On various Ramanujan's elliptic integrals, Einstein Dilaton Gauss-Bonnet Gravity and Black Hole Physics equations: mathematical connections with ϕ, $\zeta(2)$, and some parameters of High Energy Physics. VII

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Abstract

In this paper we have described several Ramanujan's elliptic integrals, Einstein Dilaton Gauss-Bonnet Gravity and Black Hole Physics equations. Furthermore, we have obtained mathematical connections with ϕ, $\zeta(2)$, and some parameters of High Energy Physics.

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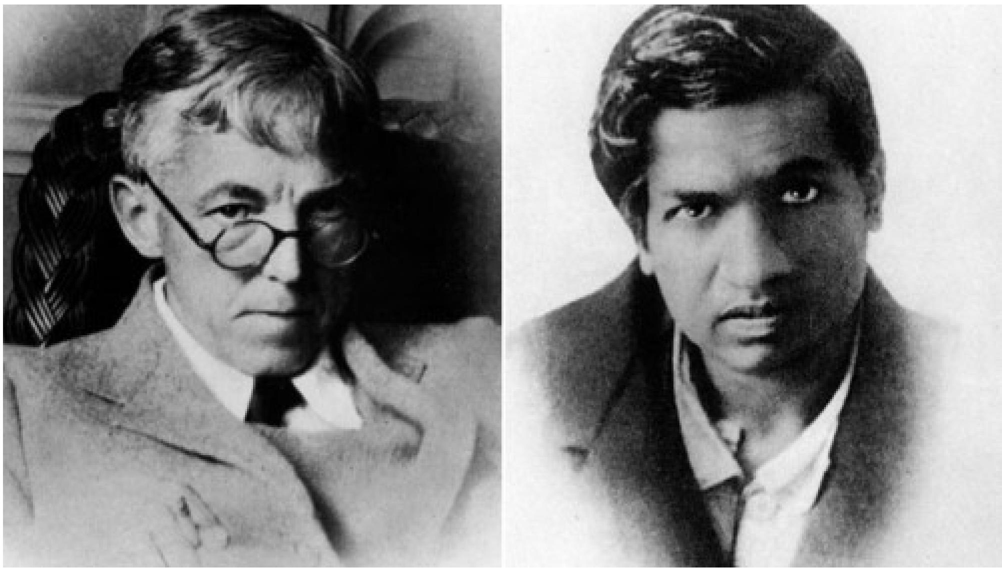


https://www.cse.iitk.ac.in/users/amit/books/hardy-1999-ramanujan-twelve-lectures.html

Introduction

Is there a connection between the so-called "bounce" of the Universe and black holes? At the end of a cycle, the final giant black hole that is formed by the fusion of all the remaining black holes, which has absorbed all the mass and energy of the cosmos, in an immeasurable, though ultra-massive, span of time, like any other black hole is subject to evaporation process. Eventually, when the black hole undergoes the final explosion, as a sort of "mirror symmetry", all the energy and mass that has been absorbed by the black hole, now reduced to quantum dimensions, is emitted from the opposite side. So there is a process of absorption-contraction / expansion-emission which can be compared to a sort of "bounce". Hence, the counterpart to the final black hole is an initial white hole, from which a new universe cycle originates.

We want to highlight that the development of the various equations was carried out according an our possible logical and original interpretation

From

George E. Andrews Bruce C. Berndt
Ramanujan's Lost Notebook Part I - 2005 Springer Science+Business Media, Inc.

We have that:

$$(PQ)^3 + \frac{125}{(PQ)^3} = \left(\frac{1}{u^4} - u^4\right) - 7\left(\frac{1}{u^3} + u^3\right) + 7\left(\frac{1}{u^2} - u^2\right) + 14\left(\frac{1}{u} + u\right)$$

(1/81-81)-7(1/27+27)+7(1/9-9)+14(1/3+3) = x^3+125/(x^3)

Input:

$$\left(\frac{1}{81} - 81\right) - 7\left(\frac{1}{27} + 27\right) + 7\left(\frac{1}{9} - 9\right) + 14\left(\frac{1}{3} + 3\right) = x^3 + \frac{125}{x^3}$$

Exact result:

$$-\frac{23150}{81} = x^3 + \frac{125}{x^3}$$

Alternate forms:

$$81 x^6 + 23150 x^3 = -10125 \quad (\text{for } x \neq 0)$$

$$-\frac{23150}{81} = \frac{x^6 + 125}{x^3}$$

$$-\frac{23150}{81} = \frac{(x^2 + 5)(x^4 - 5 x^2 + 25)}{x^3}$$

Real solutions:

$$x = -3 \times 5^{2/3} \sqrt[3]{\frac{3}{2315 + 22\sqrt{11005}}}$$

$$x = -\frac{1}{3}\sqrt[3]{\frac{11575}{3}+\frac{110\sqrt{11005}}{3}}$$

Real solutions:

$x \approx -0.75946$

$x \approx -6.5836$

-6.5836; -0.75946

Complex solutions:

$$x = 3 \times 5^{2/3}\sqrt[3]{-\frac{3}{2315+22\sqrt{11005}}}$$

$$x = -3(-5)^{2/3}\sqrt[3]{\frac{3}{2315+22\sqrt{11005}}}$$

$$x = \frac{1}{3}\sqrt[3]{-\frac{5}{3}\left(2315+22\sqrt{11005}\right)}$$

$$x = -\frac{1}{3}(-1)^{2/3}\sqrt[3]{\frac{5}{3}\left(2315+22\sqrt{11005}\right)}$$

From

$$\left(\frac{1}{v^4}-v^4\right)-7\left(\frac{1}{v^3}+v^3\right)+7\left(\frac{1}{v^2}-v^2\right)+14\left(\frac{1}{v}+v\right)$$

(1/81-81)-7(1/27+27)+7(1/9-9)+14(1/3+3)

For v = 3

Input:

$$\left(\frac{1}{81}-81\right)-7\left(\frac{1}{27}+27\right)+7\left(\frac{1}{9}-9\right)+14\left(\frac{1}{3}+3\right)$$

Exact result:

$$-\frac{23150}{81}$$

Decimal approximation:

−285.802469135802469135802469135802469135802469135802469135…

-285.802469135…

-6.5836^3-125/(6.5836^3)

for PQ = - 6.5836

Input interpretation:

$$-6.5836^3 - \frac{125}{6.5836^3}$$

Result:

−285.796214981298664548978131424834701749557109619492603804...

-285.79621498129...

or, for PQ = - 0.75946

-0.75946^3-125/(0.75946^3)

Input:

$$-0.75946^3 - \frac{125}{0.75946^3}$$

Result:

−285.799487712918671016881034807866634688336184487278450871...

-285.7994877129...

From which:

6[-((((1/81-81)-7(1/27+27)+7(1/9-9)+14(1/3+3))))]+18-(11/7-1/(1 + sqrt(2)))-e

Input:

$$6\left(-\left(\left(\frac{1}{81}-81\right)-7\left(\frac{1}{27}+27\right)+7\left(\frac{1}{9}-9\right)+14\left(\frac{1}{3}+3\right)\right)\right)+18-\left(\frac{11}{7}-\frac{1}{1+\sqrt{2}}\right)-e$$

Result:

$$\frac{327\,205}{189}+\frac{1}{1+\sqrt{2}}-e$$

Decimal approximation:

1728.939317977300293199684787496243278967055811025063231884...

1728.939317977…

Property:

$\frac{327205}{189} + \frac{1}{1+\sqrt{2}} - e$ is a transcendental number

Alternate forms:

$$\frac{1}{189}\left(327016 + 189\sqrt{2} - 189\,e\right)$$

$$\frac{327016}{189} + \sqrt{2} - e$$

$$\frac{1}{189}\left(327016 + 189\sqrt{2}\right) - e$$

Series representations:

$$6(-1)\left(\left(\frac{1}{81} - 81\right) - 7\left(\frac{1}{27} + 27\right) + 7\left(\frac{1}{9} - 9\right) + 14\left(\frac{1}{3} + 3\right)\right) + 18 - \left(\frac{11}{7} - \frac{1}{1+\sqrt{2}}\right) - e = \frac{327205}{189} - e + \frac{1}{1 + \sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$6(-1)\left(\left(\frac{1}{81} - 81\right) - 7\left(\frac{1}{27} + 27\right) + 7\left(\frac{1}{9} - 9\right) + 14\left(\frac{1}{3} + 3\right)\right) + 18 - \left(\frac{11}{7} - \frac{1}{1+\sqrt{2}}\right) - e = \frac{327205}{189} - e + \frac{1}{1 + \exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$6(-1)\left(\left(\frac{1}{81} - 81\right) - 7\left(\frac{1}{27} + 27\right) + 7\left(\frac{1}{9} - 9\right) + 14\left(\frac{1}{3} + 3\right)\right) + 18 - \left(\frac{11}{7} - \frac{1}{1+\sqrt{2}}\right) - e = \frac{327205}{189} - e + \frac{1}{1 + \left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}$$

Now, we have:

$$G(q) := 2\sqrt{\lambda}(u^5 + u^{-5}) = 2\sqrt{125\lambda + 22 + \frac{1}{\lambda}}. \qquad (15.6.5)$$

Thus, for any q_0 such that $0 < q_0 < 1$,

$$G(q) - G(q_0) = \int_{q_0}^{q} \frac{dG}{dt} dt = 125 \int_{q_0}^{q} \frac{f^8(-t^5)}{f^4(-t)} \sqrt{t}\, dt - \int_{q_0}^{q} \frac{f^8(-t)}{f^4(-t^5)} \frac{dt}{t^{3/2}},$$

Now set $q_0 = e^{-2\pi/\theta}$. Ramanujan calculated $G(e^{-2\pi/\theta})$ for three values of θ.

$$\begin{aligned} G(e^{-2\pi/5}) &= 2\sqrt{125\lambda(e^{-2\pi/5}) + 22 + 1/\lambda(e^{-2\pi/5})} \\ &= 2\sqrt{1/\lambda(e^{-2\pi}) + 22 + 125\lambda(e^{-2\pi})} = G(e^{-2\pi}). \end{aligned}$$

$$\begin{aligned} G(e^{-2\pi/5}) &= 2\sqrt{125\epsilon^3 + 22 + \epsilon^{-3}} \\ &= 2\sqrt{270 + 126\sqrt{5}} \\ &= 6 \cdot 5^{1/4}\sqrt{14 + 6\sqrt{5}} \\ &= 6 \cdot 5^{1/4}(3 + \sqrt{5}), \end{aligned}$$

thence:

