On various Ramanujan's elliptic integrals and Wormholes equations: further mathematical connections with ϕ, $\zeta(2)$, and some parameters of High Energy Physics. V

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Abstract

In this paper we have described several Ramanujan's elliptic integrals and Wormholes formulas. Furthermore, we describe new possible mathematical connections with ϕ, $\zeta(2)$, and some parameters of High Energy Physics

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https://www.cse.iitk.ac.in/users/amit/books/hardy-1999-ramanujan-twelve-lectures.html

From

George E. Andrews Bruce C. Berndt
Ramanujan's Lost Notebook Part I - 2005 Springer Science+Business Media, Inc.

Now, we have that:

Lemma 15.8.2. *Let*

$$P = \frac{1}{q}\left(\frac{f(-q)}{f(-q^7)}\right)^4 \qquad \text{and} \qquad Q = \frac{1}{q^2}\left(\frac{f(-q^2)}{f(-q^{14})}\right)^4. \tag{15.8.4}$$

Then

$$P + \frac{49}{P} = R - 1 + \frac{48}{R} + \frac{64}{R^2} \tag{15.8.5}$$

and

$$Q + \frac{49}{Q} = R^2 + 6R - 1 + \frac{8}{R}. \tag{15.8.6}$$

For P = 7 and R = 8, from (15.8.5) we obtain:

7+49/7 = 8-1+48/8+64/8^2

Input:

$$7 + \frac{49}{7} = 8 - 1 + \frac{48}{8} + \frac{64}{8^2}$$

Result:

True

Left hand side:

$$7 + \frac{49}{7} = 14$$

Right hand side:

$$8 - 1 + \frac{48}{8} + \frac{64}{8^2} = 14$$

14

For Q = 7 and R = 1, from (15.8.6), we obtain:

7+49/7 = 1+6-1+8

Input:

$$7 + \frac{49}{7} = 1 + 6 - 1 + 8$$

Result:

True

Left hand side:

$$7 + \frac{49}{7} = 14$$

Right hand side:

$$1 + 6 - 1 + 8 = 14$$

14

From

$$\begin{aligned}\sqrt{PQ} + \frac{49}{\sqrt{PQ}} &= v^{3/2} - 8v^{1/2} - 8v^{-1/2} + v^{-3/2} \\ &= \left(\frac{1}{\sqrt{v}} + \sqrt{v}\right)^3 - 11\left(\frac{1}{\sqrt{v}} + \sqrt{v}\right) \\ &= K(K^2 - 11), \qquad (15.8.7)\end{aligned}$$

sqrt49+(49/sqrt49) = x(x^2-11)

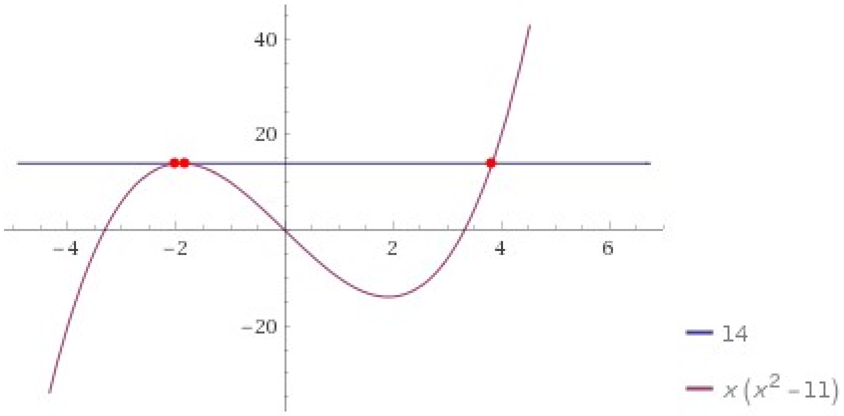
Input:

$$\sqrt{49} + \frac{49}{\sqrt{49}} = x(x^2 - 11)$$

Result:

$$14 = x(x^2 - 11)$$

Plot:



Alternate forms:

$-x^3 + 11x + 14 = 0$

$14 - x(x^2 - 11) = 0$

Expanded form:

$14 = x^3 - 11x$

Solutions:

$x = -2$

$x = 1 - 2\sqrt{2}$

$x = 1 + 2\sqrt{2}$

From

$$K(K^2 - 11)$$

for K = - 2 , we obtain:

-2((-2)^2-11)

Input:

$-2((-2)^2 - 11)$

Result:

14

14

or:

(1+2sqrt2)*((1+2sqrt2)^2-11)

Input:

$$\left(1+2\sqrt{2}\right)\left(\left(1+2\sqrt{2}\right)^2-11\right)$$

Result:

14

14

Now, from

$$K=\frac{1}{\sqrt{v}}+\sqrt{v}.$$

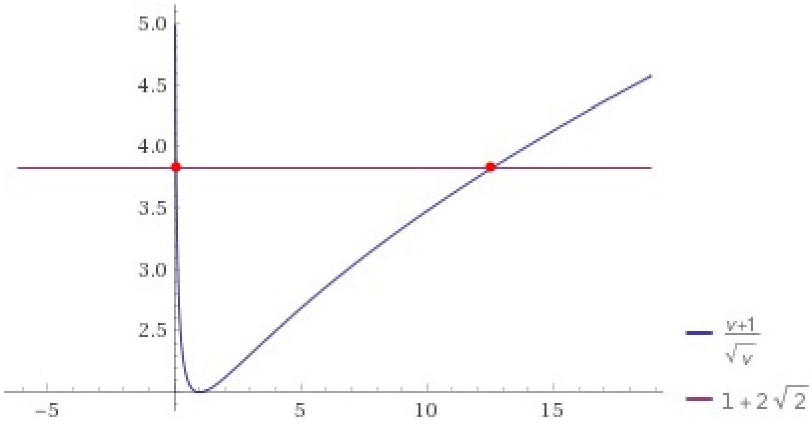
For K = 1+2√2 , we have that:

1/sqrt(v) + sqrt(v) = 1+2sqrt2

Input:

$$\frac{1}{\sqrt{v}}+\sqrt{v}=1+2\sqrt{2}$$

Plot:



Alternate form:

$$\frac{v+1}{\sqrt{v}}=1+2\sqrt{2}$$

Alternate form assuming v is positive:

$$\nu + 1 = \left(1 + 2\sqrt{2}\right)\sqrt{\nu}$$

Solutions:

$$\nu = \frac{1}{2}\left(7 + 4\sqrt{2} - \sqrt{77 + 56\sqrt{2}}\right)$$

$$\nu = \frac{1}{2}\left(7 + 4\sqrt{2} + \sqrt{77 + 56\sqrt{2}}\right)$$

Solutions:

$\nu \approx 0.079508$

$\nu \approx 12.577$

Now:

$$\left(R + \frac{8}{R} + 9\right)\left(R + \frac{8}{R} - 2\right)$$

(R+8/R+9)(R+8/R-2) = 28

Input:

$$\left(R + \frac{8}{R} + 9\right)\left(R + \frac{8}{R} - 2\right) = 28$$

Alternate form assuming R is real:

$$(R-3)\,R\,(R+10) + \frac{64}{R} + 56 = 0$$

Alternate forms:

$R^4 + 7R^3 - 30R^2 + 56R = -64$ (for $R \neq 0$)

$$\frac{(R+1)(R+8)((R-2)R+8)}{R^2} = 28$$

$$\frac{(R+1)(R+8)\left(R^2 - 2R + 8\right)}{R^2} = 28$$

Expanded form:

$$R^2 + \frac{64}{R^2} + 7R + \frac{56}{R} - 2 = 28$$

Real solutions:

$$R = \frac{1}{4}\left(-7 - \sqrt{233} - \sqrt{14\left(11 + \sqrt{233}\right)}\right)$$

$$R = \frac{1}{4}\left(-7 - \sqrt{233} + \sqrt{14\left(11 + \sqrt{233}\right)}\right)$$

Complex solutions:

$$R = \frac{1}{4}\left(-7 + \sqrt{233} - i\sqrt{14\left(\sqrt{233} - 11\right)}\right)$$

$$R = \frac{1}{4}\left(-7 + \sqrt{233} + i\sqrt{14\left(\sqrt{233} - 11\right)}\right)$$

$$\frac{(R+8)(R+1)(R^2-2R+8)}{R^2}$$

(((R+8)(R+1)(R^2-2R+8)))/R^2

Input:

$$\frac{(R+8)(R+1)\left(R^2 - 2R + 8\right)}{R^2} = 28$$

Alternate form assuming R is real:

$$(R-3)R(R+10) + \frac{64}{R} + 56 = 0$$

Alternate forms:

$$R^4 + 7R^3 - 30R^2 + 56R = -64 \quad (\text{for } R \neq 0)$$

$$\frac{(R+1)(R+8)((R-2)R+8)}{R^2} = 28$$

Expanded form:

$$R^2 + \frac{64}{R^2} + 7R + \frac{56}{R} - 2 = 28$$

Real solutions:

$$R = \frac{1}{4}\left(-7 - \sqrt{233} - \sqrt{14\left(11 + \sqrt{233}\right)}\right)$$

$$R = \frac{1}{4}\left(-7-\sqrt{233}+\sqrt{14\left(11+\sqrt{233}\right)}\right)$$

Complex solutions:

$$R = \frac{1}{4}\left(-7+\sqrt{233}-i\sqrt{14\left(\sqrt{233}-11\right)}\right)$$

$$R = \frac{1}{4}\left(-7+\sqrt{233}+i\sqrt{14\left(\sqrt{233}-11\right)}\right)$$

Thence, we have:

$$R = \frac{1}{4}\left(-7-\sqrt{233}+\sqrt{14\left(11+\sqrt{233}\right)}\right)$$

$R \approx -0.77220$

-0.77220

For the two expressions, we have:

$v \approx 0.079508$

$v \approx 12.577$

$$v = \frac{1}{2}\left(7+4\sqrt{2}+\sqrt{77+56\sqrt{2}}\right)$$

$v \approx 12.577$

12.577

1/2(7+4sqrt2+(77+56sqrt2)^0.5)

and:

$$R = \frac{1}{4}\left(-7 - \sqrt{233} + \sqrt{14\left(11 + \sqrt{233}\right)}\right)$$

$$R \approx -0.77220$$

-0.77220

1/4(-7-sqrt233+(14(11+sqrt233))^0.5)

Input:

$$\frac{1}{4}\left(-7 - \sqrt{233} + \sqrt{14\left(11 + \sqrt{233}\right)}\right)$$

Decimal approximation:

-0.7722033503711771549323793926432431178228285365363789 2878...

-0.7722033503711…

Minimal polynomial:

$$x^4 + 7x^3 - 30x^2 + 56x + 64$$

Alternate forms:

$$-\frac{7}{4} - \frac{\sqrt{233}}{4} + \frac{1}{2}\sqrt{\frac{7}{2}\left(11 + \sqrt{233}\right)}$$

root of $x^4 + 7x^3 - 30x^2 + 56x + 64$ near $x = -0.772203$

$$\frac{\sqrt{154 - 56\, i\sqrt{7}} + \sqrt{2}\left(-7 - \sqrt{233} + \sqrt{7\, i\,(4\sqrt{7} + -11\, i)}\right)}{4\sqrt{2}}$$

1/2(7+4sqrt2+(77+56sqrt2)^0.5)

Input:

$$\frac{1}{2}\left(7+4\sqrt{2}+\sqrt{77+56\sqrt{2}}\right)$$

Decimal approximation:

12.577346221135847364104800669405710230945933091008925384 74…

12.577346221135…

Minimal polynomial:

$$x^4-14x^3+19x^2-14x+1$$

Alternate forms:

$$\frac{7}{2}+2\sqrt{2}+\frac{1}{2}\sqrt{77+56\sqrt{2}}$$

$$\frac{1}{2}\left(7+4\sqrt{2}+\sqrt{7\left(11+8\sqrt{2}\right)}\right)$$

$$\frac{7}{2}+2\sqrt{2}+\frac{1}{2}\sqrt{7\left(11+8\sqrt{2}\right)}$$

We have that:

Lemma 15.8.4. *If v is defined by* (15.8.1), *then*

$$\frac{dv}{dq} = f(-q)f(-q^2)f(-q^7)f(-q^{14})\sqrt{1-14v+19v^2-14v^3+v^4}.$$

Proof. From the definition (15.8.1) of v, Lemma 15.8.3, (15.8.4), (15.8.2), and Lemma 15.8.2,

$$\begin{aligned}\frac{q}{v}\frac{dv}{dq} &= q\frac{d\log v}{dq} = q\frac{d\log\left\{q^2\frac{f^4(-q^{14})}{f^4(-q^2)}\right\}}{dq} + q\frac{d\log\left\{q^{-1}\frac{f^4(-q)}{f^4(-q^7)}\right\}}{dq}\\ &= q\left\{\frac{2}{q} + 8\sum_{n=1}^{\infty}\frac{nq^{2n-1}}{1-q^{2n}} - 56\sum_{n=1}^{\infty}\frac{nq^{14n-1}}{1-q^{14n}}\right\}\\ &\quad + q\left\{-\frac{1}{q} + 28\sum_{n=1}^{\infty}\frac{nq^{7n-1}}{1-q^{7n}} - 4\sum_{n=1}^{\infty}\frac{nq^{n-1}}{1-q^{n}}\right\}\end{aligned}$$

$$= 2\left(\frac{f^8(-q^2)+13q^2f^4(-q^2)f^4(-q^{14})+49q^4f^8(-q^{14})}{f(-q^2)f(-q^{14})}\right)^{2/3}$$
$$-\left(\frac{f^8(-q)+13qf^4(-q)f^4(-q^7)+49q^2f^8(-q^7)}{f(-q)f(-q^7)}\right)^{2/3}$$
$$= qf(-q)f(-q^2)f(-q^7)f(-q^{14})\left\{2q^{1/3}\frac{f(-q^2)f(-q^{14})}{f(-q)f(-q^7)}\right.$$
$$\left.\times\left(Q+\frac{49}{Q}+13\right)^{2/3} - q^{-1/3}\frac{f(-q)f(-q^7)}{f(-q^2)f(-q^{14})}\left(P+\frac{49}{P}+13\right)^{2/3}\right\}$$
$$= qf(-q)f(-q^2)f(-q^7)f(-q^{14})\left\{\frac{2}{R^{1/3}}\left(R^2+6R+12+\frac{8}{R}\right)^{2/3}\right.$$
$$\left.-R^{1/3}\left(R+12+\frac{48}{R}+\frac{64}{R^2}\right)^{2/3}\right\}$$
$$= qf(-q)f(-q^2)f(-q^7)f(-q^{14})\left\{\frac{2}{R^{1/3}}\left(\frac{(R+2)^6}{R^2}\right)^{1/3}\right.$$
$$\left.-R^{1/3}\left(\frac{(R+4)^6}{R^4}\right)^{1/3}\right\}$$
$$= qf(-q)f(-q^2)f(-q^7)f(-q^{14})\left(R-\frac{8}{R}\right). \qquad (15.8.12)$$

Now, by using Lemma 15.8.1, we can easily verify that

$$\left(R-\frac{8}{R}\right)^2 = \left(R+\frac{8}{R}\right)^2 + 14\left(R+\frac{8}{R}\right) + 49 - 14\left(R+7+\frac{8}{R}\right) + 17$$
$$= \left(R+7+\frac{8}{R}\right)^2 - 14\left(R+7+\frac{8}{R}\right) + 17$$
$$= \left(v+\frac{1}{v}\right)^2 - 14\left(v+\frac{1}{v}\right) + 17$$
$$= \frac{1-14v+19v^2-14v^3+v^4}{v^2}. \qquad (15.8.13)$$

