left( 1 + \left( e^{13\left(-1.33116109/(2\sqrt{3})\right)} \right)^2 \left( 14 + e^{13\left(-1.33116109/(2\sqrt{3})\right)} \right) \right) \right)$$

**Result:** 0.0208481...

 $0.0208481....\approx 1/48$ 

#### Series representations:

Series representations:  

$$\frac{-2744 + 5996 \left(e^{(13(-1.33116))/(2\sqrt{3})} + 9844 e^{(13(-1.33116))/\sqrt{3}} + e^{1/2(13\sqrt{3})(-1)(1.33116)}\right)}{12 \left(1 + \left(e^{(13(-1.33116))/(2\sqrt{3})}\right)^2 \left(14 + e^{(13(-1.33116))/(2\sqrt{3})}\right)\right)} = \\
- \left[ \left(e^{-8.65255 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} - 1499 e^{25.9576/(\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2})} - 1499 e^{25.9576/(\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2})} - 14756 156 \exp\left(\frac{8.65255}{\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2}} + 8.65255 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2}} + 14756 \frac{156}{\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2}} + 8.65255 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2}} + 1499 \exp\left(\frac{17.3051}{\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2}} + 8.65255 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2}} + 1499 \exp\left(\frac{25.9576}{\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2}} + 8.65255 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2}} + 1499 \exp\left(\frac{25.9576}{\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2}} + 8.65255 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2}} + 1499 \exp\left(\frac{25.9576}{\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2}} + 8.65255 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2}} + 1499 \exp\left(\frac{25.9576}{\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2}} + 8.65255 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2}} + 1499 \exp\left(\frac{25.9576}{\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2}} + 8.65255 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2}} + 1499 \exp\left(\frac{25.9576}{\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2}} + 8.65255 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2}} + 1499 \exp\left(\frac{25.9576}{\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2}} + 8.65255 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2}} + 1499 \exp\left(\frac{25.9576}{\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2}} + 8.65255 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2}} + 1499 \exp\left(\frac{25.9576}{\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2}} + 149 \exp\left(\frac{25.9576}{\sqrt{2} 2^{-k} \binom{1}{2}} + 149 \exp\left(\frac{25.9576}{\sqrt{2} 2^{-k} \binom{1}{2}} + 149 \exp\left(\frac{25.9576}{\sqrt{2} 2^{-k}$$

$$\frac{-2744 + 5996 \left(e^{(13(-1.33116))/(2\sqrt{3})} + 9844 e^{(13(-1.33116))/\sqrt{3}} + e^{1/2(13\sqrt{3})(-1)(1.33116)}\right)}{12 \left(1 + \left(e^{(13(-1.33116))/(2\sqrt{3})}\right)^2 \left(14 + e^{(13(-1.33116))/(2\sqrt{3})}\right)\right)} = \left(\left(e^{12 \left(1 + \left(e^{(13(-1.33116))/(2\sqrt{3})}\right)^2 + 14 + e^{(13(-1.33116))/(2\sqrt{3})}\right)}\right) - \left(e^{12 \left(1 + \left(e^{(13(-1.33116))/(2\sqrt{3})}\right)^2 + 14 + e^{1/2(13\sqrt{3})(-1)(1.33116)}\right)}\right) - 1499 e^{125.9576} + 8.65255\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right) - 1499 e^{125.255\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} + 8.65255\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right) + 1499 e^{125.9576} + 8.65255\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right) + 686 e^{125.9576} + 8.65255\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right) + 686 e^{125.9576} + 8.65255\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right) + \frac{686 e^{125.9576} + e^{125.9576} + 8.65255\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right) + \frac{686 e^{125.9576} + e^{125.9576}$$

$$\begin{split} & \frac{-2744 + 5996 \left(e^{(13(-1.33116))/(2\sqrt{3})} + 9844 e^{(13(-1.33116))/\sqrt{3}} + e^{1/2 \left(13\sqrt{3}\right)(-1)1.33116}\right)}{12 \left(1 + \left(e^{(13(-1.33116))/(2\sqrt{3})}\right)^2 \left(14 + e^{(13(-1.33116))/(2\sqrt{3})}\right)\right)} \\ & - \left(\left(\exp\left(-\frac{4.32627 \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}}\right)\right) \\ & \left(-1499 \exp\left(\frac{51.9153 \sqrt{\pi}}{\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}\right) - \\ & 14756 156 \exp\left(\frac{17.3051 \sqrt{\pi}}{\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}}\right) - \\ & 1499 \exp\left(\frac{34.6102 \sqrt{\pi}}{\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}}\right) - \\ & 1499 \exp\left(\frac{34.6102 \sqrt{\pi}}{\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}}\right) + \\ & 686 \exp\left(\frac{51.9153 \sqrt{\pi}}{\sqrt{\pi}}\right) + \\ & 686 \exp\left(\frac{51.9153 \sqrt{\pi}}{\sqrt{\pi}}\right) + \\ & \frac{4.32627 \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}}\right) + \\ & \left(3 \left(1 + 14 \exp\left(\frac{17.3051 \sqrt{\pi}}{\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}\right)\right) + \\ & \exp\left(\frac{51.9153 \sqrt{\pi}}{\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}\right) \right) \right) \right) \\ \end{array}$$

We have also that:

Input interpretation:

$$4096 \sqrt{\frac{-2744 + 5996 \left(e^{13 \left(-1.33116 / \left(2 \sqrt{3}\right)\right)} + 9844 e^{13 \left(-1.33116 / \sqrt{3}\right)} + e^{-1.33116 \left(1/2 \left(13 \sqrt{3}\right)\right)}\right)}{12 \left(1 + \left(e^{13 \left(-1.33116 / \left(2 \sqrt{3}\right)\right)}\right)^2 \left(14 + e^{13 \left(-1.33116 / \left(2 \sqrt{3}\right)\right)}\right)\right)}$$

#### **Result:**

0.9990555 result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0**. **989117352243** =  $\phi$ 

2\*sqrt(((log base 0.9990555 (0.020848078167772438))))-Pi+1/golden ratio

#### **Input interpretation:**

 $2\sqrt{\log_{0.9990555}(0.020848078167772438)} - \pi + \frac{1}{\phi}$ 

 $\log_b(x)$  is the base- b logarithm

 $\phi$  is the golden ratio

#### **Result:**

125.476...

125.476.... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

#### Alternative representation:

$$2\sqrt{\log_{0.999056}(0.0208480781677724380000)} - \pi + \frac{1}{\phi} = -\pi + \frac{1}{\phi} + 2\sqrt{\frac{\log(0.0208480781677724380000)}{\log(0.999056)}}$$

#### Series representations:

$$2\sqrt{\log_{0.999056}(0.0208480781677724380000)} - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi + 2\sqrt{-\frac{\sum_{k=1}^{\infty} \frac{(-1)^{k} (-0.979151921832227562000)^{k}}{\log(0.999056)}}}}{2\sqrt{\log_{0.999056}(0.0208480781677724380000)} - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi + 2\sqrt{-1 + \log_{0.999056}(0.0208480781677724380000)}}{\sum_{k=0}^{\infty} \left(\frac{1}{2}\right)(-1 + \log_{0.999056}(0.0208480781677724380000)) - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi + 2\sqrt{-1 + \log_{0.999056}(0.0208480781677724380000)} - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi + 2\sqrt{-1 + \log_{0.999056}(0.0208480781677724380000)} - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi + 2\sqrt{-1 + \log_{0.999056}(0.0208480781677724380000)} - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi + 2\sqrt{-1 + \log_{0.999056}(0.0208480781677724380000)} - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi + 2\sqrt{-1 + \log_{0.999056}(0.0208480781677724380000)} - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi + 2\sqrt{-1 + \log_{0.999056}(0.0208480781677724380000)} - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi + 2\sqrt{-1 + \log_{0.999056}(0.0208480781677724380000)} - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi + 2\sqrt{-1 + \log_{0.999056}(0.0208480781677724380000)} - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi + 2\sqrt{-1 + \log_{0.999056}(0.0208480781677724380000)} - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi + 2\sqrt{-1 + \log_{0.999056}(0.0208480781677724380000)} - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi + 2\sqrt{-1 + \log_{0.999056}(0.0208480781677724380000)} - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi + 2\sqrt{-1 + \log_{0.999056}(0.0208480781677724380000)} - \frac{1}{2} + \frac{1}{$$

2\*sqrt(((log base 0.9990555 (0.020848078167772438))))+11+1/golden ratio

Input interpretation:  $2\sqrt{\log_{0.9990555}(0.020848078167772438)} + 11 + \frac{1}{\phi}$ 

 $\log_{b}(x)$  is the base- b logarithm

 $\phi$  is the golden ratio

#### **Result:**

139.618...

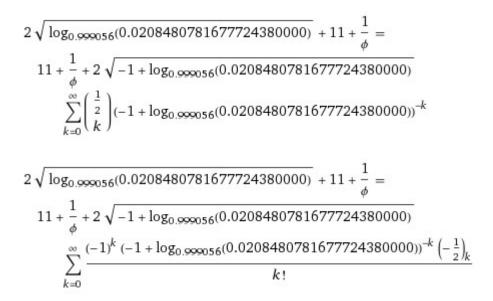
139.618... result practically equal to the rest mass of Pion meson 139.57 MeV

#### Alternative representation:

$$2\sqrt{\log_{0.999056}(0.0208480781677724380000)} + 11 + \frac{1}{\phi} = 11 + \frac{1}{\phi} + 2\sqrt{\frac{\log(0.0208480781677724380000)}{\log(0.999056)}}$$

#### Series representations:

$$2\sqrt{\log_{0.999056}(0.0208480781677724380000)} + 11 + \frac{1}{\phi}$$
$$11 + \frac{1}{\phi} + 2\sqrt{-\frac{\sum_{k=1}^{\infty} \frac{(-1)^{k} (-0.9791519218322275620000)^{k}}{k}}{\log(0.999056)}}$$



Note that the two values of the field  $-1.3312 = \phi$  and  $-1.3288 = \phi$  are very near to the value of the following 5<sup>th</sup> order Ramanujan mock theta function:

$$f(q) = 1 + \frac{q}{1+q} + \frac{q^4}{(1+q)(1+q^2)} + \frac{q^9}{(1+q)(1+q^2)(1+q^3)} + \dots$$

#### $1+0.449329/(1+0.449329)+0.449329^{4/(((1+0.449329)(1+0.449329^{2}))))$

#### **Input interpretation:**

 $1 + \frac{0.449329}{1 + 0.449329} + \frac{0.449329^4}{(1 + 0.449329)\left(1 + 0.449329^2\right)}$ 

#### **Result:**

 $1.333425959911272680899883774926957939703837145947480074487\ldots$ 

f(q) = 1.333425959...

We have also:

(4.076594584857 + 0.020848078167)\*10

#### Input interpretation:

(4.076594584857 + 0.020848078167)×10

#### **Result:**

40.97442663024

40.97442663024

#### And:

(4.076594584857)\*10 = 40.76594584857

Furthermore:

(4.076594584857 + 0.020848078167)

#### Input interpretation:

4.076594584857 + 0.020848078167

#### **Result:**

4.097442663024 4.097442663024 And:

(4.076594584857-0.020848078167)

Input interpretation: 4.076594584857 – 0.020848078167

**Result:** 4.05574650669 4.05574650669

From the sum of the two results, considering 49/12 and 1/48, we obtain: 4.104166666

We note that 10 \* 4.10416666 = 41.04166666

From:

**On a Polya functional for rhombi, isosceles triangles, and thinning convex sets.** *M. van den Berg, V. Ferone, C. Nitsch, C. Trombetti* - arXiv:1811.04503v2 [math.AP] 21 May 2019

Let  $\Omega$  be an open convex set in  $\mathbb{R}^m$  with finite width, and with boundary  $\partial\Omega$ . Let  $v_{\Omega}$  be the torsion function for  $\Omega$ , i.e. the solution of  $-\Delta v = 1, v|_{\partial\Omega} = 0$ . An upper bound is obtained for the product of  $\|v_{\Omega}\|_{L^{\infty}(\Omega)}\lambda(\Omega)$ , where  $\lambda(\Omega)$  is the bottom of the spectrum of the Dirichlet Laplacian acting in  $L^2(\Omega)$ . The upper bound is sharp in the limit of a thinning sequence of convex sets. For planar rhombi and isosceles triangles with area 1, it is shown that  $\|v_{\Omega}\|_{L^1(\Omega)}\lambda(\Omega) \geq \frac{\pi^2}{24}$ , and that this bound is sharp.

