# On some Ramanujan equations: mathematical connections with $\phi$, Particle Physics parameters and various expressions regarding Anti-de-Sitter charged black holes in f(T) gravity. V

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## Abstract

*In this paper we have described some Ramanujan equations and obtained several mathematical connections with ϕ, Particle Physics parameters and various expressions inherent Anti-de-Sitter charged black holes in f(T) gravity*

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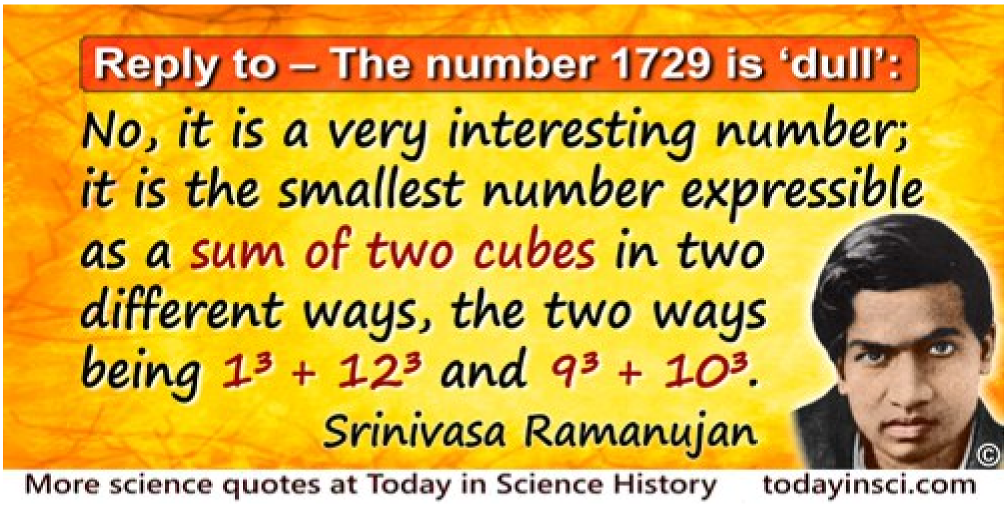
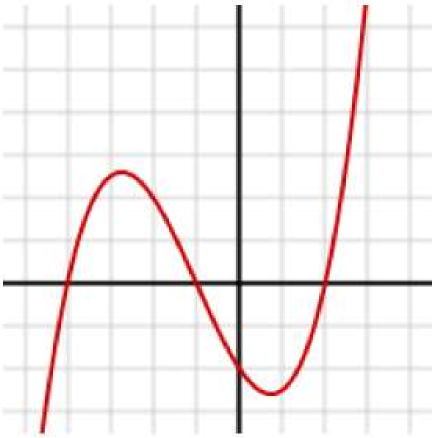
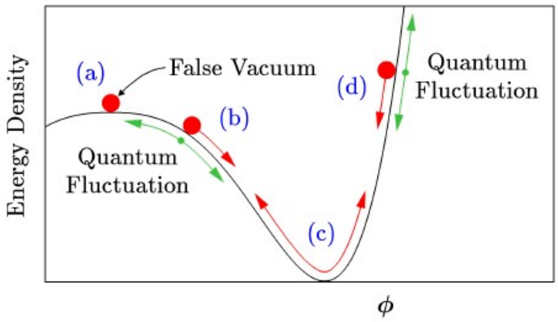



https://todayinsci.com/R/Ramanujan_Srinivasa/RamanujanSrinivasa-Quotations.htm





From:

**D-dimensional charged Anti-de-Sitter black holes in f(T) gravity**
*A. M. Awad1,2, S. Capozziello4,5,6 and G. G. L. Nashed* - arXiv:1706.01773v3 [gr-qc] 18 Jul 2017

We have that:

where $V' = \frac{dV}{dr}$ and as before we set $\Lambda = \frac{1}{24\beta}$. The general $D$-dimensional solution of the above differential equations takes the form

$$N(r) = \frac{r^2(D-3)^4{c_2}^4}{(D-1)(D-2)(2D-5)^2{c_3}^2} + \frac{c_1}{r^{D-3}} + \frac{3(D-3){c_2}^2}{(D-2)r^{2(D-3)}} + \frac{2(D-3)c_2c_3}{(D-2)r^{3D-8}},$$
$$N_1(r) = \frac{1}{f(r)N(r)}, \qquad \text{where} \qquad f(r) = -\frac{(2D-5)^2{c_3}^2\left[1+\frac{(2D-5)c_3}{c_2(D-3)r^{D-2}}\right]^2}{6\beta(D-3)^4{c_2}^4},$$
$$V(r) = \frac{c_2}{r^{D-3}} + \frac{c_3}{r^{2D-5}}. \tag{21}$$

To get an asymptotically AdS or dS solution we have to set

$${c_3}^2 = \frac{-6(D-3)^4{c_2}^4\beta}{(2D-5)^2}, \tag{22}$$

otherwise the solution have no clear asymptotic behavior. As a result, the monopole momentum is related to the quadrupole momentum of the solution. In this case, one gets

$$N(r) = \frac{r^2}{6(D-1)(D-2)\,|\beta|} - \frac{m}{r^{D-3}} + \frac{3(D-3)q^2}{(D-2)r^{2(D-3)}} + \frac{2\sqrt{6\,|\beta|}(D-3)^3q^3}{(2D-5)(D-2)r^{3D-8}},$$
$$N_1(r) = \frac{1}{f(r)N(r)}, \qquad f(r) = \left[1+\frac{(D-3)q\sqrt{6\,|\beta|}}{r^{D-2}}\right]^2, \qquad V(r) = \frac{q}{r^{D-3}} + \frac{(D-3)^2q^2\sqrt{6\,|\beta|}}{(2D-5)r^{2D-5}}, \tag{23}$$

where we set $c_1 = -m$, and $c_2 = q$, which is the monopole momentum. The quadrupole moment is $Q = \frac{(D-3)^2q^2\sqrt{6|\beta|}}{(2D-5)}$. As one can notice Eq. (22) tells us that $\beta$ have a negative value otherwise we get an unphysical solution.

For: $c_1 = -1$; $c_2 = 1/2$ ; $q = 1/2$ ; $D = 5$; $\beta = -2$; ${c_3}^2 = 0.48$

From

$${c_3}^2 = \frac{-6(D-3)^4{c_2}^4\beta}{(2D-5)^2},$$

We obtain:

((-6(5-3)^4*(1/2)^4*(-2))) / ((2*5-5)^2)

**Input:**

$$\frac{-6\,(5-3)^4\left(\frac{1}{2}\right)^4\times(-2)}{(2\times5-5)^2}$$

**Exact result:**

$$\frac{12}{25}$$

**Decimal form:**

0.48

0.48

From:

$$N(r)=\frac{r^2(D-3)^4{c_2}^4}{(D-1)(D-2)(2D-5)^2{c_3}^2}+\frac{c_1}{r^{D-3}}+\frac{3(D-3){c_2}^2}{(D-2)r^{2(D-3)}}+\frac{2(D-3)c_2c_3}{(D-2)r^{3D-8}},$$

For: $c_1 = -1$; $c_2 = 1/2$ ; $q = 1/2$ ; $D = 5$; $\beta = -2$; $c_3^2 = 0.48$; $r = 5$

((25*(2)^4*(1/2)^4))/(4*3*5^2*0.48) -1/5^2 + (3*2*1/4)/(3*5^4) + (2*2*1/2*0.48^0.5)/(3*5^7)

**Input:**

$$\frac{25\times2^4\left(\frac{1}{2}\right)^4}{4\times3\times5^2\times0.48}-\frac{1}{5^2}+\frac{3\times2\times\frac{1}{4}}{3\times5^4}+\frac{2\times2\times\frac{1}{2}\sqrt{0.48}}{3\times5^7}$$

**Result:**

0.134417023177867612878939684794623451155456942553044117210...

0.134417023…

[(1/(((((25*(2)^4*(1/2)^4))/(4*3*5^2*0.48) -1/5^2 + (3*2*1/4)/(3*5^4) + (2*2*1/2*0.48^0.5)/(3*5^7))))))]^1/4-(29+4)/10^3

**Input:**

$$\sqrt[4]{\frac{1}{\frac{25\times2^4\left(\frac{1}{2}\right)^4}{4\times3\times5^2\times0.48}-\frac{1}{5^2}+\frac{3\times2\times\frac{1}{4}}{3\times5^4}+\frac{2\times2\times\frac{1}{2}\sqrt{0.48}}{3\times5^7}}}-\frac{29+4}{10^3}$$

**Result:**

1.618529865570538548793547259272875424813140591278920886771...

1.61852986557.... result that is a very good approximation to the value of the golden ratio 1.618033988749...

From

$$f(r) = -\frac{(2D-5)^2 {c_3}^2 \left[1 + \frac{(2D-5)c_3}{c_2(D-3)r^{D-2}}\right]^2}{6\beta(D-3)^4 {c_2}^4},$$

-((((2*5-5)^2*0.48(1+(5*0.48^0.5)/(1/2*2*5^3))^2))) / ((-12(2)^4*(1/2)^4))

**Input:**

$$-\frac{\left((2\times5-5)^2\times0.48\left(1+\frac{5\sqrt{0.48}}{\frac{1}{2}\times2\times5^3}\right)^2\right)}{-12\times2^4\left(\frac{1}{2}\right)^4}$$

**Result:**

1.05619...

1.05619…

From:

$$N_1(r) = \frac{1}{f(r)N(r)},$$

1/-[((((((((2*5-5)^2*0.48(1+(5*0.48^0.5)/(1/2*2*5^3))^2)))/((-12(2)^4*(1/2)^4)))))) * ((((((25*(2)^4*(1/2)^4))/(4*3*5^2*0.48) -1/5^2 + (3*2*1/4)/(3*5^4) + (2*2*1/2*0.48^0.5)/(3*5^7)))))]

**Input:**

$$-\frac{1}{-\frac{(2\times5-5)^2\times0.48\left(1+\frac{5\sqrt{0.48}}{\frac{1}{2}\times2\times5^3}\right)^2}{12\times2^4\left(\frac{1}{2}\right)^4}\left(\frac{25\times2^4\left(\frac{1}{2}\right)^4}{4\times3\times5^2\times0.48}-\frac{1}{5^2}+\frac{3\times2\times\frac{1}{4}}{3\times5^4}+\frac{2\times2\times\frac{1}{2}\sqrt{0.48}}{3\times5^7}\right)}$$

**Result:**

7.0437216368276714732785572188479888905786479156627894069 19...

7.04372163682...

[1/-[((((((((2*5-5)^2*0.48(1+(5*0.48^0.5)/(1/2*2*5^3))^2)))/((-12(2)^4*(1/2)^4)))))) * (((((25*(2)^4*(1/2)^4))/(4*3*5^2*0.48) -1/5^2 + (3*2*1/4)/(3*5^4) + (2*2*1/2*0.48^0.5)/(3*5^7))))]]^1/4-11/10^3

**Input:**

$$\sqrt[4]{-\frac{1}{-\frac{(2\times5-5)^2\times0.48\left(1+\frac{5\sqrt{0.48}}{\frac{1}{2}\times2\times5^3}\right)^2}{12\times2^4\left(\frac{1}{2}\right)^4}\left(\frac{25\times2^4\left(\frac{1}{2}\right)^4}{4\times3\times5^2\times0.48}-\frac{1}{5^2}+\frac{3\times2\times\frac{1}{4}}{3\times5^4}+\frac{2\times2\times\frac{1}{2}\sqrt{0.48}}{3\times5^7}\right)}}-\frac{11}{10^3}$$

**Result:**

1.6181105125034445682226639910780200453298214303827951629 48...

1.61811051250344.... result that is a very good approximation to the value of the golden ratio 1.618033988749...

We have also:

$$N(r)=\frac{r^2}{6(D-1)(D-2)\,|\beta|}-\frac{m}{r^{D-3}}+\frac{3(D-3)q^2}{(D-2)r^{2(D-3)}}+\frac{2\sqrt{6\,|\beta|}(D-3)^3q^3}{(2D-5)(D-2)r^{3D-8}},$$

For: $c_1 = -1$; $c_2 = 1/2$ ; $q = 1/2$ ; $D = 5$; $\beta = -2$; $c_3^2 = 0.48$; $r = 5$; $m = 1$

25/(6*4*3*(2)) - 1/(5^2) + (6*1/4) / (3*5^4) + (2*sqrt(12)*2^3*1/8) / (5*3*5^7)

**Input:**

$$\frac{25}{6\times4\times3\times2}-\frac{1}{5^2}+\frac{6\times\frac{1}{4}}{3\times5^4}+\frac{2\sqrt{12}\times2^3\times\frac{1}{8}}{5\times3\times5^7}$$

**Result:**

$$\frac{12097}{90000}+\frac{4}{390625\sqrt{3}}$$

**Decimal approximation:**

0.134417023177867612878939684794623451155456942553044117210...

0.1344170231778676…… as the previous result

**Alternate forms:**

$$\frac{7560625+192\sqrt{3}}{56250000}$$

$$\frac{4\sqrt{3}}{1171875}+\frac{12097}{90000}$$

$$\frac{576+7560625\sqrt{3}}{56250000\sqrt{3}}$$

**Minimal polynomial:**

$$3164062500000000\,x^2-850570312500000\,x+57163050280033$$

From which:

(((1/(((25/(6*4*3*(2)) - 1/(5^2) + (6*1/4) / (3*5^4) + (2*sqrt(12)*2^3*1/8) / (5*3*5^7)))))))^1/4-(29+4)/10^3

**Input:**

$$\sqrt[4]{\frac{1}{\frac{25}{6\times4\times3\times2}-\frac{1}{5^2}+\frac{6\times\frac{1}{4}}{3\times5^4}+\frac{2\sqrt{12}\times2^3\times\frac{1}{8}}{5\times3\times5^7}}}-\frac{29+4}{10^3}$$

**Result:**

$$\frac{1}{\sqrt[4]{\frac{12097}{90000}+\frac{4}{390625\sqrt{3}}}}-\frac{33}{1000}$$

**Decimal approximation:**

1.618529865570538548793547259272875424813140591278920886771...

1.61852986557…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

**Alternate forms:**

$$\frac{50000\sqrt{3}\;57163050280033^{3/4}\sqrt[4]{7560625-192\sqrt{3}}-1886380659241089}{57163050280033000}$$

$$\frac{50\sqrt{3}}{\sqrt[4]{7560625+192\sqrt{3}}}-\frac{33}{1000}$$

$$\frac{50\times 3^{5/8}}{\sqrt[4]{576+7560625\sqrt{3}}}-\frac{33}{1000}$$

And:

18(((1/(((25/(6*4*3*(2)) - 1/(5^2) + (6*1/4) / (3*5^4) + (2*sqrt(12)*2^3*1/8) / (5*3*5^7)))))))+7-golden ratio

**Input:**

$$18\times\frac{1}{\frac{25}{6\times4\times3\times2}-\frac{1}{5^2}+\frac{6\times\frac{1}{4}}{3\times5^4}+\frac{2\sqrt{12}\times2^3\times\frac{1}{8}}{5\times3\times5^7}}+7-\phi$$

$\phi$ is the golden ratio

**Result:**

$$-\phi+7+\frac{18}{\frac{12097}{90000}+\frac{4}{390625\sqrt{3}}}$$

**Decimal approximation:**

139.29357612168575713674917844108158709214084241139889466 51...

139.29357612… result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternate forms:**

$$\frac{16053385278640429-388800000000\sqrt{3}-57163050280033\sqrt{5}}{114326100560066}$$

$$-\phi+7+\frac{1012500000}{7560625+192\sqrt{3}}$$

$$-\frac{194400000000\sqrt{3}}{57163050280033}-\frac{\sqrt{5}}{2}+\frac{16053385278640429}{114326100560066}$$

**Minimal polynomial:**

$$3267614317317580839718481089x^4-1835320939695328829363930508314x^3+386558599094162412415128713753339x^2-36184865368993807955167811775840874x+1270166555577496812957997394891710609$$

**Series representations:**

$$\frac{18}{\frac{25}{6\times4\times3\times2}-\frac{1}{5^2}+\frac{6}{4(3\times5^4)}+\frac{2\sqrt{12}\,2^3}{8(5\times3\times5^7)}}+7-\phi=7-\phi+\frac{1012500000}{7560625+96\sqrt{11}\sum_{k=0}^{\infty}11^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{18}{\frac{25}{6\times4\times3\times2}-\frac{1}{5^2}+\frac{6}{4(3\times5^4)}+\frac{2\sqrt{12}\,2^3}{8(5\times3\times5^7)}}+7-\phi=7-\phi+\frac{1012500000}{7560625+96\sqrt{11}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{11}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{18}{\frac{25}{6\times4\times3\times2}-\frac{1}{5^2}+\frac{6}{4(3\times5^4)}+\frac{2\sqrt{12}\,2^3}{8(5\times3\times5^7)}}+7-\phi=7-\phi+\frac{1012500000\sqrt{\pi}}{7560625\sqrt{\pi}+48\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}11^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}$$

18(((1/(((25/(6*4*3*(2)) - 1/(5^2) + (6*1/4) / (3*5^4) + (2*sqrt(12)*2^3*1/8) / (5*3*5^7)))))))-7-golden ratio

**Input:**

$$18\times\frac{1}{\frac{25}{6\times4\times3\times2}-\frac{1}{5^2}+\frac{6\times\frac{1}{4}}{3\times5^4}+\frac{2\sqrt{12}\times2^3\times\frac{1}{8}}{5\times3\times5^7}}-7-\phi$$

$\phi$ is the golden ratio

**Result:**

$$-\phi-7+\frac{18}{\frac{12097}{90000}+\frac{4}{390625\sqrt{3}}}$$

**Decimal approximation:**

125.2935761216857571367491784410815870921408424113988946651...

