On the Ramanujan's integral equations and Wormholes Mathematics: further connections with ϕ, $\zeta(2)$, and some Standard Model of Particle Physics parameters. VI

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Abstract

In this paper we have described several Ramanujan integral equations and Wormholes formulas. Furthermore, we obtain connections with ϕ, $\zeta(2)$, and some Standard Model of Particle Physics parameters.

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https://www.cse.iitk.ac.in/users/amit/books/hardy-1999-ramanujan-twelve-lectures.html

From

George E. Andrews Bruce C. Berndt
Ramanujan's Lost Notebook Part I - 2005 Springer Science+Business Media, Inc.

We have that:

$$\begin{aligned}
&\left(\frac{1}{qf(-q)f(-q^5)f(-q^7)f(-q^{35})}\frac{dv}{dq}\right)^2 \\
&= \frac{v^2}{36}\left\{\frac{49}{R^2}\left(Q^6+\frac{125}{Q^6}+22\right)+R^2\left(P^6+\frac{125}{P^6}+22\right)\right. \\
&\qquad\left. -14\sqrt{\left(Q^6+\frac{125}{Q^6}+22\right)\left(P^6+\frac{125}{P^6}+22\right)}\right\} \\
&= \frac{v^2}{36}\left\{\frac{(V_1-V_2)}{2}\left(\frac{U_1+U_2}{2}+22\right)+\frac{(V_1+V_2)}{2}\left(\frac{U_1-U_2}{2}+22\right)\right. \\
&\qquad\left. -14\sqrt{\left(\frac{U_1}{2}+22\right)^2-\frac{U_2^2}{4}}\right\} \\
&= \frac{v^2}{36}\left\{\frac{1}{2}(U_1V_1-U_2V_2)+22V_1-14\sqrt{\left(\frac{U_1}{2}+22\right)^2-\frac{U_2^2}{4}}\right\} \\
&= v^2(K^4-4K^3-2K^2-16K-19), \qquad (15.9.16)
\end{aligned}$$

$$\begin{aligned}
&= \frac{v^2}{36}\left\{\frac{49}{R^2}\left(Q^6+\frac{125}{Q^6}+22\right)+R^2\left(P^6+\frac{125}{P^6}+22\right)\right. \\
&\qquad\left. -14\sqrt{\left(Q^6+\frac{125}{Q^6}+22\right)\left(P^6+\frac{125}{P^6}+22\right)}\right\}
\end{aligned}$$

4/36 [49/49(25+125/25+22)+49(25+125/25+22)-14*sqrt(((25+125/25+22)(25+125/25+22)))]

Input:

$$\frac{4}{36}\left(\frac{49}{49}\left(25+\frac{125}{25}+22\right)+49\left(25+\frac{125}{25}+22\right)-14\sqrt{\left(25+\frac{125}{25}+22\right)\left(25+\frac{125}{25}+22\right)}\right)$$

Exact result:

208

208

v **= 2; R = 7; Q**^{6} **= P**^{6} **= 25**

$$= \frac{v^2}{36}\left\{\frac{(V_1 - V_2)}{2}\left(\frac{U_1 + U_2}{2} + 22\right) + \frac{(V_1 + V_2)}{2}\left(\frac{U_1 - U_2}{2} + 22\right) - 14\sqrt{\left(\frac{U_1}{2} + 22\right)^2 - \frac{U_2^2}{4}}\right\}$$

4/36[((x/2(5/2+22)))+x/2((1/2+22))-14sqrt(((3/2+22)^2-(2^2)/4))] = 208

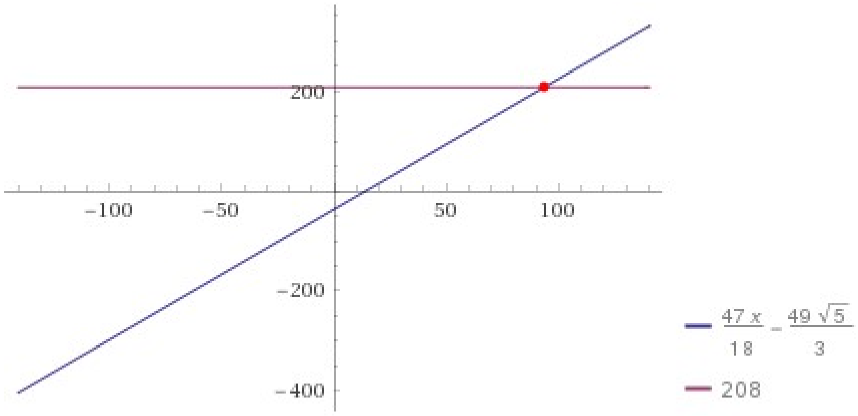
Input:

$$\frac{4}{36}\left(\frac{x}{2}\left(\frac{5}{2}+22\right)+\frac{x}{2}\left(\frac{1}{2}+22\right)-14\sqrt{\left(\frac{3}{2}+22\right)^2-\frac{2^2}{4}}\right)=208$$

Exact result:

$$\frac{1}{9}\left(\frac{47\,x}{2}-147\sqrt{5}\right)=208$$

Plot:



Alternate forms:

$$\frac{47\,x}{18}-\frac{49\sqrt{5}}{3}=208$$

$$\frac{47x}{18} - \frac{49\sqrt{5}}{3} - 208 = 0$$

$$\frac{1}{18}\left(47x - 294\sqrt{5}\right) = 208$$

Solution:

$$x = \frac{3744}{47} + \frac{294\sqrt{5}}{47}$$

Solution:

$$x \approx 93.647$$

93.647

4/36[((93.647/2((3+2)/2+22)))+x/2(((3-2)/2+22))-14sqrt(((3/2+22)^2-(2^2)/4))] = 208

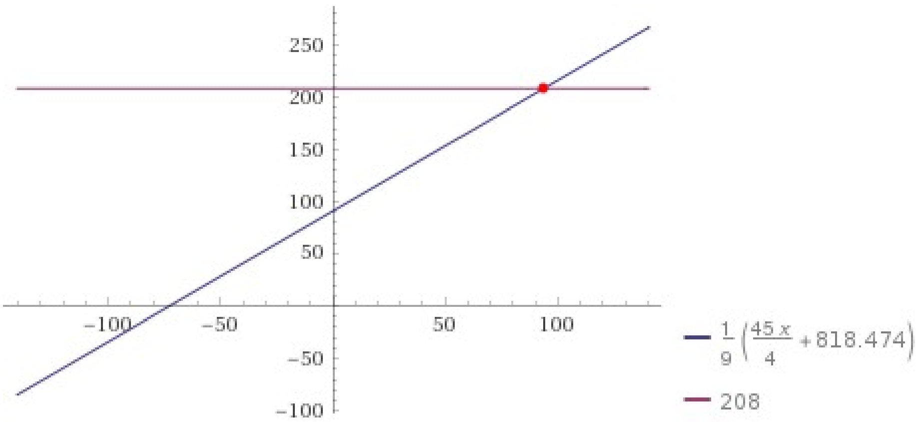
Input interpretation:

$$\frac{4}{36}\left(\frac{93.647}{2}\left(\frac{3+2}{2}+22\right)+\frac{x}{2}\left(\frac{3-2}{2}+22\right)-14\sqrt{\left(\frac{3}{2}+22\right)^2-\frac{2^2}{4}}\right) = 208$$

Result:

$$\frac{1}{9}\left(\frac{45x}{4}+818.474\right) = 208$$

Plot:



Alternate forms:

$$1.25(x + 72.7532) = 208$$

$$\frac{5x}{4} - 117.058 = 0$$

Expanded form:

$$\frac{5x}{4} + 90.9415 = 208$$

Solution:

$x \approx 93.6468$

93.6468

Indeed:

4/36[((93.647/2((3+2)/2+22)))+93.6468/2(((3-2)/2+22))-14sqrt(((3/2+22)^2-(2^2)/4))]

Input interpretation:

$$\frac{4}{36}\left(\frac{93.647}{2}\left(\frac{3+2}{2}+22\right)+\frac{93.6468}{2}\left(\frac{3-2}{2}+22\right)-14\sqrt{\left(\frac{3}{2}+22\right)^2-\frac{2^2}{4}}\right)$$

Result:

208.000...

208

$$\frac{v^2}{36}\left\{\frac{1}{2}(U_1V_1-U_2V_2)+22V_1-14\sqrt{\left(\frac{U_1}{2}+22\right)^2-\frac{U_2^2}{4}}\right\}$$

4/36 [1/2(25*93.6468)+22*46.8234-14*sqrt((((3/2+22)^2-1)))]

Input interpretation:

$$\frac{4}{36}\left(\frac{1}{2}(25\times 93.6468)+22\times 46.8234-14\sqrt{\left(\frac{3}{2}+22\right)^2-1}\right)$$

Result:

208.000...

208

$U_1 = 3$; $U_2 = 2$; $V_1 - V_2 = 93.647$; $V_1 + V_2 = 93.6468$

$U_1V_1 - U_2V_2 = 25\ (V_1 + V_2)$

From

$$v^2(K^4 - 4K^3 - 2K^2 - 16K - 19)$$

we obtain:

4(x^4 - 4x^3 -2x^2 - 16x -19) = 208

Input:

$$4\left(x^4 - 4x^3 - 2x^2 - 16x - 19\right) = 208$$

Alternate forms:

$$4(x+1)(x((x-5)x+3)-19) = 208$$

$$x^4 - 4x^3 - 2x^2 - 16x = 71$$

$$4(x+1)\left(x^3 - 5x^2 + 3x - 19\right) = 208$$

Expanded form:

$$4x^4 - 16x^3 - 8x^2 - 64x - 76 = 208$$

Real solutions:

$$x = 1 + \frac{1}{\sqrt{\frac{6}{8 - \frac{260}{\sqrt[3]{411\sqrt{129} - 2053}} + \sqrt[3]{411\sqrt{129} - 2053}}}} - \frac{1}{2}\sqrt{\left(\frac{32}{3} + \frac{520}{3\sqrt[3]{411\sqrt{129} - 2053}} - \frac{2}{3}\sqrt[3]{411\sqrt{129} - 2053} + 28\sqrt{\frac{6}{8 - \frac{260}{\sqrt[3]{411\sqrt{129} - 2053}} + \sqrt[3]{411\sqrt{129} - 2053}}}\right)}$$

$$x = 1 + \frac{1}{\sqrt{\frac{6}{8 - \frac{260}{\sqrt[3]{411\sqrt{129} - 2053}} + \sqrt[3]{411\sqrt{129} - 2053}}}} +$$

$$\frac{1}{2}\sqrt{\left(\frac{32}{3} + \frac{520}{3\sqrt[3]{411\sqrt{129} - 2053}} - \frac{2}{3}\sqrt[3]{411\sqrt{129} - 2053} + \right.}$$

$$\left. 28\sqrt{\frac{6}{8 - \frac{260}{\sqrt[3]{411\sqrt{129} - 2053}} + \sqrt[3]{411\sqrt{129} - 2053}}}\right)$$

Real solutions:

$x \approx -1.9886$

$x \approx 5.3803$

Complex solutions:

$x \approx 0.3041 - 2.5580\, i$

$x \approx 0.3041 + 2.5580\, i$

Indeed:

4(5.3803^4 - 4*5.3803^3 -2*5.3803^2 - 16*5.3803 -19)

Input interpretation:

$4(5.3803^4 - 4 \times 5.3803^3 - 2 \times 5.3803^2 + 16 \times (-5.3803) - 19)$

Result:

207.9896888257961924

207.9896888257961924 ≈ 208

K = 5.3803

Now:

$$(PQ)^3 + \frac{125}{(PQ)^3} = (K^3 - 7K^2 + 9K + 7)\sqrt{K^2 + 4}. \tag{15.9.3}$$

For K = 5.3803 , we obtain:

(5.3803^3-7*5.3803^2+9*5.3803+7) (5.3803^2+4)^1/2

Input interpretation:

$(5.3803^3 - 7\times 5.3803^2 + 9\times 5.3803 + 7)\sqrt{5.3803^2 + 4}$

Result:

48.9980...

48.9980…

x^3 + 125/x^3 = 48.9980

Input interpretation:

$x^3 + \frac{125}{x^3} = 48.9980$

Result:

$x^3 + \frac{125}{x^3} = 48.998$

Alternate forms:

$\frac{x^6 + 125}{x^3} = 48.998$

$\frac{(x^2 + 5)(x^4 - 5x^2 + 25)}{x^3} = 48.998$

Alternate form assuming x is positive:

$(x - 4.98323)x + 5 = 0$ (for $x \neq 0$)

Real solutions:

$x \approx 1.39246$

$x \approx 3.59077$

PQ = 3.59077

Indeed:

3.59077^3 + 125/(3.59077^3)

Input interpretation:

$$3.59077^3 + \frac{125}{3.59077^3}$$

Result:

48.997954222661702878954991742622960859647630635450293026 36...

48.997954222…

Now:

$$\frac{v^2}{36}\left\{\frac{49}{R^2}\left(Q^6+\frac{125}{Q^6}+22\right)+R^2\left(P^6+\frac{125}{P^6}+22\right) - 14\sqrt{\left(Q^6+\frac{125}{Q^6}+22\right)\left(P^6+\frac{125}{P^6}+22\right)}\right\}$$

For PQ = 3.59077; R = 7 and v = 2

4/36 [49/49(33.49234+125/33.49234+22)+49(33.49234+125/33.49234+22)-14*sqrt(((33.49234+125/33.49234+22)(33.49234+125/33.49234+22)))]

Input interpretation:

$$\frac{4}{36}\left(\frac{49}{49}\left(33.49234+\frac{125}{33.49234}+22\right)+49\left(33.49234+\frac{125}{33.49234}+22\right)- 14\sqrt{\left(33.49234+\frac{125}{33.49234}+22\right)\left(33.49234+\frac{125}{33.49234}+22\right)}\right)$$

Result:

236.89814670167566673454288353695203141972164381467523618 83...

236.8981467.... ≈ 237

(note that 237 – 29 = 208, where 29 is a Lucas number)

We have:

((((4/36 [49/49(33.49234+125/33.49234+22)+49(33.49234+125/33.49234+22)-14*sqrt(((33.49234+125/33.49234+22)(33.49234+125/33.49234+22)))])))))^1/11

Input interpretation:

$$\left(\frac{4}{36}\left(\frac{49}{49}\left(33.49234+\frac{125}{33.49234}+22\right)+49\left(33.49234+\frac{125}{33.49234}+22\right)-14\sqrt{\left(33.49234+\frac{125}{33.49234}+22\right)\left(33.49234+\frac{125}{33.49234}+22\right)}\right)\right)\hat{}\,(1/11)$$

Result:

1.643876708364981094535677549629922376952175249791381059285...

1.64387670836…

While:

(((4/36 [49/49(25+125/25+22)+49(25+125/25+22)-14*sqrt(((25+125/25+22)(25+125/25+22)))])))^1/11

Input:

$$\left(\frac{4}{36}\left(\frac{49}{49}\left(25+\frac{125}{25}+22\right)+49\left(25+\frac{125}{25}+22\right)-14\sqrt{\left(25+\frac{125}{25}+22\right)\left(25+\frac{125}{25}+22\right)}\right)\right)\hat{}\,(1/11)$$

Result:

$2^{4/11}\sqrt[11]{13}$

Decimal approximation:

1.624549805320111953535335931030485439487366781848077901593...

1.62454980532…

Thence:

1/2 * ((((2^(4/11) 13^(1/11) + ((((4/36 [49/49(33.49234+125/33.49234+22)+49(33.49234+125/33.49234+22)-14*sqrt(((33.49234+125/33.49234+22)(33.49234+125/33.49234+22)))])))^1/11))))-16/10^3

Input interpretation:

$$\frac{1}{2}\left(2^{4/11}\sqrt[11]{13}+\left(\frac{4}{36}\left(\frac{49}{49}\left(33.49234+\frac{125}{33.49234}+22\right)+49\left(33.49234+\frac{125}{33.49234}+22\right)-14\sqrt{\left(33.49234+\frac{125}{33.49234}+22\right)\left(33.49234+\frac{125}{33.49234}+22\right)}\right)\right)^{\wedge}(1/11)\right)-\frac{16}{10^3}$$

Result:

1.618213256842546524035506740330203908219771015819729480439...

1.61821325684…

Now:

Now, by Lemma 15.9.3, we find that

$$\frac{1}{v}\frac{dv}{dq} = \frac{d\log v}{dq} = \frac{d\log\left\{q^{7/6}\frac{f(-q^{35})}{f(-q^7)}\right\}}{dq} + \frac{d\log\left\{q^{-1/6}\frac{f(-q)}{f(-q^5)}\right\}}{dq}$$

$$= \frac{7}{6q} + 7\sum_{n=1}^{\infty}\frac{nq^{7n-1}}{1-q^{7n}} - 35\sum_{n=1}^{\infty}\frac{nq^{35n-1}}{1-q^{35n}}$$

$$-\frac{1}{6q} + 5\sum_{n=1}^{\infty}\frac{nq^{5n-1}}{1-q^{5n}} - \sum_{n=1}^{\infty}\frac{nq^{n-1}}{1-q^n}$$

$$= \frac{7}{6q}\sqrt{\frac{f^{12}(-q^7)+22q^7f^6(-q^7)f^6(-q^{35})+125q^{14}f^{12}(-q^{35})}{f^2(-q^7)f^2(-q^{35})}}$$

$$-\frac{1}{6q}\sqrt{\frac{f^{12}(-q)+22qf^6(-q)f^6(-q^5)+125q^2f^{12}(-q^5)}{f^2(-q)f^2(-q^5)}}$$

$$= qf(-q)f(-q^5)f(-q^7)f(-q^{35})$$
$$\times\left(-\frac{7}{6R}\sqrt{Q^6+\frac{125}{Q^6}+22} + \frac{R}{6}\sqrt{P^6+\frac{125}{P^6}+22}\right), \qquad (15.9.15)$$

From

$$qf(-q)f(-q^5)f(-q^7)f(-q^{35})$$
$$\times\left(-\frac{7}{6R}\sqrt{Q^6+\frac{125}{Q^6}+22} + \frac{R}{6}\sqrt{P^6+\frac{125}{P^6}+22}\right)$$

For **R = 7; Q^6 = P^6 = 25**, we obtain:

(((-7/42*(25+125/25+22)^0.5+7/6*(25+125/25+22)^0.5)))

Input:

$$-\frac{7}{42}\sqrt{25+\frac{125}{25}+22} + \frac{7}{6}\sqrt{25+\frac{125}{25}+22}$$

Exact result:

$2\sqrt{13}$

Decimal approximation:

7.211102550927978586238442534940991892502593147690492425420...