**Theorem 1.2** If  $\triangle_{\beta}$  is an isosceles triangle with angles  $\beta, \beta, \pi - 2\beta$ , and if  $0 < \beta \leq \frac{\pi}{3}$  then

$$\frac{T(\Delta_{\beta})\lambda(\Delta_{\beta})}{|\Delta_{\beta}|} \le \frac{\pi^2}{24} \left(1 + 81\left(\tan\beta\right)^{2/3}\right).$$

$$(1.14)$$

**Theorem 1.3** If  $\Diamond_{\beta}$  is a rhombus with angles  $\beta, \pi - \beta, \beta, \pi - \beta$ , and if  $\beta \leq \frac{\pi}{3}$  then

$$\frac{T(\Diamond_{\beta})\lambda(\Diamond_{\beta})}{|\Diamond_{\beta}|} \le \frac{\pi^2}{24} \left(1 + 15(\tan\beta)^{2/3}\right). \tag{1.15}$$

**Theorem 1.4** If  $\Diamond_\beta$  is as in Theorem 1.3, then

$$\frac{T(\Diamond_{\beta})\lambda(\Diamond_{\beta})}{|\Diamond_{\beta}|} \ge \frac{\pi^2}{24}.$$
(1.16)

**Theorem 1.5** If  $\triangle_{\beta}$  is an isosceles triangle with angles  $\beta, \beta, 2\pi - \beta$ , then

$$\frac{T(\Delta_{\beta})\lambda(\Delta_{\beta})}{|\wedge_{\beta}|} \ge \frac{\pi^2}{24}.$$
(1.17)

$$\begin{aligned} \frac{T(\triangle_{\beta})\lambda(\triangle_{\beta})}{||\triangle_{\beta}|} &\leq \frac{\pi^{2}}{24} \left(1 + d^{2}\right)^{2} \left(1 + 7\left(\frac{d}{2}\right)^{2/3}\right) \\ &\leq \frac{\pi^{2}}{24} \left(1 + 81d^{2/3}\right) \\ &= \frac{\pi^{2}}{24} \left(1 + 81\left(\tan\beta\right)^{2/3}\right), \ 0 < \beta \leq \frac{\pi}{3}. \end{aligned}$$

For  $\beta = \pi/4$ 

(Pi^2)/(24)\*((1+81(tan(Pi/4)^(2/3))))

Input:  $\frac{\pi^2}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right)$ 

#### **Exact result:**

 $\frac{41 \pi^2}{12}$ 

#### **Decimal approximation:**

33.72114837038864194768451091624351637898847297473936797357...

#### 33.72114837...

#### **Property:**

 $\frac{41\pi^2}{12}$  is a transcendental number

#### Alternative representations:

$$\frac{1}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 = \frac{1}{24} \pi^2 \left( 1 + 81 \left( \frac{1}{\cot\left(\frac{\pi}{4}\right)} \right)^{2/3} \right)$$
$$\frac{1}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 = \frac{1}{24} \pi^2 \left( 1 + 81 \cot^{2/3} \left( \frac{\pi}{2} - \frac{\pi}{4} \right) \right)$$
$$\frac{1}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 = \frac{1}{24} \pi^2 \left( 1 + 81 \left( -\cot\left(\frac{\pi}{2} + \frac{\pi}{4}\right) \right)^{2/3} \right)$$

#### Series representations:

$$\frac{1}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 = \frac{41}{2} \sum_{k=1}^{\infty} \frac{1}{k^2}$$
$$\frac{1}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 = -41 \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$$
$$\frac{1}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 = \frac{82}{3} \sum_{k=0}^{\infty} \frac{1}{(1+2k)^2}$$

#### Integral representations:

$$\frac{1}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 = \frac{41}{3} \left( \int_0^\infty \frac{1}{1 + t^2} dt \right)^2$$
$$\frac{1}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 = \frac{164}{3} \left( \int_0^1 \sqrt{1 - t^2} dt \right)^2$$
$$\frac{1}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 = \frac{41}{3} \left( \int_0^1 \frac{1}{\sqrt{1 - t^2}} dt \right)^2$$

#### Multiple-argument formulas:

$$\frac{1}{24} \left(1 + 81 \tan^{2/3} \left(\frac{\pi}{4}\right)\right) \pi^2 = \frac{1}{24} \pi^2 \left(1 + 81 \times 2^{2/3} \left(-\frac{\tan\left(\frac{\pi}{8}\right)}{-1 + \tan^2\left(\frac{\pi}{8}\right)}\right)^{2/3}\right)$$
$$\frac{1}{24} \left(1 + 81 \tan^{2/3} \left(\frac{\pi}{4}\right)\right) \pi^2 = \frac{1}{24} \pi^2 \left(1 + 81 \left(\frac{\tan\left(\frac{\pi}{12}\right)\left(-3 + \tan^2\left(\frac{\pi}{12}\right)\right)}{-1 + 3 \tan^2\left(\frac{\pi}{12}\right)}\right)^{2/3}\right)$$

$$\frac{1}{24} \left(1 + 81 \tan^{2/3} \left(\frac{\pi}{4}\right)\right) \pi^2 = \frac{1}{24} \pi^2 \left(1 + 81 \left(\frac{\tan\left(-\frac{3\pi}{4}\right) + \tan(\pi)}{1 - \tan\left(-\frac{3\pi}{4}\right)\tan(\pi)}\right)^{2/3}\right)$$
$$\frac{1}{24} \left(1 + 81 \tan^{2/3} \left(\frac{\pi}{4}\right)\right) \pi^2 = \frac{1}{24} \pi^2 \left(1 + 81 \left(\frac{U_{-\frac{3}{4}}(\cos(\pi))\sin(\pi)}{\frac{4}{4}}\right)^{2/3}\right)$$

$$\frac{T(\triangle_\beta)\lambda(\triangle_\beta))}{|\triangle_\beta|} \geq \frac{\pi^2}{24}, \ 0 < \beta \leq \frac{\pi}{3}.$$

$$\begin{aligned} \frac{T(\Diamond_{\beta})\lambda(\Diamond_{\beta})}{|\Diamond_{\beta}|} &\leq \frac{\pi^{2}}{24} \left(1 + \frac{d^{2}}{4}\right)^{2} \left(1 + \frac{9d^{2}}{32}\right) \left(1 + 7\left(\frac{d}{2}\right)^{2/3}\right) \\ &\leq \frac{\pi^{2}}{24} \left(1 + 15\left(\frac{d}{2}\right)^{2/3}\right) \\ &= \frac{\pi^{2}}{24} \left(1 + 15\left(\tan\beta\right)^{2/3}\right), \ 0 < \beta \leq \frac{\pi}{3}. \end{aligned}$$

(Pi^2)/(24)\*((1+15(tan(Pi/4)^(2/3))))

# $\frac{\pi^2}{24} \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right)$

#### **Exact result:**

 $\frac{2\pi^2}{3}$ 

#### **Decimal approximation:**

6.579736267392905745889660666584100756875799604827193750942...

6.579736267...

Property:  $\frac{2\pi^2}{3}$  is a transcendental number

## Alternative representations:

$$\frac{1}{24} \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 = \frac{1}{24} \pi^2 \left( 1 + 15 \left( \frac{1}{\cot \left( \frac{\pi}{4} \right)} \right)^{2/3} \right)$$
$$\frac{1}{24} \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 = \frac{1}{24} \pi^2 \left( 1 + 15 \cot^{2/3} \left( \frac{\pi}{2} - \frac{\pi}{4} \right) \right)$$
$$\frac{1}{24} \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 = \frac{1}{24} \pi^2 \left( 1 + 15 \left( -\cot \left( \frac{\pi}{2} + \frac{\pi}{4} \right) \right)^{2/3} \right)$$

#### Series representations:

$$\frac{1}{24} \left(1 + 15 \tan^{2/3} \left(\frac{\pi}{4}\right)\right) \pi^2 = 4 \sum_{k=1}^{\infty} \frac{1}{k^2}$$
$$\frac{1}{24} \left(1 + 15 \tan^{2/3} \left(\frac{\pi}{4}\right)\right) \pi^2 = -8 \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$$
$$\frac{1}{24} \left(1 + 15 \tan^{2/3} \left(\frac{\pi}{4}\right)\right) \pi^2 = \frac{16}{3} \sum_{k=0}^{\infty} \frac{1}{(1+2k)^2}$$

#### Integral representations:

$$\frac{1}{24} \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 = \frac{32}{3} \left( \int_0^1 \sqrt{1 - t^2} \, dt \right)^2$$
$$\frac{1}{24} \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 = \frac{8}{3} \left( \int_0^\infty \frac{1}{1 + t^2} \, dt \right)^2$$
$$\frac{1}{24} \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 = \frac{8}{3} \left( \int_0^1 \frac{1}{\sqrt{1 - t^2}} \, dt \right)^2$$

## Multiple-argument formulas:

$$\frac{1}{24} \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 = \frac{1}{24} \pi^2 \left( 1 + 15 \times 2^{2/3} \left( -\frac{\tan\left(\frac{\pi}{8}\right)}{-1 + \tan^2\left(\frac{\pi}{8}\right)} \right)^{2/3} \right)$$

$$\frac{1}{24} \left(1 + 15 \tan^{2/3} \left(\frac{\pi}{4}\right)\right) \pi^2 = \frac{1}{24} \pi^2 \left(1 + 15 \left(\frac{\tan\left(\frac{\pi}{12}\right)\left(-3 + \tan^2\left(\frac{\pi}{12}\right)\right)}{-1 + 3 \tan^2\left(\frac{\pi}{12}\right)}\right)^{2/3}\right)$$
$$\frac{1}{24} \left(1 + 15 \tan^{2/3} \left(\frac{\pi}{4}\right)\right) \pi^2 = \frac{1}{24} \pi^2 \left(1 + 15 \left(\frac{\tan\left(-\frac{3\pi}{4}\right) + \tan(\pi)}{1 - \tan\left(-\frac{3\pi}{4}\right) \tan(\pi)}\right)^{2/3}\right)$$
$$\frac{1}{24} \left(1 + 15 \tan^{2/3} \left(\frac{\pi}{4}\right)\right) \pi^2 = \frac{1}{24} \pi^2 \left(1 + 15 \left(\frac{U_{-\frac{3}{4}}(\cos(\pi))\sin(\pi)}{\frac{4}{4}}\right)^{2/3}\right)$$

$$\frac{\lambda(\Diamond_{\beta})T(\Diamond_{\beta})}{|\Diamond_{\beta}|} \ge \frac{\pi^2}{24} \frac{16 + 24d^2 + d^4}{(1 + \frac{3}{4}d^2)(16 + 4d^2)} \ge \frac{\pi^2}{24}, \qquad 0 \le d \le 2.$$

From the sum of the four results, we obtain:

 $(((33.72114837038864 + 6.5797362673929057 + (Pi^2)/24 + (Pi^2)/24)))$ 

#### Input interpretation:

 $33.72114837038864 + 6.5797362673929057 + \frac{\pi^2}{24} + \frac{\pi^2}{24}$ 

#### **Result:**

41.12335167120566...

41.1233516... result very near to the previous results: 1/48 = 4.104166666, from which we obtain 10 \* 4.10416666 = 41.04166666 and 40.97442663024

```
(4.076594584857 + 0.020848078167)×10
```

#### Alternative representations:

 $33.721148370388640000 + 6.57973626739290570000 + \frac{\pi^2}{24} + \frac{\pi^2}{24} = 40.300884637781545700 + \frac{2}{24} (180^\circ)^2$ 

$$33.721148370388640000 + 6.57973626739290570000 + \frac{\pi^2}{24} + \frac{\pi^2}{24} = 40.300884637781545700 + \frac{2}{24} (-i \log(-1))^2$$

$$33.721148370388640000 + 6.57973626739290570000 + \frac{\pi^2}{24} + \frac{\pi^2}{24} = 40.300884637781545700 + \frac{12\zeta(2)}{24}$$

#### Series representations:

$$33.721148370388640000 + 6.57973626739290570000 + \frac{\pi^2}{24} + \frac{\pi^2}{24} = 40.300884637781545700 + \frac{1}{3} \left( -1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}} \right)^2$$

#### **Integral representations:**

-

From the sum of the four results, performing the following calculations, we obtain: 1+sqrt729/((( $(33.72114837038864 + 6.5797362673929057 + (Pi^2)/24 + (Pi^2)/24)$ )) Where 729 = 9<sup>3</sup> (see Ramanujan cubes)

#### **Input interpretation:**

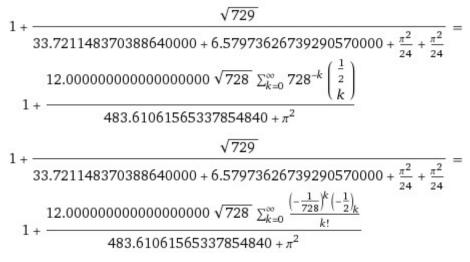
 $1 + \frac{\sqrt{729}}{33.72114837038864 + 6.5797362673929057 + \frac{\pi^2}{24} + \frac{\pi^2}{24}}$ 

#### **Result:**

1.656561270002349...

1.65656127.... result very near to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164,2696$  i.e. 1,65578...

#### Series representations:



Note that, we obtain:

41.12335167120566+(((1/60 (Fibonacci factorial constant + 67))))

Where:

Fibonacci factorial constant

 $\left(-\frac{1}{\phi^2}; -\frac{1}{\phi^2}\right)_{\infty}$ 

 $(a; q)_n$  gives the q-Pochhammer symbol

∅ is the golden ratio

1.226742010720353244417630230455361655871409690440250419643...

1.2267420107...

Input interpretation: 41.12335167120566 +  $\frac{1}{60}$  ( $\mathcal{F}_{FF}$  + 67)

 $\mathcal{T}_{FF}$  is the Fibonacci factorial constant

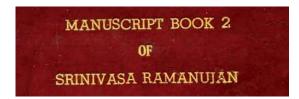
#### **Result:**

42.26046403805100...

