Numbers are naturally 3 dimensional

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Abstract

Riemann hypothesis stands proved in three different ways.To prove Riemann hypothesis from the functional equation concept of Delta function and projective harmonic conjugate of both Gamma and Delta functions are introduced similar to Gamma and Pi function. Other two proofs are derived using Eulers formula and elementary algebra. Analytically continuing zeta function to an extended domain, poles and zeros of zeta values are redefined. Other prime conjectures like Goldbach conjecture, Twin prime conjecture etc.. are also proved in the light of new understanding of primes. Numbers are proved to be three dimensional as worked out by Hamilton. Logarithm of negative and complex numbers are redefined using extended number system. Factorial of negative and complex numbers are redefined using values of Delta function and projective harmonic conjugate of both Gamma and Delta functions. Numbers are proved to be cyclic following algebraic cycles.The apparent exponential growth of numbers are primarily due to absolute pythagorean distance formula. In the background of pythagoras and quaternions , Euler's unit circle is in constant rotation not only in 2D but also in 3D unit sphere via the same Euler's formula in real sense.Hodge conjecture, BSD conjecture, Navier Stokes, Yang Mills are also numerrically proved to have solutions using zeta results. Most of physics unsolved mystries are found to be solvable in the light of new understanding. The path for unification of General relativity and Quantum mechanichs are paved now following the unification of numbers which physicists may like to spell quantities.

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Contents

1 Let us begin with a short Introduction

In this section we will have a short introduction to Riemann zeta function and Riemann Hypothesis on zeta function.

1.1 Euler the great grandfather of zeta function

In 1737, Leonard Euler published a paper where he derived a tricky formula that pointed to a wonderful connection between the infinite sum of the reciprocals of all natural integers (zeta function in its simplest form) and all prime numbers.

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+...=\frac{2.3.5.7.11....}{1.2.4.6.12....}$$

Now:

$$1+\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^4...=\frac{2}{1}$$

$$1+\left(\frac{1}{3}\right)+\left(\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)^3+\left(\frac{1}{3}\right)^4...=\frac{3}{2}$$

$\vdots$

Euler product forms of zeta function when $s>1$:

$$\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^s}=\prod_p\left(1+\frac{1}{p^s}+\frac{1}{p^{2s}}+\frac{1}{p^{3s}}+\frac{1}{p^{4s}}...\right)$$

Equivalent to:

$$\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^s}=\prod_p\frac{1}{1-P^{-s}}$$

To carry out the multiplication on the right, we need to pick up exactly one term from every sum that is a factor in the product and, since every integer admits a unique prime factorization; the reciprocal of every integer will be obtained in this manner - each exactly once. This was originally the method by which Euler discovered the formula. There is a certain sieving property that we can use to our advantage:

$$\zeta(s)=1+\frac{1}{2^s}+\frac{1}{3^s}+\frac{1}{4^s}+\frac{1}{5^s}+\dots$$

$$\frac{1}{2^s}\zeta(s)=\frac{1}{2^s}+\frac{1}{4^s}+\frac{1}{6^s}+\frac{1}{8^s}+\frac{1}{10^s}+\dots$$

Subtracting the second equation from the first we remove all elements that have a factor of 2:

$$\left(1-\frac{1}{2^s}\right)\zeta(s)=1+\frac{1}{3^s}+\frac{1}{5^s}+\frac{1}{7^s}+\frac{1}{9^s}+\frac{1}{11^s}+\frac{1}{13^s}+\dots$$

Repeating for the next term:

$$\frac{1}{3^s}\left(1-\frac{1}{2^s}\right)\zeta(s)=\frac{1}{3^s}+\frac{1}{9^s}+\frac{1}{15^s}+\frac{1}{21^s}+\frac{1}{27^s}+\frac{1}{33^s}+\dots$$

Subtracting again we get:

$$\left(1-\frac{1}{3^s}\right)\left(1-\frac{1}{2^s}\right)\zeta(s)=1+\frac{1}{5^s}+\frac{1}{7^s}+\frac{1}{11^s}+\frac{1}{13^s}+\frac{1}{17^s}+\dots$$

where all elements having a factor of 3 or 2 (or both) are removed.
It can be seen that the right side is being sieved. Repeating infinitely for $\frac{1}{p^s}$ where p is prime, we get:

$$\dots\left(1-\frac{1}{11^s}\right)\left(1-\frac{1}{7^s}\right)\left(1-\frac{1}{5^s}\right)\left(1-\frac{1}{3^s}\right)\left(1-\frac{1}{2^s}\right)\zeta(s)=1$$

Dividing both sides by everything but the $\zeta(s)$ we obtain:

$$\zeta(s)=\frac{1}{\left(1-\frac{1}{2^s}\right)\left(1-\frac{1}{3^s}\right)\left(1-\frac{1}{5^s}\right)\left(1-\frac{1}{7^s}\right)\left(1-\frac{1}{11^s}\right)\dots}$$

This can be written more concisely as an infinite product over all primes p:

$$\zeta(s)=\prod_{p\text{ prime}}\frac{1}{1-p^{-s}}$$

To make this proof rigorous, we need only to observe that when $\Re(s)>1$, the sieved right-hand side approaches 1, which follows immediately from the convergence of the Dirichlet series for $\zeta(s)$.[1, 2, 3].

1.2 Riemann the grandfather of zeta function

Riemann might had seen the following relation between zeta function and eta function (also known as alternate zeta function) which converges for all values $\mathrm{Re}(s) > 0$.

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$
$$\sum_{n=1}^{\infty} \frac{2}{(2n)^s} = \frac{1}{2^{s-1}}\zeta(s)$$

Now subtracting the latter from the former we get:

$$\left(1 - \frac{1}{2^{s-1}}\right)\zeta(s) = \frac{1}{1^s} - \frac{1}{2^s} + \frac{1}{3^s} - \ldots = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n^s} =: \eta(s)$$
$$\implies \zeta(s) = \left(1 - 2^{1-s}\right)^{-1}\eta(s)$$

Then Riemann might had realised that he could analytically continue zeta function from the above equation for $1 \neq \ \mathrm{Re}\,(s) > 0$ after re-normalizing the potential problematic points. In his seminal paper Riemann showed that zeta function have the property of analytic continuation in the whole complex plane except for s=1 where the zeta function has its pole. Zeta function satisfies Riemann's functional equation.

$$\zeta(s) = 2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(1-s)\zeta(1-s)$$

Riemann Hypothesis is all about non trivial zeros of zeta function. There are trivial zeros which occur at every negative even integer. There are no zeros for $s > 1$. All other zeros lies at a critical strip $0 < s < 1$. In this critical strip he conjectured that all non trivial zeros lies on a critical line of the form of $z = \frac{1}{2} \pm iy$ i.e. the real part of all those complex numbers equals $\frac{1}{2}$. [4].

Showing that there are no zeros with real part 1 - Jacques Hadamard and Charles Jean de la Vallée-Poussin independently prove the prime number theorem which essentially says that if there exists a limit to the ratio of primes up to a given number and that numbers natural logarithm that should be equal to 1. When I started reading about number theory I wondered that if prime number theorem is proved then what is left. The biggest job is done. I questioned myself why zeta function cannot be defined at 1. Calculus has got set of rules for checking convergence of any infinite series, sometime especially when we are encapsulating infinities into unity, those rules may fall short to check the convergence of infinite series. In spite of that Euler was successful proving sum to product form and calculated zeta values for some numbers by hand only. Leopold Kronecker proved and interpreted Euler's formulas is the outcome of passing to the right-sided limit as $s \to 1^+$. I decided I will stick to Great Grandpa Eulers approach in attacking the problem.

2 Let us work out the required lemmas

In this section we will work out required math to prove Riemann Hypothesis.

2.1 Understanding Gamma function

Euler in the year 1730 proved that the following indefinite integral gives the factorial of x for all real positive numbers,

$$x! = \Pi(x) = \int_0^\infty t^x e^{-t} dt, x > 1$$

Euler's Pi function satisfies the following recurrence relation for all positive real numbers.

$$\Pi(x+1) = (x+1)\Pi(x)$$

In 1768, Euler defined Gamma function, $\Gamma(x)$, and extended the concept of factorials to all real negative numbers, except zero and negative integers. $\Gamma(x)$, is an extension of the Pi function, with its argument shifted down by 1 unit.

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$$

Euler's Gamma function is related to Pi function as follows:

$$\Gamma(x+1) = \Pi(x) = x!$$

Euler's Gamma function have the following properties:

- $\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt$ for $z \in \mathbb{C}, \Re(z) > 0$;
- Γ is analytic on $\mathbb{C} \backslash \{0,\ -1,\ -2,\ \ldots\}$;
- Γ has a simple pole with residue $\frac{(-1)^n}{n!}$ at $z = -n$ for $n = 0, 1, 2, ...$;
- $\Gamma(z+1) = z\Gamma(z)$ for $z \in \mathbb{C} \backslash \{0,\ -1,\ -2,\ \ldots\}$;
- $\Gamma(x) = (x-1)!$ for $x \in \mathbb{Z} > 0$.

2.2 Introduction of Delta function

Now let us extend factorials of negative integers by way of shifting the argument of Gamma function further down by 1 unit. Hoping that capital Delta will not be confused with Dirac's small delta function let us define capital Delta function as follows:

Lemma 1

$$\Delta(x) = \int_0^\infty t^{x-2} e^{-t} dt$$

I am not trying to somehow imprint my name in this function as neither of my names initial can be related to Delta. Still I choose Delta because in cyclic sense Epsilon will sit exactly beween Pi and Gamma in Greek Alphabet making a semi circle, but nature has a complete cycle, my following work will complete the circle, so the very next letter after Gamma should the best candidate to represent 1+ something, 1+ something squared pattern or something - 1, something squared - 1 pattern. Hope people will recognise this biased definition of mine. Let $z \in \mathbb{C}$ with $\Re(z) > 0$. Then using integration by parts,

$$\begin{aligned}
\Delta(z+2) &= \int_0^\infty t^{z+1} e^{-t} dt \\
&= \Big[-t^{z+1} e^{-t} \Big]_0^\infty + \int_0^\infty (z+1) t^{z+1-1} e^{-t} dt \\
&= \lim_{t\to\infty} (-t^{z+1} e^{-t}) - (0e^{-0}) + (z+1) \int_0^\infty t^{z+1-1} e^{-t} dt \\
&= (z+1) \int_0^\infty t^{z} e^{-t} dt \\
&= (z+1)\Delta(z+1) \\
&= (z+1) \int_0^\infty t^{z} e^{-t} dt \\
&= (z+1) \left(\Big[-t^{z} e^{-t} \Big]_0^\infty + \int_0^\infty z t^{z-1} e^{-t} dt \right) \\
&= (z+1) \left(\lim_{t\to\infty} (-t^{z} e^{-t}) - (0e^{-0}) + z \int_0^\infty t^{z-1} e^{-t} dt \right) \\
&= (z+1) z \int_0^\infty t^{z-1} e^{-t} dt \\
&= (z+1) z \Delta(z)
\end{aligned}$$

Similar to Gamma function Delta function will then have the following properties:

- $\Delta(z) = \int_0^\infty e^{-t} t^{z-2} dt$ for $z \in \mathbb{C}, \Re(z) > 0$;

- Δ is analytic on $\mathbb{C}\backslash\{-1,\ -2,\ ...\}$;
- Δ has a simple pole with residue $\frac{(-1)^n}{n!}$ at $z = -n$ for $n = 1, 2, ...$;
- $\Delta(z+1) = z\Delta(z)$ for $z \in \mathbb{C}\backslash\{-1,\ -2,\ ...\}$;
- $\Delta(x) = (x-2)!$ for $x \in \mathbb{Z} \geq 0$.

The extended Delta function shall have the following recurrence relation.

$$\Delta(x+2) = (x+2)\Delta(x+1) = (x+2)(x+1)\Delta(x) = x!$$

2.3 Removing Poles of Gamma and Pi function

Newly defined Delta function is related to Euler's Gamma function and Pi function as follows:

Lemma 2

$$\Delta(x+2) = \Gamma(x+1) = \Pi(x)$$

To make it simple we will be taking examples on integers but we will not forget the same may be true in case other numbers such as fractions, complex numbers etc. within the extended domain. Whereever necessary we will use that extended domain without recalling. Plugging into $x = 2$ above

$$\Delta(4) = \Gamma(3) = \Pi(2) = 2$$

Plugging into $x = 1$ above

$$\Delta(3) = \Gamma(2) = \Pi(1) = 1$$

Plugging into $x = 0$ above

$$\Delta(2) = \Gamma(1) = \Pi(0) = 1$$

Now these three factorial functions are behaving like triangle of harmonic functions based on the fact they all are analytic around -1. Gamma is playing the role of the central function whereas Pi and Delta are acting like its harmonic conjugates. When we look deeper we will see following the definition of Delta function zeta function is harmonised first if not simultaneously (see the proof of Riemann hypothesis) which in turn harmonises these factorial functions.$\Gamma(z) = u + iv$ analytic means u and v which can be expressed as variants of zeta series must be harmonic conjugates and so are u and -v because $i\Gamma(z) = -v + iu$. The same kind of recursive relation is observed above, therefore harmonicity among Gamma, Pi and Delta functions are established based on the harmonicity of zeta series. The real and imaginary parts of $\Gamma(z) = \frac{1}{z}$ must be harmonic away from the origin given the fact $\Gamma(z)$ is defined on zero now.Since the real and imaginary parts of $\Gamma(z) = \frac{1}{z}$ are harmonic, the same must be true of the respective integrals, *which is limit of linear combinations of such functions. Since the circle is complete and it is finite and Gamma, Pi and Delta functions trio are continuous, interchanging the order of integration is not a problem now. We can bring two more harmonic function centered around Delta function into the picture to unify the symmetries among these fantastic five factorial functions as follows. Plugging into* $x = -1$ *above we can remove poles of Gamma and Pi function as follows:*

$$\Delta(1) = \Gamma(0) = \Pi(-1) = 1.\Delta(0) = -1.\Delta(-1) = \int_{-\infty}^{0} t^{1-1} e^t dt = \Big[e^x\Big]_{-\infty}^{0} \quad \text{Refer:2.4}$$

$$\implies \Delta(1) = \Gamma(0) = \Pi(-1) = 1.\Delta(0) = -1.\Delta(-1) = (e^0) - \lim_{x \to -\infty} e^x = 1$$

$$\implies \Delta(-1) = \Gamma(-2) = \Pi(-3) = -1 = \Delta(-2) = \Gamma(-3) = \Pi(-4)$$

Plugging into $x = -2$ *above we can remove poles of Gamma and Pi function as follows:*

$$\Delta(0) = \Gamma(-1) = \Pi(-2) = -1.\Delta(-1) = -2.\Delta(-2) = \int_{-\infty}^{0} t^{2-2} e^t dt = \Big[e^x\Big]_{-\infty}^{0} \quad \text{Refer:2.5}$$

$$\implies \Delta(0) = \Gamma(-1) = \Pi(-2) = -1.\Delta(-1) = -2.\Delta(-2) = (e^0) - \lim_{x \to -\infty} e^x = 1$$

$$\implies \Delta(-2) = \Gamma(-3) = \Pi(-4) = -1 = \Delta(-3) = \Gamma(-4) = \Pi(-5)$$

Continuing further we can remove poles of Gamma and Pi function as follows:

Plugging into $x = -3$ *above and equating with result found above*

$$\Delta(-1) = \Gamma(-2) = \Pi(-3) = -2. - 1.\Delta(-3) = -1 \implies \Delta(-3) = \Gamma(-4) = \Pi(-5) = -\frac{1}{2}$$

Plugging into $x = -4$ *above and equating with result found above*

$$\Delta(-2) = \Gamma(-3) = \Pi(-4) = -3. - 2.\Delta(-4) = -\frac{1}{2} \implies \Delta(-4) = \Gamma(-5) = \Pi(-6) = -\frac{1}{12}$$

Plugging into $x = -5$ *above and equating with result found above*

$$\Delta(-3) = \Gamma(-4) = \Pi(-5) = -4. - 3.\Delta(-5) = -\frac{1}{2} \implies \Delta(-5) = \Gamma(-6) = \Pi(-7) = -\frac{1}{24}$$

Plugging into $x = -6$ *above and equating with result found above*

$$\Delta(-4) = \Gamma(-5) = \Pi(-6) = -5. -4.\Delta(-6) = -\frac{1}{12} \implies \Delta(-6) = \Gamma(-7) = \Pi(-8) = -\frac{1}{240}$$

Plugging into $x = -7$ above and equating with result found above

$$\Delta(-5) = \Gamma(-6) = \Pi(-7) = -6.-5.\Delta(-7) = -\frac{1}{24} \implies \Delta(-7) = \Gamma(-8) = \Pi(-9) = -\frac{1}{720}$$

Plugging into $x = -8$ above and equating with result found above

$$\Delta(-6) = \Gamma(-7) = \Pi(-8) = -7.-6.\Delta(-8) = -\frac{1}{240} \implies \Delta(-8) = \Gamma(-9) = \Pi(-10) = -\frac{1}{10080}$$

$\vdots$

And the pattern continues up to negative infinity.