7.211102550927...

We note that:

4(((-7/42*(25+125/25+22)^0.5+7/6*(25+125/25+22)^0.5)))^2

Input:

$$4\left(-\frac{7}{42}\sqrt{25+\frac{125}{25}+22}+\frac{7}{6}\sqrt{25+\frac{125}{25}+22}\right)^2$$

Exact result:

208

208

or:

8sqrt13 (((-7/42*(25+125/25+22)^0.5+7/6*(25+125/25+22)^0.5)))

Input:

$$8\sqrt{13}\left(-\frac{7}{42}\sqrt{25+\frac{125}{25}+22}+\frac{7}{6}\sqrt{25+\frac{125}{25}+22}\right)$$

Exact result:

208

208

As the previous expression:

$$\frac{v^2}{36}\left\{\frac{49}{R^2}\left(Q^6+\frac{125}{Q^6}+22\right)+R^2\left(P^6+\frac{125}{P^6}+22\right)\right.$$
$$\left.-14\sqrt{\left(Q^6+\frac{125}{Q^6}+22\right)\left(P^6+\frac{125}{P^6}+22\right)}\right\}$$

4/36 [49/49(25+125/25+22)+49(25+125/25+22)-14*sqrt(((25+125/25+22)(25+125/25+22)))]

Input:

$$\frac{4}{36}\left(\frac{49}{49}\left(25+\frac{125}{25}+22\right)+49\left(25+\frac{125}{25}+22\right)-14\sqrt{\left(25+\frac{125}{25}+22\right)\left(25+\frac{125}{25}+22\right)}\right)$$

Exact result:

208

208

v **= 2; R = 7;** $\mathbf{Q^6 = P^6 = 25}$

Thence:

$$qf(-q)f(-q^5)f(-q^7)f(-q^{35}) = 8\sqrt{13} =$$

$$= 28.844410203711914344953770139763967570010372590761969701 68\ldots$$

28.84441020371...

From

Entry 15.9.1 (p. 53). *If v is defined by (15.9.1), then*

$$\int_0^q t\, f(-t)f(-t^5)f(-t^7)f(-t^{35})dt = \int_0^v \frac{t\, dt}{\sqrt{(1+t-t^2)(1-5t-9t^3-5t^5-t^6)}}.$$

we obtain:

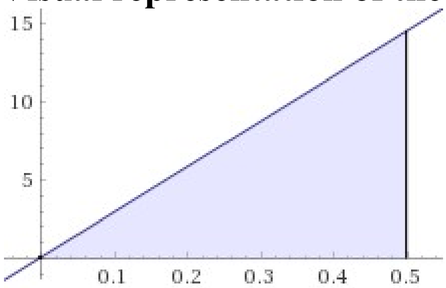
integrate (8sqrt13)x x = 0..0.5

Definite integral:

$$\int_0^{0.5} \left(8\sqrt{13}\right) x\, dx = 3.60555$$

3.60555

Visual representation of the integral:



Indefinite integral:

$$\int \left(8\sqrt{13}\right) x\, dx = 4\sqrt{13}\, x^2 + \text{constant}$$

We note that from the previous expression

$$\left(-\frac{7}{6R}\sqrt{Q^6+\frac{125}{Q^6}+22}+\frac{R}{6}\sqrt{P^6+\frac{125}{P^6}+22}\right)$$

$$-\frac{7}{42}\sqrt{25+\frac{125}{25}+22}+\frac{7}{6}\sqrt{25+\frac{125}{25}+22}$$

Performing the following integral, we obtain:

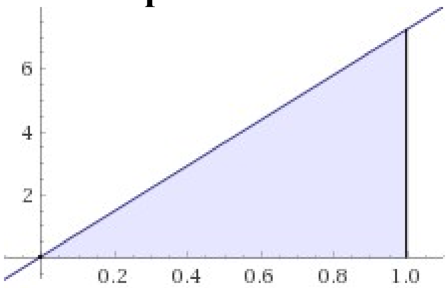
integrate ((((-7/42*(25+125/25+22)^0.5+7/6*(25+125/25+22)^0.5))))x x = 0..1

Definite integral:

$$\int_0^1 \left(-\frac{7}{42}\sqrt{25+\frac{125}{25}+22}+\frac{7}{6}\sqrt{25+\frac{125}{25}+22}\right) x\,dx = \sqrt{13} \approx 3.6056$$

3.6056 = √13

Visual representation of the integral:



Indefinite integral:

$$\int \left(-\frac{7}{42}\sqrt{25+\frac{125}{25}+22}+\frac{7}{6}\sqrt{25+\frac{125}{25}+22}\right) x\,dx = \sqrt{13}\, x^2 + \text{constant}$$

Thence, we obtain the similar result as for the previous integral.

Furthermore, we have that:

[integrate ((((-7/42*(25+125/25+22)^0.5+7/6*(25+125/25+22)^0.5))))x x = 0..1]^2

Input:

$$\left(\int_0^1 \left(-\frac{7}{42}\sqrt{25+\frac{125}{25}+22}+\frac{7}{6}\sqrt{25+\frac{125}{25}+22}\right) x\,dx\right)^2$$

Result:

13

13

Computation result:

$$\left(\int_0^1 \left(-\frac{7}{42}\sqrt{25+\frac{125}{25}+22}+\frac{7}{6}\sqrt{25+\frac{125}{25}+22}\right) x\,dx\right)^2 = 13$$

and:

21 / [integrate ((((-7/42*(25+125/25+22)^0.5+7/6*(25+125/25+22)^0.5))))x x = 0..1]^2

Input:

$$\frac{21}{\left(\int_0^1 \left(-\frac{7}{42}\sqrt{25+\frac{125}{25}+22}+\frac{7}{6}\sqrt{25+\frac{125}{25}+22}\right)x\,dx\right)^2}$$

Result:

$$\frac{21}{13} \approx 1.61538$$

1.61538 that is the ratio between 21 and 13 (Fibonacci numbers)

Now, we have that:

Put

$$\alpha = 2\tan^{-1}\left(5^{1/4}\sqrt{\frac{CB}{A}}\right)$$

and

$$\beta = 2\tan^{-1}\left(5^{3/4}\frac{A+CB}{A+5CB}\sqrt{\frac{CB}{A}}\right).$$

For B = C = 12 and A = 16 , we obtain:

2 tan^-1 [((5^(1/4) * sqrt((12*12)/(16))))]

Input:

$$2\tan^{-1}\left(\sqrt[4]{5}\sqrt{\frac{12\times 12}{16}}\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$2\tan^{-1}\left(3\sqrt[4]{5}\right)$$

(result in radians)

2 tan^(-1)(3 5^(1/4)) = α

Decimal approximation:

2.702937584613176417208026519908466181706992694728122878375...

(result in radians)

2.7029375846…

Alternate form:

$$i\log\left(1-3\,i\sqrt[4]{5}\right)-i\log\left(1+3\,i\sqrt[4]{5}\right)$$

log(x) is the natural logarithm

Alternative representations:

$$2\tan^{-1}\left(\sqrt[4]{5}\sqrt{\frac{12\times 12}{16}}\right)=2\,\mathrm{sc}^{-1}\left(\sqrt[4]{5}\sqrt{\frac{144}{16}}\,\middle|\,0\right)$$

$$2\tan^{-1}\left(\sqrt[4]{5}\sqrt{\frac{12\times 12}{16}}\right)=2\tan^{-1}\left(1,\sqrt[4]{5}\sqrt{\frac{144}{16}}\right)$$

$$2\tan^{-1}\left(\sqrt[4]{5}\sqrt{\frac{12\times 12}{16}}\right)=-2\,i\tanh^{-1}\left(i\sqrt[4]{5}\sqrt{\frac{144}{16}}\right)$$

Series representations:

$$2\tan^{-1}\left(\sqrt[4]{5}\sqrt{\frac{12\times 12}{16}}\right)=\pi-2\sum_{k=0}^{\infty}\frac{(-1)^k\,3^{-1-2k}\times 5^{-1/4-k/2}}{1+2k}$$

$$2\tan^{-1}\left(\sqrt[4]{5}\sqrt{\frac{12\times 12}{16}}\right)=2\sum_{k=0}^{\infty}\frac{(-1)^k\,5^{-k+1/4(1+2k)}\times 6^{1+2k}\left(1+\sqrt{1+\frac{36}{\sqrt{5}}}\right)^{-1-2k}F_{1+2k}}{1+2k}$$

$$2\tan^{-1}\left(\sqrt[4]{5}\sqrt{\frac{12\times 12}{16}}\right)=6\sum_{k=0}^{\infty}\frac{\left(-\frac{9}{4}\right)^k T_{1+2k}\left(\sqrt[4]{5}\right){}_2F_1\left(\frac{1}{2}+k,1+k;2+2k;-9\right)}{1+2k}$$

Integral representations:

$$2\tan^{-1}\left(\sqrt[4]{5}\sqrt{\frac{12\times 12}{16}}\right)=6\sqrt[4]{5}\int_0^1\frac{1}{1+9\sqrt{5}\,t^2}\,dt$$

$$2\tan^{-1}\left(\sqrt[4]{5}\sqrt{\frac{12\times 12}{16}}\right)=-\frac{3\,i\sqrt[4]{5}}{2\pi^{3/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(1+9\sqrt{5}\right)^{-s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds$$
$$\text{for } 0<\gamma<\frac{1}{2}$$

$$2\tan^{-1}\left(\sqrt[4]{5}\sqrt{\frac{12\times 12}{16}}\right)=-\frac{3\,i\sqrt[4]{5}}{2\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{5^{-s/2}\times 9^{-s}\,\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds$$
$$\text{for } 0<\gamma<\frac{1}{2}$$

Continued fraction representations:

$$2\tan^{-1}\left(\sqrt[4]{5}\sqrt{\frac{12\times 12}{16}}\right)=\frac{6\sqrt[4]{5}}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{9\sqrt{5}\,k^2}{1+2k}}=\frac{6\sqrt[4]{5}}{1+\frac{9\sqrt{5}}{3+\frac{36\sqrt{5}}{5+\frac{81\sqrt{5}}{7+\frac{144\sqrt{5}}{9+\dots}}}}}$$

$$2\tan^{-1}\left(\sqrt[4]{5}\sqrt{\frac{12\times 12}{16}}\right)=$$
$$6\sqrt[4]{5}-\frac{54\times 5^{3/4}}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{9\sqrt{5}\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}=6\sqrt[4]{5}-\frac{54\times 5^{3/4}}{3+\frac{81\sqrt{5}}{5+\frac{36\sqrt{5}}{7+\frac{225\sqrt{5}}{9+\frac{144\sqrt{5}}{11+\dots}}}}}$$

$$2\tan^{-1}\left(\sqrt[4]{5}\sqrt{\frac{12\times 12}{16}}\right)=\frac{6\sqrt[4]{5}}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{9\sqrt{5}\,(1-2k)^2}{1+2k-9\sqrt{5}\,(-1+2k)}}=$$
$$\frac{6\sqrt[4]{5}}{1+\frac{9\sqrt{5}}{3-9\sqrt{5}+\frac{81\sqrt{5}}{5-27\sqrt{5}+\frac{225\sqrt{5}}{7-45\sqrt{5}+\frac{441\sqrt{5}}{9-63\sqrt{5}+\dots}}}}}$$

$$2\tan^{-1}\left(\sqrt[4]{5}\sqrt{\frac{12\times 12}{16}}\right) = \frac{6\sqrt[4]{5}}{1+9\sqrt{5}+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{18\sqrt{5}\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1+\frac{9}{2}\sqrt{5}\left(1+(-1)^k\right)\right)(1+2k)}} =$$

$$\cfrac{6\sqrt[4]{5}}{1+9\sqrt{5}+-\cfrac{18\sqrt{5}}{3-\cfrac{18\sqrt{5}}{5\left(1+9\sqrt{5}\right)-\cfrac{108\sqrt{5}}{7-\cfrac{108\sqrt{5}}{9\left(1+9\sqrt{5}\right)+\ldots}}}}}$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

From which:

sqrt((2 tan^(-1)(3 5^(1/4))))

Input:

$$\sqrt{2\tan^{-1}\left(3\sqrt[4]{5}\right)}$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\sqrt{2\tan^{-1}\left(3\sqrt[4]{5}\right)}$$

(result in radians)

Decimal approximation:

1.644061308045772488067680806719818565268365762746856902250...

(result in radians)

1.64406130804…

Alternate form:

$$\sqrt{i\left(\log\left(1-3\,i\sqrt[4]{5}\right)-\log\left(1+3\,i\sqrt[4]{5}\right)\right)}$$

$\log(x)$ is the natural logarithm

All 2nd roots of 2 tan^(-1)(3 5^(1/4)):

$$e^0 \sqrt{2\tan^{-1}\left(3\sqrt[4]{5}\right)} \approx 1.64406 \text{ (real, principal root)}$$

$$e^{i\pi} \sqrt{2\tan^{-1}\left(3\sqrt[4]{5}\right)} \approx -1.6441 \text{ (real root)}$$

Alternative representations:

$$\sqrt{2\tan^{-1}\left(3\sqrt[4]{5}\right)} = \sqrt{2\,\mathrm{sc}^{-1}\left(3\sqrt[4]{5}\,\middle|\,0\right)}$$

$$\sqrt{2\tan^{-1}\left(3\sqrt[4]{5}\right)} = \sqrt{2\tan^{-1}\left(1, 3\sqrt[4]{5}\right)}$$

$$\sqrt{2\tan^{-1}\left(3\sqrt[4]{5}\right)} = \sqrt{2\,i\tanh^{-1}\left(-3\,i\sqrt[4]{5}\right)}$$

$\mathrm{sc}^{-1}(x\,|\,m)$ is the inverse of the Jacobi elliptic function sc

Series representations:

$$\sqrt{2\tan^{-1}\left(3\sqrt[4]{5}\right)} = \sqrt{\pi - 2\sum_{k=0}^{\infty} \frac{(-1)^k\, 3^{-1-2k} \times 5^{-1/4-k/2}}{1+2k}}$$

$$\sqrt{2\tan^{-1}\left(3\sqrt[4]{5}\right)} = \sqrt{2}\sqrt{\frac{\pi}{2} - \sum_{k=0}^{\infty} \frac{(-1)^k\, 3^{-1-2k} \times 5^{1/4(-1-2k)}}{1+2k}}$$

$$\sqrt{2\tan^{-1}\left(3\sqrt[4]{5}\right)} = \sqrt{6}\sqrt{\sum_{k=0}^{\infty} \frac{\left(-\frac{9}{4}\right)^k T_{1+2k}\left(\sqrt[4]{5}\right) {}_2F_1\left(\frac{1}{2}+k, 1+k; 2+2k; -9\right)}{1+2k}}$$

$T_n(x)$ is the Chebyshev polynomial of the first kind

${}_pF_q(a_1, \ldots, a_p; b_1, \ldots, b_q; z)$ is the generalized hypergeometric function

Integral representations:

$$\sqrt{2\tan^{-1}\left(3\sqrt[4]{5}\right)} = \sqrt[8]{5}\,\sqrt{6}\sqrt{\int_0^1 \frac{1}{1+9\sqrt{5}\,t^2}\,dt}$$

$$\sqrt{2\tan^{-1}\left(3\sqrt[4]{5}\right)} = \frac{\sqrt{\frac{3}{2}}\ \sqrt[8]{5}\ \sqrt{-i\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\left(1+9\sqrt{5}\right)^{-s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)^2\,ds}}{\pi^{3/4}}$$

for $0 < \gamma < \frac{1}{2}$

$$\sqrt{2\tan^{-1}\left(3\sqrt[4]{5}\right)} = \sqrt[8]{5}\ \sqrt{\frac{3}{2\pi}}\ \sqrt{-i\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{5^{-s/2}\times 9^{-s}\,\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds}$$

for $0 < \gamma < \frac{1}{2}$

Continued fraction representations:

$$\sqrt{2\tan^{-1}\left(3\sqrt[4]{5}\right)} = \sqrt[8]{5}\ \sqrt{6}\ \sqrt{\frac{1}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{9\sqrt{5}\,k^2}{1+2k}}} = \sqrt[8]{5}\ \sqrt{6}\ \sqrt{\frac{1}{1+\cfrac{9\sqrt{5}}{3+\cfrac{36\sqrt{5}}{5+\cfrac{81\sqrt{5}}{7+\cfrac{144\sqrt{5}}{9+\ldots}}}}}}$$

$$\sqrt{2\tan^{-1}\left(3\sqrt[4]{5}\right)} = \sqrt{6\sqrt[4]{5} - \frac{54\times 5^{3/4}}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{9\sqrt{5}\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}} =$$

$$\sqrt{6\sqrt[4]{5} - \cfrac{54\times 5^{3/4}}{3+\cfrac{81\sqrt{5}}{5+\cfrac{36\sqrt{5}}{7+\cfrac{225\sqrt{5}}{9+\cfrac{144\sqrt{5}}{11+\ldots}}}}}}$$

$$\sqrt{2\tan^{-1}\left(3\sqrt[4]{5}\right)} = \sqrt{2}\ \sqrt{3\sqrt[4]{5} - \frac{27\times 5^{3/4}}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{9\sqrt{5}\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}} =$$

$$\sqrt{2}\ \sqrt{3\sqrt[4]{5} - \cfrac{27\times 5^{3/4}}{3+\cfrac{81\sqrt{5}}{5+\cfrac{36\sqrt{5}}{7+\cfrac{225\sqrt{5}}{9+\cfrac{144\sqrt{5}}{11+\ldots}}}}}}$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

2 tan^-1 [((5^(3/4) * (16+144)/(16+5*144) * sqrt((12*12)/(16))))]

Input:

$$2\tan^{-1}\left(5^{3/4}\times\frac{16+144}{16+5\times144}\sqrt{\frac{12\times12}{16}}\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$2\tan^{-1}\left(\frac{15\times5^{3/4}}{23}\right)$$

(result in radians)

2 tan^(-1)((15 5^(3/4))/23) = β

Decimal approximation:

2.281670975978912275310811546231450773061382640827715351081...

