On some Ramanujan equations: further possible mathematical connections with ϕ, $\zeta(2)$, several equations of Highly Effective Actions and Modular Invariance in Superstring Theory From N = 4 Super-Yang Mills

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Abstract

In this paper we have described some Ramanujan equations and obtained new possible mathematical connections with ϕ, $\zeta(2)$, several equations of Highly Effective Actions and Modular Invariance in Superstring Theory From N = 4 Super Yang Mills

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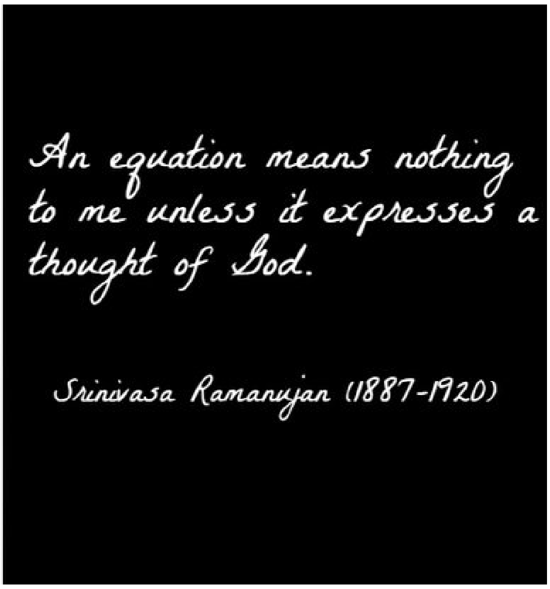


https://mobygeek.com/features/indian-mathematician-srinivasa-ramanujan-quotes-11012

We want to highlight that the development of the various equations was carried out according an our possible logical and original interpretation

From

Highly Effective Actions [1]
John H. Schwarz - arXiv:1311.0305v2 [hep-th] 22 Nov 2013

If we put:

$$\left(-\frac{N^2 c_5^3}{32\pi^3}\right) = \left(\left\{\sqrt{\left(\frac{17+\sqrt{145}}{8}\right)} + \sqrt{\left(\frac{9+\sqrt{145}}{8}\right)}\right\}\right)$$

And:

$$\left\{\left[\sqrt{-\det\left(\eta_{\mu\nu}+\frac{\partial_\mu\phi^I\partial_\nu\phi^I}{c_5\phi^3}+i\tilde{H}_{\mu\nu}\right)}-1\right]\right\}=\left[\sqrt{\left\{\frac{(2+\sqrt{5})(5+\sqrt{29})}{2}\right\}}\right]$$

We have the following action

$$S=S_1+S_2=-\frac{N^2c_5^3}{32\pi^3}\int\phi^3\left[\sqrt{-\det\left(\eta_{\mu\nu}+\frac{\partial_\mu\phi^I\partial_\nu\phi^I}{c_5\phi^3}+i\tilde{H}_{\mu\nu}\right)}-1\right]d^6x. \qquad (92)$$

where $\phi = 3$; $\phi^3 = 27$

Now, we take the following Ramanujan expression: (from: "**Modular equations and approximations to π**" – *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372) [2]

$$G_{145}^2 = \sqrt{\left\{\frac{(2+\sqrt{5})(5+\sqrt{29})}{2}\right\}}\left\{\sqrt{\left(\frac{17+\sqrt{145}}{8}\right)}+\sqrt{\left(\frac{9+\sqrt{145}}{8}\right)}\right\},$$

We obtain:

((1/2(2+sqrt5)(5+sqrt29)))^0.5 [(1/8(17+sqrt145))^0.5+(1/8(9+sqrt145))^0.5]

Input:

$$\sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)\left(5+\sqrt{29}\right)}\left(\sqrt{\frac{1}{8}\left(17+\sqrt{145}\right)}+\sqrt{\frac{1}{8}\left(9+\sqrt{145}\right)}\right)$$

Exact result:

$$\sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)\left(5+\sqrt{29}\right)}\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(9+\sqrt{145}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(17+\sqrt{145}\right)}\right)$$

Decimal approximation:

16.542097553485330822915990352914694829810037886057345799 14...

16.5420975534…

Alternate forms:

$$\frac{1}{4}\sqrt{\left(2+\sqrt{5}\right)\left(5+\sqrt{29}\right)}\left(\sqrt{9+\sqrt{145}}+\sqrt{17+\sqrt{145}}\right)$$

root of $x^8-17x^7+7x^6+12x^5-42x^4+12x^3+7x^2-17x+1$ near $x=16.5421$

$$\left(\frac{\sqrt{5}}{4}+\frac{1}{4}\sqrt{9-8i}+\frac{1}{4}\sqrt{9+8i}+\frac{\sqrt{29}}{4}\right)\sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)\left(5+\sqrt{29}\right)}$$

Minimal polynomial:

$$x^8-17x^7+7x^6+12x^5-42x^4+12x^3+7x^2-17x+1$$

From

$$S=S_1+S_2=-\frac{N^2c_5^3}{32\pi^3}\int\phi^3\left[\sqrt{-\det\left(\eta_{\mu\nu}+\frac{\partial_\mu\phi^I\partial_\nu\phi^I}{c_5\phi^3}+i\tilde{H}_{\mu\nu}\right)}-1\right]d^6x. \qquad (92)$$

we obtain:

[(1/8(17+sqrt145))^0.5+(1/8(9+sqrt145))^0.5] integrate [27*(((((1/2(2+sqrt5)(5+sqrt29)))^0.5)))] d^6x

Indefinite integral:

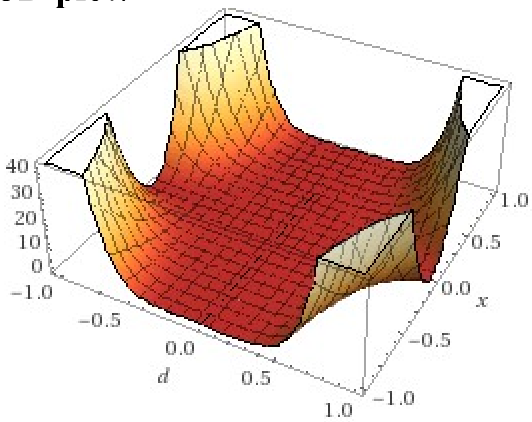
$$\sqrt{\frac{1}{8}\left(17+\sqrt{145}\right)}+\sqrt{\frac{1}{8}\left(9+\sqrt{145}\right)}\int\left(27\sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)\left(5+\sqrt{29}\right)}\right)d^6x\,dx=$$
$$\frac{27}{2}\sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)\left(5+\sqrt{29}\right)}\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(9+\sqrt{145}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(17+\sqrt{145}\right)}\right)d^6x^2+\text{constant}$$

Indefinite integral:

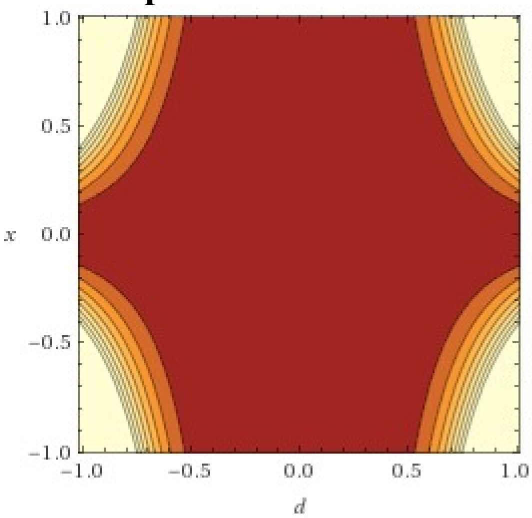
$$\sqrt{\frac{1}{8}\left(17+\sqrt{145}\right)}+\sqrt{\frac{1}{8}\left(9+\sqrt{145}\right)}\int\left(27\sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)\left(5+\sqrt{29}\right)}\right)d^6x\,dx\approx$$
$$\text{constant}+223.318\,d^6x^2$$

223.318

3D plot:



Contour plot:



Alternate forms:

$$d^6 x^2 \left(\text{root of } 256x^8 - 58752x^7 + 326592x^6 + 7558272x^5 - 357128352x^4 + 1377495072x^3 + 10847773692x^2 - 355652008902x + 282429536481 \text{ near } x = 223.318\right)$$

$$\frac{27}{4} d^6 x^2 \left((\text{root of } x^4 - 17x^2 + 36 \text{ near } x = 3.81062)(\text{root of } x^8 - 20x^6 - 43x^4 - 20x^2 + 1 \text{ near } x = 4.69) + (\text{root of } x^4 - 9x^2 - 16 \text{ near } x = 3.24358)(\text{root of } x^8 - 20x^6 - 43x^4 - 20x^2 + 1 \text{ near } x = 4.69) \right)$$

$$\frac{27}{8} \sqrt{\left(2+\sqrt{5}\right)\left(5+\sqrt{29}\right)} \left(\sqrt{9+\sqrt{145}} + \sqrt{17+\sqrt{145}}\right) d^6 x^2$$

Expanded form:

$$\frac{27}{8} \sqrt{\left(2+\sqrt{5}\right)\left(5+\sqrt{29}\right)\left(17+\sqrt{145}\right)}\, d^6 x^2 + \frac{27}{8} \sqrt{\left(2+\sqrt{5}\right)\left(5+\sqrt{29}\right)\left(9+\sqrt{145}\right)}\, d^6 x^2$$

For $d^6x^2 = 1$, we obtain 223.318, from which:

1/2(223.318)+11+3

Input interpretation:

$$\frac{1}{2} \times 223.318 + 11 + 3$$

Result:

125.659

125.659

1/2(223.318)+29-1

Input interpretation:

$$\frac{1}{2} \times 223.318 + 29 - 1$$

Result:

139.659

139.659

27*1/2(((1/2(223.318)+18-golden ratio)))+1/2

Input interpretation:

$$27 \times \frac{1}{2} \left(\frac{1}{2} \times 223.318 + 18 - \phi\right) + \frac{1}{2}$$

ϕ is the golden ratio

Result:

1729.05…

1729.05…

With regard 27 (From Wikipedia):

"The fundamental group of the complex form, compact real form, or any algebraic version of E_6 is the cyclic group $\mathbf{Z}/3\mathbf{Z}$, and its outer automorphism group is the cyclic group $\mathbf{Z}/2\mathbf{Z}$. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E_6 plays a role in some grand unified theories".

Alternative representations:

$$\frac{27}{2}\left(\frac{223.318}{2}+18-\phi\right)+\frac{1}{2}=\frac{1}{2}+\frac{27}{2}(129.659-2\sin(54°))$$

$$\frac{27}{2}\left(\frac{223.318}{2}+18-\phi\right)+\frac{1}{2}=\frac{1}{2}+\frac{27}{2}(129.659+2\cos(216°))$$

$$\frac{27}{2}\left(\frac{223.318}{2}+18-\phi\right)+\frac{1}{2}=\frac{1}{2}+\frac{27}{2}(129.659+2\sin(666°))$$

((((27*1/2(((1/2(223.318)+18-golden ratio)))+1/2))))^1/15

Input interpretation:

$$\sqrt[15]{27\times\frac{1}{2}\left(\frac{1}{2}\times 223.318+18-\phi\right)+\frac{1}{2}}$$

ϕ is the golden ratio

Result:

1.643818590561144202271599949236056511210491616842434867052...

1.64381859056…

((((27*1/2(((1/2(223.318)+18-golden ratio)))+1/2))))^1/15 - (21+5)1/10^3

Input interpretation:

$$\sqrt[15]{27\times\frac{1}{2}\left(\frac{1}{2}\times 223.318+18-\phi\right)+\frac{1}{2}}-(21+5)\times\frac{1}{10^3}$$

ϕ is the golden ratio

Result:

1.617818590561144202271599949236056511210491616842434867052...

1.61781859056…

Alternative representations:

$$\sqrt[15]{\frac{27}{2}\left(\frac{223.318}{2}+18-\phi\right)+\frac{1}{2}}-\frac{21+5}{10^3}=-\frac{26}{10^3}+\sqrt[15]{\frac{1}{2}+\frac{27}{2}\left(129.659-2\sin(54°)\right)}$$

$$\sqrt[15]{\frac{27}{2}\left(\frac{223.318}{2}+18-\phi\right)+\frac{1}{2}}-\frac{21+5}{10^3}=\sqrt[15]{\frac{1}{2}+\frac{27}{2}\left(129.659+2\cos(216°)\right)}-\frac{26}{10^3}$$

$$\sqrt[15]{\frac{27}{2}\left(\frac{223.318}{2}+18-\phi\right)+\frac{1}{2}}-\frac{21+5}{10^3}=-\frac{26}{10^3}+\sqrt[15]{\frac{1}{2}+\frac{27}{2}\left(129.659+2\sin(666°)\right)}$$

From:

Modular Invariance in Superstring Theory From N = 4 Super-Yang Mills [3]
Shai M. Chester, Michael B. Green, Silviu S. Pufu, Yifan Wang and Congkao Wen - arXiv:1912.13365v1 [hep-th] 31 Dec 2019

We know that:

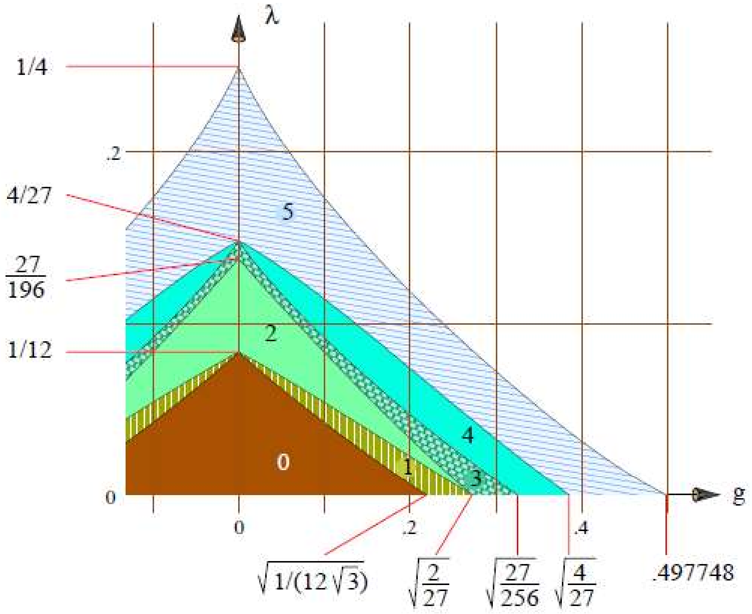
The 'field theory' is described by the action [8,9]

$$S(M)=\mathrm{Tr}\left(-\tfrac{1}{2}M^2+\tfrac{1}{3}gM^3+\tfrac{1}{4}\lambda M^4\right), \qquad (24)$$

where M is an $N\times N$-dimensional matrix, in the limit

$$N\to\infty\,,\qquad g,\lambda\to 0,\qquad N g^2=\tilde{g}^2 \text{ and } N\lambda=\tilde{\lambda} \text{ fixed.} \qquad (25)$$

And that



We have the following equation:

$$\begin{aligned}\partial_m^2 \log Z\big|_{m=0}^{\mathrm{pert}} =& 2N^2 \log g_{\mathrm{YM}} + \sqrt{N}\left[-\frac{16\zeta(3)}{g_{\mathrm{YM}}^3} - \frac{g_{\mathrm{YM}}}{3}\right] + \frac{1}{\sqrt{N}}\left[\frac{12\zeta(5)}{g_{\mathrm{YM}}^5} + \frac{g_{\mathrm{YM}}^3}{1440}\right] \\ &+ \frac{1}{N^{\frac{3}{2}}}\left[\frac{135\zeta(7)}{8g_{\mathrm{YM}}^7} + \frac{g_{\mathrm{YM}}^5}{215040} - \frac{13\zeta(3)}{32g_{\mathrm{YM}}^3} - \frac{13g_{\mathrm{YM}}}{1536}\right] \\ &+ \frac{1}{N^{\frac{5}{2}}}\left[\frac{1575\zeta(9)}{16g_{\mathrm{YM}}^9} + \frac{g_{\mathrm{YM}}^7}{6881280} - \frac{75\zeta(5)}{64g_{\mathrm{YM}}^5} - \frac{5g_{\mathrm{YM}}^3}{73728}\right] \\ &+ \frac{1}{N^{\frac{7}{2}}}\left[\frac{2480625\zeta(11)}{2048g_{\mathrm{YM}}^{11}} + \frac{25g_{\mathrm{YM}}^9}{2491416576} - \frac{80325\zeta(7)}{8192g_{\mathrm{YM}}^7} - \frac{17g_{\mathrm{YM}}^5}{6291456}\right. \\ &\left. + \frac{1533\zeta(3)}{16384g_{\mathrm{YM}}^3} + \frac{511g_{\mathrm{YM}}}{262144}\right] + O(N^{-\frac{9}{2}})\,. \end{aligned} \tag{4.3}$$

and we have:

$$\lambda = g_{\mathrm{YM}}^2 N$$

Sqrt(27/256) = g_{YM} ; N = 4

We obtain:

2*16 ln(sqrt(27/256)) + 2[(-16 zeta(3)) / ((sqrt(27/256))^3) – 1/3*(sqrt(27/256))]+1/2[(12 zeta(5)) / ((sqrt(27/256))^5) + 1/1440(sqrt(27/256))^3]

Input:

$$2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{-16\,\zeta(3)}{\sqrt{\frac{27}{256}}^{3}}-\frac{1}{3}\sqrt{\frac{27}{256}}\right)+\frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^{5}}+\frac{1}{1440}\sqrt{\frac{27}{256}}^{3}\right)$$

$\log(x)$ is the natural logarithm

$\zeta(s)$ is the Riemann zeta function

Exact result:

$$2\left(-\frac{65\,536\,\zeta(3)}{81\sqrt{3}}-\frac{\sqrt{3}}{16}\right)+\frac{1}{2}\left(\frac{4\,194\,304\,\zeta(5)}{729\sqrt{3}}+\frac{9\sqrt{3}}{655\,360}\right)+32\log\left(\frac{3\sqrt{3}}{16}\right)$$

Decimal approximation:

562.9958007367684101492447648751234705801711784682354583528...

562.9958…..

Alternate forms:

$$32\log\left(\frac{3\sqrt{3}}{16}\right)-\frac{1\,546\,188\,226\,560\,\zeta(3)-2\,748\,779\,069\,440\,\zeta(5)+358\,298\,397}{955\,514\,880\sqrt{3}}$$

$$-\frac{1}{955\,514\,880\sqrt{3}}\left(1\,546\,188\,226\,560\,\zeta(3)-2\,748\,779\,069\,440\,\zeta(5)+358\,298\,397+122\,305\,904\,640\sqrt{3}\log(2)-45\,864\,714\,240\sqrt{3}\log(3)\right)$$

$$\frac{1}{2\,866\,544\,640}\left(-1\,546\,188\,226\,560\sqrt{3}\,\zeta(3)+2\,748\,779\,069\,440\sqrt{3}\,\zeta(5)-358\,298\,397\sqrt{3}+91\,729\,428\,480\log\left(\frac{3\sqrt{3}}{16}\right)\right)$$

Alternative representations:

$$2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(-\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^{3}}-\frac{\sqrt{\frac{27}{256}}}{3}\right)+\frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^{5}}+\frac{\sqrt{\frac{27}{256}}^{3}}{1440}\right)=$$
$$32\log\left(\sqrt{\frac{27}{256}}\right)+2\left(-\frac{\sqrt{\frac{27}{256}}}{3}-\frac{16\,\zeta(3,1)}{\sqrt{\frac{27}{256}}^{3}}\right)+\frac{1}{2}\left(\frac{\sqrt{\frac{27}{256}}^{3}}{1440}+\frac{12\,\zeta(5,1)}{\sqrt{\frac{27}{256}}^{5}}\right)$$

$$2 \times 16 \log\left(\sqrt{\frac{27}{256}}\right) + 2\left(-\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^3} - \frac{\sqrt{\frac{27}{256}}}{3}\right) + \frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^5} + \frac{\sqrt{\frac{27}{256}}^3}{1440}\right) =$$

$$32 \log_e\left(\sqrt{\frac{27}{256}}\right) + 2\left(-\frac{\sqrt{\frac{27}{256}}}{3} - \frac{16\,\zeta(3,1)}{\sqrt{\frac{27}{256}}^3}\right) + \frac{1}{2}\left(\frac{\sqrt{\frac{27}{256}}^3}{1440} + \frac{12\,\zeta(5,1)}{\sqrt{\frac{27}{256}}^5}\right)$$

$$2 \times 16 \log\left(\sqrt{\frac{27}{256}}\right) + 2\left(-\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^3} - \frac{\sqrt{\frac{27}{256}}}{3}\right) + \frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^5} + \frac{\sqrt{\frac{27}{256}}^3}{1440}\right) =$$

$$32 \log(a) \log_a\left(\sqrt{\frac{27}{256}}\right) + 2\left(-\frac{\sqrt{\frac{27}{256}}}{3} - \frac{16\,\zeta(3,1)}{\sqrt{\frac{27}{256}}^3}\right) + \frac{1}{2}\left(\frac{\sqrt{\frac{27}{256}}^3}{1440} + \frac{12\,\zeta(5,1)}{\sqrt{\frac{27}{256}}^5}\right)$$

Series representations:

$$2 \times 16 \log\left(\sqrt{\frac{27}{256}}\right) + 2\left(-\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^3} - \frac{\sqrt{\frac{27}{256}}}{3}\right) + \frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^5} + \frac{\sqrt{\frac{27}{256}}^3}{1440}\right) =$$

$$\frac{1}{1\,310\,720}\left(-163\,831\sqrt{3} - 41\,943\,040 \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \frac{3\sqrt{3}}{16}\right)^k}{k} + 1\,310\,720 \sum_{n=0}^{\infty}\left(\sum_{k=0}^{n} -\frac{65\,536\,(-1)^k\,(1 + 18k + 9k^2)\binom{n}{k}}{729\sqrt{3}\,(1+k)^4\,(1+n)}\right)\right)$$

$$2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(-\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^{3}}-\frac{\sqrt{\frac{27}{256}}}{3}\right)+\frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^{5}}+\frac{\sqrt{\frac{27}{256}}^{3}}{1440}\right)=$$
$$\frac{1}{1\,310\,720}\left(-163\,831\sqrt{3}-41\,943\,040\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{3\sqrt{3}}{16}\right)^k}{k}+\right.$$
$$\left.1\,310\,720\sum_{m=1}^{\infty}\left(\sum_{k=1+m}^{\infty}\frac{131\,072\,(-16+9\,k^2)}{729\sqrt{3}\,k^4\,m}+\sum_{k=1}^{m}-\frac{131\,072\,(-16+9\,m^2)}{729\sqrt{3}\,k\,m^4}\right)\right)$$

$$2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(-\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^{3}}-\frac{\sqrt{\frac{27}{256}}}{3}\right)+\frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^{5}}+\frac{\sqrt{\frac{27}{256}}^{3}}{1440}\right)=$$
$$\frac{1}{1\,310\,720}\left(-163\,831\sqrt{3}-41\,943\,040\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{3\sqrt{3}}{16}\right)^k}{k}+\right.$$
$$\left.1\,310\,720\sum_{k=0}^{\infty}-\frac{131\,072\left(9\,(3-s_0)^k-16\,(5-s_0)^k\right)\zeta^{(k)}(s_0)}{729\sqrt{3}\,k!}\right)\quad\text{for } s_0\neq 1$$

Integral representations:

$$2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(-\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^{3}}-\frac{\sqrt{\frac{27}{256}}}{3}\right)+\frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^{5}}+\frac{\sqrt{\frac{27}{256}}^{3}}{1440}\right)=$$
$$-\frac{163\,831\sqrt{3}}{1\,310\,720}+\int_0^{\infty}\frac{65\,536\,t^2\,(-27+4\,t^2)}{2187\sqrt{3}\,(-1+e^t)}\,dt+32\log\left(\frac{3\sqrt{3}}{16}\right)$$

$$2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(-\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^{3}}-\frac{\sqrt{\frac{27}{256}}}{3}\right)+\frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^{5}}+\frac{\sqrt{\frac{27}{256}}^{3}}{1440}\right)=$$
$$-\frac{163\,831\sqrt{3}}{1\,310\,720}+\int_0^{\infty}\frac{262\,144\,t^2\,(-135+16\,t^2)}{32\,805\sqrt{3}\,(1+e^t)}\,dt+32\log\left(\frac{3\sqrt{3}}{16}\right)$$

$$2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(-\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^{3}}-\frac{\sqrt{\frac{27}{256}}}{3}\right)+\frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^{5}}+\frac{\sqrt{\frac{27}{256}}^{3}}{1440}\right)=$$

$$-\frac{163831\sqrt{3}}{1310720}+\int_0^1\frac{131072\left(45\,t^2\log^3(1-t^2)-8\log^5(1-t^4)\right)}{10935\sqrt{3}\,t^5}\,dt+32\log\left(\frac{3\sqrt{3}}{16}\right)$$

1/((4)^1.5)*[(135 zeta(7)) / (8*(sqrt(27/256))^7) + ((sqrt(27/256))^5) / (215040) – (13 zeta(3)) / (32*(sqrt(27/256))^3) – (13*(sqrt(27/256))) / (1536)]

Input:

$$\frac{1}{4^{1.5}}\left(\frac{135\,\zeta(7)}{8\sqrt{\frac{27}{256}}^{7}}+\frac{\sqrt{\frac{27}{256}}^{5}}{215040}-\frac{13\,\zeta(3)}{32\sqrt{\frac{27}{256}}^{3}}-\frac{13\sqrt{\frac{27}{256}}}{1536}\right)$$

$\zeta(s)$ is the Riemann zeta function

Result:

5580.75…

5580.75…

Alternative representations:

$$\frac{\frac{135\,\zeta(7)}{8\sqrt{\frac{27}{256}}^{7}}+\frac{\sqrt{\frac{27}{256}}^{5}}{215040}-\frac{13\,\zeta(3)}{32\sqrt{\frac{27}{256}}^{3}}-\frac{13\sqrt{\frac{27}{256}}}{1536}}{4^{1.5}}=\frac{\frac{\sqrt{\frac{27}{256}}^{5}}{215040}-\frac{13\sqrt{\frac{27}{256}}}{1536}-\frac{13\,\zeta(3,1)}{32\sqrt{\frac{27}{256}}^{3}}+\frac{135\,\zeta(7,1)}{8\sqrt{\frac{27}{256}}^{7}}}{4^{1.5}}$$

$$\frac{\frac{135\,\zeta(7)}{8\sqrt{\frac{27}{256}}^{7}}+\frac{\sqrt{\frac{27}{256}}^{5}}{215040}-\frac{13\,\zeta(3)}{32\sqrt{\frac{27}{256}}^{3}}-\frac{13\sqrt{\frac{27}{256}}}{1536}}{4^{1.5}}=$$

$$\frac{-\frac{13\,S_{2,1}(1)}{32\sqrt{\frac{27}{256}}^{3}}+\frac{135\,S_{6,1}(1)}{8\sqrt{\frac{27}{256}}^{7}}+\frac{\sqrt{\frac{27}{256}}^{5}}{215040}-\frac{13\sqrt{\frac{27}{256}}}{1536}}{4^{1.5}}$$

$$\frac{\frac{135\,\zeta(7)}{8\sqrt{\frac{27}{256}}^{7}}+\frac{\sqrt{\frac{27}{256}}^{5}}{215040}-\frac{13\,\zeta(3)}{32\sqrt{\frac{27}{256}}^{3}}-\frac{13\sqrt{\frac{27}{256}}}{1536}}{4^{1.5}}=$$

$$\frac{\frac{13\,\mathrm{Li}_3(-1)}{\frac{3}{4}\left(32\sqrt{\frac{27}{256}}^{3}\right)}-\frac{135\,\mathrm{Li}_7(-1)}{\left(1-\frac{1}{2^6}\right)\left(8\sqrt{\frac{27}{256}}^{7}\right)}+\frac{\sqrt{\frac{27}{256}}^{5}}{215040}-\frac{13\sqrt{\frac{27}{256}}}{1536}}{4^{1.5}}$$

Series representations:

$$\frac{\frac{135\,\zeta(7)}{8\sqrt{\frac{27}{256}}^{7}}+\frac{\sqrt{\frac{27}{256}}^{5}}{215040}-\frac{13\,\zeta(3)}{32\sqrt{\frac{27}{256}}^{3}}-\frac{13\sqrt{\frac{27}{256}}}{1536}}{4^{1.5}}=$$

$$\left(2.10938\left(\sum_{k=1}^{\infty}\frac{1}{k^7}-0.0240741\left(\sum_{k=1}^{\infty}\frac{1}{k^3}\right)\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^4-\right.\right.$$
$$0.000501543\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^8+$$
$$\left.\left.2.75573\times 10^{-7}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{12}\right)\right)\Big/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^7$$

$$\frac{\frac{135\zeta(7)}{8\sqrt{\frac{27}{256}}^7}+\frac{\sqrt{\frac{27}{256}}^5}{215040}-\frac{13\zeta(3)}{32\sqrt{\frac{27}{256}}^3}-\frac{13\sqrt{\frac{27}{256}}}{1536}}{4^{1.5}}=$$

$$\left(2.10938\left(\exp\left(\sum_{k=1}^{\infty}\frac{P(7k)}{k}\right)-0.0240741\exp\left(\sum_{k=1}^{\infty}\frac{P(3k)}{k}\right)\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^4-\right.\right.$$
$$0.000501543\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^8+$$
$$\left.\left.2.75573\times10^{-7}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{12}\right)\right)\Big/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^7$$

$$\frac{\frac{135\zeta(7)}{8\sqrt{\frac{27}{256}}^7}+\frac{\sqrt{\frac{27}{256}}^5}{215040}-\frac{13\zeta(3)}{32\sqrt{\frac{27}{256}}^3}-\frac{13\sqrt{\frac{27}{256}}}{1536}}{4^{1.5}}=$$

$$\left(5.81287\times10^{-7}\left(3.6288\times10^6\sum_{k=1}^{\infty}\frac{1}{k^7}-87360.\exp^4\left(i\pi\left\lfloor\frac{\arg\left(\frac{27}{256}-x\right)}{2\pi}\right\rfloor\right)\right.\right.$$
$$\sqrt{x}^4\left(\sum_{k=1}^{\infty}\frac{1}{k^3}\right)\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{27}{256}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4-$$
$$1820.\exp^8\left(i\pi\left\lfloor\frac{\arg\left(\frac{27}{256}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{27}{256}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^8+$$
$$\left.\left.\exp^{12}\left(i\pi\left\lfloor\frac{\arg\left(\frac{27}{256}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}^{12}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{27}{256}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^{12}\right)\right)\Big/$$
$$\left(\exp^7\left(i\pi\left\lfloor\frac{\arg\left(\frac{27}{256}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}^7\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{27}{256}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^7\right)$$

for $(x\in\mathbb{R}$ and $x<0)$

Integral representations:

$$\frac{\frac{135\,\zeta(7)}{8\sqrt{\frac{27}{256}}^{7}}+\frac{\sqrt{\frac{27}{256}}^{5}}{215040}-\frac{13\,\zeta(3)}{32\sqrt{\frac{27}{256}}^{3}}-\frac{13\sqrt{\frac{27}{256}}}{1536}}{4^{1.5}}=$$

$$\int_0^\infty \frac{t^2\operatorname{csch}(t)\left(\frac{1.06299\,t^4}{\Gamma(7)}-\frac{0.0290179\sqrt{\frac{27}{256}}^{4}}{\Gamma(3)}\right)}{\sqrt{\frac{27}{256}}^{7}}\,dt-$$

$$0.00105794\sqrt{\frac{27}{256}}+5.81287\times10^{-7}\sqrt{\frac{27}{256}}^{5}$$

$$\frac{\frac{135\,\zeta(7)}{8\sqrt{\frac{27}{256}}^{7}}+\frac{\sqrt{\frac{27}{256}}^{5}}{215040}-\frac{13\,\zeta(3)}{32\sqrt{\frac{27}{256}}^{3}}-\frac{13\sqrt{\frac{27}{256}}}{1536}}{4^{1.5}}=$$

$$\int_0^\infty \frac{t^3\operatorname{csch}^2(t)\left(\frac{135\,t^4}{\Gamma(8)}-\frac{0.203125\sqrt{\frac{27}{256}}^{4}}{\Gamma(4)}\right)}{\sqrt{\frac{27}{256}}^{7}}\,dt-$$

$$0.00105794\sqrt{\frac{27}{256}}+5.81287\times10^{-7}\sqrt{\frac{27}{256}}^{5}$$

$$\frac{\frac{135\,\zeta(7)}{8\sqrt{\frac{27}{256}}^{7}}+\frac{\sqrt{\frac{27}{256}}^{5}}{215040}-\frac{13\,\zeta(3)}{32\sqrt{\frac{27}{256}}^{3}}-\frac{13\sqrt{\frac{27}{256}}}{1536}}{4^{1.5}}=$$

$$\int_0^\infty \frac{2.14286\,t^6\,\Gamma(3)-0.0677083\,t^2\,\Gamma(7)\sqrt{\frac{27}{256}}^{4}}{(1+e^t)\,\Gamma(3)\,\Gamma(7)\sqrt{\frac{27}{256}}^{7}}\,dt-$$

$$0.00105794\sqrt{\frac{27}{256}}+5.81287\times10^{-7}\sqrt{\frac{27}{256}}^{5}$$

1/((4)^2.5)*[(1575 zeta(9)) / (16*(sqrt(27/256))^9) + ((sqrt(27/256))^7) / (6881280) – (75 zeta(5)) / (64*(sqrt(27/256))^5) – (5*(sqrt(27/256))^3) / (73728)]

Input:

$$\frac{1}{4^{2.5}}\left(\frac{1575\,\zeta(9)}{16\sqrt{\frac{27}{256}}^{9}}+\frac{\sqrt{\frac{27}{256}}^{7}}{6\,881\,280}-\frac{75\,\zeta(5)}{64\sqrt{\frac{27}{256}}^{5}}-\frac{5\sqrt{\frac{27}{256}}^{3}}{73\,728}\right)$$

$\zeta(s)$ is the Riemann zeta function

Result:

76694.7...