2.4 Projective harmonic conjugate of Gamma function

Lemma 3 *We can define projective Gamma function as harmonic conjugate of capital Gamma function as follows:*

$$\Gamma_p(x) = \int_{-\infty}^{0} t^{x+1} e^t dt$$

Let $z \in \mathbb{C}$ with $\Re(z) < 0$. Then using integration by parts,

$$\begin{aligned}
\Gamma_p(z-1) &= \int_{-\infty}^{0} t^z e^t dt \\
&= \Big[t^z e^t\Big]_{-\infty}^{0} - \int_{-\infty}^{0} z t^{z-1} e^t dt \\
&= (0e^0) - \lim_{t \to -\infty} (t^z e^t) + z \int_{0}^{\infty} t^{z-1} e^t dt \\
&= z \int_{0}^{\infty} t^{z-1} e^t dt \\
&= z\Gamma(z) = \Gamma(z+1) \\
&= -z\Gamma_p(z-2)
\end{aligned}$$

Similar to Gamma function projective Gamma function will then have the following properties:

- $\Gamma_p(z) = \int_{-\infty}^{0} e^t t^{z+1} dt$ for $z \in \mathbb{C}, \Re(z) < 0$;
- Γ_p is analytic on $\mathbb{C} \backslash \{0,\ +1,\ +2,\ .\ .\ .\}$;
- Γ_p has a simple pole with residue $\frac{1}{n!}$ at $z = n$ for $n = 0, 1, 2, ...$;
- $\Gamma_p(z+1) = z\Gamma_p(z)$ for $z \in \mathbb{C} \backslash \{0,\ +1,\ +2,\ .\ .\ .\}$;

The projective Gamma function shall have the following recurrence relation.

$$\Gamma_p(x+1) = x\Gamma_p(x)$$

2.5 Projective harmonic conjugate of Delta function

Lemma 4 *We can define projective Delta function as harmonic conjugate of capital Delta function as follows:*

$$\Delta_p(x) = \int_{-\infty}^{0} t^{x+2} e^t dt$$

Let $z \in \mathbb{C}$ with $\Re(z) < 0$. Then using integration by parts,

$$\begin{aligned}
\Delta_p(z-2) &= \int_{-\infty}^{0} t^{z+1} e^t dt \\
&= \left[t^{z+1} e^t\right]_{-\infty}^{0} - \int_{-\infty}^{0} (z+1) t^{z+1-1} e^t dt \\
&= (0e^0) - \lim_{t \to -\infty} (t^{(z+1)} e^t) + (z+1) \int_{0}^{\infty} t^z e^t dt \\
&= (z+1) \int_{0}^{\infty} t^z e^t dt \\
&= (z+1)\Gamma(z+1) = \Gamma(z+2) \\
&= -(z+1)\Delta_p(z-1) \\
&= -z(z+1)\Delta_p(z-2)
\end{aligned}$$

Similar to Gamma function projective Delta function will then have the following properties:

- $\Delta_p(z) = \int_{-\infty}^{0} e^t t^{z+1} dt$ for $z \in \mathbb{C}, \Re(z) < 0$;
- Δ_p is analytic on $\mathbb{C} \backslash \{\ +1,\ +2,\ +3,\ .\ .\ .\}$;
- Δ_p has a simple pole with residue $\frac{1}{n!}$ at $z = n$ for $n = 1, 2, 3, ...$;
- $\Delta_p(z+1) = z\Delta_p(z)$ for $z \in \mathbb{C} \backslash \{\ +1,\ +2,\ +3,\ .\ .\ .\}$;

The projective Delta function shall have the following recurrence relation.

$$\Delta_p(x+2) = x\Delta_p(x+1)$$

2.6 Closure of factorial operation

Now these fantastic five factorial functions shall have the following inter functional relationship.

$$\Delta(x+2) = \Gamma(x+1) = \Pi(x) = \Gamma_p(x-1) = \Delta_p(x-3)$$

Lemma 5 *We can use fantastic five factorial functions for closure of factorials. For negative even numbers the formulas are as follows. For odd numbers we just need to multiply or divide the result by the corresponding succeeding or preceding odd number.*

$$\frac{-1}{\Delta(-3)} = \frac{-1}{\Gamma(-4)} = \frac{-1}{\Pi(-5)} = \frac{-1}{-\Gamma_p(-6)} = \frac{-1}{-\Delta_p(-8)} = -2! = 2$$

$$\frac{-1}{\Delta(-5)} = \frac{-1}{\Gamma(-6)} = \frac{-1}{\Pi(-7)} = \frac{-1}{-\Gamma_p(-8)} = \frac{-1}{-\Delta_p(-10)} = -4! = 12$$

$$\frac{-1}{\Delta(-7)} = \frac{-1}{\Gamma(-8)} = \frac{-1}{\Pi(-9)} = \frac{-1}{-\Gamma_p(-10)} = \frac{-1}{-\Delta_p(-12)} = -6! = 720$$

$$\frac{-1}{\Delta(-9)} = \frac{-1}{\Gamma(-10)} = \frac{-1}{\Pi(-11)} = \frac{-1}{-\Gamma_p(-12)} = \frac{-1}{-\Delta_p(-14)} = -8! = 40320$$

$$\frac{-1}{\Delta(-11)} = \frac{-1}{\Gamma(-12)} = \frac{-1}{\Pi(-13)} = \frac{-1}{-\Gamma_p(-14)} = \frac{-1}{-\Delta_p(-16)} = -10! = 3628800$$

$$\frac{-1}{\Delta(-13)} = \frac{-1}{\Gamma(-14)} = \frac{-1}{\Pi(-15)} = \frac{-1}{-\Gamma_p(-16)} = \frac{-1}{-\Delta_p(-18)} = -12! = 479001600$$

$$\frac{-1}{\Delta(-15)} = \frac{-1}{\Gamma(-16)} = \frac{-1}{\Pi(-17)} = \frac{-1}{-\Gamma_p(-18)} = \frac{-1}{-\Delta_p(-20)} = -14! = 87178291200$$

And the pattern continues up to negative infinity.
We can evaluate factorial of all complex argument $z! = (x+iy)! = \Delta(x+iy+2) = \Gamma(x+iy+1)$

Example 1

$$\Delta(2+i) = \Gamma(1+i) = i! = i\Gamma(i) \approx 0.498 - 0.155i$$

$$\Delta(1+i) = \Gamma(i) = (i-1)! = (i-1).i! \approx -0.343 + 0.653i$$

$$\Delta(2-i) = \Gamma(1-i) = -i! = -i\Gamma(-i) \approx 0.498 + 0.155i$$

$$\Delta(1-i) = \Gamma(-i) = (-i-1)! = (-i-1).-i! \approx -0.343 - 0.653i$$

2.7 Harmonic zeta function and alternate functional equation

Multiplying both side of Riemann's functional equation by $(1-s)$ we get

$$(1-s)\zeta(s) = 2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)(1-s)\Gamma(1-s)\zeta(1-s)$$

Putting $(1-s)\Gamma(1-s) = \Gamma(2-s)$ we get:

$$(1-s)\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Gamma(2-s)\zeta(1-s)$$

$s \to 1$we get: $\because \lim_{s\to 1}(s-1)\zeta(s) = 1 \therefore (1-s)\zeta(s) = -1$

$$\boxed{2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Gamma(2-s)\zeta(1-s) = -1}$$

Similarly multiplying both numerator and denominator right hand side of Riemann's functional equation by $(1-s)(2-s)$ before applying any limit we get :

$$\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\frac{(1-s)(2-s)\Gamma(1-s)\zeta(1-s)}{(1-s)(2-s)}$$

Putting $(1-s)(2-s)\Gamma(1-s) = \Gamma(3-s)$ we get:

$$\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\frac{\Gamma(3-s)\zeta(1-s)}{(1-s)(2-s)}$$

Multiplying both side of the above equation by $(1-s)$ we get

$$(1-s)\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\frac{\Gamma(3-s)\zeta(1-s)}{(2-s)}$$

$s \to 1$we get: $\because \lim_{s\to 1}(s-1)\zeta(s) = 1 \therefore (1-s)\zeta(s) = -1$

$$-1 = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\frac{\Gamma(3-s)\zeta(1-s)}{(2-s)}$$

Multiplying both side of the above equation further by $(2-s)$ we get:

$$(s-2) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s)$$

Multiplying both side of the above equation by $\zeta(s-1)$ we get

$$(s-2)\zeta(s-1) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s)\zeta(s-1)$$

$s \to 2$ we get: $\because \lim_{s\to 2}(s-2)\zeta(s-1) = 1$

$$2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s)\zeta(s-1) = 1$$

Which can also be written as:

$$2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s) = \frac{1}{\zeta(s-1)}$$

The above equation is consistent at $s = 1$ following analytical continuation of Riemann's functional functional equation at $\zeta(1-s) = \zeta(s-1) = \zeta(0) = \frac{-1}{2}$. Following harmonic conjugate theorem the above consistency make it a harmonic conjugate function around the pole of zeta function.The real and imaginary parts of $\zeta(z) = \frac{1}{z}$ around a unit disc centered at at 1 must be harmonic away from the center given the fact $\zeta(1)$ is analytic now. The above analytic function will not have any zeros as proven by Jacques Hadamard and Charles Jean de la Vallée-Poussin. Therefore reciprocal of the above function will be an entire function. But overall we see zeta function have got zeros. That is possible only another harmonic away from the center. Due to the fact that harmonic conjugate appears in pairs the domain of unit disk area gets bifurcated. Since the real and imaginary parts of $\zeta(z) = \frac{1}{z}$ are harmonic, the same must be true of the respective functional equations. Since the circle is complete and it is finite changing the sign of the equation multiplying by -1 is not a problem. So we can set $\frac{1}{\zeta(s-1)} = 1$ due to its own inverse and we can rewrite the above equation multiplying by -1 as follows:

$$\boxed{-2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s) = -1}$$

Both the above boxed forms are equivalent to Riemann's original functional equation therefore Riemann's original functional equation can be analytically continued further which will lead us to zeros of zeta function as follows:

Lemma 6

$$\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Delta(4-s)\zeta(1-s)$$

This can be rewritten in terms of Gamma function as follows:

$$\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s)$$

This again can be rewritten in terms of Pi function as follows:

$$\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Pi(2-s)\zeta(1-s)$$

To manually define zeta function such a way that it takes value 1 or mathematically $\exists! s \in \mathbb{N}; \zeta(s-1) = 1$, Euler's induction approach was applied and it was observed that zeta function have the potential unit value as demonstrated in the results section. Justification of the definition we set for $\zeta(3-2) = 1$ and consistency of the above forms of functional equation have been cross checked and it was found that the proposition complies with all the theorems used in complex analysis. Justification of the definition we set for $\zeta(-1) = \frac{1}{2}$ and consistency of the above forms of functional equation have been cross checked in the section 4.2. $\zeta(-1) = \frac{1}{2}$ must be the second solution to $\zeta(-1)$ apart from the known Ramanujan's proof $\zeta(-1) = \frac{-1}{12}$. One has to accept that following the zeta functions analytic and its harmonic conjugal behaviour zeta values can be multivalued if seen as a continuam or alternatively it can be seen as a multi zeta function.

3 Let us prove Riemann Hypothesis

In this section we will prove Riemann Hypothesis.

3.1 An elegant proof using Euler's product form

Euler's Product form of zeta Function in Euler's exponential form of complex numbers is as follows:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_p \left(1 + re^{i\theta} + r^2e^{i2\theta} + r^3e^{i3\theta}...\right)$$

Now any such factor $\left(1 + re^{i\theta} + r^2e^{i2\theta} + r^3e^{i3\theta}...\right)$ will be zero if

$$\left(re^{i\theta} + r^2e^{i2\theta} + r^3e^{i3\theta}...\right) = -1 = e^{i\pi}$$

Comparing both side of the equation and equating left side to right side on the unit circle we can say: *

$$\boxed{\theta + 2\theta + 3\theta + 4\theta... = \pi}$$

$$\boxed{r + r^2 + r^3 + r^4.... = 1}$$

We can solve θ and r as follows:

$$\begin{aligned} \theta + 2\theta + 3\theta + 4\theta ... &= \pi \\ \theta(1+2+3+4...) &= \pi \\ \theta.\zeta(-1) &= \pi \\ \theta.\frac{-1}{12} &= \pi \\ \theta &= -12\pi \end{aligned} \qquad \begin{aligned} r + r^2 + r^3 + r^4 .... &= 1 \\ r(1 + r + r^2 + r^3 + r^4 ....) &= 1 \\ r\frac{1}{1-r} &= 1 \\ r &= 1 - r \\ r &= \frac{1}{2} \end{aligned}$$

We can determine the real part of the non trivial zeros of zeta function as follows:

$$\boxed{r\cos\theta = \frac{1}{2}\cos(-12\pi) = \frac{1}{2}}$$

Therefore Principal value of $\zeta(\frac{1}{2})$ will be zero, hence Riemann Hypothesis is proved.

Explanation 1 **We can try back the trigonometric form then the algebraic form of complex numbers do the summation algebraically and then come back to exponential form as follows:*

$$\begin{aligned} & re^{i\theta} + r^2e^{i2\theta} + r^3e^{i3\theta} ... \\ &= (r\cos\theta + ir\sin\theta) + (r^2\cos 2\theta + ir^2\sin 2\theta) + (r^3\cos 3\theta + ir^3\sin 3\theta).... \\ &= (x_1 + iy_1) + (x_2 + iy_2) + (x_3 + iy_3) + (x_4 + iy_4) + (x_5 + iy_5).... \\ &= (x_1 + x_2 + x_3 + x_4 + x_5 + ...) + i(y_1 + y_2 + y_3 + y_4 + y_5 + ...) \\ &= R\cos\Theta + iR\sin\Theta \\ &= (r + r^2 + r^3 + r^4 ....)e^{i(\theta+2\theta+3\theta+4\theta...)} \end{aligned}$$

Explanation 2 *One may attempt to show that* $(re^{i\theta} + r^2e^{i2\theta} + r^3e^{i3\theta}...) = -1$ *actually results* $\frac{re^{i\theta}}{1-re^{i\theta}}$ *which implies in absurdity of* $0 = -1$*. Correct way to evaluate* $\frac{re^{i\theta}}{1-re^{i\theta}}$ *is to apply the complex conjugate of denominator before reaching any conclusion.* $\frac{re^{i\theta}(1+re^{i\theta})}{(1-re^{i\theta})(1+re^{i\theta})}$ *then shall result to* $re^{i\theta} = -1$ *which points towards the unit circle. In the present proof we need to go deeper into the d-unit circle and come up with the interpretation which can explain the Riemann Hypothesis.*

Explanation 3 *One may attempt to show inequality of the reverse calculation* $\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3}... = 1 \neq -1$. $re^{i\pi} = -1$ *need to be interpreted as the exponent which then satisfies* $1^{-1} = 1$ *or* $2.2^{-1} = 1$ *on the unit or d-unit circle. There is nothing called t-unit circle satisfying* $3.3^{-1} = 1$.

Explanation 4 *Essentially proving* $\log_2(\frac{1}{2}) = -1$ *in a complex turned simple way is equivalent of saying* $\log(1) = 0$ *in real way. Primes other than 2 satisfy* $\log_p(\frac{1}{2}) = e^{i\theta}$ *also in a pure complex way.*

3.2 An elementary proof using alternate product form

Euler's alternate Product form of zeta Function in Euler's exponential form of complex numbers is as follows:

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_p \left(\frac{1}{1 - \frac{1}{re^{i\theta}}} \right) = \prod_p \left(\frac{re^{i\theta}}{re^{i\theta} - 1} \right)$$

Multiplying both numerator and denominator by $re^{i\theta} + 1$we get:

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_p \left(\frac{re^{i\theta}(re^{i\theta} + 1)}{(re^{i\theta} - 1)(re^{i\theta} + 1)} \right)$$

Now any such factor $\left(\frac{re^{i\theta}(re^{i\theta}+1)}{(r^2e^{i2\theta}-1)} \right)$ will be zero if $re^{i\theta}(re^{i\theta} + 1)$ is zero:

$$\boxed{\begin{aligned} re^{i\theta}(re^{i\theta} + 1) &= 0 \\ re^{i\theta}(re^{i\theta} - e^{i\pi}) &= 0 \\ r^2e^{i2\theta} - re^{i(\pi-\theta)*} &= 0 \\ r^2e^{i2\theta} &= re^{i(\pi-\theta)} \end{aligned}}$$

We can solve θ and r as follows:

$$\boxed{\begin{aligned} 2\theta &= (\pi - \theta) & r^2 &= r \\ 3\theta &= \pi & \frac{r^2}{r} &= \frac{r}{r} \\ \theta &= \frac{\pi}{3} & r &= 1 \end{aligned}}$$

We can determine the real part of the non trivial zeros of zeta function as follows:

$$\boxed{r\cos\theta = 1.\cos(\tfrac{\pi}{3}) = \tfrac{1}{2}}$$

Therefore Principal value of $\zeta(\frac{1}{2})$ will be zero, and Riemann Hypothesis is proved.

Explanation 5 ** $e^{i(-\theta)}$ is arrived as follows:*

$$e^{i\theta} = \left(e^{i\theta}\right)^1 = \left(e^{i\theta}\right)^{1^{-1}} = \left(e^{i\theta}\right)^{-1^1} = \left(\left(e^{i\theta}\right)^{i^2}\right)^1 = \left(e^{i\theta}\right)^{i^2} = e^{-i\theta}$$

3.3 An exhaustive proof using harmonic zeta function

Multiplying both side of Riemann's functional equation by $(1-s)$ we get

$$(1-s)\zeta(s) = 2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)(1-s)\Gamma(1-s)\zeta(1-s)$$

Putting $(1-s)\Gamma(1-s) = \Gamma(2-s)$ we get:

$$\zeta(1-s) = \frac{(1-s)\zeta(s)}{2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(2-s)}$$

$s \to 1$we get: $\because \lim_{s\to 1}(s-1)\zeta(s) = 1 \therefore (1-s)\zeta(s) = -1$ and $\Gamma(2-1) = \Gamma(1) = 1$

$$\zeta(0) = \frac{-1}{2^1\pi^0\sin\left(\frac{\pi}{2}\right)} = -\frac{1}{2}$$

Examining the functional equation we shall observe that the pole of zeta function at $Re(s) = 1$ is attributable to the pole of Gamma function. In the critical strip $0 < s < 1$ Delta function holds equally good if not better for factorial function. As zeta function has got the holomorphic property the act of stretching or squeezing preserves the holomorphic character. Using this property we can remove the pole of zeta function. Introducing Delta function for factorial we can remove the poles of Gamma and Pi function and rewrite the functional equation in terms of its harmonic conjugate function as follows (see above):

$$\zeta(s) = -2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Delta(4-s)\zeta(1-s)$$

This can be rewritten in terms of Gamma function as follows:

$$\zeta(s) = -2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s)$$

This again can be rewritten in terms of Pi function as follows:

$$\zeta(s) = -2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Pi(2-s)\zeta(1-s)$$

Now Putting $s = 1$we get:

$$\zeta(1) = -2^1\pi^{(1-1)}\sin\left(\frac{\pi}{2}\right)\Gamma(3-1)\zeta(0) = 1$$