42.260464... result equal to above first result 42.260464... obtained from the formula

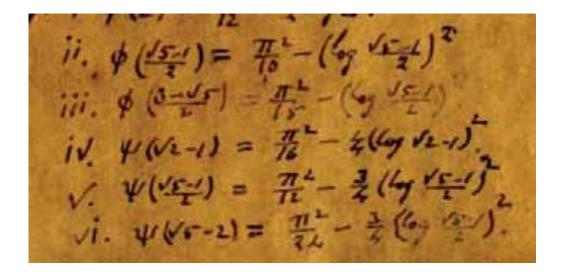
$$-3\left(-\frac{2744+5996\times42.63931648+9844\times1818.1113+77523.023543}{12\left(1+42.63931648\right)^2\left(14+42.63931648\right)}\right)$$

From:



we have that:

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Pi^2/(24) -3/4((( ln ((sqrt5-1)/2))))^2

## Input:

 $\frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right)$ 

 $\log(x)$  is the natural logarithm

## **Decimal approximation:**

 $0.237559901279160814745406988237856727292432712764725456322\ldots$ 

0.23755990127916....

#### **Alternate forms:**

$$\frac{1}{24} \left( \pi^2 - 18 \operatorname{csch}^{-1}(2)^2 \right)$$
$$\frac{1}{24} \left( \pi^2 - 18 \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right)$$

$$\frac{1}{24} \left( \pi^2 - 18 \left( \log \left( \sqrt{5} - 1 \right) - \log(2) \right)^2 \right)$$

 $\operatorname{csch}^{-1}(x)$  is the inverse hyperbolic cosecant function

Alternative representations:

$$\frac{\pi^2}{24} - \frac{1}{4} \log^2 \left(\frac{1}{2} \left(\sqrt{5} - 1\right)\right) 3 = \frac{\pi^2}{24} - \frac{3}{4} \log_e^2 \left(\frac{1}{2} \left(-1 + \sqrt{5}\right)\right)$$
$$\frac{\pi^2}{24} - \frac{1}{4} \log^2 \left(\frac{1}{2} \left(\sqrt{5} - 1\right)\right) 3 = \frac{\pi^2}{24} - \frac{3}{4} \left(\log(a) \log_a \left(\frac{1}{2} \left(-1 + \sqrt{5}\right)\right)\right)^2$$
$$\frac{\pi^2}{24} - \frac{1}{4} \log^2 \left(\frac{1}{2} \left(\sqrt{5} - 1\right)\right) 3 = \frac{\pi^2}{24} - \frac{3}{4} \left(-\text{Li}_1 \left(1 + \frac{1}{2} \left(1 - \sqrt{5}\right)\right)\right)^2$$

## Series representations:

$$\frac{\pi^2}{24} - \frac{1}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) 3 = \frac{\pi^2}{24} - \frac{3}{4} \left( \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{2} \right)^k \left( -3 + \sqrt{5} \right)^k}{k} \right)^2$$

$$\frac{\pi^2}{24} - \frac{1}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) 3 = \frac{1}{24} \left( \pi^2 - 18 \left( 2 i \pi \left[ \frac{\arg(-1 + \sqrt{5} - 2 x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{2} \right)^k \left( -1 + \sqrt{5} - 2 x \right)^k x^{-k}}{k} \right)^2 \right)$$
for  $x < 0$ 

$$\frac{\pi^2}{24} - \frac{1}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) 3 = \frac{\pi^2}{24} - \frac{3}{4} \left( 2i\pi \left[ \frac{\arg\left(\frac{1}{2} \left( -1 + \sqrt{5} \right) - x \right)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{2} \right)^k \left( -1 + \sqrt{5} - 2x \right)^k x^{-k}}{k} \right)^2}{\text{for } x < 0$$

# Integral representation: $\frac{\pi^2}{24} - \frac{1}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) 3 = \frac{\pi^2}{24} - \frac{3}{4} \left( \int_1^{\frac{1}{2} \left( -1 + \sqrt{5} \right)} \frac{1}{t} dt \right)^2$

((((Pi^2/(24) -3/4((( ln ((sqrt5-1)/2))))^2))))^1/128

## Input:

$$\frac{128}{\sqrt{\frac{\pi^2}{24} - \frac{3}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)}}$$

\_\_\_\_\_

 $\log(x)$  is the natural logarithm

## **Decimal approximation:**

0.988833628580485387235048704408866760465401974342081212010...

0.988833628580.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0**. **989117352243** =  $\phi$ 

## Alternate forms:

$$\frac{\frac{128}{3} \frac{\pi^2}{3} - 6 \operatorname{csch}^{-1}(2)^2}{2^{3/128}}$$

$$\frac{128}{\sqrt[3]{24} - \frac{3}{4} \left( \log \left( \sqrt{5} - 1 \right) - \log(2) \right)^2}{\frac{128}{3} \frac{1}{3} \left( \pi^2 - 18 \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right)}{2^{3/128}}$$

 $\operatorname{csch}^{-1}(x)$  is the inverse hyperbolic cosecant function

All 128th roots of 
$$\pi^2/24 - 3/4 \log^2(1/2 (\operatorname{sqrt}(5) - 1)):$$
  
 $e^{0} {}^{128} \sqrt{\frac{\pi^2}{24} - \frac{3}{4} \log^2(\frac{1}{2}(\sqrt{5} - 1))} \approx 0.98883 (\operatorname{real. principal root})$   
 $e^{(i\pi)/64} {}^{128} \sqrt{\frac{\pi^2}{24} - \frac{3}{4} \log^2(\frac{1}{2}(\sqrt{5} - 1))} \approx 0.98764 + 0.04852 i$   
 $e^{(i\pi)/32} {}^{128} \sqrt{\frac{\pi^2}{24} - \frac{3}{4} \log^2(\frac{1}{2}(\sqrt{5} - 1))} \approx 0.98407 + 0.09692 i$   
 $e^{(3i\pi)/64} {}^{128} \sqrt{\frac{\pi^2}{24} - \frac{3}{4} \log^2(\frac{1}{2}(\sqrt{5} - 1))} \approx 0.97813 + 0.14509 i$   
 $e^{(i\pi)/16} {}^{128} \sqrt{\frac{\pi^2}{24} - \frac{3}{4} \log^2(\frac{1}{2}(\sqrt{5} - 1))} \approx 0.96983 + 0.19291 i$ 

## Alternative representations:

$${}^{128} \sqrt{\frac{\pi^2}{24} - \frac{1}{4} \log^2 \left(\frac{1}{2} \left(\sqrt{5} - 1\right)\right) 3} = {}^{128} \sqrt{\frac{\pi^2}{24} - \frac{3}{4} \log_e^2 \left(\frac{1}{2} \left(-1 + \sqrt{5}\right)\right)}$$

$${}^{128} \sqrt{\frac{\pi^2}{24} - \frac{1}{4} \log^2 \left(\frac{1}{2} \left(\sqrt{5} - 1\right)\right) 3} = {}^{128} \sqrt{\frac{\pi^2}{24} - \frac{3}{4} \left(\log(a) \log_a \left(\frac{1}{2} \left(-1 + \sqrt{5}\right)\right)\right)^2}$$

$${}^{128} \sqrt{\frac{\pi^2}{24} - \frac{1}{4} \log^2 \left(\frac{1}{2} \left(\sqrt{5} - 1\right)\right) 3} = {}^{128} \sqrt{\frac{\pi^2}{24} - \frac{3}{4} \left(\log(a) \log_a \left(\frac{1}{2} \left(-1 + \sqrt{5}\right)\right)\right)^2}$$

## Integral representation:

$${}^{128}\sqrt{\frac{\pi^2}{24} - \frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)3} = {}^{128}\sqrt{\frac{\pi^2}{24} - \frac{3}{4}\left(\int_1^{\frac{1}{2}\left(-1 + \sqrt{5}\right)}\frac{1}{t}\,dt\right)^2}$$

# log base 0.988833628580485 ((((Pi^2/(24) –3/4((( ln ((sqrt5-1)/2))))^2))))-Pi+1/golden ratio

#### **Input interpretation:**

 $\log_{0.988833628580485} \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right) - \pi + \frac{1}{\phi}$ 

log(x) is the natural logarithm

 $\log_b(x)$  is the base- b logarithm

#### **Result:**

125.4764413352...

125.4764413352... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0

## Alternative representations:

$$\begin{split} \log_{0.9888336285804850000} & \left( \frac{\pi^2}{24} - \frac{1}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) 3 \right) - \pi + \frac{1}{\phi} = \\ & -\pi + \log_{0.9888336285804850000} \left( \frac{\pi^2}{24} - \frac{3}{4} \log_e^2 \left( \frac{1}{2} \left( -1 + \sqrt{5} \right) \right) \right) + \frac{1}{\phi} \end{split}$$

$$\begin{split} \log_{0.9888336285804850000} & \left( \frac{\pi^2}{24} - \frac{1}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) 3 \right) - \pi + \frac{1}{\phi} = \\ & -\pi + \frac{1}{\phi} + \frac{\log \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( -1 + \sqrt{5} \right) \right) \right)}{\log(0.9888336285804850000)} \end{split}$$

$$\log_{0.9888336285804850000} \left( \frac{\pi^2}{24} - \frac{1}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) 3 \right) - \pi + \frac{1}{\phi} = -\pi + \log_{0.9888336285804850000} \left( \frac{\pi^2}{24} - \frac{3}{4} \left( \log(a) \log_a \left( \frac{1}{2} \left( -1 + \sqrt{5} \right) \right) \right)^2 \right) + \frac{1}{\phi}$$

## Series representations:

$$\log_{0.9888336285804850000} \left( \frac{\pi^2}{24} - \frac{1}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) 3 \right) - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi + \log_{0.9888336285804850000} \left( \frac{1}{24} \left( \pi^2 - 18 \left( \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{2} \right)^k \left( -3 + \sqrt{5} \right)^k}{k} \right)^2 \right) \right)$$

$$\log_{0.9888336285804850000} \left( \frac{\pi^2}{24} - \frac{1}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) 3 \right) - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi - \frac{\sum_{k=1}^{\infty} \frac{\left( -\frac{1}{24} \right)^k \left( -24 + \pi^2 - 18 \log^2 \left( \frac{1}{2} \left( -1 + \sqrt{5} \right) \right) \right)^k}{\log(0.9888336285804850000)}$$

## Integral representation:

$$\begin{aligned} \log_{0.9888336285804850000} \left( \frac{\pi^2}{24} - \frac{1}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) 3 \right) - \pi + \frac{1}{\phi} = \\ \frac{1}{\phi} - \pi + \log_{0.9888336285804850000} \left( \frac{1}{24} \left( \pi^2 - 18 \left( \int_1^{\frac{1}{2} \left( -1 + \sqrt{5} \right)} \frac{1}{t} dt \right)^2 \right) \right) \end{aligned}$$

## Adding the previous analyzed expression:

$$\frac{\pi^2}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right)$$

with

$$\frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right)$$

we obtain:

$$(((Pi^2/(24) - 3/4((( ln ((sqrt5-1)/2))))^2))) + ((((Pi^2)/(24)^*((1+81(tan(Pi/4)^(2/3)))))))))$$

## Input:

$$\left(\frac{\pi^2}{24} - \frac{3}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)\right) + \frac{\pi^2}{24}\left(1 + 81\tan^{2/3}\left(\frac{\pi}{4}\right)\right)$$

 $\log(x)$  is the natural logarithm

## Exact result: $\frac{83 \pi^2}{24} - \frac{3}{4} \log^2 \left(\frac{1}{2} \left(\sqrt{5} - 1\right)\right)$

## **Decimal approximation:**

33.95870827166780276242991790448137310628090568750409342990...

 $33.95870827... \approx 34$  (Fibonacci number)

## Alternate forms:

$$\frac{83 \pi^2}{24} - \frac{3}{4} \operatorname{csch}^{-1}(2)^2$$
$$\frac{1}{24} \left( 83 \pi^2 - 18 \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right)$$
$$\frac{83 \pi^2}{24} - \frac{3}{4} \left( \log \left( \sqrt{5} - 1 \right) - \log(2) \right)^2$$

 $\operatorname{csch}^{-1}(x)$  is the inverse hyperbolic cosecant function

## Alternative representations:

$$\begin{split} &\left(\frac{\pi^2}{24} - \frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)3\right) + \frac{1}{24}\left(1 + 81\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^2 = \\ & \frac{\pi^2}{24} + \frac{1}{24}\pi^2\left(1 + 81\left(-\cot\left(-\frac{\pi}{2} + \frac{\pi}{4}\right)\right)^{2/3}\right) - \frac{3}{4}\log^2_e\left(\frac{1}{2}\left(-1 + \sqrt{5}\right)\right) \\ & \left(\frac{\pi^2}{24} - \frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)3\right) + \frac{1}{24}\left(1 + 81\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^2 = \\ & \frac{\pi^2}{24} - \frac{3}{4}\log^2\left(\frac{1}{2}\left(-1 + \sqrt{5}\right)\right) + \frac{1}{24}\pi^2\left(1 + 81\left(-i + \frac{2i}{1 + e^{(2i\pi)/4}}\right)^{2/3}\right) \\ & \left(\frac{\pi^2}{24} - \frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)3\right) + \frac{1}{24}\left(1 + 81\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^2 = \\ & \frac{\pi^2}{24} + \frac{1}{24}\log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)3\right) + \frac{1}{24}\left(1 + 81\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^2 = \\ & \frac{\pi^2}{24} + \frac{1}{24}\pi^2\left(1 + 81\left(-\cot\left(-\frac{\pi}{2} + \frac{\pi}{4}\right)\right)^{2/3}\right) - \frac{3}{4}\left(\log(a)\log_a\left(\frac{1}{2}\left(-1 + \sqrt{5}\right)\right)\right)^2 \end{split}$$