2(((125x(exp((-2Pi)/5))+22+1/x*(exp((-2Pi)/5))))^0.5 = 6*5^(1/4) (3+sqrt5)

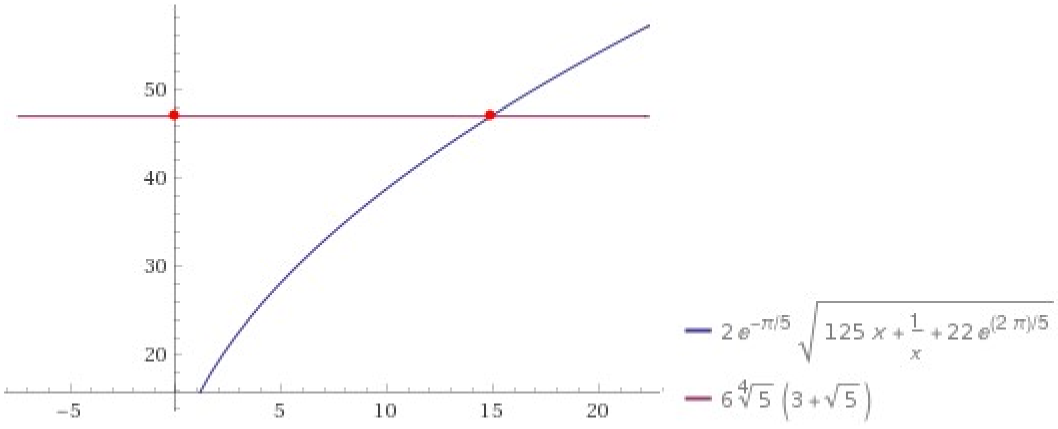
Input:

$$2\sqrt{125\, x \exp\left(\frac{1}{5}(-2\pi)\right) + 22 + \frac{1}{x}\exp\left(\frac{1}{5}(-2\pi)\right)} = 6\sqrt[4]{5}\left(3+\sqrt{5}\right)$$

Exact result:

$$2\sqrt{125\, e^{-(2\pi)/5}\, x + \frac{e^{-(2\pi)/5}}{x} + 22} = 6\sqrt[4]{5}\left(3+\sqrt{5}\right)$$

Plot:



Alternate form assuming x is real:

$$\sqrt{125x+\frac{1}{x}+22e^{(2\pi)/5}} = 3\sqrt[4]{5}\left(3+\sqrt{5}\right)e^{\pi/5}$$

Alternate forms:

$$125x^2+\left(-248e^{(2\pi)/5}-126\sqrt{5}\,e^{(2\pi)/5}\right)x=-1$$

$$2e^{-\pi/5}\sqrt{125x+\frac{1}{x}+22e^{(2\pi)/5}} = 6\sqrt[4]{5}\left(3+\sqrt{5}\right)$$

$$2\sqrt{125e^{-(2\pi)/5}x+\frac{e^{-(2\pi)/5}}{x}+22} = 6\sqrt{2\left(15+7\sqrt{5}\right)}$$

Solutions:

$$x=\frac{2}{\sqrt{(-248-126\sqrt{5})^2e^{(4\pi)/5}-500}-(-248-126\sqrt{5})e^{(2\pi)/5}}$$

$$x=\frac{1}{250}\left(\sqrt{\left(-248-126\sqrt{5}\right)^2e^{(4\pi)/5}-500}-\left(-248-126\sqrt{5}\right)e^{(2\pi)/5}\right)$$

Solutions:

$x\approx 0.000537277$

$x\approx 14.8899$

14.8899

we obtain:

[2((125*14.8899(exp((-2Pi)/5))+22+1/(14.8899)*(exp((-2Pi)/5))))^0.5]

Input interpretation:

$$2\sqrt{125\times 14.8899\exp\left(\frac{1}{5}(-2\pi)\right)+22+\frac{1}{14.8899}\exp\left(\frac{1}{5}(-2\pi)\right)}$$

Result:

46.9785...

46.9785...

and:

6*5^(1/4) (3+sqrt5)

Input:

$$6\sqrt[4]{5}\left(3+\sqrt{5}\right)$$

Decimal approximation:

46.97848721127463047451117409229587920818333138663904116239...

46.978487211…

Alternate forms:

$$6\left(3\sqrt[4]{5}+5^{3/4}\right)$$

$$\sqrt[4]{5}\left(18+6\sqrt{5}\right)$$

$$18\sqrt[4]{5}+6\times 5^{3/4}$$

Minimal polynomial:

$$x^4-2160x^2-103680$$

Thence, from

$$G(q) - G(q_0) = \int_{q_0}^{q} \frac{dG}{dt} dt = 125 \int_{q_0}^{q} \frac{f^8(-t^5)}{f^4(-t)} \sqrt{t}\, dt - \int_{q_0}^{q} \frac{f^8(-t)}{f^4(-t^5)} \frac{dt}{t^{3/2}},$$

For $G(q_0) = 46.9785$, we obtain:

$$G(q) = \left(125 \int_{q_0}^{q} \frac{f^8(-t^5)}{f^4(-t)} \sqrt{t}\, dt - \int_{q_0}^{q} \frac{f^8(-t)}{f^4(-t^5)} \frac{dt}{t^{3/2}} \right) + 46.9785$$

Now, we have:

$$G(q) := 2\sqrt{\lambda}(u^5 + u^{-5}) = 2\sqrt{125\lambda + 22 + \frac{1}{\lambda}}. \quad (15.6.5)$$

From:

$$2\sqrt{125\epsilon^3 + 22 + \epsilon^{-3}}$$

$$2\sqrt{270 + 126\sqrt{5}}$$

, where $\lambda = (1+\sqrt{5})/2$, we obtain:

2sqrt(270+126*5^0.5) = 125*x + 46.9785

Input interpretation:

$$2\sqrt{270 + 126\sqrt{5}} = 125\,x + 46.9785$$

Result:

$$2\sqrt{270 + 126\sqrt{5}} = 125\,x + 46.9785$$

Alternate forms:

$$-125\,x - 0.0000127887 = 0$$

$$6\sqrt{30 + 14\sqrt{5}} = 125\,x + 46.9785$$

$$6\sqrt{2\left(15+7\sqrt{5}\right)} = 125\,(x+0.375828)$$

Solution:

$x \approx -1.0231 \times 10^{-7}$

$-1.0231*10^{-7}$

In conclusion, we have:

125*(-1.0231e-7) + 46.9785

Input interpretation:

$125\,(-1.0231 \times 10^{-7}) + 46.9785$

Result:

46.97848721125

46.97848721125

and:

2sqrt(270+126*5^0.5)

Input:

$$2\sqrt{270+126\sqrt{5}}$$

Decimal approximation:

46.97848721127463047451117409229587920818333138663904116239...

46.97848721127…

Alternate forms:

$$6\sqrt[4]{5}\left(3+\sqrt{5}\right)$$

$$6\sqrt{30+14\sqrt{5}}$$

$$6\sqrt{2\left(15+7\sqrt{5}\right)}$$

Minimal polynomial:

$x^4 - 2160x^2 - 103680$

Thence:

$$G(q) - G(q_0) = \int_{q_0}^{q} \frac{dG}{dt} dt = 125 \int_{q_0}^{q} \frac{f^8(-t^5)}{f^4(-t)} \sqrt{t}\, dt - \int_{q_0}^{q} \frac{f^8(-t)}{f^4(-t^5)} \frac{dt}{t^{3/2}},$$

and

$125(-1.0231 \times 10^{-7}) + 46.9785 = 46.97848721125$

$46.97848721125 \approx 47$

We have that:

$$\left[\int_{q_0}^{q} \frac{f^8(-t^5)}{f^4(-t)} \sqrt{t}\, dt - \int_{q_0}^{q} \frac{f^8(-t)}{f^4(-t^5)} \frac{dt}{t^{3/2}} \right] = -1.0231 \times 10^{-7}$$

Furthermore, we note that:

76/ (((125*(-1.0231e-7) + 46.9785)))

Input interpretation:

$$\frac{76}{125(-1.0231 \times 10^{-7}) + 46.9785}$$

Result:

1.617761756742992359205669269804333348450208286611986875039...

1.6177617567429.......

Now, we have that:

$$q\frac{dR}{dq}=\frac{PR-Q^2}{2} \qquad \text{and} \qquad q\frac{dQ}{dq}=\frac{PQ-R}{3},$$

in (15.3.1), we find that

$$\frac{q}{z}\frac{dz}{dq}=\frac{R^2-Q^3}{RQ}. \tag{15.3.2}$$

Hence, by (15.3.2),

$$q\frac{d}{dq}\log\left(\frac{Q^{3/2}-R}{Q^{3/2}+R}\right)=q\frac{d}{dq}\log\left(\frac{1-\sqrt{z}}{1+\sqrt{z}}\right)$$

For P = 1; Q = 2; R = 3, we obtain:

3 – 4 / 2 = - 1/2 and 2 – 3 / 3 = - 1/3 ; 9 – 8 / 3 * 2 = 1/6 . We note that

- 1/2 * - 1/ 3 = 1/6

Thence:

$$\log\left(\frac{Q^{3/2}-R}{Q^{3/2}+R}\right)$$

ln(((2^1.5 – 3)/(2^1.5 + 3)))

Input:

$$\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)$$

log(x) is the natural logarithm

Result:

– 3.52549… +
3.14159… i

Polar coordinates:

r = 4.72215 (radius), θ = 138.296° (angle)

4.72215

Alternative representations:

$$\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right) = \log_e\left(\frac{-3+2^{1.5}}{3+2^{1.5}}\right)$$

$$\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right) = \log(a)\log_a\left(\frac{-3+2^{1.5}}{3+2^{1.5}}\right)$$

$$\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right) = -\text{Li}_1\left(1-\frac{-3+2^{1.5}}{3+2^{1.5}}\right)$$