(result in radians)

2.2816709759…

Alternate form:

$$i\log\left(1-\frac{15}{23}i\,5^{3/4}\right)-i\log\left(1+\frac{15}{23}i\,5^{3/4}\right)$$

$\log(x)$ is the natural logarithm

Alternative representations:

$$2\tan^{-1}\left(\frac{\left(5^{3/4}\sqrt{\frac{12\times12}{16}}\right)(16+144)}{16+5\times144}\right)=2\,\mathrm{sc}^{-1}\left(\frac{160}{736}\times5^{3/4}\sqrt{\frac{144}{16}}\,\middle|\,0\right)$$

$$2\tan^{-1}\left(\frac{\left(5^{3/4}\sqrt{\frac{12\times12}{16}}\right)(16+144)}{16+5\times144}\right)=2\tan^{-1}\left(1,\frac{160}{736}\times5^{3/4}\sqrt{\frac{144}{16}}\right)$$

$$2\tan^{-1}\left(\frac{\left(5^{3/4}\sqrt{\frac{12\times12}{16}}\right)(16+144)}{16+5\times144}\right) = -2\,i\tanh^{-1}\left(\frac{160}{736}\,i\,5^{3/4}\sqrt{\frac{144}{16}}\right)$$

$sc^{-1}(x\mid m)$ is the inverse of the Jacobi elliptic function sc

Series representations:

$$2\tan^{-1}\left(\frac{\left(5^{3/4}\sqrt{\frac{12\times12}{16}}\right)(16+144)}{16+5\times144}\right) = \pi - 2\sum_{k=0}^{\infty}\frac{\left(\frac{3}{23}\right)^{-1-2k}5^{-7/4-(7k)/2}\,e^{i k\pi}}{1+2k}$$

$$2\tan^{-1}\left(\frac{\left(5^{3/4}\sqrt{\frac{12\times12}{16}}\right)(16+144)}{16+5\times144}\right) =$$

$$2\sum_{k=0}^{\infty}\frac{(-1)^k\,5^{1+k+3/4(1+2k)}\times6^{1+2k}\left(23\left(1+\sqrt{1+\frac{900\sqrt{5}}{529}}\right)\right)^{-1-2k}F_{1+2k}}{1+2k}$$

$$2\tan^{-1}\left(\frac{\left(5^{3/4}\sqrt{\frac{12\times12}{16}}\right)(16+144)}{16+5\times144}\right) =$$

$$\frac{30}{23}\sum_{k=0}^{\infty}\frac{\left(-\frac{225}{2116}\right)^k T_{1+2k}\left(5^{3/4}\right)\,{}_2F_1\left(\frac{1}{2}+k,\,1+k;\,2+2k;\,-\frac{225}{529}\right)}{1+2k}$$

F_n is the n^{th} Fibonacci number

$T_n(x)$ is the Chebyshev polynomial of the first kind

${}_pF_q(a_1,\ldots,a_p;\,b_1,\ldots,b_q;\,z)$ is the generalized hypergeometric function

Integral representations:

$$2\tan^{-1}\left(\frac{\left(5^{3/4}\sqrt{\frac{12\times12}{16}}\right)(16+144)}{16+5\times144}\right)=\frac{30\times5^{3/4}}{23}\int_0^1\frac{1}{1+\frac{1125\sqrt{5}\,t^2}{529}}\,dt$$

$$2\tan^{-1}\left(\frac{\left(5^{3/4}\sqrt{\frac{12\times12}{16}}\right)(16+144)}{16+5\times144}\right)=$$
$$-\frac{15\,i\,5^{3/4}}{46\,\pi^{3/2}}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\left(1+\frac{1125\sqrt{5}}{529}\right)^{-s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds\quad\text{for }0<\gamma<\frac{1}{2}$$

$$2\tan^{-1}\left(\frac{\left(5^{3/4}\sqrt{\frac{12\times12}{16}}\right)(16+144)}{16+5\times144}\right)=$$
$$-\frac{15\,i\,5^{3/4}}{46\,\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{5^{-(7s)/2}\left(\frac{529}{9}\right)^{s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds\quad\text{for }0<\gamma<\frac{1}{2}$$

Continued fraction representations:

$$2\tan^{-1}\left(\frac{\left(5^{3/4}\sqrt{\frac{12\times12}{16}}\right)(16+144)}{16+5\times144}\right)=\frac{30\times5^{3/4}}{23\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{1125\sqrt{5}\,k^2}{529}}{1+2k}\right)}=$$

$$\frac{30\times5^{3/4}}{23\left(1+\frac{1125\sqrt{5}}{529\left(3+\frac{4500\sqrt{5}}{529\left(5+\frac{10\,125\sqrt{5}}{529\left(7+\frac{18\,000\sqrt{5}}{529\,(9+\ldots)}\right)}\right)}\right)}\right)}$$

$$2\tan^{-1}\left(\frac{\left(5^{3/4}\sqrt{\frac{12\times 12}{16}}\right)(16+144)}{16+5\times 144}\right)=$$

$$2\left(\frac{15\times 5^{3/4}}{23}-\frac{84\,375\sqrt[4]{5}}{12\,167\left(3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{1125}{529}\sqrt{5}\left(1+(-1)^{1+k}+k\right)^2}{3+2k}\right)}\right)=$$

$$2\left(\frac{15\times 5^{3/4}}{23}-\frac{84\,375\sqrt[4]{5}}{12\,167\left(3+\frac{10\,125\sqrt{5}}{529\left(5+\frac{4500\sqrt{5}}{529\left(7+\frac{28\,125\sqrt{5}}{529\left(9+\frac{18\,000\sqrt{5}}{529\,(11+\ldots)}\right)}\right)}\right)}\right)}\right)$$

$$2\tan^{-1}\left(\frac{\left(5^{3/4}\sqrt{\frac{12\times 12}{16}}\right)(16+144)}{16+5\times 144}\right)=\frac{30\times 5^{3/4}}{23\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{1125}{529}\sqrt{5}\,(1-2k)^2}{1+2k-\frac{1125}{529}\sqrt{5}\,(-1+2k)}\right)}=\left(30\times 5^{3/4}\right)\Bigg/\left(23\left(1+\left(1125\sqrt{5}\right)\Bigg/\left(529\left(3-\frac{1125\sqrt{5}}{529}+\left(10\,125\sqrt{5}\right)\Bigg/\left(529\left(5-\frac{3375\sqrt{5}}{529}+\frac{28\,125\sqrt{5}}{529\left(7-\frac{5625\sqrt{5}}{529}+\frac{55\,125\sqrt{5}}{529\left(9-\frac{7875\sqrt{5}}{529}+\ldots\right)}\right)}\right)\right)\right)\right)\right)\right)$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

Multiplying the two results:

(((2 tan^(-1)((15*5^(3/4))/23)))) * (((2 tan^(-1)(3*5^(1/4)))))

Input:

$$\left(2\tan^{-1}\left(\frac{1}{23}\left(15\times 5^{3/4}\right)\right)\right)\left(2\tan^{-1}\left(3\sqrt[4]{5}\right)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$4\tan^{-1}\left(3\sqrt[4]{5}\right)\tan^{-1}\left(\frac{15\times 5^{3/4}}{23}\right)$$

(result in radians)

Decimal approximation:

6.1672142366944300146748383790915895001324043687395105613 61...

(result in radians)

6.1672142366…

Alternate form:

$$-\left(\log\left(1-3 i \sqrt[4]{5}\right)-\log\left(1+3 i \sqrt[4]{5}\right)\right)\left(\log\left(1-\frac{15}{23} i 5^{3/4}\right)-\log\left(1+\frac{15}{23} i 5^{3/4}\right)\right)$$

$\log(x)$ is the natural logarithm

Alternative representations:

$$\left(2 \tan^{-1}\left(3 \sqrt[4]{5}\right)\right) 2 \tan^{-1}\left(\frac{15 \times 5^{3/4}}{23}\right)=4 \operatorname{sc}^{-1}\left(3 \sqrt[4]{5} \mid 0\right) \operatorname{sc}^{-1}\left(\frac{15 \times 5^{3/4}}{23} \mid 0\right)$$

$$\left(2 \tan^{-1}\left(3 \sqrt[4]{5}\right)\right) 2 \tan^{-1}\left(\frac{15 \times 5^{3/4}}{23}\right)=4 \tan^{-1}\left(1, 3 \sqrt[4]{5}\right) \tan^{-1}\left(1, \frac{15 \times 5^{3/4}}{23}\right)$$

$$\left(2 \tan^{-1}\left(3 \sqrt[4]{5}\right)\right) 2 \tan^{-1}\left(\frac{15 \times 5^{3/4}}{23}\right)=4 i^2 \tanh^{-1}\left(3 i \sqrt[4]{5}\right) \tanh^{-1}\left(\frac{15}{23} i 5^{3/4}\right)$$

$\operatorname{sc}^{-1}(x \mid m)$ is the inverse of the Jacobi elliptic function sc

$\tan^{-1}(x, y)$ is the inverse tangent function

$\tanh^{-1}(x)$ is the inverse hyperbolic tangent function

i is the imaginary unit

Series representations:

$$\left(2 \tan^{-1}\left(3 \sqrt[4]{5}\right)\right) 2 \tan^{-1}\left(\frac{15 \times 5^{3/4}}{23}\right)=$$

$$\left(\pi-2 \sum_{k=0}^{\infty} \frac{(-1)^k 3^{-1-2 k} \times 5^{-1/4-k/2}}{1+2 k}\right)\left(\pi-2 \sum_{k=0}^{\infty} \frac{\left(\frac{3}{23}\right)^{-1-2 k} 5^{-7/4-(7 k)/2} e^{i k \pi}}{1+2 k}\right)$$

$$\left(2\tan^{-1}\left(3\sqrt[4]{5}\right)\right)2\tan^{-1}\left(\frac{15\times 5^{3/4}}{23}\right) =$$
$$4\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{1}{(1+2k_1)(1+2k_2)}(-1)^{k_1+k_2}\,5^{1-k_1+1/4(1+2k_1)+k_2+3/4(1+2k_2)}\times 6^{2+2k_1+2k_2}$$
$$\left(1+\sqrt{1+\frac{36}{\sqrt{5}}}\right)^{-1-2k_1}\left(23\left(1+\sqrt{1+\frac{900\sqrt{5}}{529}}\right)\right)^{-1-2k_2} F_{1+2k_1}\,F_{1+2k_2}$$

$$\left(2\tan^{-1}\left(3\sqrt[4]{5}\right)\right)2\tan^{-1}\left(\frac{15\times 5^{3/4}}{23}\right) =$$
$$\left(2\tan^{-1}(z_0)+i\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(3\sqrt[4]{5}-z_0\right)^k}{k}\right)$$
$$\left(2\tan^{-1}(z_0)+i\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{15\times 5^{3/4}}{23}-z_0\right)^k}{k}\right)$$
for $(i z_0 \notin \mathbb{R}$ or $((\text{not } 1 \le i z_0 < \infty)$ and $(\text{not } -\infty < i z_0 \le -1)))$

F_n is the n^{th} Fibonacci number

Integral representations:

$$\left(2\tan^{-1}\left(3\sqrt[4]{5}\right)\right)2\tan^{-1}\left(\frac{15\times 5^{3/4}}{23}\right) =$$
$$529\int_0^1\int_0^1\frac{1}{\left(529+1125\sqrt{5}\,t_1^2\right)\left(1+9\sqrt{5}\,t_2^2\right)}\,dt_2\,dt_1$$

$$\left(2\tan^{-1}\left(3\sqrt[4]{5}\right)\right)2\tan^{-1}\left(\frac{15\times 5^{3/4}}{23}\right) =$$
$$-\frac{1}{92\pi^3}\,225\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(1+\frac{1125\sqrt{5}}{529}\right)^{-s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds\right)$$
$$\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(1+9\sqrt{5}\right)^{-s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds \quad \text{for } 0<\gamma<\frac{1}{2}$$

$$\left(2\tan^{-1}\left(3\sqrt[4]{5}\right)\right)2\tan^{-1}\left(\frac{15\times 5^{3/4}}{23}\right) =$$
$$-\frac{225\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{5^{-s/2}\times 9^{-s}\,\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{5^{-(7s)/2}\left(\frac{529}{9}\right)^s\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds}{92\pi^2} \quad \text{for}$$
$$0<\gamma<\frac{1}{2}$$

Continued fraction representations:

$$\left(2\tan^{-1}\left(3\sqrt[4]{5}\right)\right)2\tan^{-1}\left(\frac{15\times5^{3/4}}{23}\right)=\frac{900}{23\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{1125\sqrt{5}\,k^2}{529}}{1+2k}\right)\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{9\sqrt{5}\,k^2}{1+2k}\right)}=$$

$$\frac{900}{23\left(1+\frac{1125\sqrt{5}}{529\left(3+\frac{4500\sqrt{5}}{529\left(5+\frac{10125\sqrt{5}}{529\left(7+\frac{18000\sqrt{5}}{529(9+\ldots)}\right)}\right)}\right)}\right)\left(1+\frac{9\sqrt{5}}{3+\frac{36\sqrt{5}}{5+\frac{81\sqrt{5}}{7+\frac{144\sqrt{5}}{9+\ldots}}}}\right)}$$

$$\left(2\tan^{-1}\left(3\sqrt[4]{5}\right)\right)2\tan^{-1}\left(\frac{15\times 5^{3/4}}{23}\right)=$$

$$4\left(\frac{15\times 5^{3/4}}{23}-\frac{84\,375\sqrt[4]{5}}{12\,167\left(3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{1125}{529}\sqrt{5}\left(1+(-1)^{1+k}+k\right)^2}{3+2k}\right)}\right)$$

$$\left(3\sqrt[4]{5}-\frac{27\times 5^{3/4}}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{9\sqrt{5}\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}\right)=$$

$$4\left(\frac{15\times 5^{3/4}}{23}-\frac{84\,375\sqrt[4]{5}}{12\,167\left(3+\frac{10\,125\sqrt{5}}{529\left(5+\frac{4500\sqrt{5}}{529\left(7+\frac{28\,125\sqrt{5}}{529\left(9+\frac{18\,000\sqrt{5}}{529\,(11+\ldots)}\right)}\right)}\right)}\right)}\right)$$

$$\left(3\sqrt[4]{5}-\frac{27\times 5^{3/4}}{3+\frac{81\sqrt{5}}{5+\frac{36\sqrt{5}}{7+\frac{225\sqrt{5}}{9+\frac{144\sqrt{5}}{11+\ldots}}}}}\right)$$

$$\left(2\tan^{-1}\left(3\sqrt[4]{5}\right)\right) 2\tan^{-1}\left(\frac{15\times 5^{3/4}}{23}\right) = \frac{900}{23\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{1125}{529}\sqrt{5}\,(-1+2k)^2}{1+2k-\frac{1125}{529}\sqrt{5}\,(-1+2k)}\right)\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{9\sqrt{5}\,(-1+2k)^2}{1+2k-9\sqrt{5}\,(-1+2k)}\right)} = 900 \Big/ \left(23\left(1+\left(1125\sqrt{5}\right)\Big/\left(529\left(3-\frac{1125\sqrt{5}}{529}+\left(10\,125\sqrt{5}\right)\Big/\left(529\left(5-\frac{3375\sqrt{5}}{529}+\frac{28\,125\sqrt{5}}{529\left(7-\frac{5625\sqrt{5}}{529}+\frac{55\,125\sqrt{5}}{529\left(9-\frac{7875\sqrt{5}}{529}+\dots\right)}\right)}\right)\right)\right)\right)\right)\left(1+\frac{9\sqrt{5}}{3-9\sqrt{5}+\frac{81\sqrt{5}}{5-27\sqrt{5}+\frac{225\sqrt{5}}{7-45\sqrt{5}+\frac{441\sqrt{5}}{9-63\sqrt{5}+\dots}}}}\right)\right)$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

1/(sqrt14) (((2 tan^(-1)((15*5^(3/4))/23)))) * (((2 tan^(-1)(3*5^(1/4)))))

Input:

$$\frac{1}{\sqrt{14}}\left(2\tan^{-1}\left(\frac{1}{23}\left(15\times 5^{3/4}\right)\right)\right)\left(2\tan^{-1}\left(3\sqrt[4]{5}\right)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$2\sqrt{\frac{2}{7}}\tan^{-1}\left(3\sqrt[4]{5}\right)\tan^{-1}\left(\frac{15\times 5^{3/4}}{23}\right)$$

(result in radians)

Decimal approximation:

1.648257336038937758643378992101591236882167385243652117402...