## Series representations:

$$\begin{split} &\left(\frac{\pi^2}{24} - \frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)3\right) + \frac{1}{24}\left(1 + 81\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^2 = \\ & \frac{83\pi^2}{24} - \frac{3}{4}\left(\sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-3 + \sqrt{5}\right)^k}{k}\right)^2 \\ & \left(\frac{\pi^2}{24} - \frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)3\right) + \frac{1}{24}\left(1 + 81\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^2 = \\ & \frac{83\pi^2}{24} + \frac{3}{4}\left(2\pi\left\lfloor\frac{\arg(-1 + \sqrt{5} - 2x)}{2\pi}\right\rfloor - i\left(\log(x) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-1 + \sqrt{5} - 2x\right)^k x^{-k}}{k}\right)\right)^2 \\ & \text{for } x < 0 \end{split}$$
$$\begin{pmatrix} \left(\frac{\pi^2}{24} - \frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)3\right) + \frac{1}{24}\left(1 + 81\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^2 = \\ & \frac{83\pi^2}{24} - \frac{3}{4}\left(2i\pi\left\lfloor\frac{\arg\left(\frac{1}{2}\left(-1 + \sqrt{5}\right) - x\right)}{2\pi}\right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-1 + \sqrt{5} - 2x\right)^k x^{-k}}{k}\right)^2 \end{split}$$

for 
$$x < 0$$

## Integral representation:

$$\left(\frac{\pi^2}{24} - \frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)3\right) + \frac{1}{24}\left(1 + 81\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^2 = \frac{83\pi^2}{24} - \frac{3}{4}\left(\int_1^{\frac{1}{2}\left(-1+\sqrt{5}\right)}\frac{1}{t}\,dt\right)^2$$

## Multiple-argument formula:

$$\left(\frac{\pi^2}{24} - \frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)3\right) + \frac{1}{24}\left(1 + 81\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^2 = \frac{83\pi^2}{24} - \frac{3}{4}\left(-\log(2) + \log\left(-1 + \sqrt{5}\right)\right)^2$$

In conclusion:

 $\frac{1}{21*[(((Pi^2/(24) - 3/4((( ln ((sqrt5-1)/2))))^2))) + ((((Pi^2)/(24)*((1+81(tan(Pi/4)^(2/3)))))))]}{(((Pi^2)/(24)*((1+81(tan(Pi/4)^(2/3)))))))]}$ 

## Input:

 $\frac{1}{21} \left( \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right) + \frac{\pi^2}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \right)$ 

log(x) is the natural logarithm

#### **Exact result:**

 $\frac{1}{21} \left( \frac{83 \pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right)$ 

## **Decimal approximation:**

1.617081346269895369639519900213398719346709794643052068090...

1.61708134626... result that is a nearly approximation to the value of the golden ratio 1,618033988749...

#### **Alternate forms:**

 $\frac{1}{504} \left(83 \pi^2 - 18 \operatorname{csch}^{-1}(2)^2\right)$  $\frac{83 \pi^2}{504} - \frac{1}{28} \log^2 \left(\frac{1}{2} \left(\sqrt{5} - 1\right)\right)$  $\frac{1}{504} \left(83 \pi^2 - 18 \log^2 \left(\frac{1}{2} \left(\sqrt{5} - 1\right)\right)\right)$ 

 $\operatorname{csch}^{-1}(x)$  is the inverse hyperbolic cosecant function

#### **Alternative representations:**

$$\frac{1}{21} \left( \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right) + \frac{1}{24} \pi^2 \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \right) = \frac{1}{21} \left( \frac{\pi^2}{24} + \frac{1}{24} \pi^2 \left( 1 + 81 \left( -\cot \left( -\frac{\pi}{2} + \frac{\pi}{4} \right) \right)^{2/3} \right) - \frac{3}{4} \log_e^2 \left( \frac{1}{2} \left( -1 + \sqrt{5} \right) \right) \right)$$

$$\begin{aligned} &\frac{1}{21} \left( \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right) + \frac{1}{24} \pi^2 \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \right) = \\ &\frac{1}{21} \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( -1 + \sqrt{5} \right) \right) + \frac{1}{24} \pi^2 \left( 1 + 81 \left( -i + \frac{2i}{1 + e^{(2i\pi)/4}} \right)^{2/3} \right) \right) \\ &\frac{1}{21} \left( \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right) + \frac{1}{24} \pi^2 \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \right) = \\ &\frac{1}{21} \left( \frac{\pi^2}{24} + \frac{1}{24} \pi^2 \left( 1 + 81 \left( -\cot \left( -\frac{\pi}{2} + \frac{\pi}{4} \right) \right)^{2/3} \right) - \frac{3}{4} \left( \log(a) \log_a \left( \frac{1}{2} \left( -1 + \sqrt{5} \right) \right) \right)^2 \right) \end{aligned}$$

# Series representations:

$$\frac{1}{21} \left( \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right) + \frac{1}{24} \pi^2 \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \right) = \frac{83 \pi^2}{504} - \frac{1}{28} \left( \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{2} \right)^k \left( -3 + \sqrt{5} \right)^k}{k} \right)^2$$

$$\begin{aligned} \frac{1}{21} \left( \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right) + \frac{1}{24} \pi^2 \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \right) = \\ \frac{1}{21} \left( \frac{83 \pi^2}{24} + \frac{3}{4} \left( 2 \pi \left[ \frac{\arg(-1 + \sqrt{5} - 2x)}{2\pi} \right] - \frac{1}{2\pi} \left( \log(x) - \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{2} \right)^k \left( -1 + \sqrt{5} - 2x \right)^k x^{-k}}{k} \right) \right)^2 \right) & \text{for } x < 0 \end{aligned}$$

$$\frac{1}{21} \left( \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right) + \frac{1}{24} \pi^2 \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \right) = \frac{1}{21} \left( \frac{83 \pi^2}{24} - \frac{3}{4} \left[ 2 i \pi \left[ \frac{\arg \left( \frac{1}{2} \left( -1 + \sqrt{5} \right) - x \right)}{2 \pi} \right] + \log(x) - \frac{\sum_{k=1}^{\infty} \left( -\frac{1}{2} \right)^k \left( -1 + \sqrt{5} - 2 x \right)^k x^{-k}}{k} \right)^2 \right) \text{ for } x < 0$$

## Integral representation:

$$\frac{1}{21} \left( \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right) + \frac{1}{24} \pi^2 \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \right) = \frac{83 \pi^2}{504} - \frac{1}{28} \left( \int_1^{\frac{1}{2} \left( -1 + \sqrt{5} \right)} \frac{1}{t} dt \right)^2$$

#### **Multiple-argument formula:**

$$\frac{1}{21} \left( \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right) + \frac{1}{24} \pi^2 \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \right) = \frac{1}{21} \left( \frac{83 \pi^2}{24} - \frac{3}{4} \left( -\log(2) + \log\left( -1 + \sqrt{5} \right) \right)^2 \right)$$

Now, to the Ramanujan expression, adding to the two precedent expressions, we obtain:

 $[(Pi^2)/(24)*((1+81(tan(Pi/4)^{(2/3)})))] + [(Pi^2)/(24)*((1+15(tan(Pi/4)^{(2/3)})))] + [Pi^2/(24) - 3/4(((ln((sqrt5-1)/2))))^2]$ 

## Input:

$$\frac{\pi^2}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) + \frac{\pi^2}{24} \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) + \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right)$$

log(x) is the natural logarithm

Exact result:  $\frac{33 \pi^2}{8} - \frac{3}{4} \log^2 \left(\frac{1}{2} \left(\sqrt{5} - 1\right)\right)$ 

## **Decimal approximation:**

40.53844453906070850831957857106547386315670529233128718084...

40.538444539..... result very near to the value of the following expression:

(4.076594584857)\*10 = 40.76594584857

#### **Alternate forms:**

$$\frac{1}{8} (33 \pi^2 - 6 \operatorname{csch}^{-1}(2)^2)$$
$$\frac{3}{8} \left( 11 \pi^2 - 2 \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right)$$
$$\frac{33 \pi^2}{8} - \frac{3}{4} \left( \log \left( \sqrt{5} - 1 \right) - \log(2) \right)^2$$

 $\operatorname{csch}^{-1}(x)$  is the inverse hyperbolic cosecant function

## Alternative representations:

$$\begin{aligned} &\frac{1}{24} \left(1 + 81 \tan^{2/3} \left(\frac{\pi}{4}\right)\right) \pi^2 + \frac{1}{24} \left(1 + 15 \tan^{2/3} \left(\frac{\pi}{4}\right)\right) \pi^2 + \left(\frac{\pi^2}{24} - \frac{1}{4} \log^2 \left(\frac{1}{2} \left(\sqrt{5} - 1\right)\right) 3\right) = \\ &\frac{\pi^2}{24} + \frac{1}{24} \pi^2 \left(1 + 15 \left(-\cot \left(-\frac{\pi}{2} + \frac{\pi}{4}\right)\right)^{2/3}\right) + \\ &\frac{1}{24} \pi^2 \left(1 + 81 \left(-\cot \left(-\frac{\pi}{2} + \frac{\pi}{4}\right)\right)^{2/3}\right) - \frac{3}{4} \log_e^2 \left(\frac{1}{2} \left(-1 + \sqrt{5}\right)\right) \end{aligned}$$

$$\begin{split} &\frac{1}{24} \left(1+81 \tan^{2/3} \left(\frac{\pi}{4}\right)\right) \pi^2 + \frac{1}{24} \left(1+15 \tan^{2/3} \left(\frac{\pi}{4}\right)\right) \pi^2 + \left(\frac{\pi^2}{24} - \frac{1}{4} \log^2 \left(\frac{1}{2} \left(\sqrt{5} - 1\right)\right) 3\right) = \\ &\frac{\pi^2}{24} + \frac{1}{24} \pi^2 \left(1+15 \left(-\cot \left(-\frac{\pi}{2} + \frac{\pi}{4}\right)\right)^{2/3}\right) + \\ &\frac{1}{24} \pi^2 \left(1+81 \left(-\cot \left(-\frac{\pi}{2} + \frac{\pi}{4}\right)\right)^{2/3}\right) - \frac{3}{4} \left(\log(a) \log_a \left(\frac{1}{2} \left(-1 + \sqrt{5}\right)\right)\right)^2 \\ &\frac{1}{24} \left(1+81 \tan^{2/3} \left(\frac{\pi}{4}\right)\right) \pi^2 + \frac{1}{24} \left(1+15 \tan^{2/3} \left(\frac{\pi}{4}\right)\right) \pi^2 + \left(\frac{\pi^2}{24} - \frac{1}{4} \log^2 \left(\frac{1}{2} \left(\sqrt{5} - 1\right)\right) 3\right) = \\ &\frac{\pi^2}{24} - \frac{3}{4} \log^2 \left(\frac{1}{2} \left(-1 + \sqrt{5}\right)\right) + \frac{1}{24} \pi^2 \left(1+15 \left(-i + \frac{2i}{1+e^{(2i\pi)/4}}\right)^{2/3}\right) + \\ &\frac{1}{24} \pi^2 \left(1+81 \left(-i + \frac{2i}{1+e^{(2i\pi)/4}}\right)^{2/3}\right) \end{split}$$

## Series representations:

$$\frac{1}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 + \frac{1}{24} \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 + \left( \frac{\pi^2}{24} - \frac{1}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) 3 \right) = \frac{33 \pi^2}{8} - \frac{3}{4} \left( \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{2} \right)^k \left( -3 + \sqrt{5} \right)^k}{k} \right)^2$$

$$\begin{aligned} \frac{1}{24} \left(1 + 81 \tan^{2/3} \left(\frac{\pi}{4}\right)\right) \pi^2 + \frac{1}{24} \left(1 + 15 \tan^{2/3} \left(\frac{\pi}{4}\right)\right) \pi^2 + \left(\frac{\pi^2}{24} - \frac{1}{4} \log^2 \left(\frac{1}{2} \left(\sqrt{5} - 1\right)\right) 3\right) = \\ \frac{33 \pi^2}{8} + \frac{3}{4} \left(2 \pi \left\lfloor \frac{\arg(-1 + \sqrt{5} - 2x)}{2 \pi} \right\rfloor - i \left(\log(x) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-1 + \sqrt{5} - 2x\right)^k x^{-k}}{k}\right)\right)^2 \\ \text{for } x < 0 \end{aligned}$$

$$\frac{1}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 + \frac{1}{24} \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 + \left( \frac{\pi^2}{24} - \frac{1}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) 3 \right) = \frac{33 \pi^2}{8} - \frac{3}{4} \left[ 2 i \pi \left[ \frac{\arg \left( \frac{1}{2} \left( -1 + \sqrt{5} \right) - x \right)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{2} \right)^k \left( -1 + \sqrt{5} - 2 x \right)^k x^{-k}}{k} \right)^2 \text{ for } x < 0$$

#### **Integral representation:**

$$\frac{1}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 + \frac{1}{24} \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 + \left( \frac{\pi^2}{24} - \frac{1}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) 3 \right) = \frac{33 \pi^2}{8} - \frac{3}{4} \left( \int_1^1 \frac{1}{2} \left( -1 + \sqrt{5} \right) \frac{1}{t} dt \right)^2$$

#### **Multiple-argument formula:**

$$\frac{1}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 + \frac{1}{24} \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 + \left( \frac{\pi^2}{24} - \frac{1}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) 3 \right) = \frac{33 \pi^2}{8} - \frac{3}{4} \left( -\log(2) + \log\left( -1 + \sqrt{5} \right) \right)^2$$

From which, dividing by 10, we obtain:

 $\frac{1}{10((([(Pi^{2})/(24)*((1+81(tan(Pi/4)^{(2/3)})))] + [(Pi^{2})/(24)*((1+15(tan(Pi/4)^{(2/3)})))] + [Pi^{2}/(24) - 3/4(((ln((sqrt5-1)/2))))^{2}])))}$ 

#### **Input:**

 $\frac{1}{10} \left( \frac{\pi^2}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) + \frac{\pi^2}{24} \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) + \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right) \right)$ 

log(x) is the natural logarithm

#### **Exact result:**

 $\frac{1}{10} \left( \frac{33 \pi^2}{8} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right)$ 

#### **Decimal approximation:**

4.053844453906070850831957857106547386315670529233128718084...