Taking the square roots of both sides of (15.8.13) and substituting in (15.8.12), we complete the proof. □

Now, we want to analyze the following expression:

$$\begin{aligned}\left(R-\frac{8}{R}\right)^2 &= \left(R+\frac{8}{R}\right)^2+14\left(R+\frac{8}{R}\right)+49-14\left(R+7+\frac{8}{R}\right)+17\\ &= \left(R+7+\frac{8}{R}\right)^2-14\left(R+7+\frac{8}{R}\right)+17\\ &= \left(v+\frac{1}{v}\right)^2-14\left(v+\frac{1}{v}\right)+17\\ &= \frac{1-14v+19v^2-14v^3+v^4}{v^2}. \end{aligned} \tag{15.8.13}$$

From:

$$\left(R+7+\frac{8}{R}\right)^2-14\left(R+7+\frac{8}{R}\right)+17$$

For R = 1/4(-7-sqrt233+(14(11+sqrt233))^0.5)

we obtain:

[(((1/4(-7-sqrt233+(14(11+sqrt233))^0.5))))+7+8/(((1/4(-7-sqrt233+(14(11+sqrt233))^0.5))))]^2-14[(((1/4(-7-sqrt233+(14(11+sqrt233))^0.5))))+7+8/(((1/4(-7 sqrt233+(14(11+sqrt233))^0.5))))]+17

Input:

$$\left(\frac{1}{4}\left(-7-\sqrt{233}+\sqrt{14\left(11+\sqrt{233}\right)}\right)+7+\frac{8}{\frac{1}{4}\left(-7-\sqrt{233}+\sqrt{14(11+\sqrt{233})}\right)}\right)^2 - 14\left(\frac{1}{4}\left(-7-\sqrt{233}+\sqrt{14\left(11+\sqrt{233}\right)}\right)+7+\frac{8}{\frac{1}{4}\left(-7-\sqrt{233}+\sqrt{14(11+\sqrt{233})}\right)}\right)+17$$

Exact result:

$$17-14\left(7+\frac{32}{-7-\sqrt{233}+\sqrt{14(11+\sqrt{233})}}+\frac{1}{4}\left(-7-\sqrt{233}+\sqrt{14\left(11+\sqrt{233}\right)}\right)\right)+\left(7+\frac{32}{-7-\sqrt{233}+\sqrt{14(11+\sqrt{233})}}+\frac{1}{4}\left(-7-\sqrt{233}+\sqrt{14\left(11+\sqrt{233}\right)}\right)\right)^2$$

Decimal approximation:

91.925181328658118088395820452101678555650427175134392847 88...

91.92518132865…

Alternate forms:

$$\frac{7}{2}\left(11+\sqrt{233}\right)$$

$$\frac{1}{2}\left(77+7\sqrt{233}\right)$$

$$\frac{\left(-154-14\sqrt{233}+7\sqrt{14(11+\sqrt{233})}+\sqrt{3262(11+\sqrt{233})}\right)^2}{4\left(7+\sqrt{233}-\sqrt{14(11+\sqrt{233})}\right)^2}$$

Minimal polynomial:

$x^2-77x-1372$

From

$$\left(v+\frac{1}{v}\right)^2-14\left(v+\frac{1}{v}\right)+17 =$$

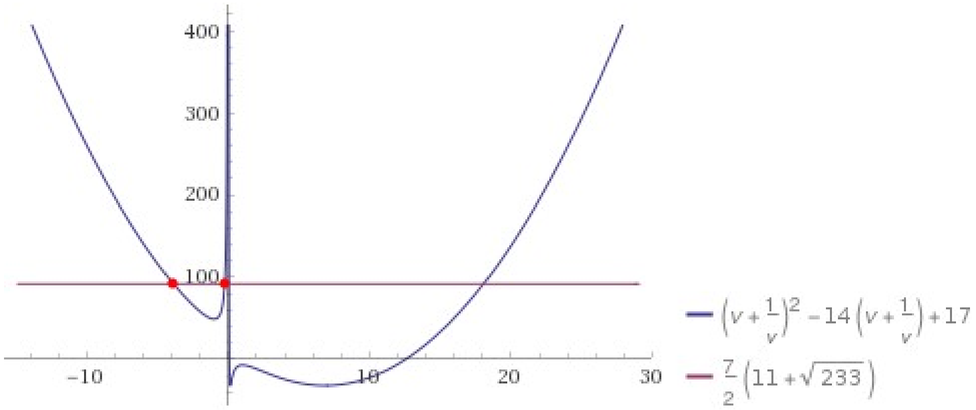
$$= \left(R+7+\frac{8}{R}\right)^2-14\left(R+7+\frac{8}{R}\right)+17 = \frac{7}{2}\left(11+\sqrt{233}\right)$$

[v+1/v]^2-14[v+1/v]+17 = 7/2 (11 + sqrt(233))

Input:

$$\left(v+\frac{1}{v}\right)^2-14\left(v+\frac{1}{v}\right)+17=\frac{7}{2}\left(11+\sqrt{233}\right)$$

Plot:



Alternate form assuming v is real:

$$\frac{2\left(v^4+1\right)}{v}=v\left(28\,v+7\sqrt{233}+39\right)+28$$

Alternate forms:

$$2\,v^4-28\,v^3+\left(-39-7\sqrt{233}\right)v^2-28\,v=-2 \quad (\text{for } v\neq 0)$$

$$\left(v+\frac{1}{v}\right)^2-14\left(v+\frac{1}{v}\right)+17=\frac{1}{2}\left(77+7\sqrt{233}\right)$$

$$\frac{v^4-14\,v^3+19\,v^2-14\,v+1}{v^2}=\frac{7}{2}\left(11+\sqrt{233}\right)$$

Alternate form assuming v>0:

$$\left(v+\frac{1}{v}\right)^2-14\left(v+\frac{1}{v}\right)+17=\frac{77}{2}+\frac{7\sqrt{233}}{2}$$

Alternate form assuming v is positive:

$$2\left(v^4+1\right)=v\left(v\left(28\,v+7\sqrt{233}+39\right)+28\right) \quad (\text{for } v \neq 0)$$

Expanded form:

$$v^2+\frac{1}{v^2}-14\,v-\frac{14}{v}+19=\frac{77}{2}+\frac{7\sqrt{233}}{2}$$

Solutions:

$$v=\frac{1}{4}\left(7-\sqrt{233}-\sqrt{266-14\sqrt{233}}\right)$$

$$v=\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)$$

$$v=\frac{4}{21+\sqrt{233}+\sqrt{658+42\sqrt{233}}}$$

$$v=\frac{1}{4}\left(21+\sqrt{233}+\sqrt{658+42\sqrt{233}}\right)$$

From

$$\left(v+\frac{1}{v}\right)^2-14\left(v+\frac{1}{v}\right)+17$$

For

$$v=\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)$$

we obtain:

((([1/4(7-sqrt(233)+sqrt(266-14sqrt(233)))]+1/[1/4(7-sqrt(233)+sqrt(266-14sqrt(233)))])))^2-14((([1/4(7-sqrt(233)+sqrt(266-14sqrt(233)))]+1/[1/4(7-sqrt(233)+sqrt(266-14sqrt(233)))])))+17

Input:

$$\left(\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)+\frac{1}{\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)}\right)^2-14\left(\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)+\frac{1}{\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)}\right)+17$$

Exact result:

$$17-14\left(\frac{4}{7-\sqrt{233}+\sqrt{266-14\sqrt{233}}}+\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)\right)+\left(\frac{4}{7-\sqrt{233}+\sqrt{266-14\sqrt{233}}}+\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)\right)^2$$

Decimal approximation:

91.925181328658118088395820452101678555650427175134392847 88...

91.925181328658…

Alternate forms:

$$\frac{7}{2}\left(11+\sqrt{233}\right)$$

$$\frac{1}{2}\left(77+7\sqrt{233}\right)$$

$$-\frac{28\left(62-30\sqrt{233}+39\sqrt{266-14\sqrt{233}}+\sqrt{3262\left(19-\sqrt{233}\right)}\right)}{\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)^2}$$

Minimal polynomial:

$$x^2-77x-1372$$

Indeed:

$$\frac{7}{2}\left(11+\sqrt{233}\right) = \frac{7}{2}\left(11+\sqrt{233}\right)$$

Input:

$$\frac{7}{2}\left(11+\sqrt{233}\right)$$

Decimal approximation:

91.92518132865811808839582045210167855565042717513439284788...

91.925181328658…

Alternate forms:

$$\frac{77}{2}+\frac{7\sqrt{233}}{2}$$

$$\frac{1}{2}\left(77+7\sqrt{233}\right)$$

Minimal polynomial:

$$x^2-77x-1372$$

From which, we obtain:

7/2 (11 + sqrt(233)) + 47 + 1/golden ratio

Input:

$$\frac{7}{2}\left(11+\sqrt{233}\right)+47+\frac{1}{\phi}$$

ϕ is the golden ratio

Decimal approximation:

139.5432153174080129366004072864673166733707363549401557100...

139.5432153174…

Alternate forms:

$$\frac{1}{2}\left(170+\sqrt{5}+7\sqrt{233}\right)$$

$$\frac{1}{\phi}+\frac{171}{2}+\frac{7\sqrt{233}}{2}$$

$$\frac{(171+7\sqrt{233})\phi+2}{2\phi}$$

Minimal polynomial:

$$x^4 - 340x^3 + 37639x^2 - 1485630x + 19078259$$

Series representations:

$$\frac{1}{2}\left(11+\sqrt{233}\right)7+47+\frac{1}{\phi} = \frac{171}{2}+\frac{1}{\phi}+\frac{7}{2}\sqrt{232}\sum_{k=0}^{\infty}232^{-k}\binom{\frac{1}{2}}{k}$$

$$\frac{1}{2}\left(11+\sqrt{233}\right)7+47+\frac{1}{\phi} = \frac{171}{2}+\frac{1}{\phi}+\frac{7}{2}\sqrt{232}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{232}\right)^k\left(-\frac{1}{2}\right)_k}{k!}$$

$$\frac{1}{2}\left(11+\sqrt{233}\right)7+47+\frac{1}{\phi} = \frac{171}{2}+\frac{1}{\phi}+\frac{7\sum_{j=0}^{\infty}\text{Res}_{s=-\frac{1}{2}+j}\,232^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{4\sqrt{\pi}}$$

7/2 (11 + sqrt(233)) + 34 - 1/golden ratio

Input:

$$\frac{7}{2}\left(11+\sqrt{233}\right)+34-\frac{1}{\phi}$$

ϕ is the golden ratio

Decimal approximation:

125.30714733990822324019123361773604043793011799532862998 57...

125.3071473399…

Alternate forms:

$$\frac{1}{2}\left(146-\sqrt{5}+7\sqrt{233}\right)$$

$$-\frac{1}{\phi}+\frac{145}{2}+\frac{7\sqrt{233}}{2}$$

$$-\frac{(-145-7\sqrt{233})\phi+2}{2\phi}$$

Minimal polynomial:

$$x^4 - 292x^3 + 26263x^2 - 722262x + 6103931$$

Series representations:

$$\frac{1}{2}\left(11+\sqrt{233}\right)7+34-\frac{1}{\phi}=\frac{145}{2}-\frac{1}{\phi}+\frac{7}{2}\sqrt{232}\sum_{k=0}^{\infty}232^{-k}\binom{\frac{1}{2}}{k}$$

$$\frac{1}{2}\left(11+\sqrt{233}\right)7+34-\frac{1}{\phi}=\frac{145}{2}-\frac{1}{\phi}+\frac{7}{2}\sqrt{232}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{232}\right)^k\left(-\frac{1}{2}\right)_k}{k!}$$

$$\frac{1}{2}\left(11+\sqrt{233}\right)7+34-\frac{1}{\phi}=\frac{145}{2}-\frac{1}{\phi}+\frac{7\sum_{j=0}^{\infty}\text{Res}_{s=-\frac{1}{2}+j}\,232^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{4\sqrt{\pi}}$$

27*1/2* ((7/2 (11 + sqrt(233)) + 34))+29

Input:

$$27\times\frac{1}{2}\left(\frac{7}{2}\left(11+\sqrt{233}\right)+34\right)+29$$

Result:

$$29+\frac{27}{2}\left(34+\frac{7}{2}\left(11+\sqrt{233}\right)\right)$$

Decimal approximation:

1728.989947936884594193343576103372660501280766864314303446...

1728.989947936...

With regard 27 (From Wikipedia):

*"The fundamental group of the complex form, compact real form, or any algebraic version of E_6 is the cyclic group **Z**/3**Z**, and its outer automorphism group is the cyclic group **Z**/2**Z**. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E_6 plays a role in some grand unified theories".*

Alternate forms:

$$\frac{4031}{4}+\frac{189\sqrt{233}}{4}$$

$$\frac{1}{4}\left(4031 + 189\sqrt{233}\right)$$

Minimal polynomial:

$$2x^2 - 4031x + 990746$$

We obtain also:

(((7/2 (11 + sqrt(233)))))^1/9-34/10^3

Input:

$$\sqrt[9]{\frac{7}{2}\left(11 + \sqrt{233}\right)} - \frac{34}{10^3}$$

Result:

$$\sqrt[9]{\frac{7}{2}\left(11 + \sqrt{233}\right)} - \frac{17}{500}$$

Decimal approximation:

1.618568188158799428673023055310857654697780459913835687446...

1.6185681881587...

Alternate form:

$$\frac{1}{500}\left(250 \times 2^{8/9}\sqrt[9]{7\left(11 + \sqrt{233}\right)} - 17\right)$$

or, from:

$$\left(\frac{1}{4}\left(7 - \sqrt{233} + \sqrt{266 - 14\sqrt{233}}\right) + \frac{1}{\frac{1}{4}\left(7 - \sqrt{233} + \sqrt{266 - 14\sqrt{233}}\right)}\right)^2 - 14\left(\frac{1}{4}\left(7 - \sqrt{233} + \sqrt{266 - 14\sqrt{233}}\right) + \frac{1}{\frac{1}{4}\left(7 - \sqrt{233} + \sqrt{266 - 14\sqrt{233}}\right)}\right) + 17$$

[(((([1/4(7-sqrt(233)+sqrt(266-14sqrt(233)))]+1/[1/4(7-sqrt(233)+sqrt(266-14sqrt(233)))])))^2-14((([1/4(7-sqrt(233)+sqrt(266-14sqrt(233)))]+1/[1/4(7-sqrt(233)+sqrt(266-14sqrt(233)))])))+17)]^1/9

Input:

$$\left(\left(\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)+\frac{1}{\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)}\right)^2-14\left(\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)+\frac{1}{\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)}\right)+17\right)\text{^}(1/9)$$

Exact result:

$$\left(17-14\left(\frac{4}{7-\sqrt{233}+\sqrt{266-14\sqrt{233}}}+\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)\right)+\left(\frac{4}{7-\sqrt{233}+\sqrt{266-14\sqrt{233}}}+\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)\right)^2\right)\text{^}(1/9)$$

Decimal approximation:

1.652568188158799428673023055310857654697780459913835687446...

