

Absorption of Spin of a Plane Circularly Polarized Wave

Radi I. Khrapko¹

Department of Physics, Moscow Aviation Institute, Moscow, Russia

Abstract:

The use of the electrodynamics spin tensor in parallel with the energy-momentum tensor proves absorption of spin and energy of a plane circularly polarized electromagnetic wave and so confirms the absorption, which was calculated in a previous paper by the dynamical way, i.e. by the use of the concept of mechanical torque.

Key Words: classical spin; circular polarization; spin tensor

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1. Introduction

According to [1-8], the electrodynamics spin density is proportional to the *gradient* of the electromagnetic energy density. See details in [9]. Therefore, in particular, plane waves do not contain spin at all, and consequently a medium that absorbs a plane electromagnetic wave does not experience a torque.

According to another concept [10-11] (see also e.g. [12-23]) the spin density is proportional to the electromagnetic energy density itself and is described now by a spin tensor $Y^{\lambda\mu\nu}$ [24-26]. Therefore, in particular, a circularly polarized plane electromagnetic wave carries an angular momentum volume *density*.

This concept does not associate spin with a moment of a linear momentum, or even with a motion of matter. **Hehl** writes about spin of an electron [27]:

“The current density in Dirac’s theory can be split into a convective part and a polarization part. The polarization part is determined by the spin distribution of the electron field. It should lead to *no* energy flux in the rest system of the electron because the genuine spin ‘motion’ take place only within a region of the order of the Compton wavelength of the electron”.

The classic Beth’s experiment [28] confirms this concept [17].

We confirmed the presence of spin in a plane circularly polarized electromagnetic wave within the framework of the concept of [10, 11] in [14]. In that paper, the torque volume density acting on the absorber during the absorption of such a wave was calculated according to the well-known formula $\tau_{\perp} = \mathbf{P} \times \mathbf{E}$ (see also [28]). In the present article, the same result is obtained directly using the spin tensor. To demonstrate the naturalness of using the spin tensor, the volume energy density released in the absorber is calculated in parallel using the energy-momentum tensor.

2. A symmetric absorber

As in [14], as the absorber, we consider a symmetric absorber. We call "symmetric absorber" a medium, which is both dielectric and magnetic with $\varepsilon = \mu$. Such a medium does not require generating a reflected wave; this simplifies formulas.

So, let a perfect plane monochromatic circularly polarized electromagnetic wave

$$F_{\alpha\beta} = \{E_x = 1, E_y = i, B^x = -i/c, B^y = 1/c\} \exp(ikz - i\omega t)E, \quad ck = \omega \quad (1)$$

¹ Email address: khrapko_ri@hotmail.com, khrapko_ri@mai.ru <http://khrapkori.wmsite.ru>

$$F^{\mu\nu} = \{D^x = \varepsilon_0, D^y = i\varepsilon_0, H_x = -i\sqrt{\varepsilon_0/\mu_0}, H_y = \sqrt{\varepsilon_0/\mu_0}\} \exp(ikz - i\omega t)E, \quad (2)$$

impinges normally on a flat x,y-surface of the absorber, which is characterized by complex permittivity and permeability $\tilde{\varepsilon} = \tilde{\mu}$ (we indicate complex numbers by the *breve* mark when necessary).

So the wave propagated in the absorber is described by the formulas

$$F_{\alpha\beta} = \{E_x = 1, E_y = i, B^x = -i\tilde{\varepsilon}/c, B^y = \tilde{\varepsilon}/c\} \exp(i\tilde{k}z - i\omega t)E, \quad (3)$$

$$c\tilde{k} = \tilde{\varepsilon}\omega, \tilde{k} = k_1 + ik_2, \tilde{\varepsilon} = \varepsilon_1 + i\varepsilon_2$$

$$F^{\mu\nu} = \{D^x = \tilde{\varepsilon}\varepsilon_0, D^y = i\tilde{\varepsilon}\varepsilon_0, H_x = -i\sqrt{\varepsilon_0/\mu_0}, H_y = \sqrt{\varepsilon_0/\mu_0}\} \exp(i\tilde{k}z - i\omega t)E, \quad (4)$$