(result in radians)

1.64825733603…

Alternate form:

$$-\frac{\left(\log\left(1-3\,i\sqrt[4]{5}\right)-\log\left(1+3\,i\sqrt[4]{5}\right)\right)\left(\log\left(1-\frac{15}{23}\,i\,5^{3/4}\right)-\log\left(1+\frac{15}{23}\,i\,5^{3/4}\right)\right)}{\sqrt{14}}$$

log(x) is the natural logarithm

Alternative representations:

$$\frac{\left(2\tan^{-1}\left(\frac{15\times 5^{3/4}}{23}\right)\right)\left(2\tan^{-1}\left(3\sqrt[4]{5}\right)\right)}{\sqrt{14}}=\frac{4\,\mathrm{sc}^{-1}\left(3\sqrt[4]{5}\,\middle|\,0\right)\mathrm{sc}^{-1}\left(\frac{15\times 5^{3/4}}{23}\,\middle|\,0\right)}{\sqrt{14}}$$

$$\frac{\left(2\tan^{-1}\left(\frac{15\times 5^{3/4}}{23}\right)\right)\left(2\tan^{-1}\left(3\sqrt[4]{5}\right)\right)}{\sqrt{14}}=\frac{4\tan^{-1}\left(1,\,3\sqrt[4]{5}\right)\tan^{-1}\left(1,\,\frac{15\times 5^{3/4}}{23}\right)}{\sqrt{14}}$$

$$\frac{\left(2\tan^{-1}\left(\frac{15\times 5^{3/4}}{23}\right)\right)\left(2\tan^{-1}\left(3\sqrt[4]{5}\right)\right)}{\sqrt{14}}=\frac{4\,i^2\tanh^{-1}\left(3\,i\sqrt[4]{5}\right)\tanh^{-1}\left(\frac{15}{23}\,i\,5^{3/4}\right)}{\sqrt{14}}$$

$\mathrm{sc}^{-1}(x\,|\,m)$ is the inverse of the Jacobi elliptic function sc

Series representations:

$$\frac{\left(2\tan^{-1}\left(\frac{15\times 5^{3/4}}{23}\right)\right)\left(2\tan^{-1}\left(3\sqrt[4]{5}\right)\right)}{\sqrt{14}}=$$

$$\frac{\left(\pi-2\sum_{k=0}^{\infty}\frac{(-1)^k\,3^{-1-2k}\times 5^{-1/4-k/2}}{1+2k}\right)\left(\pi-2\sum_{k=0}^{\infty}\frac{\left(\frac{3}{23}\right)^{-1-2k}5^{-7/4-(7k)/2}\,e^{i\,k\,\pi}}{1+2k}\right)}{\sqrt{14}}$$

$$\frac{\left(2\tan^{-1}\left(\frac{15\times 5^{3/4}}{23}\right)\right)\left(2\tan^{-1}\left(3\sqrt[4]{5}\right)\right)}{\sqrt{14}} = 2\sqrt{\frac{2}{7}}$$
$$\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{1}{(1+2k_1)(1+2k_2)}(-1)^{k_1+k_2}\,5^{1-k_1+1/4(1+2k_1)+k_2+3/4(1+2k_2)}\times 6^{2+2k_1+2k_2}$$
$$\left(1+\sqrt{1+\frac{36}{\sqrt{5}}}\right)^{-1-2k_1}\left(23\left(1+\sqrt{1+\frac{900\sqrt{5}}{529}}\right)\right)^{-1-2k_2}F_{1+2k_1}\,F_{1+2k_2}$$

$$\frac{\left(2\tan^{-1}\left(\frac{15\times 5^{3/4}}{23}\right)\right)\left(2\tan^{-1}\left(3\sqrt[4]{5}\right)\right)}{\sqrt{14}} = \frac{1}{\sqrt{14}}$$
$$\left(2\tan^{-1}(z_0)+i\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(3\sqrt[4]{5}-z_0\right)^k}{k}\right)$$
$$\left(2\tan^{-1}(z_0)+i\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{15\times 5^{3/4}}{23}-z_0\right)^k}{k}\right)$$
for $(i z_0 \notin \mathbb{R}$ or $((\text{not } 1 \le i z_0 < \infty)$ and $(\text{not } -\infty < i z_0 \le -1)))$

F_n is the n^{th} Fibonacci number

Integral representations:

$$\frac{\left(2\tan^{-1}\left(\frac{15\times 5^{3/4}}{23}\right)\right)\left(2\tan^{-1}\left(3\sqrt[4]{5}\right)\right)}{\sqrt{14}} =$$
$$529\int_0^1\int_0^1\frac{1}{(529+1125\sqrt{5}\,t_1^2)(1+9\sqrt{5}\,t_2^2)}\,dt_2\,dt_1$$

$$\frac{\left(2\tan^{-1}\left(\frac{15\times 5^{3/4}}{23}\right)\right)\left(2\tan^{-1}\left(3\sqrt[4]{5}\right)\right)}{\sqrt{14}} =$$
$$-\frac{1}{92\sqrt{14}\,\pi^3}\,225\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(1+\frac{1125\sqrt{5}}{529}\right)^{-s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds\right)$$
$$\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(1+9\sqrt{5}\right)^{-s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds \quad \text{for } 0<\gamma<\frac{1}{2}$$

$$\frac{\left(2\tan^{-1}\left(\frac{15\times 5^{3/4}}{23}\right)\right)\left(2\tan^{-1}\left(3\sqrt[4]{5}\right)\right)}{\sqrt{14}} =$$

$$-\frac{225\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{5^{-s/2}\times 9^{-s}\,\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{5^{-(7s)/2}\left(\frac{529}{9}\right)^{s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds}{92\sqrt{14}\,\pi^2} \text{ for } 0<\gamma<\frac{1}{2}$$

Continued fraction representations:

$$\frac{\left(2\tan^{-1}\left(\frac{15\times 5^{3/4}}{23}\right)\right)\left(2\tan^{-1}\left(3\sqrt[4]{5}\right)\right)}{\sqrt{14}} = \frac{450\sqrt{\frac{2}{7}}}{23\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{1125\sqrt{5}\,k^2}{529}}{1+2k}\right)\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{9\sqrt{5}\,k^2}{1+2k}\right)} =$$

$$\frac{450\sqrt{\frac{2}{7}}}{23\left(1+\frac{1125\sqrt{5}}{529\left(3+\frac{4500\sqrt{5}}{529\left(5+\frac{10125\sqrt{5}}{529\left(7+\frac{18000\sqrt{5}}{529(9+\ldots)}\right)}\right)}\right)}\right)\left(1+\frac{9\sqrt{5}}{3+\frac{36\sqrt{5}}{5+\frac{81\sqrt{5}}{7+\frac{144\sqrt{5}}{9+\ldots}}}}\right)}$$

$$\frac{\left(2\tan^{-1}\left(\frac{15\times5^{3/4}}{23}\right)\right)\left(2\tan^{-1}\left(3\sqrt[4]{5}\right)\right)}{\sqrt{14}} =$$

$$2\sqrt{\frac{2}{7}}\left(\frac{15\times5^{3/4}}{23} - \frac{84\,375\sqrt[4]{5}}{12\,167\left(3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{1125}{529}\sqrt{5}\left(1+(-1)^{1+k}+k\right)^2}{3+2k}\right)}\right)$$

$$\left(3\sqrt[4]{5} - \frac{27\times5^{3/4}}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{9\sqrt{5}\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}\right) =$$

$$2\sqrt{\frac{2}{7}}\left(\frac{15\times5^{3/4}}{23} - \frac{84\,375\sqrt[4]{5}}{12\,167\left(3+\frac{10\,125\sqrt{5}}{529\left(5+\frac{4500\sqrt{5}}{529\left(7+\frac{28\,125\sqrt{5}}{529\left(9+\frac{18\,000\sqrt{5}}{529\,(11+\ldots)}\right)}\right)}\right)}\right)}\right)$$

$$\left(3\sqrt[4]{5} - \frac{27\times5^{3/4}}{3+\frac{81\sqrt{5}}{5+\frac{36\sqrt{5}}{7+\frac{225\sqrt{5}}{9+\frac{144\sqrt{5}}{11+\ldots}}}}}\right)$$

$$\frac{\left(2\tan^{-1}\left(\frac{15\times 5^{3/4}}{23}\right)\right)\left(2\tan^{-1}\left(3\sqrt[4]{5}\right)\right)}{\sqrt{14}} =$$

$$\frac{450\sqrt{\frac{2}{7}}}{23\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{1125}{529}\sqrt{5}\,(-1+2k)^2}{1+2k-\frac{1125}{529}\sqrt{5}\,(-1+2k)}\right)\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{9\sqrt{5}\,(-1+2k)^2}{1+2k-9\sqrt{5}\,(-1+2k)}\right)} = \left(450\sqrt{\frac{2}{7}}\right)\Bigg/$$

$$\left(23\left(1+\left(1125\sqrt{5}\right)\Bigg/\left(529\left(3-\frac{1125\sqrt{5}}{529}+\left(10\,125\sqrt{5}\right)\Bigg/\left(529\left(5-\frac{3375\sqrt{5}}{529}+\frac{28\,125\sqrt{5}}{529\left(7-\frac{5625\sqrt{5}}{529}+\frac{55\,125\sqrt{5}}{529\left(9-\frac{7875\sqrt{5}}{529}+\ldots\right)}\right)}\right)\right)\right)\right)\right)\right)$$

$$\left(1+\frac{9\sqrt{5}}{3-9\sqrt{5}+\frac{81\sqrt{5}}{5-27\sqrt{5}+\frac{225\sqrt{5}}{7-45\sqrt{5}+\frac{441\sqrt{5}}{9-63\sqrt{5}+\ldots}}}}\right)\Bigg)$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

From:

On a New Approach for Constructing Wormholes in Einstein-Born-Infeld Gravity - *Jin Young Kim and Mu-In Park* - arXiv:1608.00445v3 [hep-th] 10 Oct 2016

We have that:

$$f = 1 - \frac{2C}{r} - \frac{\Lambda}{3}r^2 + \frac{2\beta^2}{r}\int_0^r r^2\left(1 - \sqrt{1 + \frac{Q^2}{\beta^2 r^4}}\right)dr, \tag{13}$$

where C is an integration constant.

With a straightforward integration, one can express the solution in a compact form in terms of the incomplete elliptic integral of the first kind,

We note that the solution is in terms of the "incomplete elliptic integral of the first kind" (see Ramanujan incomplete elliptic integrals in the next pages)

$$f = 1 - \frac{2C}{r} - \frac{\Lambda}{3}r^2 + \frac{2\beta^2}{3}r^2\left(1 - \sqrt{1 + \frac{Q^2}{\beta^2 r^4}}\right) - \frac{4}{3r}\sqrt{-i\beta Q^3}\ \mathrm{EllipticF}\left(r\sqrt{\frac{i\beta}{Q}}, i\right)$$

We have that:

$$M \equiv C + M_0,$$

$C = M - M_0 = 4.37902 \cdot 10^{31}$ -2.7638913425379 = $4.3790199999999999999999999999997236108657462 1 \times 10^{31}$; for Q = 1; β = 5 and r = 1.94973 * 10^{13}:

1-2*(4.3790199999e+31)/(1.94973e+13)- 1/3(1.1056e-52)(1.94973e+13)^2+50/3*(1.94973e+13)^2*(1-(1+1/(25*(1.94973e+13)^4))^0.5)-4/(3*1.94973e+13)*sqrt(-5i)

Input interpretation:

$$1-2\times\frac{4.3790199999\times10^{31}}{1.94973\times10^{13}}-\frac{1}{3}\times1.1056\times10^{-52}\,(1.94973\times10^{13})^2+\frac{50}{3}\,(1.94973\times10^{13})^2\left(1-\sqrt{1+\frac{1}{25\,(1.94973\times10^{13})^4}}\right)-\frac{4}{3\times1.94973\times10^{13}}\sqrt{-5\,i}$$

i is the imaginary unit

Result:

$-4.49192\ldots\times10^{18}+1.08127\ldots\times10^{-13}\,i$

Polar coordinates:

$r=4.49192\times10^{18}$ (radius), $\theta=180°$ (angle)

Alternate form:

-4.49192×10^{18}

***f* = 4.49192 * 10^18**

From which:

(((1-2*(4.3790199999e+31)/(1.94973e+13)- 1/3(1.1056e-52)(1.94973e+13)^2+50/3*(1.94973e+13)^2*(1-(1+1/(25*(1.94973e+13)^4))^0.5)-4/(3*1.94973e+13)*sqrt(-5i))))^1/89-2/10^3

Input interpretation:

$$\left(1-2\times\frac{4.3790199999\times10^{31}}{1.94973\times10^{13}}-\frac{1}{3}\times1.1056\times10^{-52}\,(1.94973\times10^{13})^2+\frac{50}{3}\,(1.94973\times10^{13})^2\left(1-\sqrt{1+\frac{1}{25\,(1.94973\times10^{13})^4}}\right)-\frac{4}{3\times1.94973\times10^{13}}\sqrt{-5\,i}\right)\hat{}\,(1/89)-\frac{2}{10^3}$$

i is the imaginary unit

Result:

1.617225... +
0.05718045... i

Polar coordinates:

$r = 1.61824$ (radius), $\theta = 2.02497°$ (angle)

1.61824

From:

Bouncing black holes in quantum gravity and the Fermi gamma-ray excess

Aurélien Barrau, Boris Bolliet, Marrit Schutten, Francesca Vidotto - Physics Letters B 772 (2017) 58–62

We have that:

$$\lambda_{obs}^{BH} \sim \frac{2Gm}{c^2}(1+z) \times \sqrt{\frac{H_0^{-1}}{6k\Omega_\Lambda^{1/2}} \sinh^{-1}\left[\left(\frac{\Omega_\Lambda}{\Omega_M}\right)^{1/2}(z+1)^{-3/2}\right]}, \quad (5)$$

where we have reinserted the Newton constant G and the speed of light c; H_0, Ω_Λ and Ω_M being the Hubble constant, the cosmological constant, and the matter density. On the other hand, for standard sources, the measured wavelength is just related to the observed wavelength by

$$\lambda_{obs}^{other} = (1+z)\lambda_{emitted}^{other}. \quad (6)$$

$\Omega_m = 0.29^{+0.05}_{-0.03}$. ; $G = 6.674 * 10^{-11}$ $\Omega_\Lambda = 0.6889 \pm 0.0056$ m = 13.446316e+39 or 13.1799199e+39 (6.76e+9 and 6.626072042e+9 solar masses) $H_0 = 74.2$ z = 3 and c = 299792458

Range SMBH87 mass = **6.3×10^9 - 7.22×10^9**

(2*6.674e-11*13.446316e+39)/(299792458)^2 (1+3) sqrt[((74.2^(-1))) / (((6*0.6889^0.5))) sinh^-1((((0.6889/0.29)^0.5 (3+1)^(-1.5)))]

Input interpretation:

$$\frac{2\times 6.674\times 10^{-11}\times 13.446316\times 10^{39}}{299\,792\,458^2}(1+3)\sqrt{\frac{1}{74.2\left(6\sqrt{0.6889}\right)}\sinh^{-1}\left(\frac{\sqrt{\frac{0.6889}{0.29}}}{(3+1)^{1.5}}\right)}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Result:

$1.8184050026386289913612432168531223210552600211535057... \times 10^{12}$

$1.818405002638...*10^{12}$

From which, we obtain:

((((2*6.674e-11*13.446316e+39)/(299792458)^2 (1+3) sqrt[((74.2^(-1))) / (((6*0.6889^0.5))) sinh^-1(((0.6889/0.29)^0.5 (3+1)^(-1.5)))])))^1/8

Input interpretation:

$$\sqrt[8]{\frac{2\times 6.674\times 10^{-11}\times 13.446316\times 10^{39}}{299\,792\,458^2}(1+3)\sqrt{\frac{1}{74.2\left(6\sqrt{0.6889}\right)}\sinh^{-1}\left(\frac{\sqrt{\frac{0.6889}{0.29}}}{(3+1)^{1.5}}\right)}}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Result:

34.0770...

34.0770... ≈ 34

From which:

55/[((((2*6.674e-11*13.446316e+39)/(299792458)^2 (1+3) sqrt[((74.2^(-1))) / (((6*0.6889^0.5))) sinh^-1(((0.6889/0.29)^0.5 (3+1)^(-1.5)))])))^1/8]

Input interpretation:

$$\frac{55}{\sqrt[8]{\frac{2\times 6.674\times 10^{-11}\times 13.446316\times 10^{39}}{299\,792\,458^2}(1+3)\sqrt{\frac{1}{74.2\left(6\sqrt{0.6889}\right)}\sinh^{-1}\left(\frac{\sqrt{\frac{0.6889}{0.29}}}{(3+1)^{1.5}}\right)}}}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Result:

1.613991942624572205797720899206678801754976838734188860177...