1.6525681881587…. result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164{,}2696$ i.e. 1,65578...

Alternate forms:

$$\sqrt[9]{\frac{7}{2}\left(11+\sqrt{233}\right)}$$

$$\frac{\left(\frac{2}{-7+\sqrt{233}-\sqrt{266-14\sqrt{233}}}\right)^{2/9}}{\sqrt[9]{7\left(-62+30\sqrt{233}-39\sqrt{266-14\sqrt{233}}-\sqrt{3262\left(19-\sqrt{233}\right)}\right)}}$$

$$\frac{\left(\frac{2}{-7+\sqrt{233}-\sqrt{14\left(19-\sqrt{233}\right)}}\right)^{2/9}}{\sqrt[9]{7\left(-62+30\sqrt{233}-39\sqrt{14\left(19-\sqrt{233}\right)}-\sqrt{3262\left(19-\sqrt{233}\right)}\right)}}$$

Minimal polynomial:

$x^{18}-77x^{9}-1372$

All 9th roots of 17 - 14 (4/(7 - sqrt(233) + sqrt(266 - 14 sqrt(233))) + 1/4 (7 - sqrt(233) + sqrt(266 - 14 sqrt(233)))) + (4/(7 - sqrt(233) + sqrt(266 - 14 sqrt(233))) + 1/4 (7 - sqrt(233) + sqrt(266 - 14 sqrt(233))))^2:

$$\left(17-14\left(\frac{4}{7-\sqrt{233}+\sqrt{266-14\sqrt{233}}}+\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)\right)+\left(\frac{4}{7-\sqrt{233}+\sqrt{266-14\sqrt{233}}}+\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)\right)^2\right)\hat{}\,(1/9)\,e^{0}\approx 1.653$$ (real, principal root)

$$\left(17-14\left(\frac{4}{7-\sqrt{233}+\sqrt{266-14\sqrt{233}}}+\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)\right)+\left(\frac{4}{7-\sqrt{233}+\sqrt{266-14\sqrt{233}}}+\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)\right)^2\right)\hat{}\,(1/9)\,e^{(2i\pi)/9}\approx 1.266+1.062\,i$$

$$\left(17-14\left(\frac{4}{7-\sqrt{233}+\sqrt{266-14\sqrt{233}}}+\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)\right)+\left(\frac{4}{7-\sqrt{233}+\sqrt{266-14\sqrt{233}}}+\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)\right)^2\right)\hat{}\,(1/9)\,e^{(4i\pi)/9}\approx 0.2870+1.627\,i$$

$$\left(17-14\left(\frac{4}{7-\sqrt{233}+\sqrt{266-14\sqrt{233}}}+\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)\right)+\left(\frac{4}{7-\sqrt{233}+\sqrt{266-14\sqrt{233}}}+\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)\right)^2\right)\hat{}\,(1/9)\,e^{(2i\pi)/3}\approx -0.8263+1.431\,i$$

$$\left(17-14\left(\frac{4}{7-\sqrt{233}+\sqrt{266-14\sqrt{233}}}+\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)\right)+\left(\frac{4}{7-\sqrt{233}+\sqrt{266-14\sqrt{233}}}+\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)\right)^2\right)^{\wedge}(1/9)\, e^{(8 i \pi)/9} \approx -1.553+0.5652\, i$$

From

$$\frac{1-14v+19v^2-14v^3+v^4}{v^2}.$$

we obtain:

(((1-14(-0.2581284442539827)+19(-0.2581284442539827)^2-14(-0.2581284442539827)^3+(-0.2581284442539827)^4))) / (-0.2581284442539827)^2

Input interpretation:

$$(1-14\times(-0.2581284442539827)+19(-0.2581284442539827)^2-14(-0.2581284442539827)^3+(-0.2581284442539827)^4)/(-0.2581284442539827)^2$$

Result:

91.92518132865814148313062965393592991923750487230358698810...

91.92518132865814…

or:

[-7/2sqrt(7/2(19-sqrt(233)))(39+sqrt(233))+7/2(-31+15sqrt(233))] / [1/4(7-sqrt(233)+sqrt(266-14sqrt(233)))]^2

Input:

$$\frac{-\frac{7}{2}\sqrt{\frac{7}{2}(19-\sqrt{233})}\,(39+\sqrt{233})+\frac{7}{2}(-31+15\sqrt{233})}{\left(\frac{1}{4}\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)\right)^2}$$

Result:

$$\frac{16\left(\frac{7}{2}\left(15\sqrt{233}-31\right)-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)\right)}{\left(7-\sqrt{233}+\sqrt{266-14\sqrt{233}}\right)^2}$$

Decimal approximation:

91.92518132865811808839582045210167855565042717513439284788...

91.9251813286…

Alternate forms:

$$\frac{4\left(-7\left(39+\sqrt{233}\right)\sqrt{14\left(19-\sqrt{233}\right)}+210\sqrt{233}-434\right)}{\left(\sqrt{266-14\sqrt{233}}-\sqrt{233}+7\right)^2}$$

$$\frac{7}{2}\left(11+\sqrt{233}\right)$$

$$\frac{1}{2}\left(77+7\sqrt{233}\right)$$

Minimal polynomial:

$$x^2-77x-1372$$

Now:

Entry 15.8.1 (p. 51). *If v is defined by* (15.8.1) *and if*

$$c=\frac{\sqrt{13+16\sqrt{2}}}{7},$$

then

$$\int_0^q f(-t)f(-t^2)f(-t^7)f(-t^{14})dt$$
$$=\frac{1}{\sqrt{8\sqrt{2}}}\int_{\cos^{-1}\left(c\frac{1+v}{1-v}\right)}^{\cos^{-1}c}\frac{d\varphi}{\sqrt{1-\frac{16\sqrt{2}-13}{32\sqrt{2}}\sin^2\varphi}}. \quad (15.8.14)$$

Proof. Let

$$\cos\varphi=c\frac{1+v(t)}{1-v(t)}, \quad (15.8.15)$$

so that at $t=0,q$, we obtain the upper and lower limits, respectively, in the integral on the right side of (15.8.14). Differentiating (15.8.15), we find that

$$-\sin\varphi\frac{d\varphi}{dt}=\frac{2c}{(1-v(t))^2}\frac{dv}{dt}. \quad (15.8.16)$$

By elementary trigonometry,

$$\sin\varphi=\frac{\sqrt{(1-v)^2-c^2(1+v)^2}}{1-v}. \quad (15.8.17)$$

Putting (15.8.17) in (15.8.16), we arrive at

$$\frac{d\varphi/dt}{dv/dt}=-\frac{2c}{(1-v)\sqrt{(1-v)^2-c^2(1+v)^2}}. \quad (15.8.18)$$

Next, by (15.8.17),

$$1-\frac{16\sqrt{2}-13}{32\sqrt{2}}\sin^2\varphi=\frac{32\sqrt{2}(1-v)^2-(16\sqrt{2}-13)\left\{(1-v)^2-c^2(1+v)^2\right\}}{32\sqrt{2}(1-v)^2}$$
$$=\frac{(16\sqrt{2}+13)(1-v)^2+7(1+v)^2}{32\sqrt{2}(1-v)^2}. \quad (15.8.19)$$

Thus, by (15.8.18) and (15.8.19),

$$\begin{aligned}
&\frac{dv/dt}{d\varphi/dt}\sqrt{1-\frac{16\sqrt{2}-13}{32\sqrt{2}}\sin^2\varphi} \\
&= -\frac{\sqrt{(1-v)^2-c^2(1+v)^2}}{2c}\,\frac{\sqrt{(16\sqrt{2}+13)(1-v)^2+7(1+v)^2}}{2\sqrt{8\sqrt{2}}} \\
&= -\frac{\sqrt{49(1-v)^2-(16\sqrt{2}+13)(1+v)^2}\sqrt{(16\sqrt{2}+13)(1-v)^2+7(1+v)^2}}{4\sqrt{16\sqrt{2}+13}\sqrt{8\sqrt{2}}} \\
&= -\frac{\sqrt{1-14v+19v^2-14v^3+v^4}}{\sqrt{8\sqrt{2}}},
\end{aligned}$$

after a calculation via *Mathematica*. Thus,

$$\begin{aligned}
&\int_0^q f(-t)f(-t^2)f(-t^7)f(-t^{14})dt \\
&= \frac{1}{\sqrt{8\sqrt{2}}}\int_{\cos^{-1}\left(c\frac{1+v}{1-v}\right)}^{\cos^{-1}c} f(-t)f(-t^2)f(-t^7)f(-t^{14}) \\
&\quad\times \frac{\sqrt{1-14v+19v^2-14v^3+v^4}}{\frac{dv}{dt}\sqrt{1-\frac{16\sqrt{2}-13}{32\sqrt{2}}\sin^2\varphi}}d\varphi \\
&= \frac{1}{\sqrt{8\sqrt{2}}}\int_{\cos^{-1}\left(c\frac{1+v}{1-v}\right)}^{\cos^{-1}c}\frac{d\varphi}{\sqrt{1-\frac{16\sqrt{2}-13}{32\sqrt{2}}\sin^2\varphi}},
\end{aligned}$$

upon the use of Lemma 15.8.4. □

We have:

$$c = \frac{\sqrt{13+16\sqrt{2}}}{7},$$

((13+16sqrt2)^0.5)/7

Input:

$$\frac{1}{7}\sqrt{13+16\sqrt{2}}$$

Decimal approximation:

0.852695809075959107197508155620489359896290562303818077173...

0.852695809075…..

Alternate forms:

$$\sqrt{\frac{13}{49}+\frac{16\sqrt{2}}{49}}$$

$$\frac{\sqrt{13-7i\sqrt{7}}+\sqrt{i(7\sqrt{7}+-13i)}}{7\sqrt{2}}$$

Minimal polynomial:

$49x^4-26x^2-7$

And

$$-\frac{\sqrt{1-14v+19v^2-14v^3+v^4}}{\sqrt{8\sqrt{2}}},$$

Where, from (15.8.13)

$$1-14v+19v^2-14v^3+v^4$$

Is equal to

[-7/2sqrt(7/2(19-sqrt(233)))(39+sqrt(233))+7/2(-31+15sqrt(233))]

Input:

$$-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)+\frac{7}{2}\left(-31+15\sqrt{233}\right)$$

Result:

$$\frac{7}{2}\left(15\sqrt{233}-31\right)-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)$$

Decimal approximation:

6.125001833386076368589036831034246543269751810068956317893...

6.125001833386…

Alternate forms:

$$\frac{1}{4}\left(-7\left(39+\sqrt{233}\right)\sqrt{14\left(19-\sqrt{233}\right)}+210\sqrt{233}-434\right)$$

$$-\frac{217}{2}+\frac{105\sqrt{233}}{2}-7\sqrt{14\left(947-17\sqrt{233}\right)}$$

$$\frac{7}{2}\left(-31+15\sqrt{233}-2\sqrt{14\left(947-17\sqrt{233}\right)}\right)$$

Minimal polynomial:

$$x^4+434x^3-2513063x^2+15068676x+1882384$$

Thence, from

[-7/2sqrt(7/2(19-sqrt(233)))(39+sqrt(233))+7/2(-31+15sqrt(233))]

$$-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)+\frac{7}{2}\left(-31+15\sqrt{233}\right)$$

We have that:

$$-\frac{\sqrt{1-14v+19v^2-14v^3+v^4}}{\sqrt{8\sqrt{2}}},$$

-(((((((([-7/2sqrt(7/2(19-sqrt(233)))(39+sqrt(233))+7/2(-31+15sqrt(233))])))^0.5)))) / (((8sqrt2)^0.5))

Input:

$$-\frac{\sqrt{-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)+\frac{7}{2}\left(-31+15\sqrt{233}\right)}}{\sqrt{8\sqrt{2}}}$$

Exact result:

$$-\frac{\sqrt{\frac{7}{2}\left(15\sqrt{233}-31\right)-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)}}{2\times 2^{3/4}}$$

Decimal approximation:

-0.73578447346754765563267417565259446305920514165655061219...

-0.735784473…

Alternate forms:

$$-\frac{1}{4}\sqrt{7}\sqrt[4]{53209-941\sqrt{233}-8\sqrt{14\left(3390017-111767\sqrt{233}\right)}}$$

$$-\frac{\sqrt{7\left(-31+15\sqrt{233}-2\sqrt{14(947-17\sqrt{233})}\right)}}{4\sqrt[4]{2}}$$

$$-\frac{\sqrt{7\left(-62+30\sqrt{233}-\sqrt{266-14\sqrt{233}}\ (39+\sqrt{233})\right)}}{4\times 2^{3/4}}$$

Minimal polynomial:

$$1048576\,x^{16}-42717036544\,x^{12}+403354226942016\,x^{8}-118263047778680\,x^{4}+13841287201$$

From:

$$-\frac{\sqrt{(1-v)^2-c^2(1+v)^2}}{2c}\,\frac{\sqrt{(16\sqrt{2}+13)(1-v)^2+7(1+v)^2}}{2\sqrt{8\sqrt{2}}}$$

For v = -0.25812844 and c = 0.852695809

-sqrt((((1+0.25812844)^2-(0.852695809)^2(1-0.25812844)^2))) / (((2(0.852695809)))) * sqrt((((16sqrt2+13)(1+0.25812844)^2+7(1-0.25812844)^2))) / (((2*sqrt(8*2^0.5)))

Input interpretation:

$$-\frac{\sqrt{(1+0.25812844)^2-0.852695809^2\,(1-0.25812844)^2}}{2\times 0.852695809}\times\frac{\sqrt{(16\sqrt{2}+13)\,(1+0.25812844)^2+7\,(1-0.25812844)^2}}{2\sqrt{8\sqrt{2}}}$$

Result:

−0.73578446673921111239880457327728431799181342105603837181...

-0.735784466739…

We note that:

-2/(((-((((((([-7/2sqrt(7/2(19-sqrt(233)))(39+sqrt(233))+7/2(-31+15sqrt(233))])))^0.5)))) / (((8sqrt2)^0.5)))))

Input:

$$\frac{-2}{-\frac{\sqrt{-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)+\frac{7}{2}\left(-31+15\sqrt{233}\right)}}{\sqrt{8\sqrt{2}}}}$$

Exact result:

$$\frac{4\times 2^{3/4}}{\sqrt{\frac{7}{2}\left(15\sqrt{233}-31\right)-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)}}$$

Decimal approximation:

2.718187284619578955841286681436945168692767212946104328402...

2.7181872846195….. ≈ ***e*** (Euler number)

Alternate forms:

$$\frac{2}{7}\sqrt[4]{2\left(2564335-167861\sqrt{233}+\sqrt{14\left(938652015995-61493118521\sqrt{233}\right)}\right)}$$

$$\frac{8\sqrt[4]{2}}{\sqrt{7\left(-31+15\sqrt{233}-2\sqrt{14\left(947-17\sqrt{233}\right)}\right)}}$$

$$\frac{8\times 2^{3/4}}{\sqrt{7\left(-62+30\sqrt{233}-\sqrt{266-14\sqrt{233}}\left(39+\sqrt{233}\right)\right)}}$$

Minimal polynomial:

$$13841287201\,x^{16}-1892208764458880\,x^{12}+103258682097156096\,x^{8}-174968981684224\,x^{4}+68719476736$$

[-2/(((-((((((([-7/2sqrt(7/2(19-sqrt(233)))(39+sqrt(233))+7/2(-31+15sqrt(233))])))^0.5)))) / (((8sqrt2)^0.5)))))]^1/2

Input:

$$\sqrt{\frac{-2}{-\frac{\sqrt{-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)+\frac{7}{2}\left(-31+15\sqrt{233}\right)}}{\sqrt{8\sqrt{2}}}}}$$

Exact result:

$$\frac{2\times 2^{3/8}}{\sqrt[4]{\frac{7}{2}\left(15\sqrt{233}-31\right)-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)}}$$

Decimal approximation:

1.648692598582155022055355346398138526394127442242330227546...