125.29357612… result very near to the Higgs boson mass 125.18 GeV

**Alternate forms:**

$$\frac{14452819870799505 - 388800000000\sqrt{3} - 57163050280033\sqrt{5}}{114326100560066}$$

$$-\phi - 7 + \frac{1012500000}{7560625 + 192\sqrt{3}}$$

$$-\frac{194400000000\sqrt{3}}{57163050280033} - \frac{\sqrt{5}}{2} + \frac{14452819870799505}{114326100560066}$$

**Minimal polynomial:**

$$3267614317317580839718481089\,x^4 - 1652334537925544302339695567330\,x^3 + 313317834064124076649352566164815\,x^2 - 26404527972151235991913448881203150\,x + 834433333847129424296777257780294225$$

**Series representations:**

$$\frac{18}{\frac{25}{6\times4\times3\times2} - \frac{1}{5^2} + \frac{6}{4(3\times5^4)} + \frac{2\sqrt{12}\,2^3}{8(5\times3\times5^7)}} - 7 - \phi = -7 - \phi + \frac{1012500000}{7560625 + 96\sqrt{11}\sum_{k=0}^{\infty} 11^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{18}{\frac{25}{6\times4\times3\times2} - \frac{1}{5^2} + \frac{6}{4(3\times5^4)} + \frac{2\sqrt{12}\,2^3}{8(5\times3\times5^7)}} - 7 - \phi = -7 - \phi + \frac{1012500000}{7560625 + 96\sqrt{11}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{11}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{18}{\frac{25}{6\times4\times3\times2} - \frac{1}{5^2} + \frac{6}{4(3\times5^4)} + \frac{2\sqrt{12}\,2^3}{8(5\times3\times5^7)}} - 7 - \phi = -7 - \phi + \frac{1012500000\sqrt{\pi}}{7560625\sqrt{\pi} + 48\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 11^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}$$

27*1/2(((18(((1/(((25/(6*4*3*(2)) - 1/(5^2) + (6*1/4) / (3*5^4) + (2*sqrt(12)*2^3*1/8) / (5*3*5^7)))))))-4-golden ratio)))-3

**Input:**

$$27\times\frac{1}{2}\left(18\times\frac{1}{\frac{25}{6\times4\times3\times2}-\frac{1}{5^2}+\frac{6\times\frac{1}{4}}{3\times5^4}+\frac{2\sqrt{12}\times2^3\times\frac{1}{8}}{5\times3\times5^7}}-4-\phi\right)-3$$

$\phi$ is the golden ratio

**Result:**

$$\frac{27}{2}\left(-\phi-4+\frac{18}{\frac{12097}{90000}+\frac{4}{390625\sqrt{3}}}\right)-3$$

**Decimal approximation:**

1728.963277642757721346113908954601425743901372553885077979...

1728.96327764…

This result is very near to the mass of candidate glueball **$f_0$(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

With regard 27 (From Wikipedia):

*"The fundamental group of the complex form, compact real form, or any algebraic version of $E_6$ is the cyclic group **Z**/3**Z**, and its outer automorphism group is the cyclic group **Z**/2**Z**. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, $E_6$ plays a role in some grand unified theories".*

**Alternate forms:**

$$\frac{398800594053591585-10497600000000\sqrt{3}-1543402357560891\sqrt{5}}{228652201120132}$$

$$-\frac{2624400000000\sqrt{3}}{57163050280033}-\frac{27\sqrt{5}}{4}+\frac{398800594053591585}{228652201120132}$$

$$-\frac{27\phi}{2}-57+\frac{13668750000\sqrt{3}}{576+7560625\sqrt{3}}$$

**Minimal polynomial:**

$$\begin{aligned}&52281829077081293435495697424x^4-\\&364746534553479763320698597156880x^3+\\&954227661335414407469728680169734540x^2-\\&1109479976507535090701719332337093080900x+\\&483734833134505782551326379905688382768225\end{aligned}$$

**Series representations:**

$$\frac{27}{2}\left(\frac{18}{\frac{25}{6\times4\times3\times2}-\frac{1}{5^2}+\frac{6}{4(3\times5^4)}+\frac{2\sqrt{12}\,2^3}{8(5\times3\times5^7)}}-4-\phi\right)-3=-57-\frac{27\phi}{2}+\frac{13668750000}{7560625+96\sqrt{11}\sum_{k=0}^{\infty}11^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{27}{2}\left(\frac{18}{\frac{25}{6\times4\times3\times2}-\frac{1}{5^2}+\frac{6}{4(3\times5^4)}+\frac{2\sqrt{12}\,2^3}{8(5\times3\times5^7)}}-4-\phi\right)-3=-57-\frac{27\phi}{2}+\frac{13668750000}{7560625+96\sqrt{11}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{11}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{27}{2}\left(\frac{18}{\frac{25}{6\times4\times3\times2}-\frac{1}{5^2}+\frac{6}{4(3\times5^4)}+\frac{2\sqrt{12}\,2^3}{8(5\times3\times5^7)}}-4-\phi\right)-3=-57-\frac{27\phi}{2}+\frac{13668750000\sqrt{\pi}}{7560625\sqrt{\pi}+48\sum_{j=0}^{\infty}\mathrm{Res}_{s=-\frac{1}{2}+j}\,11^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}$$

We have that:

$$Q=\frac{(D-3)^2q^2\sqrt{6|\beta|}}{(2D-5)}.$$

(2^2*1/4*sqrt(12)) / 5

**Input:**

$$\frac{1}{5}\left(2^2\times\frac{1}{4}\sqrt{12}\right)$$

**Result:**

$$\frac{2\sqrt{3}}{5}$$

**Decimal approximation:**

0.6928203230275509174109785366023489467771221015241522512 22...

0.692820323027…

1+1/(((sqrt(Pi)1/((((2^2*1/4*sqrt(12)) / 5))))))^1/2 - 7/10^3

**Input:**

$$1+\frac{1}{\sqrt{\sqrt{\pi}\times\frac{1}{\frac{1}{5}\left(2^2\times\frac{1}{4}\sqrt{12}\right)}}}-\frac{7}{10^3}$$

**Exact result:**

$$\frac{993}{1000}+\sqrt{\frac{2}{5}}\sqrt[4]{\frac{3}{\pi}}$$

**Decimal approximation:**

1.618205573809395862194262651377136546487058157240799460932...

1.618205573809…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

**Property:**

$\frac{993}{1000}+\sqrt{\frac{2}{5}}\sqrt[4]{\frac{3}{\pi}}$ is a transcendental number

**Alternate form:**

$$\frac{993+200\sqrt{10}\sqrt[4]{\frac{3}{\pi}}}{1000}$$

**Series representations:**

$$1+\frac{1}{\sqrt{\frac{\frac{\sqrt{\pi}}{2^2\sqrt{12}}}{4\times 5}}}-\frac{7}{10^3}=\left(993\sqrt{-1+\pi}\sum_{k=0}^{\infty}(-1+\pi)^{-k}\binom{\frac{1}{2}}{k}+200\sqrt{5}\sqrt{11}\left(\sum_{k=0}^{\infty}11^{-k}\binom{\frac{1}{2}}{k}\right)\sqrt{\frac{\sqrt{-1+\pi}\sum_{k=0}^{\infty}(-1+\pi)^{-k}\binom{\frac{1}{2}}{k}}{\sqrt{11}\sum_{k=0}^{\infty}11^{-k}\binom{\frac{1}{2}}{k}}}\right)\Bigg/\left(1000\sqrt{-1+\pi}\sum_{k=0}^{\infty}(-1+\pi)^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$1+\frac{1}{\sqrt{\frac{\frac{\sqrt{\pi}}{2^2\sqrt{12}}}{4\times 5}}}-\frac{7}{10^3}=\left(993\sqrt{-1+\pi}\sum_{k=0}^{\infty}\frac{(-1)^k(-1+\pi)^{-k}\left(-\frac{1}{2}\right)_k}{k!}+200\sqrt{5}\sqrt{11}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{11}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\sqrt{\frac{\sqrt{-1+\pi}\sum_{k=0}^{\infty}\frac{(-1)^k(-1+\pi)^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\sqrt{11}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{11}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}}\right)\Bigg/\left(1000\sqrt{-1+\pi}\sum_{k=0}^{\infty}\frac{(-1)^k(-1+\pi)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$1+\frac{1}{\sqrt{\frac{\frac{\sqrt{\pi}}{2^2\sqrt{12}}}{4\times 5}}}-\frac{7}{10^3}=\left(993\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\pi-z_0)^k z_0^{-k}}{k!}+200\sqrt{5}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(12-z_0)^k z_0^{-k}}{k!}\right)\sqrt{\frac{\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\pi-z_0)^k z_0^{-k}}{k!}}{\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(12-z_0)^k z_0^{-k}}{k!}}}\right)\Bigg/\left(1000\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\pi-z_0)^k z_0^{-k}}{k!}\right)\quad\text{for (not }(z_0\in\mathbb{R}\text{ and }-\infty<z_0\leq 0))$$

Now, from:

$$N(r)=\frac{r^2}{6(D-1)(D-2)\,|\beta|}-\frac{m}{r^{D-3}}+\frac{3(D-3)q^2}{(D-2)r^{2(D-3)}}+\frac{2\sqrt{6\,|\beta|}(D-3)^3q^3}{(2D-5)(D-2)r^{3D-8}},$$

we calculate this equation for M = 13.12806e+39; r = 1.94973e+13 and $c_1$ = -1; $c_2$ = 1/2 ; q = 1/2 ; D = 5; β = – 2; $c_3^2$ = 0.48; and obtain:

(1.94973e+13)^2/(6*4*3*(2)) – (13.12806e+39)/((1.94973e+13)^2) + (6*1/4) / (3*(1.94973e+13)^4) + (2*sqrt(12)*2^3*1/8) / (5*3*(1.94973e+13)^7)

**Input interpretation:**

$$\frac{(1.94973\times10^{13})^2}{6\times4\times3\times2}-\frac{13.12806\times10^{39}}{(1.94973\times10^{13})^2}+\frac{6\times\frac{1}{4}}{3\,(1.94973\times10^{13})^4}+\frac{2\sqrt{12}\times2^3\times\frac{1}{8}}{5\times3\,(1.94973\times10^{13})^7}$$

**Result:**

$2.63989...\times10^{24}$

$2.63989...*10^{24}$

(((((1.94973e+13)^2/(6*4*3*(2)) – (13.12806e+39)/((1.94973e+13)^2) + (6*1/4) / (3*(1.94973e+13)^4) + (2*sqrt(12)*2^3*1/8) / (5*3*(1.94973e+13)^7)))))^1/(89+21+5+2)

**Input interpretation:**

$$\left(\frac{(1.94973\times10^{13})^2}{6\times4\times3\times2}-\frac{13.12806\times10^{39}}{(1.94973\times10^{13})^2}+\frac{6\times\frac{1}{4}}{3\,(1.94973\times10^{13})^4}+\frac{2\sqrt{12}\times2^3\times\frac{1}{8}}{5\times3\,(1.94973\times10^{13})^7}\right)^\wedge\left(\frac{1}{89+21+5+2}\right)$$

**Result:**

1.617080008811218701897654853485718094900866957277021257645...

1.617080008811…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

4(((((1.94973e+13)^2/(6*4*3*(2)) – (13.12806e+39)/((1.94973e+13)^2) + (6*1/4) / (3*(1.94973e+13)^4) + (2*sqrt(12)*2^3*1/8) / (5*3*(1.94973e+13)^7)))))^1/16 + 5

**Input interpretation:**

$$4\left(\frac{(1.94973\times10^{13})^2}{6\times4\times3\times2}-\frac{13.12806\times10^{39}}{(1.94973\times10^{13})^2}+\frac{6\times\frac{1}{4}}{3\,(1.94973\times10^{13})^4}+\frac{2\sqrt{12}\times2^3\times\frac{1}{8}}{5\times3\,(1.94973\times10^{13})^7}\right)\text{^}(1/16)+5$$

**Result:**

139.403...

139.403… result practically equal to the rest mass of Pion meson 139.57 MeV

4(((((1.94973e+13)^2/(6*4*3*(2)) – (13.12806e+39)/((1.94973e+13)^2) + (6*1/4) / (3*(1.94973e+13)^4) + (2*sqrt(12)*2^3*1/8) / (5*3*(1.94973e+13)^7)))))^1/16 -7-2

**Input interpretation:**

$$4\left(\frac{(1.94973\times10^{13})^2}{6\times4\times3\times2}-\frac{13.12806\times10^{39}}{(1.94973\times10^{13})^2}+\frac{6\times\frac{1}{4}}{3\,(1.94973\times10^{13})^4}+\frac{2\sqrt{12}\times2^3\times\frac{1}{8}}{5\times3\,(1.94973\times10^{13})^7}\right)\text{^}(1/16)-7-2$$

**Result:**

125.403...

125.403… result very near to the Higgs boson mass 125.18 GeV

27*1/2((((4(((((1.94973e+13)^2/(6*4*3*(2)) – (13.12806e+39)/((1.94973e+13)^2) + (6*1/4) / (3*(1.94973e+13)^4) + (2*sqrt(12)*2^3*1/8) / (5*3*(1.94973e+13)^7)))))^1/16 -4-2))))-4

**Input interpretation:**

$$27 \times \frac{1}{2}\left(4\left(\frac{(1.94973 \times 10^{13})^2}{6 \times 4 \times 3 \times 2} - \frac{13.12806 \times 10^{39}}{(1.94973 \times 10^{13})^2} + \frac{6 \times \frac{1}{4}}{3\,(1.94973 \times 10^{13})^4} + \frac{2\sqrt{12} \times 2^3 \times \frac{1}{8}}{5 \times 3\,(1.94973 \times 10^{13})^7}\right) \hat{}\,(1/16) - 4 - 2\right) - 4$$

**Result:**

1729.44...

1729.44…

This result is very near to the mass of candidate glueball **$f_0$(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

From:

$$N_1(r) = \frac{1}{f(r)N(r)}, \qquad f(r) = \left[1 + \frac{(D-3)q\sqrt{6\,|\beta|}}{r^{D-2}}\right]^2,$$

We obtain:

[1+(2*1/2*sqrt(12))/((1.94973e+13)^3)]^2

**Input interpretation:**

$$\left(1 + \frac{2 \times \frac{1}{2}\sqrt{12}}{(1.94973 \times 10^{13})^3}\right)^2$$

**Result:**

1.0000000000000000000000000000000000000009348...

1

And:

1/ (((((([[1+(2*1/2*sqrt(12))/((1.94973e+13)^3)]^2]*[(1.94973e+13)^2/(6*4*3*(2)) – (13.12806e+39)/((1.94973e+13)^2) + (6*1/4) / (3*(1.94973e+13)^4) + (2*sqrt(12)*2^3*1/8) / (5*3*(1.94973e+13)^7)]))))

**Input interpretation:**

$$1/\left(\left(1+\frac{2\times\frac{1}{2}\sqrt{12}}{(1.94973\times10^{13})^3}\right)^2\left(\frac{(1.94973\times10^{13})^2}{6\times4\times3\times2}-\frac{13.12806\times10^{39}}{(1.94973\times10^{13})^2}+\frac{6\times\frac{1}{4}}{3\,(1.94973\times10^{13})^4}+\frac{2\sqrt{12}\times2^3\times\frac{1}{8}}{5\times3\,(1.94973\times10^{13})^7}\right)\right)$$

**Result:**

$3.78803\ldots\times10^{-25}$

3.78803…*$10^{-25}$

4/ [[[1+(2*1/2*sqrt(12))/((1.9497e+13)^3)]^2]*[(1.9497e+13)^2/(6*4*3*(2))–(13.128e+39)/((1.9497e+13)^2)+(6*1/4)/(3*(1.9497e+13)^4)+(2*sqrt(12)*2^3*1/8)/(5*3*(1.9497e+13)^7)]]^1/64-(47-4)/10^3

**Input interpretation:**

$$4/\left(\left(\left(1+\frac{2\times\frac{1}{2}\sqrt{12}}{(1.9497\times10^{13})^3}\right)^2\left(\frac{(1.9497\times10^{13})^2}{6\times4\times3\times2}-\frac{13.128\times10^{39}}{(1.9497\times10^{13})^2}+\frac{6\times\frac{1}{4}}{3\,(1.9497\times10^{13})^4}+\frac{2\sqrt{12}\times2^3\times\frac{1}{8}}{5\times3\,(1.9497\times10^{13})^7}\right)\right)^\wedge(1/64)\right)-\frac{47-4}{10^3}$$

**Result:**

1.6183950484430626077938748342793864456950692008701051399 84...

1.618395048443…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

((1/[[[1+(2*1/2*sqrt(12))/((1.9497e+13)^3)]^2]*[(1.9497e+13)^2/(6*4*3*(2))–(13.128e+39)/((1.9497e+13)^2)+(6*1/4)/(3*(1.9497e+13)^4)+(2*sqrt(12)*2^3*1/8)/(5*3*(1.9497e+13)^7)]]^-1))^1/11 - 29+1.618^2

**Input interpretation:**

$$\sqrt[11]{\frac{1}{\frac{1}{\left(1+\frac{2\times\frac{1}{2}\sqrt{12}}{(1.9497\times10^{13})^3}\right)^2\left(\frac{(1.9497\times10^{13})^2}{6\times4\times3\times2}-\frac{13.128\times10^{39}}{(1.9497\times10^{13})^2}+\frac{6\times\frac{1}{4}}{3(1.9497\times10^{13})^4}+\frac{2\sqrt{12}\times2^3\times\frac{1}{8}}{5\times3(1.9497\times10^{13})^7}\right)}}}-29+1.618^2$$

**Result:**

139.631...

139.631… result practically equal to the rest mass of Pion meson 139.57 MeV

((1/[[[1+(2*1/2*sqrt(12))/((1.9497e+13)^3)]^2]*[(1.9497e+13)^2/(6*4*3*(2))–(13.128e+39)/((1.9497e+13)^2)+(6*1/4)/(3*(1.9497e+13)^4)+(2*sqrt(12)*2^3*1/8)/(5*3*(1.9497e+13)^7)]]^-1))^1/11 - 47+4+1.618^2

**Input interpretation:**

$$\sqrt[11]{\frac{1}{\frac{1}{\left(1+\frac{2\times\frac{1}{2}\sqrt{12}}{(1.9497\times10^{13})^3}\right)^2\left(\frac{(1.9497\times10^{13})^2}{6\times4\times3\times2}-\frac{13.128\times10^{39}}{(1.9497\times10^{13})^2}+\frac{6\times\frac{1}{4}}{3(1.9497\times10^{13})^4}+\frac{2\sqrt{12}\times2^3\times\frac{1}{8}}{5\times3(1.9497\times10^{13})^7}\right)}}}-47+4+1.618^2$$

**Result:**

125.631...