4.0538444539..... result very near to the previous value of the following expression:

$$\frac{-2744 + 5996 \left(e^{13 \left(-1.3288 / \left(2 \sqrt{3}\right)\right)} + 9844 e^{13 \left(-1.3288 / \sqrt{3}\right)} + e^{-1.3288 \left(1/2 \left(13 \sqrt{3}\right)\right)}\right)}{12 \left(1 + \left(e^{13 \left(-1.3288 / \left(2 \sqrt{3}\right)\right)}\right)^2 \left(14 + e^{13 \left(-1.3288 / \left(2 \sqrt{3}\right)\right)}\right)\right)}$$

4.07659...

4.07659...≈49/12

## Alternate forms:

 $\frac{1}{80} \left( 33 \, \pi^2 - 6 \, \operatorname{csch}^{-1}(2)^2 \right)$  $\frac{33 \, \pi^2}{80} - \frac{3}{40} \, \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right)$  $\frac{3}{80} \left( 11 \, \pi^2 - 2 \, \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right)$ 

 $\operatorname{csch}^{-1}(x)$  is the inverse hyperbolic cosecant function

## Alternative representations:

$$\begin{aligned} \frac{1}{10} \left( \frac{1}{24} \pi^2 \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) + \\ & \frac{1}{24} \pi^2 \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) + \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right) \right) = \\ & \frac{1}{10} \left( \frac{\pi^2}{24} + \frac{1}{24} \pi^2 \left( 1 + 15 \left( -\cot \left( -\frac{\pi}{2} + \frac{\pi}{4} \right) \right)^{2/3} \right) + \\ & \frac{1}{24} \pi^2 \left( 1 + 81 \left( -\cot \left( -\frac{\pi}{2} + \frac{\pi}{4} \right) \right)^{2/3} \right) - \frac{3}{4} \log_e^2 \left( \frac{1}{2} \left( -1 + \sqrt{5} \right) \right) \right) \end{aligned}$$

$$\begin{aligned} \frac{1}{10} \left( \frac{1}{24} \pi^2 \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) + \\ & \frac{1}{24} \pi^2 \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) + \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right) \right) = \\ & \frac{1}{10} \left( \frac{\pi^2}{24} + \frac{1}{24} \pi^2 \left( 1 + 15 \left( -\cot \left( -\frac{\pi}{2} + \frac{\pi}{4} \right) \right)^{2/3} \right) + \frac{1}{24} \pi^2 \left( 1 + 81 \left( -\cot \left( -\frac{\pi}{2} + \frac{\pi}{4} \right) \right)^{2/3} \right) - \\ & \frac{3}{4} \left( \log(a) \log_a \left( \frac{1}{2} \left( -1 + \sqrt{5} \right) \right) \right)^2 \right) \end{aligned}$$

$$\begin{aligned} \frac{1}{10} \left( \frac{1}{24} \pi^2 \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) + \\ & \frac{1}{24} \pi^2 \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) + \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right) \right) = \\ & \frac{1}{10} \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( -1 + \sqrt{5} \right) \right) + \frac{1}{24} \pi^2 \left( 1 + 15 \left( -i + \frac{2i}{1 + e^{(2i\pi)/4}} \right)^{2/3} \right) + \\ & \frac{1}{24} \pi^2 \left( 1 + 81 \left( -i + \frac{2i}{1 + e^{(2i\pi)/4}} \right)^{2/3} \right) \right) \end{aligned}$$

## Series representations:

$$\frac{1}{10} \left( \frac{1}{24} \pi^2 \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) + \frac{1}{24} \pi^2 \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) + \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right) = \frac{33 \pi^2}{80} - \frac{3}{40} \left( \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{2} \right)^k \left( -3 + \sqrt{5} \right)^k}{k} \right)^2$$

$$\begin{aligned} \frac{1}{10} \left( \frac{1}{24} \pi^2 \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) + \frac{1}{24} \pi^2 \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) + \\ \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right) \right) &= \frac{1}{10} \left( \frac{33 \pi^2}{8} + \frac{3}{4} \left( 2 \pi \left[ \frac{\arg(-1 + \sqrt{5} - 2x)}{2\pi} \right] - i \left( \log(x) - \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{2} \right)^k \left( -1 + \sqrt{5} - 2x \right)^k x^{-k}}{k} \right) \right)^2 \right) \\ \text{for } x < 0 \end{aligned}$$

$$\begin{aligned} \frac{1}{10} \left( \frac{1}{24} \pi^2 \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) + \\ & \frac{1}{24} \pi^2 \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) + \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right) \right) = \\ & \frac{1}{10} \left( \frac{33 \pi^2}{8} - \frac{3}{4} \left[ 2 i \pi \left[ \frac{\arg \left( \frac{1}{2} \left( -1 + \sqrt{5} \right) - x \right)}{2 \pi} \right] + \log (x) - \right] \\ & \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{2} \right)^k \left( -1 + \sqrt{5} - 2 x \right)^k x^{-k}}{k} \right)^2 \right) \text{ for } x < 0 \end{aligned}$$

# **Integral representation:**

$$\frac{1}{10} \left( \frac{1}{24} \pi^2 \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) + \frac{1}{24} \pi^2 \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) + \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right) = \frac{33 \pi^2}{80} - \frac{3}{40} \left( \int_1^{\frac{1}{2} \left( -1 + \sqrt{5} \right)} \frac{1}{t} dt \right)^2$$

#### **Multiple-argument formula:**

$$\frac{1}{10} \left( \frac{1}{24} \pi^2 \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) + \frac{1}{24} \pi^2 \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) + \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right) = \frac{1}{10} \left( \frac{33 \pi^2}{8} - \frac{3}{4} \left( -\log(2) + \log\left( -1 + \sqrt{5} \right) \right)^2 \right)$$

We have also:

 $\begin{array}{l} Pi^{*}((([(Pi^{2})/(24)^{*}((1+81(tan(Pi/4)^{(2/3)})))] + [(Pi^{2})/(24)^{*}((1+15(tan(Pi/4)^{(2/3)})))] + [Pi^{2}/(24) - 3/4(((ln((sqrt5-1)/2))))^{2}]))) - 2 \end{array}$ 

#### **Input:**

$$\pi \left( \frac{\pi^2}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) + \frac{\pi^2}{24} \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) + \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right) \right) - 2\pi^2 \left( \frac{\pi^2}{4} - \frac{\pi^2}{4} \right) = 2\pi^2 \left( \frac{\pi^2}{4} - \frac{\pi^2}{4} - \frac{\pi^2}{4} \right) = 2\pi^2 \left( \frac{\pi^2}{4} - \frac{\pi^2}{4} - \frac{\pi^2}{4} \right) = 2\pi^2 \left( \frac{\pi^2}{4} - \frac{\pi^2}{4} - \frac{\pi^2}{4} \right) = 2\pi^2 \left( \frac{\pi^2}{4} - \frac{\pi^2}{4} - \frac{\pi^2}{4} \right) = 2\pi^2 \left( \frac{\pi^2}{4} - \frac{\pi^2}{4} - \frac{\pi^2}{4} \right) = 2\pi^2 \left( \frac{\pi^2}{4} - \frac{\pi^2}{4} - \frac{\pi^2}{4} \right) = 2\pi^2 \left( \frac{\pi^2}{4} - \frac{\pi^2}{4} - \frac{\pi^2}{4} \right) = 2\pi^2 \left( \frac{\pi^2}{4} - \frac{\pi^2}{4} - \frac{\pi^2}{4} \right) = 2\pi^2 \left( \frac{\pi^2}{4} - \frac{\pi^2}{4} - \frac{\pi^2}{4} \right) = 2\pi^2 \left( \frac{\pi^2}{4} - \frac{\pi^2}{4} - \frac{\pi^2}{4} - \frac{\pi^2}{4} \right) = 2\pi^2 \left( \frac{\pi^2}{4} - \frac{\pi^2}{4} - \frac{\pi^2}{4} - \frac{\pi^2}{4} - \frac{\pi^2}{4} \right) = 2\pi^2 \left( \frac{\pi^2}{4} - \frac{\pi^2}{4}$$

 $\log(x)$  is the natural logarithm

#### **Exact result:**

 $\pi \left( \frac{33 \pi^2}{8} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) \right) - 2$ 

#### **Decimal approximation:**

125.3552795518703938576422532990510192508810645896080865529...

125.3552795... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

#### **Alternate forms:**

$$-2 + \frac{33 \pi^3}{8} - \frac{3}{4} \pi \operatorname{csch}^{-1}(2)^2$$
$$-2 + \frac{33 \pi^3}{8} - \frac{3}{4} \pi \log^2 \left(\frac{1}{2} \left(\sqrt{5} - 1\right)\right)$$
$$\frac{1}{8} \left(-16 + 33 \pi^3 - 6 \pi \log^2 \left(\frac{1}{2} \left(\sqrt{5} - 1\right)\right)\right)$$

 $\operatorname{csch}^{-1}(x)$  is the inverse hyperbolic cosecant function

## Alternative representations:

$$\pi \left( \frac{1}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 + \frac{1}{24} \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 + \left( \frac{\pi^2}{24} - \frac{1}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) 3 \right) \right) - 2 = -2 + \pi \left( \frac{\pi^2}{24} + \frac{1}{24} \pi^2 \left( 1 + 15 \left( -\cot \left( -\frac{\pi}{2} + \frac{\pi}{4} \right) \right)^{2/3} \right) + \frac{1}{24} \pi^2 \left( 1 + 81 \left( -\cot \left( -\frac{\pi}{2} + \frac{\pi}{4} \right) \right)^{2/3} \right) - \frac{3}{4} \log_e^2 \left( \frac{1}{2} \left( -1 + \sqrt{5} \right) \right) \right)$$

$$\pi \left( \frac{1}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 + \frac{1}{24} \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 + \left( \frac{\pi^2}{24} - \frac{1}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) 3 \right) \right) - 2 = -2 + \pi \left( \frac{\pi^2}{24} + \frac{1}{24} \pi^2 \left( 1 + 15 \left( -\cot \left( -\frac{\pi}{2} + \frac{\pi}{4} \right) \right)^{2/3} \right) + \frac{1}{24} \pi^2 \left( 1 + 81 \left( -\cot \left( -\frac{\pi}{2} + \frac{\pi}{4} \right) \right)^{2/3} \right) - \frac{3}{4} \left( \log(a) \log_a \left( \frac{1}{2} \left( -1 + \sqrt{5} \right) \right) \right)^2 \right)$$

$$\begin{aligned} \pi \left( \frac{1}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 + \frac{1}{24} \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 + \left( \frac{\pi^2}{24} - \frac{1}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) 3 \right) \right) - \\ 2 &= -2 + \pi \left( \frac{\pi^2}{24} - \frac{3}{4} \log^2 \left( \frac{1}{2} \left( -1 + \sqrt{5} \right) \right) + \\ & \frac{1}{24} \pi^2 \left( 1 + 15 \left( -i + \frac{2i}{1 + e^{(2i\pi)/4}} \right)^{2/3} \right) + \frac{1}{24} \pi^2 \left( 1 + 81 \left( -i + \frac{2i}{1 + e^{(2i\pi)/4}} \right)^{2/3} \right) \right) \end{aligned}$$

## Series representations:

$$\pi \left( \frac{1}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 + \frac{1}{24} \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 + \left( \frac{\pi^2}{24} - \frac{1}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) 3 \right) \right) - 2 = -2 + \frac{33 \pi^3}{8} - \frac{3}{4} \pi \left( \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{2} \right)^k \left( -3 + \sqrt{5} \right)^k}{k} \right)^2$$

$$\begin{aligned} \pi \left( \frac{1}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 + \frac{1}{24} \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 + \left( \frac{\pi^2}{24} - \frac{1}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) 3 \right) \right) - \\ 2 &= -2 + \pi \left( \frac{33 \pi^2}{8} + \frac{3}{4} \left( 2 \pi \left[ \frac{\arg(-1 + \sqrt{5} - 2x)}{2\pi} \right] - \\ i \left( \log(x) - \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{2} \right)^k \left( -1 + \sqrt{5} - 2x \right)^k x^{-k}}{k} \right) \right)^2 \right) &\text{for } x < 0 \end{aligned}$$

$$\begin{aligned} \pi \left( \frac{1}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 + \frac{1}{24} \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 + \left( \frac{\pi^2}{24} - \frac{1}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) 3 \right) \right) - \\ 2 &= -2 + \pi \left( \frac{33 \pi^2}{8} - \frac{3}{4} \left( 2 i \pi \left[ \frac{\arg \left( \frac{1}{2} \left( -1 + \sqrt{5} \right) - x \right)}{2 \pi} \right] + \log(x) - \right] \right) \\ &= \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{2} \right)^k \left( -1 + \sqrt{5} - 2 x \right)^k x^{-k}}{k} \right)^2}{2 \pi} \right) + \log(x) - \end{aligned}$$