76694.7…

Alternative representations:

$$\frac{\frac{1575\,\zeta(9)}{16\sqrt{\frac{27}{256}}^{9}}+\frac{\sqrt{\frac{27}{256}}^{7}}{6881280}-\frac{75\,\zeta(5)}{64\sqrt{\frac{27}{256}}^{5}}-\frac{5\sqrt{\frac{27}{256}}^{3}}{73\,728}}{4^{2.5}}=\frac{-\frac{5\sqrt{\frac{27}{256}}^{3}}{73\,728}+\frac{\sqrt{\frac{27}{256}}^{7}}{6881\,280}-\frac{75\,\zeta(5,1)}{64\sqrt{\frac{27}{256}}^{5}}+\frac{1575\,\zeta(9,1)}{16\sqrt{\frac{27}{256}}^{9}}}{4^{2.5}}$$

$$\frac{\frac{1575\,\zeta(9)}{16\sqrt{\frac{27}{256}}^{9}}+\frac{\sqrt{\frac{27}{256}}^{7}}{6881280}-\frac{75\,\zeta(5)}{64\sqrt{\frac{27}{256}}^{5}}-\frac{5\sqrt{\frac{27}{256}}^{3}}{73\,728}}{4^{2.5}}=\frac{-\frac{75\,S_{4,1}(1)}{64\sqrt{\frac{27}{256}}^{5}}+\frac{1575\,S_{8,1}(1)}{16\sqrt{\frac{27}{256}}^{9}}-\frac{5\sqrt{\frac{27}{256}}^{3}}{73\,728}+\frac{\sqrt{\frac{27}{256}}^{7}}{6\,881\,280}}{4^{2.5}}$$

$$\frac{\frac{1575\,\zeta(9)}{16\sqrt{\frac{27}{256}}^{9}}+\frac{\sqrt{\frac{27}{256}}^{7}}{6881280}-\frac{75\,\zeta(5)}{64\sqrt{\frac{27}{256}}^{5}}-\frac{5\sqrt{\frac{27}{256}}^{3}}{73728}}{4^{2.5}}=\frac{\frac{75\,\mathrm{Li}_5(-1)}{\left(1-\frac{1}{2^4}\right)\left(64\sqrt{\frac{27}{256}}^{5}\right)}-\frac{1575\,\mathrm{Li}_9(-1)}{\left(1-\frac{1}{2^8}\right)\left(16\sqrt{\frac{27}{256}}^{9}\right)}-\frac{5\sqrt{\frac{27}{256}}^{3}}{73728}+\frac{\sqrt{\frac{27}{256}}^{7}}{6881280}}{4^{2.5}}$$

Series representations:

$$\frac{\frac{1575\,\zeta(9)}{16\sqrt{\frac{27}{256}}^{9}}+\frac{\sqrt{\frac{27}{256}}^{7}}{6881280}-\frac{75\,\zeta(5)}{64\sqrt{\frac{27}{256}}^{5}}-\frac{5\sqrt{\frac{27}{256}}^{3}}{73728}}{4^{2.5}}=$$

$$\left(3.07617\left(\sum_{k=1}^{\infty}\frac{1}{k^9}-0.0119048\left(\sum_{k=1}^{\infty}\frac{1}{k^5}\right)\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^4-6.88933\times10^{-7}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{12}+1.47628\times10^{-9}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{16}\right)\right)\Bigg/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{9}$$

$$\frac{\frac{1575\,\zeta(9)}{16\sqrt{\frac{27}{256}}^{9}}+\frac{\sqrt{\frac{27}{256}}^{7}}{6881280}-\frac{75\,\zeta(5)}{64\sqrt{\frac{27}{256}}^{5}}-\frac{5\sqrt{\frac{27}{256}}^{3}}{73728}}{4^{2.5}}=$$

$$\left(3.07617\left(\exp\left(\sum_{k=1}^{\infty}\frac{P(9\,k)}{k}\right)-0.0119048\exp\left(\sum_{k=1}^{\infty}\frac{P(5\,k)}{k}\right)\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^4-6.88933\times10^{-7}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{12}+1.47628\times10^{-9}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{16}\right)\right)\Bigg/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{9}$$

$$\frac{\frac{1575\,\zeta(9)}{16\sqrt{\frac{27}{256}}^9}+\frac{\sqrt{\frac{27}{256}}^7}{6\,881\,280}-\frac{75\,\zeta(5)}{64\sqrt{\frac{27}{256}}^5}-\frac{5\sqrt{\frac{27}{256}}^3}{73\,728}}{4^{2.5}}=$$

$$\left(4.54131\times10^{-9}\left(6.77376\times10^{8}\sum_{k=1}^{\infty}\frac{1}{k^9}-8.064\times10^{6}\exp^4\left(i\pi\left\lfloor\frac{\arg\left(\frac{27}{256}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}^4\left(\sum_{k=1}^{\infty}\frac{1}{k^5}\right)\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{27}{256}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4-466.667\exp^{12}\left(i\pi\left\lfloor\frac{\arg\left(\frac{27}{256}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}^{12}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{27}{256}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^{12}+\exp^{16}\left(i\pi\left\lfloor\frac{\arg\left(\frac{27}{256}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}^{16}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{27}{256}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^{16}\right)\right)\Bigg/\left(\exp^9\left(i\pi\left\lfloor\frac{\arg\left(\frac{27}{256}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}^9\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{27}{256}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^9\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

Integral representations:

$$\frac{\frac{1575\,\zeta(9)}{16\sqrt{\frac{27}{256}}^9}+\frac{\sqrt{\frac{27}{256}}^7}{6\,881\,280}-\frac{75\,\zeta(5)}{64\sqrt{\frac{27}{256}}^5}-\frac{5\sqrt{\frac{27}{256}}^3}{73\,728}}{4^{2.5}}=$$

$$\int_0^{\infty}\frac{t^4\operatorname{csch}(t)\left(\frac{1.5411\,t^4}{\Gamma(9)}-\frac{0.0189012\sqrt{\frac{27}{256}}^4}{\Gamma(5)}\right)}{\sqrt{\frac{27}{256}}^9}\,dt-2.11928\times10^{-6}\sqrt{\frac{27}{256}}^3+4.54131\times10^{-9}\sqrt{\frac{27}{256}}^7$$

$$\frac{\frac{1575\,\zeta(9)}{16\sqrt{\frac{27}{256}}^{9}}+\frac{\sqrt{\frac{27}{256}}^{7}}{6881280}-\frac{75\,\zeta(5)}{64\sqrt{\frac{27}{256}}^{5}}-\frac{5\sqrt{\frac{27}{256}}^{3}}{73728}}{4^{2.5}}=$$

$$\int_0^\infty \frac{3.08824\,t^8\,\Gamma(5)-0.0390625\,t^4\,\Gamma(9)\sqrt{\frac{27}{256}}^{4}}{(1+e^t)\,\Gamma(5)\,\Gamma(9)\sqrt{\frac{27}{256}}^{9}}\,dt-$$

$$2.11928\times10^{-6}\sqrt{\frac{27}{256}}^{3}+4.54131\times10^{-9}\sqrt{\frac{27}{256}}^{7}$$

$$\frac{\frac{1575\,\zeta(9)}{16\sqrt{\frac{27}{256}}^{9}}+\frac{\sqrt{\frac{27}{256}}^{7}}{6881280}-\frac{75\,\zeta(5)}{64\sqrt{\frac{27}{256}}^{5}}-\frac{5\sqrt{\frac{27}{256}}^{3}}{73728}}{4^{2.5}}=$$

$$\int_0^\infty \frac{t^5\,\text{csch}^2(t)\left(\frac{787.5\,t^4}{\Gamma(10)}-\frac{0.585938\sqrt{\frac{27}{256}}^{4}}{\Gamma(6)}\right)}{\sqrt{\frac{27}{256}}^{9}}\,dt-$$

$$2.11928\times10^{-6}\sqrt{\frac{27}{256}}^{3}+4.54131\times10^{-9}\sqrt{\frac{27}{256}}^{7}$$

csch(*x*) is the hyperbolic cosecant function

Γ(*x*) is the gamma function

1/((4)^3.5)*[(2480625 zeta(11)) / (2048*(sqrt(27/256))^11) + 25((sqrt(27/256))^9) / (2491416576) – (80325 zeta(7)) / (8192*(sqrt(27/256))^7) – (17*(sqrt(27/256))^5) / (6291456)

Input:

$$\frac{1}{4^{3.5}}\left(\frac{2480625\,\zeta(11)}{2048\sqrt{\frac{27}{256}}^{11}}+25\times\frac{\sqrt{\frac{27}{256}}^{9}}{2491416576}-\frac{80325\,\zeta(7)}{8192\sqrt{\frac{27}{256}}^{7}}-\frac{17\sqrt{\frac{27}{256}}^{5}}{6291456}\right)$$

ζ(*s*) is the Riemann zeta function

Result:

$2.23365\ldots\times10^{6}$

$2.23365\ldots*10^{6}$

2.23365 × 10^6 + 1/((4)^3.5)*[(1533 zeta(3)) / (16384*(sqrt(27/256))^3)+ (511*(sqrt(27/256))) / (262144)]+4^(-4.5)

Input interpretation:

$$2.23365\times 10^6 + \frac{1}{4^{3.5}}\left(\frac{1533\,\zeta(3)}{16\,384\sqrt{\frac{27}{256}}^{3}} + \frac{511\sqrt{\frac{27}{256}}}{262\,144}\right) + \frac{1}{4^{4.5}}$$

$\zeta(s)$ is the Riemann zeta function

Result:

$2.23365\ldots \times 10^6$

2233650

Thence, we can write also:

1/((4)^3.5)*[(2480625 zeta(11)) / (2048*(sqrt(27/256))^11) + 25((sqrt(27/256))^9) / (2491416576) – (80325 zeta(7)) / (8192*(sqrt(27/256))^7) – (17*(sqrt(27/256))^5) / (6291456)

Input:

$$\frac{1}{4^{3.5}}\left(\frac{2\,480\,625\,\zeta(11)}{2048\sqrt{\frac{27}{256}}^{11}} + 25\times\frac{\sqrt{\frac{27}{256}}^{9}}{2\,491\,416\,576} - \frac{80\,325\,\zeta(7)}{8192\sqrt{\frac{27}{256}}^{7}} - \frac{17\sqrt{\frac{27}{256}}^{5}}{6\,291\,456}\right)$$

$\zeta(s)$ is the Riemann zeta function

Result:

$2.23365\ldots \times 10^6$

$2.23365\ldots{*}10^6$

Alternative representations:

$$\frac{\frac{2\,480\,625\,\zeta(11)}{2048\sqrt{\frac{27}{256}}^{11}} + \frac{25\sqrt{\frac{27}{256}}^{9}}{2\,491\,416\,576} - \frac{80\,325\,\zeta(7)}{8192\sqrt{\frac{27}{256}}^{7}} - \frac{17\sqrt{\frac{27}{256}}^{5}}{6\,291\,456}}{4^{3.5}} = \frac{-\frac{17\sqrt{\frac{27}{256}}^{5}}{6\,291\,456} + \frac{25\sqrt{\frac{27}{256}}^{9}}{2\,491\,416\,576} - \frac{80\,325\,\zeta(7,1)}{8192\sqrt{\frac{27}{256}}^{7}} + \frac{2\,480\,625\,\zeta(11,1)}{2048\sqrt{\frac{27}{256}}^{11}}}{4^{3.5}}$$

$$\frac{\frac{2\,480\,625\,\zeta(11)}{2048\sqrt{\frac{27}{256}}^{11}} + \frac{25\sqrt{\frac{27}{256}}^{9}}{2\,491\,416\,576} - \frac{80\,325\,\zeta(7)}{8192\sqrt{\frac{27}{256}}^{7}} - \frac{17\sqrt{\frac{27}{256}}^{5}}{6\,291\,456}}{4^{3.5}} = \frac{-\frac{80\,325\,S_{6,1}(1)}{8192\sqrt{\frac{27}{256}}^{7}} + \frac{2\,480\,625\,S_{10,1}(1)}{2048\sqrt{\frac{27}{256}}^{11}} - \frac{17\sqrt{\frac{27}{256}}^{5}}{6\,291\,456} + \frac{25\sqrt{\frac{27}{256}}^{9}}{2\,491\,416\,576}}{4^{3.5}}$$

$$\frac{\frac{2\,480\,625\,\zeta(11)}{2048\sqrt{\frac{27}{256}}^{11}} + \frac{25\sqrt{\frac{27}{256}}^{9}}{2\,491\,416\,576} - \frac{80\,325\,\zeta(7)}{8192\sqrt{\frac{27}{256}}^{7}} - \frac{17\sqrt{\frac{27}{256}}^{5}}{6\,291\,456}}{4^{3.5}} = \frac{-\frac{80\,325\,\psi^{(6)}(1)(-1)^{7}}{6!\left(8192\sqrt{\frac{27}{256}}^{7}\right)} + \frac{2\,480\,625\,\psi^{(10)}(1)(-1)^{11}}{10!\left(2048\sqrt{\frac{27}{256}}^{11}\right)} - \frac{17\sqrt{\frac{27}{256}}^{5}}{6\,291\,456} + \frac{25\sqrt{\frac{27}{256}}^{9}}{2\,491\,416\,576}}{4^{3.5}}$$

Series representations:

$$\frac{\frac{2\,480\,625\,\zeta(11)}{2048\sqrt{\frac{27}{256}}^{11}} + \frac{25\sqrt{\frac{27}{256}}^{9}}{2\,491\,416\,576} - \frac{80\,325\,\zeta(7)}{8192\sqrt{\frac{27}{256}}^{7}} - \frac{17\sqrt{\frac{27}{256}}^{5}}{6\,291\,456}}{4^{3.5}} = \left(9.46283\left(\sum_{k=1}^{\infty}\frac{1}{k^{11}} - 0.00809524\left(\sum_{k=1}^{\infty}\frac{1}{k^{7}}\right)\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{4} - 2.23083\times10^{-9}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{16} + 8.28443\times10^{-12}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{20}\right)\right) \Big/ \left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{11}$$

$$\frac{\frac{2480625\,\zeta(11)}{2048\sqrt{\frac{27}{256}}^{11}}+\frac{25\sqrt{\frac{27}{256}}^{9}}{2491416576}-\frac{80325\,\zeta(7)}{8192\sqrt{\frac{27}{256}}^{7}}-\frac{17\sqrt{\frac{27}{256}}^{5}}{6291456}}{4^{3.5}}=$$

$$\left(9.46283\left(\exp\left(\sum_{k=1}^{\infty}\frac{P(11\,k)}{k}\right)-0.00809524\exp\left(\sum_{k=1}^{\infty}\frac{P(7\,k)}{k}\right)\right)\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^4-\right.$$
$$2.23083\times10^{-9}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{16}+$$
$$\left.8.28443\times10^{-12}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{20}\right)\Bigg/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{11}$$

$$\frac{\frac{2480625\,\zeta(11)}{2048\sqrt{\frac{27}{256}}^{11}}+\frac{25\sqrt{\frac{27}{256}}^{9}}{2491416576}-\frac{80325\,\zeta(7)}{8192\sqrt{\frac{27}{256}}^{7}}-\frac{17\sqrt{\frac{27}{256}}^{5}}{6291456}}{4^{3.5}}=$$

$$\left(7.83942\times10^{-11}\left(1.20708\times10^{11}\sum_{k=1}^{\infty}\frac{1}{k^{11}}-9.77163\times10^{8}\exp^4\left(i\,\pi\left\lfloor\frac{\arg\left(\frac{27}{256}-x\right)}{2\,\pi}\right\rfloor\right)\right.\right.$$
$$\sqrt{x}^4\left(\sum_{k=1}^{\infty}\frac{1}{k^7}\right)\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{27}{256}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4-269.28$$
$$\exp^{16}\left(i\,\pi\left\lfloor\frac{\arg\left(\frac{27}{256}-x\right)}{2\,\pi}\right\rfloor\right)\sqrt{x}^{16}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{27}{256}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^{16}+$$
$$\left.\left.\exp^{20}\left(i\,\pi\left\lfloor\frac{\arg\left(\frac{27}{256}-x\right)}{2\,\pi}\right\rfloor\right)\sqrt{x}^{20}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{27}{256}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^{20}\right)\right)\Bigg/$$
$$\left(\exp^{11}\left(i\,\pi\left\lfloor\frac{\arg\left(\frac{27}{256}-x\right)}{2\,\pi}\right\rfloor\right)\sqrt{x}^{11}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{27}{256}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^{11}\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

Integral representations:

$$\frac{\frac{2\,480\,625\,\zeta(11)}{2048\sqrt{\frac{27}{256}}^{11}}+\frac{25\sqrt{\frac{27}{256}}^{9}}{2\,491\,416\,576}-\frac{80\,325\,\zeta(7)}{8192\sqrt{\frac{27}{256}}^{7}}-\frac{17\sqrt{\frac{27}{256}}^{5}}{6\,291\,456}}{4^{3.5}}=$$

$$\int_0^\infty \frac{t^6\operatorname{csch}(t)\left(\frac{4.73373\,t^4}{\Gamma(11)}-\frac{0.0386035\sqrt{\frac{27}{256}}^{4}}{\Gamma(7)}\right)}{\sqrt{\frac{27}{256}}^{11}}\,dt-$$

$$2.111\times10^{-8}\sqrt{\frac{27}{256}}^{5}+7.83942\times10^{-11}\sqrt{\frac{27}{256}}^{9}$$

$$\frac{\frac{2\,480\,625\,\zeta(11)}{2048\sqrt{\frac{27}{256}}^{11}}+\frac{25\sqrt{\frac{27}{256}}^{9}}{2\,491\,416\,576}-\frac{80\,325\,\zeta(7)}{8192\sqrt{\frac{27}{256}}^{7}}-\frac{17\sqrt{\frac{27}{256}}^{5}}{6\,291\,456}}{4^{3.5}}=$$

$$\int_0^\infty \frac{t^7\operatorname{csch}^2(t)\left(\frac{9689.94\,t^4}{\Gamma(12)}-\frac{4.90265\sqrt{\frac{27}{256}}^{4}}{\Gamma(8)}\right)}{\sqrt{\frac{27}{256}}^{11}}\,dt-$$

$$2.111\times10^{-8}\sqrt{\frac{27}{256}}^{5}+7.83942\times10^{-11}\sqrt{\frac{27}{256}}^{9}$$

$$\frac{\frac{2\,480\,625\,\zeta(11)}{2048\sqrt{\frac{27}{256}}^{11}}+\frac{25\sqrt{\frac{27}{256}}^{9}}{2\,491\,416\,576}-\frac{80\,325\,\zeta(7)}{8192\sqrt{\frac{27}{256}}^{7}}-\frac{17\sqrt{\frac{27}{256}}^{5}}{6\,291\,456}}{4^{3.5}}=$$

$$\int_0^\infty \frac{9.46283\,t^{10}\,\Gamma(7)-0.0766039\,t^6\,\Gamma(11)\sqrt{\frac{27}{256}}^{4}}{\left(-1+e^t\right)\Gamma(7)\,\Gamma(11)\sqrt{\frac{27}{256}}^{11}}\,dt-$$

$$2.111\times10^{-8}\sqrt{\frac{27}{256}}^{5}+7.83942\times10^{-11}\sqrt{\frac{27}{256}}^{9}$$

In conclusion, we obtain:

$$\begin{aligned}
\partial_m^2 \log Z\Big|_{m=0}^{\text{pert}} =& 2N^2 \log g_{\text{YM}} + \sqrt{N}\left[-\frac{16\zeta(3)}{g_{\text{YM}}^3} - \frac{g_{\text{YM}}}{3}\right] + \frac{1}{\sqrt{N}}\left[\frac{12\zeta(5)}{g_{\text{YM}}^5} + \frac{g_{\text{YM}}^3}{1440}\right] \\
&+ \frac{1}{N^{\frac{3}{2}}}\left[\frac{135\zeta(7)}{8g_{\text{YM}}^7} + \frac{g_{\text{YM}}^5}{215040} - \frac{13\zeta(3)}{32g_{\text{YM}}^3} - \frac{13g_{\text{YM}}}{1536}\right] \\
&+ \frac{1}{N^{\frac{5}{2}}}\left[\frac{1575\zeta(9)}{16g_{\text{YM}}^9} + \frac{g_{\text{YM}}^7}{6881280} - \frac{75\zeta(5)}{64g_{\text{YM}}^5} - \frac{5g_{\text{YM}}^3}{73728}\right] \\
&+ \frac{1}{N^{\frac{7}{2}}}\left[\frac{2480625\zeta(11)}{2048g_{\text{YM}}^{11}} + \frac{25g_{\text{YM}}^9}{2491416576} - \frac{80325\zeta(7)}{8192g_{\text{YM}}^7} - \frac{17g_{\text{YM}}^5}{6291456}\right. \\
&\left. + \frac{1533\zeta(3)}{16384g_{\text{YM}}^3} + \frac{511g_{\text{YM}}}{262144}\right] + O(N^{-\frac{9}{2}}) \,.
\end{aligned}$$

(2233650+76694.7+5580.75+562.9958)

Input interpretation:

2 233 650 + 76 694.7 + 5580.75 + 562.9958

Result:

2.3164884458×10^6

Decimal form:

2316488.4458

2316488.4458

(2.3164884458 × 10^6)^1/3+7

Input interpretation:

$\sqrt[3]{2.3164884458 \times 10^6} + 7$

Result:

139.31529400…

139.315294…

(2.3164884458 × 10^6)^1/3-7

Input interpretation:

$$\sqrt[3]{2.3164884458\times 10^6}-7$$

Result:

125.31529400...

125.315294…

27*1/2((((2.3164884458 × 10^6)^1/3-4)))-Pi

Input interpretation:

$$27\times\frac{1}{2}\left(\sqrt[3]{2.3164884458\times 10^6}-4\right)-\pi$$

Result:

1729.1148763...

1729.1148763...

[27*1/2((((2.3164884458 × 10^6)^1/3-4)))-Pi]^1/14+29/10^3

Input interpretation:

$$\sqrt[14]{27\times\frac{1}{2}\left(\sqrt[3]{2.3164884458\times 10^6}-4\right)-\pi}+\frac{29}{10^3}$$

Result:

1.732229348826...

1.732229348826… ≈ √3

[27*1/2((((2.3164884458 × 10^6)^1/3-4)))-Pi]^1/15

Input interpretation:

$$\sqrt[15]{27\times\frac{1}{2}\left(\sqrt[3]{2.3164884458\times 10^6}-4\right)-\pi}$$

Result:

1.643822509629...

1.643822509629…

[27*1/2((((2.3164884458 × 10^6)^1/3-4)))-Pi]^1/15-(21+5)1/10^3

Input interpretation:

$$\sqrt[15]{27\times\frac{1}{2}\left(\sqrt[3]{2.3164884458\times 10^{6}}-4\right)-\pi}-(21+5)\times\frac{1}{10^{3}}$$

Result:

1.617822509629...

1.617822509629...

We have also:

(2233650+76694.7+5580.75+562.9958)^1/2

Input interpretation:

$$\sqrt{2\,233\,650+76\,694.7+5580.75+562.9958}$$

Result:

1522.0015...

1522.0015...

And:

(2233650+76694.7+5580.75+562.9958)^1/2 +13

Input interpretation:

$$\sqrt{2\,233\,650+76\,694.7+5580.75+562.9958}+13$$

Result:

1535.0015...

1535.0015.... result practically equal to the rest mass of Xi baryon 1535

We note that from:

$$\frac{1}{4^{3.5}}\left(\frac{2480625\,\zeta(11)}{2048\sqrt{\frac{27}{256}}^{11}}+25\times\frac{\sqrt{\frac{27}{256}}^{9}}{2491416576}-\frac{80325\,\zeta(7)}{8192\sqrt{\frac{27}{256}}^{7}}-\frac{17\sqrt{\frac{27}{256}}^{5}}{6291456}\right)$$

$2.23365\ldots\times10^{6}$

We have also:

((((1/((4)^3.5)*((2480625 zeta(11))/(2048*(sqrt(27/256))^11)+25((sqrt(27/256))^9)/(2491416576) - (80325 zeta(7))/((x+3)*64*(sqrt(27/256))^7) -(17*(sqrt(27/256))^5)/(6291456)))) = 2233651.7068

Input interpretation:

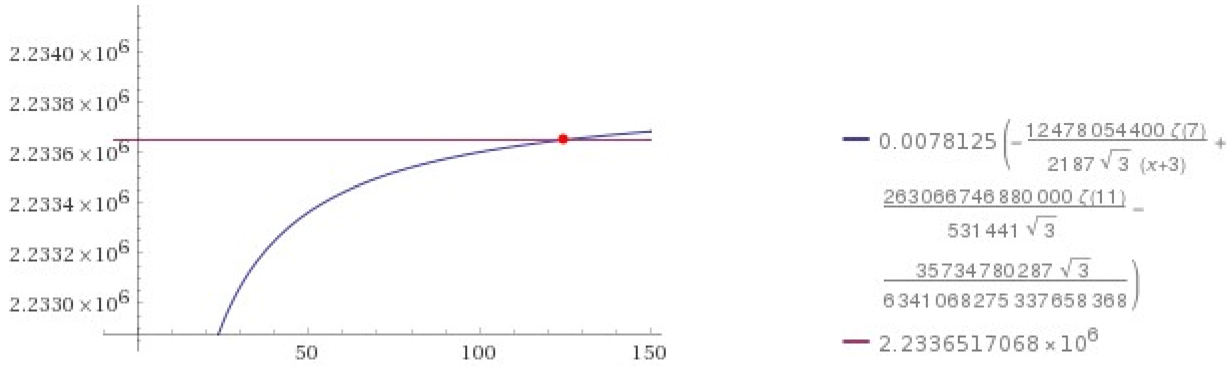
$$\frac{1}{4^{3.5}}\left(\frac{2480625\,\zeta(11)}{2048\sqrt{\frac{27}{256}}^{11}}+25\times\frac{\sqrt{\frac{27}{256}}^{9}}{2491416576}-\frac{80325\,\zeta(7)}{(x+3)\times64\sqrt{\frac{27}{256}}^{7}}-\frac{17\sqrt{\frac{27}{256}}^{5}}{6291456}\right)=$$
2.2336517068×10^{6}

$\zeta(s)$ is the Riemann zeta function

Result:

$$0.0078125\left(-\frac{12478054400\,\zeta(7)}{2187\sqrt{3}\,(x+3)}+\frac{263066746880000\,\zeta(11)}{531441\sqrt{3}}-\frac{35734780287\sqrt{3}}{6341068275337658368}\right)=2.2336517068\times10^{6}$$

Plot:



Alternate form assuming x is real:

$$\frac{128.}{x+3} = 1$$

Alternate forms:

$$\frac{2.23385\times10^{6}\, x + 6.67561\times10^{6}}{x+3} = 2.2336517068\times10^{6}$$

$$\frac{2.23275\times10^{6}\,(1.00049\, x + 2.98986)}{x+3} = 2.2336517068\times10^{6}$$

Alternate form assuming x is positive:

$x = 125.$ (for $x \neq -3$)

Expanded form:

$$2.23385\times10^{6} - \frac{25\,950.1}{x+3} = 2.2336517068\times10^{6}$$

Solution:

$x \approx 125.$

125

and:

1/((4)^3.5)[((2480625 zeta(11))/(2048(sqrt(3^3/16^2))^11)+25((sqrt(3^3/16^2))^9)/(2491416576)-(80325 zeta(7))/(((2x-2)/27)64(sqrt(3^3/16^2))^7)-(17(sqrt(3^3/16^2))^5)/(6291456))] = 2233651.7068

Input interpretation:

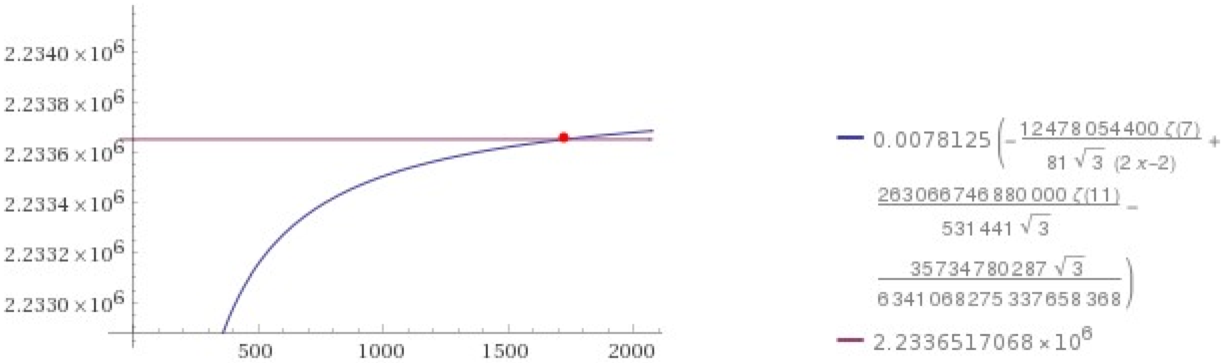
$$\frac{1}{4^{3.5}}\left(\frac{2480625\,\zeta(11)}{2048\sqrt{\frac{3^3}{16^2}}^{11}}+25\times\frac{\sqrt{\frac{3^3}{16^2}}^{9}}{2491416576}-\frac{80325\,\zeta(7)}{\left(\frac{1}{27}(2x-2)\right)\times 64\sqrt{\frac{3^3}{16^2}}^{7}}-\frac{17\sqrt{\frac{3^3}{16^2}}^{5}}{6291456}\right)=$$
$$2.2336517068\times10^6$$

$\zeta(s)$ is the Riemann zeta function

Result:

$$0.0078125\left(-\frac{12478054400\,\zeta(7)}{81\sqrt{3}\,(2x-2)}+\frac{263066746880000\,\zeta(11)}{531441\sqrt{3}}-\frac{35734780287\sqrt{3}}{6341068275337658368}\right)=2.2336517068\times10^6$$

Plot:



Alternate forms:

$$\frac{2.23385\times10^6\,x-2.58418\times10^6}{x-1}=2.2336517068\times10^6$$

$$\frac{2.23275\times10^6\,(1.00049\,x-1.1574)}{x-1}=2.2336517068\times10^6$$

Alternate form assuming x is positive:

$$\frac{1728.}{1-x}+1=0$$

Expanded form:

$$2.23385\times10^6-\frac{700652.}{2x-2}=2.2336517068\times10^6$$

Alternate form assuming x>0:

$$2.23385\times10^6-\frac{350326.}{x-1}=2.2336517068\times10^6$$

Solution:

$x \approx 1729.$

1729

Now, we have that:

Let us begin by discussing the one-instanton case. By explicitly performing the sums and products in I_1 (Eq. (3.8)) for many small values of N, we find that I_1 can be expanded for small a_i as

$$I_1 = \frac{2\Gamma(N+\frac{1}{2})}{\sqrt{\pi}\Gamma(N)} + \frac{3\Gamma(N-\frac{1}{2})}{4\sqrt{\pi}\Gamma(N+2)}C_2 - \frac{315\Gamma(N-\frac{3}{2})}{64\sqrt{\pi}\Gamma(N+4)}C_2^2 + \frac{15(3-N+4N^2)\Gamma(N-\frac{3}{2})}{32\sqrt{\pi}\Gamma(N+4)}C_4 + \cdots \tag{4.6}$$

$$\langle C_2\rangle = \lambda\left[\frac{N^2}{8\pi^2} - \frac{1}{8\pi^2}\right], \quad \langle C_2^2\rangle = \lambda^2\left[\frac{N^4}{64\pi^4} - \frac{1}{64\pi^4}\right], \quad \langle C_4\rangle = \lambda^2\left[\frac{N^2}{128\pi^4} - \frac{5}{128\pi^4}\right]. \tag{4.8}$$

(27/196)^2 [(4^4/(64Pi^4))-1/(64Pi^4)]

For N = 4, λ = 27/196 from (4.8) we obtain:

27/196[(16/(8Pi^2))-1/(8Pi^2)]

Input:

$$\frac{27}{196}\left(\frac{16}{8\pi^2} - \frac{1}{8\pi^2}\right)$$

Result:

$$\frac{405}{1568\pi^2}$$

Decimal approximation:

0.026170331234149743261971417474451335303740627386645313955...

0.026170331234149743261971

Property:

$\frac{405}{1568\pi^2}$ is a transcendental number

Alternative representations:

$$\frac{1}{196}\left(\frac{16}{8\pi^2}-\frac{1}{8\pi^2}\right)27=\frac{405}{196\left(8\left(180\,^\circ\right)^2\right)}$$

$$\frac{1}{196}\left(\frac{16}{8\pi^2}-\frac{1}{8\pi^2}\right)27=\frac{405}{196\,(48\,\zeta(2))}$$

$$\frac{1}{196}\left(\frac{16}{8\pi^2}-\frac{1}{8\pi^2}\right)27=\frac{405}{196\left(8\left(-i\log(-1)\right)^2\right)}$$

Series representations:

$$\frac{1}{196}\left(\frac{16}{8\pi^2}-\frac{1}{8\pi^2}\right)27=\frac{405}{25\,088\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^2}$$

$$\frac{1}{196}\left(\frac{16}{8\pi^2}-\frac{1}{8\pi^2}\right)27=\frac{405}{25\,088\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}\right)^2}$$

$$\frac{1}{196}\left(\frac{16}{8\pi^2}-\frac{1}{8\pi^2}\right)27=\frac{405}{1568\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^2}$$

Integral representations:

$$\frac{1}{196}\left(\frac{16}{8\pi^2}-\frac{1}{8\pi^2}\right)27=\frac{405}{6272\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^2}$$

$$\frac{1}{196}\left(\frac{16}{8\pi^2}-\frac{1}{8\pi^2}\right)27=\frac{405}{25\,088\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2}$$

$$\frac{1}{196}\left(\frac{16}{8\pi^2}-\frac{1}{8\pi^2}\right)27=\frac{405}{6272\left(\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt\right)^2}$$

(27/196)^2 [(4^4/(64Pi^4))-1/(64Pi^4)]

Input:

$$\left(\frac{27}{196}\right)^2 \left(\frac{4^4}{64\,\pi^4} - \frac{1}{64\,\pi^4}\right)$$

Result:

$$\frac{185\,895}{2\,458\,624\,\pi^4}$$

Decimal approximation:

0.000776204401825795440859422443383384221212791880979802055...