125.631… result very near to the Higgs boson mass 125.18 GeV

1+27*1/2[((1/[[[1+(2*1/2*sqrt(12))/((1.9497e+13)^3)]^2][(1.9497e+13)^2/(6*4*3*(2))–(13.128e+39)/((1.9497e+13)^2)+(6*1/4)/(3(1.9497e+13)^4)+(2*sqrt(12)*2^3*1/8)/(5*3(1.9497e+13)^7)]]^-1))^1/11 - 34-4]

**Input interpretation:**

$$1+27\times\frac{1}{2}\left(\left(1/1/\left(\left(1+\frac{2\times\frac{1}{2}\sqrt{12}}{(1.9497\times10^{13})^3}\right)^2\left(\frac{(1.9497\times10^{13})^2}{6\times4\times3\times2}-\frac{13.128\times10^{39}}{(1.9497\times10^{13})^2}+\frac{6\times\frac{1}{4}}{3\,(1.9497\times10^{13})^4}+\frac{2\sqrt{12}\times2^3\times\frac{1}{8}}{5\times3\,(1.9497\times10^{13})^7}\right)\right)\right)\hat{}\,(1/11)-34-4\right)$$

**Result:**

1729.18...

1729.18…

This result is very near to the mass of candidate glueball **$f_0$(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

We have that:

b) In the charged case, the torsion scalar has the form

$$T=\frac{r^{(D-2)}-2q^2(D-3)\sqrt{6\,|\beta|}}{6\,|\beta|\,r^{(D-2)}}, \tag{26}$$

From:

$$T=\frac{r^{(D-2)}-2q^2(D-3)\sqrt{6\,|\beta|}}{6\,|\beta|\,r^{(D-2)}},$$

for M = 13.12806e+39; r = 1.94973e+13 and $c_1$ = -1; $c_2$ = 1/2 ; q = 1/2 ; D = 5;

$\beta = -2$; $c_3^2 = 0.48$; we obtain:

(((1.94973e+13)^3-2*1/4*2*sqrt(12))) / ((12*(1.94973e+13)^3))

**Input interpretation:**

$$\frac{(1.94973\times 10^{13})^3 - 2\times\frac{1}{4}\times 2\sqrt{12}}{12\,(1.94973\times 10^{13})^3}$$

**Result:**

0.0833333...

0.0833333…

**Rational approximation:**

$$\frac{1}{12}$$

From which:

12*[1/((((((1.94973e+13)^3-2*1/4*2*sqrt(12))) / ((12*(1.94973e+13)^3)))))]^2+1

**Input interpretation:**

$$12\left(\frac{1}{\frac{(1.94973\times 10^{13})^3-2\times\frac{1}{4}\times 2\sqrt{12}}{12\,(1.94973\times 10^{13})^3}}\right)^2+1$$

**Result:**

1729.00...

1729

This result is very near to the mass of candidate glueball **$f_0$(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

(1+((((((1.94973e+13)^3-2*1/4*2*sqrt(12))) / ((12*(1.94973e+13)^3))))))^6+2/10^3

**Input interpretation:**

$$\left(1+\frac{(1.94973\times 10^{13})^3-2\times\frac{1}{4}\times 2\sqrt{12}}{12\,(1.94973\times 10^{13})^3}\right)^6+\frac{2}{10^3}$$

**Result:**

1.618488567922668038408779149519890260630652675123213078278...

1.618488567922…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

(1+((((((1.94973e+13)^3-2*1/4*2*sqrt(12))) / ((12*(1.94973e+13)^3))))))^64-29+1/golden ratio

**Input interpretation:**

$$\left(1+\frac{(1.94973\times 10^{13})^3-2\times\frac{1}{4}\times 2\sqrt{12}}{12\,(1.94973\times 10^{13})^3}\right)^{64}-29+\frac{1}{\phi}$$

$\phi$ is the golden ratio

**Result:**

139.411...

139.411… result practically equal to the rest mass of Pion meson 139.57 MeV

(1+((((((1.94973e+13)^3-2*1/4*2*sqrt(12))) / ((12*(1.94973e+13)^3))))))^64-34-8-1/golden ratio

**Input interpretation:**

$$\left(1+\frac{(1.94973\times 10^{13})^3-2\times\frac{1}{4}\times 2\sqrt{12}}{12\,(1.94973\times 10^{13})^3}\right)^{64}-34-8-\frac{1}{\phi}$$

$\phi$ is the golden ratio

**Result:**

125.175...

125.175… result very near to the Higgs boson mass 125.18 GeV

27*1/2(((((1+((((((1.94973e+13)^3-2*1/4*2*sqrt(12))) / ((12*(1.94973e+13)^3))))))^64-34-5-1/golden ratio))))-1

**Input interpretation:**

$$27\times\frac{1}{2}\left(\left(1+\frac{(1.94973\times10^{13})^3-2\times\frac{1}{4}\times2\sqrt{12}}{12\,(1.94973\times10^{13})^3}\right)^{64}-34-5-\frac{1}{\phi}\right)-1$$

$\phi$ is the golden ratio

**Result:**

1729.37...

1729.37...

This result is very near to the mass of candidate glueball **$f_0$(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Now, we have that:

The metric of the vielbein (23) takes the form

$$ds^2=\left[r^2\Lambda_{ef}-\frac{m}{r^{D-3}}+\frac{3(D-3)q^2}{(D-2)r^{2(D-3)}}+\frac{2\sqrt{6\,|\beta|}(D-3)^3q^3}{(2D-5)(D-2)r^{3D-8}}\right]dt^2$$
$$-\frac{dr^2}{\left[1+\frac{(D-3)q\sqrt{6|\beta|}}{r^{D-2}}\right]^2\left[r^2\Lambda_{ef}-\frac{m}{r^{D-3}}+\frac{3(D-3)q^2}{(D-2)r^{2(D-3)}}+\frac{2\sqrt{6|\beta|}(D-3)^3q^3}{(2D-5)(D-2)r^{3D-8}}\right]}-r^2\sum_{i=1}^{D-2}d\phi_i^2,$$

From

$$ds^2 = \left[r^2\Lambda_{ef} - \frac{m}{r^{D-3}} + \frac{3(D-3)q^2}{(D-2)r^{2(D-3)}} + \frac{2\sqrt{6\,|\beta|}(D-3)^3q^3}{(2D-5)(D-2)r^{3D-8}}\right]dt^2$$

$$-\frac{dr^2}{\left[1+\frac{(D-3)q\sqrt{6|\beta|}}{r^{D-2}}\right]^2\left[r^2\Lambda_{ef} - \frac{m}{r^{D-3}} + \frac{3(D-3)q^2}{(D-2)r^{2(D-3)}} + \frac{2\sqrt{6|\beta|}(D-3)^3q^3}{(2D-5)(D-2)r^{3D-8}}\right]} - r^2\sum_{i=1}^{D-2} d\phi_i^2,$$

For

$\Lambda_{ef} = \frac{1}{6(D-1)(D-2)|\beta|}.$

1/(6*(5-1)*(5-2)*2) = 1/144 = 0.006944444

M = 13.12806e+39; r = 1.94973e+13; $c_1$ = -1; $\Lambda_{ef}$ = 1/144 = 0.006944444;

$c_2$ = 1/2 ; q = 1/2 ; D = 5; β = – 2; $c_3^2$ = 0.48; $\phi_i = 3$ and obtain

(1.94973e+13)^2*1/144-(13.12806e+39)/((1.94973e+13)^2) + (6*1/4) / (3*(1.94973e+13)^4) + (2*sqrt(12)*2^3*1/8) / (5*3*(1.94973e+13)^7)

**Input interpretation:**

$$(1.94973\times10^{13})^2\times\frac{1}{144} - \frac{13.12806\times10^{39}}{(1.94973\times10^{13})^2} + \frac{6\times\frac{1}{4}}{3\,(1.94973\times10^{13})^4} + \frac{2\sqrt{12}\times2^3\times\frac{1}{8}}{5\times3\,(1.94973\times10^{13})^7}$$

**Result:**

$2.63989...\times10^{24}$

$2.63989...*10^{24}$

$$-\frac{dr^2}{\left[1+\frac{(D-3)q\sqrt{6|\beta|}}{r^{D-2}}\right]^2\left[r^2\Lambda_{ef}-\frac{m}{r^{D-3}}+\frac{3(D-3)q^2}{(D-2)r^{2(D-3)}}+\frac{2\sqrt{6|\beta|}(D-3)^3q^3}{(2D-5)(D-2)r^{3D-8}}\right]}-r^2\sum_{i=1}^{D-2}d\phi_i^2,$$

-(1.94973e+13)^2 / ([((1+(2*1/4*sqrt(12))/(1.94973e+13)^3))]^2 [(((1.94973e+13)^2*1/144-(13.12806e+39)/((1.94973e+13)^2) + (6*1/4)/(3(1.94973e+13)^4)+(2*sqrt(12)*2^3*1/8)/(5*3(1.94973e+13)^7)))])

**Input interpretation:**

$$-\left((1.94973\times10^{13})^2\Big/\left(\left(1+\frac{2\times\frac{1}{4}\sqrt{12}}{(1.94973\times10^{13})^3}\right)^2\left((1.94973\times10^{13})^2\times\frac{1}{144}-\frac{13.12806\times10^{39}}{(1.94973\times10^{13})^2}+\frac{6\times\frac{1}{4}}{3\,(1.94973\times10^{13})^4}+\frac{2\sqrt{12}\times2^3\times\frac{1}{8}}{5\times3\,(1.94973\times10^{13})^7}\right)\right)\right)$$

**Result:**

-144.000...

-144 result that is a Fibonacci number in absolute value

-144 – (1.94973e+13)^2 * 9

**Input interpretation:**

$-144-(1.94973\times10^{13})^2\times9$

**Result:**

-3 421 302 365 610 000 000 000 000 144

$-3.421302365610000000000000144\times10^{27}$

$-3.421302365610000000000000144\times10^{27}$

Thence:

(2.63989 × 10^24) -144 – (1.94973e+13)^2 * 9

**Input interpretation:**

$2.63989\times10^{24}-144-(1.94973\times10^{13})^2\times9$

**Result:**

$-3418662475610000000000000144$

$-3.418662475610000000000000144 \times 10^{27}$

$-3.418662475610000000000000144\times10^{27}$

From which:

[-(-3418662475610000000000000144)]^1/2

**Input:**

$\sqrt{-(-3418662475610000000000000144)}$

**Exact result:**

$4\sqrt{213666404725625000000000009}$

**Decimal approximation:**

$5.846932935830545337382956991497579931086862776335673 1\ldots \times 10^{13}$

$5.84693293583\ldots*10^{13}$

and:

[-(-3418662475610000000000000144)]^1/128

**Input:**

$\sqrt[128]{-(-3418662475610000000000000144)}$

**Result:**

$\sqrt[32]{2}\ \sqrt[128]{213666404725625000000000009}$

**Decimal approximation:**

$1.6409987557455222471971538127133460654347650654853169085 21\ldots$

$1.6409987557\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

[-(-3418662475610000000000000144)]^1/128 - (21+2)1/10^3

**Input:**

$$\sqrt[128]{-(-3\,418\,662\,475\,610\,000\,000\,000\,000\,144)} - (21+2)\times\frac{1}{10^3}$$

**Result:**

$$\sqrt[32]{2}\ \sqrt[128]{213\,666\,404\,725\,625\,000\,000\,000\,009} - \frac{23}{1000}$$

**Decimal approximation:**

1.617998755745522247197153812713346065434765065485316908521...

1.6179987557…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

**Alternate forms:**

$$\frac{1000\ \sqrt[32]{2}\ \sqrt[128]{213\,666\,404\,725\,625\,000\,000\,000\,009} - 23}{1000}$$

$$\frac{1000\ \sqrt[32]{2}\ \sqrt[128]{213\,666\,404\,725\,625\,000\,000\,000\,009} - 23}{1000}$$

log base 1.64099875574552[-(-3418662475610000000000000144)] - Pi + 1/golden ratio

**Input interpretation:**

$$\log_{1.64099875574552}(-(-3\,418\,662\,475\,610\,000\,000\,000\,000\,144)) - \pi + \frac{1}{\phi}$$

$\log_b(x)$ is the base- $b$ logarithm

$\phi$ is the golden ratio

**Result:**

125.476441335160...

125.476441335160… result very near to the Higgs boson mass 125.18 GeV

**Alternative representation:**

$$\log_{1.640998755745520000}(-(-3\,418\,662\,475\,610\,000\,000\,000\,000\,144)) - \pi + \frac{1}{\phi} = -\pi + \frac{1}{\phi} + \frac{\log(3\,418\,662\,475\,610\,000\,000\,000\,000\,144)}{\log(1.640998755745520000)}$$

**Series representation:**

$$\log_{1.640998755745520000}(-(-3\,418\,662\,475\,610\,000\,000\,000\,000\,144)) - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi + 2.0600654307619365 35 \log(3\,418\,662\,475\,610\,000\,000\,000\,000\,144) - \log(3\,418\,662\,475\,610\,000\,000\,000\,000\,144) \sum_{k=0}^{\infty} 0.640998755745520000^k \, G(k)$$

$$\text{for } \left(G(0) = 0 \text{ and } \frac{(-1)^k\, k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^{k} \frac{(-1)^{1+j}\, G(-j+k)}{1+j}\right)$$

log base 1.64099875574552[-(-3418662475610000000000000144)] +11 + 1/golden ratio

**Input interpretation:**

$$\log_{1.64099875574552}(-(-3\,418\,662\,475\,610\,000\,000\,000\,000\,144)) + 11 + \frac{1}{\phi}$$

$\log_b(x)$ is the base- $b$ logarithm

$\phi$ is the golden ratio

**Result:**

139.618033988750...

139.618033988750… result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representation:**

$$\log_{1.640998755745520000}(-(-3\,418\,662\,475\,610\,000\,000\,000\,000\,144)) + 11 + \frac{1}{\phi} = 11 + \frac{1}{\phi} + \frac{\log(3\,418\,662\,475\,610\,000\,000\,000\,000\,144)}{\log(1.640998755745520000)}$$

**Series representation:**

$$\log_{1.640998755745520000}(-(-3418662475610000000000000144))+11+\frac{1}{\phi}=$$
$$11+\frac{1}{\phi}+2.0600654307619365\,35\log(3418662475610000000000000144)-$$
$$\log(3418662475610000000000000144)\sum_{k=0}^{\infty}0.640998755745520000^k\,G(k)$$
$$\text{for }\left(G(0)=0\text{ and }G(k)=\frac{(-1)^{1+k}\,k}{2\,(1+k)\,(2+k)}+\sum_{j=1}^{k}\frac{(-1)^{1+j}\,G(-j+k)}{1+j}\right)$$

27*1/2 log base 1.64099875574552[-(-3418662475610000000000000144)] + 1

**Input interpretation:**

$$27\times\frac{1}{2}\log_{1.64099875574552}(-(-3418662475610000000000000144))+1$$

$\log_b(x)$ is the base- $b$ logarithm

**Result:**

1729.00000000000...

1729

This result is very near to the mass of candidate glueball **$f_0$(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

**Alternative representation:**

$$\frac{27}{2}\log_{1.640998755745520000}(-(-3418662475610000000000000144))+1=$$
$$1+\frac{27\log(3418662475610000000000000144)}{2\log(1.640998755745520000)}$$

## Series representation:

$$\frac{27}{2}\log_{1.640998755745520000}(-(-3418662475610000000000000144))+1 =$$

$$1.000000000000000000 + 27.8108833152861432 \log(3418662475610000000000000144) - 13.50000000000000000 \log(3418662475610000000000000144)\sum_{k=0}^{\infty} 0.640998755745520000^k\, G(k)$$

$$\text{for}\left(G(0)=0 \text{ and } G(k)=\frac{(-1)^{1+k}\,k}{2\,(1+k)\,(2+k)}+\sum_{j=1}^{k}\frac{(-1)^{1+j}\,G(-j+k)}{1+j}\right)$$

https://www.askamathematician.com/2017/11/q-how-does-12345-112-make-any-sense/

$$\begin{aligned}
\zeta(-1) &= \tfrac{1}{1-2^2}\textstyle\sum_{n=0}^{\infty}\tfrac{1}{2^{n+1}}\textstyle\sum_{k=0}^{n}\binom{n}{k}(-1)^k(k+1)\\
&= -\tfrac{1}{3}\textstyle\sum_{n=0}^{1}\tfrac{1}{2^{n+1}}\textstyle\sum_{k=0}^{n}\binom{n}{k}(-1)^k(k+1)\\
&= -\tfrac{1}{3}\cdot\tfrac{1}{2^{0+1}}\binom{0}{0}(-1)^0(0+1)-\tfrac{1}{3}\cdot\tfrac{1}{2^{1+1}}\binom{1}{0}(-1)^0(0+1)-\tfrac{1}{3}\cdot\tfrac{1}{2^{1+1}}\binom{1}{1}(-1)^1(1+1)\\
&= -\tfrac{1}{3}\cdot\tfrac{1}{2}\cdot 1\cdot 1\cdot 1-\tfrac{1}{3}\cdot\tfrac{1}{4}\cdot 1\cdot 1\cdot 1-\tfrac{1}{3}\cdot\tfrac{1}{4}\cdot 1\cdot(-1)\cdot 2\\
&= -\tfrac{1}{6}-\tfrac{1}{12}+\tfrac{1}{6}\\
&= -\tfrac{1}{12}
\end{aligned}$$

From:

*B Candelpergher*. **Ramanujan summation of divergent series**. Lectures notes in mathematics, 2185, 2017. hal-01150208v2

Non convergent series are *divergent series*. For $Re(s) \leq 1$ the Riemann series is a divergent series and does not give a finite value for the sums that appear in the Casimir effect. A possible strategy to assign a finite value to these sums is to perform an analytic continuation of the zeta function, this has been done by Riemann (cf. Edwards) who found an integral formula for $\zeta(s)$ which is valid not only for $\mathrm{Re}(s) > 1$ but also for $s \in \mathbb{C}\backslash\{1\}$. By this method we can assign to the series $\sum_{n\geq 1} n^k$ with $k > -1$ the value $\zeta(-k)$, we get for example

$$\begin{aligned}
\sum_{n\geq 1} n^0 &= 1+1+1+1+1+1+\ldots \mapsto \zeta(0) = -\frac{1}{2} \\
\sum_{n\geq 1} n^1 &= 1+2+3+4+5+6+\ldots \mapsto \zeta(-1) = -\frac{1}{12} \\
\sum_{n\geq 1} n^2 &= 1+2^2+3^2+4^2+5^2+\ldots \mapsto \zeta(-2) = 0 \\
\sum_{n\geq 1} n^3 &= 1+2^3+3^3+4^3+5^3+\ldots \mapsto \zeta(-3) = \frac{1}{120} \\
&\ldots
\end{aligned}$$

From:

*Bruce C. Berndt*
**Ramanujan's Notebooks Part 1** - 1985 by Springer-Verlag New York Inc.