## **Integral representation:**

$$\pi \left( \frac{1}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 + \frac{1}{24} \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 + \left( \frac{\pi^2}{24} - \frac{1}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right) 3 \right) \right) - 2 = -2 + \frac{33\pi^3}{8} - \frac{3}{4} \pi \left( \int_1^{\frac{1}{2} \left( -1 + \sqrt{5} \right)} \frac{1}{t} dt \right)^2$$

## Multiple-argument formula:

$$\pi \left( \frac{1}{24} \left( 1 + 81 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 + \frac{1}{24} \left( 1 + 15 \tan^{2/3} \left( \frac{\pi}{4} \right) \right) \pi^2 + \left( \frac{\pi^2}{24} - \frac{1}{4} \log^2 \left( \frac{1}{2} \left( \sqrt{5} - 1 \right) \right)^3 \right) \right) - 2 = -2 + \pi \left( \frac{33 \pi^2}{8} - \frac{3}{4} \left( -\log(2) + \log\left( -1 + \sqrt{5} \right) \right)^2 \right)$$

 $3*((([(Pi^2)/(24)*((1+81(tan(Pi/4)^(2/3))))]+[(Pi^2)/(24)*((1+15(tan(Pi/4)^(2/3))))] + [Pi^2/(24) -3/4(((ln((sqrt5-1)/2)))^2])))+13+3+golden ratio$ 

# **Input:** $3\left(\frac{\pi^2}{24}\left(1+81\tan^{2/3}\left(\frac{\pi}{4}\right)\right)+\frac{\pi^2}{24}\left(1+15\tan^{2/3}\left(\frac{\pi}{4}\right)\right)+\left(\frac{\pi^2}{24}-\frac{3}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)\right)\right)+13+3+\phi$

log(x) is the natural logarithm

 $\phi$  is the golden ratio

Exact result:  $\phi + 16 + 3\left(\frac{33\pi^2}{8} - \frac{3}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)\right)$ 

## **Decimal approximation:**

139.2333676059320203731633225475620597071904250567996244046...

139.233367... result practically equal to the rest mass of Pion meson 139.57 MeV

# Alternate forms: $QQ \pi^2 Q$

$$\phi + 16 + \frac{99\pi^{2}}{8} - \frac{9}{4} \operatorname{csch}^{-1}(2)^{2}$$

$$\phi + 16 + \frac{9}{8} \left( 11\pi^{2} - 2\log^{2}\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right) \right)$$

$$\frac{1}{8} \left( 132 + 4\sqrt{5} + 99\pi^{2} - 18\log^{2}\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right) \right)$$

 $\operatorname{csch}^{-1}(x)$  is the inverse hyperbolic cosecant function

## Alternative representations:

$$3\left(\frac{1}{24}\left(1+81\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^{2}+\frac{1}{24}\left(1+15\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^{2}+\left(\frac{\pi^{2}}{24}-\frac{1}{4}\log^{2}\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3\right)\right)+13+3+\phi=$$

$$16+\phi+3\left(\frac{\pi^{2}}{24}+\frac{1}{24}\pi^{2}\left(1+15\left(-\cot\left(-\frac{\pi}{2}+\frac{\pi}{4}\right)\right)^{2/3}\right)+\frac{1}{24}\pi^{2}\left(1+81\left(-\cot\left(-\frac{\pi}{2}+\frac{\pi}{4}\right)\right)^{2/3}\right)-\frac{3}{4}\log^{2}_{e}\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\right)$$

$$3\left(\frac{1}{24}\left(1+81\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^{2}+\frac{1}{24}\left(1+15\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^{2}+\left(\frac{\pi^{2}}{24}-\frac{1}{4}\log^{2}\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3\right)\right)+13+3+\phi=$$

$$16+\phi+3\left(\frac{\pi^{2}}{24}+\frac{1}{24}\pi^{2}\left(1+15\left(-\cot\left(-\frac{\pi}{2}+\frac{\pi}{4}\right)\right)^{2/3}\right)+\frac{1}{24}\pi^{2}\left(1+81\left(-\cot\left(-\frac{\pi}{2}+\frac{\pi}{4}\right)\right)^{2/3}\right)-\frac{3}{4}\left(\log(a)\log_{a}\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\right)^{2}\right)$$

$$\begin{split} 3\left(\frac{1}{24}\left(1+81\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^{2} + \frac{1}{24}\left(1+15\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^{2} + \\ & \left(\frac{\pi^{2}}{24} - \frac{1}{4}\log^{2}\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)3\right)\right) + 13 + 3 + \phi = \\ 16 + \phi + 3\left(\frac{\pi^{2}}{24} - \frac{3}{4}\log^{2}\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right) + \frac{1}{24}\pi^{2}\left(1+15\left(-i+\frac{2i}{1+e^{(2i\pi)/4}}\right)^{2/3}\right) + \\ & \frac{1}{24}\pi^{2}\left(1+81\left(-i+\frac{2i}{1+e^{(2i\pi)/4}}\right)^{2/3}\right) \right) \end{split}$$

Series representations:

$$3\left(\frac{1}{24}\left(1+81\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^{2}+\frac{1}{24}\left(1+15\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^{2}+\left(\frac{\pi^{2}}{24}-\frac{1}{4}\log^{2}\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3\right)\right)+13+3+\phi=16+\phi+\frac{99\pi^{2}}{8}-\frac{9}{4}\left(\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^{k}\left(-3+\sqrt{5}\right)^{k}}{k}\right)^{2}$$

$$\begin{split} 3\left(\frac{1}{24}\left(1+81\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^{2} + \frac{1}{24}\left(1+15\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^{2} + \\ & \left(\frac{\pi^{2}}{24} - \frac{1}{4}\log^{2}\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)3\right)\right) + 13 + 3 + \phi = \\ 16 + \phi + 3\left(\frac{33\pi^{2}}{8} + \frac{3}{4}\left(2\pi\left\lfloor\frac{\arg(-1+\sqrt{5}-2x)}{2\pi}\right\rfloor - \\ & i\left(\log(x) - \sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^{k}\left(-1+\sqrt{5}-2x\right)^{k}x^{-k}}{k}\right)\right)^{2}\right) \text{ for } x < 0 \end{split}$$

$$3\left(\frac{1}{24}\left(1+81\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^{2}+\frac{1}{24}\left(1+15\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^{2}+\left(\frac{\pi^{2}}{24}-\frac{1}{4}\log^{2}\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3\right)+13+3+\phi=$$

$$16+\phi+3\left(\frac{33\pi^{2}}{8}-\frac{3}{4}\left(2i\pi\left\lfloor\frac{\arg\left(\frac{1}{2}\left(-1+\sqrt{5}\right)-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^{k}\left(-1+\sqrt{5}-2x\right)^{k}x^{-k}}{k}\right)^{2}\right) \text{ for } x<0$$

Integral representation:

$$3\left(\frac{1}{24}\left(1+81\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^{2}+\frac{1}{24}\left(1+15\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^{2}+\left(\frac{\pi^{2}}{24}-\frac{1}{4}\log^{2}\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3\right)\right)+13+3+\phi=16+\phi+\frac{99\pi^{2}}{8}-\frac{9}{4}\left(\int_{1}^{\frac{1}{2}\left(-1+\sqrt{5}\right)}\frac{1}{t}\,dt\right)^{2}$$

## Multiple-argument formula:

$$3\left(\frac{1}{24}\left(1+81\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^{2}+\frac{1}{24}\left(1+15\tan^{2/3}\left(\frac{\pi}{4}\right)\right)\pi^{2}+\left(\frac{\pi^{2}}{24}-\frac{1}{4}\log^{2}\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3\right)+13+3+\phi=16+\phi+3\left(\frac{33\pi^{2}}{8}-\frac{3}{4}\left(-\log(2)+\log\left(-1+\sqrt{5}\right)\right)^{2}\right)$$

From:

# **Integrable Scalar Cosmologies II. Can they fit into Gauged Extended Supergavity or be encoded in N=1 superpotentials?** - P. Fre, A.S. Sorin and M. Trigiante - arXiv:1310.5340v1 [hep-th] 20 Oct 2013

We have that:

$$T_c \stackrel{t \to \infty}{\simeq} \frac{125e^{\frac{12t\nu}{5}}}{12\nu^5} \tag{5.65}$$

$$T_c \stackrel{t \to \infty}{\simeq} \frac{125 e^{\frac{12t\nu}{5}}}{12\nu^5}$$

 $125 e^{((12*0.25*x)/5) / ((12*0.25^5))} = y$ 

Input:  $125 \times \frac{e^{1/5} (12 \times 0.25 x)}{12 \times 0.25^5} = y$ 

**Result:**  $10\,666.7\,e^{0.6\,x} = y$ 

#### **Implicit plot:**

y 400 200 1000 x -4000-3000-2000-100900 -400

## Alternate form assuming x and y are real:

 $10\,666.7\,e^{0.6x} + 0 = y$ 

#### **Real solution:**

 $y \approx 10\,666.7 \times 2.71828^{0.6x}$ 

## Solution:

 $y = \frac{32\,000}{3} e^{(3\,x)/5}$ 

#### **Partial derivatives:**

 $\frac{\partial}{\partial r} (10\,666.7\,e^{0.6x}) = 6400.\,e^{0.6x}$  $\frac{\partial}{\partial y} \left( 10\,666.7\,e^{0.6\,x} \right) = 0$ 

## **Implicit derivatives:**

 $\frac{\partial x(y)}{\partial y} = \frac{26\,388\,279\,066\,624\,e^{-(1\,3510\,79\,888\,211\,149\,x)/2\,251\,799\,813\,685\,248}}{168\,884\,986\,926\,292\,625}$  $\frac{\partial y(x)}{\partial x} = \frac{1\,351\,079\,888\,211\,149\,y}{2\,251\,799\,813\,685\,248}$ 

#### Limit:

 $\lim_{x \to 0} 10\,666.7 \, e^{0.6x} = 0 \approx 0$ 

#### For

 $y \approx 10\,666.7 \times 2.71828^{0.6x}$ 

we obtain:

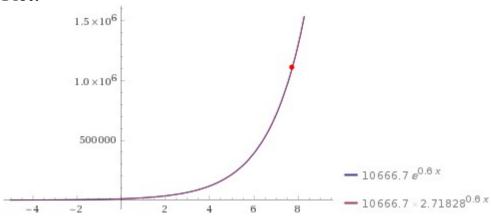
 $125 e^{((12*0.25*x)/5)} / ((12*0.25^{5})) = 10666.7 2.71828^{(0.6x)}$ 

Input interpretation:  $125 \times \frac{e^{1/5(12 \times 0.25x)}}{12 \times 0.25^5} = 10\,666.7 \times 2.71828^{0.6x}$ 

**Result:** 

 $10\,666.7\,e^{0.6\,x} = 10\,666.7 \times 2.71828^{0.6\,x}$ 

#### **Plot:**



#### **Alternate forms:**

 $e^{0.6x} = 1. \times 2.71828^{0.6x}$ 

 $10\,666.7\,e^{0.6x} = 10\,666.7\,e^{0.6x}$ 

#### Alternate form assuming x is positive:

 $e^{0.6x} = 0.999997 e^{0.6x}$ 

## Alternate form assuming x is real:

 $10\,666.7\,e^{0.6x} + 0 = 10\,666.7 \times 2.71828^{0.6x} + 0$ 

#### **Real solution:**

 $x \approx 7.74296$ 

## 7.74296

Solution:  $x \approx (2.47775 \times 10^6 i) (6.28319 n + (-3.125 \times 10^{-6} i)), \quad n \in \mathbb{Z}$ 

125\*e^((12\*0.25\*7.74296)/5) / ((12\*0.25^5))

 $\frac{125 \times \frac{e^{\frac{1}{5}(12 \times 0.25 \times 7.74296)}}{12 \times 0.25^{5}}}{12 \times 0.25^{5}}$ 

**Result:** 1.11087... × 10<sup>6</sup>

 $1.11087...*10^{6}$ 

#### Alternative representation:

 $\frac{12 \times 0.25 \times 7.74296}{5}$  (z) for z = 1125  $e^{(12 \times 0.25 \times 7.74296)/5}$ 125 exp  $12 \times 0.25^5$  $12 \times 0.25^{5}$ 

#### Series representations:

$$\frac{125 \ e^{(12 \times 0.25 \times 7.74296)/5}}{12 \times 0.25^5} = 10\ 666.7 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{4.64578}$$

$$\frac{125 \ e^{(12 \times 0.25 \times 7.74296)/5}}{12 \times 0.25^5} = 426.099 \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{4.64578}$$

$$\frac{125 \ e^{(12 \times 0.25 \times 7.74296)/5}}{12 \times 0.25^5} = 10\ 666.7 \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{4.64578}$$

$$T_c(t) \equiv \int_0^t dx \, \exp\left[\mathbf{B}(x,\nu)\right] = \frac{25t}{\nu^4} - \frac{125e^{\frac{6t\nu}{5}}}{3\nu^5} + \frac{125e^{\frac{12t\nu}{5}}}{12\nu^5} + \frac{125}{4\nu^5}$$