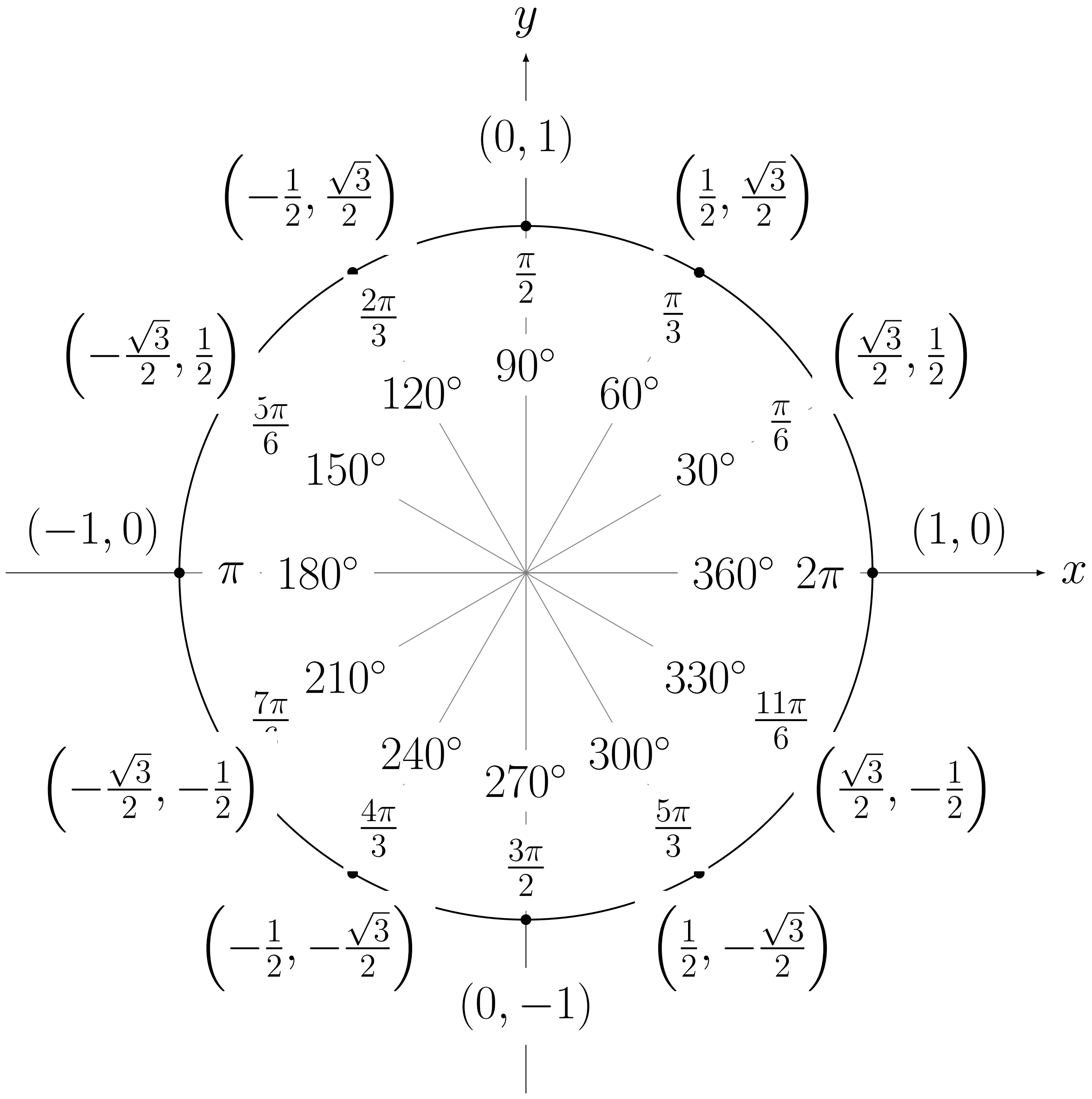
zeta function is now defined on entire $\mathbb{C}$, and as such it becomes an entire function. In complex analysis, Liouville's theorem states that every bounded entire function must be constant. That is, every holomorphic function f for which there exists a positive number M such that $|f(z)| \leq M$ for all z in $\mathbb{C}$ is constant. Being an entire function zeta function is constant as none of the values of zeta function do not exceed $M = \zeta(2) = \frac{\pi^2}{6}$.Maximum modulus principle further requires that non constant holomorphic functions attain maximum modulus on the boundary of the unit circle. Being a constant function zeta function duly complies with maximum modulus principle as it reaches maximum modulus $\frac{\pi^2}{6}$ outside the unit circle i.e. on the boundary of the double unit circle. Gauss's mean value theorem requires that in case a function is bounded in some neighbourhood, then its mean value shall occur at the centre of the unit circle drawn on the neighbourhood. $|\zeta(0)| = \frac{1}{2}$ is the mean modulus of entire zeta function. Inverse of maximum modulus principle implies points on half unit circle give the minimum modulus or zeros of zeta function. Minimum modulus principle requires holomorphic functions having all non zero values shall attain minimum modulus on the boundary of the unit circle. Having lots of zero values holomorphic zeta function do not attain minimum modulus on the boundary of the unit circle rather points on half unit circle gives the minimum modulus or zeros of zeta function. Everything put together it implies that points on the half unit circle will mostly be the zeros of the zeta function which all have $\pm\frac{1}{2}$ as real part as Riemann rightly hypothesized.

Putting $s = \frac{1}{2}$ in $\zeta(s) = -2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s)$

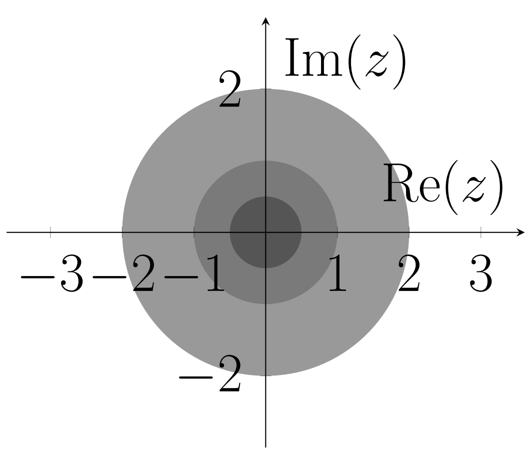
$$\boxed{\begin{aligned}
\zeta\left(\frac{1}{2}\right) &= -2^{\frac{1}{2}}\pi^{(1-\frac{1}{2})}\sin\left(\frac{\pi}{2.2}\right)\Gamma\left(\frac{5}{2}\right)\zeta\left(\frac{1}{2}\right) \\
\zeta\left(\frac{1}{2}\right)&\left(1+\frac{3\sqrt{2.\pi.\pi}}{4.\sqrt{2}}\right) = 0 \\
\zeta\left(\frac{1}{2}\right)&\left(1+\frac{3\pi}{4}\right) = 0 \\
\zeta\left(\frac{1}{2}\right) &= 0
\end{aligned}}$$

Therefore principal value of $\zeta(\frac{1}{2})$ is zero and Riemann Hypothesis holds good.

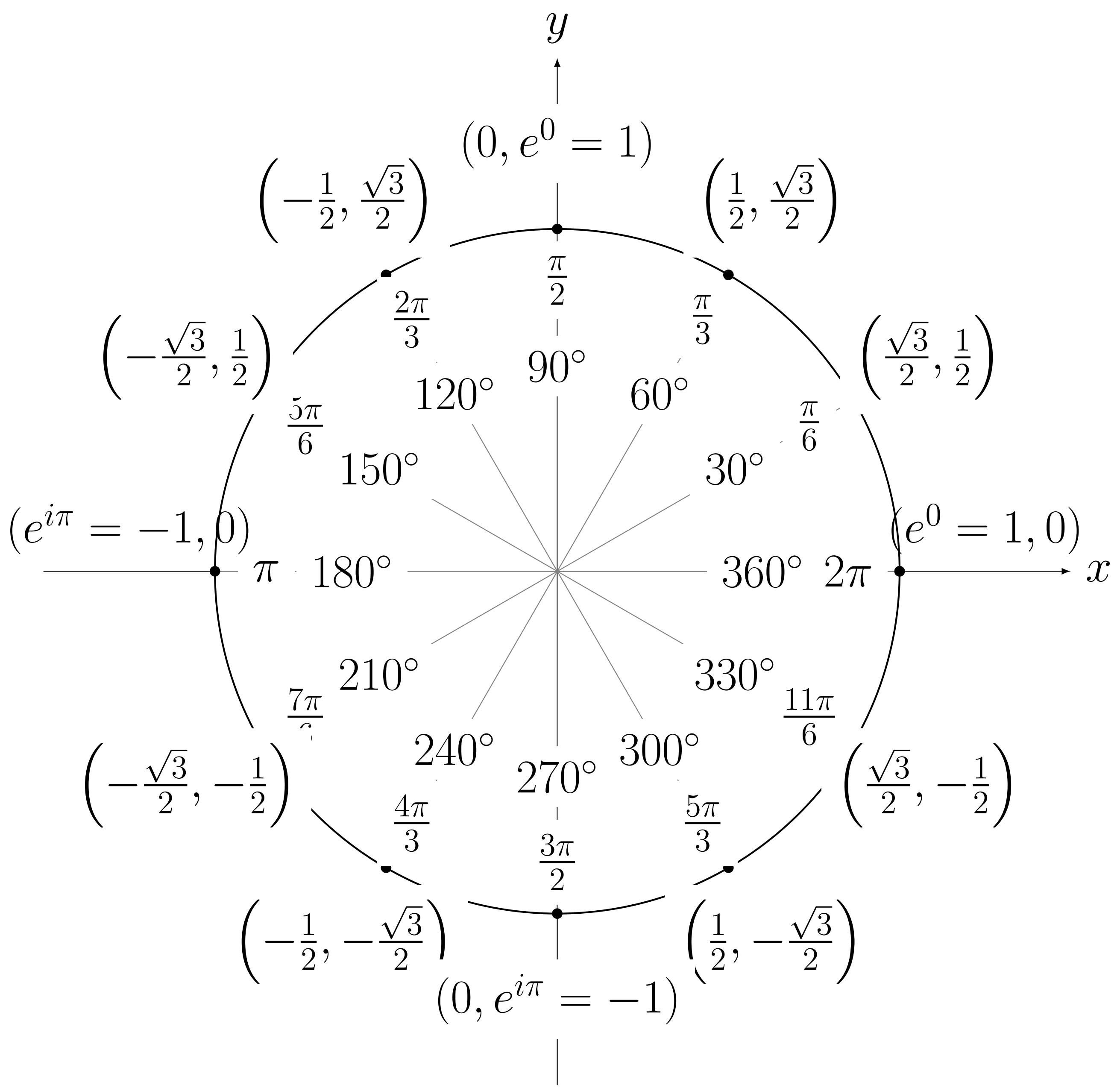
3.4 The unit circle, the unit sphere revisited



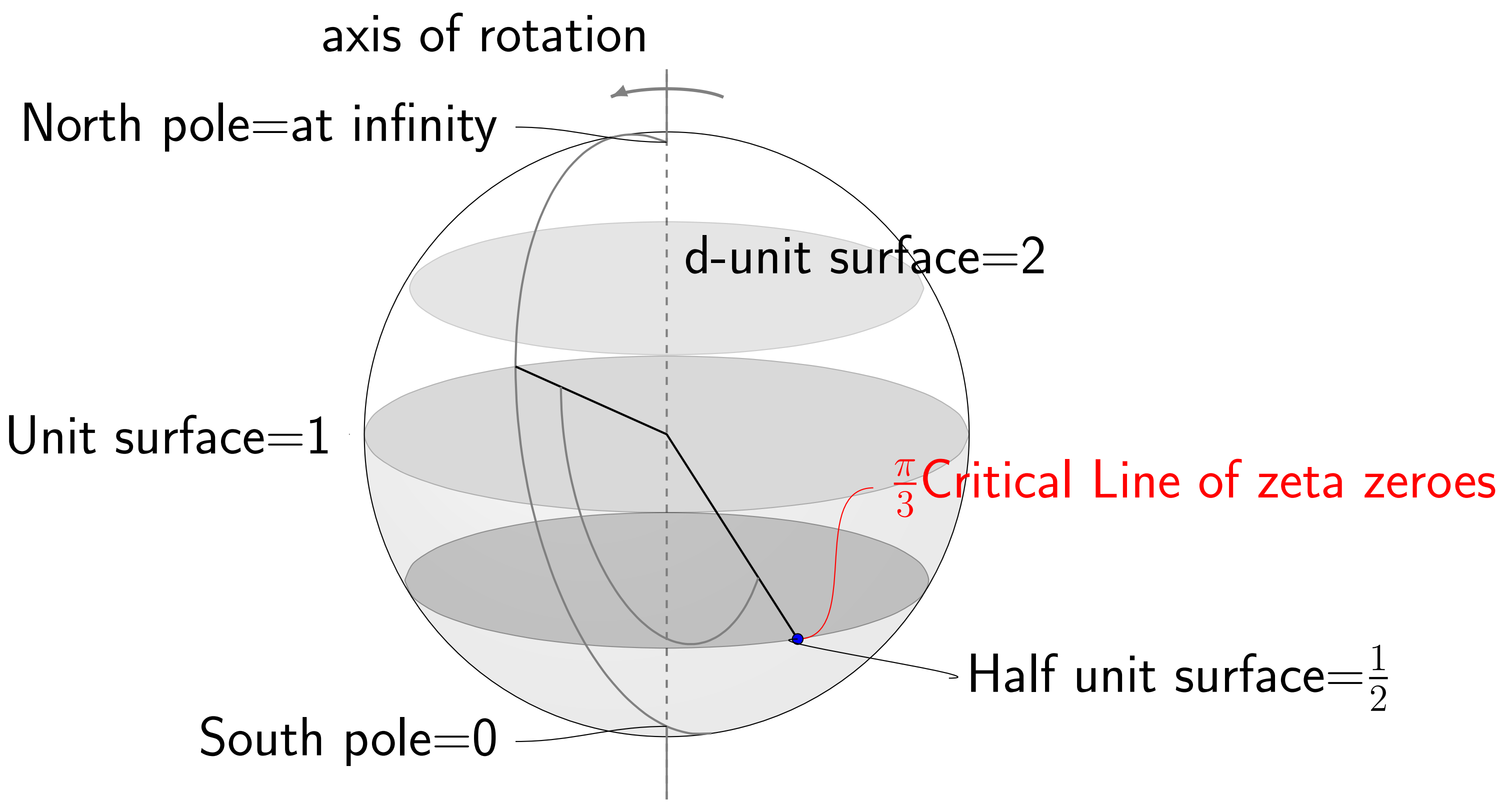
If we closely observe the above trigonometric unit circle we will see a pattern. After every third division of angle pi, value of the trigonometric ratios are same in absolute sense. So there are smaller cycles inside the unit circle. Concept of trigonometric unit circle is not the only unit circle we know. Our knowledge of mathematics is not restricted to trigonometry alone. We have got arithmetic, algebra. Four basic operation of arithmetic i.e, addition, subtraction, multiplication and division also exhibits the same cycle of 3 as three combination of those four operation on numbers can always result zero and unity. We have got algebraic functions such as polynomial, exponential, logarithmic functions etc. which also exhibits the same cycle of 3 as three appropriate combination of those functions obeys two fundamental laws of algebra i.e. the law of additive inverse and the law of multiplicative inverse.



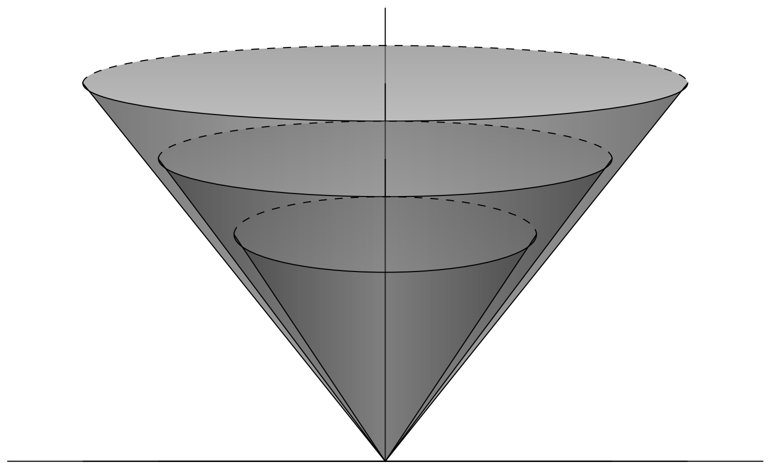
$z = r(\cos x + i \sin x)$ is the trigonometric form of complex numbers. Using Euler's formula $e^{ix} = \cos x + i \sin x$ we can write $z = re^{ix}$. Putting $x = \pi$ in Euler's formula we get , $e^{i\pi} = -1$.Putting $x = \frac{\pi}{2}$ we get $e^{\frac{i\pi}{2}} = i$. So the general equation of the points lying on unit circle $|z| = |e^{ix}| = 1$. But that's not all. If $x = \frac{\pi}{3}$ in trigonometric form then $z = cos(\frac{\pi}{3}) + i.sin(\frac{\pi}{3}) = \frac{1}{2}(\sqrt{3} + i)$.So $|z| = r = \sqrt{(\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2} = \frac{1}{2}.\sqrt{4} = \frac{1}{2}.2 = 1$.So another equation of the points lying on unit circle $|\mathbf{z}| = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{e^{ix}} = \mathbf{1}$. Although both the equation are of unit circle, usefulness of $|\mathbf{z}| = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{e^{ix}} = \mathbf{1}$ is greater than $|z| = |e^{ix}| = 1$ as $|\mathbf{z}| = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{e^{ix}} = \mathbf{1}$ bifurcates mathematical singularity and introduces unavoidable mathematical duality particularly in studies of primes and zeta function. $|\mathbf{z}| = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{e^{ix}} = \mathbf{1}$ can be regarded as d-unit circle. The d-unit circle together with the general unit circle completes the algebraic cycle via $|\mathbf{z}| = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{e^{ix}} = \mathbf{1}$. Therefore taking the imaginary number i commoned out as a constant scale factor we can visulise the imaginary axis maintaining the rotational symmetries arising out of cycle of 3 which then causes another cycle along the real axis and the dynamics keep on cycling. Five lattice points 1, i, -1, -i, 0 in [1+ i - 1 - i = 0] works as mid value to our good old decimal number system. After all the concept of adding place hoder zero to any number is just replication of completing the circle geometrically.



When Unit circle in complex plane is stereo-graphically projected to unit sphere the points within the area of unit circle gets mapped to southern hemisphere, the points on the unit circle gets mapped to equatorial plane, the points outside the unit circle gets mapped to northern hemisphere. d-unit circle can also be easily projected to Riemann sphere. Projection of d-unit circle to d-unit sphere will have three parallel disc (like three dimensions hidden in one single dimension of numbers) for three (equivalent unit values in three different sense) magnitude of $\frac{1}{2}, 1, 2$ in the southern hemisphere, on the equator, in the northern equatorial sphere respectively as shown in the following diagram. The polar sphere will be projecting points outside the d-unit circle.



Three parallel surfaces in a single cone (the one way absolute view) will look like as follows.