0.0007762044018257

Property:

$$\frac{185\,895}{2\,458\,624\,\pi^4} \text{ is a transcendental number}$$

Alternative representations:

$$\left(\frac{27}{196}\right)^2 \left(\frac{4^4}{64\,\pi^4} - \frac{1}{64\,\pi^4}\right) = \left(\frac{27}{196}\right)^2 \left(-\frac{1}{64\,(180\,°)^4} + \frac{4^4}{64\,(180\,°)^4}\right)$$

$$\left(\frac{27}{196}\right)^2 \left(\frac{4^4}{64\,\pi^4} - \frac{1}{64\,\pi^4}\right) = \left(\frac{27}{196}\right)^2 \left(-\frac{1}{64\cos^{-1}(-1)^4} + \frac{4^4}{64\cos^{-1}(-1)^4}\right)$$

$$\left(\frac{27}{196}\right)^2 \left(\frac{4^4}{64\,\pi^4} - \frac{1}{64\,\pi^4}\right) = \left(\frac{27}{196}\right)^2 \left(-\frac{1}{64\,(-i\log(-1))^4} + \frac{4^4}{64\,(-i\log(-1))^4}\right)$$

Series representations:

$$\left(\frac{27}{196}\right)^2 \left(\frac{4^4}{64\,\pi^4} - \frac{1}{64\,\pi^4}\right) = \frac{185\,895}{629\,407\,744\left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)^4}$$

$$\left(\frac{27}{196}\right)^2 \left(\frac{4^4}{64\,\pi^4} - \frac{1}{64\,\pi^4}\right) = \frac{185\,895}{629\,407\,744\left(\sum_{k=0}^{\infty} \frac{(-1)^k\, 1195^{-1-2k}\left(5^{1+2k} - 4 \times 239^{1+2k}\right)}{1+2k}\right)^4}$$

$$\left(\frac{27}{196}\right)^2 \left(\frac{4^4}{64\,\pi^4} - \frac{1}{64\,\pi^4}\right) = \frac{185\,895}{2\,458\,624\left(\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)\right)^4}$$

Integral representations:

$$\left(\frac{27}{196}\right)^2 \left(\frac{4^4}{64\pi^4} - \frac{1}{64\pi^4}\right) = \frac{185\,895}{39\,337\,984\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^4}$$

$$\left(\frac{27}{196}\right)^2 \left(\frac{4^4}{64\pi^4} - \frac{1}{64\pi^4}\right) = \frac{185\,895}{629\,407\,744\left(\int_0^1 \sqrt{1-t^2}\,dt\right)^4}$$

$$\left(\frac{27}{196}\right)^2 \left(\frac{4^4}{64\pi^4} - \frac{1}{64\pi^4}\right) = \frac{185\,895}{39\,337\,984\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt\right)^4}$$

(27/196)^2 [(16/(128Pi^4))-5/(128Pi^4)]

Input:

$$\left(\frac{27}{196}\right)^2 \left(\frac{16}{128\pi^4} - \frac{5}{128\pi^4}\right)$$

Result:

$$\frac{8019}{4\,917\,248\,\pi^4}$$

Decimal approximation:

0.000016741663568791666371477738974935738104589628805446710...

0.000016741663568791

Property:

$\frac{8019}{4\,917\,248\,\pi^4}$ is a transcendental number

Alternative representations:

$$\left(\frac{27}{196}\right)^2 \left(\frac{16}{128\pi^4} - \frac{5}{128\pi^4}\right) = \frac{11\left(\frac{27}{196}\right)^2}{128\,(180\,°)^4}$$

$$\left(\frac{27}{196}\right)^2 \left(\frac{16}{128\pi^4} - \frac{5}{128\pi^4}\right) = \frac{11\left(\frac{27}{196}\right)^2}{128\,(-i\log(-1))^4}$$

$$\left(\frac{27}{196}\right)^2\left(\frac{16}{128\pi^4}-\frac{5}{128\pi^4}\right)=\frac{11\left(\frac{27}{196}\right)^2}{128\cos^{-1}(-1)^4}$$

Series representations:

$$\left(\frac{27}{196}\right)^2\left(\frac{16}{128\pi^4}-\frac{5}{128\pi^4}\right)=\frac{8019}{1\,258\,815\,488\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^4}$$

$$\left(\frac{27}{196}\right)^2\left(\frac{16}{128\pi^4}-\frac{5}{128\pi^4}\right)=\frac{8019}{4\,917\,248\left(\sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}\right)^4}$$

$$\left(\frac{27}{196}\right)^2\left(\frac{16}{128\pi^4}-\frac{5}{128\pi^4}\right)=\frac{8019}{4\,917\,248\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^4}$$

Integral representations:

$$\left(\frac{27}{196}\right)^2\left(\frac{16}{128\pi^4}-\frac{5}{128\pi^4}\right)=\frac{8019}{78\,675\,968\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^4}$$

$$\left(\frac{27}{196}\right)^2\left(\frac{16}{128\pi^4}-\frac{5}{128\pi^4}\right)=\frac{8019}{78\,675\,968\left(\int_0^{1}\frac{1}{\sqrt{1-t^2}}\,dt\right)^4}$$

$$\left(\frac{27}{196}\right)^2\left(\frac{16}{128\pi^4}-\frac{5}{128\pi^4}\right)=\frac{8019}{1\,258\,815\,488\left(\int_0^{1}\sqrt{1-t^2}\,dt\right)^4}$$

Thence:

0.026170331234149743261971417474451335303740627386645313955...
0.000776204401825795440859422443383384221212791880979802055...
0.000016741663568791666371477738974935738104589628805446710...

$\langle C_2\rangle$ = 0.026170331234149; $\langle C_2^2\rangle$ = 0.00077620440182;

$\langle C_4\rangle$ = 0.000016741663568

From:

$$I_1 = \frac{2\Gamma(N+\frac{1}{2})}{\sqrt{\pi}\Gamma(N)} + \frac{3\Gamma(N-\frac{1}{2})}{4\sqrt{\pi}\Gamma(N+2)}C_2 - \frac{315\Gamma(N-\frac{3}{2})}{64\sqrt{\pi}\Gamma(N+4)}C_2^2 + \frac{15(3-N+4N^2)\Gamma(N-\frac{3}{2})}{32\sqrt{\pi}\Gamma(N+4)}C_4 + \cdots \quad (4.6)$$

We obtain:

((2 gamma (4+1/2))) / ((sqrt(Pi) gamma (4))) + 0.02617033123((3 gamma (4-1/2))) / ((4sqrtPi (gamma (6)))) – 0.000776204((315 gamma (4-3/2))) / ((64sqrtPi (gamma (8)))) + 0.000016741663568((15(3-4+4*16) gamma (4-3/2))) / ((32sqrtPi * (gamma (8))))

(2 gamma (9/2))/(sqrt(Pi) gamma (4)) + 0.0261703(3 gamma (7/2))/(4sqrtPi (gamma (6)))

Input interpretation:

$$\frac{2\,\Gamma\left(\frac{9}{2}\right)}{\sqrt{\pi}\,\Gamma(4)} + 0.0261703 \times \frac{3\,\Gamma\left(\frac{7}{2}\right)}{4\sqrt{\pi}\,\Gamma(6)}$$

$\Gamma(x)$ is the gamma function

Result:

2.187806683203125

2.187806683203125

2.187806683203125 - 0.000776204(315 gamma (5/2))/(64sqrtPi (gamma (8))) + 0.000016741663(15(3-4+4*16) gamma (5/2))/(32sqrtPi * (gamma (8)))

Input interpretation:

$$2.187806683203125 + \frac{315\,\Gamma\left(\frac{5}{2}\right)}{64\sqrt{\pi}\,\Gamma(8)} \times (-0.000776204) + 0.000016741663 \times \frac{15\,(3-4+4\times16)\,\Gamma\left(\frac{5}{2}\right)}{32\sqrt{\pi}\,\Gamma(8)}$$

$\Gamma(x)$ is the gamma function

Result:

2.18780618826609716796875

2.18780618826609716796875

Alternative representations:

$$2.1878066832031250000 - \frac{0.000776204\left(315\,\Gamma\left(\frac{5}{2}\right)\right)}{64\sqrt{\pi}\,\Gamma(8)} + \frac{0.0000167417\left(15\,(3-4+4\times16)\,\Gamma\left(\frac{5}{2}\right)\right)}{32\sqrt{\pi}\,\Gamma(8)} = 2.1878066832031250000 + \frac{0.0158209\times\frac{3}{2}!}{32\times7!\sqrt{\pi}} - \frac{0.244504\times\frac{3}{2}!}{64\times7!\sqrt{\pi}}$$

$$2.1878066832031250000 - \frac{0.000776204\left(315\,\Gamma\left(\frac{5}{2}\right)\right)}{64\sqrt{\pi}\,\Gamma(8)} + \frac{0.0000167417\left(15\,(3-4+4\times16)\,\Gamma\left(\frac{5}{2}\right)\right)}{32\sqrt{\pi}\,\Gamma(8)} = 2.1878066832031250000 + \frac{0.0158209\,\Gamma\left(\frac{5}{2},0\right)}{32\,\Gamma(8,0)\sqrt{\pi}} - \frac{0.244504\,\Gamma\left(\frac{5}{2},0\right)}{64\,\Gamma(8,0)\sqrt{\pi}}$$

$$2.1878066832031250000 - \frac{0.000776204\left(315\,\Gamma\left(\frac{5}{2}\right)\right)}{64\sqrt{\pi}\,\Gamma(8)} + \frac{0.0000167417\left(15\,(3-4+4\times16)\,\Gamma\left(\frac{5}{2}\right)\right)}{32\sqrt{\pi}\,\Gamma(8)} = 2.1878066832031250000 + \frac{0.0158209\,(1)_{\frac{3}{2}}}{32\,(1)_7\sqrt{\pi}} - \frac{0.244504\,(1)_{\frac{3}{2}}}{64\,(1)_7\sqrt{\pi}}$$

Series representations:

$$2.1878066832031250000 - \frac{0.000776204\left(315\,\Gamma\left(\frac{5}{2}\right)\right)}{64\sqrt{\pi}\,\Gamma(8)} + \frac{0.0000167417\left(15\,(3-4+4\times16)\,\Gamma\left(\frac{5}{2}\right)\right)}{32\sqrt{\pi}\,\Gamma(8)} = 2.1878066832031250000 - \frac{0.00332598\,\Gamma\left(\frac{5}{2}\right)}{\exp\left(i\pi\left\lfloor\frac{\arg(\pi-x)}{2\pi}\right\rfloor\right)\Gamma(8)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(\pi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}} \quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$2.1878066832031250000 - \frac{0.000776204\left(315\,\Gamma\left(\frac{5}{2}\right)\right)}{64\sqrt{\pi}\,\Gamma(8)} + \frac{0.0000167417\left(15\,(3-4+4\times16)\,\Gamma\left(\frac{5}{2}\right)\right)}{32\sqrt{\pi}\,\Gamma(8)} = 2.1878066832031250000 - \frac{0.00332598\,\Gamma\left(\frac{5}{2}\right)\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(\pi-z_0)/(2\pi)\rfloor} z_0^{-1/2-1/2\lfloor\arg(\pi-z_0)/(2\pi)\rfloor}}{\Gamma(8)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\pi-z_0)^k z_0^{-k}}{k!}}$$

$$2.1878066832031250000 - \frac{0.000776204\left(315\,\Gamma\left(\frac{5}{2}\right)\right)}{64\sqrt{\pi}\,\Gamma(8)} + \frac{0.0000167417\left(15\,(3-4+4\times16)\,\Gamma\left(\frac{5}{2}\right)\right)}{32\sqrt{\pi}\,\Gamma(8)} =$$

$$\left(2.18781\left(-0.00152023\sum_{k=0}^{\infty}\frac{\left(\frac{5}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!} + \sqrt{-1+\pi}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1+\pi)^{-k_1}\binom{\frac{1}{2}}{k_1}(8-z_0)^{k_2}\,\Gamma^{(k_2)}(z_0)}{k_2!}\right)\right) \Big/ \left(\sqrt{-1+\pi}\left(\sum_{k=0}^{\infty}(-1+\pi)^{-k}\binom{\frac{1}{2}}{k}\right)\sum_{k=0}^{\infty}\frac{(8-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}\right) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

Integral representations:

$$2.1878066832031250000 - \frac{0.000776204\left(315\,\Gamma\left(\frac{5}{2}\right)\right)}{64\sqrt{\pi}\,\Gamma(8)} + \frac{0.0000167417\left(15\,(3-4+4\times16)\,\Gamma\left(\frac{5}{2}\right)\right)}{32\sqrt{\pi}\,\Gamma(8)} =$$

$$2.1878066832031250000 - \frac{0.00332598\exp\left(\int_0^1\frac{\frac{11}{2}(-1+x)+x^{5/2}-x^8}{(-1+x)\log(x)}\,dx\right)}{\sqrt{\pi}}$$

$$2.1878066832031250000 - \frac{0.000776204\left(315\,\Gamma\left(\frac{5}{2}\right)\right)}{64\sqrt{\pi}\,\Gamma(8)} + \frac{0.0000167417\left(15\,(3-4+4\times16)\,\Gamma\left(\frac{5}{2}\right)\right)}{32\sqrt{\pi}\,\Gamma(8)} =$$

$$2.1878066832031250000 - \frac{0.00332598\exp\left(\frac{11\gamma}{2}+\int_0^1\frac{x^{5/2}-x^8-\log\left(x^{5/2}\right)+\log\left(x^8\right)}{(-1+x)\log(x)}\,dx\right)}{\sqrt{\pi}}$$

$$2.1878066832031250000 - \frac{0.000776204\left(315\,\Gamma\left(\frac{5}{2}\right)\right)}{64\sqrt{\pi}\,\Gamma(8)} + \frac{0.0000167417\left(15\,(3-4+4\times16)\,\Gamma\left(\frac{5}{2}\right)\right)}{32\sqrt{\pi}\,\Gamma(8)} =$$

$$-\frac{0.00332598\left(\int_0^1\log^{3/2}\left(\frac{1}{t}\right)dt - 657.794\sqrt{\pi}\int_0^1\log^7\left(\frac{1}{t}\right)dt\right)}{\sqrt{\pi}\int_0^1\log^7\left(\frac{1}{t}\right)dt}$$

From which:

10^3+exp^3(((2.1878066832 - 0.000776204(315 gamma (5/2))/(64sqrtPi (gamma (8))) + 0.000016741663(15(3-4+4*16) gamma (5/2))/(32sqrtPi * (gamma (8))))))+18+e-2/5

Input interpretation:

$$10^3 + \exp^3\left(2.1878066832 + \frac{315\,\Gamma\left(\frac{5}{2}\right)}{64\sqrt{\pi}\,\Gamma(8)} \times (-0.000776204) + 0.000016741663 \times \frac{15\,(3-4+4\times 16)\,\Gamma\left(\frac{5}{2}\right)}{32\sqrt{\pi}\,\Gamma(8)}\right) + 18 + e - \frac{2}{5}$$

$\Gamma(x)$ is the gamma function

Result:

1729.0085436...

1729.0085436…

Alternative representations:

$$10^3 + \exp^3\left(2.18780668320000 - \frac{0.000776204\left(315\,\Gamma\left(\frac{5}{2}\right)\right)}{64\sqrt{\pi}\,\Gamma(8)} + \frac{0.0000167417\left(15\,(3-4+4\times 16)\,\Gamma\left(\frac{5}{2}\right)\right)}{32\sqrt{\pi}\,\Gamma(8)}\right) + 18 + e - \frac{2}{5} = 18 + e - \frac{2}{5} + 10^3 + \exp^3\left(2.18780668320000 + \frac{0.0158209\,(1)_{\frac{3}{2}}}{32\,(1)_7\sqrt{\pi}} - \frac{0.244504\,(1)_{\frac{3}{2}}}{64\,(1)_7\sqrt{\pi}}\right)$$

$$10^3 + \exp^3\left(2.18780668320000 - \frac{0.000776204\left(315\,\Gamma\left(\frac{5}{2}\right)\right)}{64\sqrt{\pi}\,\Gamma(8)} + \frac{0.0000167417\left(15\,(3-4+4\times 16)\,\Gamma\left(\frac{5}{2}\right)\right)}{32\sqrt{\pi}\,\Gamma(8)}\right) + 18 + e - \frac{2}{5} = 18 + e - \frac{2}{5} + 10^3 + \exp^3\left(2.18780668320000 + \frac{0.0158209\,e^{-\log G(5/2)+\log G(7/2)}}{32\,e^{-\log(24883200)+\log(125411328000)}\sqrt{\pi}} - \frac{0.244504\,e^{-\log G(5/2)+\log G(7/2)}}{64\,e^{-\log(24883200)+\log(125411328000)}\sqrt{\pi}}\right)$$

$$10^3+\exp^3\left(2.18780668320000-\frac{0.000776204\left(315\,\Gamma\left(\frac{5}{2}\right)\right)}{64\sqrt{\pi}\,\Gamma(8)}+\frac{0.0000167417\left(15\,(3-4+4\times16)\,\Gamma\left(\frac{5}{2}\right)\right)}{32\sqrt{\pi}\,\Gamma(8)}\right)+18+e-\frac{2}{5}=$$

$$18+e-\frac{2}{5}+10^3+\exp^3\left(2.18780668320000+\frac{0.0199287}{\frac{\left(\sqrt[8]{e}\,\pi^{3/4}\right)\left(4013162496000\sqrt{\pi}\right)}{\left(2^{23/24}A^{3/2}\right)24883200}}-\frac{0.307989}{\frac{\left(\sqrt[8]{e}\,\pi^{3/4}\right)\left(8026324992000\sqrt{\pi}\right)}{\left(2^{23/24}A^{3/2}\right)24883200}}\right)$$

Series representations:

$$10^3+\exp^3\left(2.18780668320000-\frac{0.000776204\left(315\,\Gamma\left(\frac{5}{2}\right)\right)}{64\sqrt{\pi}\,\Gamma(8)}+\frac{0.0000167417\left(15\,(3-4+4\times16)\,\Gamma\left(\frac{5}{2}\right)\right)}{32\sqrt{\pi}\,\Gamma(8)}\right)+18+e-\frac{2}{5}=$$

$$\frac{1}{5}\left(5088+5e+5\exp^3\left(2.18780668320000-\frac{0.00332598\sum_{k=0}^{\infty}\frac{\left(\frac{5}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}}{\exp\left(i\pi\left\lfloor\frac{\arg(\pi-x)}{2\pi}\right\rfloor\right)\sqrt{x}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(\pi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(8-z_0)^k\Gamma^{(k)}(z_0)}{k!}}\right)\right)$$

for ($x\in\mathbb{R}$ and ($z_0\notin\mathbb{Z}$ or $z_0>0$) and $x<0$)

$$10^3+\exp^3\left(2.18780668320000-\frac{0.000776204\left(315\,\Gamma\left(\frac{5}{2}\right)\right)}{64\sqrt{\pi}\;\Gamma(8)}+\frac{0.0000167417\left(15\,(3-4+4\times 16)\,\Gamma\left(\frac{5}{2}\right)\right)}{32\sqrt{\pi}\;\Gamma(8)}\right)+18+e-\frac{2}{5}=$$

$$\frac{1}{5}\left(5088+5\,e+5\exp^3\left(2.18780668320000-\left(0.00332598\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(\pi-z_0)/(2\pi)\rfloor} z_0^{-1/2-1/2\lfloor\arg(\pi-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{\left(\frac{5}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}\right)\Big/\left(\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\pi-z_0)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(8-z_0)^k\Gamma^{(k)}(z_0)}{k!}\right)\right)\right)$$

for $(z_0 \notin \mathbb{Z}$ or $z_0>0)$

$$10^3+\exp^3\left(2.18780668320000-\frac{0.000776204\left(315\,\Gamma\left(\frac{5}{2}\right)\right)}{64\sqrt{\pi}\;\Gamma(8)}+\frac{0.0000167417\left(15\,(3-4+4\times 16)\,\Gamma\left(\frac{5}{2}\right)\right)}{32\sqrt{\pi}\;\Gamma(8)}\right)+18+e-\frac{2}{5}=$$

$$\frac{1}{5}\left(5088+5\,e+5\exp^3\left(2.18780668320000-\left(0.00332598\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(\pi-z_0)/(2\pi)\rfloor} z_0^{1/2\left(-1-\lfloor\arg(\pi-z_0)/(2\pi)\rfloor\right)}\sum_{k=0}^{\infty}\frac{\left(\frac{5}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}\right)\Big/\left(\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\pi-z_0)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(8-z_0)^k\Gamma^{(k)}(z_0)}{k!}\right)\right)\right)$$

for $(z_0 \notin \mathbb{Z}$ or $z_0>0)$

Integral representations:

$$10^3+\exp^3\left(2.18780668320000-\frac{0.000776204\left(315\,\Gamma\left(\frac{5}{2}\right)\right)}{64\sqrt{\pi}\;\Gamma(8)}+\frac{0.0000167417\left(15\,(3-4+4\times 16)\,\Gamma\left(\frac{5}{2}\right)\right)}{32\sqrt{\pi}\;\Gamma(8)}\right)+18+e-\frac{2}{5}=$$

$$\frac{1}{5}\left(5088+5\,e+5\exp^3\left(2.18780668320000-\frac{0.00332598\int_0^1\log^{3/2}\left(\frac{1}{t}\right)dt}{\sqrt{\pi}\int_0^1\log^7\left(\frac{1}{t}\right)dt}\right)\right)$$

$$10^3 + \exp^3\left(2.18780668320000 - \frac{0.000776204\left(315\,\Gamma\left(\frac{5}{2}\right)\right)}{64\sqrt{\pi}\,\Gamma(8)} + \frac{0.0000167417\left(15\,(3-4+4\times 16)\,\Gamma\left(\frac{5}{2}\right)\right)}{32\sqrt{\pi}\,\Gamma(8)}\right) + 18 + e - \frac{2}{5} = \frac{1}{5}\left(5088 + 5\,e + 5\exp^3\left(2.18780668320000 - \frac{0.00332598\int_0^\infty t^{3/2}\,\mathcal{A}^{-t}\,dt}{\sqrt{\pi}\int_0^\infty t^7\,\mathcal{A}^{-t}\,dt}\right)\right)$$

$$10^3 + \exp^3\left(2.18780668320000 - \frac{0.000776204\left(315\,\Gamma\left(\frac{5}{2}\right)\right)}{64\sqrt{\pi}\,\Gamma(8)} + \frac{0.0000167417\left(15\,(3-4+4\times 16)\,\Gamma\left(\frac{5}{2}\right)\right)}{32\sqrt{\pi}\,\Gamma(8)}\right) + 18 + e - \frac{2}{5} = \frac{5088}{5} + e + \exp^3\left(2.18780668320000 - \frac{0.00332598\,\mathcal{A}^{\int_0^1\left(\frac{11}{2}(-1+x)+x^{5/2}-x^8\right)/((-1+x)\log(x))\,dx}}{\sqrt{\pi}}\right)$$

We have that:

Instantons at higher order in $1/N$

$$I^{(0)}_{2\times 1} = -5\sqrt{\frac{2}{\pi}}\sqrt{N} + \frac{17\sqrt{\frac{1}{N}}}{8\sqrt{2\pi}} + \frac{325\left(\frac{1}{N}\right)^{3/2}}{1024\sqrt{2\pi}} + \frac{2155\left(\frac{1}{N}\right)^{5/2}}{8192\sqrt{2\pi}} + \frac{1543605\left(\frac{1}{N}\right)^{7/2}}{4194304\sqrt{2\pi}} + O(N^{-\frac{9}{2}}),$$

$$I^{(2)}_{2\times 1} = -\frac{15\sqrt{N}}{64\sqrt{2}\pi^{5/2}} + \frac{255\sqrt{\frac{1}{N}}}{1024\sqrt{2}\pi^{5/2}} + \frac{19695\left(\frac{1}{N}\right)^{3/2}}{131072\sqrt{2}\pi^{5/2}} + \frac{217365\left(\frac{1}{N}\right)^{5/2}}{1048576\sqrt{2}\pi^{5/2}} + \frac{218377215\left(\frac{1}{N}\right)^{7/2}}{536870912\sqrt{2}\pi^{5/2}} + O(N^{-\frac{9}{2}}) \quad \text{(D.4)}$$

From:

$$I^{(0)}_{2\times 1} = -5\sqrt{\frac{2}{\pi}}\sqrt{N} + \frac{17\sqrt{\frac{1}{N}}}{8\sqrt{2\pi}} + \frac{325\left(\frac{1}{N}\right)^{3/2}}{1024\sqrt{2\pi}} + \frac{2155\left(\frac{1}{N}\right)^{5/2}}{8192\sqrt{2\pi}} + \frac{1543605\left(\frac{1}{N}\right)^{7/2}}{4194304\sqrt{2\pi}} + O(N^{-\frac{9}{2}})$$

For N = 4

-5sqrt(2/Pi)*2+(17sqrt(1/4))/(8*sqrt(2Pi))+(325(1/4)^1.5)/(1024sqrt(2Pi))+(2155(1/4)^2.5)/(8192sqrt(2Pi))+(1543605(1/4)^3.5)/(4194304sqrt(2Pi))+4^(-4.5)

Input:

$$-5\sqrt{\frac{2}{\pi}}\times 2+\frac{17\sqrt{\frac{1}{4}}}{8\sqrt{2\pi}}+\frac{325\left(\frac{1}{4}\right)^{1.5}}{1024\sqrt{2\pi}}+\frac{2155\left(\frac{1}{4}\right)^{2.5}}{8192\sqrt{2\pi}}+\frac{1543605\left(\frac{1}{4}\right)^{3.5}}{4194304\sqrt{2\pi}}+\frac{1}{4^{4.5}}$$

Result:

-7.5327625...

-7.5327625....

Series representations:

$$-5\sqrt{\frac{2}{\pi}}\,2+\frac{17\sqrt{\frac{1}{4}}}{8\sqrt{2\pi}}+\frac{325\left(\frac{1}{4}\right)^{1.5}}{1024\sqrt{2\pi}}+\frac{2155\left(\frac{1}{4}\right)^{2.5}}{8192\sqrt{2\pi}}+\frac{1543605\left(\frac{1}{4}\right)^{3.5}}{4194304\sqrt{2\pi}}+\frac{1}{4^{4.5}}=$$
$$-\left(\left(10\left(-0.00507687-0.2125\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{1}{4}-z_0\right)^k z_0^{-k}}{k!}-\right.\right.\right.$$
$$0.000195313\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2\pi-z_0)^k z_0^{-k}}{k!}+\sqrt{z_0}^{\,2}$$
$$\left.\left.\left.\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}\left(\frac{2}{\pi}-z_0\right)^{k_1}(2\pi-z_0)^{k_2}z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)\right)\right/$$
$$\left.\left(\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2\pi-z_0)^k z_0^{-k}}{k!}\right)\right)\quad \text{for (not } (z_0\in\mathbb{R}\text{ and }-\infty<z_0\le 0))$$

$$-5\sqrt{\frac{2}{\pi}}\ 2+\frac{17\sqrt{\frac{1}{4}}}{8\sqrt{2\pi}}+\frac{325\left(\frac{1}{4}\right)^{1.5}}{1024\sqrt{2\pi}}+\frac{2155\left(\frac{1}{4}\right)^{2.5}}{8192\sqrt{2\pi}}+\frac{1543605\left(\frac{1}{4}\right)^{3.5}}{4194304\sqrt{2\pi}}+\frac{1}{4^{4.5}}=$$

$$-\left(\left(10\left(-0.00507687-0.2125\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{1}{4}-x\right)}{2\pi}\right\rfloor\right)\right.\right.\right.$$

$$\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{1}{4}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}-0.000195313$$

$$\exp\left(i\pi\left\lfloor\frac{\arg(2\pi-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2\pi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+$$

$$\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{2}{\pi}-x\right)}{2\pi}\right\rfloor\right)\exp\left(i\pi\left\lfloor\frac{\arg(2\pi-x)}{2\pi}\right\rfloor\right)\sqrt{x}^2$$

$$\left.\left.\left.\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(\frac{2}{\pi}-x\right)^{k_1}(2\pi-x)^{k_2}x^{-k_1-k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}}{k_1!\,k_2!}\right)\right)\right/$$

$$\left(\exp\left(i\pi\left\lfloor\frac{\arg(2\pi-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2\pi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$ for ($x \in \mathbb{R}$ and $x < 0$)

$$-5\sqrt{\frac{2}{\pi}}\,2+\frac{17\sqrt{\frac{1}{4}}}{8\sqrt{2\pi}}+\frac{325\left(\frac{1}{4}\right)^{1.5}}{1024\sqrt{2\pi}}+\frac{2155\left(\frac{1}{4}\right)^{2.5}}{8192\sqrt{2\pi}}+\frac{1543605\left(\frac{1}{4}\right)^{3.5}}{4194304\sqrt{2\pi}}+\frac{1}{4^{4.5}}=$$

$$-\left(\left(10\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2\pi-z_0)/(2\pi)\rfloor} z_0^{-1-1/2\lfloor\arg(2\pi-z_0)/(2\pi)\rfloor}\left(-0.00507687\sqrt{z_0}-0.2125\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{1}{4}-z_0\right)/(2\pi)\right\rfloor} z_0^{1+1/2\left\lfloor\arg\left(\frac{1}{4}-z_0\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{1}{4}-z_0\right)^k z_0^{-k}}{k!}-0.000195313\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2\pi-z_0)/(2\pi)\rfloor} z_0^{1+1/2\lfloor\arg(2\pi-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2\pi-z_0)^k z_0^{-k}}{k!}+\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{2}{\pi}-z_0\right)/(2\pi)\right\rfloor+1/2\lfloor\arg(2\pi-z_0)/(2\pi)\rfloor} z_0^{3/2+1/2\left\lfloor\arg\left(\frac{2}{\pi}-z_0\right)/(2\pi)\right\rfloor+1/2\lfloor\arg(2\pi-z_0)/(2\pi)\rfloor}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}\left(\frac{2}{\pi}-z_0\right)^{k_1}(2\pi-z_0)^{k_2} z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)\right)\Bigg/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2\pi-z_0)^k z_0^{-k}}{k!}\right)\right)$$

From which:

((([-(((-5sqrt(2/Pi)*2+(17sqrt(1/4))/(8*sqrt(2Pi))+(325(1/4)^1.5)/(1024sqrt(2Pi))+(2155(1/4)^2.5)/(8192sqrt(2Pi))+(1543605(1/4)^3.5)/(4194304sqrt(2Pi))+4^(-4.5))))]^1/4)))

Input:

$$\sqrt[4]{-\left(-5\sqrt{\frac{2}{\pi}}\times 2+\frac{17\sqrt{\frac{1}{4}}}{8\sqrt{2\pi}}+\frac{325\left(\frac{1}{4}\right)^{1.5}}{1024\sqrt{2\pi}}+\frac{2155\left(\frac{1}{4}\right)^{2.5}}{8192\sqrt{2\pi}}+\frac{1543605\left(\frac{1}{4}\right)^{3.5}}{4194304\sqrt{2\pi}}+\frac{1}{4^{4.5}}\right)}$$

Result:

1.65667977...

1.65667977…. result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164{,}2696$ i.e. 1,65578...

and:

((([-(((-5sqrt(2/Pi)*2+(17sqrt(1/4))/(8*sqrt(2Pi))+(325(1/4)^1.5)/(1024sqrt(2Pi))+(2155(1/4)^2.5)/(8192sqrt(2Pi))+(1543605(1/4)^3.5)/(4194304sqrt(2Pi))+4^(-4.5))))]^1/4)))^14 + 21+2zeta (2)

Input:

$$\sqrt[4]{-\left(-5\sqrt{\frac{2}{\pi}}\times 2+\frac{17\sqrt{\frac{1}{4}}}{8\sqrt{2\pi}}+\frac{325\left(\frac{1}{4}\right)^{1.5}}{1024\sqrt{2\pi}}+\frac{2155\left(\frac{1}{4}\right)^{2.5}}{8192\sqrt{2\pi}}+\frac{1543605\left(\frac{1}{4}\right)^{3.5}}{4194304\sqrt{2\pi}}+\frac{1}{4^{4.5}}\right)}^{14}+21+2\zeta(2)$$

$\zeta(s)$ is the Riemann zeta function

Result:

1197.4032...