1.6139919426…

For mass of SMBH87 = 13.1799199e+39 , we obtain:

[(((2*6.674e-11*13.1799199e+39)/(299792458)^2 (1+3) * sqrt(((((74.2^(-1)))/(((6*0.6889^0.5))) sinh^-1((((0.6889/0.29)^0.5 (3+1)^(-1.5)))))))))]^1/8

Input interpretation:

$$\left(\frac{2\times 6.674\times 10^{-11}\times 13.1799199\times 10^{39}}{299\,792\,458^2}(1+3)\sqrt{\frac{1}{74.2\left(6\sqrt{0.6889}\right)}\sinh^{-1}\left(\frac{\sqrt{\frac{0.6889}{0.29}}}{(3+1)^{1.5}}\right)}\right)\text{\textasciicircum}(1/8)$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Result:

33.9919...

33.9919…

From which, performing the following division where to the numerator put 55, we obtain:

55/[(((2*6.674e-11*13.1799199e+39)/(299792458)^2 (1+3) * sqrt(((((74.2^(-1)))/(((6*0.6889^0.5))) sinh^-1((((0.6889/0.29)^0.5 (3+1)^(-1.5)))))))))]^1/8

Input interpretation:

$$\frac{55}{\sqrt[8]{\frac{2\times 6.674\times 10^{-11}\times 13.1799199\times 10^{39}}{299\,792\,458^2}(1+3)\sqrt{\frac{1}{74.2\left(6\sqrt{0.6889}\right)}\sinh^{-1}\left(\frac{\sqrt{\frac{0.6889}{0.29}}}{(3+1)^{1.5}}\right)}}}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Result:

1.618034137264935470967634228327981974895109766623066818231...

1.618034137…

We note that, with regard 6.626072042 * 10^9 , we obtain the following interesting expression:

(6.626072042e+9)^1/47

Input interpretation:

$$\sqrt[47]{6.626072042\times 10^9}$$

Result:

1.61794180598...

1.61794180598...

We obtain also:

[(2*6.674e-11*13.446316e+39)/(299792458)^2 (1+3) sqrt((((74.2^(-1))) / (((6*0.6889^0.5))) sinh^-1(((0.6889/0.29)^0.5 (3+1)^(-1.5)))))]^1/(55+2)

Input interpretation:

$$\left(\frac{2\times 6.674\times 10^{-11}\times 13.446316\times 10^{39}}{299\,792\,458^2}\,(1+3)\sqrt{\frac{1}{74.2\left(6\sqrt{0.6889}\right)}\sinh^{-1}\left(\frac{\sqrt{\frac{0.6889}{0.29}}}{(3+1)^{1.5}}\right)}\right)\wedge\left(\frac{1}{55+2}\right)$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Result:

1.6409006669557699196394219028832439249681063219079338364 47...

1.640900666...

and:

[(2*6.674e-11*13.446316e+39)/(299792458)^2 (1+3) sqrt((((74.2^(-1))) / (((6*0.6889^0.5))) sinh^-1(((0.6889/0.29)^0.5 (3+1)^(-1.5)))))]^1/56

Input interpretation:

$$\left(\frac{2\times 6.674\times 10^{-11}\times 13.446316\times 10^{39}}{299\,792\,458^2}\,(1+3)\sqrt{\frac{1}{74.2\left(6\sqrt{0.6889}\right)}\sinh^{-1}\left(\frac{\sqrt{\frac{0.6889}{0.29}}}{(3+1)^{1.5}}\right)}\right)\wedge(1/56)$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Result:

1.655476601219360232039658978951932228269957102661080543155...

1.655476601219.... result practically equal to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164{,}2696$ i.e. 1,65578...

From:

High-redshift cosmography: auxiliary variables versus Pade polynomials
S. Capozziello, R. D'Agostino and O. Luongo - arXiv:2003.09341v2 [astro-ph.CO] 25 Mar 2020

We have that

As a relevant example, we present the method of the Padé approximations (Krantz & Parks 1992). The Padé technique is built up from the standard Taylor definition and is used to lower divergences at $z \geqslant 1$. Thus, a given function $f(z) = \sum_{i=0}^{\infty} c_i z^i$, expanded with a given set of coefficients, namely c_i, is approximated by means of a (n, m) Padé approximant by the ratio

$$P_{n,m}(z) = \frac{\sum\limits_{i=0}^{n} a_i z^i}{1 + \sum\limits_{j=1}^{m} b_j z^j}, \tag{12}$$

where the Taylor expansion matches the coefficients of the above expansion up to the highest possible order:

$$P_{n,m}(0) = f(0)\,, \tag{13}$$

$$P'_{n,m}(0) = f'(0)\,, \tag{14}$$

$$\vdots \tag{15}$$

$$P_{n,m}^{(n+m)}(0) = f^{(n+m)}(0)\,. \tag{16}$$

We have the following data:

Moreover, assuming the concordance value $\Omega_{m0} = 0.3$, one obtains the "fiducial set":

$$(q_0, j_0, s_0, l_0) = (-0.55, 1, -0.35, 0.685) . \qquad (30)$$

Now, we analyze the following equation:

$$H(z) = H_0\Big[1 + (1+q_0)z + \frac{1}{2}(j_0 - q_0^2)z^2 - \frac{1}{6}(-3q_0^2 - 3q_0^3 + j_0(3+4q_0) + s_0)z^3 + \frac{1}{24}(-4j_0^2 + l_0 - 12q_0^2 - 24q_0^3 - 15q_0^4 + j_0(12 + 32q_0 + 25q_0^2) + 8s_0 + 7q_0s_0)z^4\Big] .$$

For $z = 2$, $H_0 = 69.2$, we obtain:

69.2 [1+(1-0.55)*2+1/2(1+0.55^2)*4] = 311.746

69.2[-1/6((((-3*(-0.55)^2-3*(-0.55)^3+(3+4*(-0.55))-0.35))))*8] = -3.8406

69.2[1/24((((-4+0.685-12(-0.55)^2-24(-0.55)^3-15(-0.55)^4+((12+32*(-0.55)+25*(-0.55)^2)+8(-0.35)+7(-0.55*-0.35)))*16] = -175.979925

Thence:

311.746-3.8406+69.2[1/24((((-4+0.685-12(-0.55)^2-24(-0.55)^3-15(-0.55)^4+((12+32*(-0.55)+25*(-0.55)^2)+8(-0.35)+7(-0.55*-0.35)))*16]

Input interpretation:

$$311.746 - 3.8406 + 69.2\left(\frac{1}{24}\left(\left(-4 + 0.685 - 12(-0.55)^2 - 24(-0.55)^3 - 15(-0.55)^4 + \left(\left(12 + 32\times(-0.55) + 25(-0.55)^2\right) + 8\times(-0.35) + 7(-0.55\times(-0.35))\right)\right)\times 16\right)\right)$$

Result:

131.925475

131.925475

From:

-4+34 * 5^(3/4) integrate ((([sin(Pi/2) / sqrt(((((5sqrt5)) *((sin^2(Pi/2)))*[1-((sqrt5+1)/2)^(-5)*5^(-3/2) sin^2(Pi/2)])))])))Pi/2 x =cos^-1(golden ratio)^2.5..Pi/2

Input:

$$-4+34\times 5^{3/4}\int_{\cos^{-1}(\phi)^{2.5}}^{\frac{\pi}{2}}\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\sin^2\left(\frac{\pi}{2}\right)\right)}}\times\frac{\pi}{2}\,dx$$

$\cos^{-1}(x)$ is the inverse cosine function

ϕ is the golden ratio

Result:

$124.228+43.9959\,i$

Input interpretation:

$124.228+43.9959\,i$

i is the imaginary unit

Result:

$124.228\ldots+$
$43.9959\ldots\,i$

Polar coordinates:

$r = 131.789$ (radius), $\theta = 19.5018°$ (angle)

131.789

From

$$H(z)=H_0\Big[1+(1+q_0)z+\frac{1}{2}(j_0-q_0^2)z^2-\frac{1}{6}(-3q_0^2-3q_0^3+j_0(3+4q_0)+s_0)z^3$$
$$+\frac{1}{24}(-4j_0^2+l_0-12q_0^2-24q_0^3-15q_0^4+j_0(12+32q_0+25q_0^2)+8s_0+7q_0s_0)z^4\Big]\,.$$

For z = 1, H_0 = 69.2, we obtain:

69.2 [1+(1-0.55)+1/2(1+0.55^2)] = 145.4065

69.2[-1/6((((-3*(-0.55)^2-3*(-0.55)^3+(3+4*(-0.55))-0.35))))] = -0.480075

69.2[1/24(((((-4+0.685-12(-0.55)^2-24(-0.55)^3-15(-0.55)^4+((12+32*(-0.55)+25*(-0.55)^2)+8(-0.35)+7(-0.55*-0.35)))] = -10.9987453125

145.4065-0.480075+69.2[1/24((((-4+0.685-12(-0.55)^2-24(-0.55)^3-15(-0.55)^4+((12+32*(-0.55)+25*(-0.55)^2)+8(-0.35)+7(-0.55*-0.35)))]

Input interpretation:

$$145.4065 - 0.480075 + 69.2\left(\frac{1}{24}\left(-4 + 0.685 - 12(-0.55)^2 - 24(-0.55)^3 - 15(-0.55)^4 + \left(\left(12 + 32\times(-0.55) + 25(-0.55)^2\right) + 8\times(-0.35) + 7(-0.55\times(-0.35))\right)\right)\right)$$

Result:

133.9276796875

133.9276796875

Furthermore, for z = 8,

$$H(z) = H_0\Big[1 + (1+q_0)z + \frac{1}{2}(j_0 - q_0^2)z^2 - \frac{1}{6}(-3q_0^2 - 3q_0^3 + j_0(3+4q_0) + s_0)z^3 + \frac{1}{24}(-4j_0^2 + l_0 - 12q_0^2 - 24q_0^3 - 15q_0^4 + j_0(12 + 32q_0 + 25q_0^2) + 8s_0 + 7q_0s_0)z^4\Big] .$$

we obtain:

69.2 [1+(1-0.55)+1/2(1+0.55^2)*8] = 460.872

69.2[-1/6((((-3*(-0.55)^2-3*(-0.55)^3+(3+4*(-0.55))-0.35))))*8^3] = -245.7984

69.2[1/24((((-4+0.685-12(-0.55)^2-24(-0.55)^3-15(-0.55)^4+((12+32*(-0.55)+25*(-0.55)^2)+8(-0.35)+7(-0.55*-0.35)))*8^4] = -45050.8608

From:

460.872-245.7984+69.2[1/24((((-4+0.685-12(-0.55)^2-24(-0.55)^3-15(-0.55)^4+((12+32*(-0.55)+25*(-0.55)^2)+8(-0.35)+7(-0.55*-0.35)))*8^4]

Input interpretation:

$$460.872 - 245.7984 + 69.2\left(\frac{1}{24}\left(\left(-4 + 0.685 - 12(-0.55)^2 - 24(-0.55)^3 - 15(-0.55)^4 + \left(\left(12 + 32\times(-0.55) + 25(-0.55)^2\right) + 8\times(-0.35) + 7(-0.55\times(-0.35))\right)\right)\times 8^4\right)\right)$$

Result:

-44835.7872

-44835.7872

From which:

[-(((460.872-245.7984+69.2[1/24(-4+0.685-12(-0.55)^2-24(-0.55)^3-15(-0.55)^4+((12+32*(-0.55)+25*(-0.55)^2)+8(-0.35)+7(-0.55*-0.35)))*8^4])))]^1/22-(7+2)/10^3

Input interpretation:

$$\left(-\left(460.872-245.7984+69.2\left(\frac{1}{24}\left(-4+0.685-12(-0.55)^2-24(-0.55)^3-15(-0.55)^4+\left(\left(12+32\times(-0.55)+25(-0.55)^2\right)+8\times(-0.35)+7(-0.55\times(-0.35))\right)\right)\times 8^4\right)\right)\right)\text{^}(1/22)-\frac{7+2}{10^3}$$

Result:

$1.6181870935501984069193877438460474390576880910309283247 13...$

1.61818709355...

From the previous Ramanujan equation, we obtain:

199+10^3((((5+2*5 integrate (Pi/2) / sqrt((((1-((sqrt5+1)/2)^(-5)*5^(-3/2) sin^2(Pi/2)))))dx x =cos^-1(golden ratio)^2.5..Pi/2))))

Input:

$$199+10^3\left(5+2\times 5\int_{\cos^{-1}(\phi)^{2.5}}^{\frac{\pi}{2}}\frac{\frac{\pi}{2}}{\sqrt{1-\frac{5^{-3/2}}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\sin^2\left(\frac{\pi}{2}\right)}}\,dx\right)$$

$\cos^{-1}(x)$ is the inverse cosine function

ϕ is the golden ratio

Result:

$42913.1+12940.\,i$

Input interpretation:

42913.1 + 12940. i

i is the imaginary unit

Result:

42913.1... +
1.2940... × 10^4 i

Polar coordinates:

$r = 44821.6$ (radius), $\theta = 16.7801°$ (angle)

44821.6

From:

Exact geometric optics in a Morris-Thorne wormhole spacetime
Thomas Muller - Visualisierungsinstitut der Universitat Stuttgart, Nobelstrasse 15, 70569 Stuttgart, Germany - (Received 17 December 2007; published 26 February 2008) - PHYSICAL REVIEW D **77,** 044043 (2008)

We have that:

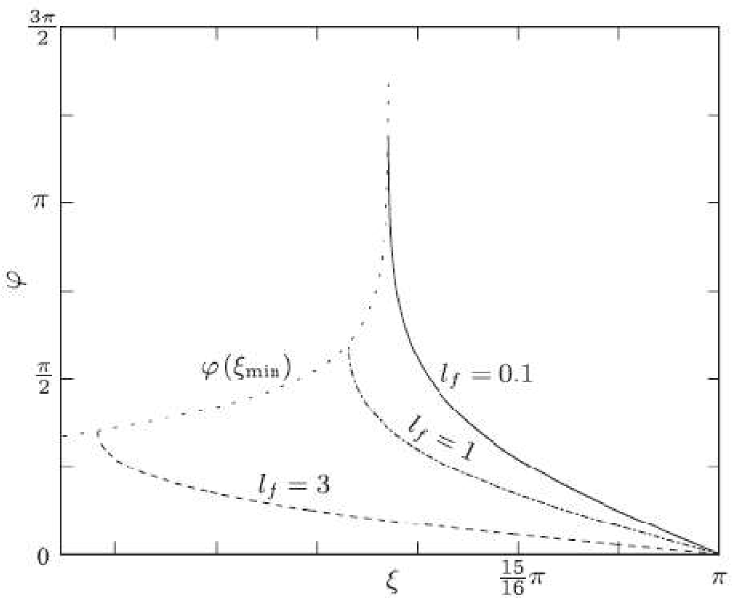


FIG. 12. Lensing $\varphi = \varphi(\xi)$ for an observer at $l_i = 6$ and rings

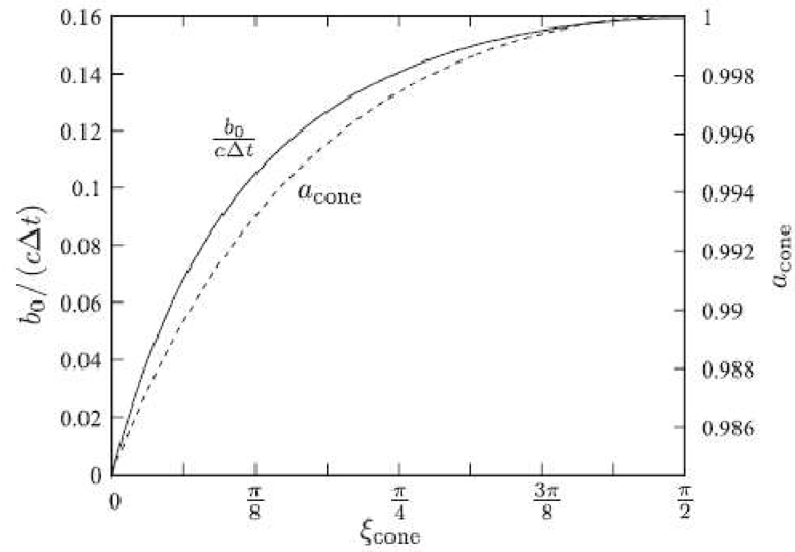


FIG. 6. The observation angle $\xi_{\rm cone}$ is plotted on the abscissa. Left axis: The solid line represents the size of the throat b_0 scaled by the time Δt between the emission of the flash and the observation of the ring. Right axis: The dashed line corresponds to the parameter $a_{\rm cone}$.

Now, the distance of the observer to the wormhole throat follows from Eq. (26),

$$l_i = \frac{b_0\sqrt{1 - a_{\rm cone}^2 \sin^2\xi_{\rm cone}}}{a_{\rm cone}\sin\xi_{\rm cone}}. \qquad (66)$$

For $a_{\rm cone} =$ 0.998136044617 that is equal to 1 / 1.0018674362, a value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5}-\varphi}} = 1 + \cfrac{e^{-2\pi}}{1+\cfrac{e^{-4\pi}}{1+\cfrac{e^{-6\pi}}{1+\cfrac{e^{-8\pi}}{1+\dots}}}} \approx 1.0018674362$$

$\xi_{cone} = \pi / 8$ and $b_0 = 2$, from

$$l_i = \frac{b_0\sqrt{1 - a_{\text{cone}}^2 \sin^2 \xi_{\text{cone}}}}{a_{\text{cone}} \sin \xi_{\text{cone}}}.$$

We obtain:

((2*sqrt(1-0.998136044617^2*sin^2(Pi/8)))) / ((0.998136044617 * sin(Pi/8)))

Input interpretation:

$$\frac{2\sqrt{1 - 0.998136044617^2 \sin^2\left(\frac{\pi}{8}\right)}}{0.998136044617 \sin\left(\frac{\pi}{8}\right)}$$

Result:

4.83898925052...