Explanation 6 *One may attempt to show that* $|\mathbf{z}| = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{e}^{\mathbf{ix}} = \mathbf{1}$ *will mean 1= 2. This may not be right interpretation. Correct way to interpret is given here under.*

We know: $e^{ix} = r(\cos\theta + i\sin\theta)$*. Taking derivative both side we get*

$$ie^{ix} = (\cos\theta + i\sin\theta)\frac{dr}{dx} + r(-\sin\theta + i\cos\theta)\frac{d\theta}{dx}.$$

Now Substituting $r(\cos\theta + i\sin\theta)$ *for* e^{ix} *and equating real and imaginary parts in this formula gives* $\frac{dr}{dx} = 0$ *and* $\frac{d\theta}{dx} = 1$*. Thus,* r *is a constant, and* θ *is* $x + C$ *for some constant* C*. Now if we assign* $r = \frac{1}{2}$ *and* $ix = \ln 2$ *then* $re^{ix} = \frac{1}{2}.e^{\ln 2} = 1$ *The initial value x=1 then gives* $i = \ln 2$*. That means in 4D the imaginary number* i *turns into a complete real number* $\ln 2$ *in logarithmic way. This proves the formula* $|\mathbf{z}| = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{e}^{\mathbf{ix}} = \mathbf{1}$*.Thus we see* $ix = \ln(\cos\theta + i\sin\theta)$ *is a multivalued function not only because of infinite rotation around* *the unit circle but also due to different real solutions to i in higher dimensional number system of quaternion discovered by Hamilton involving higher dimensional algebra like Clifford algebra, or at least involving 3 dimensional vector algebra connected to four dimensional zeros through algebraic cycles proposed by Hodge, Tate, Weil and others existence of which are not proven algebraically, numerically they actually exists, using zeta results I have outlined the minimal proof, someone has to work on the rigorous proof. Now coming back to the explanation, when we completed the circle of [1+ i - 1 - i = 0] we actually completed two semi circle [1 - 1 = 0] following* $e^{i\pi} + 1 = 0$. *Thus we cannot interpret 1=2 or 2=1, we have to interpret either 2=2 or 1=1.*

3.5 Closure of logarithmic operation under quaternion

Lemma 7 $i = \ln(2)$ *comes as a solution to indeterminacy of negative logarithm and a filler for the caveat in the definition of principal logarithm in terms of complex logarithm. We need quaternion to do the job. If we visualise principal logarithm as logarithm a set of quaternion instead of product of two pairs of i then we can arrive zero at par with the definition of logarithm and solve the issue of indeterminacy of the principal value i.e.*

$$\ln(1) = 0 = \ln(-1. - 1) = \ln(-1. - 1. - 1. - 1) = \ln(i^2.j^2.k^2.i.j.k) = 3(\ln i + \ln j + \ln k)$$

.Any guess what angle can make vector-sum of three equal vectors equal to zero? As shown in Riemann hypothesis proof, its 120 degree in 3D or 60 degree in 4D. This way numbers are very complexly 3 dimensional hidden in other hidden dimensions of quaternion although we do not feel its requirement in our everyday use of numbers. Following the above definition we can generalise the definitions further as follows.

$$\ln(-1) = \ln(1) = 0$$

$$\ln(-2) = \ln(-1) + \ln(2) = \ln(2)$$

$$\ln(-3) = \ln(-1) + \ln(3) = \ln(3)$$

And the pattern continues up to negative infinity.

But in complex sense we can do the closure of logarithmic operation under quaternion as follows:

Example 2 *Find natural logarithm of -5 using first quaternion solution of i*

$$\ln(-5) = \ln(-1) + \ln(5) = \ln(i^2) + \ln(5) = 2ln(ln(2)) + \ln(5) = 0.876412071(approx)$$

3.6 Numbers rotate in 3D via Euler's formula

Theorem 1 *The imaginary number i has real solutions.*

Proof 1 *Now let see how quaternion helps in justifying the definition for imaginary number i. For simplification let us use a single alphabet for expressing quaternion. Let us recall the power addition identity, which is,*

$$e^{(a+b)} = e^a.e^b$$

However this only applies when 'a' and 'b' commute, so it applies when a or b is a scalar for instance. The more general case where 'a' and 'b' don't necessarily commute is given by:

$$e^q = e^{q_1}.e^{q_2}$$

where:

$$q = q_1 + q_2 + q_1 \times q_2 + \frac{1}{3}(q_1 \times (q_1 \times q_2) + q_2 \times (q_2 \times q_1)) + ...$$

Where: $\times$ = *vector cross product. This shows that when a and b become close to becoming parallel then* $q_1 \times q_2$ *approaches zero and q approaches* $q_1 + q_2$ *so the rotation algebra approaches vector algebra. As we have seen all the three unit discs appear parallel to each other our life gets easier and we can do complex exponentiation and logarithm as we do it for real numbers. We have seen in Euler's formula* $e^{i\pi} = -1$ *complex exponentials turn into real numbers. Applying the same trick we can turn quaternion exponentials into real numbers following the cycle of* $e^{i\pi} + 1 = 0$ *. This time we need to complete the cycle of [1+ i - 1 - i = 0] by way of showing* $e^q = -1$ *which will prove the formula* $|\mathbf{z}| = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{e^{ix}} = \mathbf{1}$ *for generalised complex numbers. Let us consider the following infinite series.*

$$\begin{aligned}
& e^{q_1}.e^{q_2}.^{q_3}.e^{q_4}.e^{q_5}.e^{q_6}.^{q_7}... \\
=& e^{q_1+q_2+q_3+q_4+q_5+q_6+q_7...} \\
=& e^{2.\ln(2)*} \\
=& e^{\ln(2)}.e^{\ln(2)} \\
=& -e^{-\ln(2)}.e^{\ln(2)} \\
=& -\frac{1}{2}.2 \\
=& -1
\end{aligned}$$

$$\begin{aligned}
& * \, q_1 + q_2 + q_3 + q_4 + q_5 + q_6 + q_7... \\
&= i(\sum_{s=-\infty}^{0} \zeta(s)) + j(\sum_{s=-\infty}^{0} \zeta(s)) + k(\sum_{s=-\infty}^{0} \zeta(s)) \\
&+ \sum_{s=-\infty}^{\infty} \zeta(s) + \sum_{s=-\infty}^{\infty} \zeta(s) + \sum_{s=-\infty}^{\infty} \zeta(s) + \sum_{s=-\infty}^{\infty} \zeta(s) \\
&= i(\zeta(-1) + \zeta(0)) + j(\zeta(-1) + \zeta(0)) \\
&+ k(\zeta(-1) + \zeta(0)) + \eta(1) + \eta(1) ** \\
&= i.0 + j.0 + k.0 + 2.\eta(1) \\
&= 2.\ln(2)
\end{aligned}$$

There is a cycle of three in four infinite series of infinite zeta values which emerges as two infinite alternate zeta series following $\frac{1}{2}.e^{\ln 2} = 1$.

In d-unit circle we have seen $|\mathbf{z}| = \frac{1}{2}\mathbf{e^{ix}} = \mathbf{1}$ is another form of unit circle. From $i^2 = -1$ we know that i shall have at least two roots or values, one we have already defined, another we need to find out. We have seen that at $\frac{\pi}{3}$ zeta function attains zero. Let us use Euler's formula to define another possible value of i as Euler's formula deals with unity which comes from the product of exponential and its inverse i.e. logarithm.

Now its proven :

$$z = \frac{1}{2}e^{ix} = 1 = \frac{1}{2}e^{\ln 2}$$

we can say :

$$e^{ix} = e^{ln2}$$

taking logarithm both side :

$$ix = ln(2)$$

setting x=1 :

$$\boxed{ln(2) = e^{ln(ln(2))} = e^{ln(i)} = i}$$

$$\boxed{i = e^{-\frac{1}{e}} = 2^{-\frac{1}{2}} = e - 2}$$

$$\boxed{ln(2)^{\frac{1}{ln(ln(2))}} = i^{\frac{1}{ln(i)}} = e = -\frac{1}{ln(i)} = 2 + i}$$

we get two more identity like $e^{i\pi} + 1 = 0$:

$$\boxed{\frac{1}{e} + ln(i) = 0 = e + \frac{1}{ln(i)}}$$

again we know $i^2 = -1$, taking log both side

$$\boxed{\ln(-1) = 2\ln i = 2ln(ln(2))}$$

Let us assume:

$$e^{i\frac{\pi}{3}} = z$$

taking natural log both side :

$$\frac{i\pi}{3} = ln(z)$$

Let us set:$ln(z) = i + \frac{1}{3}$

$$i\pi = 1 + 3i$$

$$i(\pi - 3) = 1$$

$$\boxed{i = \frac{1}{\pi - 3}}$$

$$\boxed{\pi = 3 + \frac{1}{i}}$$

we get two more identity like $e^{i\pi} + 1 = 0$:

$$\boxed{ln(i) - 2 = 0 = \frac{1}{ln(i)} - \frac{1}{2}}$$

taking log both side of $i^2 = -1$

$$\boxed{\ln(-1) = 2\ln i = 2ln\left(\frac{1}{\pi - 3}\right)}$$

Constant 1

$$\boxed{e^{i\pi} = e^{ln(2).\pi} = 8.824977827 = e^{2.17758609}...(approx)}$$

Constant 2

$$\boxed{e^{i\pi} = e^{\frac{\pi}{\pi-3}} = 4,324,402,934 = e^{22.18753992}...(approx)}$$

Can we have partitions representing roots of unity like $\frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{5}, \frac{\pi}{6}$. Yes, We can and each of them will be some kind constant having some relevance to appropriate mathematical conditions just like sin, cos, tan, cot, sec, cosec of those angles are important in trigonometry. Unit partitions of first cyclic number.

$$\boxed{e^{i\pi} = e^{ln(2).\pi} = 8.824977827 = e^{2.17758609}...(approx)}$$

Constant 3

$$\boxed{e^{i\frac{\pi}{2}} = e^{\frac{ln(2).\pi}{2}} = 2.970686424 = e^{1.088793045}...(approx)}$$

Constant 4

$$\boxed{e^{i\frac{\pi}{3}} = e^{\frac{ln(2).\pi}{3}} = 2.066511728 = e^{0.72586203}...(approx)}$$

Constant 5

$$\boxed{e^{i\frac{\pi}{4}} = e^{\frac{ln(2).\pi}{4}} = 1.723567934 = e^{0.544396523}...(approx)}$$

Constant 6

$$\boxed{e^{i\frac{\pi}{5}} = e^{\frac{ln(2).\pi}{5}} = 1.545762348 = e^{0.435517218}...(approx)}$$

Constant 7

$$\boxed{e^{i\frac{\pi}{6}} = e^{\frac{ln(2).\pi}{6}} = 1.437536687 = e^{0.362931015}...(approx)}$$

Unit partitions of second cyclic number.

$$\boxed{e^{i\pi} = e^{\frac{\pi}{\pi-3}} = 4,324,402,934 = e^{22.18753992}...(approx)}$$

Constant 8

$$\boxed{e^{i\frac{\pi}{2}} = e^{\frac{\pi}{2(\pi-3)}} = 65,760 = e^{11.09376703}...(approx)}$$

Constant 9

$$\boxed{e^{i\frac{\pi}{3}} = e^{\frac{\pi}{3(\pi-3)}} = 1,629 = e^{7.395721609}...(approx)}$$

Constant 10

$$\boxed{e^{i\frac{\pi}{4}} = e^{\frac{\pi}{4(\pi-3)}} = 256.4375 = e^{5.54688497}...(approx)}$$

Constant 11

$$\boxed{e^{i\frac{\pi}{5}} = e^{\frac{\pi}{5(\pi-3)}} = 84.5639441 = e^{4.43750798}...(approx)}$$

Constant 12

$$\boxed{e^{i\frac{\pi}{6}} = e^{\frac{\pi}{6(\pi-3)}} = 40.36339539 = e^{3.69792332}...(approx)}$$

4 Let us analyse the post proof results

4.1 Infinite product of positive zeta values converges

In this section we will verify our results from induction approach. Let us start from there where Euler left. Let us take infinite product of positive zeta values and see what we get. We should use the product to sum form given by Euler. So we will take infinite product of positive zeta values both from the side of sum of numbers and also from the side of product of primes.

$$\zeta(1) = 1 + \frac{1}{2^1} + \frac{1}{3^1} + \frac{1}{4^1}... = \left(1 + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3}...\right)\left(1 + \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3}...\right)...$$

$$\zeta(2) = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}... = \left(1 + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6}...\right)\left(1 + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6}...\right)...$$

$$\zeta(3) = 1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3}... = \left(1 + \frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9}...\right)\left(1 + \frac{1}{3^3} + \frac{1}{3^6} + \frac{1}{3^9}...\right)...$$

$\vdots$

$\zeta(1)\zeta(2)\zeta(3)... =$

From the side of infinite sum of negative exponents of all natural integers:

$$= \left(1 + \frac{1}{2^1} + \frac{1}{3^1} + \frac{1}{4^1}...\right)\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}...\right)\left(1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3}...\right)...$$

$$= 1 + \left(\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3}...\right) + \left(\frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3}...\right) + \left(\frac{1}{4^1} + \frac{1}{4^2} + \frac{1}{4^3}...\right)...$$

$$= 1 + 1 + \frac{1}{2^1} + \frac{1}{3^1} + \frac{1}{4^1} + \frac{1}{5^1} + \frac{1}{6^1} + \frac{1}{7^1} + \frac{1}{8^1} + \frac{1}{9^1}... = 1 + \zeta(1)$$

From the side of infinite product of sum of negative exponents of all primes:

$$= \left(1 + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3}...\right)\left(1 + \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3}...\right)\left(1 + \frac{1}{5^1} + \frac{1}{5^2} + \frac{1}{5^3}...\right)...$$

$$\left(1 + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6}...\right)\left(1 + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6}...\right)\left(1 + \frac{1}{5^2} + \frac{1}{5^4} + \frac{1}{5^6}...\right)...$$

$$\left(1 + \frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9}...\right)\left(1 + \frac{1}{3^3} + \frac{1}{3^6} + \frac{1}{3^9}...\right)\left(1 + \frac{1}{5^3} + \frac{1}{5^6} + \frac{1}{5^9}...\right)...$$

$$= \left(1 + 1\right)\left(1 + \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3}...\right)\left(1 + \frac{1}{5^1} + \frac{1}{5^2} + \frac{1}{5^3}...\right)...$$

$$\left(1+\frac{1}{2^2}+\frac{1}{2^4}+\frac{1}{2^6}...\right)\left(1+\frac{1}{3^2}+\frac{1}{3^4}+\frac{1}{3^6}...\right)\left(1+\frac{1}{5^2}+\frac{1}{5^4}+\frac{1}{5^6}...\right)...$$
$$\left(1+\frac{1}{2^3}+\frac{1}{2^6}+\frac{1}{2^9}...\right)\left(1+\frac{1}{3^3}+\frac{1}{3^6}+\frac{1}{3^9}...\right)\left(1+\frac{1}{5^3}+\frac{1}{5^6}+\frac{1}{5^9}...\right)...$$

Simultaneously halving and doubling each factor and writing it as sum of two forms

$$=2\left(\frac{1}{2}\left(1+\frac{\frac{1}{3}}{1-\frac{1}{3}}+1+\frac{1}{3^1}+\frac{1}{3^2}+\frac{1}{3^3}...\right)\right)\left(\frac{1}{2}\left(1+\frac{\frac{1}{5}}{1-\frac{1}{5}}+1+\frac{1}{5^1}+\frac{1}{5^2}+\frac{1}{5^3}...\right)\right)...$$
$$\left(\frac{1}{2}\left(1+\frac{\frac{1}{4}}{1-\frac{1}{4}}+1+\frac{1}{2^2}+\frac{1}{2^4}+\frac{1}{2^6}...\right)\right)\left(\frac{1}{2}\left(1+\frac{\frac{1}{9}}{1-\frac{1}{9}}+1+\frac{1}{3^2}+\frac{1}{3^4}+\frac{1}{3^6}...\right)\right)...$$
$$\left(\frac{1}{2}\left(1+\frac{\frac{1}{8}}{1-\frac{1}{8}}+1+\frac{1}{2^3}+\frac{1}{2^6}+\frac{1}{2^9}...\right)\left(\frac{1}{2}\left(1+\frac{\frac{1}{27}}{1-\frac{1}{27}}+1+\frac{1}{3^3}+\frac{1}{3^6}+\frac{1}{3^9}...\right)\right)...$$
$$=2\left(\frac{1}{2}\left(1+\frac{1}{2}+1+\frac{1}{3^1}+\frac{1}{3^2}+\frac{1}{3^3}...\right)\right)\left(\frac{1}{2}\left(1+\frac{1}{4}+1+\frac{1}{5^1}+\frac{1}{5^2}+\frac{1}{5^3}...\right)\right)...$$
$$\left(\frac{1}{2}\left(1+\frac{1}{3}+1+\frac{1}{2^2}+\frac{1}{2^4}+\frac{1}{2^6}...\right)\left(\frac{1}{2}\left(1+\frac{1}{8}+1+\frac{1}{3^2}+\frac{1}{3^4}+\frac{1}{3^6}...\right)\right)...$$
$$\left(\frac{1}{2}\left(1+\frac{1}{7}+1+\frac{1}{2^3}+\frac{1}{2^6}+\frac{1}{2^9}...\right)\left(\frac{1}{2}\left(1+\frac{1}{26}+1+\frac{1}{3^3}+\frac{1}{3^6}+\frac{1}{3^9}...\right)\right)...$$
$$=2\left(1+\frac{1}{2}\left(\frac{1}{2}+\frac{1}{3^1}+\frac{1}{3^2}+\frac{1}{3^3}...\right)\right)\left(1+\frac{1}{2}\left(\frac{1}{4}+\frac{1}{5^1}+\frac{1}{5^2}+\frac{1}{5^3}...\right)\right)...$$
$$\left(1+\frac{1}{2}\left(\frac{1}{3}+\frac{1}{2^2}+\frac{1}{2^4}+\frac{1}{2^6}...\right)\right)\left(1+\frac{1}{2}\left(\frac{1}{8}+\frac{1}{3^2}+\frac{1}{3^4}+\frac{1}{3^6}...\right)\right)...$$
$$\left(1+\frac{1}{2}\left(\frac{1}{7}+\frac{1}{2^3}+\frac{1}{2^6}+\frac{1}{2^9}...\right)\right)\left(1+\frac{1}{2}\left(\frac{1}{26}+\frac{1}{3^3}+\frac{1}{3^6}+\frac{1}{3^9}...\right)\right)...$$
$$=2\left(1+\frac{1}{2}\left(\frac{1}{2^1}+\frac{1}{3^1}+\frac{1}{4^1}...+\frac{1}{2^1}+\frac{1}{3^1}+\frac{1}{4^1}...\right)\right)$$
$$=2\left(1+\frac{1}{2}\left(2\zeta(1)-2\right)\right)$$
$$=2(1-1+\zeta(1))$$
$$=2\zeta(1)$$

Now comparing two identities we get:

$$\boxed{1+\zeta(1) = 2\zeta(1))}$$

Which gives:

$$\boxed{\zeta(1) = 1}$$

Hence Infinite product of positive zeta values converges to 2.