1197.4032… result practically equal to the rest mass of Sigma baryon 1197.449

Alternative representations:

$$\sqrt[4]{-\left(-5\sqrt{\frac{2}{\pi}}\,2+\frac{17\sqrt{\frac{1}{4}}}{8\sqrt{2\pi}}+\frac{325\left(\frac{1}{4}\right)^{1.5}}{1024\sqrt{2\pi}}+\frac{2155\left(\frac{1}{4}\right)^{2.5}}{8192\sqrt{2\pi}}+\frac{1543605\left(\frac{1}{4}\right)^{3.5}}{4194304\sqrt{2\pi}}+\frac{1}{4^{4.5}}\right)}^{14}+21+2\zeta(2)=21+\sqrt[4]{-\frac{1}{4^{4.5}}-\frac{325\left(\frac{1}{4}\right)^{1.5}}{1024\sqrt{2\pi}}-\frac{2155\left(\frac{1}{4}\right)^{2.5}}{8192\sqrt{2\pi}}-\frac{1543605\left(\frac{1}{4}\right)^{3.5}}{4194304\sqrt{2\pi}}-\frac{17\sqrt{\frac{1}{4}}}{8\sqrt{2\pi}}+10\sqrt{\frac{2}{\pi}}}^{14}+2\zeta(2,1)$$

$$\sqrt[4]{-\left(-5\sqrt{\frac{2}{\pi}}\ 2+\frac{17\sqrt{\frac{1}{4}}}{8\sqrt{2\pi}}+\frac{325\left(\frac{1}{4}\right)^{1.5}}{1024\sqrt{2\pi}}+\frac{2155\left(\frac{1}{4}\right)^{2.5}}{8192\sqrt{2\pi}}+\frac{1543605\left(\frac{1}{4}\right)^{3.5}}{4194304\sqrt{2\pi}}+\frac{1}{4^{4.5}}\right)^{14}}+$$

$$21+2\zeta(2)=21+2S_{1,1}(1)+\sqrt[4]{\left(-\frac{1}{4^{4.5}}-\frac{325\left(\frac{1}{4}\right)^{1.5}}{1024\sqrt{2\pi}}-\frac{2155\left(\frac{1}{4}\right)^{2.5}}{8192\sqrt{2\pi}}-\frac{1543605\left(\frac{1}{4}\right)^{3.5}}{4194304\sqrt{2\pi}}-\frac{17\sqrt{\frac{1}{4}}}{8\sqrt{2\pi}}+10\sqrt{\frac{2}{\pi}}\right)^{14}}$$

$$\sqrt[4]{-\left(-5\sqrt{\frac{2}{\pi}}\ 2+\frac{17\sqrt{\frac{1}{4}}}{8\sqrt{2\pi}}+\frac{325\left(\frac{1}{4}\right)^{1.5}}{1024\sqrt{2\pi}}+\frac{2155\left(\frac{1}{4}\right)^{2.5}}{8192\sqrt{2\pi}}+\frac{1543605\left(\frac{1}{4}\right)^{3.5}}{4194304\sqrt{2\pi}}+\frac{1}{4^{4.5}}\right)^{14}}+$$

$$21+2\zeta(2)=21-\frac{2\,\mathrm{Li}_2(-1)}{\frac{1}{2}}+\sqrt[4]{\left(-\frac{1}{4^{4.5}}-\frac{325\left(\frac{1}{4}\right)^{1.5}}{1024\sqrt{2\pi}}-\frac{2155\left(\frac{1}{4}\right)^{2.5}}{8192\sqrt{2\pi}}-\frac{1543605\left(\frac{1}{4}\right)^{3.5}}{4194304\sqrt{2\pi}}-\frac{17\sqrt{\frac{1}{4}}}{8\sqrt{2\pi}}+10\sqrt{\frac{2}{\pi}}\right)^{14}}$$

Series representations:

$$\sqrt[4]{-\left(-5\sqrt{\frac{2}{\pi}}\ 2+\frac{17\sqrt{\frac{1}{4}}}{8\sqrt{2\pi}}+\frac{325\left(\frac{1}{4}\right)^{1.5}}{1024\sqrt{2\pi}}+\frac{2155\left(\frac{1}{4}\right)^{2.5}}{8192\sqrt{2\pi}}+\frac{1543605\left(\frac{1}{4}\right)^{3.5}}{4194304\sqrt{2\pi}}+\frac{1}{4^{4.5}}\right)^{14}}+$$

$$21+2\zeta(2)=21+2\sum_{k=1}^{\infty}\frac{1}{k^2}+\left(-0.00195313+10\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{2}{\pi}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{2}{\pi}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}-\frac{0.0507687}{\exp\left(i\pi\left\lfloor\frac{\arg(2\pi-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2\pi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}-\frac{17\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{1}{4}-x\right)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{1}{4}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{8\exp\left(i\pi\left\lfloor\frac{\arg(2\pi-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(2\pi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}\right)^{7/2}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\sqrt[4]{-\left(-5\sqrt{\frac{2}{\pi}}\ 2+\frac{17\sqrt{\frac{1}{4}}}{8\sqrt{2\pi}}+\frac{325\left(\frac{1}{4}\right)^{1.5}}{1024\sqrt{2\pi}}+\frac{2155\left(\frac{1}{4}\right)^{2.5}}{8192\sqrt{2\pi}}+\frac{1543605\left(\frac{1}{4}\right)^{3.5}}{4194304\sqrt{2\pi}}+\frac{1}{4^{4.5}}\right)^{14}+}$$

$$21+2\zeta(2)=21+2\exp\left(\sum_{k=1}^{\infty}\sum_{j=1}^{\infty}\frac{(p_k)^{-2j}}{j}\right)+$$

$$\left(\left(-0.0507687-2.125\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{1}{4}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{1}{4}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+\right.\right.$$

$$\exp\left(i\pi\left\lfloor\frac{\arg(2\pi-x)}{2\pi}\right\rfloor\right)\sqrt{x}\left(-0.00195313+\right.$$

$$\left.10\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{2}{\pi}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{2}{\pi}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k(2\pi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Bigg/\left(\exp\left(i\pi\left\lfloor\frac{\arg(2\pi-x)}{2\pi}\right\rfloor\right)\right.$$

$$\left.\left.\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2\pi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)^{7/2}\quad\text{for } (x\in\mathbb{R}\text{ and } x<0)$$

$$\sqrt[4]{-\left(-5\sqrt{\frac{2}{\pi}}\ 2+\frac{17\sqrt{\frac{1}{4}}}{8\sqrt{2\pi}}+\frac{325\left(\frac{1}{4}\right)^{1.5}}{1024\sqrt{2\pi}}+\frac{2155\left(\frac{1}{4}\right)^{2.5}}{8192\sqrt{2\pi}}+\frac{1543605\left(\frac{1}{4}\right)^{3.5}}{4194304\sqrt{2\pi}}+\frac{1}{4^{4.5}}\right)^{14}+}$$

$$21+2\zeta(2)=21+2\exp\left(-\sum_{k=1}^{\infty}\log\left(1-\frac{1}{(p_k)^2}\right)\right)+$$

$$\left(-0.00195313+10\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{2}{\pi}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{2}{\pi}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}-\right.$$

$$\frac{0.0507687}{\exp\left(i\pi\left\lfloor\frac{\arg(2\pi-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2\pi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}-$$

$$\left.\frac{17\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{1}{4}-x\right)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{1}{4}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{8\exp\left(i\pi\left\lfloor\frac{\arg(2\pi-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(2\pi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}\right)^{7/2}\quad\text{for } (x\in\mathbb{R}\text{ and } x<0)$$

Integral representations:

$$\sqrt[4]{-\left(-5\sqrt{\frac{2}{\pi}}\;2+\frac{17\sqrt{\frac{1}{4}}}{8\sqrt{2\pi}}+\frac{325\left(\frac{1}{4}\right)^{1.5}}{1024\sqrt{2\pi}}+\frac{2155\left(\frac{1}{4}\right)^{2.5}}{8192\sqrt{2\pi}}+\frac{1543605\left(\frac{1}{4}\right)^{3.5}}{4194304\sqrt{2\pi}}+\frac{1}{4^{4.5}}\right)^{14}+\left(\frac{-0.0507687-2.125\sqrt{\frac{1}{4}}+\left(-0.00195313+10\sqrt{\frac{2}{\pi}}\right)\sqrt{2\pi}}{\sqrt{2\pi}}\right)^{7/2}}$$

$$21+2\zeta(2)=21+\frac{2}{\Gamma(2)}\int_0^\infty\frac{t}{-1+e^t}\,dt$$

$$\sqrt[4]{-\left(-5\sqrt{\frac{2}{\pi}}\;2+\frac{17\sqrt{\frac{1}{4}}}{8\sqrt{2\pi}}+\frac{325\left(\frac{1}{4}\right)^{1.5}}{1024\sqrt{2\pi}}+\frac{2155\left(\frac{1}{4}\right)^{2.5}}{8192\sqrt{2\pi}}+\frac{1543605\left(\frac{1}{4}\right)^{3.5}}{4194304\sqrt{2\pi}}+\frac{1}{4^{4.5}}\right)^{14}+\left(\frac{-0.0507687-2.125\sqrt{\frac{1}{4}}+\left(-0.00195313+10\sqrt{\frac{2}{\pi}}\right)\sqrt{2\pi}}{\sqrt{2\pi}}\right)^{7/2}}$$

$$21+2\zeta(2)=21+\frac{4}{3\,\Gamma(2)}\int_0^\infty t\,\operatorname{csch}(t)\,dt$$

$$\sqrt[4]{-\left(-5\sqrt{\frac{2}{\pi}}\;2+\frac{17\sqrt{\frac{1}{4}}}{8\sqrt{2\pi}}+\frac{325\left(\frac{1}{4}\right)^{1.5}}{1024\sqrt{2\pi}}+\frac{2155\left(\frac{1}{4}\right)^{2.5}}{8192\sqrt{2\pi}}+\frac{1543605\left(\frac{1}{4}\right)^{3.5}}{4194304\sqrt{2\pi}}+\frac{1}{4^{4.5}}\right)^{14}+\left(\frac{-0.0507687-2.125\sqrt{\frac{1}{4}}+\left(-0.00195313+10\sqrt{\frac{2}{\pi}}\right)\sqrt{2\pi}}{\sqrt{2\pi}}\right)^{7/2}}$$

$$21+2\zeta(2)=21+\frac{1}{0!}\int_0^1\frac{\log^2(1-t)}{t^2}\,dt$$

From:

$$I^{(2)}_{2\times 1} = -\frac{15\sqrt{N}}{64\sqrt{2}\pi^{5/2}} + \frac{255\sqrt{\frac{1}{N}}}{1024\sqrt{2}\pi^{5/2}} + \frac{19695\left(\frac{1}{N}\right)^{3/2}}{131072\sqrt{2}\pi^{5/2}} + \frac{217365\left(\frac{1}{N}\right)^{5/2}}{1048576\sqrt{2}\pi^{5/2}} + \frac{218377215\left(\frac{1}{N}\right)^{7/2}}{536870912\sqrt{2}\pi^{5/2}} + O(N^{-\frac{9}{2}}) \quad \text{(D.4)}$$

We obtain:

-(15sqrt(4))/(64*sqrt(2)Pi^2.5)+(255(1/4)^0.5)/(1024sqrt(2)Pi^2.5)+(19695(1/4)^1.5)/(131072sqrt(2)Pi^2.5)+(217365(1/4)^2.5)/(1048576sqrt(2)Pi^2.5)+(218377215(1/4)^3.5)/(536870912sqrt(2)Pi^2.5)+4^-4.5

Input:

$$-\frac{15\sqrt{4}}{64\sqrt{2}\,\pi^{2.5}} + \frac{255\sqrt{\frac{1}{4}}}{1024\sqrt{2}\,\pi^{2.5}} + \frac{19\,695\left(\frac{1}{4}\right)^{1.5}}{131\,072\sqrt{2}\,\pi^{2.5}} + \frac{217\,365\left(\frac{1}{4}\right)^{2.5}}{1\,048\,576\sqrt{2}\,\pi^{2.5}} + \frac{218\,377\,215\left(\frac{1}{4}\right)^{3.5}}{536\,870\,912\sqrt{2}\,\pi^{2.5}} + \frac{1}{4^{4.5}}$$

Result:

-0.0108119...

-0.0108119…

Series representations:

$$-\frac{15\sqrt{4}}{64\sqrt{2}\,\pi^{2.5}} + \frac{255\sqrt{\frac{1}{4}}}{1024\sqrt{2}\,\pi^{2.5}} + \frac{19\,695\left(\frac{1}{4}\right)^{1.5}}{131\,072\sqrt{2}\,\pi^{2.5}} + \frac{217\,365\left(\frac{1}{4}\right)^{2.5}}{1\,048\,576\sqrt{2}\,\pi^{2.5}} + \frac{218\,377\,215\left(\frac{1}{4}\right)^{3.5}}{536\,870\,912\sqrt{2}\,\pi^{2.5}} + \frac{1}{4^{4.5}} = \left(0.00195313\left(78.3105 + \pi^{2.5}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} - 120\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (4-z_0)^k z_0^{-k}}{k!}\right)\right) \Big/ \left(\pi^{2.5}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}\right) \text{ for (not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$-\frac{15\sqrt{4}}{64\sqrt{2}\,\pi^{2.5}}+\frac{255\sqrt{\frac{1}{4}}}{1024\sqrt{2}\,\pi^{2.5}}+\frac{19\,695\left(\frac{1}{4}\right)^{1.5}}{131\,072\sqrt{2}\,\pi^{2.5}}+\frac{217\,365\left(\frac{1}{4}\right)^{2.5}}{1\,048\,576\sqrt{2}\,\pi^{2.5}}+\frac{218\,377\,215\left(\frac{1}{4}\right)^{3.5}}{536\,870\,912\sqrt{2}\,\pi^{2.5}}+\frac{1}{4^{4.5}}=$$

$$\left(0.00195313\left(78.3105+\pi^{2.5}\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}-120\exp\left(i\pi\left\lfloor\frac{\arg(4-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(4-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\Big/\left(\pi^{2.5}\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$-\frac{15\sqrt{4}}{64\sqrt{2}\,\pi^{2.5}}+\frac{255\sqrt{\frac{1}{4}}}{1024\sqrt{2}\,\pi^{2.5}}+\frac{19\,695\left(\frac{1}{4}\right)^{1.5}}{131\,072\sqrt{2}\,\pi^{2.5}}+\frac{217\,365\left(\frac{1}{4}\right)^{2.5}}{1\,048\,576\sqrt{2}\,\pi^{2.5}}+\frac{218\,377\,215\left(\frac{1}{4}\right)^{3.5}}{536\,870\,912\sqrt{2}\,\pi^{2.5}}+\frac{1}{4^{4.5}}=$$

$$\left(0.15295\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{-1-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sqrt{z_0}+0.0127697\,\pi^{2.5}\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}-1.53236\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(4-z_0)/(2\pi)\rfloor} z_0^{1+1/2\lfloor\arg(4-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(4-z_0)^k z_0^{-k}}{k!}\right)\right)\Big/\left(\pi^{2.5}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)$$

From:

$$-\frac{15\sqrt{4}}{64\sqrt{2}\,\pi^{2.5}}+\frac{255\sqrt{\frac{1}{4}}}{1024\sqrt{2}\,\pi^{2.5}}+\frac{19\,695\left(\frac{1}{4}\right)^{1.5}}{131\,072\sqrt{2}\,\pi^{2.5}}+\frac{217\,365\left(\frac{1}{4}\right)^{2.5}}{1\,048\,576\sqrt{2}\,\pi^{2.5}}+\frac{218\,377\,215\left(\frac{1}{4}\right)^{3.5}}{536\,870\,912\sqrt{2}\,\pi^{2.5}}+\frac{1}{4^{4.5}}$$

−0.010811917851336253696967628185644699679465148862361760 80...

-0.01081191785...

we obtain:

exp(-0.010811917851336253696967628185644699679465148862361)

Input interpretation:

exp(-0.010811917851336253696967628185644699679465148862361)

Result:

0.989246320852807131901378977456079492618823284546224 3...

0.98924632085…. result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684$$

and to the Omega mesons (ω/ω_3 | $5+3$ | $m_{u/d} = 255 - 390$ | $0.988 - 1.18$) Regge slope value (0.988) connected to the dilaton scalar field $\mathbf{0.989117352243} = \boldsymbol{\phi}$

A_1^{**} above the two low-lying pseudo-scalars. (bound states of gluons, or ’glueballs’).

A_1^{**}	0.943(39) [2.5]	0.988(38)	0.152(53)
A_4	1.03(10) [2.5]	0.999(32)	0.035(21)

(**Glueball Regge trajectories -** *Harvey Byron Meyer*, Lincoln College -Thesis submitted for the degree of Doctor of Philosophy at the University of Oxford Trinity Term, 2004)

and:

(((-[ln(-(-0.01081191785))]^5-144-21-8+1/2)))

Input interpretation:

$$-\log^5(-(-0.01081191785)) - 144 - 21 - 8 + \frac{1}{2}$$

$\log(x)$ is the natural logarithm

Result:

1729.031115...

1729.031115…

Alternative representations:

$$-\log^5(-(-1)\,0.0108119) - 144 - 21 - 8 + \frac{1}{2} = -\frac{345}{2} - \log_e^5(0.0108119)$$

$$-\log^5(-(-1)\,0.0108119) - 144 - 21 - 8 + \frac{1}{2} = -\frac{345}{2} - (\log(a)\log_a(0.0108119))^5$$

$$-\log^5(-(-1)\,0.0108119) - 144 - 21 - 8 + \frac{1}{2} = -\frac{345}{2} - (-\text{Li}_1(0.989188))^5$$

Series representations:

$$-\log^5(-(-1)\,0.0108119) - 144 - 21 - 8 + \frac{1}{2} = -\frac{345}{2} + \left(\sum_{k=1}^{\infty} \frac{(-1)^k (-0.989188)^k}{k}\right)^5$$

$$-\log^5(-(-1)\,0.0108119) - 144 - 21 - 8 + \frac{1}{2} = -\frac{345}{2} - \left(2 i \pi \left\lfloor \frac{\arg(0.0108119 - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.0108119 - x)^k x^{-k}}{k}\right)^5 \quad \text{for } x < 0$$

$$-\log^5(-(-1)\,0.0108119) - 144 - 21 - 8 + \frac{1}{2} = -\frac{345}{2} - \left(\log(z_0) + \left\lfloor \frac{\arg(0.0108119 - z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.0108119 - z_0)^k z_0^{-k}}{k}\right)^5$$

Integral representation:

$$-\log^5(-(-1)\,0.0108119) - 144 - 21 - 8 + \frac{1}{2} = -\frac{345}{2} - \left(\int_1^{0.0108119} \frac{1}{t}\,dt\right)^5$$

(((-[ln(-(-0.01081191785))]^5-144-21-8+1/2)))^1/15

Input interpretation:

$$\sqrt[15]{-\log^5(-(-0.01081191785)) - 144 - 21 - 8 + \frac{1}{2}}$$

log(x) is the natural logarithm

Result:

1.6438172009...

1.6438172009…

(((-[ln(-(-0.01081191785))]^5-144-21-8+1/2)))^1/15-(21+5)1/10^3

Input interpretation:

$$\sqrt[15]{-\log^5(-(-0.01081191785)) - 144 - 21 - 8 + \frac{1}{2}} - (21+5)\times\frac{1}{10^3}$$

log(x) is the natural logarithm

Result:

1.6178172009...

1.6178172009…

Alternative representations:

$$\sqrt[15]{-\log^5(-(-1)\,0.0108119) - 144 - 21 - 8 + \frac{1}{2}} - \frac{21+5}{10^3} = -\frac{26}{10^3} + \sqrt[15]{-\frac{345}{2} - \log_e^5(0.0108119)}$$

$$\sqrt[15]{-\log^5(-(-1)\,0.0108119) - 144 - 21 - 8 + \frac{1}{2}} - \frac{21+5}{10^3} = -\frac{26}{10^3} + \sqrt[15]{-\frac{345}{2} - (\log(a)\log_a(0.0108119))^5}$$

$$\sqrt[15]{-\log^5(-(-1)\,0.0108119) - 144 - 21 - 8 + \frac{1}{2}} - \frac{21+5}{10^3} = -\frac{26}{10^3} + \sqrt[15]{-\frac{345}{2} - (-\text{Li}_1(0.989188))^5}$$

Series representations:

$$\sqrt[15]{-\log^5(-(-1)\,0.0108119) - 144 - 21 - 8 + \frac{1}{2}} - \frac{21+5}{10^3} = -\frac{13}{500} + \sqrt[15]{-\frac{345}{2} + \left(\sum_{k=1}^{\infty} \frac{(-1)^k (-0.989188)^k}{k}\right)^5}$$

$$\sqrt[15]{-\log^5(-(-1)\,0.0108119) - 144 - 21 - 8 + \frac{1}{2}} - \frac{21+5}{10^3} = -\frac{13}{500} + \left(-\frac{345}{2} - \left(2 i \pi \left\lfloor \frac{\arg(0.0108119 - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.0108119 - x)^k x^{-k}}{k}\right)^5\right)^{\wedge}(1/15) \quad \text{for } x < 0$$

$$\sqrt[15]{-\log^5(-(-1)\,0.0108119) - 144 - 21 - 8 + \frac{1}{2}} - \frac{21+5}{10^3} = -\frac{13}{500} + \left(-\frac{345}{2} - \left(\log(z_0) + \left\lfloor \frac{\arg(0.0108119 - z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.0108119 - z_0)^k z_0^{-k}}{k}\right)^5\right)^{\wedge}(1/15)$$

Integral representation:

$$\sqrt[15]{-\log^5(-(-1)\,0.0108119) - 144 - 21 - 8 + \frac{1}{2}} - \frac{21+5}{10^3} = -\frac{13}{500} + \sqrt[15]{-\frac{345}{2} - \left(\int_1^{0.0108119} \frac{1}{t}\, dt\right)^5}$$

and:

(((-[ln(-(-0.01081191785))]^5-144-21-8+1/2)))^1/14+29/10^3

Input interpretation:

$$\sqrt[14]{-\log^5(-(-0.01081191785)) - 144 - 21 - 8 + \frac{1}{2}} + \frac{29}{10^3}$$

log(x) is the natural logarithm

Result:

1.7322234553...

1.7322234553…. ≈ √3

Alternative representations:

$$\sqrt[14]{-\log^5(-(-1)\,0.0108119) - 144 - 21 - 8 + \frac{1}{2}} + \frac{29}{10^3} = \frac{29}{10^3} + \sqrt[14]{-\frac{345}{2} - \log_e^5(0.0108119)}$$

$$\sqrt[14]{-\log^5(-(-1)\,0.0108119) - 144 - 21 - 8 + \frac{1}{2}} + \frac{29}{10^3} = \frac{29}{10^3} + \sqrt[14]{-\frac{345}{2} - (\log(a)\log_a(0.0108119))^5}$$

$$\sqrt[14]{-\log^5(-(-1)\,0.0108119) - 144 - 21 - 8 + \frac{1}{2}} + \frac{29}{10^3} = \frac{29}{10^3} + \sqrt[14]{-\frac{345}{2} - (-\mathrm{Li}_1(0.989188))^5}$$

Series representations:

$$\sqrt[14]{-\log^5(-(-1)\,0.0108119) - 144 - 21 - 8 + \frac{1}{2}} + \frac{29}{10^3} = \frac{29}{1000} + \sqrt[14]{-\frac{345}{2} + \left(\sum_{k=1}^{\infty} \frac{(-1)^k (-0.989188)^k}{k}\right)^5}$$

$$\sqrt[14]{-\log^5(-(-1)\,0.0108119) - 144 - 21 - 8 + \frac{1}{2}} + \frac{29}{10^3} = \frac{29}{1000} + \left(-\frac{345}{2} - \left(2\,i\,\pi \left\lfloor \frac{\arg(0.0108119 - x)}{2\,\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(0.0108119 - x)^k\,x^{-k}}{k}\right)^5\right) \hat{}\,(1/14) \quad \text{for } x < 0$$

$$\sqrt[14]{-\log^5(-(-1)\,0.0108119) - 144 - 21 - 8 + \frac{1}{2}} + \frac{29}{10^3} = \frac{29}{1000} + \left(-\frac{345}{2} - \left(\log(z_0) + \left\lfloor \frac{\arg(0.0108119 - z_0)}{2\,\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(0.0108119 - z_0)^k\,z_0^{-k}}{k}\right)^5\right) \hat{}\,(1/14)$$

Integral representation:

$$\sqrt[14]{-\log^5(-(-1)\,0.0108119) - 144 - 21 - 8 + \frac{1}{2}} + \frac{29}{10^3} = \frac{29}{1000} + \sqrt[14]{-\frac{345}{2} - \left(\int_1^{0.0108119} \frac{1}{t}\,dt\right)^5}$$

Now, we have:

$$\langle I_{p\times q}\rangle\big|_{1/\sqrt{N}} = \frac{1}{1+\delta_{pq}} \frac{16\pi^2}{\sqrt{N} g_{\rm YM}^5 k^2} \exp\left[k\frac{8\pi^2}{g_{\rm YM}^2}\right] \int_1^\infty dx \left(\exp\left[-\frac{8k\pi^2 x}{g_{\rm YM}^2 (x^2-1)^{\frac{1}{2}}}\right] \times \left[\frac{c_1 g_{\rm YM}^2 x}{(x^2-1)^{\frac{5}{2}}} - \frac{c_2 \pi^2}{(x^2-1)^3}\right]\right), \tag{3.57}$$

that is equal to

$$\langle I_{p\times q}\rangle|_{1/\sqrt{N}} = \frac{1}{1+\delta_{pq}}\frac{16\pi^2}{\sqrt{N}g_{\rm YM}^5 k^2}\exp\left[k\frac{8\pi^2}{g_{\rm YM}^2}\right]\left[\frac{c_1 g_{\rm YM}^2}{(k\frac{8\pi^2}{g_{\rm YM}^2})} - \frac{3c_2\pi^2}{(k\frac{8\pi^2}{g_{\rm YM}^2})^2}\right]K_2(k\frac{8\pi^2}{g_{\rm YM}^2})$$
$$= \frac{k^2}{1+\delta_{pq}}\left(\frac{1}{p^4}+\frac{1}{q^4}\right)\frac{2}{\sqrt{N}g_{\rm YM}}\exp\left[k\frac{8\pi^2}{g_{\rm YM}^2}\right]K_2(k\frac{8\pi^2}{g_{\rm YM}^2})\,. \qquad (3.59)$$

For k = 12; Sqrt(27/256) = g_{YM} ; N = 4; K_2 = 8.41724; K_3 = 13.0152; K_4 = 11.0647; K_5 = 18.9801 p = 3, q = 4

$$p \le q \in \mathbb{Z}_+, \quad pq = k\,,$$

Bessel K_2 function

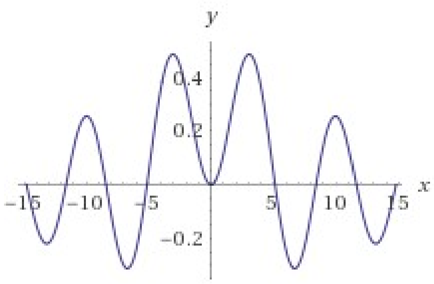
With regard the Bessel function, we have that:

Input:

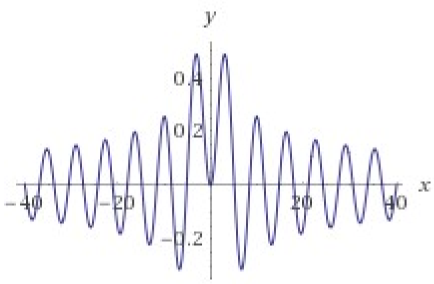
$J_2(x)$

$J_n(z)$ is the Bessel function of the first kind

Plots:



(x from -14.8 to 14.8)



(x from -40 to 40)

Real root:

$x = 0$

Roots:

$x = -j_{2,n}\,, \quad n \in \mathbb{Z}\,, \quad n \ge 1$

$x = j_{2,n}, \quad n \in \mathbb{Z}, \quad n \geq 1$

$j_{n,k}$ is the kth zero of the Bessel J-function

$\mathbb{Z}$ is the set of integers

Numerical roots:

$x \approx -14.7959517823513...$

$x \approx \pm 11.6198411721491...$

$x \approx \pm 8.41724414039986...$ $K_2 = 8.41724$

$x \approx \pm 5.13562230184068...$

$x = 0$

Series expansion at x = 0:

$$\frac{x^2}{8} - \frac{x^4}{96} + \frac{x^6}{3072} + O(x^7)$$

(Taylor series)

Series expansion at x = ∞:

$$\sin\left(x + \frac{\pi}{4}\right)\left(-\sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{x}} + \frac{105\left(\frac{1}{x}\right)^{5/2}}{64\sqrt{2\pi}} - \frac{10\,395\left(\frac{1}{x}\right)^{9/2}}{16\,384\sqrt{2\pi}} + O\left(\left(\frac{1}{x}\right)^{11/2}\right)\right) + \cos\left(x + \frac{\pi}{4}\right)\left(-\frac{15\left(\frac{1}{x}\right)^{3/2}}{4\sqrt{2\pi}} - \frac{315\left(\frac{1}{x}\right)^{7/2}}{512\sqrt{2\pi}} + O\left(\left(\frac{1}{x}\right)^{11/2}\right)\right)$$

Derivative:

$$\frac{d}{dx}(J_2(x)) = \frac{1}{2}(J_1(x) - J_3(x))$$

Indefinite integral:

$$\int J_2(x)\,dx = \frac{1}{24}x^3 \, {}_1F_2\left(\frac{3}{2}; \frac{5}{2}, 3; -\frac{x^2}{4}\right) + \text{constant}$$

${}_pF_q(a_1, \ldots, a_p; b_1, \ldots, b_q; z)$ is the generalized hypergeometric function

Limit:

$$\lim_{x \to \pm\infty} J_2(x) = 0$$

Alternative representations:

$$J_2(x) = {}_0\tilde{F}_1\left(;\, 3;\, -\frac{x^2}{4}\right)\left(\frac{x}{2}\right)^2$$

$$J_2(x) = \frac{I_2(i\,x)\,x^2}{(i\,x)^2}$$

$$J_2(x) = \frac{{}_0F_1\left(;\, 3;\, -\frac{x^2}{4}\right)\left(\frac{x}{2}\right)^2}{\Gamma(3)}$$

Series representations:

$$J_2(x) = \sum_{k=0}^{\infty} \frac{(-1)^k\, 2^{-2(1+k)}\, x^{2+2k}}{k!\,\Gamma(3+k)}$$

$$J_2(x) = \frac{1}{8}\, x^2 \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k x^{2k}}{k!\,(3)_k}$$

$$J_2(x) = \sum_{j=0}^{\infty} \operatorname{Res}_{s=-1-j} \frac{4^s\, x^{-2s}\, \Gamma(1+s)}{\Gamma(2-s)}$$

Integral representations:

$$J_2(x) = \frac{1}{\pi} \int_0^{\pi} \cos(2\,t - x\sin(t))\,dt$$

$$J_2(x) = \frac{2\,x^2}{3\,\pi} \int_0^1 \left(1-t^2\right)^{3/2} \cos(t\,x)\,dt$$

$$J_2(x) = -\frac{1}{2\,\pi} \int_{-\pi}^{\pi} e^{i\,(2\,t + x\cos(t))}\,dt$$

Hypergeometric-type representations:

$$J_2(x) = {}_0\tilde{F}_1\left(;\, 3;\, -\frac{x^2}{4}\right)\left(\frac{x}{2}\right)^2$$

$$J_2(x) = \frac{{}_0F_1\left(;\, 3;\, -\frac{x^2}{4}\right)\left(\frac{x}{2}\right)^2}{\Gamma(3)}$$

$$J_2(x) = \frac{{}_1F_1\left(\frac{5}{2};\, 5;\, 2\,i\,x\right)x^2}{4\,\Gamma(3)\,e^{i\,x}}$$

$$J_2(x) = \frac{\left(\lim_{a\to\infty}\, {}_1F_1\left(a;\, 3;\, -\frac{x^2}{4\,a}\right)\right)x^2}{4\,\Gamma(3)}$$

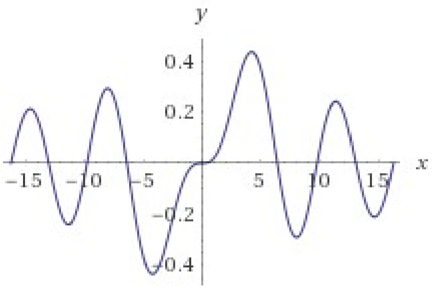
${}_p\tilde{F}_q(a_1, \ldots, a_p;\, b_1, \ldots, b_q;\, z)$ is the regularized generalized hypergeometric function

Input:

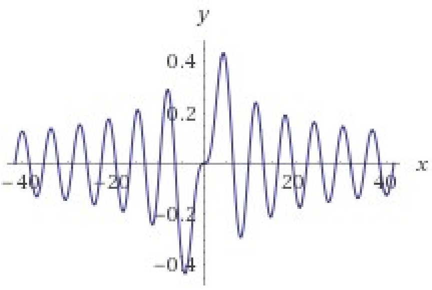
$J_3(x)$

$J_n(z)$ is the Bessel function of the first kind

Plots:



(x from −16.2 to 16.2)



(x from −41.5 to 41.5)

Real root:

$x = 0$

Roots:

$x = -j_{3,n}\,,\quad n \in \mathbb{Z}\,,\quad n \geq 1$

$x = j_{3,n}\,,\quad n \in \mathbb{Z}\,,\quad n \geq 1$

$j_{n,k}$ is the kth zero of the Bessel J-function

$\mathbb{Z}$ is the set of integers

Numerical roots:

$x \approx -16.2234661603188\ldots$

$x \approx \pm 13.0152007216984\ldots$ $K_3 = 13.0152$

$x \approx \pm 9.76102312998167\ldots$

$x \approx \pm 6.38016189592398\ldots$

$x = 0$

Series expansion at x = 0:

$$\frac{x^3}{48} - \frac{x^5}{768} + \frac{x^7}{30\,720} + O(x^8)$$

(Taylor series)

Series expansion at x = ∞:

$$\sin\left(x + \frac{\pi}{4}\right)\left(-\frac{35\left(\frac{1}{x}\right)^{3/2}}{4\sqrt{2\pi}} + \frac{3465\left(\frac{1}{x}\right)^{7/2}}{512\sqrt{2\pi}} + O\left(\left(\frac{1}{x}\right)^{11/2}\right)\right) + \cos\left(x + \frac{\pi}{4}\right)\left(\sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{x}} - \frac{945\left(\frac{1}{x}\right)^{5/2}}{64\sqrt{2\pi}} - \frac{45\,045\left(\frac{1}{x}\right)^{9/2}}{16\,384\sqrt{2\pi}} + O\left(\left(\frac{1}{x}\right)^{11/2}\right)\right)$$

Derivative:

$$\frac{d}{dx}(J_3(x)) = \frac{1}{2}(J_2(x) - J_4(x))$$

Indefinite integral:

$$\int J_3(x)\,dx = -\frac{2 J_1(x)}{x} - J_2(x) + \text{constant}$$

Limit:

$$\lim_{x \to \pm\infty} J_3(x) = 0$$

Alternative representations:

$$J_3(x) = {}_0\tilde{F}_1\left(; 4; -\frac{x^2}{4}\right)\left(\frac{x}{2}\right)^3$$

$$J_3(x) = \frac{I_3(i\,x)\,x^3}{(i\,x)^3}$$

$$J_3(x) = \frac{{}_0F_1\left(;\,4;\,-\frac{x^2}{4}\right)\left(\frac{x}{2}\right)^3}{\Gamma(4)}$$

Series representations:

$$J_3(x) = \sum_{k=0}^{\infty} \frac{(-1)^k\,2^{-3-2k}\,x^{3+2k}}{k!\,\Gamma(4+k)}$$

$$J_3(x) = \frac{1}{48}\,x^3 \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k x^{2k}}{k!\,(4)_k}$$

$$J_3(x) = \frac{3}{32}\sqrt{\pi}\;z_0^3 \sum_{k=0}^{\infty} \frac{2^k\,{}_2\tilde{F}_3\left(2,\,\frac{5}{2};\,2-\frac{k}{2},\,\frac{5}{2}-\frac{k}{2},\,4;\,-\frac{z_0^2}{4}\right)(x-z_0)^k\,z_0^{-k}}{k!}$$

for (not $(z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0))$

Integral representations:

$$J_3(x) = \frac{1}{\pi}\int_0^{\pi} \cos(3\,t - x\sin(t))\,dt$$

$$J_3(x) = \frac{2\,x^3}{15\,\pi}\int_0^1 \left(1-t^2\right)^{5/2} \cos(t\,x)\,dt$$

$$J_3(x) = \frac{i}{\pi}\int_0^{\pi} e^{i\,x\cos(t)} \cos(3\,t)\,dt$$

Hypergeometric-type representations:

$$J_3(x) = {}_0\tilde{F}_1\left(;\,4;\,-\frac{x^2}{4}\right)\left(\frac{x}{2}\right)^3$$

$$J_3(x) = \frac{{}_0F_1\left(;\,4;\,-\frac{x^2}{4}\right)\left(\frac{x}{2}\right)^3}{\Gamma(4)}$$

$$J_3(x) = \frac{{}_1F_1\left(\frac{7}{2};\,7;\,2\,i\,x\right)x^3}{8\,\Gamma(4)\,e^{i\,x}}$$

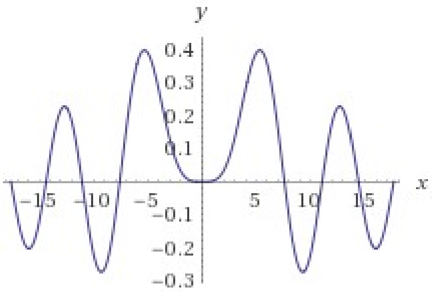
$$J_3(x) = \frac{\left(\lim_{a \to \infty} {}_1F_1\left(a; 4; -\frac{x^2}{4a}\right)\right) x^3}{8\,\Gamma(4)}$$

Input:

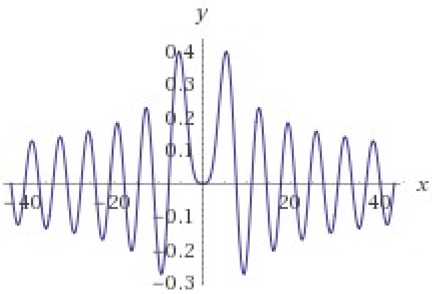
$J_4(x)$

$J_n(z)$ is the Bessel function of the first kind

Plots:



(x from −17.6 to 17.6)



(x from −43 to 43)

Real root:

$x = 0$

Roots:

$x = -j_{4,n}\,, \quad n \in \mathbb{Z}\,, \quad n \geq 1$

$x = j_{4,n}\,, \quad n \in \mathbb{Z}\,, \quad n \geq 1$

$j_{n,k}$ is the kth zero of the Bessel J-function

$\mathbb{Z}$ is the set of integers

Numerical roots:

$x \approx -17.6159660498048\ldots$

$x \approx \pm 14.3725366716176\ldots$

$x \approx \pm 11.0647094885012\ldots$ $K_4 = 11.0647$

$x \approx \pm 7.58834243450380\ldots$

$x = 0$

Series expansion at x = 0:

$$\frac{x^4}{384} - \frac{x^6}{7680} + \frac{x^8}{368\,640} + O(x^9)$$

(Taylor series)

Series expansion at x = ∞:

$$\sin\left(x + \frac{\pi}{4}\right)\left(\sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{x}} - \frac{3465\left(\frac{1}{x}\right)^{5/2}}{64\sqrt{2\pi}} + \frac{675\,675\left(\frac{1}{x}\right)^{9/2}}{16\,384\sqrt{2\pi}} + O\left(\left(\frac{1}{x}\right)^{11/2}\right)\right) +$$
$$\cos\left(x + \frac{\pi}{4}\right)\left(\frac{63\left(\frac{1}{x}\right)^{3/2}}{4\sqrt{2\pi}} - \frac{45\,045\left(\frac{1}{x}\right)^{7/2}}{512\sqrt{2\pi}} + O\left(\left(\frac{1}{x}\right)^{11/2}\right)\right)$$

Derivative:

$$\frac{d}{dx}(J_4(x)) = \frac{1}{2}(J_3(x) - J_5(x))$$

Indefinite integral:

$$\int J_4(x)\,dx = \frac{x^5\,{}_1F_2\left(\frac{5}{2}; \frac{7}{2}, 5; -\frac{x^2}{4}\right)}{1920} + \text{constant}$$

${}_pF_q(a_1, \ldots, a_p; b_1, \ldots, b_q; z)$ is the generalized hypergeometric function

Limit:

$$\lim_{x \to \pm\infty} J_4(x) = 0$$

Alternative representations:

$$J_4(x) = {}_0\tilde{F}_1\left(; 5; -\frac{x^2}{4}\right)\left(\frac{x}{2}\right)^4$$

$$J_4(x) = \frac{I_4(i\,x)\,x^4}{(i\,x)^4}$$

$$J_4(x) = \frac{{}_0F_1\left(;5;-\frac{x^2}{4}\right)\left(\frac{x}{2}\right)^4}{\Gamma(5)}$$

Series representations:

$$J_4(x) = \sum_{k=0}^{\infty} \frac{(-1)^k\,2^{-2(2+k)}\,x^{4+2k}}{k!\,\Gamma(5+k)}$$

$$J_4(x) = \frac{1}{384}\,x^4 \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k x^{2k}}{k!\,(5)_k}$$

$$J_4(x) = \sum_{j=0}^{\infty} \operatorname{Res}_{s=-2-j} \frac{4^s\,x^{-2s}\,\Gamma(2+s)}{\Gamma(3-s)}$$

Integral representations:

$$J_4(x) = \frac{1}{\pi}\int_0^{\pi} \cos(4\,t - x\sin(t))\,dt$$

$$J_4(x) = \frac{1}{2\,\pi}\int_{-\pi}^{\pi} e^{i\,(4\,t + x\cos(t))}\,dt$$

$$J_4(x) = \frac{2\,x^4}{105\,\pi}\int_0^{1} \left(1-t^2\right)^{7/2} \cos(t\,x)\,dt$$

Hypergeometric-type representations:

$$J_4(x) = {}_0\tilde{F}_1\left(;5;-\frac{x^2}{4}\right)\left(\frac{x}{2}\right)^4$$

$$J_4(x) = \frac{{}_0F_1\left(;5;-\frac{x^2}{4}\right)\left(\frac{x}{2}\right)^4}{\Gamma(5)}$$

$$J_4(x) = \frac{{}_1F_1\left(\frac{9}{2};9;2\,i\,x\right)x^4}{\Gamma(5)\,2^4\,e^{i\,x}}$$

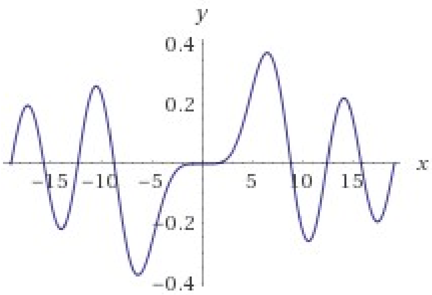
$$J_4(x) = \frac{\left(\lim_{a\to\infty} {}_1F_1\left(a; 5; -\frac{x^2}{4a}\right)\right) x^4}{\Gamma(5)\, 2^4}$$

Input:

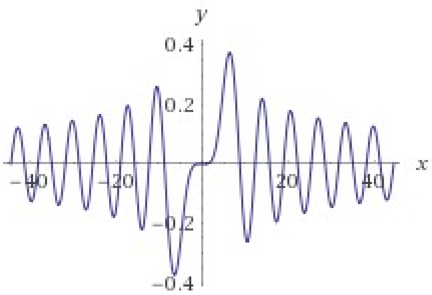
$J_5(x)$

$J_n(z)$ is the Bessel function of the first kind

Plots:



(x from −19 to 19)



(x from −44.5 to 44.5)

Real root:

$x = 0$

Roots:

$x = -j_{5,n}\,, \quad n \in \mathbb{Z}\,, \quad n \geq 1$

$x = j_{5,n}\,, \quad n \in \mathbb{Z}\,, \quad n \geq 1$

$j_{n,k}$ is the kth zero of the Bessel J-function

$\mathbb{Z}$ is the set of integers

Numerical roots:

$x \approx \pm 15.7001740797117...$

$x \approx \pm 12.3386041974669...$

$x \approx \pm 8.77148381595995...$

$x = 0$

$x \approx 18.9801338751799...$ $K_5 = 18.9801$

Series expansion at x = 0:

$$\frac{x^5}{3840} - \frac{x^7}{92\,160} + \frac{x^9}{5\,160\,960} - \frac{x^{11}}{495\,452\,160} + O\left(x^{13}\right)$$

(Taylor series)

Series expansion at x = ∞:

$$\sin\left(x + \frac{\pi}{4}\right)\left(\frac{99\left(\frac{1}{x}\right)^{3/2}}{4\sqrt{2\pi}} - \frac{225\,225\left(\frac{1}{x}\right)^{7/2}}{512\sqrt{2\pi}} + O\left(\left(\frac{1}{x}\right)^{11/2}\right)\right) +$$
$$\cos\left(x + \frac{\pi}{4}\right)\left(-\sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{x}} + \frac{9009\left(\frac{1}{x}\right)^{5/2}}{64\sqrt{2\pi}} - \frac{11\,486\,475\left(\frac{1}{x}\right)^{9/2}}{16\,384\sqrt{2\pi}} + O\left(\left(\frac{1}{x}\right)^{11/2}\right)\right)$$

Derivative:

$$\frac{d}{dx}(J_5(x)) = \frac{1}{2}(J_4(x) - J_6(x))$$

Indefinite integral:

$$\int J_5(x)\,dx = -\frac{x^2 J_4(x) + 4x J_3(x) + 8 J_2(x)}{x^2} + \text{constant}$$

Limit:

$$\lim_{x \to \pm\infty} J_5(x) = 0$$

Alternative representations:

$$J_5(x) = {}_0\tilde{F}_1\left(;6;-\frac{x^2}{4}\right)\left(\frac{x}{2}\right)^5$$

$$J_5(x) = \frac{I_5(i\,x)\,x^5}{(i\,x)^5}$$

$$J_5(x) = \frac{{}_0F_1\left(;6;-\frac{x^2}{4}\right)\left(\frac{x}{2}\right)^5}{\Gamma(6)}$$

Series representations:

$$J_5(x) = \sum_{k=0}^{\infty} \frac{(-1)^k\, 2^{-5-2k}\, x^{5+2k}}{k!\,\Gamma(6+k)}$$

$$J_5(x) = \frac{x^5 \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k x^{2k}}{k!\,(6)_k}}{3840}$$

$$J_5(x) = \frac{15}{128}\sqrt{\pi}\, z_0^5 \sum_{k=0}^{\infty} \frac{2^k\, {}_2\tilde{F}_3\left(3, \frac{7}{2}; 3-\frac{k}{2}, \frac{7}{2}-\frac{k}{2}, 6; -\frac{z_0^2}{4}\right)(x-z_0)^k\, z_0^{-k}}{k!}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Integral representations:

$$J_5(x) = \frac{1}{\pi}\int_0^{\pi} \cos(5\,t - x\sin(t))\,dt$$

$$J_5(x) = \frac{2\,x^5}{945\,\pi}\int_0^1 \left(1-t^2\right)^{9/2} \cos(t\,x)\,dt$$

$$J_5(x) = -\frac{i}{\pi}\int_0^{\pi} e^{i\,x\cos(t)} \cos(5\,t)\,dt$$

Hypergeometric-type representations:

$$J_5(x) = {}_0\tilde{F}_1\left(;6;-\frac{x^2}{4}\right)\left(\frac{x}{2}\right)^5$$

$$J_5(x) = \frac{{}_0F_1\left(;6;-\frac{x^2}{4}\right)\left(\frac{x}{2}\right)^5}{\Gamma(6)}$$

$$J_5(x) = \frac{{}_1F_1\left(\frac{11}{2};11;2\,i\,x\right)x^5}{\Gamma(6)\,2^5\,e^{i\,x}}$$

$$J_5(x) = \frac{\left(\lim_{a\to\infty}{}_1F_1\left(a;6;-\frac{x^2}{4a}\right)\right)x^5}{\Gamma(6)\,2^5}$$

Thence, from

$$\langle I_{p\times q}\rangle\big|_{1/\sqrt{N}} = \frac{1}{1+\delta_{pq}}\frac{16\pi^2}{\sqrt{N}g_{\rm YM}^5k^2}\exp\left[k\frac{8\pi^2}{g_{\rm YM}^2}\right]\left[\frac{c_1g_{\rm YM}^2}{(k\frac{8\pi^2}{g_{\rm YM}^2})}-\frac{3c_2\pi^2}{(k\frac{8\pi^2}{g_{\rm YM}^2})^2}\right]K_2(k\frac{8\pi^2}{g_{\rm YM}^2})$$
$$-\frac{k^2}{1+\delta_{pq}}\left(\frac{1}{p^4}+\frac{1}{q^4}\right)\frac{2}{\sqrt{N}g_{\rm YM}}\exp\left[k\frac{8\pi^2}{g_{\rm YM}^2}\right]K_2(k\frac{8\pi^2}{g_{\rm YM}^2}).$$

For k = 12; Sqrt(27/256) = g_{YM} ; N = 4; K_2 = 8.41724; K_3 = 13.0152; K_4 = 11.0647; K_5 = 18.9801 p = 3, q = 4

we obtain:

144*1/2*(1/81+1/256)* 2/(2*(sqrt(27/256))) * exp[((12*8*Pi^2))/((27/256))] * 8.41724*(12*8*Pi^2)*256/27

Input interpretation:

$$144\times\frac{1}{2}\left(\frac{1}{81}+\frac{1}{256}\right)\times\frac{2}{2\sqrt{\frac{27}{256}}}\exp\left(\frac{12\times8\pi^2}{\frac{27}{256}}\right)\times8.41724\,(12\times8\pi^2)\times\frac{256}{27}$$

Result:

$8.5943107502756839505654051874251547874455118793263 74\ldots\times10^{3906}$

8.594310750275…*10^{3906}

Series representations:

$$\frac{\left(144\left(\frac{1}{81}+\frac{1}{256}\right)\right)2\left(\exp\left(\frac{12\times 8\pi^2}{\frac{27}{256}}\right)8.41724\left(12\times 8\pi^2\right)256\right)}{2\left(2\sqrt{\frac{27}{256}}\right)27} = \frac{8965.09\,\pi^2\exp\left(\frac{8192\pi^2}{9}\right)}{\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{\left(144\left(\frac{1}{81}+\frac{1}{256}\right)\right)2\left(\exp\left(\frac{12\times 8\pi^2}{\frac{27}{256}}\right)8.41724\left(12\times 8\pi^2\right)256\right)}{2\left(2\sqrt{\frac{27}{256}}\right)27} = -\frac{17930.2\,\pi^2\exp\left(\frac{8192\pi^2}{9}\right)\sqrt{\pi}}{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\left(-\frac{229}{256}\right)^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}$$

$$\frac{\left(144\left(\frac{1}{81}+\frac{1}{256}\right)\right)2\left(\exp\left(\frac{12\times 8\pi^2}{\frac{27}{256}}\right)8.41724\left(12\times 8\pi^2\right)256\right)}{2\left(2\sqrt{\frac{27}{256}}\right)27} = \frac{8965.09\,\pi^2\exp\left(\frac{8192\pi^2}{9}\right)}{\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{27}{256}-z_0\right)^k z_0^{-k}}{k!}} \quad \text{for (not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\leq 0))$$

From which:

1/(8.594310750275683950565405187425154787445511879 × 10^3906)

Input interpretation:

$$\frac{1}{8.594310750275683950565405187425154787445511879\times 10^{3906}}$$

Result:

$1.163560440222530359530906052321308521597335737847 76\ldots\times 10^{-3907}$

1.16356044022253035953090605232130852159733573784776 × 10^{-3907}

From the Ramanujan formula for the calculation of golden ratio

$$\sqrt[5]{\frac{1}{\left(\frac{1}{32}\left(-1+\sqrt{5}\right)^5+5\,e^{\left(-\sqrt{5}\,\pi\right)^5}\right)-\,-\frac{1.63828987970956656772394588270120562457983147225 84}{10^{7429}}}}$$

we obtain, inserting the previous result:

((((1/(((1/32(-1+sqrt(5))^5+5*(e^((-
sqrt(5)*Pi))^5)))+(1.1635604402225303595309060523213085215973357378 4776 ×
10^-3907))))^1/5

Input interpretation:

$$\sqrt[5]{\frac{1}{\left(\frac{1}{32}\left(-1+\sqrt{5}\right)^5+5\,e^{\left(-\sqrt{5}\,\pi\right)^5}\right)+\frac{1.16356044022253035953090605232130852159733573784776}{10^{3907}}}}$$

Result:

1.618033988749894848204586834365638117720309179805762862135...

1.61803398……

that is the following spectacular number:

1.6180339887498948482045868343656381177203091798057628621354486227052
6046281890244970720720418939113748475408807538689175212663386222353 6
93179318006076672635443338908659593958290563832266131992829026788067
52087668925017116962070322210432162695486262963136144381497587012203
40805887954454749246185695364864449241044320771344947049565846788509
87433944221254487706647809158846074998871240076521705751797883416625
62494075890697040002812104276217711177780531531714101170466659914669
79873176135600670874807101317952368942752194843530567830022878569978
29778347845878228911097625003026961561700250464338243776486102838312
68330372429267526311653392473167111211588186385133162038400522216579
12866752946549068113171599343235973494985090409476213222981017261070
59611645629909816290555208524790352406020172799747175342777592778625
61943208275051312181562855122248093947123414517022373580577278616008
68838295230459264787801788992199027077690389532196819861514378031499
74110692608867429622675756052317277752035361393621076738937645560606
05921658946675955190040055590895022953094231248235521221241544400647
03405657347976639723949499465845788730396230903750339938562102423690
25138680414577995698122445747178034173126453220416397232134044449487
30231541767689375210306873788034417009395440962795589867872320951242

689355730970450959568440175551988192180206405290551893494759260073485228210108819464454422231889131929468962200230144377026992300780308526118075451928877050210968424936271359251876077788466583615023891349333312231053392321362431926372891067050339928226526355620902979864247275977256550861548754357482647181414512700060238901620777322449943530889990950168032811219432048196438767586331479857191139781539780747615077221175082694586393204565209896985556781410696837288405874610337810544439094368358358138113116899385557697548414914453415091295407005019477548616307542264172939468036731980586183391832859913039607201445595044977921207612478564591616083705949878600697018940988640076443617093341727091914336501371576601148038143062623805143211734815100559013456101180079050638142152709308588092875703450507808145458819906336129827981411745339273120809289727922213298064294687824274874017450554067787570832373109759151177629784432847479081765180977872684161176325038612112914368343767023503711163307258698832587103363222381098090121101989917684149175123313401527338438372345009347860497929459915822012581045982309255287212413704361491020547185549611808764265765110605458814756044317847985845397312863016254487611485202170644041116607669505977578325703951108782308271064789390211156910392768384538633332156582965977310343603232254574363720412440640888267375843395367959312322134373209957498894699565647360072959998391288103197426312517971414320123112795518947781726914158911779919564812558001845506563295285985910009086218029775637892599916499464281930222935523466747593269516542140210913630181947227078901220872873617073486499981562554728113734798716569527489008144384053274837813782466917444229634914708157007352545707089772675469343822619546861533120953357923801460927351021011919021836067509730895752895774681422954339438549315533963038072916917584610146099505506480367930414723657203986007355076090231731250161320484358364817704848181099160244252327167219018933459637860878752870173935930301335901123710239171265904702634940283076687674363865132710628032317406931733448234356453185058135310854973335075996677871244905836367541328908624063245639535721252426117027802865604323494283730172557440583727826799603173936401328762770124367983114464369476705312724924104716700138247831286565064934341803900410178053395058772458665575522939158239708417729833728231152569260929959422400005606266786743579239724540848176519734362652689448885527202747787473359835367277614075917120513269344837529916499809360246178442675727767900191919070380522046123248239132610432719168451230602362789354543246176997575368904176365025478513824627289984653723320982610376891509311623
5012167426636 4 e^0 ≈ 1.6180 (real, principal root)

We obtain also:

(x-1)1/12*(1/162+1/512)* 2*1/(2*(sqrt(27/256))) * exp[256((96*Pi^2))/27] * 8.41724*(96*Pi^2)*256/27 = 8.59431075e+3906

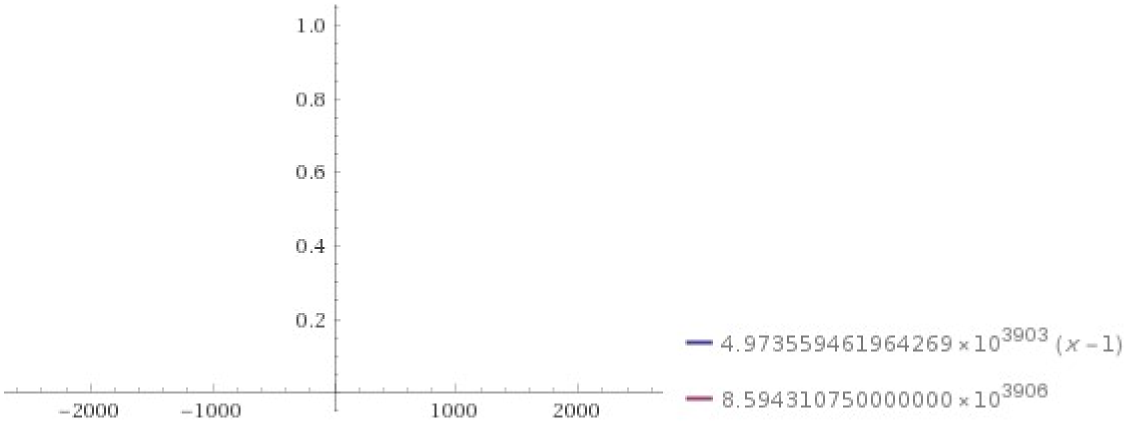
Input interpretation:

$$(x-1)\times\frac{1}{12}\left(\frac{1}{162}+\frac{1}{512}\right)\times 2\times\frac{1}{2\sqrt{\frac{27}{256}}}\exp\left(256\left(\frac{1}{27}(96\pi^2)\right)\right)\times 8.41724(96\pi^2)\times\frac{256}{27} = 8.59431075\times 10^{3906}$$

Result:

$$4.973559461964269\times 10^{3903}(x-1) = 8.594310750000000\times 10^{3906}$$

Plot:



Alternate forms:

$$4.973559461964269\times 10^{3903}\,x - 8.599284309461964\times 10^{3906} = 0$$

$$4.973559461964269\times 10^{3903}\,x - 4.973559461964269\times 10^{3903} = 8.594310750000000\times 10^{3906}$$

Solution:

$$x \approx 1728.999999944857$$

1728.99999…≈ 1729

And:

Input interpretation:

$$\left(x-\frac{1}{8^2\times 13}\right)^{15}\times\frac{1}{12}\left(\frac{1}{162}+\frac{1}{512}\right)\times 2\times\frac{1}{2\sqrt{\frac{27}{256}}}$$
$$\exp\left(256\left(\frac{1}{27}\left(96\pi^2\right)\right)\right)\times 8.41724\left(96\pi^2\right)\times\frac{256}{27}=8.59431075\times 10^{3906}$$

Result:

$$4.973559461964269\times 10^{3903}\left(x-\frac{1}{832}\right)^{15}=8.594310750000000\times 10^{3906}$$

Alternate forms:

$$4.973559461964269\times 10^{3903}\left(1.000000000000000\,x-0.001201923076923077\right)^{15}=8.594310750000000\times 10^{3906}$$

$$4.973559461964269\times 10^{3903}x^{15}-8.966753837675966\times 10^{3901}x^{14}+7.544143853813913\times 10^{3899}x^{13}-3.929241590528080\times 10^{3897}x^{12}+1.416793842738490\times 10^{3895}x^{11}-3.746329872625816\times 10^{3892}x^{10}+7.504667212792099\times 10^{3889}x^{9}-1.159718490987790\times 10^{3887}x^{8}+1.393892417052632\times 10^{3884}x^{7}-1.303051137736168\times 10^{3881}x^{6}+9.39700339713583\times 10^{3877}x^{5}-5.133852380428227\times 10^{3874}x^{4}+2.056831883184386\times 10^{3871}x^{3}-5.704970090193378\times 10^{3867}x^{2}+9.79562172079907\times 10^{3863}x-8.594310750000000\times 10^{3906}=0$$

$$8.279893990921081\times 10^{3833}\left(6.006791231165261\times 10^{69}x^{15}-1.082955149849506\times 10^{68}x^{14}+9.11140150113767\times 10^{65}x^{13}-4.745521615175871\times 10^{63}x^{12}+1.711125582395146\times 10^{61}x^{11}-4.524610914987165\times 10^{58}x^{10}+9.06372378803519\times 10^{55}x^{9}-1.400644129332360\times 10^{53}x^{8}+1.683466501601395\times 10^{50}x^{7}-1.573753406945748\times 10^{47}x^{6}+1.134918322316645\times 10^{44}x^{5}-6.200384190978176\times 10^{40}x^{4}+2.484128281641898\times 10^{37}x^{3}-6.890148710175381\times 10^{33}x^{2}+1.183061248313081\times 10^{30}x-9.47965743840609\times 10^{25}\right)=8.594310750000000\times 10^{3906}$$

Real solution:

$x\approx 1.64495375259065$

1.64495375259065

Complex solutions:

$x=-1.6066299851637-0.3417552221590\,i$

$x=-1.6066299851637+0.3417552221590\,i$

$x=-1.32862124153459-0.96617308381694\,i$

$x=-1.32862124153459+0.96617308381694\,i$

$x = -0.82067399167994 - 1.42353084187604\,i$

and again:

(((x-1/(5*8))^16)) * 1/12*(1/162+1/512)* 2*1/(2*(sqrt(27/256))) * exp[256((96*Pi^2))/27] * 8.41724*(96*Pi^2)*256/27 = 8.59431075e+3906

Input interpretation:

$$\left(x-\frac{1}{5\times 8}\right)^{16}\times\frac{1}{12}\left(\frac{1}{162}+\frac{1}{512}\right)\times 2\times\frac{1}{2\sqrt{\frac{27}{256}}}\exp\left(256\left(\frac{1}{27}(96\pi^2)\right)\right)\times 8.41724(96\pi^2)\times\frac{256}{27} = 8.59431075\times 10^{3906}$$

Result:

$$4.973559461964269\times 10^{3903}\left(x-\frac{1}{40}\right)^{16} = 8.594310750000000\times 10^{3906}$$

Alternate forms:

$$4.973559461964269\times 10^{3903}(0.02500000000000000 - 1.000000000000000\,x)^{16} = 8.594310750000000\times 10^{3906}$$

$$1.157997050779934\times 10^{3878}(1.000000000000000 - 40.00000000000000\,x)^{16} = 8.594310750000000\times 10^{3906}$$

$$4.973559461964269\times 10^{3903}x^{16} - 1.989423784785708\times 10^{3903}x^{15} + 3.730169596473202\times 10^{3902}x^{14} - 4.351864529218735\times 10^{3901}x^{13} + 3.535889929990222\times 10^{3900}x^{12} - 2.121533957994133\times 10^{3899}x^{11} + 9.72369730747311\times 10^{3897}x^{10} - 3.472749038383254\times 10^{3896}x^{9} + 9.76710667045290\times 10^{3894}x^{8} - 2.170468148989534\times 10^{3893}x^{7} + 3.798319260731684\times 10^{3891}x^{6} - 5.179526264634115\times 10^{3889}x^{5} + 5.395339858993870\times 10^{3887}x^{4} - 4.150261429995284\times 10^{3885}x^{3} + 2.223354337497474\times 10^{3883}x^{2} - 7.411181124991579\times 10^{3880}x - 8.594310750000000\times 10^{3906} = 0$$

Real solutions:

$x \approx -1.56847951084180$

$x \approx 1.61847951084180$

1.61847951084180

Complex solutions:

$x = -1.44718310554284 - 0.60979820861238\,i$

$x = -1.44718310554284 + 0.60979820861238\,i$

$x = -1.10176016779806 - 1.12676016779806\,i$

$x = -1.10176016779806 + 1.12676016779806\,i$

$x = -0.58479820861238 - 1.47218310554284\,i$

(12(x+5-1/2)) * 1/12*(1/162+1/512)* 2*1/(2*(sqrt(27/256))) * exp[256((96*Pi^2))/27] * 8.41724*(96*Pi^2)*256/27 = 8.59431075e+3906

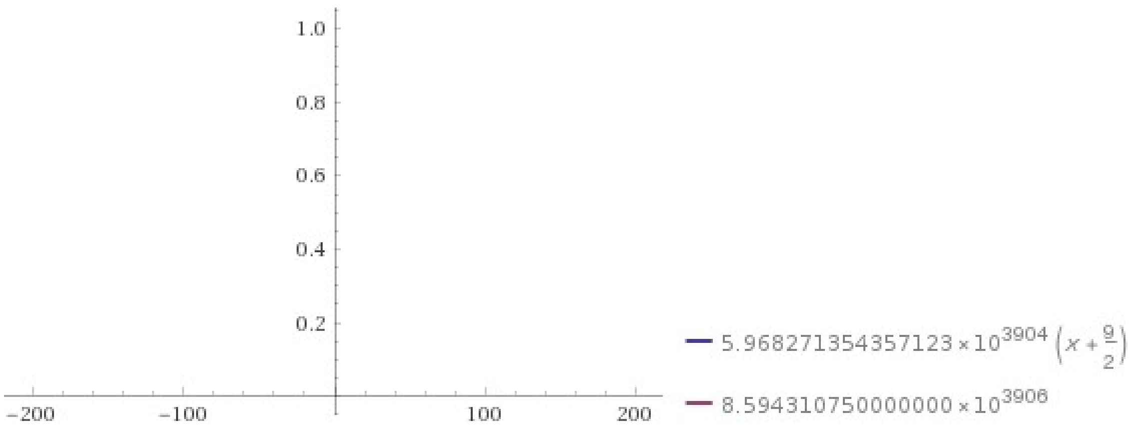
Input interpretation:

$$\left(12\left(x+5-\frac{1}{2}\right)\right)\times\frac{1}{12}\left(\frac{1}{162}+\frac{1}{512}\right)\times 2\times\frac{1}{2\sqrt{\frac{27}{256}}}\exp\left(256\left(\frac{1}{27}(96\pi^2)\right)\right)\times 8.41724(96\pi^2)\times\frac{256}{27} = 8.59431075\times 10^{3906}$$

Result:

$$5.968271354357123\times 10^{3904}\left(x+\frac{9}{2}\right) = 8.594310750000000\times 10^{3906}$$

Plot:



Alternate forms:

$$5.968271354357123\times 10^{3904}\,x - 8.3257385390539 29\times 10^{3906} = 0$$

$$5.968271354357123\times 10^{3904}\,x + 2.685722109460705\times 10^{3905} = 8.594310750000000\times 10^{3906}$$

Solution:

$x \approx 139.4999999954048$

139.499999….

(12(x+18+1/2)) * 1/12*(1/162+1/512)* 2*1/(2*(sqrt(27/256))) * exp[256((96*Pi^2))/27] * 8.41724*(96*Pi^2)*256/27 = 8.59431075e+3906

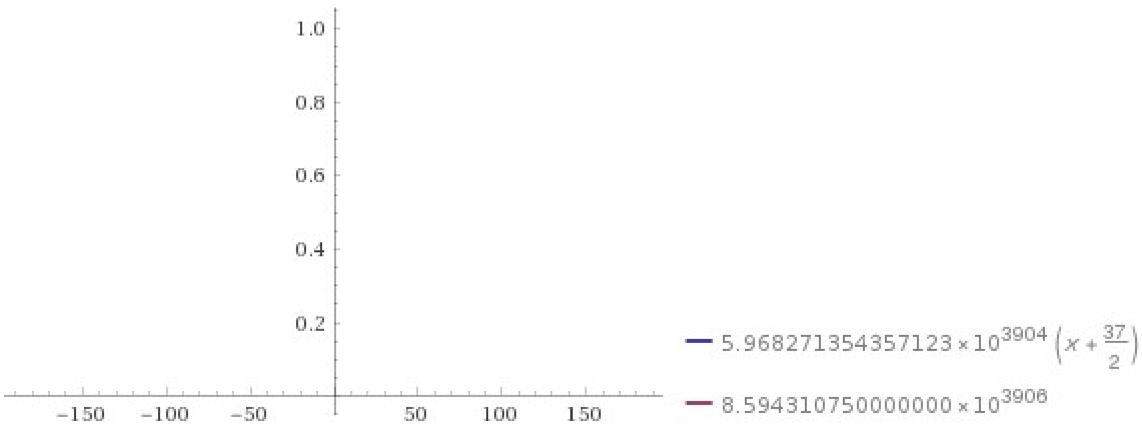
Input interpretation:

$$\left(12\left(x+18+\frac{1}{2}\right)\right)\times\frac{1}{12}\left(\frac{1}{162}+\frac{1}{512}\right)\times 2\times\frac{1}{2\sqrt{\frac{27}{256}}}$$
$$\exp\left(256\left(\frac{1}{27}\left(96\pi^2\right)\right)\right)\times 8.41724\left(96\pi^2\right)\times\frac{256}{27}=8.59431075\times 10^{3906}$$

Result:

$$5.968271354357123\times 10^{3904}\left(x+\frac{37}{2}\right)=8.594310750000000\times 10^{3906}$$

Plot:



Alternate forms:

$$5.968271354357123\times 10^{3904}\,x-7.49018054944393\times 10^{3906}=0$$

$$5.968271354357123\times 10^{3904}\,x+1.104130200556068\times 10^{3906}=8.594310750000000\times 10^{3906}$$

Solution:

$x \approx 125.4999999954048$

125.49999….

We have also:

ln(((144*1/2*(1/81+1/256)* 2/(2*(sqrt(27/256))) * exp[((12*8*Pi^2))/((27/256))] * 8.41724*(12*8*Pi^2)*256/27)))

Input interpretation:

$$\log\left(144\times\frac{1}{2}\left(\frac{1}{81}+\frac{1}{256}\right)\times\frac{2}{2\sqrt{\frac{27}{256}}}\exp\left(\frac{12\times8\pi^2}{\frac{27}{256}}\right)\times8.41724\left(12\times8\pi^2\right)\times\frac{256}{27}\right)$$

log(x) is the natural logarithm

Result:

8996.048474...

8996.048474….

Alternative representations:

$$\log\left(\frac{\left(144\left(\frac{1}{81}+\frac{1}{256}\right)\right)2\left(\exp\left(\frac{12\times8\pi^2}{\frac{27}{256}}\right)8.41724\left(12\times8\pi^2\right)256\right)}{2\left(2\sqrt{\frac{27}{256}}\right)27}\right)=$$

$$\log_e\left(\frac{2.97881\times10^7\exp\left(\frac{96\pi^2}{\frac{27}{256}}\right)\left(\frac{1}{81}+\frac{1}{256}\right)\pi^2}{27\left(2\sqrt{\frac{27}{256}}\right)}\right)$$

$$\log\left(\frac{\left(144\left(\frac{1}{81}+\frac{1}{256}\right)\right)2\left(\exp\left(\frac{12\times 8\pi^2}{\frac{27}{256}}\right)8.41724\,(12\times 8\pi^2)\,256\right)}{2\left(2\sqrt{\frac{27}{256}}\right)27}\right) = \log(a)\log_a\left(\frac{2.97881\times 10^7\exp\left(\frac{96\pi^2}{\frac{27}{256}}\right)\left(\frac{1}{81}+\frac{1}{256}\right)\pi^2}{27\left(2\sqrt{\frac{27}{256}}\right)}\right)$$

Series representation:

$$\log\left(\frac{\left(144\left(\frac{1}{81}+\frac{1}{256}\right)\right)2\left(\exp\left(\frac{12\times 8\pi^2}{\frac{27}{256}}\right)8.41724\,(12\times 8\pi^2)\,256\right)}{2\left(2\sqrt{\frac{27}{256}}\right)27}\right) = \log\left(-1+\frac{8965.09\,\pi^2\exp\left(\frac{8192\pi^2}{9}\right)}{\sqrt{\frac{27}{256}}}\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{8965.09\,\pi^2\exp\left(\frac{8192\pi^2}{9}\right)}{\sqrt{\frac{27}{256}}}\right)^{-k}}{k}$$

Integral representations:

$$\log\left(\frac{\left(144\left(\frac{1}{81}+\frac{1}{256}\right)\right)2\left(\exp\left(\frac{12\times 8\pi^2}{\frac{27}{256}}\right)8.41724\,(12\times 8\pi^2)\,256\right)}{2\left(2\sqrt{\frac{27}{256}}\right)27}\right) = \int_1^{\frac{8965.09\,\pi^2\exp\left(\frac{8192\pi^2}{9}\right)}{\sqrt{\frac{27}{256}}}}\frac{1}{t}\,dt$$

$$\log\left(\frac{\left(144\left(\frac{1}{81}+\frac{1}{256}\right)\right)2\left(\exp\left(\frac{12\times 8\pi^2}{\frac{27}{256}}\right)8.41724\,(12\times 8\pi^2)\,256\right)}{2\left(2\sqrt{\frac{27}{256}}\right)27}\right) = \frac{1}{2i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)\left(-1+\frac{8965.09\,\pi^2\exp\left(\frac{8192\pi^2}{9}\right)}{\sqrt{\frac{27}{256}}}\right)^{-s}}{\Gamma(1-s)}\,ds \quad \text{for } -1<\gamma<0$$

Now, we have that:

$$\partial_m^2 \log Z\Big|_{m=0}^{\text{pert}} = 2N^2 \log g_{\text{YM}} + \sqrt{N}\left[\frac{16\zeta(3)}{g_{\text{YM}}^3} + \frac{g_{\text{YM}}}{3}\right] - \frac{1}{\sqrt{N}}\left[\frac{12\zeta(5)}{g_{\text{YM}}^5} + \frac{g_{\text{YM}}^3}{1440}\right] + \ldots, \quad (3.5)$$

For: sqrt(27/256) = g_{YM} ; N = 4

2*16 ln(sqrt(27/256))+2 [(16 zeta(3)) / (sqrt(27/256))^3 + (sqrt(27/256))/3] – 1/2 [(12 zeta(5)) / (sqrt(27/256))^5 + (sqrt(27/256))^3 / 1440]

Input:

$$2\times 16 \log\left(\sqrt{\frac{27}{256}}\right) + 2\left(\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^3} + \frac{\sqrt{\frac{27}{256}}}{3}\right) - \frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^5} + \frac{\sqrt{\frac{27}{256}}^3}{1440}\right)$$

log(x) is the natural logarithm

ζ(s) is the Riemann zeta function

Exact result:

$$2\left(\frac{65536\,\zeta(3)}{81\sqrt{3}} + \frac{\sqrt{3}}{16}\right) + \frac{1}{2}\left(-\frac{4194304\,\zeta(5)}{729\sqrt{3}} - \frac{9\sqrt{3}}{655360}\right) + 32\log\left(\frac{3\sqrt{3}}{16}\right)$$

Decimal approximation:

-634.9746992479758789861126452238542043613401193134768560 41...