Series representations:

$$\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right) = \log(-1.02944) - \sum_{k=1}^{\infty} \frac{e^{-0.0290123\,k}}{k}$$

$$\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right) = 2\,i\,\pi \left\lfloor \frac{\arg(-0.0294373-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(-0.0294373-x)^k\,x^{-k}}{k} \quad \text{for } x<0$$

$$\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right) = \left\lfloor \frac{\arg(-0.0294373-z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(-0.0294373-z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(-0.0294373-z_0)^k\,z_0^{-k}}{k}$$

Integral representation:

$$\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right) = \int_1^{-0.0294373} \frac{1}{t}\,dt$$

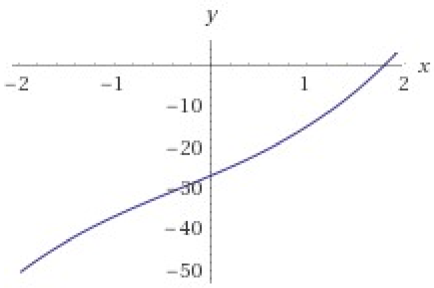
Now, we have the following cubic equation:

x^3 + x^2 + 10 x – 27

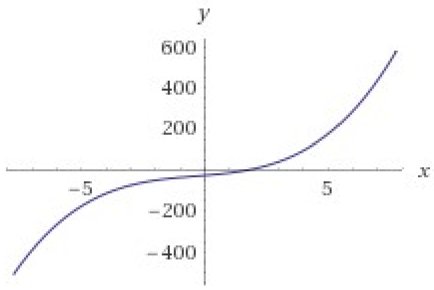
Input:

$$x^3 + x^2 + 10\,x - 27$$

Plots:



(x from −2 to 1.9)



(x from −7.7 to 7.7)

Alternate forms:

$x(x^2+x+10)-27$

$x(x(x+1)+10)-27$

$\left(x+\frac{1}{3}\right)^3+\frac{29}{3}\left(x+\frac{1}{3}\right)-\frac{817}{27}$

Real root:

$x \approx 1.7969$

1.7969

Complex roots:

$x \approx -1.3985-3.6153\,i$

$x \approx -1.3985+3.6153\,i$

Polynomial discriminant:

$\Delta = -28\,335$

Properties as a real function:

Domain

$\mathbb{R}$ (all real numbers)

Range

$\mathbb{R}$ (all real numbers)

Bijectivity

bijective from its domain to ℝ

ℝ is the set of real numbers

Derivative:

$$\frac{d}{dx}(x^3+x^2+10x-27)=3x^2+2x+10$$

Indefinite integral:

$$\int(-27+10x+x^2+x^3)\,dx=\frac{x^4}{4}+\frac{x^3}{3}+5x^2-27x+\text{constant}$$

We take the real root 1.7969 and performing the following calculation

1.7969 * 1/6 * ln(((2^1.5 – 3)/(2^1.5 + 3)))

Input interpretation:

$$1.7969\times\frac{1}{6}\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)$$

log(x) is the natural logarithm

Result:

$-1.05583... +$
$0.940855...\,i$

Polar coordinates:

$r = 1.41421$ (radius), $\theta = 138.296°$ (angle)

1.41421 ≈ √2

Alternative representations:

$$\frac{1}{6}\times1.7969\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)=\frac{1}{6}\times1.7969\log_e\left(\frac{-3+2^{1.5}}{3+2^{1.5}}\right)$$

$$\frac{1}{6}\times1.7969\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)=\frac{1}{6}\times1.7969\log(a)\log_a\left(\frac{-3+2^{1.5}}{3+2^{1.5}}\right)$$

$$\frac{1}{6}\times 1.7969\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)=\frac{1}{6}(-1.7969)\,\text{Li}_1\left(1-\frac{-3+2^{1.5}}{3+2^{1.5}}\right)$$

Series representations:

$$\frac{1}{6}\times 1.7969\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)=0.299483\log(-1.02944)-0.299483\sum_{k=1}^{\infty}\frac{e^{-0.0290123\,k}}{k}$$

$$\frac{1}{6}\times 1.7969\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)=0.598967\,i\,\pi\left\lfloor\frac{\arg(-0.0294373-x)}{2\pi}\right\rfloor+0.299483\log(x)-0.299483\sum_{k=1}^{\infty}\frac{(-1)^k(-0.0294373-x)^k x^{-k}}{k}\quad\text{for } x<0$$

$$\frac{1}{6}\times 1.7969\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)=0.299483\left\lfloor\frac{\arg(-0.0294373-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+0.299483\log(z_0)+0.299483\left\lfloor\frac{\arg(-0.0294373-z_0)}{2\pi}\right\rfloor\log(z_0)-0.299483\sum_{k=1}^{\infty}\frac{(-1)^k(-0.0294373-z_0)^k z_0^{-k}}{k}$$

Integral representation:

$$\frac{1}{6}\times 1.7969\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)=0.299483\int_1^{-0.0294373}\frac{1}{t}\,dt$$

We have that:

Entry 15.3.1 (p. 51). *Let $P(q)$, $Q(q)$, and $R(q)$ be the Eisenstein series defined by* (15.2.8)–(15.2.10). *Then*

$$\int_{e^{-2\pi}}^{q} \sqrt{Q(t)}\frac{dt}{t} = \log\left(\frac{Q^{3/2}(q) - R(q)}{Q^{3/2}(q) + R(q)}\right).$$

Proof. Following Ramanujan's suggestion, let $z = R^2(t)/Q^3(t)$. Then

$$\frac{1}{z}\frac{dz}{dq} = \frac{2}{R}\frac{dR}{dq} - \frac{3}{Q}\frac{dQ}{dq}. \tag{15.3.1}$$

Using Ramanujan's differential equations [223, equation (30)], [228, p. 142], [61, p. 330]

$$q\frac{dR}{dq} = \frac{PR - Q^2}{2} \qquad \text{and} \qquad q\frac{dQ}{dq} = \frac{PQ - R}{3},$$

in (15.3.1), we find that

$$\frac{q}{z}\frac{dz}{dq} = \frac{R^2 - Q^3}{RQ}. \tag{15.3.2}$$

Hence, by (15.3.2),

$$q\frac{d}{dq}\log\left(\frac{Q^{3/2} - R}{Q^{3/2} + R}\right) = q\frac{d}{dq}\log\left(\frac{1 - \sqrt{z}}{1 + \sqrt{z}}\right)$$

$$= q\frac{d}{dz}\log\left(\frac{1 - \sqrt{z}}{1 + \sqrt{z}}\right)\frac{dz}{dq}$$

$$= \frac{1}{\sqrt{z}(z - 1)}q\frac{dz}{dq}$$

$$= \sqrt{Q}.$$

It follows that

$$\int_{e^{-2\pi}}^{q} \sqrt{Q(t)}\frac{dt}{t} = \int_{e^{-2\pi}}^{q} \frac{d}{dt}\log\left(\frac{Q^{3/2} - R}{Q^{3/2} + R}\right)dt$$

$$= \log\left(\frac{Q^{3/2}(q) - R(q)}{Q^{3/2}(q) + R(q)}\right) - \log\left(\frac{Q^{3/2}(e^{-2\pi}) - R(e^{-2\pi})}{Q^{3/2}(e^{-2\pi}) + R(e^{-2\pi})}\right).$$

But it is well known that $R(e^{-2\pi}) = 0$ [123, p. 88], and so Entry 15.3.1 follows. ☐

With regard $\sqrt{Q} = \sqrt{2} = 1.414213562373....$, while

$$q\frac{d}{dq}\log\left(\frac{Q^{3/2}-R}{Q^{3/2}+R}\right)$$

is equal to

$$\left[1.7969\times\frac{1}{6}\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)\right] = 1.41421 \approx \sqrt{2}$$

Possible closed forms:

$\sqrt{2} \approx 1.4142135623$

$\frac{40}{9\pi} \approx 1.41471060$

$1-\frac{3}{\pi}-\sqrt{\pi}+\pi \approx 1.414209144$

Thence, in conclusion:

$$\int_{e^{-2\pi}}^{q}\sqrt{Q(t)}\frac{dt}{t} = \int_{e^{-2\pi}}^{q}\frac{d}{dt}\log\left(\frac{Q^{3/2}-R}{Q^{3/2}+R}\right)dt$$
$$= \log\left(\frac{Q^{3/2}(q)-R(q)}{Q^{3/2}(q)+R(q)}\right)-\log\left(\frac{Q^{3/2}(e^{-2\pi})-R(e^{-2\pi})}{Q^{3/2}(e^{-2\pi})+R(e^{-2\pi})}\right).$$

For

$$R(e^{-2\pi})=0$$

We obtain:

ln(((2^1.5 – 3)/(2^1.5+3))) – ln[(((2^1.5(exp(-2*Pi)))))/((2^1.5(exp(-2*Pi))))]

Input:

$$\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)$$

log(x) is the natural logarithm

Result:

$-3.52549\ldots +$
$3.14159\ldots i$

Polar coordinates:

$r = 4.72215$ (radius), $\theta = 138.296°$ (angle)

4.72215 as the previous result

Alternative representations:

$$\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right) - \log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right) = -\log_e\left(\frac{\exp(-2\pi)\,2^{1.5}}{\exp(-2\pi)\,2^{1.5}}\right) + \log_e\left(\frac{-3+2^{1.5}}{3+2^{1.5}}\right)$$

$$\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right) - \log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right) = -\log(a)\log_a\left(\frac{\exp(-2\pi)\,2^{1.5}}{\exp(-2\pi)\,2^{1.5}}\right) + \log(a)\log_a\left(\frac{-3+2^{1.5}}{3+2^{1.5}}\right)$$