4.2 Infinite product of negative zeta values converges

Similarly we will take infinite product of negative zeta values both from the side of sum of numbers and also from the side of product of primes.

$$\zeta(-1) = 1 + 2^1 + 3^1 + 4^1 + 5^1 ... = \left(1 + 2 + 2^2 + 2^3 ...\right)\left(1 + 3 + 3^2 + 3^3 ...\right)...$$

$$\zeta(-2) = 1 + 2^2 + 3^2 + 4^2 + 5^2 ... = \left(1 + 2^2 + 2^4 + 2^6 ...\right)\left(1 + 3^2 + 3^4 + 3^6 ...\right)...$$

$$\zeta(-3) = 1 + 2^3 + 3^3 + 4^3 + 5^3 ... = \left(1 + 2^3 + 2^6 + 2^9 ...\right)\left(1 + 3^3 + 3^6 + 3^9 ...\right)...$$

$\vdots$

From the side of infinite sum of negative exponents of all natural integers:

$\zeta(-1)\zeta(-2)\zeta(-3)...$

$$= \left(1 + 2^1 + 3^1 + 4^1 + 5^1 ...\right)\left(1 + 2^2 + 3^2 + 4^2 + 5^2 ...\right)\left(1 + 2^3 + 3^3 + 4^3 + 5^3 ...\right)...$$

$$= 1 + \left(2 + 2^2 + 2^3 ...\right) + \left(3 + 3^2 + 3^3 ...\right) + \left(4 + 4^2 + 4^3 ...\right)...$$

$$= 1 + \left(1 + 2 + 2^2 + 2^3 ... - 1\right) + \left(1 + 3 + 3^2 + 3^3 ... - 1\right) + \left(1 + 4 + 4^2 + 4^3 ... - 1\right)...$$

$$= 1 + \left(-\frac{1}{1} - 1\right) + \left(-\frac{1}{2} - 1\right) + \left(-\frac{1}{3} - 1\right) + \left(-\frac{1}{4} - 1\right)...$$

$$= 1 - \left(\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} ...\right) + 1 + 1 + 1 + 1 ...\right)$$

$$= 1 - \left(\zeta(1) + \zeta(0)\right) = 1 - \left(1 - \frac{1}{2}\right) = \frac{1}{2}$$

From the side of infinite product of sum of negative exponents of all primes:
$\zeta(-1)\zeta(-2)\zeta(-3)...$

$$= \left(1+2+2^2+2^3...\right)\left(1+3+3^2+3^3...\right)\left(1+5+5^2+5^3...\right)...$$
$$= \left(1+2^2+2^4+2^6...\right)\left(1+3^2+3^4+3^6...\right)\left(1+5^2+5^4+5^6...\right)...$$
$$= \left(1+2^3+2^6+2^9...\right)\left(1+3^3+3^6+3^9...\right)\left(1+5^3+5^6+5^9...\right)...$$
$$= 1+2^1+3^1+4^1+5^1... \implies \zeta(-1) = \frac{1}{2}$$

Therefore $\boxed{\zeta(-1) = \frac{1}{2}}$ must be the second solution apart from $\zeta(-1) = \frac{-1}{12}$.

Using Delta function instead of Gamma function we can rewrite the functional equation applicable as follows:

$$\zeta(s) = -2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Delta(4-s)\zeta(1-s)$$

This can be rewritten in terms of Gamma function as follows:

$$\zeta(s) = -2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s)$$

This again can be rewritten in terms of Pi function as follows:

$$\zeta(s) = -2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Pi(2-s)\zeta(1-s)$$

Putting $s = -1$we get:

$$\zeta(-1) = -2^{-1}\pi^{(-1-1)}\sin\left(\frac{-\pi}{2}\right)\Gamma(3-s)\zeta(2) = \frac{1}{2}$$

To proof Ramanujan's Way

$$\sigma = \boxed{1+2+3+4+5+6+7+8+9.....}$$
$$2\sigma = \boxed{0+1+2+3+4+5+6+7+8+9...}+1+1+1+1+1+1+1...*$$

Subtracting the bottom from the top one we get:

$$
\begin{aligned}
-\sigma &= 0+1+1+1+1+1+1+1+1... +1+1+1+1+1+1+1...\\
\sigma &= -(1+1+1+1+1+1+1+1+1+1........)\\
\sigma &= \frac{1}{2}
\end{aligned}
$$

*The second part is calculated subtracting bottom from the top before doubling.

4.3 Infinite sum of Positive zeta values converges

$$
\begin{aligned}
&\zeta(1) = 1+\frac{1}{2^1}+\frac{1}{3^1}+\frac{1}{4^1}...\\
&\zeta(2) = 1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}...\\
&\zeta(3) = 1+\frac{1}{2^3}+\frac{1}{3^3}+\frac{1}{4^3}...\\
&\vdots\\
&\zeta(1)+\zeta(2)+\zeta(3)...\\
&\ = \left(1+\frac{1}{2^1}+\frac{1}{3^1}+\frac{1}{4^1}...\right)+\left(1+1+1+1+...\right)\\
&\ = \zeta(1)+\zeta(0) = 1-\frac{1}{2} = \frac{1}{2}
\end{aligned}
$$

4.4 Infinite sum of Negative zeta values converges

$$
\begin{aligned}
&\zeta(-1) = 1+2^1+3^1+4^1+5^1...\\
&\zeta(-2) = 1+2^2+3^2+4^2+5^2...\\
&\zeta(-3) = 1+2^3+3^3+4^3+5^3...\\
&\vdots\\
&\zeta(-1)+\zeta(-2)+\zeta(-3)...\\
&\ = \left(1+2^1+3^1+4^1+5^1...\right)+\left(1+1+1+1+...\right)\\
&\ = \zeta(-1)+\zeta(0) = \frac{1}{2}-\frac{1}{2} = 0
\end{aligned}
$$

4.5 Infinite product of All zeta values converges

$$
\boxed{\zeta(-1)\zeta(-2)\zeta(-3)...\zeta(1)\zeta(2)\zeta(3)... = 2.\zeta(-1).\zeta(1) = 2.1.\frac{1}{2} = 1}
$$

4.6 Infinite sum of All zeta values converges

$$\boxed{\zeta(-1)+\zeta(-2)+\zeta(-3)...\zeta(1)+\zeta(2)+\zeta(3)...=0+\frac{1}{2}=\frac{1}{2}}$$

4.7 Integral representation of $\zeta(1)$ from Bose integral

$$-\zeta(1)=\int_0^\infty \frac{dx}{e^x-1}=\int_0^\infty \frac{e^{-x}}{1-e^{-x}}dx=\int_0^\infty \sum_{n=1}^\infty e^{-x}.(e^{-x})^{n-1}dx=\sum_{n=1}^\infty \int_0^\infty e^{-nx}dx$$

$$\text{substituting } nx=u \text{ we get } \implies \sum_{n=1}^\infty \int_0^\infty \frac{e^{-u}}{n}du=\sum_{n=1}^\infty \frac{1}{n}\int_0^\infty e^{-u}du=-\sum_{n=1}^\infty \frac{1}{n}$$

$$\text{substituting } x=\ln(2) \text{ in } -\zeta(1)=\int_0^\infty \frac{dx}{e^x-1} \text{ and changing the limit we get}$$

$$\zeta(1)=\int_0^1 \frac{dx}{e^{\ln(2)}-1}=\left[\frac{x}{2-1}\right]_0^1=1$$

4.8 Integral representation of $\zeta(-1)$ from Ramanujan summation

$$\zeta(-1)=i^3\int_0^\infty \frac{f(6it)-f(-6it)}{e^{2\pi t}-1}dt=i^3\int_0^\infty \frac{12it}{e^{2\pi t}-1}dt=12i^4\int_0^\infty \frac{t}{e^{2\pi t}-1}dt$$

Substituting $u=2\pi t$ we get

$$\zeta(-1)=12\int_0^\infty \frac{\frac{u}{2\pi}}{e^u-1}.\frac{du}{2\pi}=\frac{12}{4\pi^2}\int_0^\infty \frac{udu}{e^u-1}=\frac{12}{4\pi^2}.\frac{\pi^2}{6}=\frac{1}{2}$$

4.9 Counter to Nicole Oresme's logic of divergence

Nicole Oresme in around 1350 proved divergence of harmonic series by comparing the harmonic series with another divergent series. He replaced each denominator with the next-largest power of two.

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}...>1+\frac{1}{2}+\frac{1}{4}+\frac{1}{4}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}...$$

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}...>1+\left(\frac{1}{2}\right)+\left(\frac{1}{4}+\frac{1}{4}\right)+\left(\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}\right)+...$$

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}...>1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}...$$

He then concluded that the harmonic series must diverge as the above series diverges. It was too quick to conclude as we can go ahead and show:

$$R.H.S = 1 + \frac{1}{2}\left(1+1+1+1+1+1+1+...\right) = 1 + \frac{1}{2}.\frac{-1}{2} = 1 - \frac{1}{4}$$

If we consider $\zeta(1) = 1$ then also it passes the comparison test. Therefore We need to come out of the belief that harmonic series diverges.

$$\begin{aligned}
&= 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}\left(1+1+1...\right)\\
&= 1 + \frac{3}{2} + \frac{1}{2}.\frac{-1}{2}\\
&= 1 + \frac{3}{2} - \frac{1}{4}\\
&= 1 + \frac{3}{2} - \left(1-2+3-4+...\right)\\
&= 1 + \frac{3}{2} - \left(\left(1+2+3...\right) - 2\left(1+2+4...\right)\right)\\
&= 1 + \frac{3}{2} - \left(\frac{1}{2} - 2\left(1+1+1...\right)\right)\\
&= 1 + \frac{3}{2} - \left(\frac{1}{2} - 2\frac{-1}{2}\right)\\
&= 1 + \frac{3}{2} - \left(\frac{1}{2} + 1\right)\\
&= 1 + \frac{3}{2} - \frac{3}{2}\\
&= 1
\end{aligned}$$

$$\begin{aligned}
&= 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}\left(1+1+1...\right)\\
&= 1 + \frac{5}{2} + \frac{1}{2}.\frac{-1}{2}\\
&= 1 + \frac{5}{2} - \frac{1}{4}\\
&= 1 + \frac{5}{2} - \left(1-2+3-4+...\right)\\
&= 1 + \frac{5}{2} - \left(\left(1+2+3...\right) - 2\left(1+2+4...\right)\right)\\
&= 1 + \frac{5}{2} - \left(\frac{1}{2} - 2\left(\frac{1}{1-2}\right)\right)\\
&= 1 + \frac{5}{2} - \left(\frac{1}{2} + 2\right)\\
&= 1 + \frac{5}{2} - \left(\frac{1+4}{2}\right)\\
&= 1 + \frac{5}{2} - \frac{5}{2}\\
&= 1
\end{aligned}$$

According to residue theorem we can have a Laurent expansion of an analytic function at the pole $f(s) = \sum_{n=-\infty}^{\infty} a_n(s-s_0)^n$ of f in a punctured disk around s_0, and therefrom we can have $\operatorname{Res}(f(s); s_0) = a_{-1}$, i.e. the residue is the coefficient of $(s-s_0)^{-1}$ in that expansion. For the pole $\zeta(1)$, we know the start of the Laurent series (since it is a simple pole, there is only one term with a negative exponent), namely $\zeta(s) = \frac{1}{s-1} + \gamma + \dots$ so we have $\operatorname{Res}(\zeta(s); 1) = 1$. At the pole zeta function have zero radius of convergence (also known as infinite radius of convergence). Interpreting zeta function at the pole to be divergent is extreme arbitrary, contradictory and void of rationality. The pole neither falls outside the radius of convergence resulting $\zeta(1) = \infty$ nor inside the radius of convergence resulting $\zeta(1) = 1$ to be absolutely true, its just on the zero radius of convergence suggesting both values to be equally good. Since none of the above value is more natural than the others, both of them can be incorporated into a multivalued zeta function (We should not say, it's not a function at all as it does not give unique results, we should keep it in mind that ultimately it's two different zeta function seen as a continuum) which is again totally consistent with harmonic conjugate theorem and allows us to interpret $\Rightarrow 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}\ldots=1$

4.10 Negative even zeta values redefining trivial zeros

We can apply Euler's reflection formula for Gamma function

$$\Gamma(1-s)\Gamma(s)=\frac{\pi}{\sin(\pi s)}, s\notin\mathbb{Z}$$

in Riemann's functional equation

$$\zeta(s)=2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(1-s)\zeta(1-s)$$

to get another representation of zeta function as follows:

$$\zeta(s)=2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\frac{\pi}{\Gamma(s)\sin(\pi s)}\zeta(1-s)$$

$$\implies \zeta(s)=2^s\pi^{(s)}\sin\left(\frac{\pi s}{2}\right)\frac{1}{\Gamma(s)2\sin(\frac{\pi s}{2})\cos(\frac{\pi s}{2})}\zeta(1-s)$$

$$\implies \zeta(s)=2^{s-1}\pi^{(s)}\frac{1}{\Gamma(s)\cos(\frac{\pi s}{2})}\zeta(1-s)$$

$$\text{When x=-2,}\qquad \zeta(-2)=2^{-2-1}\pi^{(-2)}\frac{1}{\Gamma(-2)\cos(\frac{-2\pi}{2})}\zeta(1+2)=\frac{\zeta(3)}{8\pi^2}$$

$$\text{When x=-4,}\qquad \zeta(-4)=2^{-4-1}\pi^{(-4)}\frac{1}{\Gamma(-4)\cos(\frac{-4\pi}{2})}\zeta(1+4)=-\frac{5\zeta(5)}{32\pi^4}$$

$$\text{When x=-6,}\qquad \zeta(-6)=2^{-6-1}\pi^{(-6)}\frac{1}{\Gamma(-6)\cos(\frac{-6\pi}{2})}\zeta(1+6)=\frac{3\zeta(7)}{16\pi^6}$$

$$\text{When x=-8,}\qquad \zeta(-8)=2^{-8-1}\pi^{(-8)}\frac{1}{\Gamma(-8)\cos(\frac{-8\pi}{2})}\zeta(1+8)=-\frac{45\zeta(9)}{32\pi^8}$$

$\vdots$

And the pattern continues for all negative even numbers upto negative infinity.

4.11 Negative odd zeta values from harmonic conjugate

Earlier we found that following harmonic conjugate theorem Riemann's functional equation which is an extension of real valued zeta function can also be represented as its harmonic conjugate function which mimic the extended function.

$$\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \Gamma(3-s)\zeta(1-s)$$

We can get the harmonic conjugates of negative zeta values as follows:

$$\text{When s=-1} \qquad \zeta(-1) = -2^{-1}\pi^{(-1-1)} \sin\left(\frac{-1\pi}{2}\right)\Gamma(3+1)\zeta(1+1) = \frac{1}{2}$$

$$\text{When s=-3} \qquad \zeta(-3) = -2^{-3}\pi^{(-3-1)} \sin\left(\frac{-3\pi}{2}\right)\Gamma(3+3)\zeta(1+3) = \frac{-1}{6}$$

$$\text{When s=-5} \qquad \zeta(-5) = -2^{-5}\pi^{(-5-1)} \sin\left(\frac{-5\pi}{2}\right)\Gamma(3+5)\zeta(1+5) = \frac{1}{6}$$

$$\text{When s=-7} \qquad \zeta(-7) = -2^{-7}\pi^{(-7-1)} \sin\left(\frac{-7\pi}{2}\right)\Gamma(3+7)\zeta(1+7) = \frac{-3}{10}$$

$\vdots$

And the pattern continues for all negative odd numbers upto negative infinity.

4.12 Negative even values from harmonic conjugate

We can apply Euler's reflection formula for Gamma function

$$\Gamma(2-s)\Gamma(s-1) = \frac{\pi}{\sin(\pi s - \pi)}, s \notin \mathbb{Z}$$

in alternate Riemann's functional equation

$$\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \Gamma(3-s)\zeta(1-s)$$

to get another representation of zeta function as follows:

$$\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) (2-s)\Gamma(2-s)\zeta(1-s)$$

$$\zeta(s) = -2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\frac{\pi(2-s)}{\Gamma(s-1)\sin(\pi s-\pi)}\zeta(1-s)$$

$$\implies \zeta(s) = -2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\frac{\pi(2-s)}{\Gamma(s-1)\sin(\pi s)}\zeta(1-s)$$

$$\implies \zeta(s) = -2^s\pi^{(s)}\sin\left(\frac{\pi s}{2}\right)\frac{2-s}{\Gamma(s-1)2\sin(\frac{\pi s}{2})\cos(\frac{\pi s}{2})}\zeta(1-s)$$

$$\implies \zeta(s) = -2^{s-1}\pi^{(s)}\frac{2-s}{\Gamma(s-1)\cos(\frac{\pi s}{2})}\zeta(1-s)$$

When x=-2, $$\zeta(-2) = -2^{-2-1}\pi^{(-2)}\frac{2+2}{\Gamma(-3)\cos(\frac{-2\pi}{2})}\zeta(1+2) = -\frac{\zeta(3)}{2\pi^2}$$

When x=-4, $$\zeta(-4) = -2^{-4-1}\pi^{(-4)}\frac{2+4}{\Gamma(-5)\cos(\frac{-4\pi}{2})}\zeta(1+4) = +\frac{9\zeta(5)}{4\pi^4}$$

When x=-6, $$\zeta(-6) = -2^{-6-1}\pi^{(-6)}\frac{2+6}{\Gamma(-7)\cos(\frac{-6\pi}{2})}\zeta(1+6) = -\frac{15\zeta(7)}{\pi^6}$$

When x=-8, $$\zeta(-8) = -2^{-8-1}\pi^{(-8)}\frac{2+8}{\Gamma(-9)\cos(\frac{-8\pi}{2})}\zeta(1+8) = +\frac{315\zeta(9)}{8\pi^8}$$

⋮

And the pattern continues for all negative even numbers upto negative infinity.