-634.9746992479…

Alternate forms:

$$\frac{1546188226560\,\zeta(3) - 2748779069440\,\zeta(5) + 358298397}{955514880\sqrt{3}} + 32\log\left(\frac{3\sqrt{3}}{16}\right)$$

$$-\frac{1}{955514880\sqrt{3}}\left(-1546188226560\,\zeta(3) + 2748779069440\,\zeta(5) - 358298397 + 122305904640\sqrt{3}\log(2) - 45864714240\sqrt{3}\log(3)\right)$$

$$\frac{1}{2\,866\,544\,640}\left(1\,546\,188\,226\,560\sqrt{3}\,\zeta(3)-2\,748\,779\,069\,440\sqrt{3}\,\zeta(5)+358\,298\,397\sqrt{3}+91\,729\,428\,480\log\left(\frac{3\sqrt{3}}{16}\right)\right)$$

Alternative representations:

$$2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^3}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^5}+\frac{\sqrt{\frac{27}{256}}^3}{1440}\right)=$$
$$32\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{\sqrt{\frac{27}{256}}}{3}+\frac{16\,\zeta(3,1)}{\sqrt{\frac{27}{256}}^3}\right)+\frac{1}{2}\left(-\frac{\sqrt{\frac{27}{256}}^3}{1440}-\frac{12\,\zeta(5,1)}{\sqrt{\frac{27}{256}}^5}\right)$$

$$2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^3}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^5}+\frac{\sqrt{\frac{27}{256}}^3}{1440}\right)=$$
$$32\log_e\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{\sqrt{\frac{27}{256}}}{3}+\frac{16\,\zeta(3,1)}{\sqrt{\frac{27}{256}}^3}\right)+\frac{1}{2}\left(-\frac{\sqrt{\frac{27}{256}}^3}{1440}-\frac{12\,\zeta(5,1)}{\sqrt{\frac{27}{256}}^5}\right)$$

$$2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^3}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^5}+\frac{\sqrt{\frac{27}{256}}^3}{1440}\right)=$$
$$32\log(a)\log_a\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{\sqrt{\frac{27}{256}}}{3}+\frac{16\,\zeta(3,1)}{\sqrt{\frac{27}{256}}^3}\right)+\frac{1}{2}\left(-\frac{\sqrt{\frac{27}{256}}^3}{1440}-\frac{12\,\zeta(5,1)}{\sqrt{\frac{27}{256}}^5}\right)$$

Series representations:

$$2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^{3}}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^{5}}+\frac{\sqrt{\frac{27}{256}}^{3}}{1440}\right)=$$
$$\frac{1}{1\,310\,720}\left(163\,831\sqrt{3}-41\,943\,040\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{3\sqrt{3}}{16}\right)^k}{k}+1\,310\,720\sum_{n=0}^{\infty}\left(\sum_{k=0}^{n}\frac{65\,536\,(-1)^k\,(1+18\,k+9\,k^2)\binom{n}{k}}{729\sqrt{3}\,(1+k)^4\,(1+n)}\right)\right)$$

$$2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^{3}}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^{5}}+\frac{\sqrt{\frac{27}{256}}^{3}}{1440}\right)=$$
$$\frac{1}{1\,310\,720}\left(163\,831\sqrt{3}-41\,943\,040\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{3\sqrt{3}}{16}\right)^k}{k}+1\,310\,720\sum_{m=1}^{\infty}\left(\sum_{k=1+m}^{\infty}-\frac{131\,072\,(-16+9\,k^2)}{729\sqrt{3}\,k^4\,m}+\sum_{k=1}^{m}\frac{131\,072\,(-16+9\,m^2)}{729\sqrt{3}\,k\,m^4}\right)\right)$$

$$2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^{3}}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^{5}}+\frac{\sqrt{\frac{27}{256}}^{3}}{1440}\right)=$$
$$\frac{1}{1\,310\,720}\left(163\,831\sqrt{3}-41\,943\,040\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{3\sqrt{3}}{16}\right)^k}{k}+1\,310\,720\sum_{k=0}^{\infty}\frac{131\,072\left(9\,(3-s_0)^k-16\,(5-s_0)^k\right)\zeta^{(k)}(s_0)}{729\sqrt{3}\,k!}\right)\quad\text{for } s_0\neq 1$$

Integral representations:

$$2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\zeta(3)}{\sqrt{\frac{27}{256}}^3}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\zeta(5)}{\sqrt{\frac{27}{256}}^5}+\frac{\sqrt{\frac{27}{256}}^3}{1440}\right)=$$
$$\frac{163831\sqrt{3}}{1310720}+\int_0^\infty -\frac{65536t^2(-27+4t^2)}{2187\sqrt{3}(-1+e^t)}\,dt+32\log\left(\frac{3\sqrt{3}}{16}\right)$$

$$2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\zeta(3)}{\sqrt{\frac{27}{256}}^3}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\zeta(5)}{\sqrt{\frac{27}{256}}^5}+\frac{\sqrt{\frac{27}{256}}^3}{1440}\right)=$$
$$\frac{163831\sqrt{3}}{1310720}+\int_0^\infty -\frac{262144t^2(-135+16t^2)}{32805\sqrt{3}(1+e^t)}\,dt+32\log\left(\frac{3\sqrt{3}}{16}\right)$$

$$2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\zeta(3)}{\sqrt{\frac{27}{256}}^3}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\zeta(5)}{\sqrt{\frac{27}{256}}^5}+\frac{\sqrt{\frac{27}{256}}^3}{1440}\right)=$$
$$\frac{163831\sqrt{3}}{1310720}+\int_0^1 -\frac{131072(45t^2\log^3(1-t^2)-8\log^5(1-t^4))}{10935\sqrt{3}\,t^5}\,dt+32\log\left(\frac{3\sqrt{3}}{16}\right)$$

We obtain also:

-golden ratio^2*(((2*16 ln(sqrt(27/256))+2 [(16 zeta(3)) / (sqrt(27/256))^3 + (sqrt(27/256))/3] – 1/2 [(12 zeta(5)) / (sqrt(27/256))^5 + (sqrt(27/256))^3 / 1440])))+64+e

Input:

$$-\phi^2\left(2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\zeta(3)}{\sqrt{\frac{27}{256}}^3}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\zeta(5)}{\sqrt{\frac{27}{256}}^5}+\frac{\sqrt{\frac{27}{256}}^3}{1440}\right)\right)+64+e$$

$\log(x)$ is the natural logarithm

$\zeta(s)$ is the Riemann zeta function

ϕ is the golden ratio

Exact result:

$$-\phi^2\left(2\left(\frac{65536\,\zeta(3)}{81\sqrt{3}}+\frac{\sqrt{3}}{16}\right)+\frac{1}{2}\left(-\frac{4194304\,\zeta(5)}{729\sqrt{3}}-\frac{9\sqrt{3}}{655360}\right)+32\log\left(\frac{3\sqrt{3}}{16}\right)\right)+64+e$$

Decimal approximation:

1729.103626455902192331492222365362693896648824558401784542...

1729.103626455…

Alternate forms:

$$\phi^2\left(-\frac{131072\,\zeta(3)}{81\sqrt{3}}+\frac{2097152\,\zeta(5)}{729\sqrt{3}}-\frac{163831\sqrt{3}}{1310720}+128\log(2)-48\log(3)\right)+64+e$$

$$-\phi^2\left(\frac{131072\,\zeta(3)}{81\sqrt{3}}-\frac{2097152\,\zeta(5)}{729\sqrt{3}}+\frac{163831\sqrt{3}}{1310720}+32\log\left(\frac{3\sqrt{3}}{16}\right)\right)+64+e$$

$$\frac{1}{955514880\sqrt{3}}\left(\phi^2\left(-1546188226560\,\zeta(3)+2748779069440\,\zeta(5)-358298397+122305904640\sqrt{3}\log(2)-45864714240\sqrt{3}\log(3)\right)+61152952320\sqrt{3}+955514880\sqrt{3}\,e\right)$$

Alternative representations:

$$-\phi^2\left(2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^3}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^5}+\frac{\sqrt{\frac{27}{256}}^3}{1440}\right)\right)+64+e=$$

$$64+e-\phi^2\left(32\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{\sqrt{\frac{27}{256}}}{3}+\frac{16\,\zeta(3,1)}{\sqrt{\frac{27}{256}}^3}\right)+\frac{1}{2}\left(-\frac{\sqrt{\frac{27}{256}}^3}{1440}-\frac{12\,\zeta(5,1)}{\sqrt{\frac{27}{256}}^5}\right)\right)$$

$$-\phi^2\left(2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^3}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^5}+\frac{\sqrt{\frac{27}{256}}^3}{1440}\right)\right)+64+e=$$

$$64+e-\phi^2\left(32\log_e\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{\sqrt{\frac{27}{256}}}{3}+\frac{16\,\zeta(3,1)}{\sqrt{\frac{27}{256}}^3}\right)+\frac{1}{2}\left(-\frac{\sqrt{\frac{27}{256}}^3}{1440}-\frac{12\,\zeta(5,1)}{\sqrt{\frac{27}{256}}^5}\right)\right)$$

$$-\phi^2\left(2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^3}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^5}+\frac{\sqrt{\frac{27}{256}}^3}{1440}\right)\right)+64+e=$$

$$64+e-\phi^2\left(32\log(a)\log_a\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{\sqrt{\frac{27}{256}}}{3}+\frac{16\,\zeta(3,1)}{\sqrt{\frac{27}{256}}^3}\right)+\frac{1}{2}\left(-\frac{\sqrt{\frac{27}{256}}^3}{1440}-\frac{12\,\zeta(5,1)}{\sqrt{\frac{27}{256}}^5}\right)\right)$$

Series representations:

$$-\phi^2\left(2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^3}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^5}+\frac{\sqrt{\frac{27}{256}}^3}{1440}\right)\right)+64+e=$$

$$\frac{1}{2\,621\,440}\left(167\,772\,160-491\,493\sqrt{3}-163\,831\sqrt{15}+2\,621\,440\,e+125\,829\,120\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{3\sqrt{3}}{16}\right)^k}{k}+41\,943\,040\sqrt{5}\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{3\sqrt{3}}{16}\right)^k}{k}-3\,932\,160\sum_{n=0}^{\infty}\left(\sum_{k=0}^{n}\frac{65\,536\,(-1)^k\left(1+18\,k+9\,k^2\right)\binom{n}{k}}{729\sqrt{3}\,(1+k)^4\,(1+n)}\right)-1\,310\,720\sqrt{5}\sum_{n=0}^{\infty}\left(\sum_{k=0}^{n}\frac{65\,536\,(-1)^k\left(1+18\,k+9\,k^2\right)\binom{n}{k}}{729\sqrt{3}\,(1+k)^4\,(1+n)}\right)\right)$$

$$-\phi^2\left(2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^3}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^5}+\frac{\sqrt{\frac{27}{256}}^3}{1440}\right)\right)+64+e=$$

$$\frac{1}{2\,621\,440}\left(167\,772\,160-491\,493\sqrt{3}-163\,831\sqrt{15}+2\,621\,440\,e+125\,829\,120\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{3\sqrt{3}}{16}\right)^k}{k}+41\,943\,040\sqrt{5}\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{3\sqrt{3}}{16}\right)^k}{k}-3\,932\,160\sum_{k=0}^{\infty}\frac{131\,072\left(9\,(3-s_0)^k-16\,(5-s_0)^k\right)\zeta^{(k)}(s_0)}{729\sqrt{3}\,k!}-1\,310\,720\sqrt{5}\sum_{k=0}^{\infty}\frac{131\,072\left(9\,(3-s_0)^k-16\,(5-s_0)^k\right)\zeta^{(k)}(s_0)}{729\sqrt{3}\,k!}\right)\quad\text{for } s_0\neq 1$$

$$-\phi^2\left(2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^3}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^5}+\frac{\sqrt{\frac{27}{256}}^3}{1440}\right)\right)+64+e=$$

$$\frac{1}{2\,621\,440}\left(167\,772\,160-491\,493\sqrt{3}-163\,831\sqrt{15}+2\,621\,440\,e+125\,829\,120\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{3\sqrt{3}}{16}\right)^k}{k}+41\,943\,040\sqrt{5}\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{3\sqrt{3}}{16}\right)^k}{k}-3\,932\,160\sum_{m=1}^{\infty}\frac{131\,072\left(16\,m^3\sum_{k=1+m}^{\infty}\frac{1}{k^4}-9\,m^3\sum_{k=1+m}^{\infty}\frac{1}{k^2}+\left(-16+9\,m^2\right)\sum_{k=1}^{m}\frac{1}{k}\right)}{729\sqrt{3}\,m^4}-1\,310\,720\sqrt{5}\sum_{m=1}^{\infty}\frac{131\,072\left(16\,m^3\sum_{k=1+m}^{\infty}\frac{1}{k^4}-9\,m^3\sum_{k=1+m}^{\infty}\frac{1}{k^2}+\left(-16+9\,m^2\right)\sum_{k=1}^{m}\frac{1}{k}\right)}{729\sqrt{3}\,m^4}\right)$$

Integral representations:

$$-\phi^2\left(2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\zeta(3)}{\sqrt{\frac{27}{256}}^3}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\zeta(5)}{\sqrt{\frac{27}{256}}^5}+\frac{\sqrt{\frac{27}{256}}^3}{1440}\right)\right)+64+e=$$

$$64-\frac{163831\sqrt{\frac{3}{5}}}{524288}-\frac{491493\sqrt{3}}{2621440}+e+\int_0^1\frac{65536(3+\sqrt{5})(45t^2\log^3(1-t^2)-8\log^5(1-t^4))}{10935\sqrt{3}\,t^5}\,dt-48\log\left(\frac{3\sqrt{3}}{16}\right)-16\sqrt{5}\log\left(\frac{3\sqrt{3}}{16}\right)$$

$$-\phi^2\left(2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\zeta(3)}{\sqrt{\frac{27}{256}}^3}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\zeta(5)}{\sqrt{\frac{27}{256}}^5}+\frac{\sqrt{\frac{27}{256}}^3}{1440}\right)\right)+64+e=$$

$$\frac{1}{17199267840}\left(1100753141760-3224685573\sqrt{3}-1074895191\sqrt{15}+17199267840\,e-6957847019520\sqrt{3}\int_0^\infty\frac{t^2}{-1+e^t}\,dt-2319282339840\sqrt{15}\int_0^\infty\frac{t^2}{-1+e^t}\,dt+1030792151040\sqrt{3}\int_0^\infty\frac{t^4}{-1+e^t}\,dt+343597383680\sqrt{15}\int_0^\infty\frac{t^4}{-1+e^t}\,dt-825564856320\log\left(\frac{3\sqrt{3}}{16}\right)-275188285440\sqrt{5}\log\left(\frac{3\sqrt{3}}{16}\right)\right)$$

$$-\phi^2\left(2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\zeta(3)}{\sqrt{\frac{27}{256}}^3}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\zeta(5)}{\sqrt{\frac{27}{256}}^5}+\frac{\sqrt{\frac{27}{256}}^3}{1440}\right)\right)+64+e=$$

$$\frac{1}{51597803520}\left(3302259425280-9674056719\sqrt{3}-3224685573\sqrt{15}+51597803520\,e-27831388078080\sqrt{3}\int_0^\infty\frac{t^2}{1+e^t}\,dt-9277129359360\sqrt{15}\int_0^\infty\frac{t^2}{1+e^t}\,dt+3298534883328\sqrt{3}\int_0^\infty\frac{t^4}{1+e^t}\,dt+1099511627776\sqrt{15}\int_0^\infty\frac{t^4}{1+e^t}\,dt-2476694568960\log\left(\frac{3\sqrt{3}}{16}\right)-825564856320\sqrt{5}\log\left(\frac{3\sqrt{3}}{16}\right)\right)$$

and again:

-sqrtPi * (((2*16 ln(sqrt(27/256))+2 [(16 zeta(3)) / (sqrt(27/256))^3 + (sqrt(27/256))/3] – 1/2 [(12 zeta(5)) / (sqrt(27/256))^5 + (sqrt(27/256))^3 / 1440])))+24*3

Input:

$$-\sqrt{\pi}\left(2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^{3}}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^{5}}+\frac{\sqrt{\frac{27}{256}}^{3}}{1440}\right)\right)+24\times 3$$

log(x) is the natural logarithm

ζ(s) is the Riemann zeta function

Exact result:

$$72-\sqrt{\pi}\left(2\left(\frac{65\,536\,\zeta(3)}{81\sqrt{3}}+\frac{\sqrt{3}}{16}\right)+\frac{1}{2}\left(-\frac{4\,194\,304\,\zeta(5)}{729\sqrt{3}}-\frac{9\sqrt{3}}{655\,360}\right)+32\log\left(\frac{3\sqrt{3}}{16}\right)\right)$$

Decimal approximation:

1197.4633509096467185139452683521115275652805901830536424290...

1197.46335… result practically equal to the rest mass of Sigma baryon 1197.449

Alternate forms:

$$\sqrt{\pi}\left(-\frac{131\,072\,\zeta(3)}{81\sqrt{3}}+\frac{2\,097\,152\,\zeta(5)}{729\sqrt{3}}-\frac{163\,831\sqrt{3}}{1\,310\,720}+128\log(2)-48\log(3)\right)+72$$

$$-\frac{131\,072}{729}\sqrt{\frac{\pi}{3}}\,(9\,\zeta(3)-16\,\zeta(5))+72-\frac{163\,831\sqrt{3\pi}}{1\,310\,720}-32\sqrt{\pi}\log\left(\frac{3\sqrt{3}}{16}\right)$$

$$\frac{1}{955\,514\,880\sqrt{3}}\left(\sqrt{\pi}\left(-1\,546\,188\,226\,560\,\zeta(3)+2\,748\,779\,069\,440\,\zeta(5)-358\,298\,397+122\,305\,904\,640\sqrt{3}\log(2)-45\,864\,714\,240\sqrt{3}\log(3)\right)+68\,797\,071\,360\sqrt{3}\right)$$

Alternative representations:

$$-\sqrt{\pi}\left(2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\zeta(3)}{\sqrt{\frac{27}{256}}^3}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\zeta(5)}{\sqrt{\frac{27}{256}}^5}+\frac{\sqrt{\frac{27}{256}}^3}{1440}\right)\right)+24\times 3 =$$

$$72-\sqrt{\pi}\left(32\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{\sqrt{\frac{27}{256}}}{3}+\frac{16\zeta(3,1)}{\sqrt{\frac{27}{256}}^3}\right)+\frac{1}{2}\left(-\frac{\sqrt{\frac{27}{256}}^3}{1440}-\frac{12\zeta(5,1)}{\sqrt{\frac{27}{256}}^5}\right)\right)$$

$$-\sqrt{\pi}\left(2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\zeta(3)}{\sqrt{\frac{27}{256}}^3}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\zeta(5)}{\sqrt{\frac{27}{256}}^5}+\frac{\sqrt{\frac{27}{256}}^3}{1440}\right)\right)+24\times 3 =$$

$$72-\sqrt{\pi}\left(32\log_e\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{\sqrt{\frac{27}{256}}}{3}+\frac{16\zeta(3,1)}{\sqrt{\frac{27}{256}}^3}\right)+\frac{1}{2}\left(-\frac{\sqrt{\frac{27}{256}}^3}{1440}-\frac{12\zeta(5,1)}{\sqrt{\frac{27}{256}}^5}\right)\right)$$

$$-\sqrt{\pi}\left(2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\zeta(3)}{\sqrt{\frac{27}{256}}^3}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\zeta(5)}{\sqrt{\frac{27}{256}}^5}+\frac{\sqrt{\frac{27}{256}}^3}{1440}\right)\right)+24\times 3 =$$

$$72-\sqrt{\pi}\left(32\log(a)\log_a\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{\sqrt{\frac{27}{256}}}{3}+\frac{16\zeta(3,1)}{\sqrt{\frac{27}{256}}^3}\right)+\frac{1}{2}\left(-\frac{\sqrt{\frac{27}{256}}^3}{1440}-\frac{12\zeta(5,1)}{\sqrt{\frac{27}{256}}^5}\right)\right)$$

Series representations:

$$-\sqrt{\pi}\left(2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\zeta(3)}{\sqrt{\frac{27}{256}}^3}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\zeta(5)}{\sqrt{\frac{27}{256}}^5}+\frac{\sqrt{\frac{27}{256}}^3}{1440}\right)\right)+24\times 3=$$

$$\frac{1}{1310720}\left(94371840-163831\sqrt{3\pi}+41943040\sqrt{\pi}\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{3\sqrt{3}}{16}\right)^k}{k}+1310720\sum_{n=0}^{\infty}\left(\sum_{k=0}^{n}-\frac{65536(-1)^k(1+18k+9k^2)\sqrt{\frac{\pi}{3}}\binom{n}{k}}{729(1+k)^4(1+n)}\right)\right)$$

$$-\sqrt{\pi}\left(2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\zeta(3)}{\sqrt{\frac{27}{256}}^3}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\zeta(5)}{\sqrt{\frac{27}{256}}^5}+\frac{\sqrt{\frac{27}{256}}^3}{1440}\right)\right)+24\times 3=$$

$$\frac{1}{1310720}\left(94371840-163831\sqrt{3\pi}+41943040\sqrt{\pi}\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{3\sqrt{3}}{16}\right)^k}{k}+1310720\sum_{m=1}^{\infty}\left(\sum_{k=1+m}^{\infty}\frac{131072(-16+9k^2)\sqrt{\frac{\pi}{3}}}{729k^4m}+\sum_{k=1}^{m}-\frac{131072(-16+9m^2)\sqrt{\frac{\pi}{3}}}{729km^4}\right)\right)$$

$$-\sqrt{\pi}\left(2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\zeta(3)}{\sqrt{\frac{27}{256}}^3}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\zeta(5)}{\sqrt{\frac{27}{256}}^5}+\frac{\sqrt{\frac{27}{256}}^3}{1440}\right)\right)+24\times 3=$$

$$\frac{1}{1310720}\left(94371840-163831\sqrt{3\pi}+41943040\sqrt{\pi}\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{3\sqrt{3}}{16}\right)^k}{k}+1310720\sum_{k=0}^{\infty}-\frac{131072\sqrt{\frac{\pi}{3}}\left(9(3-s_0)^k-16(5-s_0)^k\right)\zeta^{(k)}(s_0)}{729k!}\right)\quad\text{for } s_0\neq 1$$

Integral representations:

$$-\sqrt{\pi}\left(2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^{3}}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^{5}}+\frac{\sqrt{\frac{27}{256}}^{3}}{1440}\right)\right)+24\times 3=$$

$$72-\frac{163831\sqrt{3\pi}}{1310720}+\int_0^\infty \frac{65536\sqrt{\frac{\pi}{3}}\;t^2\left(-27+4t^2\right)}{2187\left(-1+e^t\right)}\,dt-32\sqrt{\pi}\,\log\left(\frac{3\sqrt{3}}{16}\right)$$

$$-\sqrt{\pi}\left(2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^{3}}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^{5}}+\frac{\sqrt{\frac{27}{256}}^{3}}{1440}\right)\right)+24\times 3=$$

$$72-\frac{163831\sqrt{3\pi}}{1310720}+\int_0^\infty \frac{262144\sqrt{\frac{\pi}{3}}\;t^2\left(-135+16t^2\right)}{32805\left(1+e^t\right)}\,dt-32\sqrt{\pi}\,\log\left(\frac{3\sqrt{3}}{16}\right)$$

$$-\sqrt{\pi}\left(2\times 16\log\left(\sqrt{\frac{27}{256}}\right)+2\left(\frac{16\,\zeta(3)}{\sqrt{\frac{27}{256}}^{3}}+\frac{\sqrt{\frac{27}{256}}}{3}\right)-\frac{1}{2}\left(\frac{12\,\zeta(5)}{\sqrt{\frac{27}{256}}^{5}}+\frac{\sqrt{\frac{27}{256}}^{3}}{1440}\right)\right)+24\times 3=$$

$$72-\frac{163831\sqrt{3\pi}}{1310720}+\int_0^1 \frac{131072\sqrt{\frac{\pi}{3}}\left(45t^2\log^3\left(1-t^2\right)-8\log^5\left(1-t^4\right)\right)}{10935\,t^5}\,dt-32\sqrt{\pi}\,\log\left(\frac{3\sqrt{3}}{16}\right)$$

Now, we have the following equation:

$$\langle \partial_m^2 Z_{\rm inst}^{p\times q}(m,a_{ij})\rangle\Big|_{m=0} = \frac{e^{\frac{8pq\pi^2}{g_{\rm YM}^2}}}{1+\delta_{p,q}}\Big[-\sqrt{N}\frac{16K_1(\frac{8pq\pi^2}{g_{\rm YM}^2})}{g_{\rm YM}}\left(\frac{p}{q}+\frac{q}{p}\right)+\frac{2K_2(\frac{8pq\pi^2}{g_{\rm YM}^2})}{g_{\rm YM}\sqrt{N}}\left(\frac{p^2}{q^2}+\frac{q^2}{p^2}\right)$$
$$+\frac{1}{32g_{\rm YM}N^{\frac{3}{2}}}\left[-13K_1\left(\frac{8pq\pi^2}{g_{\rm YM}^2}\right)\left(\frac{p}{q}+\frac{q}{p}\right)+9K_3\left(\frac{8pq\pi^2}{g_{\rm YM}^2}\right)\left(\frac{p^3}{q^3}+\frac{q^3}{p^3}\right)\right]$$
$$+\frac{1}{128g_{\rm YM}N^{\frac{5}{2}}}\left[-25K_2\left(\frac{8pq\pi^2}{g_{\rm YM}^2}\right)\left(\frac{p^2}{q^2}+\frac{q^2}{p^2}\right)+15K_4\left(\frac{8pq\pi^2}{g_{\rm YM}^2}\right)\left(\frac{p^4}{q^4}+\frac{q^4}{p^4}\right)\right]$$
$$+\frac{1}{g_{\rm YM}N^{\frac{7}{2}}}\left[\frac{1533K_1\left(\frac{8pq\pi^2}{g_{\rm YM}^2}\right)}{16384}\left(\frac{p}{q}+\frac{q}{p}\right)-\frac{5355K_3\left(\frac{8pq\pi^2}{g_{\rm YM}^2}\right)}{32768}\left(\frac{p^3}{q^3}+\frac{q^3}{p^3}\right)+\frac{2625K_5\left(\frac{8pq\pi^2}{g_{\rm YM}^2}\right)}{32768}\left(\frac{p^5}{q^5}+\frac{q^5}{p^5}\right)\right]$$
$$+O(N^{-\frac{9}{2}})\Big], \tag{D.5}$$

we perform these computations:

$$: \frac{e^{\frac{8pq\pi^2}{g_{\rm YM}^2}}}{1+\delta_{p,q}}\Big[-\sqrt{N}\frac{16K_1(\frac{8pq\pi^2}{g_{\rm YM}^2})}{g_{\rm YM}}\left(\frac{p}{q}+\frac{q}{p}\right)+\frac{2K_2(\frac{8pq\pi^2}{g_{\rm YM}^2})}{g_{\rm YM}\sqrt{N}}\left(\frac{p^2}{q^2}+\frac{q^2}{p^2}\right)$$

$$+\frac{1}{32g_{\rm YM}N^{\frac{3}{2}}}\left[-13K_1\left(\frac{8pq\pi^2}{g_{\rm YM}^2}\right)\left(\frac{p}{q}+\frac{q}{p}\right)+9K_3\left(\frac{8pq\pi^2}{g_{\rm YM}^2}\right)\left(\frac{p^3}{q^3}+\frac{q^3}{p^3}\right)\right]$$

$$+\frac{1}{128g_{\rm YM}N^{\frac{5}{2}}}\left[-25K_2\left(\frac{8pq\pi^2}{g_{\rm YM}^2}\right)\left(\frac{p^2}{q^2}+\frac{q^2}{p^2}\right)+15K_4\left(\frac{8pq\pi^2}{g_{\rm YM}^2}\right)\left(\frac{p^4}{q^4}+\frac{q^4}{p^4}\right)\right]$$

$$+\frac{1}{g_{\rm YM}N^{\frac{7}{2}}}\left[\frac{1533K_1\left(\frac{8pq\pi^2}{g_{\rm YM}^2}\right)}{16384}\left(\frac{p}{q}+\frac{q}{p}\right)-\frac{5355K_3\left(\frac{8pq\pi^2}{g_{\rm YM}^2}\right)}{32768}\left(\frac{p^3}{q^3}+\frac{q^3}{p^3}\right)+\frac{2625K_5\left(\frac{8pq\pi^2}{g_{\rm YM}^2}\right)}{32768}\left(\frac{p^5}{q^5}+\frac{q^5}{p^5}\right)\right]$$
$$+O(N^{-\frac{9}{2}})\Big],$$

For k = 12; Sqrt(27/256) = g_{YM} ; N = 4; K_1 = 13.3236; K_2 = 8.41724;

K_3 = 13.0152; K_4 = 11.0647; K_5 = 18.9801 p = 3, q = 4

We have:

a)

$$\left[-\sqrt{N}\frac{16K_1(\frac{8pq\pi^2}{g_{YM}^2})}{g_{YM}}\left(\frac{p}{q}+\frac{q}{p}\right)+\frac{2K_2(\frac{8pq\pi^2}{g_{YM}^2})}{g_{YM}\sqrt{N}}\left(\frac{p^2}{q^2}+\frac{q^2}{p^2}\right)\right.$$

exp(((96*Pi^2)/(27/256)))*1/2

[-2*16*13.3236*(((96*Pi^2)/(27/256)))*1/(sqrt(27/256))*(3/4+4/3)+2*8.41724*(((96*Pi^2)/(27/256)))* 1/(2sqrt(27/256))*(9/16+16/9)]

Input interpretation:

$$-2\times 16\times 13.3236\times\frac{96\,\pi^2}{\frac{27}{256}}\times\frac{1}{\sqrt{\frac{27}{256}}}\left(\frac{3}{4}+\frac{4}{3}\right)+2\times 8.41724\times\frac{96\,\pi^2}{\frac{27}{256}}\times\frac{1}{2\sqrt{\frac{27}{256}}}\left(\frac{9}{16}+\frac{16}{9}\right)$$

Result:

$-2.40257\ldots\times 10^7$

-2.40257…*10^7

b)

$$+\frac{1}{32g_{YM}N^{\frac{3}{2}}}\left[-13K_1\left(\frac{8pq\pi^2}{g_{YM}^2}\right)\left(\frac{p}{q}+\frac{q}{p}\right)+9K_3\left(\frac{8pq\pi^2}{g_{YM}^2}\right)\left(\frac{p^3}{q^3}+\frac{q^3}{p^3}\right)\right]$$

1/((32*sqrt(27/256)*4^1.5))[-13*13.3236*(((96*Pi^2)/(27/256)))*(3/4+4/3)+9*13.0152*(((96*Pi^2)/(27/256)))*(27/64+64/27)]

Input interpretation:

$$\frac{1}{32\sqrt{\frac{27}{256}}\times 4^{1.5}}\left(-13\times 13.3236\times\frac{96\pi^2}{\frac{27}{256}}\left(\frac{3}{4}+\frac{4}{3}\right)+9\times 13.0152\times\frac{96\pi^2}{\frac{27}{256}}\left(\frac{27}{64}+\frac{64}{27}\right)\right)$$

Result:

-3649.33...

-3649.33…

c)

$$+\frac{1}{128g_{\rm YM}N^{\frac{5}{2}}}\left[-25K_2\left(\frac{8pq\pi^2}{g_{\rm YM}^2}\right)\left(\frac{p^2}{q^2}+\frac{q^2}{p^2}\right)+15K_4\left(\frac{8pq\pi^2}{g_{\rm YM}^2}\right)\left(\frac{p^4}{q^4}+\frac{q^4}{p^4}\right)\right]$$

1/((128*sqrt(27/256)*4^2.5))[-25*8.41724*(((96*Pi^2)/(27/256)))*(9/16+16/9)+15*11.0647*(((96*Pi^2)/(27/256)))* (81/256+256/81)]

Input interpretation:

$$\frac{1}{128\sqrt{\frac{27}{256}}\times 4^{2.5}}\left(-25\times 8.41724\times\frac{96\pi^2}{\frac{27}{256}}\left(\frac{9}{16}+\frac{16}{9}\right)+15\times 11.0647\times\frac{96\pi^2}{\frac{27}{256}}\left(\frac{81}{256}+\frac{256}{81}\right)\right)$$

Result:

571.313...