For t = 7.74296 and v = 1/4 = 0.25, we obtain:

 $(25*7.74296)/0.25^4 - ((125*e^{(6*7.74296*0.25)/5)))/(3*0.25^5) +$  $((125*e^{((12*7.74296*0.25)/5))})/(12*0.25^{5}) + 125/(4*0.25^{5})))$ 

#### **Input interpretation:**

$25 \times 7.74296$	125 $e^{1/5(6 \times 7.74296 \times 0.25)}$		125 e <sup>1/5 (12×7.74296×0.25)</sup>	125
0.254	$3 \times 0.25^{5}$	+	$12 \times 0.25^{5}$	$+ \frac{1}{4 \times 0.25^5}$

**Result:** 

 $7.57008...\times10^5$ 

 $7.57008...*10^{5}$ 

## Alternative representation:

$$\frac{25 \times 7.74296}{0.25^4} - \frac{125 \ e^{(6 \times 7.74296 \times 0.25)/5}}{3 \times 0.25^5} + \frac{125 \ e^{(12 \times 7.74296 \times 0.25)/5}}{12 \times 0.25^5} + \frac{125}{4 \times 0.25^5} = \frac{25 \times 7.74296}{0.25^4} - \frac{125 \ \exp^{5} \ (z)}{3 \times 0.25^5} + \frac{125 \ \exp^{5} \ (z)}{3 \times 0.25^5} + \frac{125 \ \exp^{5} \ (z)}{12 \times 0.25^5} +$$

## Series representations:

$$\frac{25 \times 7.74296}{0.25^4} - \frac{125 \ e^{(6 \times 7.74296 \times 0.25)/5}}{3 \times 0.25^5} + \frac{125 \ e^{(12 \times 7.74296 \times 0.25)/5}}{12 \times 0.25^5} + \frac{125}{4 \times 0.25^5} = -42\ 666.7 \left( -1.91144 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{2.32289} - 0.25 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{4.64578} \right)$$

$$\frac{25 \times 7.74296}{0.25^4} - \frac{125 \ e^{(6 \times 7.74296 \times 0.25)/5}}{3 \times 0.25^5} + \frac{125 \ e^{(12 \times 7.74296 \times 0.25)/5}}{12 \times 0.25^5} + \frac{125}{4 \times 0.25^5} = -8527.66 \left(-9.56358 + \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{2.32289} - 0.0499667 \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{4.64578}\right)$$

$$\frac{25 \times 7.74296}{0.25^4} - \frac{125 \ e^{(6 \times 7.74296 \times 0.25)/5}}{3 \times 0.25^5} + \frac{125 \ e^{(12 \times 7.74296 \times 0.25)/5}}{12 \times 0.25^5} + \frac{125}{4 \times 0.25^5} = -42\ 666.7 \left( -1.91144 + \left( \sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!} \right)^{2.32289} - 0.25 \left( \sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!} \right)^{4.64578} \right)$$

To work out the behavior at very early times it is more complicated, yet we can predict it by inspecting the behavior of the energy density and of the pressure. Inserting the form of the solution and of the potential in eq.(1.5) we obtain the parametric time behavior of the energy density and of the pressure <sup>8</sup>:

$$\rho = \frac{3\nu^8 \left(-4\nu^2 + 2e^{\frac{6t\nu}{5}} \left(2\nu^2 + 5\right) + e^{\frac{12t\nu}{5}} \left(3\nu^2 - 5\right) - 5\right)}{15625 \left(-1 + e^{\frac{6t\nu}{5}}\right)^6}$$
(5.68)

$$p = \frac{3\nu^8 \left(4\nu^2 - 2e^{\frac{6t\nu}{5}} \left(2\nu^2 + 5\right) + e^{\frac{12t\nu}{5}} \left(3\nu^2 + 5\right) + 5\right)}{15625 \left(-1 + e^{\frac{6t\nu}{5}}\right)^6}$$
(5.69)

We have that:

$$\rho = \frac{3\nu^8 \left(-4\nu^2 + 2e^{\frac{6t\nu}{5}} \left(2\nu^2 + 5\right) + e^{\frac{12t\nu}{5}} \left(3\nu^2 - 5\right) - 5\right)}{15625 \left(-1 + e^{\frac{6t\nu}{5}}\right)^6}$$
(5.68)

$$p = \frac{3\nu^8 \left(4\nu^2 - 2e^{\frac{6t\nu}{5}} \left(2\nu^2 + 5\right) + e^{\frac{12t\nu}{5}} \left(3\nu^2 + 5\right) + 5\right)}{15625 \left(-1 + e^{\frac{6t\nu}{5}}\right)^6}$$
(5.69)

For t = 7.74296 and v = 1/4 = 0.25, we obtain:

## Input interpretation:

$$\begin{cases} (-4 \times 0.25^{\circ} + 2 e^{1/5} (6 \times 7.74296 \times 0.25) (2 \times 0.25^{2} + 5) + e^{1/5} (12 \times 7.74296 \times 0.25) (3 \times 0.25^{2} - 5) - 5) \\ 5) \times \frac{1}{15625 (-1 + e^{1/5} (6 \times 7.74296 \times 0.25))^{6}} \end{cases}$$

## **Result:**

 $-1.93510...\times 10^{-12}$ 

 $-1.93510...*10^{-12} = \rho$ 

## Alternative representation:

$$\begin{pmatrix} (3 \times 0.25^8) \left( -4 \times 0.25^2 + 2 e^{(6 \times 7.74296 \times 0.25)/5} \left( 2 \times 0.25^2 + 5 \right) + e^{(12 \times 7.74296 \times 0.25)/5} \\ (3 \times 0.25^2 - 5) - 5 \end{pmatrix} \end{pmatrix} / \begin{pmatrix} (15 \ 625 \left( -1 + e^{(6 \times 7.74296 \times 0.25)/5} \right)^6) = \\ \begin{pmatrix} (3 \times 0.25^8) \left( -4 \times 0.25^2 + 2 \exp^{5} (z) \left( 2 \times 0.25^2 + 5 \right) + \\ \exp^{5} (z) \left( 2 \times 0.25^2 + 5 \right) + \\ \exp^{5} (z) \left( 3 \times 0.25^2 - 5 \right) - 5 \end{pmatrix} \end{pmatrix} \end{pmatrix} / \\ \begin{pmatrix} (15 \ 625 \left( -1 + \exp^{\frac{6 \times 7.74296 \times 0.25}{5}} (z) \right)^6 \end{pmatrix} \text{ for } z = 1 \end{cases}$$

## Series representations:

$$\begin{split} & \left( (3 \times 0.25^8) \left( -4 \times 0.25^2 + 2 \ e^{(6 \times 7.74296 \times 0.25)/5} \ (2 \times 0.25^2 + 5) + \\ & e^{(12 \times 7.74296 \times 0.25)/5} \ (3 \times 0.25^2 - 5) - 5) \right) \right/ \\ & \left( 15\ 625\ \left( -1 + e^{(6 \times 7.74296 \times 0.25)/5} \right)^6 \right) = \\ & \frac{3.00293 \times 10^{-8} \ \left( -0.512195 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{2.32289} - 0.469512 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{4.64578} \right) }{\left( -1 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{2.32289} \right)^6} \end{split}$$

$$\begin{split} & \left( (3 \times 0.25^8) \left( -4 \times 0.25^2 + 2 \ e^{(6 \times 7.74296 \times 0.25)/5} \ (2 \times 0.25^2 + 5) + \\ & e^{(12 \times 7.74296 \times 0.25)/5} \ (3 \times 0.25^2 - 5) - 5) \right) \right/ \\ & \left( 15\ 625\ \left( -1 + e^{(6 \times 7.74296 \times 0.25)/5} \right)^6 \right) = \\ & 0.0000941543\ \left( -2.56268 + \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^{2.32289} - 0.09384\ \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^{4.64578} \right) \\ & \left( -5.00333 + \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^{2.32289} \right)^6 \end{split}$$

$$\begin{split} & \left( (3 \times 0.25^8) \left( -4 \times 0.25^2 + 2 \ e^{(6 \times 7.74296 \times 0.25)/5} \left( 2 \times 0.25^2 + 5 \right) + \\ & e^{(12 \times 7.74296 \times 0.25)/5} \left( 3 \times 0.25^2 - 5 \right) - 5 \right) \right) / \\ & \left( 15\ 625\ \left( -1 + e^{(6 \times 7.74296 \times 0.25)/5} \right)^6 \right) = \\ & \frac{3.00293 \times 10^{-8} \left( -0.512195 + \left( \sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!} \right)^{2.32289} - 0.469512 \left( \sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!} \right)^{4.64578} \right) }{\left( -1 + \left( \sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!} \right)^{2.32289} \right)^6} \end{split}$$

$$p = \frac{3\nu^8 \left(4\nu^2 - 2e^{\frac{6t\nu}{5}} \left(2\nu^2 + 5\right) + e^{\frac{12t\nu}{5}} \left(3\nu^2 + 5\right) + 5\right)}{15625 \left(-1 + e^{\frac{6t\nu}{5}}\right)^6}$$

3\*0.25^8((((4\*0.25^2- $2e^{((6*7.74296*0.25)/5)*(2*0.25^{2}+5)+e^{((12*7.74296*0.25)/5)*(3*0.25^{2}+5)+5))}$ 

## Input interpretation:

)) \* ((1/((((15625((-1+e^((6\*7.74296\*0.25)/5)))^6)))))

$$\begin{array}{l} 3 \times 0.25^8 \\ \left( \left( 4 \times 0.25^2 - 2 \ e^{1/5 \ (6 \times 7.74296 \times 0.25)} \left( 2 \times 0.25^2 + 5 \right) + e^{1/5 \ (12 \times 7.74296 \times 0.25)} \left( 3 \times 0.25^2 + 5 \right) + 5 \right) \right) \\ 5 \\ \times \frac{1}{15 \ 625 \ \left( -1 + e^{1/5 \ (6 \times 7.74296 \times 0.25)} \right)^6} \end{array} \right)$$

## **Result:**

 $2.12317... \times 10^{-12}$ 

 $2.12317...*10^{-12} = p$ 

## Alternative representation:

Anternative representation.  

$$((3 \times 0.25^8) (4 \times 0.25^2 - 2 e^{(6 \times 7.74296 \times 0.25)/5} (2 \times 0.25^2 + 5) + e^{(12 \times 7.74296 \times 0.25)/5} (3 \times 0.25^2 + 5) + 5)) / (15 625 (-1 + e^{(6 \times 7.74296 \times 0.25)/5})^6) = ((3 \times 0.25^8) (4 \times 0.25^2 - 2 \exp^{-\frac{6 \times 7.74296 \times 0.25}{5}} (z) (2 \times 0.25^2 + 5) + \frac{12 \times 7.74296 \times 0.25}{5} (z) (3 \times 0.25^2 + 5) + 5)) / (15 625 (-1 + \exp^{-\frac{6 \times 7.74296 \times 0.25}{5}} (z) (3 \times 0.25^2 + 5) + 5))) / (15 625 (-1 + \exp^{-\frac{6 \times 7.74296 \times 0.25}{5}} (z) (3 \times 0.25^2 + 5) + 5))) / (15 625 (-1 + \exp^{-\frac{6 \times 7.74296 \times 0.25}{5}} (z) (3 \times 0.25^2 + 5) + 5))) / (15 625 (-1 + \exp^{-\frac{6 \times 7.74296 \times 0.25}{5}} (z) (3 \times 0.25^2 + 5) + 5))) / (15 625 (-1 + \exp^{-\frac{6 \times 7.74296 \times 0.25}{5}} (z) (3 \times 0.25^2 + 5) + 5))) / (15 625 (-1 + \exp^{-\frac{6 \times 7.74296 \times 0.25}{5}} (z) (3 \times 0.25^2 + 5) + 5))) / (15 625 (-1 + \exp^{-\frac{6 \times 7.74296 \times 0.25}{5}} (z) (3 \times 0.25^2 + 5) + 5))) / (15 625 (-1 + \exp^{-\frac{6 \times 7.74296 \times 0.25}{5}} (z) (3 \times 0.25^2 + 5) + 5))) / (15 625 (-1 + \exp^{-\frac{6 \times 7.74296 \times 0.25}{5}} (z) (3 \times 0.25^2 + 5) + 5))) / (15 625 (-1 + \exp^{-\frac{6 \times 7.74296 \times 0.25}{5}} (z) (3 \times 0.25^2 + 5) + 5))) / (15 625 (-1 + \exp^{-\frac{6 \times 7.74296 \times 0.25}{5}} (z) (3 \times 0.25^2 + 5) + 5))) / (15 625 (-1 + \exp^{-\frac{6 \times 7.74296 \times 0.25}{5}} (z) (3 \times 0.25^2 + 5) + 5))) / (15 625 (-1 + \exp^{-\frac{6 \times 7.74296 \times 0.25}{5}} (z) (2 \times 0.25^2 + 5) + 5))) / (15 625 (-1 + \exp^{-\frac{6 \times 7.74296 \times 0.25}{5}} (z) (2 \times 0.25^2 + 5) + 5))) / (15 625 (-1 + \exp^{-\frac{6 \times 7.74296 \times 0.25}{5} (z) (2 \times 0.25^2 + 5) + 5))) / (15 625 (-1 + \exp^{-\frac{6 \times 7.74296 \times 0.25}{5} (z) (2 \times 0.25^2 + 5) + 5))) / (15 625 (-1 + \exp^{-\frac{6 \times 7.74296 \times 0.25}{5} (z) (2 \times 0.25^2 + 5) + 5))) / (15 625 (-1 + \exp^{-\frac{6 \times 7.74296 \times 0.25}{5} (z) (2 \times 0.25^2 + 5) + 5))) / (15 625 (-1 + \exp^{-\frac{6 \times 7.74296 \times 0.25}{5} (z) (2 \times 0.25^2 + 5) + 5))) / (15 625 (-1 + \exp^{-\frac{6 \times 7.74296 \times 0.25}{5} (z) (2 \times 0.25^2 + 5) + 5))) / (15 625 (-1 + \exp^{-\frac{6 \times 7.74296 \times 0.25}{5} (z) (2 \times 0.25^2 + 5) + 5)))) / (15 625 (-1 + \exp^{-\frac{6 \times 7.74296 \times 0.25}{5} (z) (2 \times 0.25^2 + 5))))) / (15 625 (-1 + \exp^{-\frac{6 \times 7.74296 \times 0.25}{5} (z) (2 \times 0.25^2 + 5))))))) / (15 625 (-1$$