4.13 Zeta results confirms PNT

In number theory, the prime number theorem (PNT) describes the asymptotic distribution of the prime numbers among the positive integers. It formalizes the intuitive idea that primes become less common as they become larger by precisely quantifying the rate at which this occurs. The theorem was proved independently by Jacques Hadamard and Charles Jean de la Vallée Poussin in 1896 using ideas introduced by Bernhard Riemann (in particular, the Riemann zeta function). The first such distribution found is $\pi(N) \sim \frac{N}{\log N}$, where $\pi(N)$ is the prime-counting function and $\log N$ is the natural logarithm of N. This means that for large enough N, the probability that a random integer not greater than N is prime is very close to $\frac{1}{\log N}$. The prime number theorem then states that $\frac{N}{\log N}$ is a good approximation to $\pi(N)$ (where log here means the natural logarithm), in the sense that the limit of the quotient of the two functions $\pi(N)$ and $\frac{N}{\log N}$ as N increases without bound is 1:

$\lim_{N\to\infty} \frac{\pi(N)}{\left[\frac{N}{\log(N)}\right]} = 1$ known as the asymptotic law of distribution of prime numbers. Using asymptotic notation this result can be restated as $\pi(N) \sim \frac{N}{\log N}$ Wherever logarithm is there we can take it guaranteed $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ is working in the background. Now we have got one more formula for euler's number e in the form of:

$$e = \sqrt{\lim_{n\to\infty}\left(2+\frac{2}{n}\right)^n . \lim_{n\to\infty}\left(2^{-1}+\frac{2^{-1}}{n}\right)^n}$$

For this reason prime number theorem works as nicely as primes appear through zeta zeros on critical half line in analytic continuation of zeta function.

4.14 Properties of Simplex logarithm

Thanks to Roger Cotes who first time used i in complex logarithm. Thanks to Euler who extended it to exponential function and tied i, pi and exponential function to unity in his famous formula. Now taking lead from both of their work and applying results of zeta function and quaternion algebra we can define quaternion logarithm as follows. If $q_1 = a_1+ib_1+ic_1+id_1$ and $q_2 = a_2 + ib_2 + ic_2 + id_2$ then simplified complex logarithm has the following property.

Theorem 2

$$|\ln(q_1.q_2)| = |\ln(\Re(q_1)) + \ln(\Re(q_2)) + i(\ln(\Im(q_1)) + \ln(\Im(q_2))) + ...|$$

Proof 2

$$\begin{aligned}
&|\ln(q_1.q_2.q_3.q_4.q_5.q_6.q_7....)| \\
&= |\ln(\Re(1.2.3.4.5.6.7...)) + i\ln(\Im(1.2.3.4.5.6.7...)) + ...| \\
&= |\ln(1) + \ln(2) + \ln(3) + ... + i\ln(\ln(1) + \ln(2) + ...) + ...| \\
&= |\ln(\Re(q_1)) + \ln(\Re(q_2)) + +i(\ln(\Im(q_1)) + \ln(\Im(q_2))+) + ...|
\end{aligned}$$

Following zeta functions analytic continuation, we can write :

$$|\ln(q_1.q_2)| = |\ln(\Re(q_1)) + \ln(\Re(q_2)) + i(\ln(\Im(q_1)) + \ln(\Im(q_2))) + ...|$$

Corollary 1

$$\left|\ln\left(\frac{q_1}{q_2}\right)\right| = |\ln(\Re(q_1)) - \ln(\Re(q_2)) + i(\ln(\Im(q_1)) - \ln(\Im(q_2))) + ...|$$

Corollary 2

$$\left|e^{(q_1+q_2)}\right| = \left|e^{(\Re(q_1))}.e^{(\Re(q_2))} + i(e^{(\Im(q_1))}.e^{(\Im(q_2))}) + j(e^{(\Im(q_1))}.e^{(\Im(q_2))}) + ...\right|$$

Corollary 3

$$\left|e^{(q_1-q_2)}\right| = \left|\frac{e^{(\Re(q_1))}}{e^{(\Re(q_2))}} + i(\frac{e^{(\Im(q_1))}}{e^{(\Im(q_2))}}) + j(\frac{e^{(\Im(q_1))}}{e^{(\Im(q_2))}}) + ...\right|$$

Corollary 4

$$|\ln(q_1+q_2)| = |\ln(\Re(q_1+q_2)) + i(\ln(\Im(q_1+q_2))) + ...|$$

Corollary 5

$$|\ln(q_1-q_2)| = |\ln(\Re(q_1-q_2)) + i(\ln(\Im(q_1-q_2))) + ...|$$

Corollary 6

$$\ln(q_1+q_2) = \ln(\mathrm{Re}(q_1)) + \ln(\mathrm{Re}(q_2)) + i\Bigg(\ln(\mathrm{Im}(q_1)) + \ln(\mathrm{Im}(q_2))\Bigg) + ...$$

Corollary 7

$$\ln(q_1-q_2) = \ln(\mathrm{Re}(q_1)) - \ln(\mathrm{Re}(q_2)) + i\Bigg(\ln(\mathrm{Im}(q_1)) - \ln(\mathrm{Im}(q_2))\Bigg) + ...$$

Corollary 8

$$\exp(q_1+q_2) = q_1.q_2$$

Corollary 9

$$\exp(q_1-q_2) = \frac{q_1}{q_2}$$

Corollary 10

$$\ln(q_1.q_2) = q_1+q_2$$

Corollary 11

$$\ln(\frac{q_1}{q_2}) = q_1-q_2$$

Corollary 12

$$\ln(q) = \ln(\mathrm{Re}(q)) + i\Bigg(\ln(\mathrm{Im}(q))\Bigg) + j\Bigg(\ln(\mathrm{Im}(q))\Bigg) + k\Bigg(\ln(\mathrm{Im}(q))\Bigg)$$

Theorem 3 *If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ then simplified Complex Logarithm has the following property.*

$$|\ln(z_1.z_2)| = |\ln(\Re(z_1)) + \ln(\Re(z_2)) + i(\ln(\Im(z_1) + \ln(\Im(z_2))|$$

Proof 3

$$|\ln(z_1.z_2.z_3.z_4.z_5.z_6.z_7....)|$$
$$= \left|\ln\left(1.2.3.4...\right) + i\ln\left(1.2.3.4...\right)\right|$$
$$= \left|\ln(1) + \ln(2) + \ln(3) + ... + i\ln\left(\ln(1) + \ln(2) + \ln(3) + ...\right)\right|$$
$$= \left|\ln(\Re(z_1)) + \ln(\Re(z_2)) + \ln(\Re(z_3)) + ... + i\left(\ln(\Im(z_1)) + \ln(\Im(z_2)) + ...\right)\right|$$

Following zeta functions analytic continuation, we can write:

$$|\ln(z_1.z_2)| = |\ln(\Re(z_1)) + \ln(\Re(z_2)) + i(\ln(\Im(z_1) + \ln(\Im(z_2))|$$

Example 3 *Find the modulus of* $|\ln((5+13i).(12+17i))|$ *using product to sum formula. And show that the result is same orders of magnitude that of actual product.*

$$|\ln((5+13i).(12+17i))| = |\ln(5) + \ln(12) + i(\ln(13 + \ln(17)| = 6.775235638$$

$$|\ln((5+13i).(12+17i))| = |\ln((-161+241i))| = 7.476875532$$

In natural logarithmic scale both the values are of same orders of magnitude.

Corollary 13

$$\left|\ln(\frac{z_1}{z_2})\right| = \left|\ln(\Re(z_1)) - \ln(\Re(z_2)) + i\left(\ln(\Im(z_1)) - \ln(\Im(z_2))\right)\right|$$

Corollary 14

$$\left|e^{(z_1+z_2)}\right| = \left|e^{(\Re(z_1))}.e^{(\Re(z_2))} + i\left(e^{(\Im(z_1))}.e^{(\Im(z_2))}\right)\right|$$

Corollary 15

$$\left|e^{(z_1-z_2)}\right| = \left|\frac{e^{(\Re(z_1))}}{e^{(\Re(z_2))}} + i\left(\frac{e^{(\Im(z_1))}}{e^{(\Im(z_2))}}\right)\right|$$

Corollary 16

$$|\ln(z_1+z_2)| = \left|\ln(\Re(z_1+z_2)) + i\left(\ln(\Im(z_1+z_2))\right)\right|$$

Corollary 17

$$|\ln(z_1-z_2)| = \left|\ln(\Re(z_1-z_2)) + i\left(\ln(\Im(z_1-z_2))\right)\right|$$

Corollary 18

$$\ln(z_1 + z_2) = \ln(\mathrm{Re}(z_1)) + \ln(\mathrm{Re}(z_2)) + i\Bigg(\ln(\mathrm{Im}(z_1)) + \ln(\mathrm{Im}(z_2))\Bigg)$$

Corollary 19

$$\ln(z_1 - z_2) = \ln(\mathrm{Re}(z_1)) - \ln(\mathrm{Re}(z_2)) + i\Bigg(\ln(\mathrm{Im}(z_1)) - \ln(\mathrm{Im}(z_2))\Bigg)$$

Corollary 20

$$\exp(z_1 + z_2) = z_1.z_2$$

Corollary 21

$$\exp(z_1 - z_2) = \frac{z_1}{z_2}$$

Corollary 22

$$\ln(z_1.z_2) = z_1 + z_2$$

Corollary 23

$$\ln(\frac{z_1}{z_2}) = z_1 - z_2$$

Corollary 24

$$\ln(z) = \ln(\mathrm{Re}(z)) + i\Bigg(\ln(\mathrm{Im}(z))\Bigg)$$

Example 4 *Find natural logarithm of -5i using first quaternion solution of i*

$$\ln(-5i) = \ln(i^2) + \ln(5) + \ln(i) = 2ln(ln(2)) + \ln(5) + ln(ln(2)) = 0.509899151(approx)$$

Example 5 *Find natural logarithm of 5-5i using first quaternion solution of i*

$$\ln(5-5i) = \ln(5)+\ln(i^2)+\ln(5)+\ln(i) = \ln(5)+2ln(ln(2))+\ln(5)+ln(ln(2)) = 2.119337063(approx$$

Example 6 *Transform the complex number 2+9i using first quaternion solution of i.*

$$e^{2+9i} = e^{2+9\mathrm{X}0.693147181} = e^{8.238324625} = 3783.196723(approx)$$

4.15 Pi based logarithm

One thing to notice is that pi is intricately associated with e. We view pi mostly associated to circles, what it has to do with logarithm? Can it also be a base to complex logarithm? Although pi based logarithm are not common, but they can be handy in complex logarithm. We know:

$$
\begin{aligned}
&ln(2).\frac{\pi}{4}\\
&=\left(\frac{1}{1}-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\cdots\right)\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{11}-\frac{1}{13}+\cdots\right)\\
&=\left(1+\frac{1}{\lfloor 3}-\frac{1}{\lfloor 5}+\frac{1}{\lfloor 7}-\cdots\right)+\left(1+\frac{1}{\lfloor 2}+\frac{1}{\lfloor 4}+\frac{1}{\lfloor 6}+\cdots\right)-\left(1+\frac{1}{2}+\frac{1}{4}+\cdots\right)\\
&=\left(1-\frac{i^3}{\lfloor 3}+\frac{i^5}{\lfloor 5}-\frac{i^7}{\lfloor 7}-\cdots\right)+\left(1-\frac{i^2}{\lfloor 2}+\frac{i^4}{\lfloor 4}-\frac{i^6}{\lfloor 6}+\cdots\right)-\frac{1}{1-\frac{1}{2}}\\
&=\sin(i)+\cos(i)-2
\end{aligned}
$$

Let us set:$\pi=\sin(i)+\cos(i)$ and replacing $\pi-2=\ln(\pi)$ we can write

$$\frac{\ln\left(e^{\frac{ln(2)}{4}}\right)}{\ln(\pi)}=\frac{1}{\pi}=\pi^{-1}\text{Let us set:}e^{\frac{ln(2)}{4}}=\pi^{\pi^{je}} \text{ we can write } \boxed{\pi^{je}=-1}$$

4.16 Fermat's last theorem

Theorem 4 *There cannot be any integer solution for $n>2$ which satisfies $x^n+y^n=z^n$*

Proof 4 *Let there be some value $n>2$ which satisfies $x^n+y^n=z^n$. Case $n=2$ is known since Greeks (Pythagoras) if not earlier since Indians (Boudhayana). Now to search for greater values where from the search should begin. Obviously it should be 3 because if it discontinues at 3 then we should stop the search. At zero $x^3+y^3=z^3$ should satisfy the special case $x^3+y^3=(x+y)^3$ which can also be written as $x^3+y^3=(x+y)(x^2+xy+y^2)$. Replacing $(x^2+y^2)=1$ will bring zeta function into the picture analogous to Euler's sum to product form involving primes in the form of $x^3+y^3=(x+y)(1+xy)$.The idea is factoring out this (1+ something) terms one side and take their products to relate it universal zeta zeros related to primes. Similarly every higher order Diophantine equations of this form can be equated to zeros of higher order polynomials. Expanding those polynomials by Newton's binomial expansion we shall get terms like $(x^2+y^2)=1$. For example $x^4+y^4=(x^2+2xy+y^2).(x^2+2xy+y^2)$, replacing $(x^2+y^2)=1$ which can be written $x^4+y^4=(1+2xy).(1+2xy)$. Even higher order cases will not be an exception. Now this (1+ something) form is reserved for primes in Euler's sum to product form of zeta function. If we try to unify these terms arising from zeros of all higher order* *polynomials we fall into a serious conflict as the first term $(1 + xy)$ itself suggesting xy cannot be prime. Therefore there cannot be any integer solution for any value of $n > 2$ which satisfies $x^n + y^n = z^n$ as Fermat claimed. Fermat's last theorem holds good. He was not bluffing when he said he had the proof after all his claims was found to be true. Andrew Wiles has given 500+ pages proof. Proving Fermat's last theorem in more elementary way is a subject of academic interest now. I am presenting my signature proof which not to be construed as an attempt to undermine Andrew Wiles's work. His work has given the world of mathematics something new called Taniyama-Shimura theorem (I don't know what it is all about). It's always good to keep on adding new theorems as they always proves to be handy in attacking novel problems. Let z^n be a arbitrarily large number. Let two numbers $x > \ln(z^n)$ and $y > \ln(z^n)$ also satisfies $x^n + y^n = z^n$. Now to find the limit $n = \frac{(z^n)}{\ln(z^n)}$ we can look into its zeros in unit circle which satisfies $1 = \frac{1}{n}.e^{\ln(n)}$. The only value $n = 2$ satisfies such conditions, therefore there cannot be any solution for any value of $n > 2$ which satisfies $x^n + y^n = z^n$ as Fermat claimed.*

4.17 Fundamental formula of numbers

Also I got a nice relationship between sum of numbers and the product of primes which can be regarded as the second fundamental formula of arithmetic.

Theorem 5

$$2.\sum_{N=1}^{\infty} N = \prod_{i=1}^{\infty} P_i \implies \sum_{N=1}^{\infty} N = \prod_{i=2}^{\infty} P_i$$

$$\begin{aligned}
&\text{We know :}\\
&\zeta(-1) = \zeta(1) + \zeta(0)\\
\implies &\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}...\right) + \left(1 + 1 + 1 + 1 + ...\right) = \frac{1}{2}\\
\implies &\left(1 + 1\right) + \left(1 + \frac{1}{2}\right) + \left(1 + \frac{1}{3}\right) + \left(1 + \frac{1}{4}\right) + ... = \frac{1}{2}\\
\implies &\left(\frac{2}{1} + \frac{3}{2} + \frac{4}{3} + \frac{5}{4} + \frac{6}{5}...\right) = \frac{1}{2}\\
\implies &\left(\frac{1 + 2 + 3 + 4 + 5 + 6 + 7...*}{2.3.5.7.11...**}\right) = \frac{1}{2}\\
\implies &2.\sum_{N=1}^{\infty} N = \prod_{i=1}^{\infty} P_i
\end{aligned}$$

*Series in reverse order.

** LCM steps :

$$LCM = \prod_{i=1}^{\infty} P_i^1.P_i^2.P_i^3...P_i^1.P_i^2.P_i^3...$$

$$LCM = \prod_{i=1}^{\infty} P_i^{(1+2+3+...)+(1+2+3+...)}$$

$$LCM = \prod_{i=1}^{\infty} P_i^{\frac{1}{2}+\frac{1}{2}}$$

$$LCM = 2.3.5.7.11...$$

4.18 Goldbach's prime theorem

Theorem 6 *Every even integer greater than 2 can be expressed as the sum of two primes.*

Proof 5 *Let N be a arbitrarily large number. Sum of all the natural numbers up to N shall be* $\frac{N(1+N)}{2}$ *which includes sum of all the primes up to N too. Double of such sum shall be* $N(1+N)$ *which shall include double of sum of all the primes up to N too. According to PNT we know that there shall be* $\frac{N}{\ln(N)}$ *number of primes up to N with an average prime gap of* $\ln(N)$. *We can visualise* $\frac{N}{\ln(N)}$ *as a prime number itself and we can allow the prime gaps to change equivalently and complete the number sequence. Now if we take logarithm of* $N(1+N)$ *with respect to the base of* $\frac{N}{\ln(N)}$ *the result shall give us the lower bound of powers that can comfortably be applied on that prime to reach double of the sum of all the natural numbers up to N i.e.* $N(1+N)$. *In other words if we consolidate the average prime gaps into a relatively large prime having approximate value of* $P_1 \leq \frac{N}{\ln(N)}$ *then that will lead us also to lower limit of Goldbach partitions (as our push action shall have squeeze reaction on the prime gaps) which will satisfy the following set of equations* $P_2 \leq \log_{\frac{N}{\ln(N)}} N(1+N), P_1{+}P_2 = E, P_1^{\log_{\frac{N}{\ln(N)}} N(1+N)} \leq N(1{+}N)$. *If our resultant exponent is greater than 2 (ideally it should be greater than or equal to 2 as we have ensured all primes are summed up 2 times) then that will be greater than the lower bound of prime gaps 2 which eventually ensures continuity of the pattern up to infinity. Clearly due to infinitude of primes, the result* $\log_{\frac{N}{\ln(N)}} N(1+N) = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (1+N)$ *shall be always greater than 2. As 2 Goldbach partition is always lesser than the exponent as calculated above, all the even numbers greater than 2 can be expressed as sum of two primes* $P_1 + P_2$. *Hence Goldbach conjecture stands proved and it can be called as Goldbach theorem. The weaker version of Goldbach conjecture (ternary Goldbach conjecture) immediately follows from the stronger version (binary Goldbach conjecture) proved above.*

4.19 Twin prime theorem

Theorem 7 *There are infinitely many primes p such that p + 2 is also prime.*

Proof 6 *As the above result* $\log_{\frac{N}{\ln(N)}} N(1+N) = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (1+N)$ *shall be always greater than 2 there shall be infinitely many twin primes with a prime gap of 2.Hence Twin prime conjecture stands proved and it can be called as Twin prime theorem.*