4.83898925052…

Alternative representations:

$$\frac{2\sqrt{1 - 0.9981360446170000^2 \sin^2\left(\frac{\pi}{8}\right)}}{0.9981360446170000 \sin\left(\frac{\pi}{8}\right)} = \frac{2\sqrt{1 - 0.9981360446170000^2 \left(\frac{1}{\csc\left(\frac{\pi}{8}\right)}\right)^2}}{\frac{0.9981360446170000}{\csc\left(\frac{\pi}{8}\right)}}$$

$$\frac{2\sqrt{1 - 0.9981360446170000^2 \sin^2\left(\frac{\pi}{8}\right)}}{0.9981360446170000 \sin\left(\frac{\pi}{8}\right)} = \frac{2\sqrt{1 - 0.9981360446170000^2 \cos^2\left(\frac{\pi}{2} - \frac{\pi}{8}\right)}}{0.9981360446170000 \cos\left(\frac{\pi}{2} - \frac{\pi}{8}\right)}$$

$$\frac{2\sqrt{1 - 0.9981360446170000^2 \sin^2\left(\frac{\pi}{8}\right)}}{0.9981360446170000 \sin\left(\frac{\pi}{8}\right)} = -\frac{2\sqrt{1 - 0.9981360446170000^2 \left(-\cos\left(\frac{\pi}{2} + \frac{\pi}{8}\right)\right)^2}}{0.9981360446170000 \cos\left(\frac{\pi}{2} + \frac{\pi}{8}\right)}$$

Series representations:

$$\frac{2\sqrt{1-0.9981360446170000^2 \sin^2\left(\frac{\pi}{8}\right)}}{0.9981360446170000 \sin\left(\frac{\pi}{8}\right)} = \frac{1.001867436200759 \sum_{k=0}^{\infty} \frac{(-0.996275563563670)^k \left(-\frac{1}{2}\right)_k \left(-\sin^2\left(\frac{\pi}{8}\right)\right)^k}{k!}}{\sum_{k=0}^{\infty} (-1)^k J_{1+2k}\left(\frac{\pi}{8}\right)}$$

$$\frac{2\sqrt{1-0.9981360446170000^2 \sin^2\left(\frac{\pi}{8}\right)}}{0.9981360446170000 \sin\left(\frac{\pi}{8}\right)} = \frac{1}{\sum_{k=0}^{\infty} (-1)^k J_{1+2k}\left(\frac{\pi}{8}\right)} 1.001867436200759 \exp\left(i\pi \left\lfloor \frac{\arg\left(1-x-0.996275563563670 \sin^2\left(\frac{\pi}{8}\right)\right)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k x^{-k} \left(-\frac{1}{2}\right)_k \left(1-x-0.996275563563670 \sin^2\left(\frac{\pi}{8}\right)\right)^k}{k!} \quad \text{for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{2\sqrt{1-0.9981360446170000^2 \sin^2\left(\frac{\pi}{8}\right)}}{0.9981360446170000 \sin\left(\frac{\pi}{8}\right)} = \frac{1}{\sum_{k=0}^{\infty} \frac{\left(-\frac{9}{64}\right)^k (-\pi)^{2k}}{(2k)!}} 2.003734872401518 \exp\left(i\pi \left\lfloor \frac{\arg\left(1-x-0.996275563563670 \sin^2\left(\frac{\pi}{8}\right)\right)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k x^{-k} \left(-\frac{1}{2}\right)_k \left(1-x-0.996275563563670 \sin^2\left(\frac{\pi}{8}\right)\right)^k}{k!} \quad \text{for } (x \in \mathbb{R} \text{ and } x < 0)$$

Multiple-argument formulas:

$$\frac{2\sqrt{1-0.9981360446170000^2 \sin^2\left(\frac{\pi}{8}\right)}}{0.9981360446170000 \sin\left(\frac{\pi}{8}\right)} = \frac{1.001867436200759 \sqrt{1-3.985102254254679 \cos^2\left(\frac{\pi}{16}\right) \sin^2\left(\frac{\pi}{16}\right)}}{\cos\left(\frac{\pi}{16}\right) \sin\left(\frac{\pi}{16}\right)}$$

$$\frac{2\sqrt{1-0.9981360446170000^2\sin^2\left(\frac{\pi}{8}\right)}}{0.9981360446170000\sin\left(\frac{\pi}{8}\right)}=\frac{2.003734872401518\sqrt{1-0.996275563563670\left(3\sin\left(\frac{\pi}{24}\right)-4\sin^3\left(\frac{\pi}{24}\right)\right)^2}}{3\sin\left(\frac{\pi}{24}\right)-4\sin^3\left(\frac{\pi}{24}\right)}$$

$$\frac{2\sqrt{1-0.9981360446170000^2\sin^2\left(\frac{\pi}{8}\right)}}{0.9981360446170000\sin\left(\frac{\pi}{8}\right)}=\frac{2.003734872401518\sqrt{1-0.996275563563670\,U_{-\frac{7}{8}}(\cos(\pi))^2\sin^2(\pi)}}{U_{-\frac{7}{8}}(\cos(\pi))\sin(\pi)}$$

From which, we obtain:

(((((2*sqrt(1-0.998136044617^2*sin^2(Pi/8)))) / ((3((0.998136044617 * sin(Pi/8))))))))+5/10^3

Input interpretation:

$$\frac{2\sqrt{1-0.998136044617^2\sin^2\left(\frac{\pi}{8}\right)}}{3\left(0.998136044617\sin\left(\frac{\pi}{8}\right)\right)}+\frac{5}{10^3}$$

Result:

1.617996416839273748468665625519341084146844270426300911631...

1.6179964168392…

Alternative representations:

$$\frac{2\sqrt{1-0.9981360446170000^2\sin^2\left(\frac{\pi}{8}\right)}}{3\left(0.9981360446170000\sin\left(\frac{\pi}{8}\right)\right)}+\frac{5}{10^3}=\frac{5}{10^3}+\frac{2\sqrt{1-0.9981360446170000^2\left(\frac{1}{\csc\left(\frac{\pi}{8}\right)}\right)^2}}{\frac{2.994408133851000}{\csc\left(\frac{\pi}{8}\right)}}$$

$$\frac{2\sqrt{1-0.9981360446170000^2\sin^2\left(\frac{\pi}{8}\right)}}{3\left(0.9981360446170000\sin\left(\frac{\pi}{8}\right)\right)}+\frac{5}{10^3}=$$
$$\frac{5}{10^3}+\frac{2\sqrt{1-0.9981360446170000^2\cos^2\left(\frac{\pi}{2}-\frac{\pi}{8}\right)}}{2.994408133851000\cos\left(\frac{\pi}{2}-\frac{\pi}{8}\right)}$$

$$\frac{2\sqrt{1-0.9981360446170000^2\sin^2\left(\frac{\pi}{8}\right)}}{3\left(0.9981360446170000\sin\left(\frac{\pi}{8}\right)\right)}+\frac{5}{10^3}=$$
$$\frac{5}{10^3}+-\frac{2\sqrt{1-0.9981360446170000^2\left(-\cos\left(\frac{\pi}{2}+\frac{\pi}{8}\right)\right)^2}}{2.994408133851000\cos\left(\frac{\pi}{2}+\frac{\pi}{8}\right)}$$

Series representations:

$$\frac{2\sqrt{1-0.9981360446170000^2\sin^2\left(\frac{\pi}{8}\right)}}{3\left(0.9981360446170000\sin\left(\frac{\pi}{8}\right)\right)}+\frac{5}{10^3}=\frac{1}{\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{8}\right)}$$
$$0.005000000000000000\left(1.000000000000000\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{8}\right)+\right.$$
$$\left.66.7911624133839\sum_{k=0}^{\infty}\frac{(-0.996275563563670)^k\left(-\frac{1}{2}\right)_k\left(-\sin^2\left(\frac{\pi}{8}\right)\right)^k}{k!}\right)$$

$$\frac{2\sqrt{1-0.9981360446170000^2\sin^2\left(\frac{\pi}{8}\right)}}{3\left(0.9981360446170000\sin\left(\frac{\pi}{8}\right)\right)}+\frac{5}{10^3}=$$
$$\frac{1}{\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{8}\right)}\,0.333955812066920$$
$$\left(0.0149720406692550\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{8}\right)+1.000000000000000\right.$$
$$\exp\left(i\pi\left\lfloor\frac{\arg\left(1-x-0.996275563563670\sin^2\left(\frac{\pi}{8}\right)\right)}{2\pi}\right\rfloor\right)\sqrt{x}$$
$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(1-x-0.996275563563670\sin^2\left(\frac{\pi}{8}\right)\right)^k}{k!}\right)$$

for ($x\in\mathbb{R}$ and $x<0$)

$$\frac{2\sqrt{1-0.9981360446170000^2\sin^2\left(\frac{\pi}{8}\right)}}{3\left(0.9981360446170000\sin\left(\frac{\pi}{8}\right)\right)}+\frac{5}{10^3}=$$

$$\frac{1}{\sum_{k=0}^{\infty}\frac{\left(-\frac{9}{64}\right)^k(-\pi)^{2k}}{(2k)!}}\,0.66791162413384\left(0.0074860203346275\sum_{k=0}^{\infty}\frac{\left(-\frac{9}{64}\right)^k(-\pi)^{2k}}{(2k)!}+\right.$$
$$1.00000000000000\exp\left(i\pi\left\lfloor\frac{\arg\left(1-x-0.996275563563670\sin^2\left(\frac{\pi}{8}\right)\right)}{2\pi}\right\rfloor\right)$$
$$\left.\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(1-x-0.996275563563670\sin^2\left(\frac{\pi}{8}\right)\right)^k}{k!}\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

Multiple-argument formulas:

$$\frac{2\sqrt{1-0.9981360446170000^2\sin^2\left(\frac{\pi}{8}\right)}}{3\left(0.9981360446170000\sin\left(\frac{\pi}{8}\right)\right)}+\frac{5}{10^3}=$$
$$\frac{1}{200}+\frac{0.3339558120669196\sqrt{1-3.985102254254679\cos^2\left(\frac{\pi}{16}\right)\sin^2\left(\frac{\pi}{16}\right)}}{\cos\left(\frac{\pi}{16}\right)\sin\left(\frac{\pi}{16}\right)}$$

$$\frac{2\sqrt{1-0.9981360446170000^2\sin^2\left(\frac{\pi}{8}\right)}}{3\left(0.9981360446170000\sin\left(\frac{\pi}{8}\right)\right)}+\frac{5}{10^3}=$$
$$\frac{1}{200}+\frac{0.6679116241338392\sqrt{1-0.996275563563670\left(3\sin\left(\frac{\pi}{24}\right)-4\sin^3\left(\frac{\pi}{24}\right)\right)^2}}{3\sin\left(\frac{\pi}{24}\right)-4\sin^3\left(\frac{\pi}{24}\right)}$$

$$\frac{2\sqrt{1-0.9981360446170000^2\sin^2\left(\frac{\pi}{8}\right)}}{3\left(0.9981360446170000\sin\left(\frac{\pi}{8}\right)\right)}+\frac{5}{10^3}=$$
$$\frac{1}{200}+\frac{0.6679116241338392\sqrt{1-0.996275563563670\,U_{-\frac{7}{8}}(\cos(\pi))^2\sin^2(\pi)}}{U_{-\frac{7}{8}}(\cos(\pi))\sin(\pi)}$$

From

$$\frac{da}{d\xi} = -\frac{b_0 \cos\xi}{\sin^2\xi\sqrt{b_0^2 + l_i^2}}. \qquad (85)$$

For $b_0 = 2$, $\xi = 15\pi/16$ and $l_i = 6$, we obtain:

-((2 cos (15Pi/16))) / ((sin^2 (15Pi/16) sqrt(4+36)))

Input:

$$-\frac{2\cos\left(15\times\frac{\pi}{16}\right)}{\sin^2\left(15\times\frac{\pi}{16}\right)\sqrt{4+36}}$$

Exact result:

$$\frac{\cot\left(\frac{\pi}{16}\right)\csc\left(\frac{\pi}{16}\right)}{\sqrt{10}}$$

$\cot(x)$ is the cotangent function

$\csc(x)$ is the cosecant function

Decimal approximation:

8.1489656697155527913887766459165734880276438982967354445 50...

8.14896566971…

Alternate forms:

$$-\frac{\sqrt{\frac{2}{5}}\cos\left(\frac{\pi}{16}\right)}{\cos\left(\frac{\pi}{8}\right)-1}$$

$$\sqrt{\frac{1}{5}\left(84+58\sqrt{2}+\sqrt{2\left(6890+4871\sqrt{2}\right)}\right)}$$

$$\frac{\sqrt{\frac{2}{5}\left(2+\sqrt{2+\sqrt{2}}\right)}}{2-\sqrt{2+\sqrt{2}}}$$

Minimal polynomial:

$625x^8 - 42000x^6 + 33000x^4 - 2080x^2 + 8$

Alternative representations:

$$-\frac{2\cos\left(\frac{15\pi}{16}\right)}{\sin^2\left(\frac{15\pi}{16}\right)\sqrt{4+36}} = -\frac{2\cosh\left(\frac{15i\pi}{16}\right)}{\cos^2\left(\frac{\pi}{2}-\frac{15\pi}{16}\right)\sqrt{40}}$$

$$-\frac{2\cos\left(\frac{15\pi}{16}\right)}{\sin^2\left(\frac{15\pi}{16}\right)\sqrt{4+36}} = -\frac{2\cosh\left(-\frac{15i\pi}{16}\right)}{\cos^2\left(\frac{\pi}{2}-\frac{15\pi}{16}\right)\sqrt{40}}$$

$$-\frac{2\cos\left(\frac{15\pi}{16}\right)}{\sin^2\left(\frac{15\pi}{16}\right)\sqrt{4+36}} = -\frac{2\cosh\left(-\frac{15i\pi}{16}\right)}{\left(-\cos\left(\frac{\pi}{2}+\frac{15\pi}{16}\right)\right)^2\sqrt{40}}$$

Series representations:

$$-\frac{2\cos\left(\frac{15\pi}{16}\right)}{\sin^2\left(\frac{15\pi}{16}\right)\sqrt{4+36}} = -\sqrt{\frac{2}{5}}\left(1+2\sum_{k=1}^{\infty}q^{2k}\right)\sum_{k=1}^{\infty}q^{-1+2k} \quad \text{for } q=\sqrt[16]{-1}$$

$$-\frac{2\cos\left(\frac{15\pi}{16}\right)}{\sin^2\left(\frac{15\pi}{16}\right)\sqrt{4+36}} = -16\,i\sqrt{\frac{2}{5}}\sum_{k_1=-\infty}^{\infty}\sum_{k_2=1}^{\infty}\frac{q^{-1+2k_2}}{\pi-256\,\pi k_1^2} \quad \text{for } q=\sqrt[16]{-1}$$

$$-\frac{2\cos\left(\frac{15\pi}{16}\right)}{\sin^2\left(\frac{15\pi}{16}\right)\sqrt{4+36}} = \frac{8\,i\sqrt{\frac{2}{5}}\left(-1+2\sum_{k=1}^{\infty}\frac{(-1)^k}{-1+256k^2}\right)\left(1+2\sum_{k=1}^{\infty}q^{2k}\right)}{\pi} \quad \text{for } q=\sqrt[16]{-1}$$

Integral representations:

$$-\frac{2\cos\left(\frac{15\pi}{16}\right)}{\sin^2\left(\frac{15\pi}{16}\right)\sqrt{4+36}} = \frac{\sqrt{\frac{2}{5}}\left(\int_0^{\infty}\frac{1}{t^{15/16}(1+t)}\,dt\right)\int_0^{\infty}\frac{-1+t^{7/8}}{-1+t^2}\,dt}{\pi^2}$$

$$-\frac{2\cos\left(\frac{15\pi}{16}\right)}{\sin^2\left(\frac{15\pi}{16}\right)\sqrt{4+36}} = -\frac{\left(\int_0^{\infty}\frac{1}{t^{15/16}(1+t)}\,dt\right)\int_{\frac{\pi}{2}}^{\frac{\pi}{16}}\csc^2(t)\,dt}{\sqrt{10}\,\pi}$$

1/5 [-((2 cos (15Pi/16))) / ((sin^2 (15Pi/16) sqrt(4+36)))] - 11/10^3

Input:

$$\frac{1}{5}\left(-\frac{2\cos\left(15\times\frac{\pi}{16}\right)}{\sin^2\left(15\times\frac{\pi}{16}\right)\sqrt{4+36}}\right)-\frac{11}{10^3}$$

Exact result:

$$\frac{\cot\left(\frac{\pi}{16}\right)\csc\left(\frac{\pi}{16}\right)}{5\sqrt{10}}-\frac{11}{1000}$$

cot(x) is the cotangent function

csc(x) is the cosecant function

Decimal approximation:

1.618793133943110558277553291833146976055287796593470889 10...