1.648692598582155….

Alternate forms:

$$\frac{2^{5/8}\sqrt[8]{2564335-167861\sqrt{233}+\sqrt{14\left(938652015995-61493118521\sqrt{233}\right)}}}{\sqrt{7}}$$

$$\frac{2\times 2^{5/8}}{\sqrt[4]{7\left(-31+15\sqrt{233}-2\sqrt{14\left(947-17\sqrt{233}\right)}\right)}}$$

$$\frac{2\times 2^{7/8}}{\sqrt[4]{7\left(-62+30\sqrt{233}-\sqrt{266-14\sqrt{233}}\left(39+\sqrt{233}\right)\right)}}$$

Minimal polynomial:

$$13841287201\,x^{32}-1892208764458880\,x^{24}+103258682097156096\,x^{16}-174968981684224\,x^{8}+68719476736$$

All 2nd roots of (4 2^(3/4))/sqrt(7/2 (15 sqrt(233) - 31) - 7/2 sqrt(7/2 (19 - sqrt(233))) (39 + sqrt(233))):

Polar form

$$\frac{2\times 2^{3/8}\, e^{0}}{\sqrt[4]{\frac{7}{2}\left(15\sqrt{233}-31\right)-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)}} \approx 1.65 \quad \text{(real, principal root)}$$

$$\frac{2\times 2^{3/8}\, e^{i\pi}}{\sqrt[4]{\frac{7}{2}\left(15\sqrt{233}-31\right)-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)}} \approx -1.65 \quad \text{(real root)}$$

and, in conclusion:

-(((1/3 e ζ(3) log(3) log(2 π))))(((-((((((([-7/2sqrt(7/2(19-sqrt(233)))(39+sqrt(233))+7/2(-31+15sqrt(233))])))^0.5)))) / (((8sqrt2)^0.5)))))

Where

$$\frac{1}{3}\, e\, \zeta(3) \log(3) \log(2\pi) \approx 2.199171836$$

Input:

$$-\left(\frac{1}{3}\, e\, \zeta(3) \log(3) \log(2\pi)\right)\left(-\frac{\sqrt{-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)+\frac{7}{2}\left(-31+15\sqrt{233}\right)}}{\sqrt{8\sqrt{2}}}\right)$$

$\zeta(s)$ is the Riemann zeta function

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{\sqrt{\frac{7}{2}\left(15\sqrt{233}-31\right)-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)}\; e\, \zeta(3) \log(3) \log(2\pi)}{6\times 2^{3/4}}$$

Decimal approximation:

1.6181164918857812719809027172559662296285315906197587111101...

1.61811649188578….

Alternate forms:

$$\frac{1}{12}\sqrt{7}\sqrt[4]{53209-941\sqrt{233}-8\sqrt{14\left(3390017-111767\sqrt{233}\right)}}\; e\,\zeta(3)\log(3)\log(2\pi)$$

$$\frac{\sqrt{7\left(-31+15\sqrt{233}-2\sqrt{14(947-17\sqrt{233})}\right)}\; e\,\zeta(3)\log(3)\log(2\pi)}{12\sqrt[4]{2}}$$

$$\frac{1}{48}\sqrt{\left(7\left(\sqrt{2\left(133-56\sqrt{2}\right)}\left(39+\sqrt{233}\right)-6\sqrt{133-7\sqrt{233}}\left(39+\sqrt{233}\right)-\sqrt{2}\left(\frac{\sqrt{7(16+19\sqrt{2})}\,(39+\sqrt{233})}{\sqrt[4]{2}}-8\left(15\sqrt{233}-31\right)\right)\right)\right)}\; e\,\zeta(3)\log(3)(\log(2)+\log(\pi))$$

Alternative representations:

$$-\frac{1}{\sqrt{8\sqrt{2}}\;3}-\sqrt{-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)+\frac{7}{2}\left(-31+15\sqrt{233}\right)}\; e\,(\zeta(3)\,(\log(3)\log(2\pi)))=$$

$$\frac{e\log(3)\log(2\pi)\sqrt{\frac{7}{2}(-31+15\sqrt{233})-\frac{7}{2}(39+\sqrt{233})\sqrt{\frac{7}{2}(19-\sqrt{233})}}\;\zeta(3,1)}{3\sqrt{8\sqrt{2}}}$$

$$-\frac{1}{\sqrt{8\sqrt{2}}\ 3}-\sqrt{-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)+\frac{7}{2}\left(-31+15\sqrt{233}\right)}\ e\ (\zeta(3)\ (\log(3)\log(2\pi)))=$$

$$\frac{e\log(3)\log(2\pi)\sqrt{\frac{7}{2}\left(-31+15\sqrt{233}\right)-\frac{7}{2}\left(39+\sqrt{233}\right)\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}}\ \zeta\left(3,\frac{1}{2}\right)}{3\times 7\sqrt{8\sqrt{2}}}$$

$$-\frac{1}{\sqrt{8\sqrt{2}}\ 3}-\sqrt{-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)+\frac{7}{2}\left(-31+15\sqrt{233}\right)}\ e\ (\zeta(3)\ (\log(3)\log(2\pi)))=$$

$$\frac{e\log_e(3)\log_e(2\pi)\sqrt{\frac{7}{2}\left(-31+15\sqrt{233}\right)-\frac{7}{2}\left(39+\sqrt{233}\right)\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}}\ \zeta(3,1)}{3\sqrt{8\sqrt{2}}}$$

Series representations:

$$-\frac{1}{\sqrt{8\sqrt{2}}\ 3}-\sqrt{-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)+\frac{7}{2}\left(-31+15\sqrt{233}\right)}\ e\ (\zeta(3)\ (\log(3)\log(2\pi)))=$$

$$\frac{1}{24\times 2^{3/4}}\sqrt{7\left(-62+30\sqrt{233}-39\sqrt{14\left(19-\sqrt{233}\right)}-\sqrt{3262\left(19-\sqrt{233}\right)}\right)}\ e\left(\log(2)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k}{k}\right)\left(\log(-1+2\pi)-\sum_{k=1}^{\infty}\frac{\left(\frac{1}{1-2\pi}\right)^k}{k}\right)\sum_{n=0}^{\infty}\frac{\sum_{k=0}^{n}\frac{(-1)^k\binom{n}{k}}{(1+k)^2}}{1+n}$$

$$-\frac{1}{\sqrt{8\sqrt{2}}\ 3}-\sqrt{-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)+\frac{7}{2}\left(-31+15\sqrt{233}\right)}\ e\ (\zeta(3)\ (\log(3)\log(2\pi)))=\frac{1}{12\times 2^{3/4}}$$

$$\sqrt{7\left(-62+30\sqrt{233}-39\sqrt{14\left(19-\sqrt{233}\right)}-\sqrt{3262\left(19-\sqrt{233}\right)}\right)}\ e\left(\log(2)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k}{k}\right)\left(\log(-1+2\pi)-\sum_{k=1}^{\infty}\frac{\left(\frac{1}{1-2\pi}\right)^k}{k}\right)\sum_{m=1}^{\infty}\frac{-m\sum_{k=1+m}^{\infty}\frac{1}{k^2}+\sum_{k=1}^{m}\frac{1}{k}}{m^2}$$

$$-\frac{1}{\sqrt{8\sqrt{2}}\ 3}-\sqrt{-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)+\frac{7}{2}\left(-31+15\sqrt{233}\right)}$$

$$e\ (\zeta(3)\ (\log(3)\log(2\pi))) = \frac{1}{12\times 2^{3/4}}$$

$$\sqrt{7\left(-62+30\sqrt{233}-39\sqrt{14\left(19-\sqrt{233}\right)}-\sqrt{3262\left(19-\sqrt{233}\right)}\right)}\ e$$

$$\left(\log(2)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k}{k}\right)\left(\log(-1+2\pi)-\sum_{k=1}^{\infty}\frac{\left(\frac{1}{1-2\pi}\right)^k}{k}\right)\sum_{k=0}^{\infty}\frac{(3-s_0)^k\,\zeta^{(k)}(s_0)}{k!} \quad \text{for } s_0 \neq 1$$

Integral representations:

$$-\frac{1}{\sqrt{8\sqrt{2}}\ 3}-\sqrt{-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)+\frac{7}{2}\left(-31+15\sqrt{233}\right)}\ e$$

$$(\zeta(3)\ (\log(3)\log(2\pi))) = \int_0^1\int_0^1\int_0^1 \frac{\log^3\left(1-t_3^2\right)}{(1+2t_1)(1+(-1+2\pi)t_2)\,t_3^3}\,dt_3\,dt_2\,dt_1$$

$$-\frac{1}{\sqrt{8\sqrt{2}}\ 3}-\sqrt{-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)+\frac{7}{2}\left(-31+15\sqrt{233}\right)}$$

$$e\ (\zeta(3)\ (\log(3)\log(2\pi))) = -\frac{\sqrt{-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)+\frac{7}{2}\left(-31+15\sqrt{233}\right)}\ e\log(3)\log(2\pi)}{18\times 2^{3/4}}$$

$$\int_0^1 \frac{\log^3\left(1-t^2\right)}{t^3}\,dt$$

$$-\frac{1}{\sqrt{8\sqrt{2}}\ 3}-\sqrt{-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)+\frac{7}{2}\left(-31+15\sqrt{233}\right)}$$

$$e\ (\zeta(3)\ (\log(3)\log(2\pi))) = \frac{\sqrt{-\frac{7}{2}\sqrt{\frac{7}{2}\left(19-\sqrt{233}\right)}\left(39+\sqrt{233}\right)+\frac{7}{2}\left(-31+15\sqrt{233}\right)}\ e\log(3)\log(2\pi)}{12\times 2^{3/4}}$$

$$\int_0^{\infty}\frac{t^2}{-1+e^t}\,dt$$

From:

Exact geometric optics in a Morris-Thorne wormhole spacetime
Thomas Muller - Visualisierungsinstitut der Universitat Stuttgart, Nobelstrasse 15, 70569 Stuttgart, Germany - (Received 17 December 2007; published 26 February 2008) - PHYSICAL REVIEW D **77,** 044043 (2008)

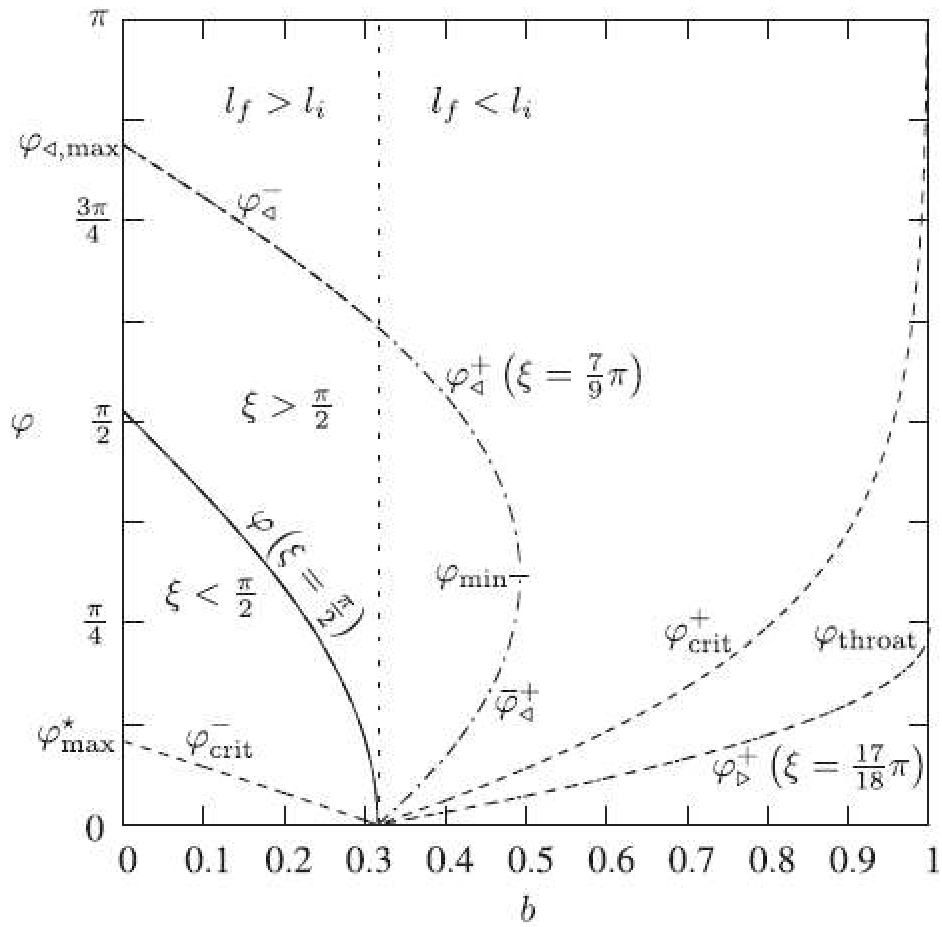


FIG. 4. The angle φ (ordinate) is plotted over the scaled distance $b = b_0/\sqrt{b_0^2 + l^2}$ (abscissa), $l \geq 0$. The throat size is $b_0 = 2$ and the observer is located at $l_i = 6$ ($b_i \approx 0.316$). The dotted line separates the destination points $l_f \gtrless l_i$ and the solid line separates the initial angle $\xi \gtrless \pi/2$. The dash-dotted line for $\xi = \frac{7}{9}\pi$ is composed of $\varphi_{\triangleleft}^{-}(b < b_i)$, $\varphi_{\triangleleft}^{+}(\varphi > \varphi_{\min})$ and $\bar{\varphi}_{\triangleleft}^{+}(\varphi < \varphi_{\min})$.

$$a = \frac{b_0\sqrt{\kappa c^2 + k^2/c^2}}{h} = \frac{b_0}{\sin\xi\sqrt{b_0^2 + l_i^2}} \qquad (26)$$

From

$\varphi_{\triangleleft}^{+}\ (\xi = \frac{7}{9}\pi)$

we obtain:

2/(((sin(7*Pi / 9))*sqrt(4+36)))

Input:

$$\frac{2}{\sin\left(7\times\frac{\pi}{9}\right)\sqrt{4+36}}$$

Exact result:

$$\frac{\csc\left(\frac{2\pi}{9}\right)}{\sqrt{10}}$$

csc(x) is the cosecant function

Decimal approximation:

0.4919630703072341556844502920763637056783920717165699780 79...