4.20 Sophie Germain's prime theorem

Theorem 8 *There are infinitely many prime numbers of the form* $2P + 1$

Proof 7 *As there shall be infinitely many twin primes with a prime gap of 2, there shall also be infinitely many prime numbers of the form* $2P + 1$ *with prime gap of* $P + 1$ *. Hence Sophie Germain conjecture stands proved and it can be called as Sophie Germain's prime theorem.*

4.21 Polignac's prime theorem

Theorem 9 *For every natural number k, there are infinitely many primes p such that p + 2k is also prime.*

Proof 8 *As there shall be infinitely many twin primes with a prime gap of 2, there shall also be infinitely many prime numbers of the form* $P + 2k$ *with prime gap of* $2k$ *. Hence Polignac's conjecture stands proved and it can be called as Polignac's prime theorem.*

4.22 Bertrands prime theorem

Theorem 10 *There is at least one prime between N and 2N.*

Proof 9 *According to PNT* $\frac{N}{\ln(N)}$ *shall be number of primes up to N. As the result* $\log_{\frac{N}{\ln(N)}} 2N = \log_{\frac{N}{\ln(N)}} 2 + \log_{\frac{N}{\ln(N)}} N$ *shall be always greater than 1, there shall be at least one prime between N and 2N. Hence Bertrands postulates stands proved and it can be called as Bertrands theorem. This proof is most elementary compared to Chebyshev, Paul Erdos's proof.*

4.23 Legendre's prime theorem

Theorem 11 *There is always a prime number between* n^2 *and* $(n + 1)^2$ *provided that* $n \neq -1$*, 0.*

Proof 10 *Let N be a arbitrarily large number. Sum of squares of all the natural numbers up to N shall be* $\frac{N(N+1)(2N+1)}{6}$*. Double of the sum shall be* $\frac{N(N+1)(2N+1)}{3}$*. According to PNT we know that there shall be* $\frac{N}{\ln(N)}$ *number of primes up to N with an average prime gap of* $\ln(N)$*. We can visualise* $\frac{N}{\ln(N)}$ *as a prime number itself and we can allow the prime gaps to change equivalently and complete the number sequence. Now if we take logarithm of* $\frac{N(N+1)(2N+1)}{3}$ *with respect to the base of* $\frac{N}{\ln(N)}$ *the result shall give us the lower bound of powers that can comfortably be applied on that prime to reach double of the sum of squares* *of all the natural numbers up to N i.e.* $\frac{N(N+1)(2N+1)}{3}$. *In other words if we consolidate the average prime gaps into a relatively large prime having approximate value of* $P_1 \leq \frac{N}{\ln(N)}$ *then that will lead us also to lower limit of Legendre's primes (as our push action shall have squeeze reaction on the prime gaps) which will satisfy the following set of equations* $P_g \leq \log_{\frac{N}{\ln(N)}} \frac{N(N+1)(2N+1)}{3}, P_1 + P_g = P_2, P_1^{\log_{\frac{N}{\ln(N)}} \frac{N(N+1)(2N+1)}{3}} \leq \frac{N(N+1)(2N+1)}{3}$. *Similarly replacing sum of* N^2 *by sum of* $(N+1)^2$ *we get* $P_g \leq \log_{\frac{N}{\ln(N)}} \frac{(N+1)(N+2)(2N+3)}{3}, P_1 + P_g = P_3, P_1^{\log_{\frac{N}{\ln(N)}} \frac{(N+1)(N+2)(2N+3)}{3}} \leq \frac{(N+1)(N+2)(2N+3)}{3}$. *If our resultant exponent is greater than 2 both the cases (ideally it should be greater than or equal to 2 as we have ensured all primes are summed up 2 times) then that will be greater than the lower bound of prime gaps 2 which eventually ensures continuity of the pattern up to infinity. Clearly due to infinitude of primes, both the results*

$$\log_{\frac{N}{\ln(N)}} \frac{N(N+1)(2N+1)}{3} = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (N+1) + \log_{\frac{N}{\ln(N)}} (2N+1) - \log_{\frac{N}{\ln(N)}} 3$$

and $\log_{\frac{N}{\ln(N)}} \frac{(N+1)(N+2)(2N+3)}{3} = \log_{\frac{N}{\ln(N)}} (N+1)((N+1)+1)((2N+1)+2) = \log_{\frac{N}{\ln(N)}} (N+1) + \log_{\frac{N}{\ln(N)}} ((N+1)+1) + \log_{\frac{N}{\ln(N)}} ((2N+2)+1)) - \log_{\frac{N}{\ln(N)}} 3$ *are greater than 2.And due to complete pattern of extra little quantity of +1 another prime can occur in the interval meaning that the lower limit of number of primes in the interval between* n^2 *and* $(n+1)^2$. *So the limit of Legendre's primes would be greater than 1. Thus there shall be at least one prime between* n^2 *and* $(n+1)^2$ *as Legendre conjectured. Hence Legendre's prime conjecture stands proved and it can be called as Legendre's theorem.*

4.24 Landau's prime theorem

Theorem 12 *There are infinitely many prime numbers of the form* $N^2 + 1$.

Proof 11 *We know*

$$P_1 + P_2 = \text{Even number followed from Goldbach theorem}$$

$$2N + 1 = \text{Odd Prime if N= Odd prime followed from Sophie Germain theorem}$$

Now lets consider this following statement:

$$(N+1)^2 + 1 = N^2 + 2N + 1 + 1 = N^2 + 1 + 2N + 1 = \text{Even number obviously if N= Even}$$

For infinite number of cases $2N+1 =$ N=Odd Sophie Prime *in the above equation ,* N^2+1 *must have infinite number of prime solution applying Goldbach theorem.*

Proof 12 *Let N be a arbitrarily large number. Sum of squares of all the natural numbers up to N shall be* $\frac{N(N+1)(2N+1)}{6}$. *Double of the sum shall be* $\frac{N(N+1)(2N+1)}{3}$. *According to PNT we know that there shall be* $\frac{N}{\ln(N)}$ *number of primes up to N with an average prime gap of* $\ln(N)$. *We can visualise* $\frac{N}{\ln(N)}$ *as a prime number itself and we can allow the prime gaps to change equivalently and complete the number sequence. Now if we take logarithm of* $\frac{N(N+1)(2N+1)}{3}$ *with respect to the base of* $\frac{N}{\ln(N)}$ *the result shall give us the lower bound of prime powers that can comfortably be applied on that prime to reach double of the sum of squares of all the natural numbers up to N i.e.* $\frac{N(N+1)(2N+1)}{3}$. *In other words if we consolidate the average prime gaps into a relatively large prime having approximate value of* $P_1 \leq \frac{N}{\ln(N)}$ *then that will lead us also to lower limit of Landau's primes (as our push action shall have squeeze reaction on the prime gaps) which will satisfy the following set of equations* $P_g \leq \log_{\frac{N}{\ln(N)}} \frac{N(N+1)(2N+1)}{3}, P_1 + P_g = P_2, P_1^{\log_{\frac{N}{\ln(N)}} \frac{N(N+1)(2N+1)}{3}} \leq \frac{N(N+1)(2N+1)}{3}$. *If our resultant exponent is greater than 2 (ideally it should be greater than or equal to 2 as we have ensured all primes are summed up 2 times) then that will be greater than the lower bound of prime gaps 2 which eventually ensures continuity of the pattern up to infinity. Clearly due to infinitude of primes, the result* $\log_{\frac{N}{\ln(N)}} \frac{N(N+1)(2N+1)}{3} = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (N+1) + \log_{\frac{N}{\ln(N)}} (2N+1)$ *is greater than 2. As exponent 2 is always lesser than the exponent as calculated above, there shall be infinitely many prime numbers of the form* $N^2 + 1$. *Hence Landau's prime conjecture stands proved and it can be called as Landau's prime theorem.*

4.25 Brocard's prime theorem

Theorem 13 *With the exception of 4, there are always at least four primes between the square of a prime and the square of the next prime.*

Proof 13 *Let N be a arbitrarily large number. Sum of squares of all the natural numbers up to N shall be* $\frac{N(N+1)(2N+1)}{6}$. *Double of the sum shall be* $\frac{N(N+1)(2N+1)}{3}$. *According to PNT we know that there shall be* $\frac{N}{\ln(N)}$ *number of primes up to N with an average prime gap of* $\ln(N)$. *We can visualise* $\frac{N}{\ln(N)}$ *as a prime number itself and we can allow the prime gaps to change equivalently and complete the number sequence. Now if we take logarithm of* $\frac{N(N+1)(2N+1)}{3}$ *with respect to the base of* $\frac{N}{\ln(N)}$ *the result shall give us the lower bound of prime powers that can comfortably be applied on that prime to reach double of the sum of squares of all the natural numbers up to N i.e.* $\frac{N(N+1)(2N+1)}{3}$. *In other words if we consolidate the average prime gaps into a relatively large prime having approximate value of* $P_1 \leq \frac{N}{\ln(N)}$ *then that will lead us also to lower limit of primes between two successive squares (as our push action shall have squeeze reaction on the prime gaps)* *which will satisfy the following set of equations* $P_g \leq \log_{\frac{N}{\ln(N)}} \frac{N(N+1)(2N+1)}{3}, P_1 + P_g = P_2, P_1^{\log_{\frac{N}{\ln(N)}} \frac{N(N+1)(2N+1)}{3}} \leq \frac{N(N+1)(2N+1)}{3}$. *If our resultant exponent is greater than 2 (ideally it should be greater than or equal to 2 as we have ensured all primes are summed up 2 times) then that will be greater than the lower bound of prime gaps 2 which eventually ensures continuity of the pattern up to infinity. Clearly due to infinitude of primes, the result* $\log_{\frac{N}{\ln(N)}} \frac{N(N+1)(2N+1)}{3} = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (N+1) + \log_{\frac{N}{\ln(N)}} (2N+1)$ *is greater than 2. In case of interval between two consecutive primes the above limit get raised to the power of its own value meaning that there shall be at least 4 primes between the square of a prime and the square of the next prime. Hence Brocard's prime conjecture stands proved and it can be called as Brocard's prime theorem.*

4.26 Opperman's prime theorem

Theorem 14 *For every integer $x > 1$, there is at least one prime number between $x(x-1)$ and x^2,and at least another prime between x^2 and $x(x+1)$*

Proof 14 *Let N be a arbitrarily large number. Sum of square of all the natural numbers up to N shall be* $\frac{N(N+1)(2N+1)}{6}$. *Double of the sum shall be* $\frac{N(N+1)(2N+1)}{3}$. *Sum of all the natural numbers up to N shall be* $\frac{N(1+N)}{2}$. *Double of the sum shall be* $N(1+N)$ *which shall include double of sum of all the primes up to N too. According to PNT we know that there shall be* $\frac{N}{\ln(N)}$ *number of primes up to N with an average prime gap of* $\ln(N)$. *We can visualise* $\frac{N}{\ln(N)}$ *as a prime number itself and we can allow the prime gaps to change equivalently and complete the number sequence. Subtracting* $N(1+N)$ *from* $\frac{N(N+1)(2N+1)}{3}$ *we get* $N(N+1).\frac{2N-2}{3}$. *Now if we take logarithm of* $N(N+1).\frac{2N-2}{3}$ *with respect to the base of* $\frac{N}{\ln(N)}$ *the result shall give us the lower bound of prime powers that can comfortably be applied on that prime to reach* $N(N+1).\frac{2N-2}{3}$. *In other words if we consolidate the average prime gaps into a relatively large prime having approximate value of* $P_1 \leq \frac{N}{\ln(N)}$ *then that will lead us also to lower limit of primes between* x^2 *and* $x^2 - x$ *(as our push action shall have squeeze reaction on the prime gaps) which will satisfy the following set of equations* $P_g \leq \log_{\frac{N}{\ln(N)}} \frac{N(N+1)(2N-2)}{3}, P_1 + P_g = P_2, P_1^{\log_{\frac{N}{\ln(N)}} \frac{N(N+1)(2N-2)}{3}} \leq \frac{N(N+1)(2N-2)}{3}$. *Clearly due to infinitude of primes, the result* $\log_{\frac{N}{\ln(N)}} N(N+1).\frac{2N-2}{3} = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (1+N) + \log_{\frac{N}{\ln(N)}} \frac{2N-2}{3}$ *shall be greater than 2 meaning that there shall be at least one prime between* $x(x-1)$ *and* x^2. *Again adding* $N(1+N)$ *with* $\frac{N(N+1)(2N+1)}{3}$ *we get* $N(N+1).\frac{(2N+4)}{3}$. *Now if we take logarithm of* $N(N+1).\frac{(2N+4)}{3}$ *with respect to the* *base of $\frac{N}{\ln(N)}$ the result shall give us the lower bound of prime powers that can comfortably be applied on that prime to reach $N(N+1).\frac{(2N+4)}{3}$. In other words if we consolidate the average prime gaps into a relatively large prime having approximate value of $P_1 \leq \frac{N}{\ln(N)}$ then that will lead us also to lower limit of primes between x^2+x and x^2 (as our push action shall have squeeze reaction on the prime gaps) which will satisfy the following set of equations $P_1 \leq \frac{N}{\ln(N)}$ to reach $N(N+1).\frac{(2N+4)}{3}$ satisfying the following set of equations $P_g \leq \log_{\frac{N}{\ln(N)}} \frac{N(N+1)(2N+4)}{3}, P_1 + P_g = P_3, P_1^{\log_{\frac{N}{\ln(N)}} \frac{N(N+1)(2N+4)}{3}} \leq \frac{N(N+1)(2N+4)}{3}$. Clearly due to infinitude of primes, the result $\log_{\frac{N}{\ln(N)}} N(N+1).\frac{(2N+4)}{3} = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (1+N) + \log_{\frac{N}{\ln(N)}} \frac{(2N+4)}{3}$ shall be greater than 2 meaning that there shall be at least one prime between x^2 and $x(x+1)$. Altogether Opperman's conjecture stands proved and it can be called as Opperman's theorem.*

4.27 Firozbakht's prime theorem

Theorem 15 *$p_n^{\frac{1}{n}}$ (where p_n is the nth prime) is a strictly decreasing function of n, i.e., $\sqrt[n+1]{p_{n+1}} < \sqrt[n]{p_n}$ Equivalently: $p_{n+1} < p_n^{1+\frac{1}{n}}$ for all $n \geq 1$*

Proof 15 *According to PNT we know that there shall be $\frac{N}{\ln(N)}$ number of primes up to N with an average prime gap of $\ln(N)$. The same is also true in case of primes i.e. there shall be $\frac{P}{\ln(P)}$ number of primes up to P with an average prime gap of $\ln(P)$. Now let us consider the following logarithmic expression.*

$$\log_{\frac{P}{\ln(P)}} (P+2) < \log_{\frac{P}{\ln(P)}} P + \frac{1}{P} \log_{\frac{P}{\ln(P)}} P$$

Which we can show by the following steps

$$\begin{aligned}
&\log_{\frac{P}{\ln(P)}} (P+2) < \log_{\frac{P}{\ln(P)}} P + \frac{1}{P} \log_{\frac{P}{\ln(P)}} P \\
\implies &\log_{\frac{P}{\ln(P)}} (P+2) < \log_{\frac{P}{\ln(P)}} P + \log_{\frac{P}{\ln(P)}} P^{\frac{1}{P}} \\
\implies &\log_{\frac{P}{\ln(P)}} (P+2) < \log_{\frac{P}{\ln(P)}} P.P^{\frac{1}{P}} \\
\implies &\log_{\frac{P}{\ln(P)}} (P+2) < \log_{\frac{P}{\ln(P)}} P^{1+\frac{1}{P}} \\
\implies &\log_{\frac{P_n}{\ln(P_n)}} (P+2) < \log_{\frac{P_n}{\ln(P_n)}} P_n^{1+\frac{1}{P_n}} \\
P_n \to \infty \implies &P_{n+1} < P_n^{1+\frac{1}{n}}
\end{aligned}$$

Hence Firozbakht's prime conjecture stands proved and it can be called as Firozbakht's prime theorem.

Theorem 16 *The Collatz conjecture is: This process will eventually reach the number 1, regardless of which positive integer is chosen initially. That smallest i such that ai = 1 is called the total stopping time of n. The conjecture asserts that every n has a well-defined total stopping time. If, for some n, such an i doesn't exist, we say that n has infinite total stopping time and the conjecture is false. If the conjecture is false, it can only be because there is some starting number which gives rise to a sequence that does not contain 1. Such a sequence would either enter a repeating cycle that excludes 1, or increase without bound.*

Proof 16 *Collatz conjectured operations on any number (i.e. halving the even numbers or simultaneously tripling and adding 1 to odd numbers) may blow up to infinity or come down to singularity or may get stuck in a loop in between. Tripling and adding 1 to odd numbers will always land on an even number. Now to end the game we just need to step upon an even number which is of the form* 2^n*. Will that happen always up to infinity when odd primes are tripled and added to 1? Let us call n as Collatz exponent. We have seen that odd primes are kind of descendants of sole even prime 2 via zeta zeros which again appear infinitely in cycles of 3 divisions. Multiplying by 3 simply connects it back to the sequence up to infinity. This small bias turns the game of equal probability into one sided game. Let N be a arbitrarily large number. Sum of squares of all the natural numbers up to N shall be* $\frac{N(N+1)(2N+1)}{6}$*. Double of the sum shall be* $\frac{N(N+1)(2N+1)}{3}$*. According to PNT we know that there shall be* $\frac{N}{\ln(N)}$ *number of primes up to N with an average prime gap of* $\ln(N)$*. We can visualise* $\frac{N}{\ln(N)}$ *as a prime number itself and we can allow the prime gaps to change equivalently and complete the number sequence. Now if we take logarithm of* $\frac{N(N+1)(2N+1)}{3}$ *with respect to the base of* $\frac{N}{\ln(N)}$ *the result shall give us the lower bound of prime powers that can comfortably be applied on that prime to reach double of the sum of squares of all the natural numbers up to N i.e.* $\frac{N(N+1)(2N+1)}{3}$*. In other words if we consolidate the average prime gaps into a relatively large prime having approximate value of* $P_1 \leq \frac{N}{\ln(N)}$ *then that will lead us also to lower limit of the collatz exponent (as our push action shall have squeeze reaction on the prime gaps) which will satisfy the following set of equations* $P_g \leq \log_{\frac{N}{\ln(N)}} \frac{N(N+1)(2N+1)}{3}, P_1 + P_g = P_2, P_1^{\log_{\frac{N}{\ln(N)}} \frac{N(N+1)(2N+1)}{3}} \leq \frac{N(N+1)(2N+1)}{3}$*. If our resultant exponent is greater than 2 (ideally it should be greater than or equal to 2 as we have ensured all primes are summed up 2 times) then that will be greater than the lower bound of prime gaps 2 which eventually ensures continuity of the pattern up to infinity. Clearly due to infinitude of primes, the* *result* $\log_{\frac{N}{\ln(N)}} \frac{N(N+1)(2N+1)}{3} = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (N+1) + \log_{\frac{N}{\ln(N)}} (2N+1)$ *is greater than 2 . As resultant exponent is always greater than 2 Collatz conjecture neither blows up to infinity nor it get stuck in a loop, it always lands on an even number of the form* 2^n *and one last step before the final whistle bring it down to singularity 1 as Collatz conjectured. Hence Collatz conjecture stands proved and it can be called as Collatz prime theorem.*

5 Let us draw the conclusion

Riemann hypothesis stands proved in different ways primarily involving the concept of duality in d-unit circle, the concept of harmonic conjugation in complex analysis, the concept quaternions in 4-dimensional number system. It has given a lot of new mathematics to research further.