**Example 1.** *The constant of* $\sum 1$ *is* $-\frac{1}{2}$.

*Proof*. Let $f(t) \equiv 1$ in (1.2).

We remark that the Abel and Cesaro sums of $\sum 1$ are both $\infty$. (See Hardy's book [15, p. 9].)

**Example 2.** *The constant of* $\sum k$ *is* $-\frac{1}{12}$.

*Proof*. Set $f(t) = t$ in (1.2) and use the value $B_2 = \frac{1}{6}$.

$$c = -\tfrac{1}{2}f(0) - \sum_{k=1}^{\infty} \frac{B_{2k}}{(2k)!} f^{(2k-1)}(0). \tag{1.2}$$

we have that:

$$c = 1 + 2 + 3 + 4 + \ldots,$$
$$4c = \quad 4 \quad + 8 + \ldots,$$

and so

$$-3c = 1 - 2 + 3 - 4 + \cdots = \frac{1}{(1+1)^2} = \frac{1}{4}. \tag{1.4}$$

Hence, $c = -\frac{1}{12}$. Note that in (1.4) Ramanujan is finding the Abel sum of $\sum(-1)^{k+1}k$. Both Example 2 and (1.4) were communicated by Ramanujan [15, p. 351] in his first letter to Hardy and were discussed by Watson [1].

The constants in Examples 1 and 2 can also be determined from (1.3).

Entry 2 is simply another form of the Euler–Maclaurin summation formula.

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From

(iii) $$\int_0^{2-\sqrt{3}} \frac{\tan^{-1} t}{t}\, dt = \frac{\pi}{12} \operatorname{Log}(2-\sqrt{3}) + \frac{2}{3}\int_0^1 \frac{\tan^{-1} t}{t}\, dt,$$

*where*

$$\int_0^1 \frac{\tan^{-1} t}{t}\, dt = G = 0.915965594177\ldots,$$

we obtain:

Pi/12 ln(2-sqrt3)+2/3*0.915965594177

**Input interpretation:**

$$\frac{\pi}{12} \log\left(2-\sqrt{3}\right) + \frac{2}{3} \times 0.915965594177$$

log(x) is the natural logarithm

**Result:**

0.265864958279...

0.265864958279…

**Alternative representations:**

$$\frac{1}{12}\log\left(2-\sqrt{3}\right)\pi+\frac{0.9159655941770000\times 2}{3}= 0.6106437294513333+\frac{1}{12}\pi\log_e\left(2-\sqrt{3}\right)$$

$$\frac{1}{12}\log\left(2-\sqrt{3}\right)\pi+\frac{0.9159655941770000\times 2}{3}= 0.6106437294513333+\frac{1}{12}\pi\log(a)\log_a\left(2-\sqrt{3}\right)$$

$$\frac{1}{12}\log\left(2-\sqrt{3}\right)\pi+\frac{0.9159655941770000\times 2}{3}= 0.6106437294513333-\frac{1}{12}\pi\,\mathrm{Li}_1\left(-1+\sqrt{3}\right)$$

**Series representations:**

$$\frac{1}{12}\log\left(2-\sqrt{3}\right)\pi+\frac{0.9159655941770000\times 2}{3}= 0.610643729451333-0.0833333333333333\,\pi\sum_{k=1}^{\infty}\frac{(-1)^k\left(1-\sqrt{3}\right)^k}{k}$$

$$\frac{1}{12}\log\left(2-\sqrt{3}\right)\pi+\frac{0.9159655941770000\times 2}{3}= 0.610643729451333+0.0833333333333333\,\pi\log\left(2-\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{1}{12}\log\left(2-\sqrt{3}\right)\pi+\frac{0.9159655941770000\times 2}{3}= 0.6106437294513333+\frac{1}{6}i\pi^2\left\lfloor\frac{\arg\left(2-x-\sqrt{3}\right)}{2\pi}\right\rfloor+ \frac{1}{12}\pi\log(x)-\frac{1}{12}\pi\sum_{k=1}^{\infty}\frac{(-1)^k x^{-k}\left(2-x-\sqrt{3}\right)^k}{k} \quad \text{for } x<0$$

**Integral representation:**

$$\frac{1}{12}\log\left(2-\sqrt{3}\right)\pi+\frac{0.9159655941770000\times 2}{3}=$$
$$0.610643729451333+0.08333333333333333\,\pi\int_1^{2-\sqrt{3}}\frac{1}{t}\,dt$$

2Pi((Pi/12 ln(2-sqrt3)+2/3*0.915965594177))

**Input interpretation:**

$$2\pi\left(\frac{\pi}{12}\log\left(2-\sqrt{3}\right)+\frac{2}{3}\times 0.915965594177\right)$$

log(x) is the natural logarithm

**Result:**

1.67047879955…

1.67047879955…

**Alternative representations:**

$$2\pi\left(\frac{1}{12}\log\left(2-\sqrt{3}\right)\pi+\frac{0.9159655941770000\times 2}{3}\right)=$$
$$2\pi\left(0.6106437294513333+\frac{1}{12}\pi\log_e\left(2-\sqrt{3}\right)\right)$$

$$2\pi\left(\frac{1}{12}\log\left(2-\sqrt{3}\right)\pi+\frac{0.9159655941770000\times 2}{3}\right)=$$
$$2\pi\left(0.6106437294513333+\frac{1}{12}\pi\log(a)\log_a\left(2-\sqrt{3}\right)\right)$$

$$2\pi\left(\frac{1}{12}\log\left(2-\sqrt{3}\right)\pi+\frac{0.9159655941770000\times 2}{3}\right)=$$
$$2\pi\left(0.6106437294513333-\frac{1}{12}\pi\,\mathrm{Li}_1\left(-1+\sqrt{3}\right)\right)$$

**Series representations:**

$$2\pi\left(\frac{1}{12}\log\left(2-\sqrt{3}\right)\pi+\frac{0.9159655941770000\times 2}{3}\right)=$$
$$1.221287458902667\,\pi-0.1666666666666667\,\pi^2\sum_{k=1}^{\infty}\frac{(-1)^k\left(1-\sqrt{3}\right)^k}{k}$$

$$2\pi\left(\frac{1}{12}\log\left(2-\sqrt{3}\right)\pi+\frac{0.9159655941770000\times 2}{3}\right)=$$
$$1.221287458902667\pi+0.1666666666666667\pi^2\log\left(2-\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$2\pi\left(\frac{1}{12}\log\left(2-\sqrt{3}\right)\pi+\frac{0.9159655941770000\times 2}{3}\right)=$$
$$2\pi\left(0.6106437294513333+\frac{1}{12}\pi\left(\log(z_0)+\left\lfloor\frac{\arg(2-\sqrt{3}-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-\sqrt{3}-z_0)^k\,z_0^{-k}}{k}\right)\right)$$

**Integral representation:**

$$2\pi\left(\frac{1}{12}\log\left(2-\sqrt{3}\right)\pi+\frac{0.9159655941770000\times 2}{3}\right)=$$
$$1.221287458902667\pi+0.1666666666666667\pi^2\int_1^{2-\sqrt{3}}\frac{1}{t}\,dt$$

-(Pi*(12Pi/(2*12+(Pi-(6/PI))))) ((ln(2-sqrt3)+2/3*0.915965594177)) - 1.67047879955

**Input interpretation:**

$$-\left(\pi\left(12\times\frac{\pi}{2\times 12+\left(\pi-\frac{6}{\pi}\right)}\right)\right)\left(\log\left(2-\sqrt{3}\right)+\frac{2}{3}\times 0.915965594177\right)-1.67047879955$$

log(x) is the natural logarithm

**Result:**

1.64488982276...

1.64488982276…

**Possible closed form:**

$$\frac{\pi^2}{6}\approx 1.64493406$$

## Alternative representations:

$$-\frac{\left(\log(2-\sqrt{3})+\frac{2\times 0.9159655941770000}{3}\right)\pi\,(12\,\pi)}{2\times 12+\left(\pi-\frac{6}{\pi}\right)}-1.670478799550000=$$
$$-1.670478799550000-\frac{12\,\pi^2\,(0.6106437294513333+\log_e(2-\sqrt{3}))}{24+\pi-\frac{6}{\pi}}$$

$$-\frac{\left(\log(2-\sqrt{3})+\frac{2\times 0.9159655941770000}{3}\right)\pi\,(12\,\pi)}{2\times 12+\left(\pi-\frac{6}{\pi}\right)}-1.670478799550000=$$
$$-1.670478799550000-\frac{12\,\pi^2\,(0.6106437294513333+\log(a)\log_a(2-\sqrt{3}))}{24+\pi-\frac{6}{\pi}}$$

$$-\frac{\left(\log(2-\sqrt{3})+\frac{2\times 0.9159655941770000}{3}\right)\pi\,(12\,\pi)}{2\times 12+\left(\pi-\frac{6}{\pi}\right)}-1.670478799550000=$$
$$-1.670478799550000-\frac{12\,\pi^2\,(0.6106437294513333-\mathrm{Li}_1(-1+\sqrt{3}))}{24+\pi-\frac{6}{\pi}}$$

## Series representations:

$$-\frac{\left(\log(2-\sqrt{3})+\frac{2\times 0.9159655941770000}{3}\right)\pi\,(12\,\pi)}{2\times 12+\left(\pi-\frac{6}{\pi}\right)}-1.670478799550000=$$
$$-1.670478799550000-\frac{7.32772475341600\,\pi^3}{-6.000000000000000+24.00000000000000\,\pi+\pi^2}+\frac{12.00000000000000\,\pi^3\sum_{k=1}^{\infty}\frac{(-1)^k\left(1-\sqrt{3}\right)^k}{k}}{-6.000000000000000+24.00000000000000\,\pi+\pi^2}$$

$$-\frac{\left(\log(2-\sqrt{3})+\frac{2\times 0.9159655941770000}{3}\right)\pi\,(12\,\pi)}{2\times 12+\left(\pi-\frac{6}{\pi}\right)}-1.670478799550000=$$
$$-1.670478799550000-\frac{7.32772475341600\,\pi^3}{-6.000000000000000+24.00000000000000\,\pi+\pi^2}-\frac{12.00000000000000\,\pi^3\log\left(2-\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)}{-6.000000000000000+24.00000000000000\,\pi+\pi^2}$$

$$-\frac{\left(\log(2-\sqrt{3})+\frac{2\times 0.9159655941770000}{3}\right)\pi\,(12\,\pi)}{2\times 12+\left(\pi-\frac{6}{\pi}\right)}-1.670478799550000 =$$

$$-1.670478799550000-\frac{1}{24-\frac{6}{\pi}+\pi}\,12\,\pi^2\left(0.6106437294513333+2\,i\,\pi\left\lfloor\frac{\arg(2-x-\sqrt{3})}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,x^{-k}\,(2-x-\sqrt{3})^k}{k}\right) \text{ for } x<0$$

**Integral representation:**

$$-\frac{\left(\log(2-\sqrt{3})+\frac{2\times 0.9159655941770000}{3}\right)\pi\,(12\,\pi)}{2\times 12+\left(\pi-\frac{6}{\pi}\right)}-1.670478799550000 =$$

$$-1.670478799550000-\frac{7.32772475341600\,\pi^3}{-6.000000000000000+24.00000000000000\,\pi+\pi^2}-\frac{12.00000000000000\,\pi^3}{-6.000000000000000+24.00000000000000\,\pi+\pi^2}\int_1^{2-\sqrt{3}}\frac{1}{t}\,dt$$

Pi*x ln(2-sqrt3)+2/3*0.915965594177 = 0.265864958279

**Input interpretation:**

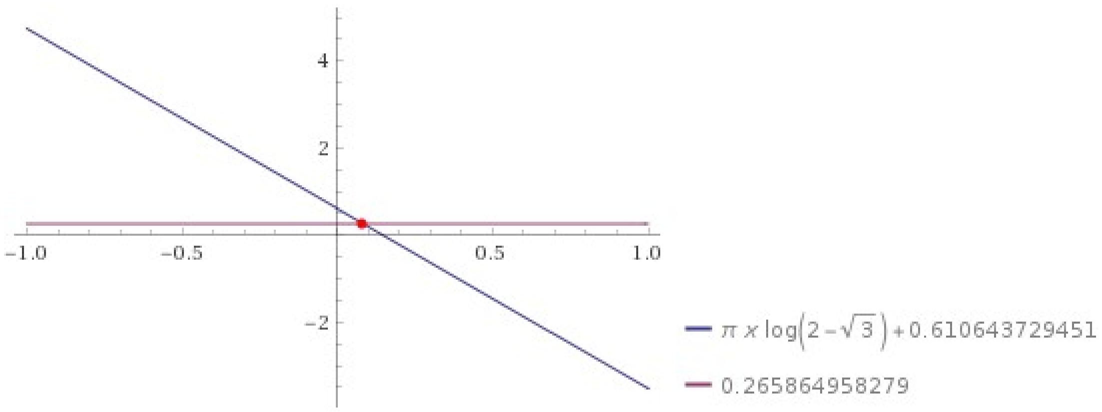
$$\pi\,x\log\left(2-\sqrt{3}\right)+\frac{2}{3}\times 0.915965594177 = 0.265864958279$$

$\log(x)$ is the natural logarithm

**Result:**

$$\pi\,x\log\left(2-\sqrt{3}\right)+0.610643729451 = 0.265864958279$$

**Plot:**




**Alternate forms:**

$$0.61064372945-4.1373452541\,x = 0.265864958279$$

$1.000000000000\,(0.61064372945 - 4.13734525407\,x) = 0.265864958279$

$\pi\,x\,\log\left(2 - \sqrt{3}\right) + 0.344778771172 = 0$

**Solution:**

$x \approx 0.083333333333$

0.083333... = 1/12

From (v),

$$\chi_2\left(\frac{\sqrt{5}-1}{2}\right) = \frac{\pi^2}{12} - \tfrac{3}{4}\,\mathrm{Log}^2\left(\frac{\sqrt{5}-1}{2}\right)$$

we obtain:

(Pi^2) / (12) – 3/4 ln^2 ((sqrt5-1)/2)

**Input:**

$$\frac{\pi^2}{12} - \frac{3}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)$$

$\log(x)$ is the natural logarithm

**Decimal approximation:**

0.648793417991217423863510779899363024597170188066425065756...

0.64879341799…

**Alternate forms:**

$$\frac{1}{12}\left(\pi^2 - 9\,\mathrm{csch}^{-1}(2)^2\right)$$

$$\frac{1}{12}\left(\pi^2 - 9\log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)\right)$$

$$\frac{1}{12}\left(\pi^2 - 9\left(\log\left(\sqrt{5} - 1\right) - \log(2)\right)^2\right)$$

$\mathrm{csch}^{-1}(x)$ is the inverse hyperbolic cosecant function

**Alternative representations:**

$$\frac{\pi^2}{12}-\frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3=\frac{\pi^2}{12}-\frac{3}{4}\log_e^2\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)$$

$$\frac{\pi^2}{12}-\frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3=\frac{\pi^2}{12}-\frac{3}{4}\left(\log(a)\log_a\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\right)^2$$

$$\frac{\pi^2}{12}-\frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3=\frac{\pi^2}{12}-\frac{3}{4}\left(-\text{Li}_1\left(1+\frac{1}{2}\left(1-\sqrt{5}\right)\right)\right)^2$$

**Series representations:**

$$\frac{\pi^2}{12}-\frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3=\frac{\pi^2}{12}-\frac{3}{4}\left(\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-3+\sqrt{5}\right)^k}{k}\right)^2$$

$$\frac{\pi^2}{12}-\frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3=$$
$$\frac{1}{12}\left(\pi^2-9\left(2i\pi\left\lfloor\frac{\arg(-1+\sqrt{5}-2x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-1+\sqrt{5}-2x\right)^k x^{-k}}{k}\right)^2\right)$$
for $x<0$

$$\frac{\pi^2}{12}-\frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3=$$
$$\frac{\pi^2}{12}-\frac{3}{4}\left(2i\pi\left\lfloor\frac{\arg\left(\frac{1}{2}\left(-1+\sqrt{5}\right)-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-1+\sqrt{5}-2x\right)^k x^{-k}}{k}\right)^2$$
for $x<0$

**Integral representation:**

$$\frac{\pi^2}{12}-\frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3=\frac{\pi^2}{12}-\frac{3}{4}\left(\int_1^{\frac{1}{2}\left(-1+\sqrt{5}\right)}\frac{1}{t}\,dt\right)^2$$

1 + (Pi^2) / (12) – 3/4 ln^2 ((sqrt5-1)/2)

**Input:**

$$1+\frac{\pi^2}{12}-\frac{3}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)$$

$\log(x)$ is the natural logarithm

**Decimal approximation:**

1.648793417991217423863510779899363024597170188066425065756...