0.491963070307….= a

Alternate forms:

$$\frac{\csc\left(\frac{\pi}{9}\right)\sec\left(\frac{\pi}{9}\right)}{2\sqrt{10}}$$

root of $375x^6 - 450x^4 + 120x^2 - 8$ near $x = 0.491963$

$$-\frac{i\sqrt{\frac{2}{5}}}{e^{-(2i\pi)/9} - e^{(2i\pi)/9}}$$

sec(x) is the secant function

Minimal polynomial:

$375x^6 - 450x^4 + 120x^2 - 8$

Alternative representations:

$$\frac{2}{\sin\left(\frac{7\pi}{9}\right)\sqrt{4+36}} = \frac{2}{\frac{\sqrt{40}}{\csc\left(\frac{7\pi}{9}\right)}}$$

$$\frac{2}{\sin\left(\frac{7\pi}{9}\right)\sqrt{4+36}} = \frac{2}{\cos\left(\frac{\pi}{2} - \frac{7\pi}{9}\right)\sqrt{40}}$$

$$\frac{2}{\sin\left(\frac{7\pi}{9}\right)\sqrt{4+36}} = -\frac{2}{\cos\left(\frac{\pi}{2} + \frac{7\pi}{9}\right)\sqrt{40}}$$

Series representations:

$$\frac{2}{\sin\left(\frac{7\pi}{9}\right)\sqrt{4+36}} = -i\sqrt{\frac{2}{5}}\sum_{k=1}^{\infty} q^{-1+2k} \quad \text{for } q = (-1)^{2/9}$$

$$\frac{2}{\sin\left(\frac{7\pi}{9}\right)\sqrt{4+36}} = -\frac{9\sqrt{\frac{2}{5}}\sum_{k=-\infty}^{\infty}\frac{(-1)^k}{-4+81k^2}}{\pi}$$

$$\frac{2}{\sin\left(\frac{7\pi}{9}\right)\sqrt{4+36}} = \frac{9}{2\sqrt{10}\,\pi} - \frac{18\sqrt{\frac{2}{5}}\sum_{k=1}^{\infty}\frac{(-1)^k}{-4+81k^2}}{\pi}$$

Integral representation:

$$\frac{2}{\sin\left(\frac{7\pi}{9}\right)\sqrt{4+36}} = \frac{1}{\sqrt{10}\,\pi}\int_0^{\infty}\frac{1}{t^{7/9}(1+t)}\,dt$$

Multiple-argument formulas:

$$\frac{2}{\sin\left(\frac{7\pi}{9}\right)\sqrt{4+36}} = \frac{\csc\left(\frac{\pi}{9}\right)\sec\left(\frac{\pi}{9}\right)}{2\sqrt{10}}$$

$$\frac{2}{\sin\left(\frac{7\pi}{9}\right)\sqrt{4+36}} = \frac{\csc^3\left(\frac{2\pi}{27}\right)}{\sqrt{10}\left(-4+3\csc^2\left(\frac{2\pi}{27}\right)\right)}$$

From

$$\tilde{\varphi}_{\triangleright}^{+}\left(\xi = \frac{17}{18}\pi\right)$$

we obtain:

2/(((sin(17Pi/18))*sqrt(4+36)))

Input:

$$\frac{2}{\sin\left(17\times\frac{\pi}{18}\right)\sqrt{4+36}}$$

Exact result:

$$\frac{\csc\left(\frac{\pi}{18}\right)}{\sqrt{10}}$$

csc(x) is the cosecant function

Decimal approximation:

1.821083124888217682989071044675012260670211389561147143070...

1.8210831248….. = a

Alternate forms:

$$-\frac{i\sqrt{\frac{2}{5}}}{e^{-(i\pi)/18}-e^{(i\pi)/18}}$$

$$\frac{(-1)^{5/9}\sqrt{\frac{2}{5}}}{\left(\sqrt[18]{-1}-1\right)\left(1+\sqrt[18]{-1}\right)}$$

$$\sqrt{\frac{1}{5}\left(6+\frac{14}{\sqrt[3]{\frac{1}{2}\left(37+i\sqrt{3}\right)}}+2^{2/3}\sqrt[3]{37+i\sqrt{3}}\right)}$$

Minimal polynomial:

$$125x^6-450x^4+120x^2-8$$

Alternative representations:

$$\frac{2}{\sin\left(\frac{17\pi}{18}\right)\sqrt{4+36}}=\frac{2}{\frac{\sqrt{40}}{\csc\left(\frac{17\pi}{18}\right)}}$$

$$\frac{2}{\sin\left(\frac{17\pi}{18}\right)\sqrt{4+36}}=\frac{2}{\cos\left(\frac{\pi}{2}-\frac{17\pi}{18}\right)\sqrt{40}}$$

$$\frac{2}{\sin\left(\frac{17\pi}{18}\right)\sqrt{4+36}} = -\frac{2}{\cos\left(\frac{\pi}{2}+\frac{17\pi}{18}\right)\sqrt{40}}$$

Series representations:

$$\frac{2}{\sin\left(\frac{17\pi}{18}\right)\sqrt{4+36}} = -i\sqrt{\frac{2}{5}}\sum_{k=1}^{\infty} q^{-1+2k} \quad \text{for } q = \sqrt[18]{-1}$$

$$\frac{2}{\sin\left(\frac{17\pi}{18}\right)\sqrt{4+36}} = \frac{9\sqrt{\frac{2}{5}}}{\pi} - \frac{18\sqrt{\frac{2}{5}}\sum_{k=1}^{\infty}\frac{(-1)^k}{-1+324k^2}}{\pi}$$

$$\frac{2}{\sin\left(\frac{17\pi}{18}\right)\sqrt{4+36}} = 9\sqrt{\frac{2}{5}}\sum_{k=-\infty}^{\infty}\frac{e^{ik\pi}}{\pi-324k^2\pi}$$

Integral representation:

$$\frac{2}{\sin\left(\frac{17\pi}{18}\right)\sqrt{4+36}} = \frac{1}{\sqrt{10}\,\pi}\int_0^{\infty}\frac{1}{t^{17/18}(1+t)}\,dt$$

Multiple-argument formulas:

$$\frac{2}{\sin\left(\frac{17\pi}{18}\right)\sqrt{4+36}} = \frac{\csc\left(\frac{\pi}{36}\right)\sec\left(\frac{\pi}{36}\right)}{2\sqrt{10}}$$

$$\frac{2}{\sin\left(\frac{17\pi}{18}\right)\sqrt{4+36}} = \frac{\csc^3\left(\frac{\pi}{54}\right)}{\sqrt{10}\left(-4+3\csc^2\left(\frac{\pi}{54}\right)\right)}$$

We have:

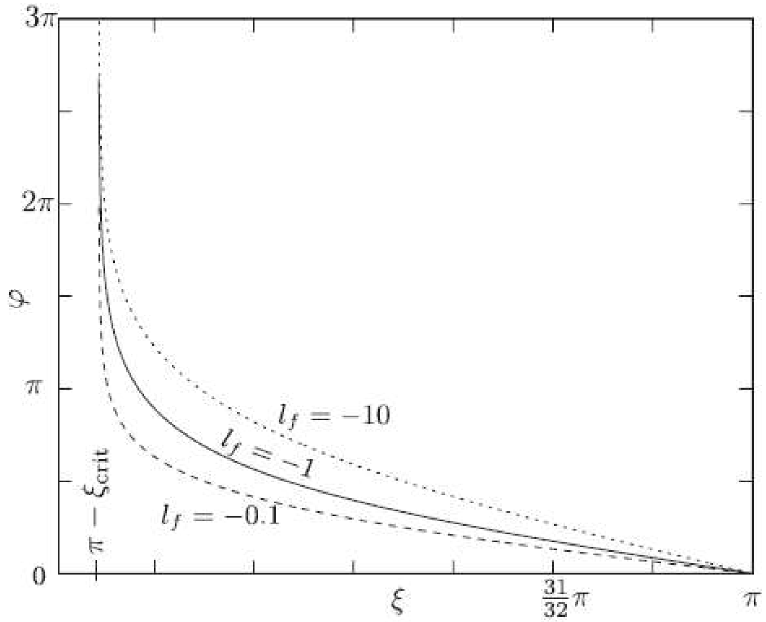


FIG. 11. Lensing $\varphi = \varphi(\xi)$ for an observer located at $l_i = 6$ and rings with radius $l_f = -0.1$ (dashed line), $l_f = -1$ (solid line), and $l_f = -10$ (dotted line).

2/(((sin(31Pi/32))*sqrt(4+36)))

Input:

$$\frac{2}{\sin\left(31\times\frac{\pi}{32}\right)\sqrt{4+36}}$$

Exact result:

$$\frac{\csc\left(\frac{\pi}{32}\right)}{\sqrt{10}}$$

csc(x) is the cosecant function

Decimal approximation:

3.226249663615905869882149991739466123732576796219893353084...

3.226249663615…. = a

Minimal polynomial:

$$390625x^{16} - 5000000x^{14} + 10500000x^{12} - 8400000x^{10} + 3300000x^8 - 704000x^6 + 83200x^4 - 5120x^2 + 128$$

Alternate forms:

$$-\frac{\sqrt{\frac{2}{5}}\sin\left(\frac{\pi}{32}\right)}{\cos\left(\frac{\pi}{16}\right)-1}$$

$$-\frac{i\sqrt{\frac{2}{5}}}{e^{-(i\pi)/32}-e^{(i\pi)/32}}$$

$$\frac{(-1)^{17/32}\sqrt{\frac{2}{5}}}{\left(\sqrt[32]{-1}-1\right)\left(1+\sqrt[32]{-1}\right)}$$

Alternative representations:

$$\frac{2}{\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}}=\frac{2}{\frac{\sqrt{40}}{\csc\left(\frac{31\pi}{32}\right)}}$$

$$\frac{2}{\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}}=\frac{2}{\cos\left(\frac{\pi}{2}-\frac{31\pi}{32}\right)\sqrt{40}}$$

$$\frac{2}{\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}}=-\frac{2}{\cos\left(\frac{\pi}{2}+\frac{31\pi}{32}\right)\sqrt{40}}$$

Series representations:

$$\frac{2}{\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}}=-i\sqrt{\frac{2}{5}}\sum_{k=1}^{\infty}q^{-1+2k}\quad\text{for } q=\sqrt[32]{-1}$$

$$\frac{2}{\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}}=\frac{16\sqrt{\frac{2}{5}}}{\pi}-\frac{32\sqrt{\frac{2}{5}}\sum_{k=1}^{\infty}\frac{(-1)^k}{-1+1024k^2}}{\pi}$$

$$\frac{2}{\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}}=16\sqrt{\frac{2}{5}}\sum_{k=-\infty}^{\infty}\frac{e^{ik\pi}}{\pi-1024k^2\pi}$$

Integral representation:

$$\frac{2}{\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}} = \frac{1}{\sqrt{10}\,\pi}\int_0^\infty \frac{1}{t^{31/32}\,(1+t)}\,dt$$

We have that:

$$\left(\frac{d\rho}{d\varphi}\right)^2 = (1 - a^2\rho^2)(1 - \rho^2). \qquad (27)$$

Case 1: If $a < 1$, which means that all geodesics rest in the universe they start from, Eq. (27) leads to the standard form of an elliptic integral of the first kind [30]

$$\pm \int_{\rho_i}^{\rho} \frac{d\rho'}{\sqrt{(1 - a^2\rho'^2)(1 - \rho'^2)}} = \varphi, \qquad (29)$$

where the left-hand side is given by the elliptic integral function $\mathcal{F}$, [31]

$$\pm [\mathcal{F}(\rho, a) - \mathcal{F}_i] = \varphi \qquad (30)$$

with $\mathcal{F}_i \equiv \mathcal{F}(\rho_i, a) = \mathcal{F}(\sin\xi, a)$ and

$$\rho = \frac{\sin\xi\sqrt{b_0^2 + l_i^2}}{\sqrt{b_0^2 + l^2}} = \frac{b_0}{a\sqrt{b_0^2 + l^2}}. \qquad (31)$$

For the previous result 0.491963070307….= a , $b_0 = 2$ and $l = 6$, we obtain:

2/(0.491963070307*sqrt(4+36))

Input interpretation:

$$\frac{2}{0.491963070307\sqrt{4+36}}$$

Result:

0.642787609687...

0.642787609687…

From which:

1+2/(0.491963070307*sqrt(4+36))

Input interpretation:

$$1+\frac{2}{0.491963070307\sqrt{4+36}}$$

Result:

1.64278760969...

1.64278760969...

Case 2: If $a > 1$, either the geodesic traverses the wormhole throat or it recedes to infinity. In both cases we have to use the inverse value $\alpha = 1/a$ and the integral in Eq. (29) transforms to

$$\pm \int_{\rho_i/\alpha}^{\rho/\alpha} \frac{d\rho'}{\sqrt{(1-\rho'^2)(1-\alpha^2\rho'^2)}} = \frac{\varphi}{\alpha}. \qquad (40)$$

Here, we again have an elliptic integral of the first kind,

$$\pm [\mathcal{F}(a\rho, \alpha) - \mathcal{F}(b_i, \alpha)] = a\varphi. \qquad (41)$$

For α = 1 / a and φ = π / 2 (a > 1), we obtain:

2/(((sin(17Pi/18))*sqrt(4+36))) = a

Pi/2 * 1/[1/(((2/(((sin(17Pi/18))*sqrt(4+36))))))]

$$\pm [\mathcal{F}(a\rho, \alpha) - \mathcal{F}(b_i, \alpha)] = a\varphi.$$

Pi/2 * [(((2/(((sin(17Pi/18))*sqrt(4+36))))))]

Input:

$$\frac{\pi}{2}\times\frac{2}{\sin\left(17\times\frac{\pi}{18}\right)\sqrt{4+36}}$$

Exact result:

$$\frac{\pi \csc\left(\frac{\pi}{18}\right)}{2\sqrt{10}}$$

csc(x) is the cosecant function

Decimal approximation:

2.860550683362584316440320513338264766162455681204995196593...