1.61879313394…

Alternate forms:

$$\frac{20\sqrt{10}\,\cot\left(\frac{\pi}{16}\right)\csc\left(\frac{\pi}{16}\right)-11}{1000}$$

$$\frac{40\sqrt{5\left(84+58\sqrt{2}+\sqrt{2\left(6890+4871\sqrt{2}\right)}\right)}-11}{1000}$$

$$\frac{\csc^2\left(\frac{\pi}{16}\right)\left(200\cos\left(\frac{\pi}{16}\right)-11\sqrt{10}\,\sin^2\left(\frac{\pi}{16}\right)\right)}{1000\sqrt{10}}$$

Minimal polynomial:

$$1\,000\,000\,000\,000\,000\,000\,000\,000\,x^8+88\,000\,000\,000\,000\,000\,000\,000\,000\,x^7-2\,684\,612\,000\,000\,000\,000\,000\,000\,x^6-177\,333\,464\,000\,000\,000\,000\,000\,x^5+79\,602\,304\,870\,000\,000\,000\,000\,x^4+3\,645\,574\,458\,856\,000\,000\,000\,x^3-152\,249\,795\,516\,292\,000\,000\,x^2-4\,238\,649\,754\,630\,632\,000\,x+8\,228\,077\,938\,390\,881$$

Alternative representations:

$$-\frac{2\cos\left(\frac{15\pi}{16}\right)}{\left(\sin^2\left(\frac{15\pi}{16}\right)\sqrt{4+36}\right)5}-\frac{11}{10^3}=-\frac{11}{10^3}-\frac{2\cosh\left(\frac{15 i\pi}{16}\right)}{5\left(\cos^2\left(\frac{\pi}{2}-\frac{15\pi}{16}\right)\sqrt{40}\right)}$$

$$-\frac{2\cos\left(\frac{15\pi}{16}\right)}{\left(\sin^2\left(\frac{15\pi}{16}\right)\sqrt{4+36}\right)5}-\frac{11}{10^3}=-\frac{11}{10^3}-\frac{2\cosh\left(-\frac{15 i\pi}{16}\right)}{5\left(\cos^2\left(\frac{\pi}{2}-\frac{15\pi}{16}\right)\sqrt{40}\right)}$$

$$-\frac{2\cos\left(\frac{15\pi}{16}\right)}{\left(\sin^2\left(\frac{15\pi}{16}\right)\sqrt{4+36}\right)5}-\frac{11}{10^3}=-\frac{11}{10^3}-\frac{2\cosh\left(-\frac{15 i\pi}{16}\right)}{5\left(\left(-\cos\left(\frac{\pi}{2}+\frac{15\pi}{16}\right)\right)^2\sqrt{40}\right)}$$

Series representations:

$$-\frac{2\cos\left(\frac{15\pi}{16}\right)}{\left(\sin^2\left(\frac{15\pi}{16}\right)\sqrt{4+36}\right)5}-\frac{11}{10^3}=-\frac{i\left(-11 i+640\sqrt{10}\sum_{k_1=-\infty}^{\infty}\sum_{k_2=1}^{\infty}\frac{q^{-1+2k_2}}{\pi-256\pi k_1^2}\right)}{1000}$$

for $q=\sqrt[16]{-1}$

$$-\frac{2\cos\left(\frac{15\pi}{16}\right)}{\left(\sin^2\left(\frac{15\pi}{16}\right)\sqrt{4+36}\right)5}-\frac{11}{10^3}=$$
$$\frac{-11-40\sqrt{10}\sum_{k=1}^{\infty}q^{-1+2k}-80\sqrt{10}\sum_{k_1=1}^{\infty}\sum_{k_2=1}^{\infty}q^{-1+2k_1+2k_2}}{1000}\quad\text{for } q=\sqrt[16]{-1}$$

$$-\frac{2\cos\left(\frac{15\pi}{16}\right)}{\left(\sin^2\left(\frac{15\pi}{16}\right)\sqrt{4+36}\right)5}-\frac{11}{10^3}=\frac{-11+5120\sqrt{10}\sum_{k_1=-\infty}^{\infty}\sum_{k_2=-\infty}^{\infty}\frac{e^{i\pi k_2}}{\left(\pi-256\pi k_1^2\right)\left(\pi-256\pi k_2^2\right)}}{1000}$$

Integral representations:

$$-\frac{2\cos\left(\frac{15\pi}{16}\right)}{\left(\sin^2\left(\frac{15\pi}{16}\right)\sqrt{4+36}\right)5}-\frac{11}{10^3}=\frac{-11\pi^2+40\sqrt{10}\left(\int_0^{\infty}\frac{1}{t^{15/16}(1+t)}\,dt\right)\int_0^{\infty}\frac{-1+t^{7/8}}{-1+t^2}\,dt}{1000\pi^2}$$

$$-\frac{2\cos\left(\frac{15\pi}{16}\right)}{\left(\sin^2\left(\frac{15\pi}{16}\right)\sqrt{4+36}\right)5}-\frac{11}{10^3}=-\frac{11\pi+20\sqrt{10}\left(\int_0^{\infty}\frac{1}{t^{15/16}(1+t)}\,dt\right)\int_{\frac{\pi}{2}}^{\frac{\pi}{16}}\csc^2(t)\,dt}{1000\pi}$$

From:

Space-time slicing in Horndeski theories and its implications for non-singular bouncing solutions

Anna Ijjas - arXiv:1710.05990v2 [gr-qc] 27 Jan 2018

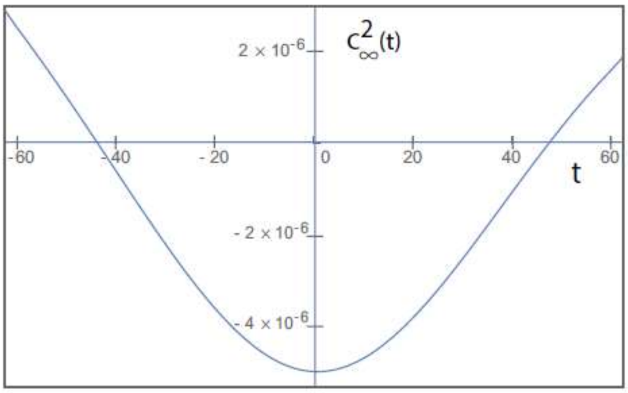


Figure 1: Evolution of c_∞^2 around γ-crossing as a function of time for the parameter values given in the caption of Figure 2. The time coordinate is given in reduced Planck units; the y-axis has dimensionless units. The γ-crossing point is at $t = 0$. Notice that, even though $|c_\infty^2| \ll 1$ around γ-crossing in this example, the true effective sound speed c_S^2 is generally much greater than $|c_\infty^2|$ and can be of order one.

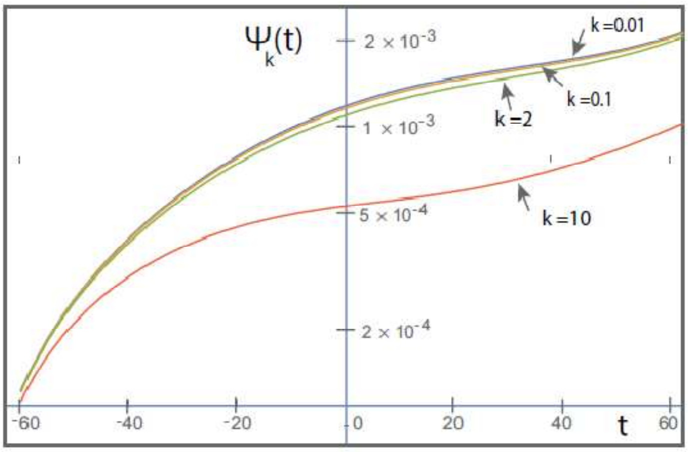


Figure 2: Numerical solution of Eq. (4.3) for the modes Ψ_k as a function of time corresponding to the background around γ-crossing as described in Eqs. (4.14-4.12) for the wave-numbers $k = 0.01$ (blue curve), $k = 0.1$ (orange curve), $k = 2$ (green curve), and $k = 10$ (red curve); and the parameters $t_\gamma = -100, p = 1/10, A_0 = 10^{-3}, A_1 = 2/10, V_0 = -5 \times 10^{-3}$, and $\gamma_0 = 10^{-6}$. The x-axis has reduced Planck units and the y-axis has dimensionless units. The graph verifies that all modes pass through γ-crossing undisturbed.

From:

$$c_\infty^2(\Delta t) \simeq \frac{\gamma_0}{(-4V_0)}\left(-1+3\frac{A_1}{A_0}(-t_\gamma)^2\left(\frac{\Delta t}{t_\gamma}\right)^2\right) \simeq -\frac{\gamma_0}{(-4V_0)}\,. \qquad (4.28)$$

(10^-6)/(-4*(-5*10^-3))

Input interpretation:

$$\frac{1}{10^6\,(-4\,(-5\times 10^{-3}))}$$

Exact result:

$$\frac{1}{20\,000}$$

Decimal form:

0.00005

0.00005

From:

$$u_H^2(\Delta t) \simeq \frac{A_0}{(-4V_0)p}\left(p+\frac{\gamma_0 t_\gamma^2}{A_0}\frac{\Delta t}{(-t_\gamma)}\right)^2 c_\infty^2(t)\,. \qquad (4.27)$$

(((10^-3)/((-4*(-5*10^-3))1/10)))*(((1/10+(10^-6)*(10^4)/(10^-3)*(100)/(100))))^2 *(2*10^-6)

Input interpretation:

$$\frac{1}{10^3\left((-4\,(-5\times 10^{-3}))\times\frac{1}{10}\right)}\left(\frac{1}{10}+\frac{\frac{10^4}{\frac{1}{10^3}}\times\frac{100}{100}}{10^6}\right)^2\times 2\times 10^{-6}$$

Exact result:

$$\frac{10\,201}{100\,000\,000}$$

Decimal form:

0.00010201

0.00010201

or:

(((1e-3)/((-4*(-5e-3))1/10)))*(((1/10+(1e-6*10^4)/(1e-3)*(100)/(100))))^2 *(2e-6)

Input interpretation:

$$\frac{1\times 10^{-3}}{(-4(-5\times 10^{-3}))\times \frac{1}{10}}\left(\frac{1}{10}+\frac{1\times 10^{-6}\times 10^{4}}{1\times 10^{-3}}\times \frac{100}{100}\right)^2\times 2\times 10^{-6}$$

Exact result:

$$\frac{10\,201}{100\,000\,000}$$

Decimal form:

0.00010201

0.00010201

or:

(((1e-3)/((-4*(-5e-3))1/10)))*(((1/10+(1e-6*10^4)/(1e-3)*(100)/(100))))^2 *(-4e-6)

Input interpretation:

$$\frac{1\times 10^{-3}}{(-4(-5\times 10^{-3}))\times \frac{1}{10}}\left(\frac{1}{10}+\frac{1\times 10^{-6}\times 10^{4}}{1\times 10^{-3}}\times \frac{100}{100}\right)^2\left(-4\times 10^{-6}\right)$$

Exact result:

$$-\frac{10\,201}{50\,000\,000}$$

Decimal form:

−0.00020402

-0.00020402

For

$$\left(c_{\infty}^{2}(t) \right) = \left(-3.7443977185 \times 10^{-6}\right)$$

we obtain:

((((((1e-3)/((-4*(-5e-3))1/10)))*(((1/10+(1e-6*10^4)/(1e-3)*(100)/(100))))^2 *(-3.7443977185e-6))))

Input interpretation:

$$\frac{1 \times 10^{-3}}{(-4(-5 \times 10^{-3})) \times \frac{1}{10}} \left(\frac{1}{10} + \frac{1 \times 10^{-6} \times 10^{4}}{1 \times 10^{-3}} \times \frac{100}{100}\right)^2 \left(-3.7443977185 \times 10^{-6}\right)$$

Result:

−0.0001909830056320925

-0.0001909830056320925

From the ratio between the two results, −0.0005 and −0.0001909830056320925 ,

we obtain:

[-0.0005/((((((1e-3)/((-4*(-5e-3))1/10)))*(((1/10+(1e-6*10^4)/(1e-3)*(100)/(100))))^2 *(-3.7443977185e-6))))]

Input interpretation:

$$-\frac{0.0005}{\frac{1 \times 10^{-3}}{(-4(-5 \times 10^{-3})) \times \frac{1}{10}} \left(\frac{1}{10} + \frac{1 \times 10^{-6} \times 10^{4}}{1 \times 10^{-3}} \times \frac{100}{100}\right)^2 \left(-3.7443977185 \times 10^{-6}\right)}$$

Result:

2.618033988653390132944588432838423167632528370329266004991...

2.618033988653…

sqrt[-0.0005/((((((1e-3)/((-4*(-5e-3))1/10)))*(((1/10+(1e-6*10^4)/(1e-3)*(100)/(100))))^2 *(-3.7443977185e-6)))))]

Input interpretation:

$$\sqrt{-\frac{0.0005}{\frac{1\times 10^{-3}}{(-4(-5\times 10^{-3}))\times\frac{1}{10}}\left(\frac{1}{10}+\frac{1\times 10^{-6}\times 10^{4}}{1\times 10^{-3}}\times\frac{100}{100}\right)^{2}(-3.7443977185\times 10^{-6})}}$$

Result:

1.6180339887200732511516571881227706316980544156009648595 21...

1.61803398872…

From

$$\frac{1\times 10^{-3}}{(-4(-5\times 10^{-3}))\times\frac{1}{10}}\left(\frac{1}{10}+\frac{1\times 10^{-6}\times 10^{4}}{1\times 10^{-3}}\times\frac{100}{100}\right)^{2}(-3.7443977185\times 10^{-6}) =$$

$= -0.0001909830056320925$

We obtain also:

(-0.11*3)/((((((1e-3)/((-4*(-5e-3))1/10)))*(((1/10+(1e-6*10^4)/(1e-3)*(100)/(100))))^2 *(-3.7443977185e-6))))

Input interpretation:

$$\frac{-0.11\times 3}{\frac{1\times 10^{-3}}{(-4(-5\times 10^{-3}))\times\frac{1}{10}}\left(\frac{1}{10}+\frac{1\times 10^{-6}\times 10^{4}}{1\times 10^{-3}}\times\frac{100}{100}\right)^{2}(-3.7443977185\times 10^{-6})}$$

Result:

1727.9024325112374877434283656733592906374687244173155632 94...

1727.9024325… ≈ 1728

-0.024/((((((1e-3)/((-4*(-5e-3))1/10)))*(((1/10+(1e-6*10^4)/(1e-3)*(100)/(100))))^2 *(-3.7443977185e-6))))

Input interpretation:

$$-\frac{0.024}{\frac{1\times 10^{-3}}{(-4(-5\times 10^{-3}))\times\frac{1}{10}}\left(\frac{1}{10}+\frac{1\times 10^{-6}\times 10^{4}}{1\times 10^{-3}}\times\frac{100}{100}\right)^{2}(-3.7443977185\times 10^{-6})}$$

Result:

125.6656314553627263813402447762443120463613617758047682395...

125.66563…

Pi-0.026/((((((1e-3)/((-4*(-5e-3))1/10)))*(((1/10+(1e-6*10^4)/(1e-3)*(100)/(100))))^2 *(-3.7443977185e-6))))

Input interpretation:

$$\pi - \frac{0.026}{\frac{1\times 10^{-3}}{(-4(-5\times 10^{-3}))\times \frac{1}{10}}\left(\frac{1}{10}+\frac{1\times 10^{-6}\times 10^{4}}{1\times 10^{-3}}\times \frac{100}{100}\right)^2 (-3.7443977185\times 10^{-6})}$$

Result:

139.279...

139.279...

For:

$$\left(c_{\infty}^{2}(t) \right) = (-1.23\times 10^{-6})$$

we obtain:

[-0.0005/((((((1e-3)/((-4*(-5e-3))1/10)))*(((1/10+(1e-6*10^4)/(1e-3)*(100)/(100))))^2 *(-1.230e-6))))]

Input interpretation:

$$-\frac{0.0005}{\frac{1\times 10^{-3}}{(-4(-5\times 10^{-3}))\times \frac{1}{10}}\left(\frac{1}{10}+\frac{1\times 10^{-6}\times 10^{4}}{1\times 10^{-3}}\times \frac{100}{100}\right)^2 (-1.23\times 10^{-6})}$$

Result:

7.969886580544072277307421638082668445545351444103598961683...

7.96988658...

From the ratio between the $-0.000190983005632092$5 and -0.0005 , we obtain:

((((((1e-3)/((-4*(-5e-3))1/10)))*(((1/10+(1e-6*10^4)/(1e-3)*(100)/(100))))^2 *(-3.7443977185e-6)))) / ((-(10e-6)/(-4*(-5e-3))))

Input interpretation:

$$-\frac{\frac{1\times10^{-3}}{(-4(-5\times10^{-3}))\times\frac{1}{10}}\left(\frac{1}{10}+\frac{1\times10^{-6}\times10^{4}}{1\times10^{-3}}\times\frac{100}{100}\right)^2(-3.7443977185\times10^{-6})}{-\frac{10\times10^{-6}}{4(-5\times10^{-3})}}$$

Result:

0.381966011264185

0.381966011264185

From which:

[((((((1e-3)/((-4*(-5e-3))1/10)))*(((1/10+(1e-6*10^4)/(1e-3)*(100)/(100))))^2 *(-3.7443977185e-6)))) / ((-(10e-6)/(-4*(-5e-3))))]^1/1088

Input interpretation:

$$\sqrt[1088]{-\frac{\frac{1\times10^{-3}}{(-4(-5\times10^{-3}))\times\frac{1}{10}}\left(\frac{1}{10}+\frac{1\times10^{-6}\times10^{4}}{1\times10^{-3}}\times\frac{100}{100}\right)^2(-3.7443977185\times10^{-6})}{-\frac{10\times10^{-6}}{4(-5\times10^{-3})}}}$$

Result:

0.99911581056531...