## Series representations:

$$\frac{\left((3 \times 0.25^{8})\left(4 \times 0.25^{2} - 2 e^{(6 \times 7.74296 \times 0.25)/5} \left(2 \times 0.25^{2} + 5\right) + e^{(12 \times 7.74296 \times 0.25)/5} \left(3 \times 0.25^{2} + 5\right) + 5\right)\right)}{\left(15625\left(-1 + e^{(6 \times 7.74296 \times 0.25)/5}\right)^{6}\right)} = \frac{3.00293 \times 10^{-8} \left(-0.512195 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2.32289} - 0.506098 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{4.64578}\right)}{\left(-1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2.32289}\right)^{6}}$$

$$\begin{split} & \left( (3 \times 0.25^8) \left( 4 \times 0.25^2 - 2 \; e^{(6 \times 7.74296 \times 0.25)/5} \left( 2 \times 0.25^2 + 5 \right) + \right. \\ & \left. e^{(12 \times 7.74296 \times 0.25)/5} \left( 3 \times 0.25^2 + 5 \right) + 5 \right) \right) \right/ \\ & \left. \left( 15\; 625 \left( -1 + e^{(6 \times 7.74296 \times 0.25)/5} \right)^6 \right) = \right. \\ & \left. - \frac{0.0000941543 \left( -2.56268 + \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^{2.32289} - 0.101152 \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^{4.64578} \right) \right. \\ & \left. \left( -5.00333 + \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^{2.32289} \right)^6 \right. \end{split}$$

$$\begin{split} & \left( \left( 3 \times 0.25^8 \right) \left( 4 \times 0.25^2 - 2 \; e^{\left( 6 \times 7.74296 \times 0.25 \right)/5} \left( 2 \times 0.25^2 + 5 \right) + \right. \\ & \left. e^{\left( 12 \times 7.74296 \times 0.25 \right)/5} \left( 3 \times 0.25^2 + 5 \right) + 5 \right) \right) \right/ \\ & \left( 15\; 625 \left( -1 + e^{\left( 6 \times 7.74296 \times 0.25 \right)/5} \right)^6 \right) = \\ & \left. - \frac{3.00293 \times 10^{-8} \left( -0.512195 + \left( \sum_{k=0}^{\infty} \frac{\left( -1 + k \right)^2}{k!} \right)^{2.32289} - 0.506098 \left( \sum_{k=0}^{\infty} \frac{\left( -1 + k \right)^2}{k!} \right)^{4.64578} \right) \\ & \left( -1 + \left( \sum_{k=0}^{\infty} \frac{\left( -1 + k \right)^2}{k!} \right)^{2.32289} \right)^6 \end{split}$$

From the ratio between p and  $\rho$ , after some calculations, we obtain:

1/(2.123169628766854516 × 10^-12 / 1.935101496001104582 × 10^-12)

# Input interpretation:

 $\frac{2.123169628766854516 \times 10^{-12}}{1.935101496001104582 \times 10^{-12}}$ 

#### **Result:**

0.911421051706084989513479994595973573934537306508437006585...

#### 0.9114210517...

We know that  $\alpha$ ' is the Regge slope (string tension). With regard the Omega mesons, a value is also:

 $\omega$  6  $m_{u/d} = 0 - 60$  0.910 - 0.918

(see ref. **Rotating strings confronting PDG mesons** - *Jacob Sonnenschein and Dorin Weissman* - arXiv:1402.5603v1 [hep-ph] 23 Feb 2014)

(((1/(2.123169628766854516 × 10<sup>-12</sup> / 1.935101496001104582 × 10<sup>-12</sup>))))<sup>1</sup>/128

#### Input interpretation:

 $\stackrel{128}{\sqrt{\frac{2.123169628766854516\times10^{-12}}{1.935101496001104582\times10^{-12}}}}}$ 

#### **Result:**

0.999275650731654233824...

0.9992756507... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{\sqrt{9^{5}\sqrt{5^{3}}} - 1}} \approx 0.9991104684$$

$$\frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}$$

and to the dilaton value **0**. **989117352243** =  $\phi$ 

log base 0.99927565(((1/(2.123169628766854516 × 10^-12 / 1.935101496001104582 × 10^-12))))-Pi+1/golden ratio

#### **Input interpretation:**

 $\log_{0.99927565} \left( \frac{1}{\frac{2.123169628766854516 \times 10^{-12}}{1.935101496001104582 \times 10^{-12}}} \right) - \pi + \frac{1}{\phi}$ 

 $\log_b(x)$  is the base- b logarithm  $\phi$  is the golden ratio

#### **Result:**

125.476...

125.476... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

log base 0.99927565(((1/(2.123169628766854516 × 10^-12 / 1.935101496001104582 × 10^-12))))+11+1/golden ratio

#### Input interpretation:

 $\log_{0.99927565} \left( \frac{1}{\frac{2.123169628766854516 \times 10^{-12}}{1.935101496001104582 \times 10^{-12}}} \right) + 11 + \frac{1}{\phi}$ 

 $\log_b(x)$  is the base– b logarithm  $\phi$  is the golden ratio

## **Result:**

139.618...

139.618... result practically equal to the rest mass of Pion meson 139.57 MeV

Appendix

## **DILATON VALUE CALCULATIONS 0.989117352243**

from:

Modular equations and approximations to  $\pi$  - Srinivasa Ramanujan Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

5. Since  $G_n$  and  $g_n$  can be expressed as roots of algebraical equations with rational coefficients, the same is true of  $G_n^{24}$  or  $g_n^{24}$ . So let us suppose that

$$1 = ag_n^{-24} - bg_n^{-48} + \cdots,$$

or

$$g_n^{24} = a - bg_n^{-24} + \cdots$$

But we know that

$$\begin{split} 64e^{-\pi\sqrt{n}}g_n^{24} &= 1-24e^{-\pi\sqrt{n}}+276e^{-2\pi\sqrt{n}}-\cdots,\\ 64g_n^{24} &= e^{\pi\sqrt{n}}-24+276e^{-\pi\sqrt{n}}-\cdots,\\ 64a-64bg_n^{-24}+\cdots &= e^{\pi\sqrt{n}}-24+276e^{-\pi\sqrt{n}}-\cdots,\\ 64a-4096be^{-\pi\sqrt{n}}+\cdots &= e^{\pi\sqrt{n}}-24+276e^{-\pi\sqrt{n}}-\cdots, \end{split}$$

that is

$$e^{\pi\sqrt{n}} = (64a + 24) - (4096b + 276)e^{-\pi\sqrt{n}} + \cdots$$
(13)

Similarly, if

$$1 = aG_n^{-24} - bG_n^{-48} + \cdots,$$

then

$$e^{\pi\sqrt{n}} = (64a - 24) - (4096b + 276)e^{-\pi\sqrt{n}} + \cdots$$
(14)

From (13) and (14) we can find whether  $e^{\pi\sqrt{n}}$  is very nearly an integer for given values of n, and ascertain also the number of 9's or 0's in the decimal part. But if  $G_n$  and  $g_n$  be simple quadratic surds we may work independently as follows. We have, for example,

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$64g_{22}^{24} = e^{\pi\sqrt{22}} \quad 24 + 276e^{-\pi\sqrt{22}} \cdots,$$
  

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots,$$

so that

$$64(g_{22}^{24}+g_{22}^{-24})=e^{\pi\sqrt{22}}-24+4372e^{-\pi\sqrt{22}}+\cdots=64\{(1+\sqrt{2})^{12}+(1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots$$

Again

$$G_{37} = (6 + \sqrt{37})^{\frac{1}{4}},$$

$$\begin{array}{rcl} 64G_{37}^{24} & = & e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} & = & 4096e^{-\pi\sqrt{37}} - \cdots, \end{array}$$

so that

$$64(G_{37}^{24}+G_{37}^{24})=e^{\pi\sqrt{37}}+24+4372e^{-\pi\sqrt{37}}-\cdots=64\{(6+\sqrt{37})^6+(6-\sqrt{37})^6\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978...$$

Similarly, from

$$g_{58} - \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} \quad 24 + 4372e^{-\pi\sqrt{58}} + \dots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.09999982...$$

From:

## An Update on Brane Supersymmetry Breaking

J. Mourad and A. Sagnotti - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

Now, we have that:

From the following vacuum equations:

$$T e^{\gamma_E \phi} = -\frac{\beta_E^{(p)} h^2}{\gamma_E} e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

$$16 \, k' e^{-2C} = \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)} \phi}}{(7-p)}$$

$$(A')^2 - k e^{-2A} + \frac{h^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

We have obtained, from the results almost equals of the equations, putting

4096  $e^{-\pi\sqrt{18}}$  instead of

$$e^{-2(8-p)C+2\beta_E^{(p)}\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning p, C,  $\beta_E$  and  $\phi$  correspond to the exponents of e (i.e. of exp). Thence we obtain for p = 5 and  $\beta_E = 1/2$ :

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

Therefore with respect to the exponential of the vacuum equation, the Ramanujan's exponential has a coefficient of 4096 which is equal to  $64^2$ , while  $-6C+\phi$  is equal to  $\pi\sqrt{18}$ . From this it follows that it is possible to establish mathematically, the dilaton value.

phi = -Pi\*sqrt(18) + 6C, for C = 1, we obtain:

exp((-Pi\*sqrt(18))

Input:  $\exp(-\pi\sqrt{18})$ 

## **Exact result:**

e<sup>-3√2</sup>л

#### **Decimal approximation:**

 $1.6272016226072509292942156739117979541838581136954016...\times 10^{-6}$ 

1.6272016... \* 10<sup>-6</sup>

Now:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016... * 10^{-6}$$

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016... * 10^{-6}$$

$$0.000244140625 e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016... * 10^{-6}$$

$$\ln(e^{-\pi\sqrt{18}}) = -13.328648814475 = -\pi\sqrt{18}$$

(1.6272016\* 10^-6) \*1/ (0.000244140625)

# Input interpretation: 1.6272016 1

 $\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$ 

#### **Result:**

0.0066650177536 0.006665017...

 $0.000244140625 \ e^{-6C+\phi} = e^{-\pi\sqrt{18}}$ 

Dividing both sides by 0.000244140625, we obtain:

 $\frac{0.000244140625}{0.000244140625}e^{-6C+\phi} = \frac{1}{0.000244140625}e^{-\pi\sqrt{18}}$ 

 $e^{-6C+\phi} = 0.0066650177536$ 

((((exp((-Pi\*sqrt(18))))))\*1/0.000244140625

## Input interpretation:

 $\exp(-\pi\sqrt{18}) \times \frac{1}{0.000244140625}$ 

#### **Result:**

0.00666501785...

0.00666501785...

 $e^{-6C+\phi} = 0.0066650177536$ 

 $\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625} =$ 

 $e^{-\pi\sqrt{18}}\times \frac{1}{0.000244140625}$ 

= 0.00666501785...

ln(0.00666501784619)

## Input interpretation:

log(0.00666501784619)

#### **Result:**

-5.010882647757...

-5.010882647757...

Now:

 $-6C + \phi = -5.010882647757 \dots$ 

For C = 1, we obtain:

 $\phi = -5.010882647757 + 6 = 0.989117352243 = \phi$ 

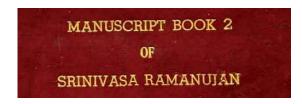
## Conclusions

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants  $\pi$ ,  $\phi$ ,  $1/\phi$ , the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

References

Axial Symmetric Kahler manifolds, the D-map of Inflaton Potentials and the Picard-Fuchs Equation - *Pietro Fre, Alexander S. Sorin* – arXiv:1310.5278v2 [hep-th] 26 Oct 2013

**On a Polya functional for rhombi, isosceles triangles, and thinning convex sets.** *M. van den Berg, V. Ferone, C. Nitsch, C. Trombetti* - arXiv:1811.04503v2 [math.AP] 21 May 2019



**Integrable Scalar Cosmologies II. Can they fit into Gauged Extended Supergavity or be encoded in N=1 superpotentials?** - P. Fre, A.S. Sorin and M. Trigiante - arXiv:1310.5340v1 [hep-th] 20 Oct 2013