2.86055068336…

Property:

$\frac{\pi \csc\left(\frac{\pi}{18}\right)}{2\sqrt{10}}$ is a transcendental number

Alternate forms:

$$-\frac{i\pi}{\sqrt{10}\left(e^{-(i\pi)/18} - e^{(i\pi)/18}\right)}$$

$$\frac{(-1)^{5/9}\pi}{\sqrt{10}\left(\sqrt[18]{-1} - 1\right)\left(1 + \sqrt[18]{-1}\right)}$$

$$\frac{1}{2}\sqrt{\frac{1}{5}\left(6 + \frac{14}{\sqrt[3]{\frac{1}{2}\left(37 + i\sqrt{3}\right)}} + 2^{2/3}\sqrt[3]{37 + i\sqrt{3}}\right)}\,\pi$$

Alternative representations:

$$\frac{2\pi}{\left(\sin\left(\frac{17\pi}{18}\right)\sqrt{4+36}\right)2} = \frac{\pi}{\frac{\sqrt{40}}{\csc\left(\frac{17\pi}{18}\right)}}$$

$$\frac{2\pi}{\left(\sin\left(\frac{17\pi}{18}\right)\sqrt{4+36}\right)2} = \frac{\pi}{\cos\left(\frac{\pi}{2} - \frac{17\pi}{18}\right)\sqrt{40}}$$

$$\frac{2\pi}{\left(\sin\left(\frac{17\pi}{18}\right)\sqrt{4+36}\right)2} = -\frac{\pi}{\cos\left(\frac{\pi}{2} + \frac{17\pi}{18}\right)\sqrt{40}}$$

Series representations:

$$\frac{2\pi}{\left(\sin\left(\frac{17\pi}{18}\right)\sqrt{4+36}\right)2} = -\frac{i\pi\sum_{k=1}^{\infty} q^{-1+2k}}{\sqrt{10}} \quad \text{for } q = \sqrt[18]{-1}$$

$$\frac{2\pi}{\left(\sin\left(\frac{17\pi}{18}\right)\sqrt{4+36}\right)2} = \frac{9}{\sqrt{10}} - 9\sqrt{\frac{2}{5}}\sum_{k=1}^{\infty}\frac{(-1)^k}{-1+324k^2}$$

$$\frac{2\pi}{\left(\sin\left(\frac{17\pi}{18}\right)\sqrt{4+36}\right)2} = \frac{9\pi\sum_{k=-\infty}^{\infty}\frac{e^{ik\pi}}{\pi-324k^2\pi}}{\sqrt{10}}$$

Integral representation:

$$\frac{2\pi}{\left(\sin\left(\frac{17\pi}{18}\right)\sqrt{4+36}\right)2} = \frac{1}{2\sqrt{10}}\int_0^{\infty}\frac{1}{t^{17/18}(1+t)}\,dt$$

Multiple-argument formulas:

$$\frac{2\pi}{\left(\sin\left(\frac{17\pi}{18}\right)\sqrt{4+36}\right)2} = \frac{\pi\csc\left(\frac{\pi}{36}\right)\sec\left(\frac{\pi}{36}\right)}{4\sqrt{10}}$$

$$\frac{2\pi}{\left(\sin\left(\frac{17\pi}{18}\right)\sqrt{4+36}\right)2} = \frac{\pi\csc^3\left(\frac{\pi}{54}\right)}{\sqrt{10}\left(-8+6\csc^2\left(\frac{\pi}{54}\right)\right)}$$

For a = 2/(((sin(31Pi/32))*sqrt(4+36)))

(((Pi/2 * [(((2/(((sin(31Pi/32))*sqrt(4+36))))))])))

Input:

$$\frac{\pi}{2}\times\frac{2}{\sin\left(31\times\frac{\pi}{32}\right)\sqrt{4+36}}$$

Exact result:

$$\frac{\pi\csc\left(\frac{\pi}{32}\right)}{2\sqrt{10}}$$

$\csc(x)$ is the cosecant function

Decimal approximation:

5.067781120931135765977187680436669088431487064037941963037...

5.0677811209…

Property:

$\frac{\pi \csc\left(\frac{\pi}{32}\right)}{2\sqrt{10}}$ is a transcendental number

Alternate forms:

$$-\frac{\pi \sin\left(\frac{\pi}{32}\right)}{\sqrt{10}\left(\cos\left(\frac{\pi}{16}\right)-1\right)}$$

$$-\frac{i\pi}{\sqrt{10}\left(e^{-(i\pi)/32}-e^{(i\pi)/32}\right)}$$

$$\frac{(-1)^{17/32}\pi}{\sqrt{10}\left(\sqrt[32]{-1}-1\right)\left(1+\sqrt[32]{-1}\right)}$$

Alternative representations:

$$\frac{2\pi}{\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)2} = \frac{\pi}{\frac{\sqrt{40}}{\csc\left(\frac{31\pi}{32}\right)}}$$

$$\frac{2\pi}{\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)2} = \frac{\pi}{\cos\left(\frac{\pi}{2}-\frac{31\pi}{32}\right)\sqrt{40}}$$

$$\frac{2\pi}{\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)2} = -\frac{\pi}{\cos\left(\frac{\pi}{2}+\frac{31\pi}{32}\right)\sqrt{40}}$$

Series representations:

$$\frac{2\pi}{\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)2} = -\frac{i\pi\sum_{k=1}^{\infty} q^{-1+2k}}{\sqrt{10}} \quad \text{for } q = \sqrt[32]{-1}$$

$$\frac{2\pi}{\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)2} = 8\sqrt{\frac{2}{5}} - 16\sqrt{\frac{2}{5}}\sum_{k=1}^{\infty}\frac{(-1)^k}{-1+1024k^2}$$

$$\frac{2\pi}{\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)2} = 8\sqrt{\frac{2}{5}}\,\pi\sum_{k=-\infty}^{\infty}\frac{e^{i k \pi}}{\pi - 1024 k^2 \pi}$$

Integral representation:

$$\frac{2\pi}{\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)2} = \frac{1}{2\sqrt{10}}\int_0^\infty \frac{1}{t^{31/32}(1+t)}\,dt$$

We observe that multiplying by 1 / π the above expression, we obtain:

1/Pi * (((Pi/2 * [(((2/(((sin(31Pi/32))*sqrt(4+36))))))])))

Input:

$$\frac{1}{\pi}\left(\frac{\pi}{2}\times\frac{2}{\sin\left(31\times\frac{\pi}{32}\right)\sqrt{4+36}}\right)$$

Exact result:

$$\frac{\csc\left(\frac{\pi}{32}\right)}{2\sqrt{10}}$$

$\csc(x)$ is the cosecant function

Decimal approximation:

1.613124831807952934941074995869733061866288398109946676542...

1.6131248318…

Minimal polynomial:

$$200\,000\,000\,x^{16} - 640\,000\,000\,x^{14} + 336\,000\,000\,x^{12} - 67\,200\,000\,x^{10} + 6\,600\,000\,x^8 - 352\,000\,x^6 + 10\,400\,x^4 - 160\,x^2 + 1$$

Alternate forms:

$$-\frac{\sin\left(\frac{\pi}{32}\right)}{\sqrt{10}\left(\cos\left(\frac{\pi}{16}\right)-1\right)}$$

$$\frac{1}{\sqrt{10\left(2-\sqrt{2+\sqrt{2+\sqrt{2}}}\right)}}$$

$$-\frac{i}{\sqrt{10}\left(e^{-(i\pi)/32}-e^{(i\pi)/32}\right)}$$

Alternative representations:

$$\frac{\pi 2}{\left(2\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)\right)\pi}=\frac{\pi}{\frac{\pi\sqrt{40}}{\csc\left(\frac{31\pi}{32}\right)}}$$

$$\frac{\pi 2}{\left(2\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)\right)\pi}=\frac{\pi}{\pi\left(\cos\left(\frac{\pi}{2}-\frac{31\pi}{32}\right)\sqrt{40}\right)}$$

$$\frac{\pi 2}{\left(2\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)\right)\pi}=\frac{\pi}{\pi\left(-\cos\left(\frac{\pi}{2}+\frac{31\pi}{32}\right)\sqrt{40}\right)}$$

Series representations:

$$\frac{\pi 2}{\left(2\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)\right)\pi}=-\frac{i\sum_{k=1}^{\infty}q^{-1+2k}}{\sqrt{10}}\quad\text{for } q=\sqrt[32]{-1}$$

$$\frac{\pi 2}{\left(2\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)\right)\pi}=\frac{8\sqrt{\frac{2}{5}}}{\pi}-\frac{16\sqrt{\frac{2}{5}}\sum_{k=1}^{\infty}\frac{(-1)^k}{-1+1024k^2}}{\pi}$$

$$\frac{\pi 2}{\left(2\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)\right)\pi}=8\sqrt{\frac{2}{5}}\sum_{k=-\infty}^{\infty}\frac{e^{ik\pi}}{\pi-1024k^2\pi}$$

Integral representation:

$$\frac{\pi 2}{\left(2\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)\right)\pi}=\frac{1}{2\sqrt{10}\,\pi}\int_0^{\infty}\frac{1}{t^{31/32}(1+t)}\,dt$$

But also:

1/2* 2/(((sin(31Pi/32))*sqrt(4+36)))

Input:

$$\frac{1}{2}\times\frac{2}{\sin\left(31\times\frac{\pi}{32}\right)\sqrt{4+36}}$$

Exact result:

$$\frac{\csc\left(\frac{\pi}{32}\right)}{2\sqrt{10}}$$

$\csc(x)$ is the cosecant function

Decimal approximation:

1.613124831807952934941074995869733061866288398109946676542...

1.6131248318…

Minimal polynomial:

$$200\,000\,000\,x^{16}-640\,000\,000\,x^{14}+336\,000\,000\,x^{12}-67\,200\,000\,x^{10}+6\,600\,000\,x^{8}-352\,000\,x^{6}+10\,400\,x^{4}-160\,x^{2}+1$$

Alternate forms:

$$-\frac{\sin\left(\frac{\pi}{32}\right)}{\sqrt{10}\left(\cos\left(\frac{\pi}{16}\right)-1\right)}$$

$$\frac{1}{\sqrt{10\left(2-\sqrt{2+\sqrt{2+\sqrt{2}}}\right)}}$$

$$-\frac{i}{\sqrt{10}\left(e^{-(i\pi)/32}-e^{(i\pi)/32}\right)}$$

Alternative representations:

$$\frac{2}{\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)2}=\frac{1}{\frac{\sqrt{40}}{\csc\left(\frac{31\pi}{32}\right)}}$$

$$\frac{2}{\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)2}=\frac{1}{\cos\left(\frac{\pi}{2}-\frac{31\pi}{32}\right)\sqrt{40}}$$

$$\frac{2}{\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)2} = -\frac{1}{\cos\left(\frac{\pi}{2}+\frac{31\pi}{32}\right)\sqrt{40}}$$

Series representations:

$$\frac{2}{\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)2} = -\frac{i\sum_{k=1}^{\infty} q^{-1+2k}}{\sqrt{10}} \quad \text{for } q = \sqrt[32]{-1}$$

$$\frac{2}{\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)2} = \frac{8\sqrt{\frac{2}{5}}}{\pi} - \frac{16\sqrt{\frac{2}{5}}\sum_{k=1}^{\infty}\frac{(-1)^k}{-1+1024k^2}}{\pi}$$

$$\frac{2}{\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)2} = 8\sqrt{\frac{2}{5}}\sum_{k=-\infty}^{\infty}\frac{e^{ik\pi}}{\pi-1024k^2\pi}$$

Integral representation:

$$\frac{2}{\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)2} = \frac{1}{2\sqrt{10}\,\pi}\int_0^{\infty}\frac{1}{t^{31/32}(1+t)}\,dt$$

From which:

1/2 * 2/(((sin(31Pi/32))*sqrt(4+36))) + 5/10^3

Input:

$$\frac{1}{2}\times\frac{2}{\sin\left(31\times\frac{\pi}{32}\right)\sqrt{4+36}}+\frac{5}{10^3}$$

Exact result:

$$\frac{1}{200}+\frac{\csc\left(\frac{\pi}{32}\right)}{2\sqrt{10}}$$

$\csc(x)$ is the cosecant function

Decimal approximation:

1.618124831807952934941074995869733061866288398109946676542...

1.6181248318…

Alternate forms:

$$\frac{1}{200}\left(1+10\sqrt{10}\csc\left(\frac{\pi}{32}\right)\right)$$

$$\frac{1}{200}-\frac{\sin\left(\frac{\pi}{32}\right)}{\sqrt{10}\left(\cos\left(\frac{\pi}{16}\right)-1\right)}$$

$$\frac{\left(100+\sqrt{10}\sin\left(\frac{\pi}{32}\right)\right)\csc\left(\frac{\pi}{32}\right)}{200\sqrt{10}}$$

Alternative representations:

$$\frac{2}{\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)2}+\frac{5}{10^3}=\frac{5}{10^3}+\frac{1}{\frac{\sqrt{40}}{\csc\left(\frac{31\pi}{32}\right)}}$$

$$\frac{2}{\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)2}+\frac{5}{10^3}=\frac{5}{10^3}+\frac{1}{\cos\left(\frac{\pi}{2}-\frac{31\pi}{32}\right)\sqrt{40}}$$

$$\frac{2}{\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)2}+\frac{5}{10^3}=\frac{5}{10^3}+-\frac{1}{\cos\left(\frac{\pi}{2}+\frac{31\pi}{32}\right)\sqrt{40}}$$

Series representations:

$$\frac{2}{\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)2}+\frac{5}{10^3}=\frac{1}{200}-\frac{i\sum_{k=1}^{\infty}q^{-1+2k}}{\sqrt{10}}\quad \text{for } q=\sqrt[32]{-1}$$

$$\frac{2}{\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)2}+\frac{5}{10^3}=\frac{1}{200}+\frac{8\sqrt{\frac{2}{5}}}{\pi}-\frac{16\sqrt{\frac{2}{5}}\sum_{k=1}^{\infty}\frac{(-1)^k}{-1+1024k^2}}{\pi}$$

$$\frac{2}{\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)2}+\frac{5}{10^3}=\frac{1}{200}+8\sqrt{\frac{2}{5}}\sum_{k=-\infty}^{\infty}\frac{e^{ik\pi}}{\pi-1024k^2\pi}$$

Integral representation:

$$\frac{2}{\left(\sin\left(\frac{31\pi}{32}\right)\sqrt{4+36}\right)2}+\frac{5}{10^3}=\frac{1}{200}+\frac{1}{2\sqrt{10}\,\pi}\int_0^\infty \frac{1}{t^{31/32}(1+t)}\,dt$$

We have that:

Case 3: In the limiting case $a = 1$, the integral Eq. (29) simplifies to

$$\varphi_{\text{crit}}^{\pm} = \frac{1}{2}\ln\frac{(b_i \mp 1)(\rho \pm 1)}{(b_i \pm 1)(\rho \mp 1)} \tag{50}$$

with $b_i = b_0/\sqrt{b_0^2 + l_i^2}$. The upper sign has to be used for initial angles $\xi > \pi/2$, $l_f \leq l_i$. The orbital equation $\rho = \rho(\varphi)$ reads

$$\rho_{\text{crit}}^{\pm} = \frac{\pm\sinh\varphi\cosh\varphi(1 - b_i^2) + b_i}{\cosh^2\varphi - b_i^2\sinh^2\varphi} \tag{51}$$

and thus,

$$l_{\text{crit}}^{\pm} = b_0\sqrt{\frac{1}{(\rho_{\text{crit}}^{\pm})^2} - 1}. \tag{52}$$

In that case, a geodesic either starts with the critical angle $\xi = \xi_{\text{crit}}$ (lower sign) and recedes to infinity or it starts with $\xi = \pi - \xi_{\text{crit}}$ and approaches the throat asymptotically (upper sign). For $\xi = \pi - \xi_{\text{crit}}$ the angle φ might grow unlimited, while for $\xi = \xi_{\text{crit}}$ the maximum angle $\varphi_{\max}^{\star}$ reads

$$\varphi_{\max}^{\star} = \frac{1}{2}\ln\frac{1 + b_i}{1 - b_i} = \operatorname{arsinh}\frac{b_0}{l_i} \tag{53}$$

From (53), we obtain:

1/2 ln((1+0.316)/(1-0.316))

Input:

$$\frac{1}{2}\log\left(\frac{1+0.316}{1-0.316}\right)$$

log(x) is the natural logarithm

Result:

0.327197097131356024563392753741224246000387427328728749826...