$1.64879341799.... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$

**Alternate forms:**

$$\frac{1}{12}\left(12+\pi^2-9\operatorname{csch}^{-1}(2)^2\right)$$

$$\frac{1}{12}\left(12+\pi^2-9\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)\right)$$

$$\frac{1}{12}\left(12+\pi^2\right)-\frac{3}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)$$

$\operatorname{csch}^{-1}(x)$ is the inverse hyperbolic cosecant function

**Alternative representations:**

$$1+\frac{\pi^2}{12}-\frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3 = 1+\frac{\pi^2}{12}-\frac{3}{4}\log_e^2\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)$$

$$1+\frac{\pi^2}{12}-\frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3 = 1+\frac{\pi^2}{12}-\frac{3}{4}\left(\log(a)\log_a\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\right)^2$$

$$1+\frac{\pi^2}{12}-\frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3 = 1+\frac{\pi^2}{12}-\frac{3}{4}\left(-\operatorname{Li}_1\left(1+\frac{1}{2}\left(1-\sqrt{5}\right)\right)\right)^2$$

**Series representations:**

$$1+\frac{\pi^2}{12}-\frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3 = 1+\frac{\pi^2}{12}-\frac{3}{4}\left(\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-3+\sqrt{5}\right)^k}{k}\right)^2$$

$$1+\frac{\pi^2}{12}-\frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3 = \frac{1}{12}\left(12+\pi^2-9\left(2i\pi\left\lfloor\frac{\arg\left(-1+\sqrt{5}-2x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-1+\sqrt{5}-2x\right)^k x^{-k}}{k}\right)^2\right)$$

for $x < 0$

$$1+\frac{\pi^2}{12}-\frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3 =$$

$$1+\frac{\pi^2}{12}-\frac{3}{4}\left(2\,i\,\pi\left\lfloor\frac{\arg\left(\frac{1}{2}\left(-1+\sqrt{5}\right)-x\right)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-1+\sqrt{5}-2\,x\right)^k x^{-k}}{k}\right)^2$$

for $x < 0$

**Integral representation:**

$$1+\frac{\pi^2}{12}-\frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3 = 1+\frac{\pi^2}{12}-\frac{3}{4}\left(\int_1^{\frac{1}{2}\left(-1+\sqrt{5}\right)}\frac{1}{t}\,dt\right)^2$$

2* (((Pi^2) / (12) – 3/4 ln^2 ((sqrt5-1)/2)))

**Input:**

$$2\left(\frac{\pi^2}{12}-\frac{3}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)\right)$$

$\log(x)$ is the natural logarithm

**Decimal approximation:**

1.297586835982434847727021559798726049194340376132850131512...

1.29758683598…

**Alternate forms:**

$$\frac{1}{6}\left(\pi^2-9\operatorname{csch}^{-1}(2)^2\right)$$

$$\frac{1}{6}\left(\pi^2-9\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)\right)$$

$$\frac{\pi^2}{6}-\frac{3}{2}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)$$

$\operatorname{csch}^{-1}(x)$ is the inverse hyperbolic cosecant function

**Alternative representations:**

$$2\left(\frac{\pi^2}{12}-\frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3\right) = 2\left(\frac{\pi^2}{12}-\frac{3}{4}\log_e^2\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\right)$$

$$2\left(\frac{\pi^2}{12}-\frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3\right)=2\left(\frac{\pi^2}{12}-\frac{3}{4}\left(\log(a)\log_a\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\right)^2\right)$$

$$2\left(\frac{\pi^2}{12}-\frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3\right)=2\left(\frac{\pi^2}{12}-\frac{3}{4}\left(-\text{Li}_1\left(1+\frac{1}{2}\left(1-\sqrt{5}\right)\right)\right)^2\right)$$

**Series representations:**

$$2\left(\frac{\pi^2}{12}-\frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3\right)=\frac{\pi^2}{6}-\frac{3}{2}\left(\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-3+\sqrt{5}\right)^k}{k}\right)^2$$

$$2\left(\frac{\pi^2}{12}-\frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3\right)=$$
$$\frac{1}{6}\left(\pi^2-9\left(2\,i\,\pi\left\lfloor\frac{\arg\left(-1+\sqrt{5}-2\,x\right)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-1+\sqrt{5}-2\,x\right)^k x^{-k}}{k}\right)^2\right)$$
for $x<0$

$$2\left(\frac{\pi^2}{12}-\frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3\right)=$$
$$2\left(\frac{\pi^2}{12}-\frac{3}{4}\left(2\,i\,\pi\left\lfloor\frac{\arg\left(\frac{1}{2}\left(-1+\sqrt{5}\right)-x\right)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-1+\sqrt{5}-2\,x\right)^k x^{-k}}{k}\right)^2\right)$$
for $x<0$

**Integral representation:**

$$2\left(\frac{\pi^2}{12}-\frac{1}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3\right)=\frac{\pi^2}{6}-\frac{3}{2}\left(\int_1^{\frac{1}{2}\left(-1+\sqrt{5}\right)}\frac{1}{t}\,dt\right)^2$$

x - 3/2 log^2(1/2 (-1 + sqrt(5))) = 1.297586835982434

**Input interpretation:**

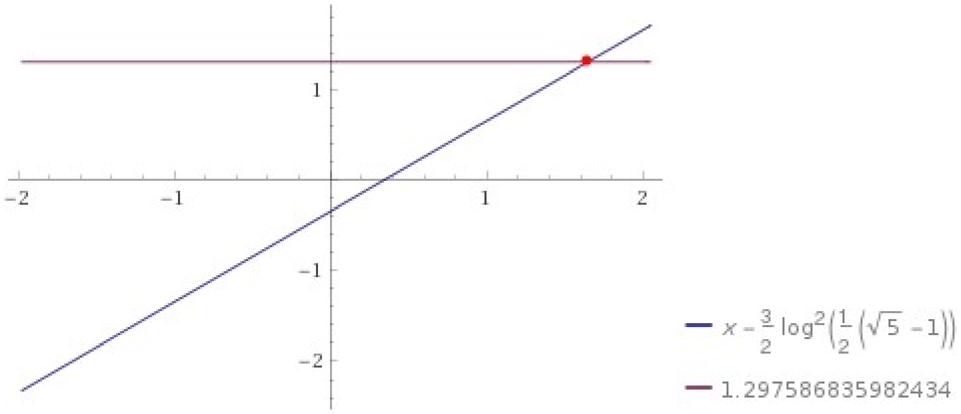
$$x-\frac{3}{2}\log^2\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)=1.297586835982434$$

$\log(x)$ is the natural logarithm

**Result:**

$$x - \frac{3}{2}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right) = 1.297586835982434$$

**Plot:**




**Alternate forms:**

$$x - 1.644934066848226 = 0$$

$$x - \frac{3}{2}\operatorname{csch}^{-1}(2)^2 = 1.297586835982434$$

$$x - \frac{3}{2}\left(\log\left(\sqrt{5}-1\right) - \log(2)\right)^2 = 1.297586835982434$$

$\operatorname{csch}^{-1}(x)$ is the inverse hyperbolic cosecant function

**Alternate form assuming x>0:**

$$x - \frac{3}{2}\log^2\left(\frac{2}{\sqrt{5}-1}\right) = 1.297586835982434$$

**Solution:**

$$x \approx 1.644934066848226$$

$1.644934066848226 = \zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

(Pi^2)/(12) – 3/4 ln^2 (x) = 0.6487934179912174

**Input interpretation:**

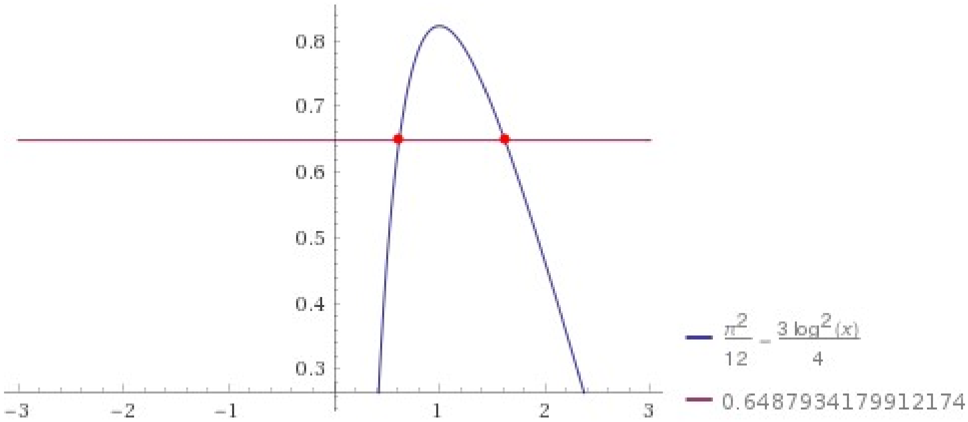
$$\frac{\pi^2}{12} - \frac{3}{4}\log^2(x) = 0.6487934179912174$$

$\log(x)$ is the natural logarithm

**Result:**

$$\frac{\pi^2}{12} - \frac{3\log^2(x)}{4} = 0.6487934179912174$$

**Plot:**




**Alternate forms:**

$$\frac{1}{12}\left(\pi^2 - 9\log^2(x)\right) = 0.6487934179912174$$

$$\frac{1}{12}(\pi - 3\log(x))(3\log(x) + \pi) = 0.6487934179912174$$

**Alternate form assuming x is positive:**

$$1.00000000000000\log^2(x) = 0.231564820577194$$

**Solutions:**

$x \approx 0.61803398874989 48$

0.6180339887498948

$x \approx 1.6180339887498949$

1.6180339887498949

(Pi^2) * x – 3/4 ln^2 ((sqrt5-1)/2) = 0.6487934179912174

**Input interpretation:**

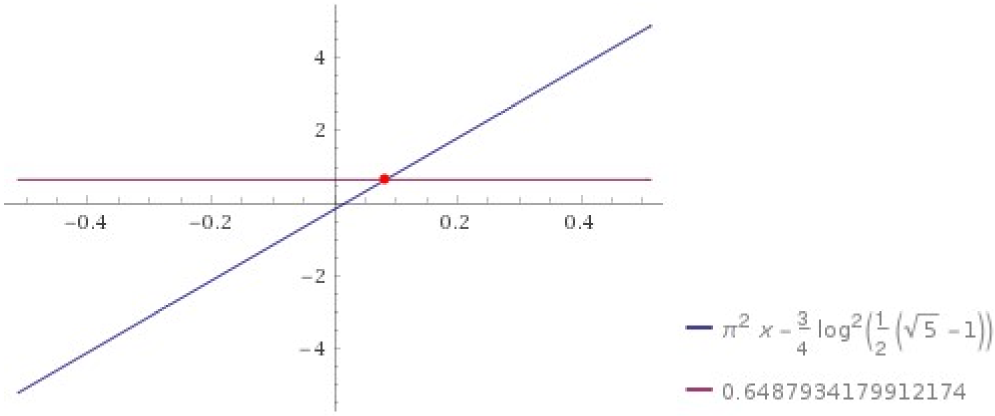
$$\pi^2 x - \frac{3}{4}\log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right) = 0.6487934179912174$$

log(x) is the natural logarithm

**Result:**

$$\pi^2 x - \frac{3}{4} \log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right) = 0.6487934179912174$$

**Plot:**




**Alternate forms:**

$$\pi^2 x - 0.8224670334241132 = 0$$

$$\pi^2 x - \frac{3}{4} \operatorname{csch}^{-1}(2)^2 = 0.6487934179912174$$

$$\pi^2 x - \frac{3}{4} \left(\log\left(\sqrt{5} - 1\right) - \log(2)\right)^2 = 0.6487934179912174$$

$\operatorname{csch}^{-1}(x)$ is the inverse hyperbolic cosecant function

**Alternate form assuming x>0:**

$$\pi^2 x - \frac{3}{4} \log^2\left(\frac{2}{\sqrt{5} - 1}\right) = 0.6487934179912174$$

**Solution:**

$x \approx 0.0833333333333333$

0.0833333.... = 1/12

From:

**1 + 2 + 3 + 4 + ...**

*Frank Thorne*, University of South Carolina - Dartmouth College - May 9, 2013

Theorem (Riemann, 1859)

*The zeta function has analytic continuation to all complex numbers $s \neq 1$, with*

$$\zeta(s) = \zeta(1-s)\frac{\Gamma\left(\frac{1-s}{2}\right)\pi^{-\frac{1-s}{2}}}{\Gamma\left(\frac{s}{2}\right)\pi^{-\frac{s}{2}}}.$$

Therefore,

$$\zeta(-1) = \zeta(2)\frac{\Gamma(1)\pi^{-1}}{\Gamma(-\frac{1}{2})\pi^{1/2}} = \frac{\pi^2}{6}\cdot\frac{1\times\pi^{-1}}{(-2\sqrt{\pi})\pi^{1/2}} = -\frac{1}{12}.$$

zeta(2)*((gamma (1)*1/Pi))*1/((gamma (-1/2))*sqrtPi)

**Input:**

$$\zeta(2)\left(\Gamma(1)\times\frac{1}{\pi}\right)\times\frac{1}{\Gamma\left(-\frac{1}{2}\right)\sqrt{\pi}}$$

$\zeta(s)$ is the Riemann zeta function

$\Gamma(x)$ is the gamma function

**Exact result:**

$$-\frac{1}{12}$$

**Decimal approximation:**

−0.0833333333333333333333333333333333333333333333333333333...

-0.0833333…. = -1/12

**Alternative representations:**

$$\frac{\zeta(2)\,\Gamma(1)}{\left(\Gamma\left(-\frac{1}{2}\right)\sqrt{\pi}\right)\pi} = \frac{e^0\,\zeta(2,\,1)}{\pi\left(e^{-\log G(-1/2)+\log G(1/2)}\,\sqrt{\pi}\right)}$$

$$\frac{\zeta(2)\,\Gamma(1)}{\left(\Gamma\left(-\frac{1}{2}\right)\sqrt{\pi}\right)\pi} = \frac{e^0\,\zeta\left(2, \frac{1}{2}\right)}{3\,\pi\left(e^{-\log G(-1/2)+\log G(1/2)}\,\sqrt{\pi}\right)}$$

$$\frac{\zeta(2)\,\Gamma(1)}{\left(\Gamma\left(-\frac{1}{2}\right)\sqrt{\pi}\right)\pi} = \frac{(1)_0\,\zeta(2, 1)}{\pi\left((1)_{-\frac{3}{2}}\,\sqrt{\pi}\right)}$$

**Series representations:**

$$\frac{\zeta(2)\,\Gamma(1)}{\left(\Gamma\left(-\frac{1}{2}\right)\sqrt{\pi}\right)\pi} = \frac{\exp\left(-\sum_{k=1}^{\infty}\log\left(1-\frac{1}{(p_k)^2}\right)\right)\sum_{k=0}^{\infty}\frac{(1-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}}{\pi\exp\left(i\,\pi\left\lfloor\frac{\arg(\pi-x)}{2\,\pi}\right\rfloor\right)\sqrt{x}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(\pi-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}-z_0\right)^k\,\Gamma^{(k)}(z_0)}{k!}}$$

for ($x \in \mathbb{R}$ and ($z_0 \notin \mathbb{Z}$ or $z_0 > 0$) and $x < 0$)

$$\frac{\zeta(2)\,\Gamma(1)}{\left(\Gamma\left(-\frac{1}{2}\right)\sqrt{\pi}\right)\pi} = \frac{\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(1-z_0)^{k_2}\,\Gamma^{(k_2)}(z_0)}{k_2!\,k_1^2}}{\pi\exp\left(i\,\pi\left\lfloor\frac{\arg(\pi-x)}{2\,\pi}\right\rfloor\right)\sqrt{x}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(\pi-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}-z_0\right)^k\,\Gamma^{(k)}(z_0)}{k!}}$$

for ($x \in \mathbb{R}$ and ($z_0 \notin \mathbb{Z}$ or $z_0 > 0$) and $x < 0$)

$$\frac{\zeta(2)\,\Gamma(1)}{\left(\Gamma\left(-\frac{1}{2}\right)\sqrt{\pi}\right)\pi} = \frac{\exp\left(\sum_{k=1}^{\infty}\frac{P(2\,k)}{k}\right)\sum_{k=0}^{\infty}\frac{(1-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}}{\pi\exp\left(i\,\pi\left\lfloor\frac{\arg(\pi-x)}{2\,\pi}\right\rfloor\right)\sqrt{x}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(\pi-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}-z_0\right)^k\,\Gamma^{(k)}(z_0)}{k!}}$$

for ($x \in \mathbb{R}$ and ($z_0 \notin \mathbb{Z}$ or $z_0 > 0$) and $x < 0$)

**Integral representations:**

$$\frac{\zeta(2)\,\Gamma(1)}{\left(\Gamma\left(-\frac{1}{2}\right)\sqrt{\pi}\right)\pi} = \frac{\Gamma(1)}{2\,\pi\,0!\,\Gamma\left(-\frac{1}{2}\right)\sqrt{\pi}}\int_0^1\frac{\log^2(1-t)}{t^2}\,dt$$

$$\frac{\zeta(2)\,\Gamma(1)}{\left(\Gamma\left(-\frac{1}{2}\right)\sqrt{\pi}\right)\pi} = \frac{\sqrt{\pi}}{6\oint_L\frac{e^t}{t}\,dt}\oint_L e^t\,\sqrt{t}\,dt$$

$$\frac{\zeta(2)\,\Gamma(1)}{\left(\Gamma\left(-\frac{1}{2}\right)\sqrt{\pi}\right)\pi} = \frac{\sqrt{\pi}}{6\oint_L -\frac{e^{-t}}{t}\,dt}\oint_L e^{-t}\,\sqrt{-t}\,dt$$

**Occurrence in convergents:**

$\frac{8}{\pi^4} \approx 0, \frac{1}{12}, \frac{5}{61}, \frac{6}{73}, \ldots$

(simple continued fraction convergent sequence)

From which:

12[1/(((zeta(2)*((gamma (1)*1/Pi))*1/((gamma (-1/2))*sqrtPi))))]^2 +1

**Input:**

$$12\left(\frac{1}{\zeta(2)\left(\Gamma(1)\times\frac{1}{\pi}\right)\times\frac{1}{\Gamma\left(-\frac{1}{2}\right)\sqrt{\pi}}}\right)^2+1$$