3. Energy and spin flux

Using the Maxwell tensor $T^{\mu\nu} = -g^{\mu\lambda}F_{\lambda\alpha}F^{\nu\alpha} + g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}/4$ yields the Poynting vector in the vacuum

$$\langle c^2 T^{Tz} \rangle = \Re\{-(\bar{F}_x F^{zx} + \bar{F}_y F^{zy})\}/2 = \mathbf{E} \times \mathbf{H} = \sqrt{\varepsilon_0/\mu_0} E^2 \quad [\text{J/m}^2\text{s}] \quad (5)$$

and the Poynting vector in the absorber

$$\mathbf{E} \times \mathbf{H} = \sqrt{\varepsilon_0/\mu_0} \exp(-2k_2 z) E^2 \quad (6)$$

(the bar means complex conjugation). Power volume density of the released energy in the absorber is

$$w = -\partial_z (\mathbf{E} \times \mathbf{H}) = 2k_2 \sqrt{\varepsilon_0/\mu_0} \exp(-2k_2 z) E^2 \quad [\text{J/m}^3\text{s}]. \quad (7)$$

Using the spin tensor [23-25] $Y^{\lambda\mu\nu} = -2A^{[\lambda} F^{\mu]\nu}$ and $A_i = -\int E_i dt = -iE_i/\omega$ yields the spin flux density in the vacuum

$$\langle Y_c^{xyz} \rangle = \Re\{-\bar{A}^x F^{yz} + \bar{A}^y F^{xz}\}/2 = \Re\{\bar{A}_x H_x + \bar{A}_y H_y\}/2 = \sqrt{\varepsilon_0/\mu_0} E^2 / \omega \quad [\text{Js/m}^2\text{s}]. \quad (8)$$

and the spin flux density in the absorber

$$\langle Y_c^{xyz} \rangle = \Re\{-\bar{A}^x F^{yz} + \bar{A}^y F^{xz}\}/2 = \Re\{\bar{A}_x H_x + \bar{A}_y H_y\}/2 = \sqrt{\varepsilon_0/\mu_0} \exp(-2k_2 z) E^2 / \omega. \quad (9)$$

Torque volume density from the absorbed spin angular momentum in the absorber is

$$\tau_z = -\partial_z \langle Y_c^{xyz} \rangle = 2k_2 \sqrt{\varepsilon_0/\mu_0} \exp(-2k_2 z) E^2 / \omega \quad [\text{J/m}^3]. \quad (10)$$

You see $w = \omega\tau_z$, as energy $\hbar\omega$ and spin \hbar of a photon. We found that such a torque density induces specific mechanical stresses in the absorber [18].

4. Conclusion

The new use of the spin tensor, this time to calculate the spin absorption of a plane wave in matter, confirms the conclusion [9,14-23] that the concept of spin by Sadowsky & Poynting [10,11], according to which the spin density is proportional to the energy density, takes precedence over the nowadays spin concept [1-8], which links spin to the energy density *gradient*.

We are eternally grateful to Professor Robert Romer, having courageously published the question: "Does a plane wave really not carry spin?" [29].

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However, the article was rejected by some journals.

Physical Review Letters. LH16556

We find that your claim:

"the need to replace the modern spin concept [1-6], which links spin to the energy density gradient, with the concept of Sadowsky & Poynting [8,9], according to which the spin density is proportional to the energy density itself"

is unsubstantiated. Therefore, this paper does not amount to a viable submission to PRL and cannot be made so by revision. We maintain our decision and we are not considering this paper further for publication in PRL. Please note that due to the constraints on our resources, we may not be able to

respond to further queries that are not directly connected to editorial aspects of a paper under active consideration.

Yours sincerely, Stojan Rebic Associate Editor

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Many thanks for sending us your manuscript. I have discussed this paper at some length with my associate editors. We agree that there are some very interesting points of discussion contained in the paper. However the style is a bit polemical and this leads to a tendency toward exaggeration. Also, it would be good to avoid single-sentence paragraphs.

Professor Thomas Brown Editor in Chief.

Applied Optics (375113)

I am sorry to inform you that your recent submission to Applied Optics will not be considered for publication. Sincerely, Applied Optics Manuscripts Office

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Unfortunately, this manuscript on **a fundamental topic** does not fall under the range of topics that are covered by AIP Advances, which focuses on the applied physical sciences.

Ben Slater Deputy Editor