0.327197097131356….= φ_{max}^*

Alternative representations:

$$\frac{1}{2}\log\left(\frac{1+0.316}{1-0.316}\right)=\frac{\log_e\left(\frac{1.316}{0.684}\right)}{2}$$

$$\frac{1}{2}\log\left(\frac{1+0.316}{1-0.316}\right)=\frac{1}{2}\log(a)\log_a\left(\frac{1.316}{0.684}\right)$$

$$\frac{1}{2}\log\left(\frac{1+0.316}{1-0.316}\right)=-\frac{1}{2}\operatorname{Li}_1\left(1-\frac{1.316}{0.684}\right)$$

Series representations:

$$\frac{1}{2}\log\left(\frac{1+0.316}{1-0.316}\right)=-\frac{1}{2}\sum_{k=1}^{\infty}\frac{(-0.923977)^k}{k}$$

$$\frac{1}{2}\log\left(\frac{1+0.316}{1-0.316}\right)=i\pi\left\lfloor\frac{\arg(1.92398-x)}{2\pi}\right\rfloor+\frac{\log(x)}{2}-\frac{1}{2}\sum_{k=1}^{\infty}\frac{(-1)^k(1.92398-x)^k x^{-k}}{k}$$

for $x<0$

$$\frac{1}{2}\log\left(\frac{1+0.316}{1-0.316}\right)=\frac{1}{2}\left\lfloor\frac{\arg(1.92398-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\frac{\log(z_0)}{2}+\frac{1}{2}\left\lfloor\frac{\arg(1.92398-z_0)}{2\pi}\right\rfloor\log(z_0)-\frac{1}{2}\sum_{k=1}^{\infty}\frac{(-1)^k(1.92398-z_0)^k z_0^{-k}}{k}$$

Integral representations:

$$\frac{1}{2}\log\left(\frac{1+0.316}{1-0.316}\right)=\frac{1}{2}\int_1^{1.92398}\frac{1}{t}\,dt$$

$$\frac{1}{2}\log\left(\frac{1+0.316}{1-0.316}\right)=\frac{1}{4i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{0.0790685s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1<\gamma<0$$

That is about equal to:

asinh(2/6)

Input:

$$\sinh^{-1}\left(\frac{2}{6}\right)$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Exact result:

$$\sinh^{-1}\left(\frac{1}{3}\right)$$

Decimal approximation:

0.327450150237258443322535259988258127700524528990767451275...

0.327450150237…

Property:

$\sinh^{-1}\left(\frac{1}{3}\right)$ is a transcendental number

Alternate forms:

$$\text{csch}^{-1}(3)$$

$$\log\left(\frac{1}{3} + \frac{\sqrt{10}}{3}\right)$$

$\text{csch}^{-1}(x)$ is the inverse hyperbolic cosecant function

$\log(x)$ is the natural logarithm

Alternative representations:

$$\sinh^{-1}\left(\frac{2}{6}\right) = \text{sc}^{-1}\left(\frac{2}{6} \,\middle|\, 1\right)$$

$$\sinh^{-1}\left(\frac{2}{6}\right) = \text{sd}^{-1}\left(\frac{2}{6} \,\middle|\, 1\right)$$

$$\sinh^{-1}\left(\frac{2}{6}\right) = \log\left(\frac{2}{6} + \sqrt{1 + \left(\frac{2}{6}\right)^2}\right)$$

$\text{sc}^{-1}(x \mid m)$ is the inverse of the Jacobi elliptic function sc

$\text{sd}^{-1}(x \mid m)$ is the inverse of the Jacobi elliptic function sd

Series representations:

$$\sinh^{-1}\left(\frac{2}{6}\right) = \sum_{k=0}^{\infty} \frac{(-1)^k \, 3^{-1-2k} \left(\frac{1}{2}\right)_k}{(1+2k)\, k!}$$

$$\sinh^{-1}\left(\frac{2}{6}\right) = \frac{i\pi}{2} - i\sqrt{2+\frac{2i}{3}} \sum_{k=0}^{\infty} \frac{\left(\frac{i}{6}\right)^k (1-3i)^k \left(\frac{1}{2}\right)_k}{(1+2k)\, k!}$$

$$\sinh^{-1}\left(\frac{2}{6}\right) = -\frac{i\pi}{2} + i\sqrt{2-\frac{2i}{3}} \sum_{k=0}^{\infty} \frac{\left(-\frac{i}{6}\right)^k (1+3i)^k \left(\frac{1}{2}\right)_k}{(1+2k)\, k!}$$

Integral representations:

$$\sinh^{-1}\left(\frac{2}{6}\right) = \int_0^1 \frac{1}{\sqrt{9+t^2}}\, dt$$

$$\sinh^{-1}\left(\frac{2}{6}\right) = -\frac{i}{12\,\pi^{3/2}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \left(\frac{9}{10}\right)^s \Gamma\left(\frac{1}{2}-s\right)^2 \Gamma(s)\, \Gamma\left(\frac{1}{2}+s\right) ds \quad \text{for } 0 < \gamma < \frac{1}{2}$$

$$\sinh^{-1}\left(\frac{2}{6}\right) = -\frac{i}{12\,\pi^{3/2}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{9^s\, \Gamma\left(\frac{1}{2}-s\right)^2 \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\, ds \quad \text{for } 0 < \gamma < \frac{1}{2}$$

We note that, from

$$\frac{1}{2} \log\left(\frac{1+0.316}{1-0.316}\right)$$

we obtain:

7/10^3+1+1/5*1/(((1/2 ln((1+0.316)/(1-0.316)))))

Input:

$$\frac{7}{10^3} + 1 + \frac{1}{5} \times \frac{1}{\frac{1}{2}\log\left(\frac{1+0.316}{1-0.316}\right)}$$

log(x) is the natural logarithm

Result:

1.618252366703327808268683133913510843360830994746373582257...

1.6182523667…

Alternative representations:

$$\frac{7}{10^3}+1+\frac{1}{\frac{1}{2}\log\left(\frac{1+0.316}{1-0.316}\right)5}=1+\frac{1}{\frac{5\log_e\left(\frac{1.316}{0.684}\right)}{2}}+\frac{7}{10^3}$$

$$\frac{7}{10^3}+1+\frac{1}{\frac{1}{2}\log\left(\frac{1+0.316}{1-0.316}\right)5}=1+\frac{1}{\frac{5}{2}\left(\log(a)\log_a\left(\frac{1.316}{0.684}\right)\right)}+\frac{7}{10^3}$$

$$\frac{7}{10^3}+1+\frac{1}{\frac{1}{2}\log\left(\frac{1+0.316}{1-0.316}\right)5}=1+\frac{1}{\frac{5}{2}\left(-\text{Li}_1\left(1-\frac{1.316}{0.684}\right)\right)}+\frac{7}{10^3}$$

Series representations:

$$\frac{7}{10^3}+1+\frac{1}{\frac{1}{2}\log\left(\frac{1+0.316}{1-0.316}\right)5}=\frac{1007}{1000}-\frac{2}{5\sum_{k=1}^{\infty}\frac{(-0.923977)^k}{k}}$$

$$\frac{7}{10^3}+1+\frac{1}{\frac{1}{2}\log\left(\frac{1+0.316}{1-0.316}\right)5}=$$
$$\frac{1007}{1000}+\frac{2}{5\left(2\,i\,\pi\left\lfloor\frac{\arg(1.92398-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(1.92398-x)^k x^{-k}}{k}\right)}\quad\text{for } x<0$$

$$\frac{7}{10^3}+1+\frac{1}{\frac{1}{2}\log\left(\frac{1+0.316}{1-0.316}\right)5}=$$
$$\frac{1007}{1000}+\frac{2}{5\left(\log(z_0)+\left\lfloor\frac{\arg(1.92398-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(1.92398-z_0)^k z_0^{-k}}{k}\right)}$$

Integral representations:

$$\frac{7}{10^3}+1+\frac{1}{\frac{1}{2}\log\left(\frac{1+0.316}{1-0.316}\right)5}=\frac{1007}{1000}+\frac{2}{5\int_1^{1.92398}\frac{1}{t}\,dt}$$

$$\frac{7}{10^3}+1+\frac{1}{\frac{1}{2}\log\left(\frac{1+0.316}{1-0.316}\right)5}=\frac{1007}{1000}+\frac{4\,i\,\pi}{5\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{0.0790685\,s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\quad\text{for } -1<\gamma<0$$

From:

$$\rho_{\text{crit}}^{\pm} = \frac{\pm \sinh\varphi \cosh\varphi (1 - b_i^2) + b_i}{\cosh^2\varphi - b_i^2 \sinh^2\varphi}$$

We obtain, for 0.327197097131356….= φ_{max}^{*} and b_i = 0.316

(((sinh(0.327197097131356)cosh(0.327197097131356)(1-0.316^2)+0.316))) / (((cosh^2(0.327197097131356)-0.316^2*sinh^2(0.327197097131356))))

Input interpretation:

$$\frac{\sinh(0.327197097131356)\cosh(0.327197097131356)(1-0.316^2)+0.316}{\cosh^2(0.327197097131356)-0.316^2\sinh^2(0.327197097131356)}$$

sinh(x) is the hyperbolic sine function

cosh(x) is the hyperbolic cosine function

Result:

0.574620677615978802592569755587082437008154852301414723536...

0.5746206776159788……

Rational approximation:

$$\frac{39500}{68741}$$

Alternative representations:

$$\frac{\sinh(0.3271970971313560000)\cosh(0.3271970971313560000)(1-0.316^2)+0.316}{\cosh^2(0.3271970971313560000)-0.316^2\sinh^2(0.3271970971313560000)} = \left(0.316+\frac{1}{2}\cos(-0.3271970971313560000\,i)(1-0.316^2)\left(-\frac{1}{e^{0.3271970971313560000}}+e^{0.3271970971313560000}\right)\right)\Big/\left(\cos^2(-0.3271970971313560000\,i)-0.316^2\left(\frac{1}{2}\left(-\frac{1}{e^{0.3271970971313560000}}+e^{0.3271970971313560000}\right)\right)^2\right)$$

$$\frac{\sinh(0.3271970971313560000)\cosh(0.3271970971313560000)(1-0.316^2)+0.316}{\cosh^2(0.3271970971313560000)-0.316^2\sinh^2(0.3271970971313560000)} = \left(0.316+\frac{1}{2}\cos(0.3271970971313560000\,i)(1-0.316^2)\left(-\frac{1}{e^{0.3271970971313560000}}+e^{0.3271970971313560000}\right)\right) \Big/ \left(\cos^2(0.3271970971313560000\,i) - 0.316^2\left(\frac{1}{2}\left(-\frac{1}{e^{0.3271970971313560000}}+e^{0.3271970971313560000}\right)\right)^2\right)$$

$$\frac{\sinh(0.3271970971313560000)\cosh(0.3271970971313560000)(1-0.316^2)+0.316}{\cosh^2(0.3271970971313560000)-0.316^2\sinh^2(0.3271970971313560000)} = \left(0.316+i\cos(-0.3271970971313560000\,i)\cos\left(0.3271970971313560000\,i+\frac{\pi}{2}\right)(1-0.316^2)\right) \Big/ \left(\cos^2(-0.3271970971313560000\,i) - 0.316^2\left(i\cos\left(0.3271970971313560000\,i+\frac{\pi}{2}\right)\right)^2\right)$$

Series representations:

$$\frac{\sinh(0.3271970971313560000)\cosh(0.3271970971313560000)(1-0.316^2)+0.316}{\cosh^2(0.3271970971313560000)-0.316^2\sinh^2(0.3271970971313560000)} = -\left(\left(4.50721\left(0.175527+\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{e^{-2.234385092281668246 k_2} I_{1+2k_1}(0.3271970971313560000)}{(2k_2)!}\right)\right) \Big/ \left(\left(\sum_{k=0}^{\infty} I_{1+2k}(0.3271970971313560000)\right)^2 - 2.50361\left(\sum_{k=0}^{\infty}\frac{e^{-2.234385092281668246 k}}{(2k)!}\right)^2\right)\right)$$

$$\frac{\sinh(0.3271970971313560000)\cosh(0.3271970971313560000)(1-0.316^2)+0.316}{\cosh^2(0.3271970971313560000)-0.316^2\sinh^2(0.3271970971313560000)} = \left(0.900144\left(0.351055+\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{0.3271970971313560000^{1+2k_2} e^{-2.234385092281668246 k_1}}{(2k_1)!(1+2k_2)!}\right)\right) \Big/ \left(\left(\sum_{k=0}^{\infty}\frac{e^{-2.234385092281668246 k}}{(2k)!}\right)^2 - 0.099856\left(\sum_{k=0}^{\infty}\frac{0.3271970971313560000^{1+2k}}{(1+2k)!}\right)^2\right)$$

$$\frac{\sinh(0.3271970971313560000)\cosh(0.3271970971313560000)(1-0.316^2)+0.316}{\cosh^2(0.3271970971313560000)-0.316^2\sinh^2(0.3271970971313560000)}$$

$$= -\left(\left(9.01442\left(0.351055 + i\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(0.3271970971313560000^{1+2k_1}\left(0.3271970971313560000-\frac{i\pi}{2}\right)^{1+2k_2}\right)\Big/\left((1+2k_1)!\,(1+2k_2)!\right)\right)\right)\Big/\left(\left(\sum_{k=0}^{\infty}\frac{0.3271970971313560000^{1+2k}}{(1+2k)!}\right)^2 - 10.0144\,i^2\left(\sum_{k=0}^{\infty}\frac{\left(0.3271970971313560000-\frac{i\pi}{2}\right)^{1+2k}}{(1+2k)!}\right)^2\right)\right)$$

From which, we obtain:

((cot^2(1 - π/16)))1/((((((sinh(0.327197097131356)cosh(0.327197097131356)(1-0.316^2)+0.316))) / (((cosh^2(0.327197097131356)-0.316^2*sinh^2(0.327197097131356)))))))

where

$$\cot^2\left(1-\frac{\pi}{16}\right) \approx 0.929577$$

Note that, this value is very near to the range of α’, the Regge slope (string tension) of K meson:

$$K^* \;\big|\; 5 \;\big|\; m_{u/d} = 0-240 \quad m_s = 0-390 \;\big|\; 0.848-0.927$$

Input interpretation:

$$\cot^2\left(1-\frac{\pi}{16}\right) \times \frac{1}{\frac{\sinh(0.327197097131356)\cosh(0.327197097131356)(1-0.316^2)+0.316}{\cosh^2(0.327197097131356)-0.316^2\sinh^2(0.327197097131356)}}$$

$\cot(x)$ is the cotangent function

$\sinh(x)$ is the hyperbolic sine function

$\cosh(x)$ is the hyperbolic cosine function

Result:

1.617723223947977654863329904464980258323387005416024876903...

1.617723223947...