0.99911581056531…. result practically equal to the following Rogers-Ramanujan continued fraction value:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\dots}}}}\approx 0.9991104684$$

We have that:

$$\frac{k_T}{a} = \frac{c_S}{u_H} \simeq 2\sqrt{\frac{(-V_0)}{A_0}\left(\frac{2A_1}{\gamma_0} + \frac{p+2}{p}\right)}. \tag{4.29}$$

2*sqrt(((((5*10^-3)/(10^-3)*((((2*2/10*1/(10^-6))+(1/10+2)/(1/10)))))))

Input interpretation:

$$2\sqrt{\frac{5\times 10^{-3}}{\frac{1}{10^3}}\left(2\times\frac{2}{10}\times\frac{1}{\frac{1}{10^6}}+\frac{\frac{1}{10}+2}{\frac{1}{10}}\right)}$$

Result:

$$2\sqrt{2000105}$$

Decimal approximation:

2828.501369983758731571647819495282445398389788179539594988...

2828.5013699837...

Or:

2*sqrt(((((5e-3)/(1e-3)*((((2*2/10*1/(1e-6))+(1/10+2)/(1/10)))))))

Input interpretation:

$$2\sqrt{\frac{5\times 10^{-3}}{1\times 10^{-3}}\left(2\times\frac{2}{10}\times\frac{1}{1\times 10^{-6}}+\frac{\frac{1}{10}+2}{\frac{1}{10}}\right)}$$

Result:

$$2\sqrt{2000105}$$

Decimal approximation:

2828.501369983758731571647819495282445398389788179539594988...

2828.5013699837…

From which:

Input interpretation:

$$2\sqrt{\frac{5\times10^{-3}}{1\times10^{-3}}\left(2\times\frac{2}{10}\times\frac{1}{1\times10^{-6}}+\frac{\frac{1}{10}+2}{\frac{1}{10}}\right)}-322-29-7$$

Result:

$$2\sqrt{2000105}-358$$

Decimal approximation:

2470.501369983758731571647819495282445398389788179539594988...

2470.501369983… result practically equal to the rest mass of charmed Xi baryon 2470.88

Alternate form:

$$2\left(\sqrt{2000105}-179\right)$$

and:

(((2*sqrt(((((5e-3)/(1e-3)*((((2*2/10*1/(1e-6))+(1/10+2)/(1/10))))))))))^1/16

Input interpretation:

$$\sqrt[16]{2\sqrt{\frac{5\times10^{-3}}{1\times10^{-3}}\left(2\times\frac{2}{10}\times\frac{1}{1\times10^{-6}}+\frac{\frac{1}{10}+2}{\frac{1}{10}}\right)}}$$

Result:

$$\sqrt[16]{2}\ \sqrt[32]{2000105}$$

Decimal approximation:

1.643320506031719620500374029460204062533961646738821776558...

1.643320506…

Alternate form:

root of $x^{32}-8000420$ near $x=1.64332$

sqrt[6((((((2*sqrt(((((5e-3)/(1e-3)*((((2*2/10*1/(1e-6))+(1/10+2)/(1/10)))))))))))^1/16)))]

Input interpretation:

$$\sqrt{6\sqrt[16]{2\sqrt{\frac{5\times10^{-3}}{1\times10^{-3}}\left(2\times\frac{2}{10}\times\frac{1}{1\times10^{-6}}+\frac{\frac{1}{10}+2}{\frac{1}{10}}\right)}}}$$

Result:

$$2^{17/32}\sqrt{3}\sqrt[64]{2000105}$$

Decimal approximation:

3.140051438462484442713188496023850142331062180841409134445...

3.14005143846… ≈ π

and:

(((2*sqrt(((((5e-3)/(1e-3)*((((2*2/10*1/(1e-6))+(1/10+2)/(1/10)))))))))))^1/16-(29-4)1/10^3

Input interpretation:

$$\sqrt[16]{2\sqrt{\frac{5\times10^{-3}}{1\times10^{-3}}\left(2\times\frac{2}{10}\times\frac{1}{1\times10^{-6}}+\frac{\frac{1}{10}+2}{\frac{1}{10}}\right)}}-(29-4)\times\frac{1}{10^3}$$

Result:

$$\sqrt[16]{2}\sqrt[32]{2000105}-\frac{1}{40}$$

Decimal approximation:

1.618320506031719620500374029460204062533961646738821776558...

1.618320506…

Alternate forms:

$$\text{root of } x^{32}-8000420 \text{ near } x=1.64332-\frac{1}{40}$$

$$\frac{1}{40}\left(40\sqrt[16]{2}\sqrt[32]{2000105}-1\right)$$

and again:

(((2*sqrt(((((5e-3)/(1e-3)*((((2*2/10*1/(1e-6))+(1/10+2)/(1/10))))))))))^5 * 25

Input interpretation:

$$\left(2\sqrt{\frac{5\times 10^{-3}}{1\times 10^{-3}}\left(2\times\frac{2}{10}\times\frac{1}{1\times 10^{-6}}+\frac{\frac{1}{10}+2}{\frac{1}{10}}\right)}\right)^5\times 25$$

Result:

$$3\,200\,336\,008\,820\,000\sqrt{2\,000\,105}$$

Decimal approximation:

$$4.5260773926778622836099165542570940040624461747785764\ldots\times 10^{18}$$

$4.526077392\ldots*10^{18}$

Now, we have that:

Figure 2: Numerical solution of Eq. (4.3) for the modes Ψ_k as a function of time corresponding to the background around γ-crossing as described in Eqs. (4.14-4.12) for the wave-numbers $k = 0.01$ (blue curve), $k = 0.1$ (orange curve), $k = 2$ (green curve), and $k = 10$ (red curve); and the parameters $t_\gamma = -100, p = 1/10, A_0 = 10^{-3}, A_1 = 2/10, V_0 = -5\times 10^{-3}$, and $\gamma_0 = 10^{-6}$. The x-axis has reduced Planck units and the y-axis has dimensionless units. The graph verifies that all modes pass through γ-crossing undisturbed.

Finally, to simplify the term $\propto \Psi$ in Eq. (4.3), we use the following approximations:

$$AH-\gamma \simeq -\frac{A_0}{(-t_\gamma)}\left(p+\frac{\gamma_0 t_\gamma^2}{A_0}\left(-\frac{\Delta t}{t_\gamma}\right)\right), \tag{C.11}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(AH-\gamma)^2 \simeq 2\frac{A_0}{(-t_\gamma)}\left(p+\frac{\gamma_0 t_\gamma^2}{A_0}\left(-\frac{\Delta t}{t_\gamma}\right)\right), \tag{C.12}$$

$$-\dot{H}+\frac{\dot{A}_h}{A_h}H \simeq \frac{p}{t_\gamma^2}\left(1+2\frac{A_1}{A_0}t_\gamma^2\left(\frac{\Delta t}{t_\gamma}+\left(-\frac{\Delta t}{t_\gamma}\right)^2\right)\right), \tag{C.13}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(-\dot{H}+\frac{\dot{A}_h}{A_h}H\right) \simeq 2\frac{p}{t_\gamma^3}\left(\frac{A_1}{A_0}t_\gamma^2\right)\left(1-\left(-\frac{\Delta t}{t_\gamma}\right)\right); \tag{C.14}$$

where

$$\Delta t \equiv t - t_\gamma$$

i.e., t < 0 runs towards zero. t = - 1/24

$$AH - \gamma \simeq -\frac{A_0}{(-t_\gamma)}\left(p + \frac{\gamma_0 t_\gamma^2}{A_0}\left(-\frac{\Delta t}{t_\gamma}\right)\right),$$

-(10^-3)/(100) [(1/10+((10^-6)*(10^4))/(10^-3))*(-(-1/24+100)/(-100))]

Input:

$$-\frac{1}{10^3 \times 100}\left(\left(\frac{1}{10}+\frac{\frac{10^4}{10^6}}{\frac{1}{10^3}}\right)\left(\frac{-1}{-100}\left(-\frac{1}{24}+100\right)\right)\right)$$

Exact result:

$$-\frac{242\,299}{2\,400\,000\,000}$$

Decimal approximation:

−0.000100957916666666666666666666666666666666666666666666...

-0.000100957916666…

$$\frac{\mathrm{d}}{\mathrm{d}t}(AH - \gamma)^2 \simeq 2\frac{A_0}{(-t_\gamma)}\left(p + \frac{\gamma_0 t_\gamma^2}{A_0}\left(-\frac{\Delta t}{t_\gamma}\right)\right),$$

2(10^-3)/(100) (1/10+[(((10^-6*(((10^4))/(10^-3))))))]*(-(-1/24+100)/(-100))

Input:

$$2 \times \frac{1}{10^3 \times 100}\left(\frac{1}{10}+\frac{\frac{10^4}{\frac{1}{10^3}}}{10^6}\left(\frac{-1}{-100}\left(-\frac{1}{24}+100\right)\right)\right)$$

Exact result:

$$\frac{2423}{12\,000\,000}$$

Decimal approximation:

0.000201916666666666666666666666666666666666666666666666...

0.000201916666....

$$-\dot{H}+\frac{\dot{A}_h}{A_h}H \simeq \frac{p}{t_\gamma^2}\left(1+2\frac{A_1}{A_0}t_\gamma^2\left(\frac{\Delta t}{t_\gamma}+\left(-\frac{\Delta t}{t_\gamma}\right)^2\right)\right),$$

(1/10) / (10^4) [1+(2*(2/10)*10^4)/(10^-3)*((((((-1/24+100)/(-100)))+((((-(-1/24+100)/(-100)))))^2)))]

Input:

$$\frac{\frac{1}{10}}{10^4}\left(1+\frac{2\times\frac{2}{10}\times 10^4}{\frac{1}{10^3}}\left(-\frac{1}{100}\left(-\frac{1}{24}+100\right)+\left(\frac{-1}{-100}\left(-\frac{1}{24}+100\right)\right)^2\right)\right)$$

Exact result:

$$-\frac{59\,939}{3\,600\,000}$$

Decimal approximation:

-0.016649722222222222222222222222222222222222222222222222...

-0.01664972222….

From

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(-\dot{H}+\frac{\dot{A}_h}{A_h}H\right)\simeq 2\frac{p}{t_\gamma^3}\left(\frac{A_1}{A_0}t_\gamma^2\right)\left(1-\left(-\frac{\Delta t}{t_\gamma}\right)\right);$$

we obtain:

2((1/10)/(10^6)) [((2/10)*(10^4))/(10^-3)] ((((1-(-(-1/24+100)/(-100))))))

Input:

$$\left(2\times\frac{\frac{1}{10}}{10^6}\right)\times\frac{\frac{2}{10}\times 10^4}{\frac{1}{10^3}}\left(1-\frac{-1}{-100}\left(-\frac{1}{24}+100\right)\right)$$

Exact result:

$$\frac{1}{6000}$$

Decimal approximation:

0.000166666666666666666666666666666666666666666666666666...

0.00016666...

From the four results, we obtain

(-0.000100957916+ 0.000201916 -0.01664972 + 0.00016666)

Input interpretation:

$-0.000100957916 + 0.000201916 - 0.01664972 + 0.00016666$

Result:

-0.016382101916

-0.016382101916

From which:

-10^2*(-0.000100957916+ 0.000201916 -0.01664972 + 0.00016666)

Input interpretation:

$-10^2 (-0.000100957916 + 0.000201916 - 0.01664972 + 0.00016666)$

Result:

1.6382101916

1.6382101916

((-(89+13)(-0.000100957916+ 0.000201916 -0.01664972 + 0.00016666)))-(55-2)1/10^3

Input interpretation:

$$-(89+13)(-0.000100957916 + 0.000201916 - 0.01664972 + 0.00016666) - (55-2)\times\frac{1}{10^3}$$

Result:

1.617974395432

1.617974395432

and:

-1/(-0.000100957916+ 0.000201916 -0.01664972 + 0.00016666)+3

Input interpretation:

$$-\frac{1}{-0.000100957916+0.000201916-0.01664972+0.00016666}+3$$

Result:

64.04222798317011764340680347651183541812852679850218556044...

64.042227983… ≈ 64

-2/(-0.000100957916+ 0.000201916 -0.01664972 + 0.00016666)+Pi

Input interpretation:

$$-\frac{2}{-0.000100957916+0.000201916-0.01664972+0.00016666}+\pi$$

Result:

125.2260...

125.2260…

Alternative representations:

$$-\frac{2}{-0.000100958+0.000201916-0.0166497+0.00016666}+\pi = 180\,° - -\frac{2}{0.0163821}$$

$$-\frac{2}{-0.000100958+0.000201916-0.0166497+0.00016666}+\pi = -i\log(-1) - -\frac{2}{0.0163821}$$

$$-\frac{2}{-0.000100958+0.000201916-0.0166497+0.00016666}+\pi = \cos^{-1}(-1) - -\frac{2}{0.0163821}$$

Series representations:

$$-\frac{2}{-0.000100958+0.000201916-0.0166497+0.00016666}+\pi = 122.084+4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$-\frac{2}{-0.000100958+0.000201916-0.0166497+0.00016666}+\pi = 120.084+2\sum_{k=1}^{\infty}\frac{2^k}{\binom{2k}{k}}$$

$$-\frac{2}{-0.000100958+0.000201916-0.0166497+0.00016666}+\pi = 122.084+\sum_{k=0}^{\infty}\frac{2^{-k}(-6+50k)}{\binom{3k}{k}}$$

Integral representations:

$$-\frac{2}{-0.000100958+0.000201916-0.0166497+0.00016666}+\pi = 122.084+2\int_0^{\infty}\frac{1}{1+t^2}\,dt$$

$$-\frac{2}{-0.000100958+0.000201916-0.0166497+0.00016666}+\pi = 122.084+4\int_0^{1}\sqrt{1-t^2}\,dt$$

$$-\frac{2}{-0.000100958+0.000201916-0.0166497+0.00016666}+\pi = 122.084+2\int_0^{\infty}\frac{\sin(t)}{t}\,dt$$

-2/(-0.000100957916+ 0.000201916 -0.01664972 + 0.00016666)+2Pi+11

Input interpretation:

$$-\frac{2}{-0.000100957916+0.000201916-0.01664972+0.00016666}+2\pi+11$$

Result:

139.3676...

139.3676…

Alternative representations:

$$-\frac{2}{-0.000100958+0.000201916-0.0166497+0.00016666}+2\pi+11 = 11+360^\circ--\frac{2}{0.0163821}$$

$$-\frac{2}{-0.000100958+0.000201916-0.0166497+0.00016666}+2\pi+11 = 11-2\,i\log(-1)--\frac{2}{0.0163821}$$

$$-\frac{2}{-0.000100958+0.000201916-0.0166497+0.00016666}+2\pi+11 = 11+2\cos^{-1}(-1)--\frac{2}{0.0163821}$$

Series representations:

$$-\frac{2}{-0.000100958+0.000201916-0.0166497+0.00016666}+2\pi+11 = 133.084+8\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$-\frac{2}{-0.000100958+0.000201916-0.0166497+0.00016666}+2\pi+11 = 129.084+4\sum_{k=1}^{\infty}\frac{2^k}{\binom{2k}{k}}$$

$$-\frac{2}{-0.000100958+0.000201916-0.0166497+0.00016666}+2\pi+11 = 133.084+2\sum_{k=0}^{\infty}\frac{2^{-k}(-6+50k)}{\binom{3k}{k}}$$

Integral representations:

$$-\frac{2}{-0.000100958+0.000201916-0.0166497+0.00016666}+2\pi+11 = 133.084+4\int_0^{\infty}\frac{1}{1+t^2}\,dt$$

$$-\frac{2}{-0.000100958+0.000201916-0.0166497+0.00016666}+2\pi+11 = 133.084+8\int_0^{1}\sqrt{1-t^2}\,dt$$

$$-\frac{2}{-0.000100958+0.000201916-0.0166497+0.00016666}+2\pi+11 = 133.084+4\int_0^{\infty}\frac{\sin(t)}{t}\,dt$$

27*(((-1/(-0.000100957916+ 0.000201916 -0.01664972 + 0.00016666)+3)))

Input interpretation:

$$27\left(-\frac{1}{-0.000100957916+0.000201916-0.01664972+0.00016666}+3\right)$$

Result:

1729.140155545593176371983693865819556289470223559559010132...

1729.1401555455...

With regard 27 (From Wikipedia):

*"The fundamental group of the complex form, compact real form, or any algebraic version of E_6 is the cyclic group **Z**/3**Z**, and its outer automorphism group is the cyclic group **Z**/2**Z**. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E_6 plays a role in some grand unified theories".*

Conclusions

Is there a connection between the so-called "bounce" of the Universe and black holes? At the end of a cycle, the final giant black hole that is formed by the fusion of all the remaining black holes, which has absorbed all the mass and energy of the cosmos, in an immeasurable, though ultra-massive, span of time, like any other black hole is subject to evaporation process. Eventually, when the black hole undergoes the final explosion, as a sort of "mirror symmetry", all the energy and mass that has been absorbed by the black hole, now reduced to quantum dimensions, is emitted from the opposite side. So there is a process of absorption-contraction / expansion-emission which can be compared to a sort of "bounce". Hence, the counterpart to the final black hole is an initial white hole, from which a new universe cycle originates.

Appendix

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125.
In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson:

125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64 g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276 e^{-\pi\sqrt{22}} - \cdots, \\ 64 g_{22}^{-24} &= \qquad\qquad 4096 e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372 e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64 g_{22}^{-24} = \qquad 4096 e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372 e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.

Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] *That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies*[3] *- golden spirals are one special case of these logarithmic spirals*

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to ζ(2) = $\frac{\pi^2}{6} = 1.644934 \ldots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

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