Series representations:

$$\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right) - \log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right) = 2 i \pi \left\lfloor \frac{\arg(-0.0294373 - x)}{2\pi} \right\rfloor - 2 i \pi \left\lfloor \frac{\arg(1-x)}{2\pi} \right\rfloor + \sum_{k=1}^{\infty} \frac{(-1)^{1+k}\left((-0.0294373 - x)^k - (1-x)^k\right) x^{-k}}{k} \quad \text{for } x < 0$$

$$\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right) - \log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right) = \left\lfloor \frac{\arg(-0.0294373 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) - \left\lfloor \frac{\arg(1-z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \left\lfloor \frac{\arg(-0.0294373 - z_0)}{2\pi} \right\rfloor \log(z_0) - \left\lfloor \frac{\arg(1-z_0)}{2\pi} \right\rfloor \log(z_0) + \sum_{k=1}^{\infty} \frac{(-1)^{1+k}\left((-0.0294373 - z_0)^k - (1-z_0)^k\right) z_0^{-k}}{k}$$

$$\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right) - \log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right) = 2 i \pi \left\lfloor -\frac{-\pi + \arg\left(-\frac{0.0294373}{z_0}\right) + \arg(z_0)}{2\pi} \right\rfloor - 2 i \pi \left\lfloor -\frac{-\pi + \arg\left(\frac{1}{z_0}\right) + \arg(z_0)}{2\pi} \right\rfloor + \sum_{k=1}^{\infty} \frac{(-1)^{1+k}\left((-0.0294373 - z_0)^k - (1-z_0)^k\right) z_0^{-k}}{k}$$

Integral representation:

$$\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)=\int_1^{-0.0294373}\left(0+\frac{1}{t}\right)dt$$

We have also:

1/(((((1/((((1/3((((ln(((2^1.5 – 3)/(2^1.5+3))) – ln[(((2^1.5(exp(-2*Pi)))))/((2^1.5(exp(-2*Pi))))])))))))))+0.026i)))))

Input:

$$\frac{1}{\frac{1}{\frac{1}{3}\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)}+0.026\,i}$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

Result:

$-1.24065\ldots+1.03754\ldots\,i$

Polar coordinates:

$r = 1.61731$ (radius), $\theta = 140.095°$ (angle)

1.61731

Alternative representations:

$$\frac{1}{\frac{1}{\frac{1}{3}\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)}+0.026\,i}=\frac{1}{0.026\,i+\frac{1}{\frac{1}{3}\left(-\log_e\left(\frac{\exp(-2\pi)\,2^{1.5}}{\exp(-2\pi)\,2^{1.5}}\right)+\log_e\left(\frac{-3+2^{1.5}}{3+2^{1.5}}\right)\right)}}$$

$$\frac{1}{\frac{1}{\frac{1}{3}\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)}+0.026\,i}=\frac{1}{0.026\,i+\frac{1}{\frac{1}{3}\left(-\log(a)\log_a\left(\frac{\exp(-2\pi)\,2^{1.5}}{\exp(-2\pi)\,2^{1.5}}\right)+\log(a)\log_a\left(\frac{-3+2^{1.5}}{3+2^{1.5}}\right)\right)}}$$

Series representations:

$$\frac{1}{\frac{1}{\frac{1}{3}\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)}+0.026\,i} = \frac{1}{0.026\,i+\frac{3}{2\pi\mathcal{A}\left\lfloor\frac{\arg(-0.0294373-x)}{2\pi}\right\rfloor-2\pi\mathcal{A}\left\lfloor\frac{\arg(1-x)}{2\pi}\right\rfloor+\sum_{k=1}^{\infty}\frac{(-1)^{1+k}\left((-0.0294373-x)^k-(1-x)^k\right)x^{-k}}{k}}} \quad \text{for } x<0$$

$$\frac{1}{\frac{1}{\frac{1}{3}\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)}+0.026\,i} = 1\Bigg/\left(0.026\,i+3\Bigg/\left(\left\lfloor\frac{\arg(-0.0294373-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\left\lfloor\frac{\arg(1-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)+\sum_{k=1}^{\infty}\frac{(-1)^{1+k}\left((-0.0294373-z_0)^k-(1-z_0)^k\right)z_0^{-k}}{k}\right)\right)$$

$$\frac{1}{\frac{1}{\frac{1}{3}\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)}+0.026\,i} = 1\Bigg/\left(0.026\,i+3\Bigg/\left(2\pi\mathcal{A}\left\lfloor-\frac{-\pi+\arg\left(-\frac{0.0294373}{z_0}\right)+\arg(z_0)}{2\pi}\right\rfloor-2\pi\mathcal{A}\left\lfloor-\frac{-\pi+\arg\left(\frac{1}{z_0}\right)+\arg(z_0)}{2\pi}\right\rfloor+\sum_{k=1}^{\infty}\frac{(-1)^{1+k}\left((-0.0294373-z_0)^k-(1-z_0)^k\right)z_0^{-k}}{k}\right)\right)$$

Integral representation:

$$\frac{1}{\frac{1}{\frac{1}{3}\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)}+0.026\,i} = \frac{1}{0.026\,i+\frac{3}{\int_1^{-0.0294373}\left(0+\frac{1}{t}\right)dt}}$$

and:

(144+21+5)-Pi*((((ln(((2^1.5 – 3)/(2^1.5+3))) – ln[(((2^1.5(exp(-2*Pi)))))/((2^1.5(exp(-2*Pi))))]))))^4

Input:

$$(144+21+5)-\pi\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)^4$$

log(x) is the natural logarithm

Result:

1690.94... +
356.234... i

Polar coordinates:

$r = 1728.06$ (radius), $\theta = 11.8967°$ (angle)

1728.06

Alternative representations:

$$(144+21+5)-\pi\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)^4 = 170-\pi\left(-\log_e\left(\frac{\exp(-2\pi)\,2^{1.5}}{\exp(-2\pi)\,2^{1.5}}\right)+\log_e\left(\frac{-3+2^{1.5}}{3+2^{1.5}}\right)\right)^4$$

$$(144+21+5)-\pi\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)^4 = 170-\pi\left(-\log(a)\log_a\left(\frac{\exp(-2\pi)\,2^{1.5}}{\exp(-2\pi)\,2^{1.5}}\right)+\log(a)\log_a\left(\frac{-3+2^{1.5}}{3+2^{1.5}}\right)\right)^4$$

Series representations:

$$(144+21+5)-\pi\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)^4 = 170-\pi\left(2i\pi\left\lfloor\frac{\arg(-0.0294373-x)}{2\pi}\right\rfloor-2i\pi\left\lfloor\frac{\arg(1-x)}{2\pi}\right\rfloor+\sum_{k=1}^{\infty}\frac{(-1)^{1+k}\left((-0.0294373-x)^k-(1-x)^k\right)x^{-k}}{k}\right)^4 \quad \text{for } x<0$$

$$(144+21+5)-\pi\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)^4 = 170-\pi\left(\left\lfloor\frac{\arg(-0.0294373-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\left\lfloor\frac{\arg(1-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)+\sum_{k=1}^{\infty}\frac{(-1)^{1+k}\left((-0.0294373-z_0)^k-(1-z_0)^k\right)z_0^{-k}}{k}\right)^4$$

$$(144+21+5)-\pi\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)^4 =$$

$$170-\pi\left(2 i \pi \left\lfloor\frac{\pi-\arg\left(-\frac{0.0294373}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor - 2 i \pi \left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor + \sum_{k=1}^{\infty}\frac{(-1)^{1+k}\left((-0.0294373-z_0)^k-(1-z_0)^k\right)z_0^{-k}}{k}\right)^4$$

Integral representation:

$$(144+21+5)-\pi\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)^4 = 170-\pi\left(\int_1^{-0.0294373}\left(0+\frac{1}{t}\right)dt\right)^4$$

18+golden ratio+1/Pi*((((ln(((2^1.5 – 3)/(2^1.5+3))) – ln[(((2^1.5(exp(-2*Pi)))))/((2^1.5(exp(-2*Pi))))]))))^4

Input:

$$18+\phi+\frac{1}{\pi}\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)^4$$

log(x) is the natural logarithm

ϕ is the golden ratio

Result:

$-134.486\ldots - 36.0941\ldots i$

Polar coordinates:

$r = 139.245$ (radius), $\theta = -164.977°$ (angle)

139.245

Alternative representations:

$$18+\phi+\frac{\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)^4}{\pi} = 18+\phi+\frac{\left(-\log_e\left(\frac{\exp(-2\pi)2^{1.5}}{\exp(-2\pi)2^{1.5}}\right)+\log_e\left(\frac{-3+2^{1.5}}{3+2^{1.5}}\right)\right)^4}{\pi}$$

$$18+\phi+\frac{\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)^4}{\pi} = 18+\phi+\frac{\left(-\log(a)\log_a\left(\frac{\exp(-2\pi)2^{1.5}}{\exp(-2\pi)2^{1.5}}\right)+\log(a)\log_a\left(\frac{-3+2^{1.5}}{3+2^{1.5}}\right)\right)^4}{\pi}$$

Series representations:

$$18+\phi+\frac{\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)^4}{\pi} = 18+\phi+\frac{\left(2i\pi\left\lfloor\frac{\arg(-0.0294373-x)}{2\pi}\right\rfloor-2i\pi\left\lfloor\frac{\arg(1-x)}{2\pi}\right\rfloor+\sum_{k=1}^{\infty}\frac{(-1)^{1+k}\left((-0.0294373-x)^k-(1-x)^k\right)x^{-k}}{k}\right)^4}{\pi} \quad \text{for } x<0$$

$$18+\phi+\frac{\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)^4}{\pi} = 18+\phi+\frac{1}{\pi}\left(\left\lfloor\frac{\arg(-0.0294373-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\left\lfloor\frac{\arg(1-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)+\sum_{k=1}^{\infty}\frac{(-1)^{1+k}\left((-0.0294373-z_0)^k-(1-z_0)^k\right)z_0^{-k}}{k}\right)^4$$

$$18+\phi+\frac{\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)^4}{\pi} = 18+\phi+\frac{1}{\pi}\left(2i\pi\left\lfloor\frac{\pi-\arg\left(-\frac{0.0294373}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor-2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\sum_{k=1}^{\infty}\frac{(-1)^{1+k}\left((-0.0294373-z_0)^k-(1-z_0)^k\right)z_0^{-k}}{k}\right)^4$$

Integral representation:

$$18+\phi+\frac{\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)^4}{\pi} = 18+\phi+\frac{\left(\int_1^{-0.0294373}\left(0+\frac{1}{t}\right)dt\right)^4}{\pi}$$