$\zeta(s)$ is the Riemann zeta function

$\Gamma(x)$ is the gamma function

**Exact result:**

1729

1729

This result is very near to the mass of candidate glueball **$f_0$(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

**Alternative representations:**

$$12\left(\frac{1}{\frac{\zeta(2)\Gamma(1)}{\left(\Gamma\left(-\frac{1}{2}\right)\sqrt{\pi}\right)\pi}}\right)^2+1=1+12\left(\frac{1}{\frac{e^0\,\zeta(2,1)}{\pi\left(e^{-\log G(-1/2)+\log G(1/2)}\sqrt{\pi}\right)}}\right)^2$$

$$12\left(\frac{1}{\frac{\zeta(2)\Gamma(1)}{\left(\Gamma\left(-\frac{1}{2}\right)\sqrt{\pi}\right)\pi}}\right)^2+1=1+12\left(\frac{1}{\frac{(1)_0\,\zeta(2,1)}{\pi\left((1)_{-\frac{3}{2}}\sqrt{\pi}\right)}}\right)^2$$

$$12\left(\frac{1}{\frac{\zeta(2)\Gamma(1)}{\left(\Gamma\left(-\frac{1}{2}\right)\sqrt{\pi}\right)\pi}}\right)^2+1=1+12\left(\frac{1}{\frac{e^0\,\zeta\left(2,\frac{1}{2}\right)}{3\pi\left(e^{-\log G(-1/2)+\log G(1/2)}\sqrt{\pi}\right)}}\right)^2$$

**Integral representations:**

$$12\left(\frac{1}{\frac{\zeta(2)\Gamma(1)}{\left(\Gamma\left(-\frac{1}{2}\right)\sqrt{\pi}\right)\pi}}\right)^2+1=1+\frac{48\pi^2(0!)^2\,\Gamma\left(-\frac{1}{2}\right)^2\sqrt{\pi}^2}{\Gamma(1)^2\left(\int_0^1\frac{\log^2(1-t)}{t^2}\,dt\right)^2}$$

$$12\left(\frac{1}{\frac{\zeta(2)\Gamma(1)}{\left(\Gamma\left(-\frac{1}{2}\right)\sqrt{\pi}\right)\pi}}\right)^2+1=1+\frac{48\pi^2(0!)^2\left(\oint_L\frac{e^t}{t}\,dt\right)^2\sqrt{\pi}^2}{\left(\int_0^1\frac{\log^2(1-t)}{t^2}\,dt\right)^2\left(\oint_L e^t\sqrt{t}\,dt\right)^2}$$

$$12\left(\frac{1}{\frac{\zeta(2)\Gamma(1)}{\left(\Gamma\left(-\frac{1}{2}\right)\sqrt{\pi}\right)\pi}}\right)^2+1=1+\frac{48\pi^2(0!)^2\left(\oint_L-\frac{e^{-t}}{t}\,dt\right)^2\sqrt{\pi}^2}{\left(\int_0^1\frac{\log^2(1-t)}{t^2}\,dt\right)^2\left(\oint_L e^{-t}\sqrt{-t}\,dt\right)^2}$$

(((Pi^2)/6 * 1/Pi))*1/(((-2sqrtPi)*sqrtPi))

**Input:**

$$\left(\frac{\pi^2}{6}\times\frac{1}{\pi}\right)\times\frac{1}{\left(-2\sqrt{\pi}\right)\sqrt{\pi}}$$

**Exact result:**

$$-\frac{1}{12}$$

**Decimal approximation:**

−0.083333333333333333333333333333333333333333333333333333333333...

-0.08333333…. = -1/12

## Series representations:

$$-\frac{\pi^2}{((2\sqrt{\pi})\sqrt{\pi})6\pi} = -\frac{\pi}{12\sqrt{-1+\pi}^2\left(\sum_{k=0}^{\infty}(-1+\pi)^{-k}\binom{\frac{1}{2}}{k}\right)^2}$$

$$-\frac{\pi^2}{((2\sqrt{\pi})\sqrt{\pi})6\pi} = -\frac{\pi}{12\sqrt{-1+\pi}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k(-1+\pi)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$-\frac{\pi^2}{((2\sqrt{\pi})\sqrt{\pi})6\pi} = -\frac{\pi\sqrt{\pi}^2}{3\left(\sum_{j=0}^{\infty}\mathrm{Res}_{s=-\frac{1}{2}+j}(-1+\pi)^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^2}$$

## Occurrence in convergents:

$$\frac{8}{\pi^4} \approx 0, \frac{1}{12}, \frac{5}{61}, \frac{6}{73}, \ldots$$

(simple continued fraction convergent sequence)

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**Example.** *With $f$ defined by (8.1), we have*

(i) $$f(\tfrac{1}{3}) = \frac{\pi^2}{24} - \tfrac{1}{8}\operatorname{Log}^2 2,$$

(ii) $$f\left(\frac{1}{\sqrt{5}}\right) = \frac{\pi^2}{20},$$

*and*

(iii) $$f(\sqrt{5}-2) = \frac{\pi^2}{30} - \tfrac{3}{8}\operatorname{Log}^2\left(\frac{\sqrt{5}-1}{2}\right).$$

From (i), we obtain:

(Pi^2)/24 -1/8 ln^2(2)

**Input:**

$$\frac{\pi^2}{24} - \frac{1}{8} \log^2(2)$$

log(x) is the natural logarithm

**Exact result:**

$$\frac{\pi^2}{24} - \frac{\log^2(2)}{8}$$

**Decimal approximation:**

0.3511768899722814310347159758706731758384183563523814110 75...

0.351176889972…

**Alternate form:**

$$\frac{1}{24}\left(\pi^2 - 3\log^2(2)\right)$$

**Alternative representations:**

$$\frac{\pi^2}{24} - \frac{\log^2(2)}{8} = \frac{\pi^2}{24} - \frac{1}{8}\log_e^2(2)$$

$$\frac{\pi^2}{24} - \frac{\log^2(2)}{8} = \frac{\pi^2}{24} - \frac{1}{8}\left(\log(a)\log_a(2)\right)^2$$

$$\frac{\pi^2}{24} - \frac{\log^2(2)}{8} = \frac{\pi^2}{24} - \frac{1}{8}\left(2\coth^{-1}(3)\right)^2$$

**Series representations:**

$$\frac{\pi^2}{24} - \frac{\log^2(2)}{8} = \frac{\pi^2}{24} - \frac{1}{8}\left(2 i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}\right)^2 \quad \text{for } x < 0$$

$$\frac{\pi^2}{24} - \frac{\log^2(2)}{8} =$$
$$\frac{\pi^2}{24} - \frac{1}{8}\left(\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}\right)^2$$

$$\frac{\pi^2}{24} - \frac{\log^2(2)}{8} = \frac{\pi^2}{24} - \frac{1}{8}\left(2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - z_0)^k z_0^{-k}}{k}\right)^2$$

**Integral representations:**

$$\frac{\pi^2}{24} - \frac{\log^2(2)}{8} = \frac{\pi^2}{24} - \frac{1}{8}\left(\int_1^2 \frac{1}{t}\, dt\right)^2$$

$$\frac{\pi^2}{24} - \frac{\log^2(2)}{8} = \frac{\pi^2}{24} + \frac{\left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\, ds\right)^2}{32\pi^2} \quad \text{for } -1 < \gamma < 0$$

From

$$\frac{\pi^2}{24} - \frac{\log^2(2)}{8} = \frac{\pi^2}{24} - \frac{1}{8}\left(\int_1^2 \frac{1}{t}\, dt\right)^2$$

We obtain:

π^2/24 - 1/8 ( integral_1^2 1/t dt)^2

**Input:**

$$\frac{\pi^2}{24} - \frac{1}{8}\left(\int_1^2 \frac{1}{t}\, dt\right)^2$$

**Result:**

$$\frac{\pi^2}{24} - \frac{\log^2(2)}{8} \approx 0.351177$$

0.351177

log(x) is the natural logarithm

From which:

4((0.351177+ 1/8 ( integral_1^2 1/t dt)^2))

**Input interpretation:**

$$4\left(0.351177 + \frac{1}{8}\left(\int_1^2 \frac{1}{t}\, dt\right)^2\right)$$

**Result:**

1.64493

1.64493

$$\frac{\pi^2}{6} \approx 1.6449340668$$

and:

4((0.351177+ 1/8 ( integral_1^2 1/t dt)^2))-(29-2)1/10^3

**Input interpretation:**

$$4\left(0.351177 + \frac{1}{8}\left(\int_1^2 \frac{1}{t}\,dt\right)^2\right) - (29-2)\times\frac{1}{10^3}$$

**Result:**

1.61793

1.61793 result that is a very good approximation to the value of the golden ratio 1.618033988749...

We have also:

(Pi^2)/(x) -1/8 ln^2(2) = 0.351176889972281431

**Input interpretation:**

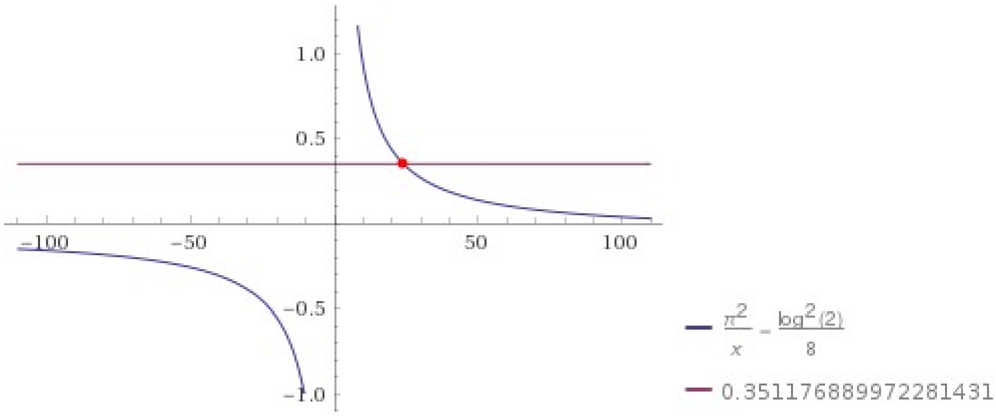
$$\frac{\pi^2}{x} - \frac{1}{8}\log^2(2) = 0.351176889972281431$$

$\log(x)$ is the natural logarithm

**Result:**

$$\frac{\pi^2}{x} - \frac{\log^2(2)}{8} = 0.351176889972281431$$

**Plot:**




**Alternate form assuming x is real:**

$$\frac{24.0000000000000}{x} = 1.000000000000000$$

**Alternate form:**

$$\frac{8\pi^2 - x\log^2(2)}{8x} = 0.351176889972281431$$

**Alternate form assuming x is positive:**

$1.00000000000000000\,x = 24.000000000000000$ (for $x \neq 0$)

**Solution:**

$x = 24$

24

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

**Integer solution:**

$x = 24$

and:

1/2(Pi^2) (x) -1/8 ln^2(2) = 0.351176889972281431

**Input interpretation:**

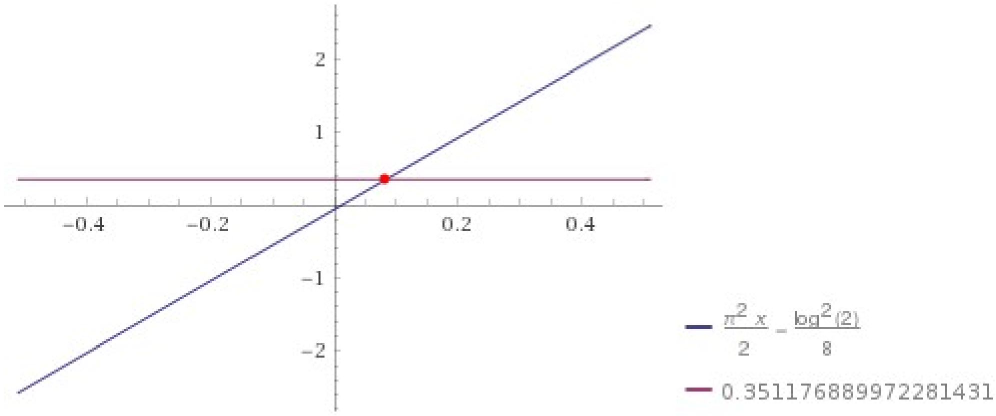
$$\frac{1}{2}\pi^2 x - \frac{1}{8}\log^2(2) = 0.351176889972281431$$

log(x) is the natural logarithm

**Result:**

$$\frac{\pi^2 x}{2} - \frac{\log^2(2)}{8} = 0.351176889972281431$$

**Plot:**

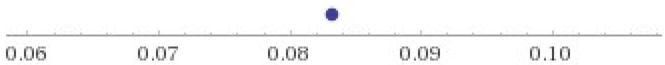



**Alternate forms:**

$$\frac{\pi^2 x}{2} - 0.411233516712056609 = 0$$

$$\frac{1}{8}\left(4\pi^2 x - \log^2(2)\right) = 0.351176889972281431$$

**Number line:**




**Solution:**

$$x \approx 0.0833333333333333333$$

0.083333333... = 1/12

[(((1/2(Pi^2) ((1/(1/12*(x-1))^0.5)) -1/8 ln^2(2) - 0.351176889972281431)))]

**Input interpretation:**

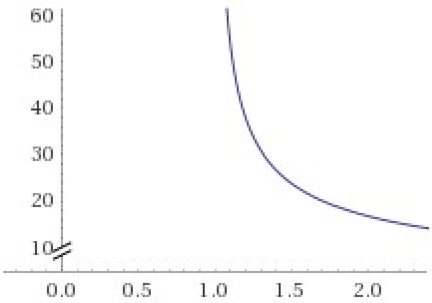
$$\frac{1}{2}\pi^2 \times \frac{1}{\sqrt{\frac{1}{12}(x-1)}} - \frac{1}{8}\log^2(2) - 0.351176889972281431$$

log(x) is the natural logarithm

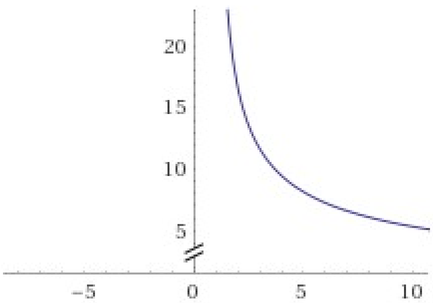
**Result:**

$$\frac{\sqrt{3}\,\pi^2}{\sqrt{x-1}} - 0.411233516712056609$$

**Plots:**




(x from -0.4 to 2.4)




(x from -8.7 to 10.7)

**Alternate forms:**

$$\frac{\sqrt{3}\,\pi^2 - 0.411233516712056609\sqrt{x-1}}{\sqrt{x-1}}$$

$$\left(17.094656273292169\left(-0.024056261216234407\,x + 1.00000000000000000\sqrt{x-1} + 0.024056261216234407\right)\right) / (1.0000000000000000\,x - 1.0000000000000000)$$

**Root:**

$x = 1729$

1729

This result is very near to the mass of candidate glueball **$f_0$(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

**Properties as a real function:**

**Domain**

$\{x \in \mathbb{R} : x > 1\}$

**Range**

$\{y \in \mathbb{R} : y > -\frac{411233516712056609}{1000000000000000000}\}$

**Injectivity**

injective (one-to-one)

$\mathbb{R}$ is the set of real numbers

**Series expansion at x = 0:**

$$\begin{cases} -(0.411233516712056609 + 17.0946562732921691147429728514233473\, i) - \frac{1}{2} i \sqrt{3}\, \pi^2 x - \frac{3}{8} i \sqrt{3}\, \pi^2 x^2 - \frac{5}{16} i \sqrt{3}\, \pi^2 x^3 - \frac{35}{128} i \sqrt{3}\, \pi^2 x^4 - \frac{63}{256} i \sqrt{3}\, \pi^2 x^5 + O(x^6) & \mathrm{Im}(x) \geq 0 \\ -(0.411233516712056609 - 17.0946562732921691147429728514233473\, i) + \frac{1}{2} i \sqrt{3}\, \pi^2 x + \frac{3}{8} i \sqrt{3}\, \pi^2 x^2 + \frac{5}{16} i \sqrt{3}\, \pi^2 x^3 + \frac{35}{128} i \sqrt{3}\, \pi^2 x^4 + \frac{63}{256} i \sqrt{3}\, \pi^2 x^5 + O(x^6) & \text{(otherwise)} \end{cases}$$

**Series expansion at x = ∞:**

$$-0.411233516712056609 + \sqrt{3}\, \pi^2 \sqrt{\frac{1}{x}} + \frac{1}{2} \sqrt{3}\, \pi^2 \left(\frac{1}{x}\right)^{3/2} + \frac{3}{8} \sqrt{3}\, \pi^2 \left(\frac{1}{x}\right)^{5/2} + \frac{5}{16} \sqrt{3}\, \pi^2 \left(\frac{1}{x}\right)^{7/2} + O\left(\left(\frac{1}{x}\right)^4\right)$$

(Puiseux series)

**Derivative:**

$$\frac{d}{dx}\left(\frac{\sqrt{3}\,\pi^2}{\sqrt{x-1}} - 0.411233516712056609\right) = -\frac{\sqrt{3}\,\pi^2}{2\,(x-1)^{3/2}}$$

**Indefinite integral:**

$$\int\left(\frac{\pi^2}{2\sqrt{\frac{x-1}{12}}} - \frac{\log^2(2)}{8} - 0.3511768899722814\,31\right)dx =$$

$$34.1893125465843382\sqrt{x - 1.000000000000000000} - 0.411233516712056609\,x + \text{constant}$$

**Limit:**

$$\lim_{x\to\pm\infty}\left(-0.411233516712056609 + \frac{\sqrt{3}\,\pi^2}{\sqrt{-1+x}}\right) = -0.41123351671205661$$

From (iii), we obtain:

(Pi^2)/30 – 3/8 ln^2((sqrt5-1)/2)

**Input:**

$$\frac{\pi^2}{30} - \frac{3}{8}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)$$

$\log(x)$ is the natural logarithm

**Decimal approximation:**

0.2421500056531973901081346316173802528376375989728726109 91...