571.313…

d)

$$+\frac{1}{g_{\rm YM}N^{\frac{7}{2}}}\left[\frac{1533K_1\left(\frac{8pq\pi^2}{g_{\rm YM}^2}\right)}{16384}\left(\frac{p}{q}+\frac{q}{p}\right)-\frac{5355K_3\left(\frac{8pq\pi^2}{g_{\rm YM}^2}\right)}{32768}\left(\frac{p^3}{q^3}+\frac{q^3}{p^3}\right)+\frac{2625K_5\left(\frac{8pq\pi^2}{g_{\rm YM}^2}\right)}{32768}\left(\frac{p^5}{q^5}+\frac{q^5}{p^5}\right)\right]$$
$$+O(N^{-\frac{9}{2}})\Big],$$

[(1/16384)*1533*13.3236*(((96*Pi^2)/(27/256)))*(3/4+4/3)-(1/32768)*5355*13.0152*(((96*Pi^2)/(27/256)))*(27/64+64/27)+(1/32768)*2625*18.9801*(((96*Pi^2)/(27/256)))*(243/1024+1024/243)]+4^(-4.5)

Input interpretation:

$$\left(\frac{1}{16\,384}\times 1533\times 13.3236\times \frac{96\,\pi^2}{\frac{27}{256}}\left(\frac{3}{4}+\frac{4}{3}\right)+\frac{1}{32\,768}\times 5355\times \frac{96\,\pi^2}{\frac{27}{256}}\left(\frac{27}{64}+\frac{64}{27}\right)\times(-13.0152)+\frac{1}{32\,768}\times 2625\times 18.9801\times \frac{96\,\pi^2}{\frac{27}{256}}\left(\frac{243}{1024}+\frac{1024}{243}\right)\right)+\frac{1}{4^{4.5}}$$

Result:

30779.7...

30779.7…

1/((sqrt(27/256)*4^3.5)) 30779.7181920352927

Input interpretation:

$$\frac{1}{\sqrt{\frac{27}{256}}\times 4^{3.5}}\times 30\,779.7181920352927$$

Result:

740.445...

740.445…

Thence, in conclusion:

$$\left\langle \partial_m^2 Z_{\text{inst}}^{p\times q}(m,a_{ij})\right\rangle\Big|_{m=0}=\frac{e^{\frac{8pq\pi^2}{g_{\text{YM}}^2}}}{1+\delta_{p,q}}\Big[-\sqrt{N}\frac{16K_1(\frac{8pq\pi^2}{g_{\text{YM}}^2})}{g_{\text{YM}}}\left(\frac{p}{q}+\frac{q}{p}\right)+\frac{2K_2(\frac{8pq\pi^2}{g_{\text{YM}}^2})}{g_{\text{YM}}\sqrt{N}}\left(\frac{p^2}{q^2}+\frac{q^2}{p^2}\right)$$
$$+\frac{1}{32g_{\text{YM}}N^{\frac{3}{2}}}\left[-13K_1\left(\frac{8pq\pi^2}{g_{\text{YM}}^2}\right)\left(\frac{p}{q}+\frac{q}{p}\right)+9K_3\left(\frac{8pq\pi^2}{g_{\text{YM}}^2}\right)\left(\frac{p^3}{q^3}+\frac{q^3}{p^3}\right)\right]$$
$$+\frac{1}{128g_{\text{YM}}N^{\frac{5}{2}}}\left[-25K_2\left(\frac{8pq\pi^2}{g_{\text{YM}}^2}\right)\left(\frac{p^2}{q^2}-\frac{q^2}{p^2}\right)+15K_4\left(\frac{8pq\pi^2}{g_{\text{YM}}^2}\right)\left(\frac{p^4}{q^4}+\frac{q^4}{p^4}\right)\right]$$
$$+\frac{1}{g_{\text{YM}}N^{\frac{7}{2}}}\left[\frac{1533K_1\left(\frac{8pq\pi^2}{g_{\text{YM}}^2}\right)}{16384}\left(\frac{p}{q}+\frac{q}{p}\right)-\frac{5355K_3\left(\frac{8pq\pi^2}{g_{\text{YM}}^2}\right)}{32768}\left(\frac{p^3}{q^3}+\frac{q^3}{p^3}\right)+\frac{2625K_5\left(\frac{8pq\pi^2}{g_{\text{YM}}^2}\right)}{32768}\left(\frac{p^5}{q^5}+\frac{q^5}{p^5}\right)\right]$$
$$+O(N^{-\frac{9}{2}})\Big],$$

is equal to:

(((exp(((96*Pi^2)/(27/256)))*1/2))) *(((-2.40257e+7 – 3649.33 + 571.313 + 740.445)))

Input interpretation:

$$\left(\exp\left(\frac{96\pi^2}{\frac{27}{256}}\right)\times\frac{1}{2}\right)(-2.40257\times10^7 - 3649.33 + 571.313 + 740.445)$$

Result:

$$-3.78972\ldots\times10^{3908}$$

-3.78972...*10^{3908}

Or:

(((exp(((96*Pi^2)/(27/256)))*1/2))) [-2*16*13.3236*(((96*Pi^2)/(27/256)))*1/(sqrt(27/256))*(3/4+4/3)+2*8.41724*(((96*Pi^2)/(27/256)))* 1/(2sqrt(27/256))*(9/16+16/9)]-4.85467 × 10^3904

Input interpretation:

$$\left(\exp\left(\frac{96\pi^2}{\frac{27}{256}}\right)\times\frac{1}{2}\right)\left(-2\times16\times13.3236\times\frac{96\pi^2}{\frac{27}{256}}\times\frac{1}{\sqrt{\frac{27}{256}}}\left(\frac{3}{4}+\frac{4}{3}\right)+2\times8.41724\times\frac{96\pi^2}{\frac{27}{256}}\times\frac{1}{2\sqrt{\frac{27}{256}}}\left(\frac{9}{16}+\frac{16}{9}\right)\right)-4.85467\times10^{3904}$$

Result:

$$-3.78983\ldots\times10^{3908}$$

-3.78983…*10^{3908}

Series representations:

$$\frac{1}{2}\left(-\frac{2\left(96\pi^2\right)16\left(13.3236\left(\frac{3}{4}+\frac{4}{3}\right)\right)}{\frac{27\sqrt{\frac{27}{256}}}{256}}+\frac{2\left(96\pi^2\right)8.41724\left(\frac{9}{16}+\frac{16}{9}\right)}{\frac{27}{256}\left(2\sqrt{\frac{27}{256}}\right)}\right)\exp\left(\frac{96\pi^2}{\frac{27}{256}}\right)-4.85467\times 10^{3904}=-4.854670000000000\times 10^{3904}-\frac{395\,283.\,\pi^2\exp\left(\frac{8192\pi^2}{9}\right)}{\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{229}{256}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{1}{2}\left(-\frac{2\left(96\pi^2\right)16\left(13.3236\left(\frac{3}{4}+\frac{4}{3}\right)\right)}{\frac{27\sqrt{\frac{27}{256}}}{256}}+\frac{2\left(96\pi^2\right)8.41724\left(\frac{9}{16}+\frac{16}{9}\right)}{\frac{27}{256}\left(2\sqrt{\frac{27}{256}}\right)}\right)\exp\left(\frac{96\pi^2}{\frac{27}{256}}\right)-4.85467\times 10^{3904}=-4.854670000000000\times 10^{3904}+\frac{790\,566.\,\pi^2\exp\left(\frac{8192\pi^2}{9}\right)\sqrt{\pi}}{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\left(-\frac{229}{256}\right)^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}$$

$$\frac{1}{2}\left(-\frac{2\left(96\pi^2\right)16\left(13.3236\left(\frac{3}{4}+\frac{4}{3}\right)\right)}{\frac{27\sqrt{\frac{27}{256}}}{256}}+\frac{2\left(96\pi^2\right)8.41724\left(\frac{9}{16}+\frac{16}{9}\right)}{\frac{27}{256}\left(2\sqrt{\frac{27}{256}}\right)}\right)\exp\left(\frac{96\pi^2}{\frac{27}{256}}\right)-4.85467\times 10^{3904}=-4.854670000000000\times 10^{3904}-\frac{395\,283.\,\pi^2\exp\left(\frac{8192\pi^2}{9}\right)}{\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{27}{256}-z_0\right)^k z_0^{-k}}{k!}}$$

for (not ($z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0$))

From the Ramanujan fundamental formula for obtain a beautiful and highly precise golden ratio:

$$\sqrt[5]{\left(\frac{1}{\frac{1}{32}\left(-1+\sqrt{5}\right)^5+5e^{\left(-\sqrt{5}\pi\right)^5}}-\frac{11\times 5e^{\left(-\sqrt{5}\pi\right)^5}}{2\left(\frac{1}{32}\left(-1+\sqrt{5}\right)^5+5e^{\left(-\sqrt{5}\pi\right)^5}\right)}-\frac{5\sqrt{5}\times 5e^{\left(-\sqrt{5}\pi\right)^5}}{2\left(\frac{1}{32}\left(-1+\sqrt{5}\right)^5+5e^{\left(-\sqrt{5}\pi\right)^5}\right)}\right)}$$

i.e.

$$\sqrt[5]{\frac{1}{\left(\frac{1}{32}\left(-1+\sqrt{5}\right)^5+5\,e^{\left(-\sqrt{5}\,\pi\right)^5}\right)--\frac{1.6382898797095665677239458827012056245798314722584}{10^{7429}}}}$$

from the inverse of the previous expression:

1/[(((exp(((96*Pi^2)/(27/256)))*1/2)))*(((-2.40257e+7 – 3649.33 + 571.313 + 740.445)))]

Input interpretation:

$$\frac{1}{\left(\exp\left(\frac{96\pi^2}{\frac{27}{256}}\right)\times\frac{1}{2}\right)\left(-2.40257\times10^7-3649.33+571.313+740.445\right)}$$

Result:

$-2.6387183924417537293017597956465554801570429713859... \times 10^{-3909}$

-2.63871839244...*10^-3909

inserting this value in the above Ramanujan formula, we obtain:

((((1/(((1/32(-1+sqrt(5))^5+5*(e^((-sqrt(5)*Pi))^5)))+(2.63871839244175372930175979564655548015704297 × 10^-3909))))))^1/5

Input interpretation:

$$\sqrt[5]{\frac{1}{\left(\frac{1}{32}\left(-1+\sqrt{5}\right)^5+5\,e^{\left(-\sqrt{5}\,\pi\right)^5}\right)+\frac{2.63871839244175372930175979564655548015704297}{10^{3909}}}}$$

Result:

1.6180339887498948482045868343656381177203091798057628621354...

1.61803398…..

Now:

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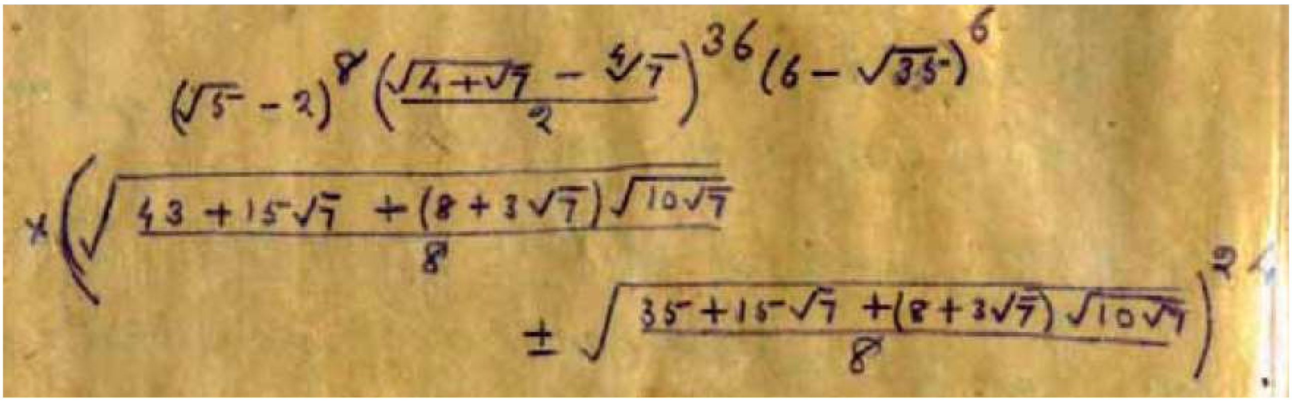


(((sqrt5-2)^8 (1/2((4+sqrt7)^0.5-(7)^(1/4)))^36 (6-sqrt35)^6 ((((sqrt((1/8(43+15sqrt7+(8+3sqrt7)(10sqrt7)^0.5)))+sqrt((1/8*(35+15sqrt7+(8+3sqrt7)(10sqrt7)^0.5)))))))^2))

Input:

$$\left(\sqrt{5}-2\right)^8\left(\frac{1}{2}\left(\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)\right)^{36}\left(6-\sqrt{35}\right)^6\left(\sqrt{\frac{1}{8}\left(43+15\sqrt{7}+\left(8+3\sqrt{7}\right)\sqrt{10\sqrt{7}}\right)}+\sqrt{\frac{1}{8}\left(35+15\sqrt{7}+\left(8+3\sqrt{7}\right)\sqrt{10\sqrt{7}}\right)}\right)^2$$

Exact result:

$$\frac{1}{68\,719\,476\,736}\left(\sqrt{5}-2\right)^8\left(6-\sqrt{35}\right)^6\left(\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)^{36}\left(\frac{1}{2\sqrt{\frac{2}{35+15\sqrt{7}+\sqrt[4]{7}\sqrt{10}\left(8+3\sqrt{7}\right)}}}+\frac{1}{2\sqrt{\frac{2}{43+15\sqrt{7}+\sqrt[4]{7}\sqrt{10}\left(8+3\sqrt{7}\right)}}}\right)^2$$

Decimal approximation:

$6.5408125118250116027393097514034064674753219499497434\ldots\times 10^{-22}$

$6.540812511825\ldots{*}10^{-22}$

From which:

(1/728)^6 / (((sqrt5-2)^8 (1/2((4+sqrt7)^0.5-(7)^(1/4)))^36 (6-sqrt35)^6 ((((sqrt((1/8(43+15sqrt7+(8+3sqrt7)(10sqrt7)^0.5))+sqrt((1/8*(35+15sqrt7+(8+3sqrt 7)(10sqrt7)^0.5)))))))^2))-1024-256+6

Input:

$$\left(\frac{1}{728}\right)^6 / \left(\left(\sqrt{5}-2\right)^8 \left(\frac{1}{2}\left(\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)\right)^{36} \left(6-\sqrt{35}\right)^6 \left(\sqrt{\frac{1}{8}\left(43+15\sqrt{7}+\left(8+3\sqrt{7}\right)\sqrt{10\sqrt{7}}\right)} + \sqrt{\frac{1}{8}\left(35+15\sqrt{7}+\left(8+3\sqrt{7}\right)\sqrt{10\sqrt{7}}\right)}\right)^2\right) - 1024 - 256 + 6$$

Exact result:

$$262\,144 / \left(567\,869\,252\,041 \left(\sqrt{5}-2\right)^8 \left(6-\sqrt{35}\right)^6 \left(\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)^{36} \left(\frac{1}{2\sqrt{\frac{2}{35+15\sqrt{7}+\sqrt[4]{7}\sqrt{10}\left(8+3\sqrt{7}\right)}}} + \frac{1}{2\sqrt{\frac{2}{43+15\sqrt{7}+\sqrt[4]{7}\sqrt{10}\left(8+3\sqrt{7}\right)}}}\right)^2\right) - 1274$$

Decimal approximation:

8996.226545675034203923478298925074748822873905785048171067...

8996.226545675…..

and also:

1/27 1/(((((sqrt5-2)^8 (1/2((4+sqrt7)^0.5-(7)^(1/4)))^36 (6-sqrt35)^6 ((((sqrt((1/8(43+15sqrt7+(8+3sqrt7)(10sqrt7)^0.5)))+sqrt((1/8*(35+15sqrt7+(8+3sqrt7)(10sqrt7)^0.5)))))))^2))))^1/5-4

Input:

$$\frac{1}{27}\times 1/\left(\left(\left(\sqrt{5}-2\right)^8\left(\frac{1}{2}\left(\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)\right)^{36}\left(6-\sqrt{35}\right)^6\left(\sqrt{\frac{1}{8}\left(43+15\sqrt{7}+\left(8+3\sqrt{7}\right)\sqrt{10\sqrt{7}}\right)}+\sqrt{\frac{1}{8}\left(35+15\sqrt{7}+\left(8+3\sqrt{7}\right)\sqrt{10\sqrt{7}}\right)}\right)^2\right)\wedge(1/5)\right)-4$$

Exact result:

$$\left(128\sqrt[5]{2}\right)/\left(27\left(\sqrt{5}-2\right)^{8/5}\left(6-\sqrt{35}\right)^{6/5}\left(\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)^{36/5}\left(\frac{1}{2\sqrt{\frac{2}{35+15\sqrt{7}+\sqrt[4]{7}\sqrt{10}\left(8+3\sqrt{7}\right)}}}+\frac{1}{2\sqrt{\frac{2}{43+15\sqrt{7}+\sqrt[4]{7}\sqrt{10}\left(8+3\sqrt{7}\right)}}}\right)^{2/5}\right)-4$$

Decimal approximation:

635.0132975910101118751145951933779892827810868021425968201...

635.013297591....

We have also:

$$\left(\frac{\sqrt{4+\sqrt{7}}-\sqrt[4]{7}}{2}\right)^{24}\left(\frac{\sqrt{7}-\sqrt{3}}{2}\right)^{12}\left(2-\sqrt{3}\right)^{4}\times\left(\frac{\sqrt{3+\sqrt{7}}-\sqrt[4]{6\sqrt{7}}}{\sqrt{3+\sqrt{7}}+\sqrt[4]{6\sqrt{7}}}\right)^{12}$$

((1/2(4+sqrt7)^0.5-(7)^(1/4)))^24 (1/2(sqrt7-sqrt3))^12 (2-sqrt3)^4 [((sqrt(3+7^0.5))-(6sqrt7)^(1/4))/((sqrt(3+7^0.5))+(6sqrt7)^(1/4))]^12

Input:

$$\left(\frac{1}{2}\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)^{24}\left(\frac{1}{2}\left(\sqrt{7}-\sqrt{3}\right)\right)^{12}\left(2-\sqrt{3}\right)^{4}\left(\frac{\sqrt{3+\sqrt{7}}-\sqrt[4]{6\sqrt{7}}}{\sqrt{3+\sqrt{7}}+\sqrt[4]{6\sqrt{7}}}\right)^{12}$$

Exact result:

$$\frac{(2-\sqrt{3})^4(\sqrt{7}-\sqrt{3})^{12}\left(\sqrt{3+\sqrt{7}}-\sqrt[4]{6}\sqrt[8]{7}\right)^{12}\left(\frac{\sqrt{4+\sqrt{7}}}{2}-\sqrt[4]{7}\right)^{24}}{4096\left(\sqrt[4]{6}\sqrt[8]{7}+\sqrt{3+\sqrt{7}}\right)^{12}}$$

Decimal approximation:

$3.8076936653286636541096070702737285701017658195906599... \times 10^{-31}$

$3.8076936653286....*10^{-31}$

From which:

1/4*(1/1729)^8 1/(((((1/2(4+sqrt7)^0.5-(7)^(1/4)))^24 (1/2(sqrt7-sqrt3))^12 (2-sqrt3)^4 [((sqrt(3+7^0.5))-(6sqrt7)^(1/4))/((sqrt(3+7^0.5))+(6sqrt7)^(1/4))]^12)))+728+47+1/4

Input:

$$\frac{1}{4}\left(\frac{1}{1729}\right)^8\times\frac{1}{\left(\frac{1}{2}\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)^{24}\left(\frac{1}{2}\left(\sqrt{7}-\sqrt{3}\right)\right)^{12}\left(2-\sqrt{3}\right)^{4}\left(\frac{\sqrt{3+\sqrt{7}}-\sqrt[4]{6\sqrt{7}}}{\sqrt{3+\sqrt{7}}+\sqrt[4]{6\sqrt{7}}}\right)^{12}}+$$

$$728+47+\frac{1}{4}$$

Exact result:

$$\frac{3101}{4} + \left(1024\left(\sqrt[4]{6}\,\sqrt[8]{7} + \sqrt{3+\sqrt{7}}\right)^{12}\right) \Big/ \left(79\,865\,634\,479\,415\,290\,771\,535\,361\,(2-\sqrt{3})^4 \left(\sqrt{7}-\sqrt{3}\right)^{12}\left(\sqrt{3+\sqrt{7}} - \sqrt[4]{6}\,\sqrt[8]{7}\right)^{12}\left(\frac{\sqrt{4+\sqrt{7}}}{2} - \sqrt[4]{7}\right)^{24}\right)$$

Decimal approximation:

8996.1253082134191806816765129704444215073542036750287730 45...

8996.125308213….

and also:

1/5(((1/4*(1/1729)^8 1/(((((1/2(4+sqrt7)^0.5-(7)^(1/4)))^24 (1/2(sqrt7-sqrt3))^12 (2-sqrt3)^4 [((sqrt(3+7^0.5))-(6sqrt7)^(1/4))/((sqrt(3+7^0.5))+(6sqrt7)^(1/4))]^12)))+728+47+1/4)))-76+7-2*0.618

Input:

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{1729}\right)^8 \times \frac{1}{\left(\frac{1}{2}\sqrt{4+\sqrt{7}} - \sqrt[4]{7}\right)^{24}\left(\frac{1}{2}\left(\sqrt{7}-\sqrt{3}\right)\right)^{12}(2-\sqrt{3})^4\left(\frac{\sqrt{3+\sqrt{7}} - \sqrt[4]{6\sqrt{7}}}{\sqrt{3+\sqrt{7}} + \sqrt[4]{6\sqrt{7}}}\right)^{12}} + 728 + 47 + \frac{1}{4}\right) - 76 + 7 - 2\times 0.618$$

Result:

1728.98906...

1728.98906… ≈ 1729

From the two expressions, we obtain:

(((sqrt5-2)^8 (1/2((4+sqrt7)^0.5-(7)^(1/4)))^36 (6-sqrt35)^6 ((((sqrt((1/8(43+15sqrt7+(8+3sqrt7)(10sqrt7)^0.5)))+sqrt((1/8*(35+15sqrt7+(8+3sqrt7)(10sqrt7)^0.5)))))))^2)) 1/3.807693665 × 10^-31

Input interpretation:

$$\left(\left(\sqrt{5}-2\right)^8\left(\frac{1}{2}\left(\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)\right)^{36}\left(6-\sqrt{35}\right)^6\left(\sqrt{\frac{1}{8}\left(43+15\sqrt{7}+\left(8+3\sqrt{7}\right)\sqrt{10\sqrt{7}}\right)}+\sqrt{\frac{1}{8}\left(35+15\sqrt{7}+\left(8+3\sqrt{7}\right)\sqrt{10\sqrt{7}}\right)}\right)^2\right)\times\frac{1}{3.807693665\times10^{-31}}$$

Result:

$1.717788532... \times 10^9$

1.717788532...*10^9

From which:

[(((sqrt5-2)^8 (1/2((4+sqrt7)^0.5-(7)^(1/4)))^36 (6-sqrt35)^6 ((((sqrt((1/8(43+15sqrt7+(8+3sqrt7)(10sqrt7)^0.5)))+sqrt((1/8*(35+15sqrt7+(8+3sqrt7)(10sqrt7)^0.5)))))))^2)) 1/3.807693665 × 10^-31]^1/3

Input interpretation:

$$\left(\left(\left(\sqrt{5}-2\right)^8\left(\frac{1}{2}\left(\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)\right)^{36}\left(6-\sqrt{35}\right)^6\left(\sqrt{\frac{1}{8}\left(43+15\sqrt{7}+\left(8+3\sqrt{7}\right)\sqrt{10\sqrt{7}}\right)}+\sqrt{\frac{1}{8}\left(35+15\sqrt{7}+\left(8+3\sqrt{7}\right)\sqrt{10\sqrt{7}}\right)}\right)^2\right)\times\frac{1}{3.807693665\times10^{-31}}\right)\hat{}\,(1/3)$$

Result:

1197.631563...

1197.631563... result practically equal to the rest mass of Sigma baryon 1197.449

and:

[((sqrt5-2)^8 (1/2((4+sqrt7)^0.5-(7)^(1/4)))^36 (6-sqrt35)^6 [(sqrt((1/8(43+15sqrt7+(8+3sqrt7)(10sqrt7)^0.5)))+sqrt((1/8*(35+15sqrt7+(8+3sqrt7)(10sqrt7)^0.5))))]^2) 1/3.8076936e-31]^1/(Pi)-233-2

Input interpretation:

$$\left(\left(\left(\sqrt{5}-2\right)^8\left(\frac{1}{2}\left(\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)\right)^{36}\left(6-\sqrt{35}\right)^6\left(\sqrt{\frac{1}{8}\left(43+15\sqrt{7}+\left(8+3\sqrt{7}\right)\sqrt{10\sqrt{7}}\right)}+\sqrt{\frac{1}{8}\left(35+15\sqrt{7}+\left(8+3\sqrt{7}\right)\sqrt{10\sqrt{7}}\right)}\right)^2\right)\times\frac{1}{3.8076936\times10^{-31}}\right)\hat{}\,(1/\pi)-233-2$$

Result:

635.12602...

635.12602...

and again:

[(((sqrt5-2)^8 (1/2((4+sqrt7)^0.5-(7)^(1/4)))^36 (6-sqrt35)^6 [(sqrt((1/8(43+15sqrt7+(8+3sqrt7)(10sqrt7)^0.5)))+sqrt((1/8(35+15sqrt7+(8+3sqrt7)(10sqrt7)^0.5))))]^2)) 1/3.80769e-31]^1/3+521+11-0.618

Input interpretation:

$$\left(\left(\left(\sqrt{5}-2\right)^8\left(\frac{1}{2}\left(\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)\right)^{36}\left(6-\sqrt{35}\right)^6\left(\sqrt{\frac{1}{8}\left(43+15\sqrt{7}+\left(8+3\sqrt{7}\right)\sqrt{10\sqrt{7}}\right)}+\sqrt{\frac{1}{8}\left(35+15\sqrt{7}+\left(8+3\sqrt{7}\right)\sqrt{10\sqrt{7}}\right)}\right)^2\right)\times\frac{1}{3.80769\times10^{-31}}\right)\hat{}\,(1/3)+521+11-0.618$$

Result:

1729.014...

1729.014...

Appendix

1729 The Ramanujan Taxicab Number - Integral

(https://www.youtube.com/watch?v=eHvAxIqXHV4)

$$I = 2\left[F\left(\frac{1}{1729}\right) - F\left(-\frac{1}{1729}\right)\right]$$

$$= 2\pi \ln\left(\frac{1+\sqrt{1+1/1729}}{1+\sqrt{1-\frac{1}{1729}}}\right) = 2\pi \ln\left(\frac{\sqrt{1729}+\sqrt{1730}}{\sqrt{1729}+\sqrt{1728}}\right)$$

$$\int_0^1 \ln\left(\frac{1729+x}{1729-x}\right)\frac{dx}{\sqrt{x(1-x)}} = 2\pi \ln\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)$$

We have that:

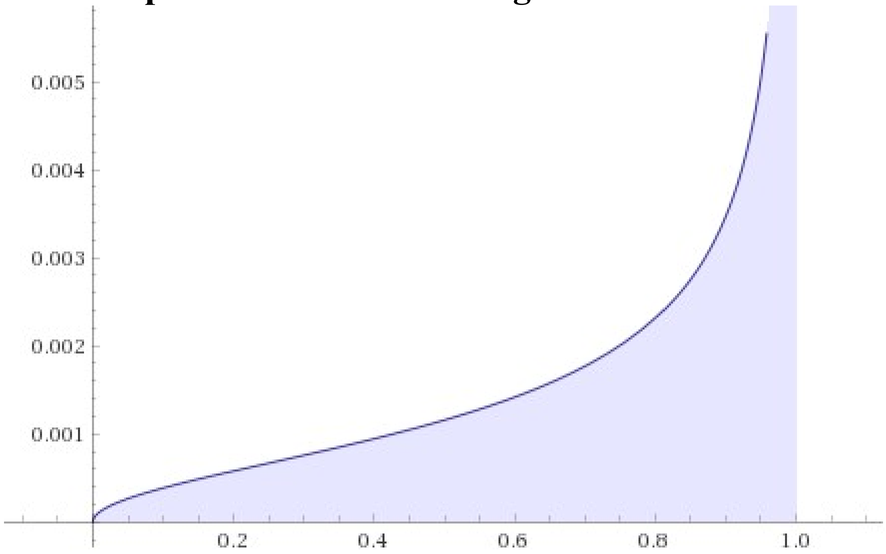
integrate [ln (((1729+x)/(1729-x))) * dx/(sqrt((x(1-x))))] from x=0 to 1

Definite integral:

$$\int_0^1 \frac{\log\left(\frac{1729+x}{1729-x}\right)}{\sqrt{(1-x)\,x}}\,dx = 0.001817$$

$\log(x)$ is the natural logarithm

Visual representation of the integral:



2Pi ln((sqrt1730 + sqrt1729)/(sqrt1729+sqrt1728))

Input:

$$2\pi \log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)$$

log(x) is the natural logarithm

Exact result:

$$2\pi \log\left(\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}\right)$$

Decimal approximation:

0.001816999926273319080951529960002622095007331804637921759...

0.0018169999….

Alternate forms:

$$2\pi\left(\sinh^{-1}\left(\sqrt{1729}\right)-\sinh^{-1}\left(24\sqrt{3}\right)\right)$$

$$2\pi \log\left(1729+\sqrt{2991170}-24\sqrt{3\left(3459+2\sqrt{2991170}\right)}\right)$$

$$-2\pi\left(\log\left(24\sqrt{3}+\sqrt{1729}\right)-\log\left(\sqrt{1729}+\sqrt{1730}\right)\right)$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Alternative representations:

$$2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)=2\pi\log_e\left(\frac{\sqrt{1729}+\sqrt{1730}}{\sqrt{1728}+\sqrt{1729}}\right)$$

$$2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)=2\pi\log(a)\log_a\left(\frac{\sqrt{1729}+\sqrt{1730}}{\sqrt{1728}+\sqrt{1729}}\right)$$

$$2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)=-2\pi\,\text{Li}_1\left(1-\frac{\sqrt{1729}+\sqrt{1730}}{\sqrt{1728}+\sqrt{1729}}\right)$$

Series representations:

$$2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)=-2\pi\sum_{k=1}^{\infty}\frac{\left(\frac{24\sqrt{3}-\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}\right)^k}{k}$$

$$2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)=4i\pi^2\left\lfloor\frac{\arg\left(\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}-x\right)}{2\pi}\right\rfloor+2\pi\log(x)-2\pi\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}-x\right)^k x^{-k}}{k}\quad\text{for } x<0$$

$$2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)=4i\pi^2\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+2\pi\log(z_0)-2\pi\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}-z_0\right)^k z_0^{-k}}{k}$$

Integral representations:

$$2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)=2\pi\int_1^{\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}}\frac{1}{t}\,dt$$

$$2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)=-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\left(-1+\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}\right)^{-s}\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\quad\text{for } -1<\gamma<0$$

We have:

$$\left(\int_0^1 \frac{\log\left(\frac{1729+x}{1729-x}\right)}{\sqrt{(1-x)\,x}}\,dx = 0.001817\right) = \left(2\pi \log\left(\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}\right)\right) =$$

= 0.0018169999926273319

$$\left[2\pi \log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right) = -i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(-1+\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}\right)^{-s} \Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1 < \gamma < 0\right] =$$

= 0.0018169999926273319

We note that:

1+(((2Pi ln((sqrt1730 + sqrt1729)/(sqrt1729+sqrt1728)))))

Input:

$$1 + 2\pi \log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$1 + 2\pi \log\left(\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}\right)$$

Decimal approximation:

1.001816999926273319080951529960002622095007331804637921759...

1.0018169999….result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5}-\varphi}} = 1+\frac{e^{-2\pi}}{1+\frac{e^{-4\pi}}{1+\frac{e^{-6\pi}}{1+\frac{e^{-8\pi}}{1+\ldots}}}} \approx 1.0018674362$$

Alternate forms:

$$1+2\pi\left(\sinh^{-1}\left(\sqrt{1729}\right)-\sinh^{-1}\left(24\sqrt{3}\right)\right)$$

$$1+2\pi\log\left(1729+\sqrt{2991170}-24\sqrt{3\left(3459+2\sqrt{2991170}\right)}\right)$$

$$1-2\pi\left(\log\left(24\sqrt{3}+\sqrt{1729}\right)-\log\left(\sqrt{1729}+\sqrt{1730}\right)\right)$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Alternative representations:

$$1+2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right) = 1+2\pi\log_e\left(\frac{\sqrt{1729}+\sqrt{1730}}{\sqrt{1728}+\sqrt{1729}}\right)$$

$$1+2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right) = 1+2\pi\log(a)\log_a\left(\frac{\sqrt{1729}+\sqrt{1730}}{\sqrt{1728}+\sqrt{1729}}\right)$$

$$1+2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right) = 1-2\pi\,\mathrm{Li}_1\left(1-\frac{\sqrt{1729}+\sqrt{1730}}{\sqrt{1728}+\sqrt{1729}}\right)$$

Series representations:

$$1+2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right) = 1-2\pi\sum_{k=1}^{\infty}\frac{\left(\frac{24\sqrt{3}-\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}\right)^k}{k}$$

$$1 + 2\pi \log\left(\frac{\sqrt{1730} + \sqrt{1729}}{\sqrt{1729} + \sqrt{1728}}\right) = 1 + 4 i \pi^2 \left\lfloor \frac{\arg\left(\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}} - x\right)}{2\pi} \right\rfloor + 2\pi \log(x) - 2\pi \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}} - x\right)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$1 + 2\pi \log\left(\frac{\sqrt{1730} + \sqrt{1729}}{\sqrt{1729} + \sqrt{1728}}\right) = 1 + 4 i \pi^2 \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + 2\pi \log(z_0) - 2\pi \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}} - z_0\right)^k z_0^{-k}}{k}$$

Integral representations:

$$1 + 2\pi \log\left(\frac{\sqrt{1730} + \sqrt{1729}}{\sqrt{1729} + \sqrt{1728}}\right) = 1 + 2\pi \int_1^{\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}} \frac{1}{t}\, dt$$

$$1 + 2\pi \log\left(\frac{\sqrt{1730} + \sqrt{1729}}{\sqrt{1729} + \sqrt{1728}}\right) = 1 - i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(-1 + \frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}\right)^{-s} \Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds \quad \text{for } -1 < \gamma < 0$$

and:

1/(((2Pi ln((sqrt1730 + sqrt1729)/(sqrt1729+sqrt1728)))))

Input:

$$\frac{1}{2\pi \log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)}$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{1}{2\pi \log\left(\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}\right)}$$

Decimal approximation:

550.35775485748518640710430346539594238211020953046539293 60...