29+2+golden ratio^2+1/Pi*((((ln(((2^1.5 – 3)/(2^1.5+3))) – ln[(((2^1.5(exp(-2*Pi)))))/((2^1.5(exp(-2*Pi))))]))))^4

Input:

$$29+2+\phi^2+\frac{1}{\pi}\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)^4$$

log(x) is the natural logarithm

ϕ is the golden ratio

Result:

$-120.486\ldots-36.0941\ldots i$

Polar coordinates:

$r = 125.776$ (radius), $\theta = -163.323°$ (angle)

125.776

Alternative representations:

$$29+2+\phi^2+\frac{\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)^4}{\pi}=31+\phi^2+\frac{\left(-\log_e\left(\frac{\exp(-2\pi)2^{1.5}}{\exp(-2\pi)2^{1.5}}\right)+\log_e\left(\frac{-3+2^{1.5}}{3+2^{1.5}}\right)\right)^4}{\pi}$$

$$29+2+\phi^2+\frac{\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)^4}{\pi}=31+\phi^2+\frac{\left(-\log(a)\log_a\left(\frac{\exp(-2\pi)2^{1.5}}{\exp(-2\pi)2^{1.5}}\right)+\log(a)\log_a\left(\frac{-3+2^{1.5}}{3+2^{1.5}}\right)\right)^4}{\pi}$$

Series representations:

$$29+2+\phi^2+\frac{\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)^4}{\pi}=31+\phi^2+\frac{\left(2i\pi\left\lfloor\frac{\arg(-0.0294373-x)}{2\pi}\right\rfloor-2i\pi\left\lfloor\frac{\arg(1-x)}{2\pi}\right\rfloor+\sum_{k=1}^{\infty}\frac{(-1)^{1+k}\left((-0.0294373-x)^k-(1-x)^k\right)x^{-k}}{k}\right)^4}{\pi}$$

for $x < 0$

$$29+2+\phi^2+\frac{\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)^4}{\pi}=31+\phi^2+\frac{1}{\pi}$$
$$\left(\left\lfloor\frac{\arg(-0.0294373-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\left\lfloor\frac{\arg(1-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)+\right.$$
$$\left.\sum_{k=1}^{\infty}\frac{(-1)^{1+k}\left((-0.0294373-z_0)^k-(1-z_0)^k\right)z_0^{-k}}{k}\right)^4$$

$$29+2+\phi^2+\frac{\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)^4}{\pi}=$$
$$31+\phi^2+\frac{1}{\pi}\left(2i\pi\left\lfloor\frac{\pi-\arg\left(-\frac{0.0294373}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor-2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\right.$$
$$\left.\sum_{k=1}^{\infty}\frac{(-1)^{1+k}\left((-0.0294373-z_0)^k-(1-z_0)^k\right)z_0^{-k}}{k}\right)^4$$

Integral representation:

$$29+2+\phi^2+\frac{\left(\log\left(\frac{2^{1.5}-3}{2^{1.5}+3}\right)-\log\left(\frac{2^{1.5}\exp(-2\pi)}{2^{1.5}\exp(-2\pi)}\right)\right)^4}{\pi}=31+\phi^2+\frac{\left(\int_1^{-0.0294373}\left(0+\frac{1}{t}\right)dt\right)^4}{\pi}$$

Now:

Gravitational Collapse in Einstein Dilaton Gauss-Bonnet Gravity

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We have that:

Equation $(3a)$ for the EdGB scalar is

$$E^{(P,\phi)} \equiv \partial_t P - \frac{1}{r^2}\left(r^2 e^{A-B}\partial_r Q\right) - 8\lambda e^{-A-B}\frac{1+e^{2B}}{r^2}\left(\partial_t B\right)^2 + 8\lambda e^{A-3B}\frac{3-e^{2B}}{r^2}\partial_r A\partial_r B + 8\lambda e^{-A-B}\frac{1-e^{2B}}{r^2}\left(\partial_r^2 B - \partial_t A\partial_t B - e^{-2B}\partial_r A - e^{-2B}\left(\partial_r A\right)^2\right) = 0, \quad \text{(B.5)}$$

and the evolution equation for the constraint $\partial_r\phi = Q$ is

From:

Modular equations and approximations to π – *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have:

$$e^{\frac{1}{2}\pi\sqrt{46}} = 144(147 + 104\sqrt{2})$$

From which:

exp((Pi/2)*sqrt46)

Input:

$$\exp\left(\frac{\pi}{2}\sqrt{46}\right)$$

Exact result:

$$e^{\sqrt{23/2}\,\pi}$$

Decimal approximation:

42347.26108215699922855859544926430987516136752699531538765...

42347.2610821…

Property:

$e^{\sqrt{23/2}\,\pi}$ is a transcendental number

Series representations:

$$e^{\left(\sqrt{46}\,\pi\right)/2} = e^{1/2\,\pi\sqrt{45}\,\sum_{k=0}^{\infty} 45^{-k}\binom{1/2}{k}}$$

$$e^{\left(\sqrt{46}\,\pi\right)/2} = \exp\left(\frac{1}{2}\pi\sqrt{45}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{45}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{\left(\sqrt{46}\,\pi\right)/2} = \exp\left(\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 45^{-s}\, \Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{4\sqrt{\pi}}\right)$$

Integral representation:

$$(1+z)^a = \frac{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\,\pi\,i)\,\Gamma(-a)} \quad \text{for } (0 < \gamma < -\operatorname{Re}(a) \text{ and } |\arg(z)| < \pi)$$

exp -((Pi/2)*sqrt46)

Input:

$$\exp\left(-\left(\frac{\pi}{2}\sqrt{46}\right)\right)$$

Exact result:

$$e^{-\sqrt{23/2}\,\pi}$$

Decimal approximation:

0.000023614278100770715071560643021895369213292650902831012...

0.0000236142781…

Property:

$e^{-\sqrt{23/2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\left(\pi\sqrt{46}\right)/2} = e^{-1/2\,\pi\sqrt{45}\,\sum_{k=0}^{\infty} 45^{-k}\binom{1/2}{k}}$$

$$e^{-\left(\pi\sqrt{46}\right)/2} = \exp\left(-\frac{1}{2}\,\pi\sqrt{45}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{45}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\left(\pi\sqrt{46}\right)/2} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 45^{-s}\, \Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{4\sqrt{\pi}}\right)$$

Integral representation:

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\Gamma(-a)} \quad \text{for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$$

144(147+104sqrt2)

Input:

$$144\left(147+104\sqrt{2}\right)$$

Decimal approximation:

42347.262310099471450854090333764438424659406005645174343 89...

42347.26231…

Alternate form:

$$21168 + 14976\sqrt{2}$$

Minimal polynomial:

$$x^2 - 42336\,x - 476928$$

1 / ((144(147+104sqrt2)))

Input:

$$\frac{1}{144\left(147+104\sqrt{2}\right)}$$

Decimal approximation:

0.0000236142774160281024684865090001812111249622702906 39978...

0.000023614277…

Alternate forms:

$$\frac{104\sqrt{2}-147}{3312}$$

$$\frac{1}{21168+14976\sqrt{2}}$$

$$\frac{13\sqrt{2}}{414} - \frac{49}{1104}$$

Minimal polynomial:

$476\,928\,x^2 + 42\,336\,x - 1$

Thence:

((Pi/2)*sqrt46)

Input:

$\frac{\pi}{2}\sqrt{46}$

Exact result:

$\sqrt{\frac{23}{2}}\,\pi$

Decimal approximation:

10.65365902460406436470496489685050977165072774816927903524...

10.653659.... = 2B

-10.653659.... = -2B

For:

$A \to -\infty$ here, while $B \to \infty$ on the horizon.

$\lambda = 50$, $r_{max} = 100$, $r/m \sim 12.5$.

$\partial_r = \partial_t = 2$; A = B = 5.3268295 ; e^{2B} = 42347.261 ; e^{-2B} = 0.00002361427 ;

10.653659.... = 2B ; -10.653659.... = -2B ; ϕ is the dilaton field = 0.9991104684

$$Q \equiv \partial_r \phi,$$
$$P \equiv e^{-A+B} \partial_t \phi.$$

(with regard the dilaton field see: "*On some new possible mathematical connections between some equations of the Ramanujan's manuscripts, the Rogers-Ramanujan continued fractions and some sectors of Particle Physics, String Theory and D-branes*").

We note that 0.9991104684 is equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\dots}}}} \approx 0.9991104684$$

Thence, from

$$E^{(P,\phi)} \equiv \partial_t P - \frac{1}{r^2}\left(r^2 e^{A-B}\partial_r Q\right) - 8\lambda e^{-A-B}\frac{1+e^{2B}}{r^2}\left(\partial_t B\right)^2 + 8\lambda e^{A-3B}\frac{3-e^{2B}}{r^2}\partial_r A \partial_r B$$
$$+ 8\lambda e^{-A-B}\frac{1-e^{2B}}{r^2}\left(\partial_r^2 B - \partial_t A \partial_t B - e^{-2B}\partial_r A - e^{-2B}\left(\partial_r A\right)^2\right) = 0, \qquad \text{(B.5)}$$

$\lambda = 50,\ r_{max} = 100,\ r/m \sim 12.5.$

$\partial_r = \partial_t = 2$;   A = B = 5.3268295 ; e^{2B} = 42347.261 ; e^{-2B} = 0.00002361427 ;

10.653659.... = 2B ;   -10.653659.... = -2B ; ϕ is the dilaton field = 0.9991104684

Q = P = 2*0.9991104684

we obtain:

4*0.9991104684 – 1/(100^2)*(100^2*4*0.9991104684)-8*50*(0.00002361427)*1/(100^2)*(1+42347.261)*(10.653659)^2+8*50*(0.00002361427)*1/(100^2)*(3-42347.261)*(10.653659)^2

$$\partial_t P - \frac{1}{r^2}\left(r^2 e^{A-B}\partial_r Q\right) - 8\lambda e^{-A-B}\frac{1+e^{2B}}{r^2}\left(\partial_t B\right)^2 + 8\lambda e^{A-3B}\frac{3-e^{2B}}{r^2}\partial_r A \partial_r B$$