0.2421500056531…

**Alternate forms:**

$$\frac{\pi^2}{30} - \frac{3}{8}\,\text{csch}^{-1}(2)^2$$

$$\frac{1}{120}\left(4\pi^2 - 45\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)\right)$$

$$\frac{\pi^2}{30} - \frac{3}{8}\left(\log\left(\sqrt{5}-1\right) - \log(2)\right)^2$$

$\text{csch}^{-1}(x)$ is the inverse hyperbolic cosecant function

**Alternative representations:**

$$\frac{\pi^2}{30}-\frac{1}{8}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3=\frac{\pi^2}{30}-\frac{3}{8}\log_e^2\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)$$

$$\frac{\pi^2}{30}-\frac{1}{8}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3=\frac{\pi^2}{30}-\frac{3}{8}\left(\log(a)\log_a\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\right)^2$$

$$\frac{\pi^2}{30}-\frac{1}{8}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3=\frac{\pi^2}{30}-\frac{3}{8}\left(-\text{Li}_1\left(1+\frac{1}{2}\left(1-\sqrt{5}\right)\right)\right)^2$$

**Series representations:**

$$\frac{\pi^2}{30}-\frac{1}{8}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3=\frac{\pi^2}{30}-\frac{3}{8}\left(\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-3+\sqrt{5}\right)^k}{k}\right)^2$$

$$\frac{\pi^2}{30}-\frac{1}{8}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3=$$
$$\frac{\pi^2}{30}+\frac{3}{8}\left(2\pi\left\lfloor\frac{\arg\left(-1+\sqrt{5}-2x\right)}{2\pi}\right\rfloor-i\left(\log(x)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-1+\sqrt{5}-2x\right)^k x^{-k}}{k}\right)\right)^2$$

for $x<0$

$$\frac{\pi^2}{30}-\frac{1}{8}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3=$$
$$\frac{\pi^2}{30}-\frac{3}{8}\left(2i\pi\left\lfloor\frac{\arg\left(\frac{1}{2}\left(-1+\sqrt{5}\right)-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-1+\sqrt{5}-2x\right)^k x^{-k}}{k}\right)^2$$

for $x<0$

**Integral representation:**

$$\frac{\pi^2}{30}-\frac{1}{8}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3=\frac{\pi^2}{30}-\frac{3}{8}\left(\int_1^{\frac{1}{2}\left(-1+\sqrt{5}\right)}\frac{1}{t}\,dt\right)^2$$

(Pi^2)/30 – 3/8 ln^2(x) = 0.24215000565319739

**Input interpretation:**

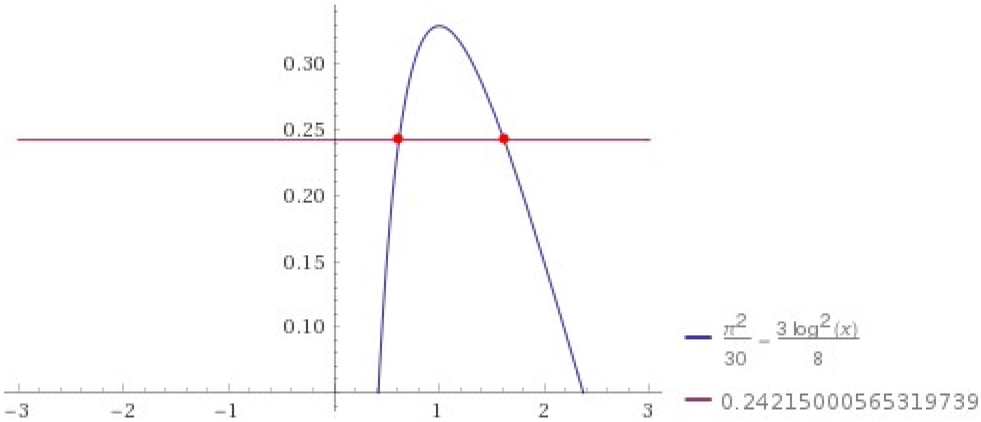
$$\frac{\pi^2}{30}-\frac{3}{8}\log^2(x)=0.24215000565319739$$

$\log(x)$ is the natural logarithm

**Result:**

$$\frac{\pi^2}{30} - \frac{3 \log^2(x)}{8} = 0.24215000565319739$$

**Plot:**




**Alternate form:**

$$\frac{1}{120}\left(4\pi^2 - 45 \log^2(x)\right) = 0.24215000565319739$$

**Alternate form assuming x is positive:**

$$1.000000000000000 \log^2(x) = 0.231564820577194$$

**Solutions:**

$x \approx 0.6180339887498948$

0.6180339887498948

$x \approx 1.6180339887498948$

1.6180339887498948

5(((Pi^2)/30 – 3/8 ln^2((sqrt5-1)/2)))

**Input:**

$$5\left(\frac{\pi^2}{30} - \frac{3}{8} \log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)\right)$$

$\log(x)$ is the natural logarithm

**Decimal approximation:**

1.210750028265986950540673158086901264188187994864363054956...

1.2107500282659…

**Alternate forms:**

$$\frac{\pi^2}{6} - \frac{15}{8} \operatorname{csch}^{-1}(2)^2$$

$$\frac{\pi^2}{6} - \frac{15}{8} \log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)$$

$$\frac{1}{24}\left(4\pi^2 - 45\log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)\right)$$

$\operatorname{csch}^{-1}(x)$ is the inverse hyperbolic cosecant function

**Alternative representations:**

$$5\left(\frac{\pi^2}{30} - \frac{1}{8}\log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)3\right) = 5\left(\frac{\pi^2}{30} - \frac{3}{8}\log_e^2\left(\frac{1}{2}\left(-1 + \sqrt{5}\right)\right)\right)$$

$$5\left(\frac{\pi^2}{30} - \frac{1}{8}\log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)3\right) = 5\left(\frac{\pi^2}{30} - \frac{3}{8}\left(\log(a)\log_a\left(\frac{1}{2}\left(-1 + \sqrt{5}\right)\right)\right)^2\right)$$

$$5\left(\frac{\pi^2}{30} - \frac{1}{8}\log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)3\right) = 5\left(\frac{\pi^2}{30} - \frac{3}{8}\left(-\operatorname{Li}_1\left(1 + \frac{1}{2}\left(1 - \sqrt{5}\right)\right)\right)^2\right)$$

**Series representations:**

$$5\left(\frac{\pi^2}{30} - \frac{1}{8}\log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)3\right) = \frac{\pi^2}{6} - \frac{15}{8}\left(\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-3 + \sqrt{5}\right)^k}{k}\right)^2$$

$$5\left(\frac{\pi^2}{30} - \frac{1}{8}\log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)3\right) = 5\left(\frac{\pi^2}{30} + \frac{3}{8}\left(2\pi\left\lfloor\frac{\arg(-1 + \sqrt{5} - 2x)}{2\pi}\right\rfloor - i\left(\log(x) - \sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-1 + \sqrt{5} - 2x\right)^k x^{-k}}{k}\right)\right)^2\right)$$

for $x < 0$

$$5\left(\frac{\pi^2}{30} - \frac{1}{8}\log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)3\right) = 5\left(\frac{\pi^2}{30} - \frac{3}{8}\left(2i\pi\left\lfloor\frac{\arg\left(\frac{1}{2}\left(-1 + \sqrt{5}\right) - x\right)}{2\pi}\right\rfloor + \log(x) - \sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-1 + \sqrt{5} - 2x\right)^k x^{-k}}{k}\right)^2\right)$$

for $x < 0$

**Integral representation:**

$$5\left(\frac{\pi^2}{30}-\frac{1}{8}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)3\right)=\frac{\pi^2}{6}-\frac{15}{8}\left(\int_1^{\frac{1}{2}\left(-1+\sqrt{5}\right)}\frac{1}{t}\,dt\right)^2$$

From

$$\frac{\pi^2}{6}-\frac{15}{8}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)$$

We obtain:

x - 15/8 log^2(1/2 (-1 + sqrt(5))) =1.21075002826598695

**Input interpretation:**

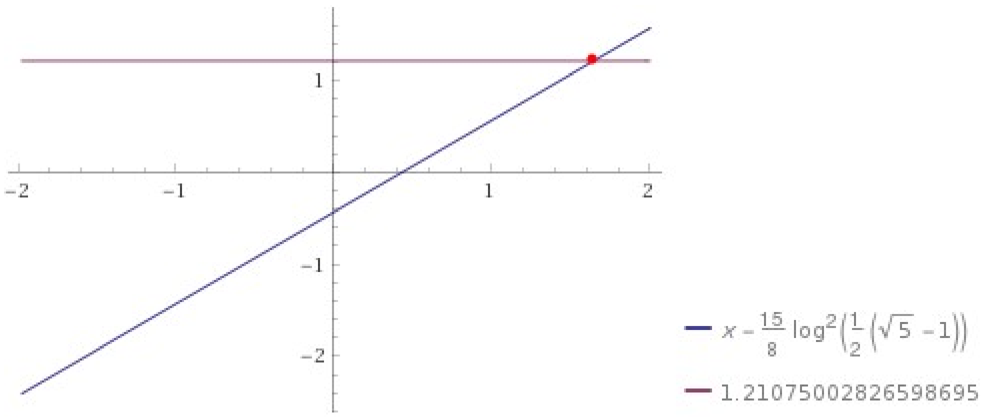
$$x-\frac{15}{8}\log^2\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)=1.21075002826598695$$

$\log(x)$ is the natural logarithm

**Result:**

$$x-\frac{15}{8}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)=1.21075002826598695$$

**Plot:**




**Alternate forms:**

$$x-1.64493406684822644=0$$

$$x-\frac{15}{8}\operatorname{csch}^{-1}(2)^2=1.21075002826598695$$

$$x-\frac{15}{8}\left(\log\left(\sqrt{5}-1\right)-\log(2)\right)^2=1.21075002826598695$$

$\operatorname{csch}^{-1}(x)$ is the inverse hyperbolic cosecant function

**Alternate form assuming x>0:**

$$x - \frac{15}{8} \log^2\left(\frac{2}{\sqrt{5} - 1}\right) = 1.21075002826598695$$

**Solution:**

$$x \approx 1.64493406684822644$$

$1.64493406684822644 = \zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

1/2(((π^2/6 - 15/8 log^2(1/2 (-1 + sqrt(5))))))

**Input:**

$$\frac{1}{2}\left(\frac{\pi^2}{6} - \frac{15}{8} \log^2\left(\frac{1}{2}\left(-1 + \sqrt{5}\right)\right)\right)$$

log(x) is the natural logarithm

**Decimal approximation:**

0.6053750141329934752703365790434506320940939974321815274 78...

0.605375014132…

**Alternate forms:**

$$\frac{1}{48}\left(4\pi^2 - 45 \operatorname{csch}^{-1}(2)^2\right)$$

$$\frac{\pi^2}{12} - \frac{15}{16} \log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)$$

$$\frac{1}{48}\left(4\pi^2 - 45 \log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)\right)$$

$\operatorname{csch}^{-1}(x)$ is the inverse hyperbolic cosecant function

**Alternative representations:**

$$\frac{1}{2}\left(\frac{\pi^2}{6} - \frac{15}{8} \log^2\left(\frac{1}{2}\left(-1 + \sqrt{5}\right)\right)\right) = \frac{1}{2}\left(\frac{\pi^2}{6} - \frac{15}{8} \log_e^2\left(\frac{1}{2}\left(-1 + \sqrt{5}\right)\right)\right)$$

$$\frac{1}{2}\left(\frac{\pi^2}{6}-\frac{15}{8}\log^2\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\right)=\frac{1}{2}\left(\frac{\pi^2}{6}-\frac{15}{8}\left(\log(a)\log_a\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\right)^2\right)$$

$$\frac{1}{2}\left(\frac{\pi^2}{6}-\frac{15}{8}\log^2\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\right)=\frac{1}{2}\left(\frac{\pi^2}{6}-\frac{15}{8}\left(-\mathrm{Li}_1\left(1+\frac{1}{2}\left(1-\sqrt{5}\right)\right)\right)^2\right)$$

**Series representations:**

$$\frac{1}{2}\left(\frac{\pi^2}{6}-\frac{15}{8}\log^2\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\right)=\frac{\pi^2}{12}-\frac{15}{16}\left(\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-3+\sqrt{5}\right)^k}{k}\right)^2$$

$$\frac{1}{2}\left(\frac{\pi^2}{6}-\frac{15}{8}\log^2\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\right)=\frac{1}{2}\left(\frac{\pi^2}{6}+\frac{15}{8}\left(2\pi\left\lfloor\frac{\arg\left(-1+\sqrt{5}-2x\right)}{2\pi}\right\rfloor-i\left(\log(x)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-1+\sqrt{5}-2x\right)^k x^{-k}}{k}\right)\right)^2\right)$$

for $x<0$

$$\frac{1}{2}\left(\frac{\pi^2}{6}-\frac{15}{8}\log^2\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\right)=\frac{1}{2}\left(\frac{\pi^2}{6}-\frac{15}{8}\left(2i\pi\left\lfloor\frac{\arg\left(\frac{1}{2}\left(-1+\sqrt{5}\right)-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-1+\sqrt{5}-2x\right)^k x^{-k}}{k}\right)^2\right)$$

for $x<0$

**Integral representation:**

$$\frac{1}{2}\left(\frac{\pi^2}{6}-\frac{15}{8}\log^2\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\right)=\frac{\pi^2}{12}-\frac{15}{16}\left(\int_1^{\frac{1}{2}\left(-1+\sqrt{5}\right)}\frac{1}{t}\,dt\right)^2$$

From

$$\frac{\pi^2}{12}-\frac{15}{16}\log^2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)$$

We obtain:

π^2*x - 15/16 log^2(1/2 (-1 + sqrt(5))) = 0.6053750141329

**Input interpretation:**

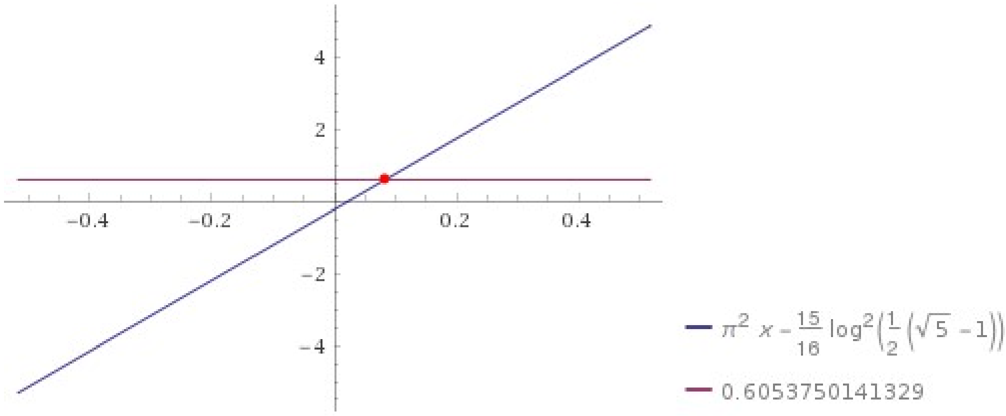
$$\pi^2 x - \frac{15}{16} \log^2\left(\frac{1}{2}\left(-1 + \sqrt{5}\right)\right) = 0.6053750141329$$

$\log(x)$ is the natural logarithm

**Result:**

$$\pi^2 x - \frac{15}{16} \log^2\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right) = 0.6053750141329$$

**Plot:**




**Alternate forms:**

$$\pi^2 x - 0.8224670334240 = 0$$

$$\pi^2 x - \frac{15}{16} \operatorname{csch}^{-1}(2)^2 = 0.6053750141329$$

$$\pi^2 x - \frac{15}{16} \left(\log\left(\sqrt{5} - 1\right) - \log(2)\right)^2 = 0.6053750141329$$

$\operatorname{csch}^{-1}(x)$ is the inverse hyperbolic cosecant function

**Alternate form assuming x>0:**

$$\pi^2 x - \frac{15}{16} \log^2\left(\frac{2}{\sqrt{5} - 1}\right) = 0.6053750141329$$

**Solution:**

$x \approx 0.0833333333333$

0.0833333... = 1/12

Page 275-276

(i) $$\int_0^{1/\sqrt{3}} \frac{\tan^{-1} t}{t}\, dt = -\frac{\pi}{12}\,\mathrm{Log}\,3 - \frac{5\pi^2}{18\sqrt{3}} + \frac{5\sqrt{3}}{4}\sum_{k=0}^{\infty} \frac{1}{(3k+1)^2},$$

(-Pi/12) ln(3) – (5Pi^2) / (18sqrt3) + (5sqrt3) / 4  *  sum (1/(3k+1)^2), k = 0..infinity

**Input interpretation:**

$$-\frac{\pi}{12}\log(3) - \frac{5\pi^2}{18\sqrt{3}} + \left(\frac{1}{4}\left(5\sqrt{3}\right)\right)\sum_{k=0}^{\infty} \frac{1}{(3k+1)^2}$$

log(x) is the natural logarithm

**Result:**

$$-\frac{5\pi^2}{18\sqrt{3}} - \frac{1}{12}\pi\log(3) + \frac{5\,\psi^{(1)}\left(\frac{1}{3}\right)}{12\sqrt{3}} \approx 0.558169$$

0.558169

$\psi^{(n)}(x)$ is the $n^{\text{th}}$ derivative of the digamma function

**Alternate forms:**

$$-\frac{10\pi^2 + 3\sqrt{3}\,\pi\log(3) - 15\,\psi^{(1)}\left(\frac{1}{3}\right)}{36\sqrt{3}}$$

$$\frac{1}{108}\left(-10\sqrt{3}\,\pi^2 - 9\pi\log(3) + 15\sqrt{3}\,\psi^{(1)}\left(\frac{1}{3}\right)\right)$$

(((-Pi*x) ln(3) – (5Pi^2) / (18sqrt3) + (5sqrt3) / 4  *  sum (1/(3k+1)^2), k = 0..infinity))  = 0.558169