Alternative representations:

$$\frac{\cot^2\left(1-\frac{\pi}{16}\right)}{\frac{\sinh(0.327197097131356000)\cosh(0.327197097131356000)(1-0.316^2)+0.316}{\cosh^2(0.327197097131356000)-0.316^2\sinh^2(0.327197097131356000)}} = \left(i+\frac{2i}{-1+e^{2i(1-\pi/16)}}\right)^2 \Big/ \left(0.316+\frac{1}{2}\cos(-0.327197097131356000\,i)(1-0.316^2)\left(-\frac{1}{e^{0.327197097131356000}}+e^{0.327197097131356000}\right)\right) \Big/ \left(\cos^2(-0.327197097131356000\,i)-0.316^2\left(\frac{1}{2}\left(-\frac{1}{e^{0.327197097131356000}}+e^{0.327197097131356000}\right)\right)^2\right)$$

$$\frac{\cot^2\left(1-\frac{\pi}{16}\right)}{\frac{\sinh(0.327197097131356000)\cosh(0.327197097131356000)(1-0.316^2)+0.316}{\cosh^2(0.327197097131356000)-0.316^2\sinh^2(0.327197097131356000)}} = \left(-i\coth\left(-i\left(1-\frac{\pi}{16}\right)\right)\right)^2 \Big/ \left(0.316+\frac{1}{4}(1-0.316^2)\left(-\frac{1}{e^{0.327197097131356000}}+e^{0.327197097131356000}\right)\left(\frac{1}{e^{0.327197097131356000}}+e^{0.327197097131356000}\right)\right) \Big/ \left(-0.316^2\left(\frac{1}{2}\left(-\frac{1}{e^{0.327197097131356000}}+e^{0.327197097131356000}\right)\right)^2+\left(\frac{1}{2}\left(\frac{1}{e^{0.327197097131356000}}+e^{0.327197097131356000}\right)\right)^2\right)$$

$$\frac{\cot^2\left(1-\frac{\pi}{16}\right)}{\frac{\sinh(0.327197097131356000)\cosh(0.327197097131356000)(1-0.316^2)+0.316}{\cosh^2(0.327197097131356000)-0.316^2\sinh^2(0.327197097131356000)}} = \left(i+\frac{2i}{-1+e^{2i(1-\pi/16)}}\right)^2 \Big/ \left(0.316+\frac{1}{4}(1-0.316^2)\left(-\frac{1}{e^{0.327197097131356000}}+e^{0.327197097131356000}\right)\left(\frac{1}{e^{0.327197097131356000}}+e^{0.327197097131356000}\right)\right) \Big/ \left(-0.316^2\left(\frac{1}{2}\left(-\frac{1}{e^{0.327197097131356000}}+e^{0.327197097131356000}\right)\right)^2+\left(\frac{1}{2}\left(\frac{1}{e^{0.327197097131356000}}+e^{0.327197097131356000}\right)\right)^2\right)$$

Series representations:

$$\frac{\cot^2\left(1-\frac{\pi}{16}\right)}{\frac{\sinh(0.327197097131356000)\cosh(0.327197097131356000)\left(1-0.316^2\right)+0.316}{\cosh^2(0.327197097131356000)-0.316^2\sinh^2(0.327197097131356000)}} = \left(\left(i+2\,i\sum_{k=1}^{\infty} q^{2k}\right)^2 \left(\left(I_0(0.327197097131356000)+2\sum_{k=1}^{\infty} I_{2k}(0.327197097131356000)\right)^2 - 0.399424\left(\sum_{k=0}^{\infty} I_{1+2k}(0.327197097131356000)\right)^2\right)\right) \Big/ \left(0.316+1.80029\left(I_0(0.327197097131356000)+2\sum_{k=1}^{\infty} I_{2k}(0.327197097131356000)\right)\sum_{k=0}^{\infty} I_{1+2k}(0.327197097131356000)\right) \text{ for } \sqrt[16]{-1}\; q = e^i$$

$$\frac{\cot^2\left(1-\frac{\pi}{16}\right)}{\frac{\sinh(0.327197097131356000)\cosh(0.327197097131356000)\left(1-0.316^2\right)+0.316}{\cosh^2(0.327197097131356000)-0.316^2\sinh^2(0.327197097131356000)}} = \left(\left(i+2\,i\sum_{k=1}^{\infty} q^{2k}\right)^2 \left(\left(I_0(0.327197097131356000)+2\sum_{k=1}^{\infty} I_{2k}(0.327197097131356000)\right)^2 - 0.099856\left(\sum_{k=0}^{\infty} \frac{0.327197097131356000^{1+2k}}{(1+2k)!}\right)^2\right)\right) \Big/ \left(0.316+0.900144\left(I_0(0.327197097131356000)+2\sum_{k=1}^{\infty} I_{2k}(0.327197097131356000)\right)\sum_{k=0}^{\infty} \frac{0.327197097131356000^{1+2k}}{(1+2k)!}\right) \text{ for } \sqrt[16]{-1}\; q = e^i$$

$$\frac{\cot^2\left(1-\frac{\pi}{16}\right)}{\frac{\sinh(0.327197097131356000)\cosh(0.327197097131356000)\left(1-0.316^2\right)+0.316}{\cosh^2(0.327197097131356000)-0.316^2\sinh^2(0.327197097131356000)}} = \left(\left(1-\frac{\pi}{16}\right)^2 \left(\sum_{k=-\infty}^{\infty} \frac{1}{\left(1-\frac{\pi}{16}\right)^2-k^2\pi^2}\right)^2 \left(\left(I_0(0.327197097131356000)+2\sum_{k=1}^{\infty} I_{2k}(0.327197097131356000)\right)^2 - 0.399424\left(\sum_{k=0}^{\infty} I_{1+2k}(0.327197097131356000)\right)^2\right)\right) \Big/ \left(0.316+1.80029\left(I_0(0.327197097131356000)+2\sum_{k=1}^{\infty} I_{2k}(0.327197097131356000)\right)\sum_{k=0}^{\infty} I_{1+2k}(0.327197097131356000)\right)$$

From:

$$\varphi^{\pm}_{\text{crit}} = \frac{1}{2} \ln \frac{(b_i \mp 1)(\rho \pm 1)}{(b_i \pm 1)(\rho \mp 1)}$$

we obtain, for ρ = 0.5746206776159788 and b_i = 0.316 :

1/2 ln [(((0.316+1)(0.5746206776159788 - 1))) / (((0.316-1)(0.5746206776159788+1)))]

Input interpretation:

$$\frac{1}{2} \log\left(\frac{(0.316+1)(0.5746206776159788-1)}{(0.316-1)(0.5746206776159788+1)}\right)$$

log(x) is the natural logarithm

Result:

−0.327197097131355996129401490707423712413097079498940967 30...

-0.32719709713…

Alternative representations:

$$\frac{1}{2} \log\left(\frac{(0.316+1)(0.57462067761597880000-1)}{(0.316-1)(0.57462067761597880000+1)}\right) = \frac{1}{2} \log_e\left(\frac{-0.559799}{-1.07704}\right)$$

$$\frac{1}{2} \log\left(\frac{(0.316+1)(0.57462067761597880000-1)}{(0.316-1)(0.57462067761597880000+1)}\right) = \frac{1}{2} \log(a) \log_a\left(\frac{-0.559799}{-1.07704}\right)$$

$$\frac{1}{2} \log\left(\frac{(0.316+1)(0.57462067761597880000-1)}{(0.316-1)(0.57462067761597880000+1)}\right) = -\frac{1}{2} \text{Li}_1\left(1 + -\frac{0.559799}{1.07704}\right)$$

Series representations:

$$\frac{1}{2} \log\left(\frac{(0.316+1)(0.57462067761597880000-1)}{(0.316-1)(0.57462067761597880000+1)}\right) = -\frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k (-0.480243)^k}{k}$$

$$\frac{1}{2} \log\left(\frac{(0.316+1)(0.57462067761597880000-1)}{(0.316-1)(0.57462067761597880000+1)}\right) = i \pi \left\lfloor \frac{\arg(0.519757 - x)}{2 \pi} \right\rfloor + \frac{\log(x)}{2} - \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k (0.519757 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\frac{1}{2}\log\left(\frac{(0.316+1)(0.57462067761597880000-1)}{(0.316-1)(0.57462067761597880000+1)}\right)=$$
$$\frac{1}{2}\left\lfloor\frac{\arg(0.519757-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\frac{\log(z_0)}{2}+$$
$$\frac{1}{2}\left\lfloor\frac{\arg(0.519757-z_0)}{2\pi}\right\rfloor\log(z_0)-\frac{1}{2}\sum_{k=1}^{\infty}\frac{(-1)^k(0.519757-z_0)^k z_0^{-k}}{k}$$

Integral representation:

$$\frac{1}{2}\log\left(\frac{(0.316+1)(0.57462067761597880000-1)}{(0.316-1)(0.57462067761597880000+1)}\right)=\frac{1}{2}\int_1^{0.519757}\frac{1}{t}\,dt$$

Or, inverting the sign:

Input interpretation:

$$\frac{1}{2}\log\left(\frac{(0.316-1)(0.5746206776159788+1)}{(0.316+1)(0.5746206776159788-1)}\right)$$

$\log(x)$ is the natural logarithm

Result:

0.327197097131355996129401490707423712413097079498940967301...

0.32719709713…

Alternative representations:

$$\frac{1}{2}\log\left(\frac{(0.316-1)(0.57462067761597880000+1)}{(0.316+1)(0.57462067761597880000-1)}\right)=\frac{1}{2}\log_e\left(\frac{-1.07704}{-0.559799}\right)$$

$$\frac{1}{2}\log\left(\frac{(0.316-1)(0.57462067761597880000+1)}{(0.316+1)(0.57462067761597880000-1)}\right)=\frac{1}{2}\log(a)\log_a\left(\frac{-1.07704}{-0.559799}\right)$$

$$\frac{1}{2}\log\left(\frac{(0.316-1)(0.57462067761597880000+1)}{(0.316+1)(0.57462067761597880000-1)}\right)=-\frac{1}{2}\operatorname{Li}_1\left(1+-\frac{1.07704}{0.559799}\right)$$

Series representations:

$$\frac{1}{2}\log\left(\frac{(0.316-1)(0.57462067761597880000+1)}{(0.316+1)(0.57462067761597880000-1)}\right)=-\frac{1}{2}\sum_{k=1}^{\infty}\frac{(-0.923977)^k}{k}$$

$$\frac{1}{2}\log\left(\frac{(0.316-1)(0.57462067761597880000+1)}{(0.316+1)(0.57462067761597880000-1)}\right) = i\pi\left\lfloor\frac{\arg(1.92398-x)}{2\pi}\right\rfloor + \frac{\log(x)}{2} - \frac{1}{2}\sum_{k=1}^{\infty}\frac{(-1)^k (1.92398-x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\frac{1}{2}\log\left(\frac{(0.316-1)(0.57462067761597880000+1)}{(0.316+1)(0.57462067761597880000-1)}\right) = \frac{1}{2}\left\lfloor\frac{\arg(1.92398-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right) + \frac{\log(z_0)}{2} + \frac{1}{2}\left\lfloor\frac{\arg(1.92398-z_0)}{2\pi}\right\rfloor\log(z_0) - \frac{1}{2}\sum_{k=1}^{\infty}\frac{(-1)^k (1.92398-z_0)^k z_0^{-k}}{k}$$

Integral representations:

$$\frac{1}{2}\log\left(\frac{(0.316-1)(0.57462067761597880000+1)}{(0.316+1)(0.57462067761597880000-1)}\right) = \frac{1}{2}\int_1^{1.92398}\frac{1}{t}\,dt$$

$$\frac{1}{2}\log\left(\frac{(0.316-1)(0.57462067761597880000+1)}{(0.316+1)(0.57462067761597880000-1)}\right) = \frac{1}{4i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{0.0790685\,s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1 < \gamma < 0$$

From which:

1+(1/(3 sqrt(e))) 1/(((1/2 ln [(((0.316-1)(0.5746206776159788 + 1))) / (((0.316+1)(0.5746206776159788-1)))])))

Input interpretation:

$$1 + \frac{1}{3\sqrt{e}} \times \frac{1}{\frac{1}{2}\log\left(\frac{(0.316-1)(0.5746206776159788+1)}{(0.316+1)(0.5746206776159788-1)}\right)}$$

log(x) is the natural logarithm

Result:

1.617905502045796620050298406557096782985609782750190666936...

1.6179055020457…

Alternative representations:

$$1+\frac{1}{\frac{1}{2}\log\left(\frac{(0.316-1)(0.57462067761597880000+1)}{(0.316+1)(0.57462067761597880000-1)}\right)\left(3\sqrt{e}\right)}=1+\frac{1}{\frac{1}{2}\log_e\left(\frac{-1.07704}{-0.559799}\right)\left(3\sqrt{e}\right)}$$

$$1+\frac{1}{\frac{1}{2}\log\left(\frac{(0.316-1)(0.57462067761597880000+1)}{(0.316+1)(0.57462067761597880000-1)}\right)\left(3\sqrt{e}\right)}=$$
$$1+\frac{1}{\frac{1}{2}\left(\log(a)\log_a\left(\frac{-1.07704}{-0.559799}\right)\right)\left(3\sqrt{e}\right)}$$

$$1+\frac{1}{\frac{1}{2}\log\left(\frac{(0.316-1)(0.57462067761597880000+1)}{(0.316+1)(0.57462067761597880000-1)}\right)\left(3\sqrt{e}\right)}=1+-\frac{1}{\frac{1}{2}\operatorname{Li}_1\left(1+-\frac{1.07704}{0.559799}\right)\left(3\sqrt{e}\right)}$$

Series representations:

$$1+\frac{1}{\frac{1}{2}\log\left(\frac{(0.316-1)(0.57462067761597880000+1)}{(0.316+1)(0.57462067761597880000-1)}\right)\left(3\sqrt{e}\right)}=$$
$$\frac{-2+3\sqrt{-1+e}\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-0.923977)^{k_1}(-1+e)^{-k_2}\binom{\frac{1}{2}}{k_2}}{k_1}}{3\sqrt{-1+e}\left(\sum_{k=1}^{\infty}\frac{(-0.923977)^k}{k}\right)\sum_{k=0}^{\infty}(-1+e)^{-k}\binom{\frac{1}{2}}{k}}$$

$$1+\frac{1}{\frac{1}{2}\log\left(\frac{(0.316-1)(0.57462067761597880000+1)}{(0.316+1)(0.57462067761597880000-1)}\right)\left(3\sqrt{e}\right)}=$$
$$1+\frac{2}{3\exp\left(i\pi\left\lfloor\frac{\arg(e-x)}{2\pi}\right\rfloor\right)\log(1.92398)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(e-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$
for ($x\in\mathbb{R}$ and $x<0$)

$$1+\frac{1}{\frac{1}{2}\log\left(\frac{(0.316-1)(0.57462067761597880000+1)}{(0.316+1)(0.57462067761597880000-1)}\right)\left(3\sqrt{e}\right)}=$$
$$1+\frac{2\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(e-z_0)/(2\pi)\rfloor}z_0^{-1/2-1/2\lfloor\arg(e-z_0)/(2\pi)\rfloor}}{3\log(1.92398)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(e-z_0)^k z_0^{-k}}{k!}}$$

Integral representations:

$$1+\frac{1}{\frac{1}{2}\log\left(\frac{(0.316-1)(0.57462067761597880000+1)}{(0.316+1)(0.57462067761597880000-1)}\right)\left(3\sqrt{e}\right)}=1+\frac{2}{3\sqrt{e}\int_1^{1.92398}\frac{1}{t}\,dt}$$

$$1+\frac{1}{\frac{1}{2}\log\left(\frac{(0.316-1)(0.5746206776159788000+1)}{(0.316+1)(0.5746206776159788000-1)}\right)\left(3\sqrt{e}\right)} = 1+\frac{4\,i\,\pi}{3\sqrt{e}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{0.0790685\,s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,d\,s} \quad \text{for } -1<\gamma<0$$

Conclusion

We note that many results of the Ramanujan's elliptic integrals are very nears to the various solutions of the Wormhole equations. Especially, the values concerning the golden ratio, are those that are most frequent.

Observations

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or *by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.*

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} \quad = \qquad 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that *sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.*

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to ζ(2) = $\frac{\pi^2}{6} = 1.644934 \ldots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

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Exact geometric optics in a Morris-Thorne wormhole spacetime
Thomas Muller - Visualisierungsinstitut der Universitat Stuttgart, Nobelstrasse 15, 70569 Stuttgart, Germany - (Received 17 December 2007; published 26 February 2008) - PHYSICAL REVIEW D **77,** 044043 (2008)