550.357754857485……

Alternate forms:

$$\frac{1}{2\pi\sinh^{-1}(\sqrt{1729}) - 2\pi\sinh^{-1}(24\sqrt{3})}$$

$$\frac{1}{2\pi\log\left(1729+\sqrt{2991170} - 24\sqrt{3(3459+2\sqrt{2991170})}\right)}$$

$$-\frac{1}{2\pi\log\left(\frac{24\sqrt{3}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1730}}\right)}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Alternative representations:

$$\frac{1}{2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)} = \frac{1}{2\pi\log_e\left(\frac{\sqrt{1729}+\sqrt{1730}}{\sqrt{1728}+\sqrt{1729}}\right)}$$

$$\frac{1}{2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)} = \frac{1}{2\pi\log(a)\log_a\left(\frac{\sqrt{1729}+\sqrt{1730}}{\sqrt{1728}+\sqrt{1729}}\right)}$$

$$\frac{1}{2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)} = -\frac{1}{2\pi\,\text{Li}_1\left(1-\frac{\sqrt{1729}+\sqrt{1730}}{\sqrt{1728}+\sqrt{1729}}\right)}$$

Series representations:

$$\frac{1}{2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)} = -\frac{1}{2\pi\sum_{k=1}^{\infty}\frac{\left(\frac{24\sqrt{3}-\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}\right)^k}{k}}$$

$$\frac{1}{2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)} =$$

$$1\Big/\left(2\pi\left(2i\pi\left\lfloor\frac{\arg\left(\sqrt{1729}+\sqrt{1730}-\left(24\sqrt{3}+\sqrt{1729}\right)x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}-x\right)^k x^{-k}}{k}\right)\right) \text{ for } x<0$$

$$\frac{1}{2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)} =$$

$$\frac{1}{2\pi\left(2i\pi\left\lfloor\frac{\arg\left(\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}-x\right)^k x^{-k}}{k}\right)} \text{ for } x<0$$

Integral representations:

$$\frac{1}{2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)} = \frac{1}{2\pi\int_1^{\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}}\frac{1}{t}\,dt}$$

$$\frac{1}{2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)} = \frac{i}{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\left(-1+\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}\right)^{-s}\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \text{ for } -1<\gamma<0$$

We note that:

Pi / (((integrate [ln (((1729+x)/(1729-x))) * dx/(sqrt((x(1-x))))] from x=0 to 1)))

Input:

$$\frac{\pi}{\int_0^1 \frac{\log\left(\frac{1729+x}{1729-x}\right)}{\sqrt{x(1-x)}}\,dx}$$

$\log(x)$ is the natural logarithm

Computation result:

$$\frac{\pi}{\int_0^1 \frac{\log\left(\frac{1729+x}{1729-x}\right)}{\sqrt{x(1-x)}}\,dx} = 1729.$$

Decimal approximation:

1728.99987950644780622331593159181985014359444362324490317 8...

1728.99987….≈ 1729

Indeed:

Pi/(((2Pi ln((sqrt1730 + sqrt1729)/(sqrt1729+sqrt1728)))))

Input:

$$\frac{\pi}{2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)}$$

log(x) is the natural logarithm

Exact result:

$$\frac{1}{2\log\left(\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}\right)}$$

Decimal approximation:

1728.999879506447806223320137121028175002953423450641607316...

1728.99987….≈ 1729

Property:

$$\frac{1}{2\log\left(\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}\right)}$$ is a transcendental number

Alternate forms:

$$\frac{1}{2\sinh^{-1}(\sqrt{1729}) - 2\sinh^{-1}(24\sqrt{3})}$$

$$\frac{1}{2\log\left(1729+\sqrt{2991170}-24\sqrt{3\left(3459+2\sqrt{2991170}\right)}\right)}$$

$$-\frac{1}{2\log\left(\frac{24\sqrt{3}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1730}}\right)}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Alternative representations:

$$\frac{\pi}{2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)}=\frac{\pi}{2\pi\log_e\left(\frac{\sqrt{1729}+\sqrt{1730}}{\sqrt{1728}+\sqrt{1729}}\right)}$$

$$\frac{\pi}{2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)}=\frac{\pi}{2\pi\log(a)\log_a\left(\frac{\sqrt{1729}+\sqrt{1730}}{\sqrt{1728}+\sqrt{1729}}\right)}$$

$$\frac{\pi}{2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)}=-\frac{\pi}{2\pi\operatorname{Li}_1\left(1-\frac{\sqrt{1729}+\sqrt{1730}}{\sqrt{1728}+\sqrt{1729}}\right)}$$

Series representations:

$$\frac{\pi}{2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)}=-\frac{1}{2\sum_{k=1}^{\infty}\frac{\left(\frac{24\sqrt{3}-\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}\right)^k}{k}}$$

$$\frac{\pi}{2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)}=$$

$$\frac{1}{2\left(2i\pi\left\lfloor\frac{\arg\left(\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}-x\right)^k x^{-k}}{k}\right)}\quad\text{for } x<0$$

$$\frac{\pi}{2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)}=$$

$$\frac{1}{2\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}-z_0\right)^k z_0^{-k}}{k}\right)}$$

Integral representations:

$$\frac{\pi}{2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)} = \frac{1}{2\int_1^{\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}} \frac{1}{t}\,dt}$$

$$\frac{\pi}{2\pi\log\left(\frac{\sqrt{1730}+\sqrt{1729}}{\sqrt{1729}+\sqrt{1728}}\right)} = \frac{i\pi}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(-1+\frac{\sqrt{1729}+\sqrt{1730}}{24\sqrt{3}+\sqrt{1729}}\right)^{-s}\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \quad \text{for } -1<\gamma<0$$

[Pi / (((integrate [ln (((1729+x)/(1729-x))) * dx/(sqrt((x(1-x))))] from x=0 to 1)))]^1/15

Input:

$$\sqrt[15]{\frac{\pi}{\int_0^1 \frac{\log\left(\frac{1729+x}{1729-x}\right)}{\sqrt{x(1-x)}}\,dx}}$$

log(x) is the natural logarithm

Computation result:

$$\sqrt[15]{\frac{\pi}{\int_0^1 \frac{\log\left(\frac{1729+x}{1729-x}\right)}{\sqrt{x(1-x)}}\,dx}} = 1.64382$$

Decimal approximation:

1.643815221111591346167514667300234923986886898289283232677...

1.643815221……

[Pi / (((integrate [ln (((1729+x)/(1729-x))) * dx/(sqrt((x(1-x))))] from x=0 to 1)))]^1/15 - (21+5)1/10^3

Input:

$$\sqrt[15]{\frac{\pi}{\int_0^1 \frac{\log\left(\frac{1729+x}{1729-x}\right)}{\sqrt{x(1-x)}}\,dx}} - (21+5)\times\frac{1}{10^3}$$

log(x) is the natural logarithm

Computation result:

$$\sqrt[15]{\frac{\pi}{\int_0^1 \frac{\log\left(\frac{1729+x}{1729-x}\right)}{\sqrt{x(1-x)}}\,dx}} - \frac{21+5}{10^3} = 1.61782$$

Decimal approximation:

1.617815221115913461675146673002349239868868982892832326 77...

1.617815221…..

With regard 550.357754857485……, we observe that from the formula of coefficients of the '5th order' mock theta function ψ_1(q): (A053261 OEIS Sequence)

sqrt(golden ratio) * exp(Pi*sqrt(n/15)) / (2*5^(1/4)*sqrt(n)) for n = 141, we obtain:

sqrt(golden ratio) * exp(Pi*sqrt(141/15)) / (2*5^(1/4)*sqrt(141)) + 4

Input:

$$\sqrt{\phi} \times \frac{\exp\left(\pi\sqrt{\frac{141}{15}}\right)}{2\sqrt[4]{5}\sqrt{141}} + 4$$

ϕ is the golden ratio

Exact result:

$$\frac{e^{\sqrt{47/5}\,\pi}\sqrt{\frac{\phi}{141}}}{2\sqrt[4]{5}} + 4$$

Decimal approximation:

550.02239655608437498273740261503472213722841726157819920 41...

550.0223965…..

Property:

$$4+\frac{e^{\sqrt{47/5}\,\pi}\sqrt{\frac{\phi}{141}}}{2\sqrt[4]{5}} \text{ is a transcendental number}$$

Alternate forms:

$$4+\frac{1}{2}\sqrt{\frac{5+\sqrt{5}}{1410}}\;e^{\sqrt{47/5}\,\pi}$$

$$4+\frac{\sqrt{\frac{1}{282}\left(1+\sqrt{5}\right)}\;e^{\sqrt{47/5}\,\pi}}{2\sqrt[4]{5}}$$

$$\frac{11\,280+5^{3/4}\sqrt{282\left(1+\sqrt{5}\right)}\;e^{\sqrt{47/5}\,\pi}}{2820}$$

Series representations:

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{141}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{141}}+4=\left(40\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(141-z_0)^k z_0^{-k}}{k!}+\right.$$
$$\left.5^{3/4}\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{47}{5}-z_0\right)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right)\Big/$$
$$\left(10\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(141-z_0)^k z_0^{-k}}{k!}\right)\quad\text{for (not } (z_0\in\mathbb{R}\text{ and }-\infty<z_0\leq 0))$$

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{141}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{141}}+4=$$
$$\left(40\exp\left(i\pi\left\lfloor\frac{\arg(141-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(141-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+5^{3/4}\exp\left(i\pi\left\lfloor\frac{\arg(\phi-x)}{2\pi}\right\rfloor\right)\right.$$
$$\exp\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{47}{5}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{47}{5}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$
$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k(\phi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Big/$$
$$\left(10\exp\left(i\pi\left\lfloor\frac{\arg(141-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(141-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\quad\text{for } (x\in\mathbb{R}\text{ and }x<0)$$

$$\frac{\sqrt{\phi}\ \exp\left(\pi\sqrt{\frac{141}{15}}\right)}{2\sqrt[4]{5}\ \sqrt{141}}+4=\left(\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(141-z_0)/(2\pi)\rfloor} z_0^{-1/2\lfloor\arg(141-z_0)/(2\pi)\rfloor}\right.$$

$$\left(40\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(141-z_0)/(2\pi)\rfloor} z_0^{1/2\lfloor\arg(141-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(141-z_0)^k z_0^{-k}}{k!}+\right.$$

$$5^{3/4}\exp\left(\pi\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{47}{5}-z_0\right)/(2\pi)\right\rfloor} z_0^{1/2\left(1+\left\lfloor\arg\left(\frac{47}{5}-z_0\right)/(2\pi)\right\rfloor\right)}\right.$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{47}{5}-z_0\right)^k z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor}$$

$$\left.\left.z_0^{1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right)\right)\Big/$$

$$\left(10\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(141-z_0)^k z_0^{-k}}{k!}\right)$$

Furthermore, we have:

2 π log((sqrt(1729) + sqrt(1730))/(24 (x) + sqrt(1729))) = 0.001816999926273

Input interpretation:

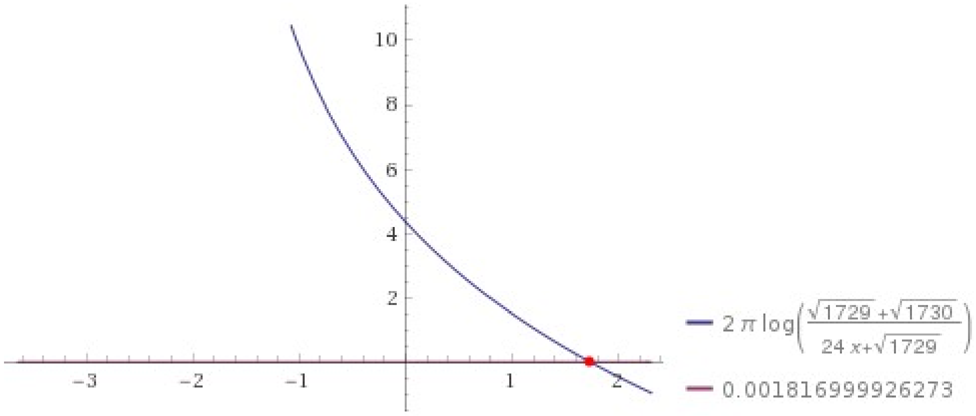
$$2\pi\log\left(\frac{\sqrt{1729}+\sqrt{1730}}{24x+\sqrt{1729}}\right)=0.001816999926273$$

log(x) is the natural logarithm

Result:

$$2\pi\log\left(\frac{\sqrt{1729}+\sqrt{1730}}{24x+\sqrt{1729}}\right)=0.001816999926273$$

Plot:



Alternate form assuming x is real:

$$1.000000000000 \log\left(\frac{1}{24 x + \sqrt{1729}}\right) + 4.42065179978 = 0$$

Alternate forms:

$$2 \pi \left(\log\left(\frac{1}{24 x + \sqrt{1729}}\right) + \sinh^{-1}\left(\sqrt{1729}\right)\right) = 0.001816999926273$$

$$2 \pi \left(\log\left(\frac{1}{24 x + \sqrt{1729}}\right) + \log\left(\sqrt{1729} + \sqrt{1730}\right)\right) = 0.001816999926273$$

$$2 \pi \log\left(\frac{\sqrt{3459 + 2\sqrt{2991170}}}{24 x + \sqrt{1729}}\right) = 0.001816999926273$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Alternate form assuming x is positive:

$$1.000000000000 \log\left(24 x + \sqrt{1729}\right) = 4.42065179978$$

Alternate forms assuming x>0:

$$2 \pi \left(\log\left(\sqrt{1729} + \sqrt{1730}\right) - \log\left(24 x + \sqrt{1729}\right)\right) = 0.001816999926273$$

$$2 \pi \log\left(\sqrt{1729} + \sqrt{1730}\right) - 2 \pi \log\left(24 x + \sqrt{1729}\right) = 0.001816999926273$$

Solution:

$x \approx 1.7320508076$

1.7320508076 = √3

2 π log((sqrt(x) + sqrt(1730))/(24 sqrt(3) + sqrt(x))) = 0.001816999926273

Input interpretation:

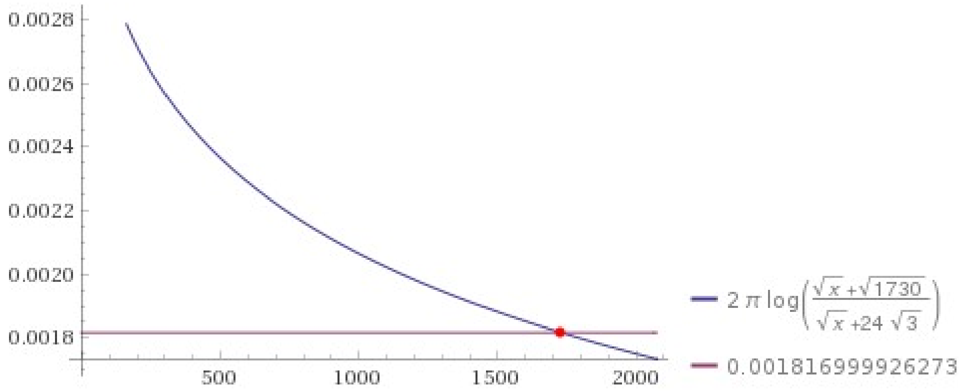
$$2\pi \log\left(\frac{\sqrt{x}+\sqrt{1730}}{24\sqrt{3}+\sqrt{x}}\right) = 0.001816999926273$$

log(x) is the natural logarithm

Result:

$$2\pi \log\left(\frac{\sqrt{x}+\sqrt{1730}}{\sqrt{x}+24\sqrt{3}}\right) = 0.001816999926273$$

Plot:



Alternate form assuming x is real:

$$1.000000000000 \log\left(\frac{\sqrt{x}+\sqrt{1730}}{\sqrt{x}+24\sqrt{3}}\right) = 0.000289184519864$$

Alternate forms assuming x>0:

$$2\pi\left(\log\left(\sqrt{x}+\sqrt{1730}\right)-\log\left(\sqrt{x}+24\sqrt{3}\right)\right) = 0.001816999926273$$

$$2\pi\log\left(\sqrt{x}+\sqrt{1730}\right)-2\pi\log\left(\sqrt{x}+24\sqrt{3}\right) = 0.001816999926273$$

Solution:

$x \approx 1729.0000000012227$

Integer solution:

$x = 1729$

1729

2 π log((sqrt(x^15) + sqrt(1730))/(24 sqrt(3) + sqrt(x^15))) = 0.001816999926273

Input interpretation:

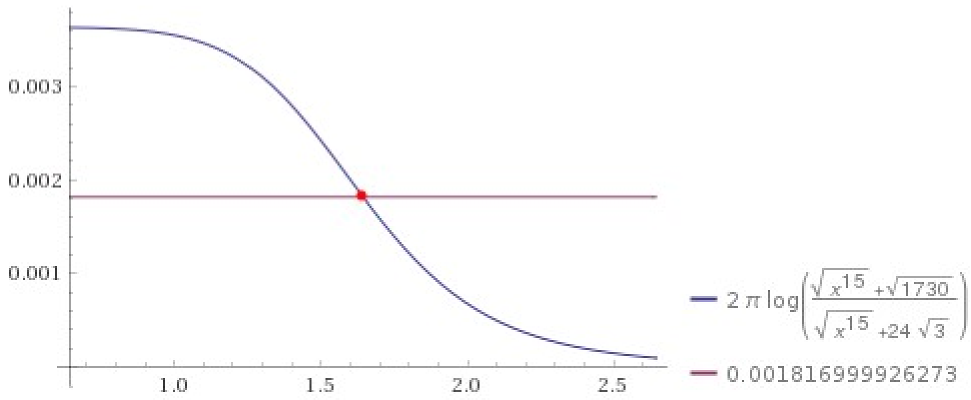
$$2\pi \log\left(\frac{\sqrt{x^{15}}+\sqrt{1730}}{24\sqrt{3}+\sqrt{x^{15}}}\right)=0.001816999926273$$

log(x) is the natural logarithm

Result:

$$2\pi \log\left(\frac{\sqrt{x^{15}}+\sqrt{1730}}{\sqrt{x^{15}}+24\sqrt{3}}\right)=0.001816999926273$$

Plot:



Alternate form assuming x is real:

$$1.000000000000 \log\left(\frac{\sqrt{x^{15}}+\sqrt{1730}}{\sqrt{x^{15}}+24\sqrt{3}}\right)=0.000289184519864$$

Alternate forms assuming x>0:

$$2\pi\left(\log\left(x^{15/2}+\sqrt{1730}\right)-\log\left(x^{15/2}+24\sqrt{3}\right)\right)=0.001816999926273$$

$$2\pi\log\left(x^{15/2}+\sqrt{1730}\right)-2\pi\log\left(x^{15/2}+24\sqrt{3}\right)=0.001816999926273$$

Solutions:

$x = -1.607893922050417 - 0.341768403601163\, i$

$x = -1.607893922050417 + 0.341768403601163\, i$

$x = -1.329874455670192 - 0.966210348952374\, i$

$x = -1.329874455670192 + 0.966210348952374\, i$

$x = -0.821907614374444 - 1.423585747224265\, i$

$x = -0.821907614374444 + 1.423585747224265\, i$

$x = -0.171825479757088 - 1.634810236930576\, i$

$x = -0.171825479757088 + 1.634810236930576\, i$

$x = 0.507966841295748 - 1.563361184886838\, i$

$x = 0.507966841295748 + 1.563361184886838\, i$

$x = 1.099927080754669 - 1.221592781285675\, i$

$x = 1.099927080754669 + 1.221592781285675\, i$

$x = 1.501699935427279 - 0.668599887978201\, i$

$x = 1.501699935427279 + 0.668599887978201\, i$

$x = 1.643815228748888$

x = 1.643815228748888

2 π log((sqrt((11+sqrt2)x+1) + sqrt(1730))/(24 sqrt(3) + sqrt((11+sqrt2)x+1))) = 0.001816999926273

Input interpretation:

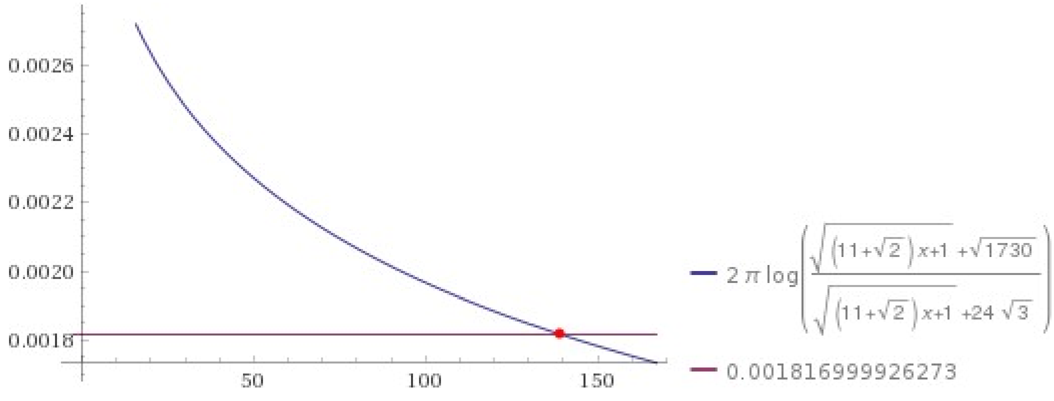
$$2\pi \log\left(\frac{\sqrt{(11+\sqrt{2})x+1}+\sqrt{1730}}{24\sqrt{3}+\sqrt{(11+\sqrt{2})x+1}}\right) = 0.001816999926273$$

log(x) is the natural logarithm

Result:

$$2\pi \log\left(\frac{\sqrt{(11+\sqrt{2})x+1}+\sqrt{1730}}{\sqrt{(11+\sqrt{2})x+1}+24\sqrt{3}}\right) = 0.001816999926273$$

Plot:



Alternate form assuming x>0:

$$2\pi\log\left(\sqrt{\left(11+\sqrt{2}\right)x+1}+\sqrt{1730}\right)-2\pi\log\left(\sqrt{\left(11+\sqrt{2}\right)x+1}+24\sqrt{3}\right)=0.001816999926273$$

Solution:

$x \approx 139.19528541370599$

139.19528541370599

2 π log((sqrt((12+sqrt3)x+1) + sqrt(1730))/(24 sqrt(3) + sqrt((12+sqrt3)x+1))) = 0.001816999926273

Input interpretation:

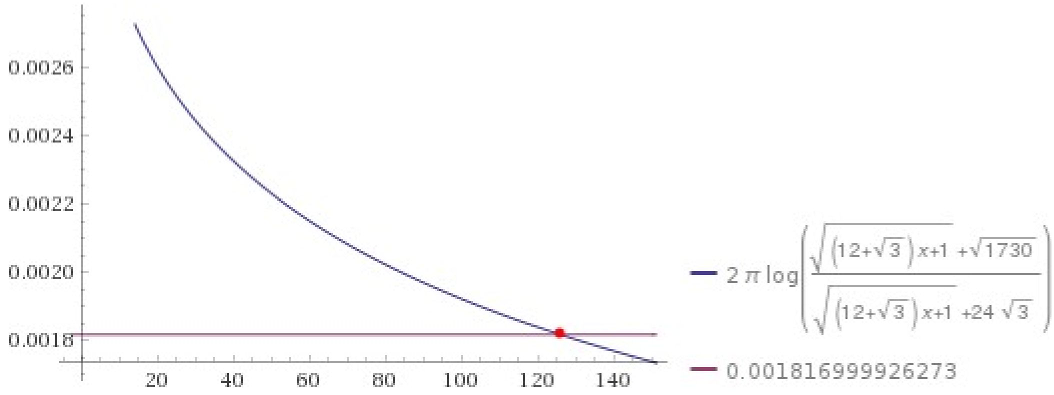
$$2\pi\log\left(\frac{\sqrt{\left(12+\sqrt{3}\right)x+1}+\sqrt{1730}}{24\sqrt{3}+\sqrt{\left(12+\sqrt{3}\right)x+1}}\right)=0.001816999926273$$

log(x) is the natural logarithm

Result:

$$2\pi\log\left(\frac{\sqrt{\left(12+\sqrt{3}\right)x+1}+\sqrt{1730}}{\sqrt{\left(12+\sqrt{3}\right)x+1}+24\sqrt{3}}\right)=0.001816999926273$$

Plot:



Alternate form assuming x>0:

$$2\pi\log\left(\sqrt{\left(12+\sqrt{3}\right)x+1}+\sqrt{1730}\right)-2\pi\log\left(\sqrt{\left(12+\sqrt{3}\right)x+1}+24\sqrt{3}\right)=$$
$$0.001816999926273$$

Solution:

$x \approx 125.83699435839386$

125.83699435839386

Observations

Figs.

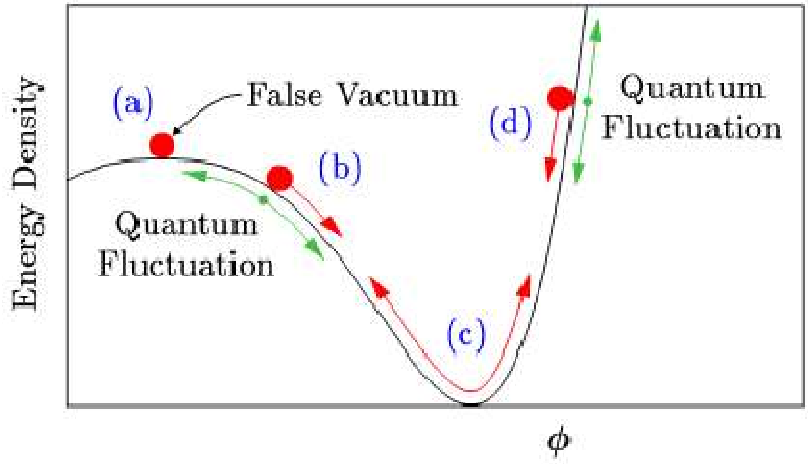
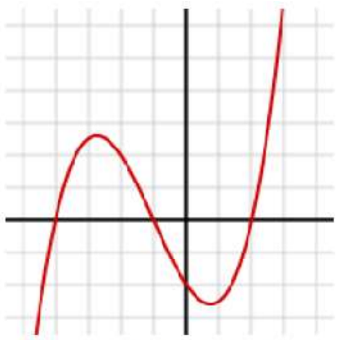


FIG. 1: In simple inflationary models, the universe at early times is dominated by the potential energy density of a scalar field, ϕ. Red arrows show the classical motion of ϕ. When ϕ is near region (a), the energy density will remain nearly constant, $\rho \cong \rho_f$, even as the universe expands. Moreover, cosmic expansion acts like a frictional drag, slowing the motion of ϕ: Even near regions (b) and (d), ϕ behaves more like a marble moving in a bowl of molasses, slowly creeping down the side of its potential, rather than like a marble sliding down the inside of a polished bowl. During this period of "slow roll," ρ remains nearly constant. Only after ϕ has slid most of the way down its potential will it begin to oscillate around its minimum, as in region (c), ending inflation.



Graph of a cubic function with 3 real roots (where the curve crosses the horizontal axis at $y = 0$). The case shown has two critical points. Here the function is $f(x) = (x^3 + 3x^2 - 6x - 8)/4$.

The ratio between M_0 and q

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3})\, M_s}{2}.$$

i.e. the gravitating mass M_0 and the Wheelerian mass q of the wormhole, is equal to:

$$\frac{\sqrt{3\,(2.17049 \times 10^{37})^2 - 0.001^2}}{\frac{1}{2}\,((3\sqrt{3})\,(4.2 \times 10^6 \times 1.9891 \times 10^{30}))}$$

1.7320507879051944207039476256710181600835665488020824605 20...

1.7320507879

1.7320507879 ≈ $\sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q of the wormhole

We note that:

$$\left(-\frac{1}{2} + \frac{i}{2}\sqrt{3}\right) - \left(-\frac{1}{2} - \frac{i}{2}\sqrt{3}\right)$$

i is the imaginary unit

$$i\sqrt{3}$$

1.7320508075688772935274463415058723669428052538103806280 55... i

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

1.73205

This result is very near to the ratio between M_0 and q, that is equal to 1.7320507879 ≈ $\sqrt{3}$

With regard $\sqrt{3}$, we note that is a fundamental value of the formula structure that we need to calculate a Cubic Equation

We have that the previous result

$$\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right) = i\sqrt{3} =$$

$$= 1.7320508075688772935274463415058723669428052538103806280558... i$$

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

can be related with:

$$u^2(-u)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)+v^2(-v)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)=q$$

Considering:

$$(-1)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)-(-1)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=q$$

$$= i\sqrt{3} = 1.7320508075688772935274463415058723669428052538103806280558... i$$

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

Thence:

$$\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right) \Rightarrow$$

$$\Rightarrow (-1)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)-(-1)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=q=1.73205\approx\sqrt{3}$$

We observe how the graph above, concerning the cubic function, is very similar to the graph that represent the scalar field (in red). It is possible to hypothesize that cubic functions and cubic equations, with their roots, are connected to the scalar field.

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden

ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] *The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.*

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

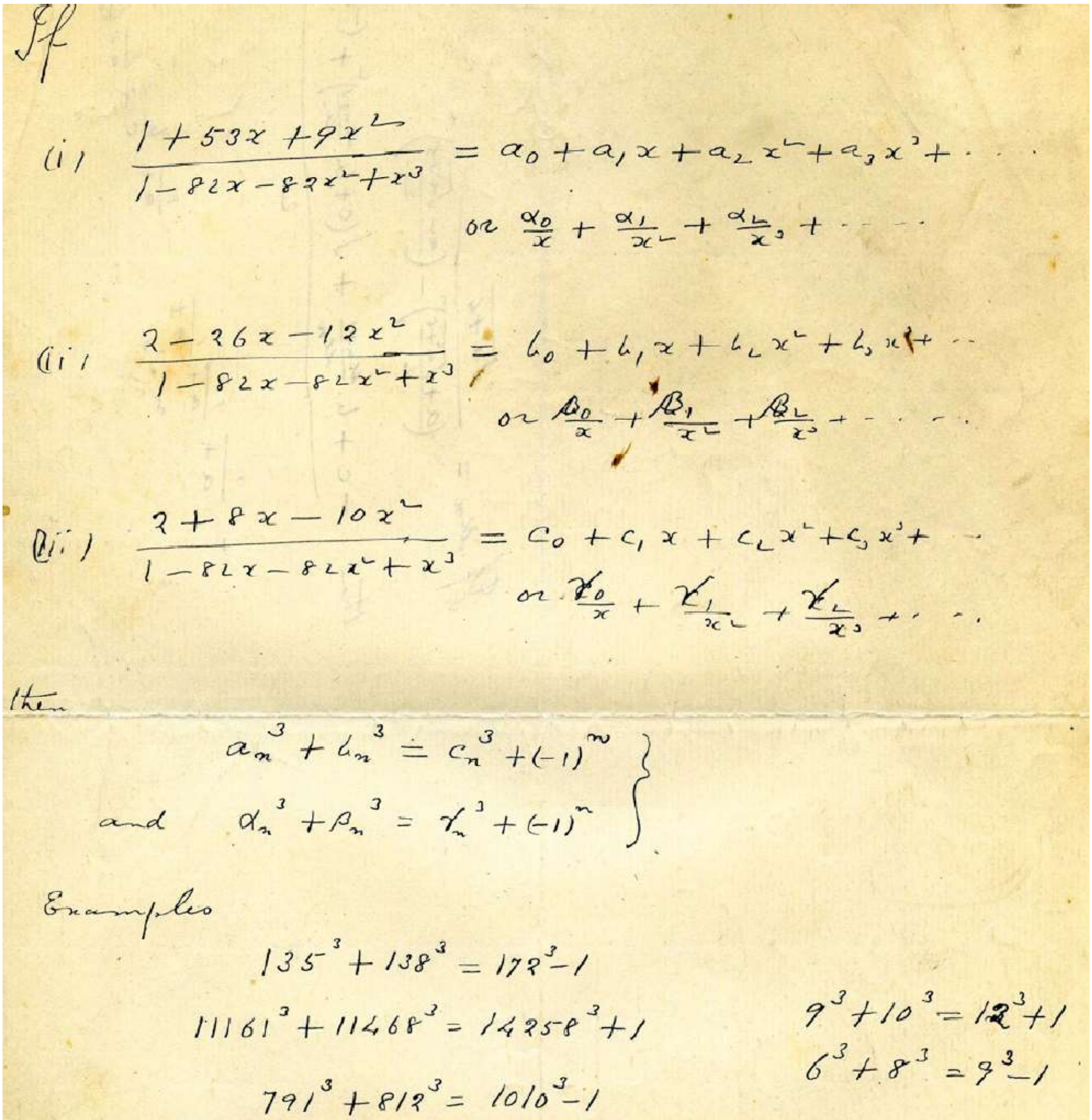
In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

https://plus.maths.org/content/ramanujan



If

(i) $$\frac{1+53x+9x^2}{1-82x-82x^2+x^3} = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

$$\text{or } \frac{\alpha_0}{x} + \frac{\alpha_1}{x^2} + \frac{\alpha_2}{x^3} + \cdots$$

(ii) $$\frac{2-26x-12x^2}{1-82x-82x^2+x^3} = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + \cdots$$

$$\text{or } \frac{\beta_0}{x} + \frac{\beta_1}{x^2} + \frac{\beta_2}{x^3} + \cdots$$

(iii) $$\frac{2+8x-10x^2}{1-82x-82x^2+x^3} = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$$

$$\text{or } \frac{\gamma_0}{x} + \frac{\gamma_1}{x^2} + \frac{\gamma_2}{x^3} + \cdots$$

then

$$a_n^3 + b_n^3 = c_n^3 + (-1)^n$$

and

$$\alpha_n^3 + \beta_n^3 = \gamma_n^3 + (-1)^n$$

Examples

$$135^3 + 138^3 = 172^3 - 1$$

$$11161^3 + 11468^3 = 14258^3 + 1$$

$$791^3 + 812^3 = 1010^3 - 1$$

$$9^3 + 10^3 = 12^3 + 1$$

$$6^3 + 8^3 = 9^3 - 1$$

Ramanujan's manuscript. The representations of 1729 as the sum of two cubes appear in the bottom right corner. The equation expressing the near counter examples to Fermat's last theorem appears further up: $\alpha^3 + \beta^3 = \gamma^3 + (-1)^n$. Image courtesy Trinity College library.

References

[1] **Highly Effective Actions**
John H. Schwarz - arXiv:1311.0305v2 [hep-th] 22 Nov 2013

[2] "**Modular equations and approximations to π**" – *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

[3] **Modular Invariance in Superstring Theory From N = 4 Super-Yang Mills**
Shai M. Chester, Michael B. Green, Silviu S. Pufu, Yifan Wang and Congkao Wen - arXiv:1912.13365v1 [hep-th] 31 Dec 2019

[4] **Manuscript Book 1 of Srinivasa Ramanujan**