**Input interpretation:**

$$(-\pi x)\log(3) - \frac{5\pi^2}{18\sqrt{3}} + \left(\frac{1}{4}\left(5\sqrt{3}\right)\right)\sum_{k=0}^{\infty} \frac{1}{(3k+1)^2} = 0.558169$$

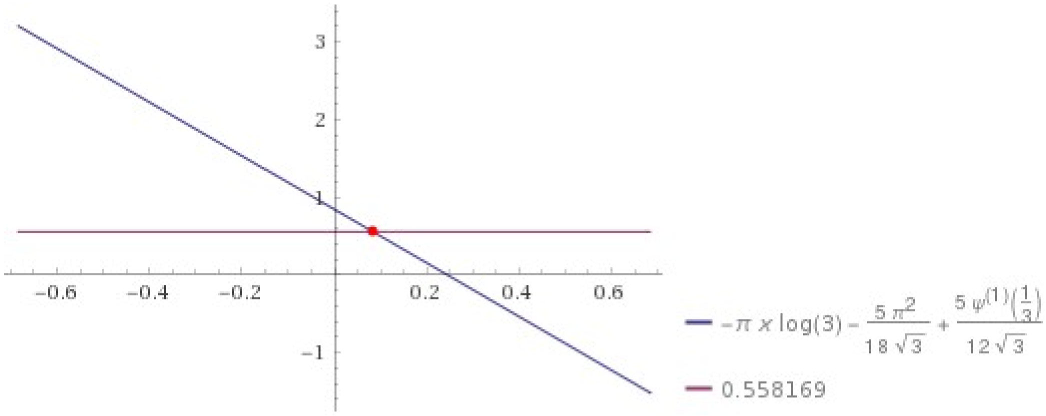
log(x) is the natural logarithm

**Result:**

$$-\pi x \log(3) - \frac{5\pi^2}{18\sqrt{3}} + \frac{5\psi^{(1)}\left(\frac{1}{3}\right)}{12\sqrt{3}} = 0.558169$$

$\psi^{(n)}(x)$ is the $n^{\text{th}}$ derivative of the digamma function

**Plot:**




**Alternate forms:**

$$0.287616 - \pi x \log(3) = 0$$

$$\frac{1}{108}\left(-108\pi x \log(3) - 10\sqrt{3}\,\pi^2 + 15\sqrt{3}\,\psi^{(1)}\left(\frac{1}{3}\right)\right) = 0.558169$$

**Solution:**

$x \approx 0.0833332$

0.0833332

**Possible closed form:**

$$\frac{1}{12} \approx 0.08333333333$$

2((((-Pi/12) ln(3) – (5Pi^2) / (18sqrt3) + (5sqrt3) / 4 * sum (1/(3k+1)^2), k = 0..infinity)))

**Input interpretation:**

$$2\left(-\frac{\pi}{12}\log(3) - \frac{5\pi^2}{18\sqrt{3}} + \left(\frac{1}{4}\left(5\sqrt{3}\right)\right)\sum_{k=0}^{\infty}\frac{1}{(3k+1)^2}\right)$$

$\log(x)$ is the natural logarithm

**Result:**

$$2\left(-\frac{5\pi^2}{18\sqrt{3}} - \frac{1}{12}\pi\log(3) + \frac{5\,\psi^{(1)}\left(\frac{1}{3}\right)}{12\sqrt{3}}\right) \approx 1.11634$$

$\psi^{(n)}(x)$ is the $n^{\text{th}}$ derivative of the digamma function

**Alternate forms:**

$$-\frac{10\pi^2 + 3\sqrt{3}\,\pi\log(3) - 15\,\psi^{(1)}\left(\frac{1}{3}\right)}{18\sqrt{3}}$$

$$\frac{1}{54}\left(-10\sqrt{3}\,\pi^2 - 9\pi\log(3) + 15\sqrt{3}\,\psi^{(1)}\left(\frac{1}{3}\right)\right)$$

$$-\frac{5\pi^2}{9\sqrt{3}} - \frac{1}{6}\pi\log(3) + \frac{5\,\psi^{(1)}\left(\frac{1}{3}\right)}{6\sqrt{3}}$$

and:

-(5 π^2)/(9 sqrt(3)) - (x/Pi) log(3) + (5 polygamma(1, 1/3))/(6 sqrt(3)) = 1.116337294812

**Input interpretation:**

$$-\frac{5\pi^2}{9\sqrt{3}} - \frac{x}{\pi}\log(3) + \frac{5\,\psi^{(1)}\left(\frac{1}{3}\right)}{6\sqrt{3}} = 1.116337294812$$

$\log(x)$ is the natural logarithm

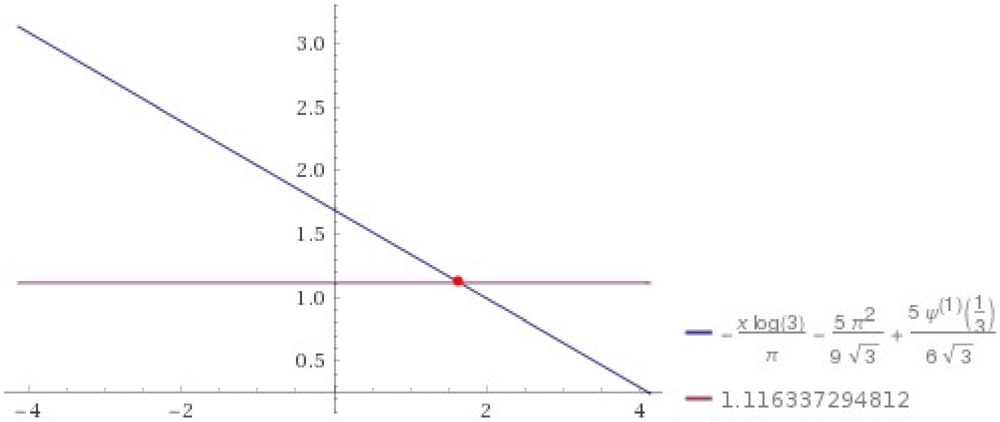
$\psi^{(n)}(x)$ is the $n^{\text{th}}$ derivative of the digamma function

**Result:**

$$-\frac{x\log(3)}{\pi} - \frac{5\pi^2}{9\sqrt{3}} + \frac{5\,\psi^{(1)}\left(\frac{1}{3}\right)}{6\sqrt{3}} = 1.116337294812$$

1.116337294812

**Plot:**



**Alternate forms:**

$$0.575232049204 - \frac{x \log(3)}{\pi} = 0$$

$$\frac{5\left(3\,\psi^{(1)}\left(\frac{1}{3}\right) - 2\,\pi^2\right)}{18\,\sqrt{3}} - \frac{x \log(3)}{\pi} = 1.116337294812$$

$$-\frac{54\,x \log(3) + 10\,\sqrt{3}\,\pi^3 - 15\,\sqrt{3}\,\pi\,\psi^{(1)}\left(\frac{1}{3}\right)}{54\,\pi} = 1.116337294812$$

**Solution:**

$x \approx 1.64493406685$

$1.64493406685 = \zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

-(5 π^2)/(9 sqrt(3)) - (x/Pi+(8+1/2)1/10^3) log(3) + (5 polygamma(1, 1/3))/(6 sqrt(3)) = 1.116337294812

**Input interpretation:**

$$-\frac{5\,\pi^2}{9\,\sqrt{3}} - \left(\frac{x}{\pi} + \left(8 + \frac{1}{2}\right) \times \frac{1}{10^3}\right)\log(3) + \frac{5\,\psi^{(1)}\left(\frac{1}{3}\right)}{6\,\sqrt{3}} = 1.116337294812$$

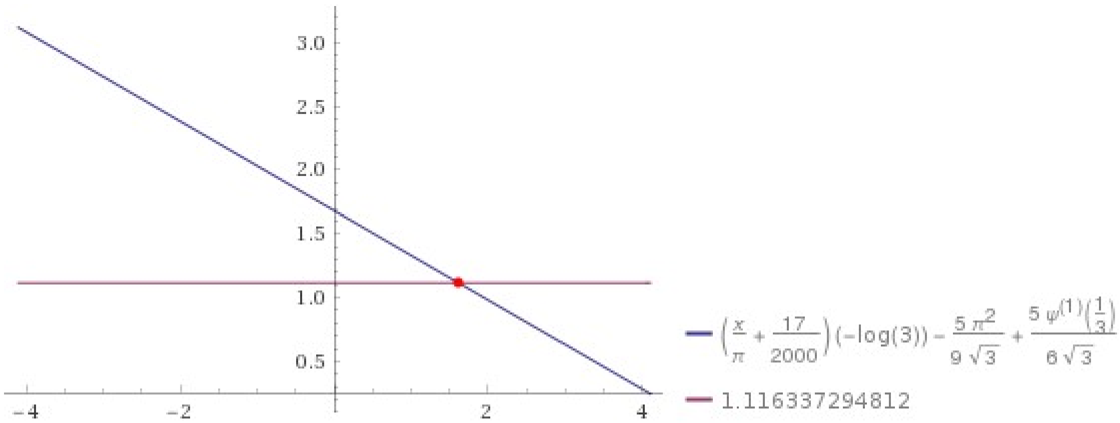
$\log(x)$ is the natural logarithm

$\psi^{(n)}(x)$ is the $n^{\text{th}}$ derivative of the digamma function

**Result:**

$$\left(\frac{x}{\pi} + \frac{17}{2000}\right)(-\log(3)) - \frac{5\,\pi^2}{9\,\sqrt{3}} + \frac{5\,\psi^{(1)}\left(\frac{1}{3}\right)}{6\,\sqrt{3}} = 1.116337294812$$

**Plot:**



**Alternate forms:**

$$0.565893844750-\frac{x\log(3)}{\pi}=0$$

$$-\frac{54000\,x\log(3)+10000\sqrt{3}\,\pi^3+459\,\pi\log(3)-15000\sqrt{3}\,\pi\,\psi^{(1)}\left(\frac{1}{3}\right)}{54000\,\pi}=1.116337294812$$

$$-\frac{x\log(3)}{\pi}-\frac{5\pi^2}{9\sqrt{3}}+\frac{5000\sqrt{3}\,\psi^{(1)}\left(\frac{1}{3}\right)-153\log(3)}{18000}=1.116337294812$$

**Expanded form:**

$$-\frac{x\log(3)}{\pi}-\frac{5\pi^2}{9\sqrt{3}}-\frac{17\log(3)}{2000}+\frac{5\,\psi^{(1)}\left(\frac{1}{3}\right)}{6\sqrt{3}}=1.116337294812$$

**Solution:**

$x\approx 1.61823052929$

1.61823052929 result that is a very good approximation to the value of the golden ratio 1.618033988749...

7/((((-Pi/12) ln(3) – (5Pi^2) / (18sqrt3) + (5sqrt3) / 4 * sum (1/(3k+1)^2), k = 0..infinity)))

**Input interpretation:**

$$\frac{7}{-\frac{\pi}{12}\log(3)-\frac{5\pi^2}{18\sqrt{3}}+\left(\frac{1}{4}\left(5\sqrt{3}\right)\right)\sum_{k=0}^{\infty}\frac{1}{(3k+1)^2}}$$

log(x) is the natural logarithm

**Result:**

$$\frac{7}{-\frac{5\pi^2}{18\sqrt{3}}-\frac{1}{12}\pi\log(3)+\frac{5\,\psi^{(1)}\left(\frac{1}{3}\right)}{12\sqrt{3}}}\approx 12.541$$

12.541 result very near to 4π and to the value of black hole entropy 12.5664, that is equal to ln(196884)

$\psi^{(n)}(x)$ is the $n^{\text{th}}$ derivative of the digamma function

**Alternate forms:**

$$-\frac{252\sqrt{3}}{10\pi^2 + 3\sqrt{3}\ \pi\log(3) - 15\,\psi^{(1)}\left(\frac{1}{3}\right)}$$

$$-\frac{756}{10\sqrt{3}\ \pi^2 + 9\pi\log(3) - 15\sqrt{3}\ \psi^{(1)}\left(\frac{1}{3}\right)}$$

$$\frac{1}{-\frac{5\pi^2}{126\sqrt{3}} - \frac{1}{84}\pi\log(3) + \frac{5\psi^{(1)}\left(\frac{1}{3}\right)}{84\sqrt{3}}}$$

**Observations**

Figs.

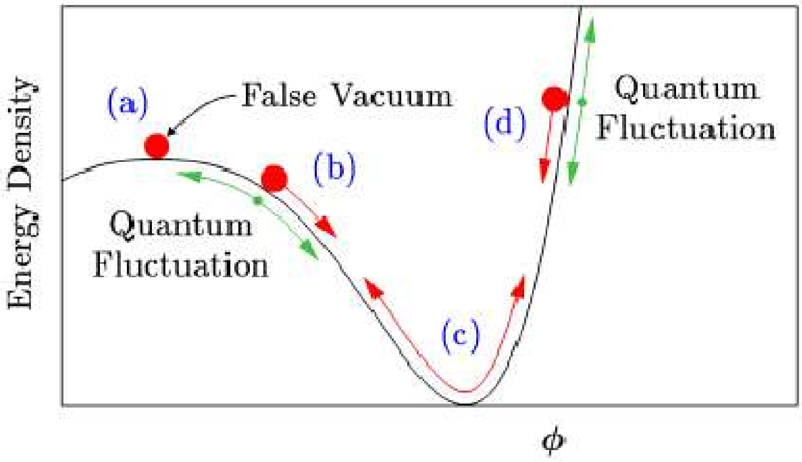
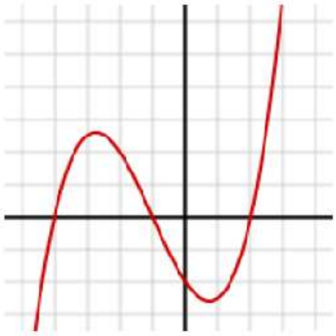



FIG. 1: In simple inflationary models, the universe at early times is dominated by the potential energy density of a scalar field, $\phi$. Red arrows show the classical motion of $\phi$. When $\phi$ is near region (a), the energy density will remain nearly constant, $\rho \cong \rho_f$, even as the universe expands. Moreover, cosmic expansion acts like a frictional drag, slowing the motion of $\phi$: Even near regions (b) and (d), $\phi$ behaves more like a marble moving in a bowl of molasses, slowly creeping down the side of its potential, rather than like a marble sliding down the inside of a polished bowl. During this period of "slow roll," $\rho$ remains nearly constant. Only after $\phi$ has slid most of the way down its potential will it begin to oscillate around its minimum, as in region (c), ending inflation.



Graph of a cubic function with 3 real roots (where the curve crosses the horizontal axis at $y = 0$). The case shown has two critical points. Here the function is $f(x) = (x^3 + 3x^2 - 6x - 8)/4$.

The ratio between $M_0$ and q

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{\left(3\sqrt{3}\right) M_s}{2}.$$

i.e. the gravitating mass $M_0$ and the Wheelerian mass q of the wormhole, is equal to:

$$\frac{\sqrt{3\,(2.17049 \times 10^{37})^2 - 0.001^2}}{\frac{1}{2}\,((3\sqrt{3})\,(4.2 \times 10^6 \times 1.9891 \times 10^{30}))}$$

1.7320507879051944207039476256710181600835665488020824605 20...

1.7320507879

$1.7320507879 \approx \sqrt{3}$ that is the ratio between the gravitating mass $M_0$ and the Wheelerian mass q of the wormhole

We note that:

$$\left(-\frac{1}{2} + \frac{i}{2}\sqrt{3}\right) - \left(-\frac{1}{2} - \frac{i}{2}\sqrt{3}\right)$$

$i$ is the imaginary unit

$i\sqrt{3}$

$1.7320508075688772935274463415058723669428052538103806280558\ldots\, i$

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

1.73205

This result is very near to the ratio between $M_0$ and q, that is equal to 1.7320507879 $\approx \sqrt{3}$

With regard $\sqrt{3}$ , we note that is a fundamental value of the formula structure that we need to calculate a Cubic Equation

We have that the previous result

$$\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right) = i\sqrt{3} =$$

$$= 1.7320508075688772935274463415058723669428052538103806280558\ldots\, i$$

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

can be related with:

$$u^2(-u)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)+v^2(-v)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)=q$$

Considering:

$$(-1)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)-(-1)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=q$$

$$= i\sqrt{3} = 1.7320508075688772935274463415058723669428052538103806280558\ldots\, i$$

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

Thence:

$$\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right) \Rightarrow$$

$$\Rightarrow (-1)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)-(-1)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=q=1.73205\approx\sqrt{3}$$

**We observe how the graph above, concerning the cubic function, is very similar to the graph that represent the scalar field (in red). It is possible to hypothesize that cubic functions and cubic equations, with their roots, are connected to the scalar field.**

*From:*
*https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA*

*Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.*

*Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.*

*Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125.*
*In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.*

*From Wikipedia*

*In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field $\phi$ and a Dirac field $\psi$. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.*

*Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV*

*Note that:*

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

*Thence:*

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots$$

*And*

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

*That are connected with 64, 128, 256, 512, 1024 and 4096 = $64^2$*

*(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)*

*All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.*

*In mathematics, the Fibonacci numbers, commonly denoted $F_n$, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.*
*Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences*

*The beginning of the sequence is thus:*

*0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...*

*The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.*

*The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.*

*The sequence of Lucas numbers is:*

*2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……*

*All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.*

*A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:*

*2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).*

*In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals*

**We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.**

## References


**D-dimensional charged Anti-de-Sitter black holes in f(T) gravity**
*A. M. Awad1,2, S. Capozziello4,5,6 and G. G. L. Nashed* - arXiv:1706.01773v3 [gr-qc] 18 Jul 2017

*B Candelpergher*. **Ramanujan summation of divergent series**. Lectures notes in mathematics, 2185, 2017. hal-01150208v2

*Bruce C. Berndt*
**Ramanujan's Notebooks Part 1** - 1985 by Springer-Verlag New York Inc.

**1 + 2 + 3 + 4 + ...**

*Frank Thorne*, University of South Carolina - Dartmouth College - May 9, 2013