Input interpretation:

$$4\times 0.9991104684 - \frac{1}{100^2}\left(100^2\times 4\times 0.9991104684\right) + 8\times 50\times\frac{1}{100^2}(1+42\,347.261)\times 10.653659^2\times(-0.00002361427) + 8\times 50\times 0.00002361427\times\frac{1}{100^2}(3-42\,347.261)\times 10.653659^2$$

Result:

$-9.0798184561595651891570044856$

-9.0798184561595651891570044856

$$+8\lambda e^{-A-B}\frac{1-e^{2B}}{r^2}\left(\partial_r^2 B-\partial_t A\partial_t B-e^{-2B}\partial_r A-e^{-2B}\left(\partial_r A\right)^2\right)=0,$$

8*50*(0.00002361427)*1/(100^2)*(1-42347.261)* [(((4*5.3268295-4*5.3268295^2-0.00002361427*10.653659-0.00002361427(10.653659)^2)))]

Input interpretation:

$$8\times 50\times 0.00002361427\times\frac{1}{100^2}(1-42\,347.261)\left(4\times 5.3268295-4\times 5.3268295^2+10.653659\times(-0.00002361427)+10.653659^2\times(-0.00002361427)\right)$$

Result:

$3.68775419791523413028890714478684108475 6$

3.687754197915234130288907144786841084756

Thence, in conclusion, we have:

4*0.9991104684 – 1/(100^2)*(100^2*4*0.9991104684)-8*50*(0.00002361427)*1/(100^2)*(1+42347.261)*(10.653659)^2+8*50*(0.00002361427)*1/(100^2)*(3-42347.261)*(10.653659)^2 + 3.687754197915234130288907

Input interpretation:

$$4\times 0.9991104684-\frac{1}{100^2}(100^2\times 4\times 0.9991104684)+8\times 50\times\frac{1}{100^2}(1+42\,347.261)\times 10.653659^2\times(-0.00002361427)+8\times 50\times 0.00002361427\times\frac{1}{100^2}(3-42\,347.261)\times 10.653659^2+3.687754197915234130288907$$

Result:

$-5.3920642582443310588680974856$

-5.3920642582443310588680974856

From which:

1+1/(((-(4*0.9991104684 – 1/(100^2)*(100^2*4*0.9991104684)-8*50*(0.00002361427)*1/(100^2)*(1+42347.261)*(10.653659)^2+8*50*(0.00002361427)*1/(100^2)*(3-42347.261)*(10.653659)^2 + 3.68775419791))))^1/4

Input interpretation:

$$1+1\Big/\left(\left(-\left(4\times 0.9991104684-\frac{1}{100^2}\left(100^2\times 4\times 0.9991104684\right)+8\times 50\times\frac{1}{100^2}(1+42\,347.261)\times 10.653659^2\times(-0.00002361427)+8\times 50\times 0.00002361427\times\frac{1}{100^2}(3-42\,347.261)\times 10.653659^2+3.68775419791\right)\right)\hat{}\,(1/4)\right)$$

Result:

1.656237788849051604056533179328511283169098258086288890250...

1.656237788849… result very near to the 14th root of the following Ramanujan’s class invariant $Q=\left(G_{505}/G_{101/5}\right)^3=1164.2696$ i.e. 1.65578...

and:

1+1/(((-(4*0.99911046 -1/(100^2)(100^2*4*0.99911046)-8*50(0.00002361427)1/(100^2)(1+42347.261)(10.653659)^2+8*50(0.00002361427)1/(100^2)(3-42347.261)(10.653659)^2 +3.68775411))))^1/4 - (47-7-2)1/10^3

Input interpretation:

$$1+1\Big/\left(\left(-\left(4\times 0.99911046-\frac{1}{100^2}\left(100^2\times 4\times 0.99911046\right)+8\times 50\times\frac{1}{100^2}(1+42\,347.261)\times 10.653659^2\times(-0.00002361427)+8\times 50\times 0.00002361427\times\frac{1}{100^2}(3-42\,347.261)\times 10.653659^2+3.68775411\right)\right)\hat{}\,(1/4)\right)-(47-7-2)\times\frac{1}{10^3}$$

Result:

1.618237786174293818037343941906222107298604660929223282310...

1.61823778617…

e*((((4*0.99911046-1/(100^2)*(100^2*4*0.99911046)-8*50*(0.00002361427)1/(100^2)(1+42347.261)(10.653659)^2+8*50(0.00002361427)1/(100^2)(3-42347.261)(10.653659)^2 + 3.6877541979))))^4-521-47-0.61803398

Input interpretation:

$$e\left(4\times 0.99911046 - \frac{1}{100^2}(100^2\times 4\times 0.99911046) + 8\times 50\times \frac{1}{100^2}(1+42\,347.261)\times 10.653659^2\times(-0.00002361427) + 8\times 50\times 0.00002361427\times \frac{1}{100^2}(3-42\,347.261)\times 10.653659^2 + 3.6877541979\right)^4 - 521 - 47 - 0.61803398$$

Result:

1729.20…

1729.20…

Alternative representation:

$$e\left(4\times 0.99911 - \frac{100^2\times 4\times 0.99911}{100^2} - \frac{8\times 50\,(0.0000236143\,(1+42\,347.3)\,10.6537^2)}{100^2} + \frac{(8\times 50)\,0.0000236143\,((3-42\,347.3)\,10.6537^2)}{100^2} + 3.68775419790000\right)^4 - 521 - 47 - 0.618034 = \exp(z)\left(4\times 0.99911 - \frac{100^2\times 4\times 0.99911}{100^2} - \frac{8\times 50\,(0.0000236143\,(1+42\,347.3)\,10.6537^2)}{100^2} + \frac{(8\times 50)\,0.0000236143\,((3-42\,347.3)\,10.6537^2)}{100^2} + 3.68775419790000\right)^4 - 521 - 47 - 0.618034 \quad \text{for } z = 1$$

Series representations:

$$e\left(4\times 0.99911-\frac{100^2\times 4\times 0.99911}{100^2}-\frac{8\times 50\,(0.0000236143\,(1+42\,347.3)\,10.6537^2)}{100^2}+\frac{(8\times 50)\,0.0000236143\,((3-42\,347.3)\,10.6537^2)}{100^2}+3.68775419790000\right)^4-521-47-0.618034=-568.618+845.318\sum_{k=0}^{\infty}\frac{1}{k!}$$

$$e\left(4\times 0.99911-\frac{100^2\times 4\times 0.99911}{100^2}-\frac{8\times 50\,(0.0000236143\,(1+42\,347.3)\,10.6537^2)}{100^2}+\frac{(8\times 50)\,0.0000236143\,((3-42\,347.3)\,10.6537^2)}{100^2}+3.68775419790000\right)^4-521-47-0.618034=-568.618+422.659\sum_{k=0}^{\infty}\frac{1+k}{k!}$$

$$e\left(4\times 0.99911-\frac{100^2\times 4\times 0.99911}{100^2}-\frac{8\times 50\,(0.0000236143\,(1+42\,347.3)\,10.6537^2)}{100^2}+\frac{(8\times 50)\,0.0000236143\,((3-42\,347.3)\,10.6537^2)}{100^2}+3.68775419790000\right)^4-521-47-0.618034=-568.618+\frac{845.318\sum_{k=0}^{\infty}\frac{-1+k+z}{k!}}{z}$$

We have that:

With regard the Einstein dilaton Gauss Bonnet (EdGB) gravity

In the coordinates (4), the nontrivial components of the EdGB equations of motion (3a) are

$$E_{tt}^{(g)} \propto \left(1+4\lambda\left(1-3e^{-2B}\right)\frac{Q}{r}\right)\partial_r B+\frac{e^{2B}-1}{2r}-\frac{1}{2}r\left(Q^2+P^2\right)+4\lambda\frac{-1+e^{-2B}}{r}\left(\partial_r Q+e^{-A-B}P\partial_t B\right)=0, \qquad \text{(B.1)}$$

$$E_{tr}^{(g)} \propto \left(1 + 4\lambda\left(1 - 3e^{-2B}\right)\frac{Q}{r}\right)\partial_t B - \frac{1}{2} r e^{A-B} QP$$
$$+ 4\lambda e^{A-B}\frac{1 - e^{-2B}}{r}\left(P\partial_r B - \partial_r P\right) = 0, \tag{B.2}$$

$$E_{rr}^{(g)} \propto \left(1 + 4\lambda\left(1 - 3e^{-2B}\right)\frac{Q}{r}\right)\partial_r A + \frac{1 - e^{2B}}{2r} - \frac{1}{2} r\left(Q^2 + P^2\right)$$
$$+ 4\lambda e^{-A-B}\frac{e^{2B} - 1}{r}\left(P\partial_t B - \partial_t P\right) = 0, \tag{B.3}$$

$$E_{\vartheta\vartheta}^{(g)} \propto \left(-1 + 8\lambda e^{-2B}\frac{Q}{r}\right)\left(\partial_t^2 B - e^{2A-2B}\partial_r^2 A + e^{2A-2B}\left(\partial_r A\right)^2 + \partial_t A \partial_t B\right)$$
$$- \left(1 + 8\lambda e^{-2B}\frac{Q}{r}\right)\left(\partial_t B\right)^2 + 8\lambda e^{A-3B} P\left(\frac{\partial_r A}{r} - \frac{\partial_r B}{r} + 2\frac{\partial_r P}{r}\right)\partial_t B$$

$$+ e^{2A-2B}\left(\frac{1 - e^{-4B}}{r} + 24\lambda e^{-2B}\frac{Q}{r}\partial_r B\right)\partial_r A + e^{2A-2B}\frac{\partial_r B}{r}$$
$$+ 8\lambda e^{2A-4B}\frac{\partial_r Q \partial_r A}{r} + 8\lambda e^{A-3B}\frac{\partial_r B \partial_t P}{r} + \frac{1}{2} e^{2A-2B}\left(Q^2 - P^2\right) = 0. \tag{B.4}$$

Now:

For

$\lambda = 50,\ r_{max} = 100,\ r/m \sim 12.5.$

$\partial_r = \partial_t = 2$; A = B = 5.3268295 ; e^{2B} = 42347.261 ; e^{-2B} = 0.00002361427 ;

10.653659.... = 2B ; -10.653659.... = -2B ; ϕ is the dilaton field = 0.9991104684

Q = P = 2*0.9991104684

From

$$E_{tt}^{(g)} \propto \left(1 + 4\lambda\left(1 - 3e^{-2B}\right)\frac{Q}{r}\right)\partial_r B + \frac{e^{2B} - 1}{2r} - \frac{1}{2} r\left(Q^2 + P^2\right)$$
$$+ 4\lambda\frac{-1 + e^{-2B}}{r}\left(\partial_r Q + e^{-A-B} P\partial_t B\right) = 0,$$

we obtain:

((1+200(1-3*0.00002361427)(2*0.9991104684)/100))10.653659+(42347.261-1)/200-50(((2*0.9991104684)^2+(2*0.9991104684)^2))

Input interpretation:

$$\left(1+200\,(1+3\times(-0.00002361427))\times\frac{2\times0.9991104684}{100}\right)\times10.653659+\frac{42347.261-1}{200}-50\,((2\times0.9991104684)^2+(2\times0.9991104684)^2)$$

Result:

−134.330014547049796326171977744

-134.330014547049796326171977744+200*((-1+0.00002361427)/100)(4*0.9991104684+0.00002361427*10.653659*2*0.9991104684)

Input interpretation:

$$-134.330014547049796326171977744+200\left(\frac{1}{100}\,(-1+0.00002361427)\right)(4\times0.9991104684+0.00002361427\times10.653659\times2\times0.9991104684)$$

Result:

−142.32371494276547018323655929177703973104

-142.3237149427......

From

$$E_{tr}^{(g)}\propto\left(1+4\lambda\left(1-3e^{-2B}\right)\frac{Q}{r}\right)\partial_t B-\frac{1}{2}re^{A-B}QP+4\lambda e^{A-B}\frac{1-e^{-2B}}{r}\left(P\partial_r B-\partial_r P\right)=0,\qquad\text{(B.2)}$$

$\lambda = 50,\ r_{max} = 100,\ r/m \sim 12.5.$

$\partial_r = \partial_t = 2$;   A = B = 5.3268295 ; e^{2B} = 42347.261 ; e^{-2B} = 0.00002361427 ;

10.653659.... = 2B ;   -10.653659.... = -2B ; ϕ is the dilaton field = 0.9991104684

Q = P = 2*0.9991104684

(1+200(1-3*0.00002361427)(2*0.9991104684)/100)10.653659-50*(2*0.9991104684)^2+200*((1-0.00002361427)/100)*(10.653659*2*0.9991104684-4*0.9991104684)

Input interpretation:

$$\left(1+200\,(1+3\times(-0.00002361427))\times\frac{2\times0.9991104684}{100}\right)\times10.653659-50\,(2\times0.9991104684)^2+200\times\frac{1-0.00002361427}{100}\,(10.653659\times2\times0.9991104684+4\times(-0.9991104684))$$

Result:

−111.833945418558708989018636992

-111.833945418...

From

$$E_{rr}^{(g)} \propto \left(1+4\lambda\left(1-3e^{-2B}\right)\frac{Q}{r}\right)\partial_r A+\frac{1-e^{2B}}{2r}-\frac{1}{2}r\left(Q^2+P^2\right)+4\lambda e^{-A-B}\frac{e^{2B}-1}{r}\left(P\partial_t B-\partial_t P\right)=0, \quad (B.3)$$

$\lambda = 50,\ r_{max} = 100,\ r/m \sim 12.5.$

$\partial_r = \partial_t = 2$; A = B = 5.3268295 ; e^{2B} = 42347.261 ; e^{-2B} = 0.00002361427 ;

10.653659.... = 2B ; -10.653659.... = -2B ; ϕ is the dilaton field = 0.9991104684

Q = P = 2*0.9991104684

(((1+200(1-3*0.00002361427)(2*0.99911046)/100)))10.653659+(1-42347.261)/200-50(((2*0.99911046)^2+(2*0.99911046)^2))+200*((0.00002361427*42347.261-1)/100)(10.653659*2*0.99911046-4*0.99911046)

Input interpretation:

$$\left(1+200\,(1+3\times(-0.00002361427))\times\frac{2\times0.99911046}{100}\right)\times10.653659+\frac{1-42\,347.261}{200}-50\left((2\times0.99911046)^2+(2\times0.99911046)^2\right)+200\left(\frac{1}{100}\,(0.00002361427\times42\,347.261-1)\right)(10.653659\times2\times0.99911046+4\times(-0.99911046))$$

Result:

-557.7926301218911213247345522568

-557.79263....

From

$$E^{(g)}_{\vartheta\vartheta}\propto\left(-1+8\lambda e^{-2B}\frac{Q}{r}\right)\left(\partial_t^2B-e^{2A-2B}\partial_r^2A+e^{2A-2B}\left(\partial_rA\right)^2+\partial_tA\partial_tB\right)-\left(1+8\lambda e^{-2B}\frac{Q}{r}\right)\left(\partial_tB\right)^2+8\lambda e^{A-3B}P\left(\frac{\partial_rA}{r}-\frac{\partial_rB}{r}+2\frac{\partial_rP}{r}\right)\partial_tB$$

(-1+400*0.00002361427*0.9991104684/50)(4*5.3268295-4*5.3268295+10.653659^2+4*5.3268295^2)-(1+400*0.00002361427*0.9991104684/50)(10.653659)^2

Input interpretation:

$$\left(-1+400\times0.00002361427\times\frac{0.9991104684}{50}\right)(4\times5.3268295+4\times(-5.3268295)+10.653659^2+4\times5.3268295^2)-\left(1+400\times0.00002361427\times\frac{0.9991104684}{50}\right)\times10.653659^2$$

Result:

-340.479927495851138952241193173223136

Input interpretation:

$$-340.479927495851138952241193173223136+400\times0.00002361427\times2\times0.9991104684\left(\frac{10.653659}{100}-\frac{10.653659}{100}+2\times2\times\frac{0.9991104684}{50}\right)\times10.653659$$

Result:

−340.4638551115092836049175994603579877888

-340.4638551115...

From

$$+ e^{2A-2B}\left(\frac{1-e^{-4B}}{r} + 24\lambda e^{-2B}\frac{Q}{r}\partial_r B\right)\partial_r A + e^{2A-2B}\frac{\partial_r B}{r}$$
$$+ 8\lambda e^{2A-4B}\frac{\partial_r Q \partial_r A}{r} + 8\lambda e^{A-3B}\frac{\partial_r B \partial_t P}{r} + \frac{1}{2}e^{2A-2B}\left(Q^2 - P^2\right) = 0. \quad \text{(B.4)}$$

$\lambda = 50,\ r_{max} = 100,\ r/m \sim 12.5.$

$\partial_r = \partial_t = 2$;   A = B = 5.3268295 ; e^{2B} = 42347.261 ; e^{-2B} = 0.00002361427 ;

10.653659.... = 2B ;   -10.653659.... = -2B ; ϕ is the dilaton field = 0.9991104684

Q = P = 2*0.9991104684

((((((1-exp(-4*5.3268))/100)+1200*0.000023614(0.02*0.9991104*10.6536)))))10.653659+0.10653659+(32*0.000023614*0.9991104*5.3268)+(32*0.000023614*0.9991104*5.3268)+1/2(((2*0.9991104)^2-(2*0.9991104)^2))

Input interpretation:

$$\left(\frac{1}{100}(1-\exp(-4\times 5.3268)) + 1200\times 0.000023614\,(0.02\times 0.9991104\times 10.6536)\right)\times 10.653659 + 0.10653659 + 32\times 0.000023614\times 0.9991104\times 5.3268 + 32\times 0.000023614\times 0.9991104\times 5.3268 + \frac{1}{2}\left((2\times 0.9991104)^2 - (2\times 0.9991104)^2\right)$$

Result:

0.285383601695945011604238623532638750243967824978161296304...

Result:

−340.1784715098133385933133608368253490385560321750218388

Final result:

-340.17847150981333859....

In conclusion, we obtain the following expressions:

(-(-340.1784715 -557.79263 -111.833945418 -142.3237149427))

Input interpretation:

$-(-340.1784715 - 557.79263 - 111.833945418 - 142.3237149427)$

Result:

1152.1287618607

1152.1287618607

From which:

(-(-340.1784715 -557.79263 -111.833945418 -142.3237149427))+34+Pi

Input interpretation:

$-(-340.1784715 - 557.79263 - 111.833945418 - 142.3237149427) + 34 + \pi$

Result:

1189.2704...

1189.2704... result practically equal to the rest mass of Sigma baryon 1189.37

((-340.1784715 * -557.79263 * -111.833945418 * -142.3237149427))^1/44

Input interpretation:

$\sqrt[44]{-340.1784715 \times (-557.79263) \times (-111.833945418) \times (-142.3237149427)}$

Result:

1.642310384...

1.642310384...

((-340.1784715 * -557.79263 * -111.833945418 * -142.3237149427))^1/44 - 24/10^3

Input interpretation:

$$\sqrt[44]{-340.1784715\times(-557.79263)\times(-111.833945418)\times(-142.3237149427)}-\frac{24}{10^3}$$

Result:

1.618310384...

1.618310384…

((-340.1784715 * -557.79263 * -111.833945418 * -142.3237149427))^1/3 +233+55-5+1/golden ratio

Input interpretation:

$$\sqrt[3]{-340.1784715\times(-557.79263)\times(-111.833945418)\times(-142.3237149427)}+233+55-5+\frac{1}{\phi}$$

ϕ is the golden ratio

Result:

1729.09166...

1729.09166…

Alternative representations:

$$\sqrt[3]{(-340.178\,(-111.8339454180000)\,(-142.32371494270000))\,(-1)\,557.793}+233+55-5+\frac{1}{\phi}=283+\sqrt[3]{3.02016\times10^9}+\frac{1}{2\sin(54\,°)}$$

$$\sqrt[3]{(-340.178\,(-111.8339454180000)\,(-142.32371494270000))\,(-1)\,557.793}+233+55-5+\frac{1}{\phi}=283+\sqrt[3]{3.02016\times10^9}+-\frac{1}{2\cos(216\,°)}$$

$$\sqrt[3]{(-340.178\,(-111.8339454180000)\,(-142.32371494270000))\,(-1)\,557.793}+233+55-5+\frac{1}{\phi}=283+\sqrt[3]{3.02016\times10^9}+-\frac{1}{2\sin(666\,°)}$$

((-340.1784715 * -557.79263 * -111.833945418 * -142.3237149427))^1/4-89-8

Input interpretation:

$$\sqrt[4]{-340.1784715\times(-557.79263)\times(-111.833945418)\times(-142.3237149427)}-89-8$$

Result:

137.42700...

137.427...

This result is very near to the inverse of fine-structure constant 137.035

((-340.1784715 * -557.79263 * -111.833945418 * -142.3237149427))^1/4-76-18-1/golden ratio

Input interpretation:

$$\sqrt[4]{-340.1784715\times(-557.79263)\times(-111.833945418)\times(-142.3237149427)}-76-18-\frac{1}{\phi}$$

ϕ is the golden ratio

Result:

139.80897...

139.80897…

Alternative representations:

$$\sqrt[4]{(-340.178\,(-111.8339454180000)\,(-142.32371494270000))\,(-1)\,557.793}-76-18-\frac{1}{\phi}=-94+\sqrt[4]{3.02016\times10^9}-\frac{1}{2\sin(54^\circ)}$$

$$\sqrt[4]{(-340.178\,(-111.8339454180000)\,(-142.32371494270000))\,(-1)\,557.793}-76-18-\frac{1}{\phi}=-94+\sqrt[4]{3.02016\times10^9}--\frac{1}{2\cos(216^\circ)}$$

$$\sqrt[4]{(-340.178\,(-111.8339454180000)\,(-142.32371494270000))\,(-1)\,557.793}-76-18-\frac{1}{\phi}=-94+\sqrt[4]{3.02016\times10^9}--\frac{1}{2\sin(666^\circ)}$$

From:

White Holes as Remnants: A Surprising Scenario for the End of a Black Hole
Eugenio Bianchi, Marios Christodoulou, Fabio D'Ambrosio and Carlo Rovelli, Hal M. Haggard - arXiv:1802.04264v2 [gr-qc] 17 Mar 2018

$$v_+ u_+ = v_- u_- \equiv \left(1 - \frac{r_P}{2m}\right) e^{\frac{r_P}{2m}}. \qquad (20)$$

For

$r_P \sim 7/3\ m.$

((1-(7/3*13.12806e+39)/(2*13.12806e+39)))*exp(((7/3*13.12806e+39)/(2*13.12806e+39)))

Input interpretation:

$$\left(1 - \frac{\frac{7}{3} \times 13.12806 \times 10^{39}}{2 \times 13.12806 \times 10^{39}}\right) \exp\left(\frac{\frac{7}{3} \times 13.12806 \times 10^{39}}{2 \times 13.12806 \times 10^{39}}\right)$$

Result:

-0.53521175719226012375231151633761839075239843073263553222...

-0.5352117571922...

From which:

sqrt[-sqrt2/(((((1-(7/3*13.12806e+39)/(2*13.12806e+39)))*exp(((7/3*13.12806e+39)/(2*13.12806e+39))))))]-8/10^3

Input interpretation:

$$\sqrt{-\frac{\sqrt{2}}{\left(1 - \frac{\frac{7}{3} \times 13.12806 \times 10^{39}}{2 \times 13.12806 \times 10^{39}}\right) \exp\left(\frac{\frac{7}{3} \times 13.12806 \times 10^{39}}{2 \times 13.12806 \times 10^{39}}\right)}} - \frac{8}{10^3}$$

Result:

1.617528829569127836157963433967419761761520639551016090549...

1.6175288295691…

For

$$l \sim (m\hbar)^{\frac{1}{3}}, \qquad (9)$$

$$t_s = x, \quad \text{and} \quad r_s = \tau^2. \qquad (8)$$

$$K(\tau) \approx \frac{9\,l^2 - 24\,l\tau^2 + 48\,\tau^4}{(l+\tau^2)^8} m^2 \qquad (5)$$

$$K(0) \approx \frac{9\,m^2}{l^6}. \qquad (6)$$

We know that

$\hbar$ = 1.054571817 * 10^-34 and m = 13.12806e+39 , thence:

l = (13.12806e+39 * 1.054571817e-34)^1/3

Input interpretation:

$\sqrt[3]{13.12806 \times 10^{39} \times 1.054571817 \times 10^{-34}}$

Result:

111.4531218888670286228753437470873901933156229644735167293...

111.4531218.....

$\tau^2 = r_s$ = 1.94973e+13

thence, from

$$K(\tau) \approx \frac{9\,l^2 - 24\,l\tau^2 + 48\,\tau^4}{(l+\tau^2)^8} m^2 \qquad (5)$$

We obtain:

((((9*(111.4531218)^2-24(111.4531218*1.94973e+13)+48(1.94973e+13)^2*(13.12806e+39)^2)))) / ((((111.4531218+1.94973e+13)^8)))

Input interpretation:

$$\left(9\times 111.4531218^2-24\,(111.4531218\times 1.94973\times 10^{13})+48\,(1.94973\times 10^{13})^2\,(13.12806\times 10^{39})^2\right)\times \frac{1}{(111.4531218+1.94973\times 10^{13})^8}$$

Result:

150.5897798952398090516407512976260053855187796542170804536...

150.58977989523...

((((9*(111.4531218)^2-24(111.4531218*1.94973e+13)+48(1.94973e+13)^2*(13.12806e+39)^2)))) / ((((111.4531218+1.94973e+13)^8)))+111.4531218-2Pi+(sqrt2/2)^4

Input interpretation:

$$\left(9\times 111.4531218^2-24\,(111.4531218\times 1.94973\times 10^{13})+48\,(1.94973\times 10^{13})^2\,(13.12806\times 10^{39})^2\right)/(111.4531218+1.94973\times 10^{13})^8+111.4531218-2\pi+\left(\frac{\sqrt{2}}{2}\right)^4$$

Result:

256.010...

256.010…

From which:

27/4*(((((((9*(111.4531218)^2-24(111.4531218*1.94973e+13)+48(1.94973e+13)^2*(13.12806e+39)^2)))) / ((((111.4531218+1.94973e+13)^8)))+111.4531218-2Pi+(sqrt2/2)^4))+1

Input interpretation:

$$\frac{27}{4}\left(\left(9\times 111.4531218^2-24\,(111.4531218\times 1.94973\times 10^{13})+48\,(1.94973\times 10^{13})^2\,(13.12806\times 10^{39})^2\right)/(111.4531218+1.94973\times 10^{13})^8+111.4531218-2\pi+\left(\frac{\sqrt{2}}{2}\right)^4\right)+1$$

Result:

1729.07...

1729.07…

Now, from

$$(x_{max} - x_{min}) \sim m^3/\hbar$$

$$V = 4\pi l^2 \sqrt{\frac{2m}{l}} (x_{max} - x_{min}).$$

For $\hbar$ = 1.054571817 * 10^-34 and m = 13.12806e+39 , thence:

we obtain:

4*Pi*111.4531218^2*sqrt[((2*13.12806e+39)/(111.4531218))] * (((13.12806e+39)^3)) / (((1.054571817 * 10^-34)))

Input interpretation:

$$4\pi \times 111.4531218^2 \sqrt{\frac{2 \times 13.12806 \times 10^{39}}{111.4531218}} \times \frac{(13.12806 \times 10^{39})^3}{\frac{1.054571817}{10^{34}}}$$

Result:

$5.14031... \times 10^{178}$

$5.14031...*10^{178}$

(((((4*Pi*111.4531218^2*sqrt[((2*13.12806e+39)/(111.4531218))] * (((13.12806e+39)^3)) / (((1.054571817 * 10^-34))))))))^1/856

where 856 = 107*8

Input interpretation:

$$\sqrt[856]{4\pi \times 111.4531218^2 \sqrt{\frac{2 \times 13.12806 \times 10^{39}}{111.4531218}} \times \frac{(13.12806 \times 10^{39})^3}{\frac{1.054571817}{10^{34}}}}$$

Result:

1.617240169194891971046108081892324982270149177353186630958...

1.61724016919...

(((((4*Pi*111.4531218^2*sqrt[((2*13.12806e+39)/(111.4531218))] * (((13.12806e+39)^3)) / (((1.054571817 * 10^-34))))))))^1/55-34-13+(sqrt2)/2)

Input interpretation:

$$\sqrt[55]{4\pi\times 111.4531218^2\sqrt{\frac{2\times 13.12806\times 10^{39}}{111.4531218}}\times\frac{(13.12806\times 10^{39})^3}{\frac{1.054571817}{10^{34}}}}-34-13+\frac{\sqrt{2}}{2}$$

Result:

1729.085...

1729.085…

((((((((4*Pi*111.4531218^2*sqrt[((2*13.12806e+39)/(111.4531218))] * (((13.12806e+39)^3)) / (((1.054571817 * 10^-34))))))))^1/55-34-13+(sqrt2)/2)))))^1/15

Input interpretation:

$$\left(\sqrt[55]{4\pi\times 111.4531218^2\sqrt{\frac{2\times 13.12806\times 10^{39}}{111.4531218}}\times\frac{(13.12806\times 10^{39})^3}{\frac{1.054571817}{10^{34}}}}-34-13+\frac{\sqrt{2}}{2}\right)\text{^}(1/15)$$

Result:

1.6438206...

1.6438206…

Observations

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125.
In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson:

125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64 g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276 e^{-\pi\sqrt{22}} - \cdots, \\ 64 g_{22}^{-24} &= 4096 e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372 e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64 g_{22}^{-24} = 4096 e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372 e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to ζ(2) = $\frac{\pi^2}{6} = 1.644934 \ldots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

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Justin L Ripley and Frans Pretorius - Department of Physics, Princeton University, Princeton, New Jersey 08544, USA. - E-mail: jripley@princeton.edu and fpretori@princeton.edu -February 2019

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