6 Discussion Notes on Riemann applied math

Frequently appeared Questions and Answers 1 *Here goes list of some frequently appeared questions related to Riemann hypothesis and its application while undergoing the concept building process and answers to that based on my understanding of Riemann Hypothesis, my proofs thereof and post proof analysis of the same.*

Now the question may arise do we need further higher dimensional number system such as octonion, sedenions etc. to solve further higher dimensional problems

We invented complex numbers to solve all kinds of quadratic equations. Quaternion although invented long back remained unused as complex numbers did its job excellently. Now zeros of zeta functions are asking us to go beyond the complex numbers and bring quaternion. Also it introduces algebraic cycles through cycles of zeros (not the place holder zero in decimal number system) so that we do not need octonion, sedenions etc. in general math. However any specific problem related to eight dimensional number system may still require octonion and so on.

If so how do we manage non commutativity faced in quaternion

Non commutativity will not be an issue as we will be doing only linear things.

What will happen to non linear things

We shall turn it into linear by way of passing it through the cyclic math.

Cyclic math! negative quantities will be annihilating positive quantities, what remain absolute then, a zero, It's ugly

Nope, not exactly, it's beautiful as we are allowed to apply Pythagoras, take modulus etc. to work with the absolute value, distance from origin in the normal way.

Then what will be the added advantage of using cycles
We can fit everything into 3 Euclidean spatial dimensions using Descartes coordinates.

Does that mean higher dimensions do not exist at all
We need to differentiate between higher dimensions and higher degrees of freedom, we can have as many degrees of freedom as we wish but mathematically all of those higher degrees of freedom can be fitted into 3D because nature is 3 dimensional and its self evident from the value of $\pi = 3.14....$

What happens to the decimal part
Thats the residue which cannot be renormalised.

What the value of e then suggest
Value of e suggest half of a cycle involving another cycle of three.

How cycle of three
$e^{i\pi} + e^0 = 1 - 1 = 0$ *way and total count of lattice points is 3 as clear from [1 - 1 = 0]*

Hmm! $e^{i\theta} = e^{-i\theta}$ **will be true when both the exponent results zero which will give** $e^0 = 1$ **and subsequently** $e^{i\pi} + e^0 = 0$ **. What is so cyclic about it**
It's the sum total of one complete rotation in orthogonal representation of complex numbers [1+ i - 1 - i = 0] .

Here the count of lattice points are 5, then how its a cycle of 3.
It happens at an angle of $\frac{\pi}{3}$ *and that way its a cycle of three.*

Now we have got Figenbaum constant with approximate value of 4.67 representing infinite fractals, what that will suggest
That actually projects to nonlinear 6D math without cycles, which we can also fit into 3D linear math with cycles.

What about 12 D
That will be mere symmetrical copy of one complete cycle.

What if we show resistance in accepting this kind of deadly math
We will not be able to close the operation of logarithm, factorials algebraically or arithmetically. The day we accept this definitions, logarithm will be an algebraically or arithmetically closed operation under quaternion based number system and factorials will be an algebraically or arithmetically closed operation under complex number system .

Can all operation be closed algebraically or arithmetically under quaternion based number system. What shall be next

Next can be any higher order operations like multifactorial will have cycles of double factorial and primorials, chain arrow notation will have cycles of mutifactorials and primorials, up arrow notation will have cycles of chain arrow notation and mutifactorials, Tree(3) notation will have cycles of up arrow notation and chain arrow notation and so on. I have not checked the math as these were out of scope of my current paper, I may be wrong somewhere but it will be cyclic, that's guaranteed. I am sure in future somebody interested in cycles will pick up this area for research. Come on buddies show your balance; show how quick you can close how many cycles. I was very sloppy; it took almost 3 years for me as I did not know the rules of the game. You have rules of the game now, Its cycle of three. Although quantitatively it's going to be more complex you need to simplify it. I have a feel that you may have to define infinite number of factorial functions similar to the fantastic five, pass it through the zeros of negative logarithm or zeros of harmonic conjugates of those infinite number factorial functions and knock down the functions having sky scrapers in their exponent floor by floor or flatten down those functions having horizontal growth. Warning! You will get frequent quitting tendencies, do not get bogged down. Keep yourself tightly tied to zero, find the cycle of nonlinear linear combination of three's, make it linear by finding its zeros you will catch them all. Once you complete the cycles you will feel to be on top of the world. You will not find anybody nearby to share your feeling that's sad part of the story.

We can have infinite cycles like this, does that mean i shall have infinite number of real solutions

Yes, that's true but this infinites must be very smaller compared to the infinites of the number line itself.

Why, why it will be so

Because this infinities are sitting in the exponent of e via Euler's formula e^{ix}. *Even smaller values of i will be growing exponentially fast. For example with* $i = \ln 2$ *and a relatively smaller value for* $x = 271$ *will give us a number close to the number of particles in the observable universe. For higher values of i it will be growing even faster.*

Is there any general formula to calculate the cycles

Yes there can be as many formulas as many mathematicians on the earth can work out their own, all road goes to cycles. For n number of rotations, dimensions, etc. whatever name we call it the cycle will be completing near $e^{i \cdot \sum_{n=1}^{\infty}(1+\pi^n)}$. *I have used pi as the rotator because it's natural, it can be anything else as well. Also we can take partial sum or a truncated partial sum or a single term anywhere from the series to predict some particular cyclic behaviour. In case of a single iteration we can use* e^{n^2}. *For more* *accuracy we need to take smaller and smaller values of the rotator. For n=3 in the above series formula, it will be accurate enough to predict the cycle of big bounces happening periodically in cosmos.*

What will be the math
Refer 3.6 for the math behind the first cycle.

What is so special about the number $e^{22.18} = 4,324,402,934$ do that appear in nature somewhere
Yes, they are special. They are there in physics

$$\frac{2 \times \text{Mass of electron} \times \text{Speed of light squared} \times \text{Charles ideal gas constant}}{\text{Boltzmann constant}} \approx e^{\frac{\pi}{(\pi-3)}}$$

, they are there in biology in the form of average number of heartbeats in human life time, they are there in cosmology in the form of Black hole alignment, they are everywhere.

What is the next special cycle number, do that also appear in nature somewhere
Its $e^{33.38} = 3.27 \times 10^{15}$. Yes, they also appear. They are there in physics

$$\frac{\text{Speed of light} \times \text{Planck Constant}}{\text{Graviational constant} \times \text{Mass of electron}} \approx e^{2+\frac{4\pi-3}{2\pi(\pi-3)}} \approx e^{35.72} \approx 3.27 \times 10^{15}$$

, they are there in biology in the form of average number of cell divisions in human life time, they are there in cosmology in the form of Large number hypothesis, Multiverse they are everywhere.

But Large number hypothesis if of 10^{43} orders of magnitude
Its 3 cycles again $e^{100} \approx 10^{43}$.

What charles constant is doing there
Its kind of coupling constant although not dimensionless but complexly dimensionless and its reciprocal of 100 times of eulers number e.

Wherefrom 100 is coming here
π^4 *approximates to hundred. When pi is squared the decimal part results to 1 as follows*

$$\frac{1.10^n + 2.10^{n-1} + 3.10^{n-2} + 4.10^{n-3} + 5.10^{n-4} + ...}{10^n}$$

when n tends to infinity factoring out

$$\frac{10^n(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + ...)}{10^n}$$

which finally gives $\pi^2 = 9 + \zeta(1) = 9 + 1 = 10$

Dimensionless physical constant: At the present time, the values of the dimensionless physical constants cannot be calculated; they are determined only by physical measurement. What is the minimum number of dimensionless physical constants from which all other dimensionless physical constants can be derived? Are dimensional physical constants necessary at all?
For each cycle there can be different dimensionless physical constants. Even in the planck cycle alpha is not enough, we need to half it to describe the cyclicity among physical events. Dimensional physical constants are not necessary at all to describe reality in simple terms. When it comes to detailing, physical constants are required.

Fine-tuned universe: The values of the fundamental physical constants are in a narrow range necessary to support carbon-based life. Is this because there exist other universes with different constants, or are our universe's constants the result of chance, or some other factor or process? In particular, Tegmark's mathematical multiverse hypothesis of abstract mathematical parallel universe formalized models, and the landscape multiverse hypothesis of spacetime regions having different formalized sets of laws and physical constants from that of the surrounding space — require formalization.
The apparent fine tuning is due to renormalising infinities arising out of math to some finite result to match our experiment or observation. Doing so we miss the cycles of physical events and we settle with a narrow bound which is not the reality. The reality is everywhere its infinite and within that infinity rising of carbon based life forms is not at all a miracle. Zeta results and numerical relativity coming out of zeta results directly proves all the four levels of Tegmark's mathematical multiverse hypothesis. We need to formally accept it.

What is the most prominent number theoretic signature in physics
Its dark energy, 69 % $= \ln 2 = i$.

What should be the solution to dark energy
Pass it through cyclic math and let the universe go older and older, cycle after cycles.

What should be the math
There are two independent Friedmann equations for modeling a homogeneous, isotropic universe. The first is:

$$\frac{\dot{a}^2 + kc^2}{a^2} = \frac{8\pi G\rho + \Lambda c^2}{3}$$

which is derived from the 00 component of Einstein's field equations. The second is:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}$$

which is derived from the first, together with the trace of Einstein's field equations. Using the first equation, the second equation can be re-expressed as $\dot{\rho} = -3H\left(\rho + \frac{p}{c^2}\right)$ *which eliminates* Λ *and expresses the conservation of mass-energy* $T^{\alpha\beta}{}_{;\beta} = 0$. *These equations are sometimes simplified by replacing* $\rho \to \rho - \frac{\Lambda c^2}{8\pi G}$ *and* $p \to p + \frac{\Lambda c^4}{8\pi G}$ *to get the following:*

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right).$$

The simplified form of the second equation is invariant under this transformation. We can bring cycles as follows:

$$\frac{\ddot{a}}{a\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)} = -1 \implies \dot{H} + H^2 = 0$$

That's applied math, what it has to do with Number theory
It's a zero, for every branch of mathematics it is equally applicable.

This kills SR and GR, speed of light, gravitational constant all become variable; it can take zero / infinite values
No, it does not kill SR and GR, Its ultimate relativity. Its numerical relativity which paves the path for unification of GR and QM.

Does that mean both GR representing classical physics and QM representing quantum physics involves cycles and they are also Number theory stuff
Yes, if physical dimensions are flattened, what left is number theory.

What is wrong in QM
The probabilistic view of nature, heisenberg uncertainty principle is not always true. QM can be turned into a deterministic theory once we start following the cycle of 3. Uncertainty in $\Delta p \Delta x \geq \frac{1}{2}\hbar$ *can be transformed to* $\Delta p \Delta x \Delta y = 1$ *using the techniques of Fourier transformation provided we recognise the cycle of 3 in QM. Dimensionally* $\Delta p \Delta x \geq \frac{1}{2}\hbar$ *has got one extra dimension of length after completing a cycle of 3 which creates problem causing the uncertainty. To get back our determinism we need to complete the cycle of 3 same as we complete the cycle of 3 in CPT symmetry.*

This kills QM, space time become absolutely continuous, no discrete nature remains then

No, it does not kill QM, we are just adding cycles, we are not stooping anywhere, we will just keep on cycling and these cycles are itself discrete in nature.

Theory of everything: Is there a theory which explains the values of all fundamental physical constants, i.e., of all coupling constants, all elementary particle masses and all mixing angles of elementary particles? Is there a theory which explains why the gauge groups of the standard model are as they are, and why observed spacetime has 3 spatial dimensions and 1 temporal dimension? Are "fundamental physical constants" really fundamental or do they vary over time? Are any of the fundamental particles in the standard model of particle physics actually composite particles too tightly bound to observe as such at current experimental energies? Are there elementary particles that have not yet been observed, and, if so, which ones are they and what are their properties? Are there unobserved fundamental forces?
Yes, Its Number Theory. space-time has 3 spatial dimensions and 1 temporal dimension because that 3+1 pattern is natural. When we try to unify everything we need to solve the zeros of physical equations which will bring cycles of physical phenomenon in our math. The results we get will match the observed values and that will be the proof of correctness of our theories. "Fundamental physical constants" are really fundamental but numerically we need to apply scale factor adjustments as we move along the infinite space-time. There are no elementary particles that have not yet been observed, we need to sum up the anti matter version of it to get a zero state. There are no unobserved fundamental forces as it has completed the cycles of three. Gravity is a pseudo force, its curvature of space time therefore we have to exclude it from unification. There will be no conflict between GR and QM at zero gravity and zero time, there after QM can take on. GR 's job will be renormalising the infinities arising in QM and vice versa.

Arrow of time (e.g. entropy's arrow of time): Why does time have a direction? Why did the universe have such low entropy in the past, and time correlates with the universal (but not local) increase in entropy, from the past and to the future, according to the second law of thermodynamics? Why are CP violations observed in certain weak force decays, but not elsewhere? Are CP violations somehow a product of the second law of thermodynamics, or are they a separate arrow of time? Are there exceptions to the principle of causality? Is there a single possible past? Is the present moment physically distinct from the past and future, or is it merely an emergent property of consciousness? What links the quantum arrow of time to the thermodynamic arrow?
Because time does not exist or in other words negative proper time and positive proper time always sum up to zero. However existence itself is the result of amplification of

some implied quantities by way of squaring/ doubling it. The act of squaring gives the notion of one way time, however when we take logarithm, it becomes cyclic and the apparent straight line bends back to the origin. The universal entropy is always zero and it does not increase which is not a violation of second law of thermodynamics. CP violation occurs due to obey CPT symmetry which is 3 dimensional as nature. No there is no exception to the principal of causality. The only single possible past is zero or nothing. The present moment is physically distinct from the past and future as the notion of time is an emergent property of consciousness and a conscious mind neither can follow cycles nor it can comprehend the infinities in cycles. Time zero links the quantum arrow of time to the thermodynamic arrow.

Physical information: Are there physical phenomena, such as wave function collapse or black holes that irrevocably destroy information about their prior states? How is quantum information stored as a state of a quantum system?
Information cannot be destroyed, they may undergo physical changes cyclically but they reappear elsewhere coming in contact with its conjugal pair.

Interpretation of quantum mechanics: How does the quantum description of reality, which includes elements such as the superposition of states and wavefunction collapse or quantum decoherence, give rise to the reality we perceive? Another way of stating this question regards the measurement problem: What constitutes a "measurement" which apparently causes the wave function to collapse into a definite state? Unlike classical physical processes, some quantum mechanical processes (such as quantum teleportation arising from quantum entanglement) cannot be simultaneously "local", "causal", and "real", but it is not obvious which of these properties must be sacrificed, or if an attempt to describe quantum mechanical processes in these senses is a category error such that a proper understanding of quantum mechanics would render the question meaningless. Can a multiverse resolve it?
Wave function never collapses, we can restore wave pattern just by detecting the spin simultaneously in both of the detector in the famous double slit experiment. We will see the wave pattern comes back. The act of measurement do not play any role in a scalable experiment, still we need little bit approximations corresponding to desired level of accuracy due to uncertainties involved. Quantum entanglement is local and not subject to causality as the event itself is its cause. We can do an experiment to prove its locality. 3 lab situated in 3 cities anywhere in the globe will entangle pair of photons among themselves and make a triangle as such. Now a fourth lab will not be able to entangle any further photons with any of the three above. Which will prove entanglement is local also do not violate causality, but its real. Multiverse will not be required to explain

entanglement.

Problem of time: In quantum mechanics time is a classical background parameter and the flow of time is universal and absolute. In general relativity time is one component of four-dimensional spacetime, and the flow of time changes depending on the curvature of spacetime and the spacetime trajectory of the observer. How can these two concepts of time be reconciled?
They can be reconciled at T=0. There from local quantum time line can begin giving the impression that local time is flowing. But after completion of a half time cycle the local time will start flow backward through cyclic changes and that will cause Big Bounce which will again go back to T=0 meaning "Time does not exist".

Cosmic inflation: Is the theory of cosmic inflation in the very early universe correct, and, if so, what are the details of this epoch? What is the hypothetical inflaton scalar field that gave rise to this cosmic inflation? If inflation happened at one point, is it self-sustaining through inflation of quantum-mechanical fluctuations, and thus ongoing in some extremely distant place?
Yes, its mathematically plausible. Once it happens, always it happens, therefore eternal inflation is more correct.

Horizon problem: Why is the distant universe so homogeneous when the Big Bang theory seems to predict larger measurable anisotropies of the night sky than those observed? Cosmological inflation is generally accepted as the solution, but are other possible explanations such as a variable speed of light more appropriate?
The local anisotropies gets exponentially smoothened out by eternal inflation so the homogeneousness. Variable speed of light has greater domain to work. Einstein realised that in a rotating frame of reference speed of light may not remain constant following Mach principal, he took the relativity route instead to keep the speed of light constant. His choice was wise. Now we just need a scale factor to scale up or scale down while moving between two rotating frame of reference. Overall speed of light remain constant.

Origin and future of the universe: How did the conditions for anything to exist arise? Is the universe heading towards a Big Freeze, a Big Rip, a Big Crunch, or a Big Bounce? Or is it part of an infinitely recurring cyclic model?
Universe existed and it will exist forever, Big bounce keep on happening cyclically across the universe meaning every structures that forms in an universe has got an age but the universe itself is timeless.

Size of universe: The diameter of the observable universe is about 93 billion light-years, but what is the size of the whole universe?
Its truly infinite even bigger than 10^{500} *multiverses as string theories suggest. Yet It can be unified into an eternal omniverse having zero entropy in total thermal equilibrium which can be seen as one boltzman brain where every event whether possible or impossible can happen over infinite time cycles.*

Baryon asymmetry: Why is there far more matter than antimatter in the observable universe?
Because antimatters are moving backward in time through cycle of changes, Its like one particle universe.

Cosmological constant problem: Why does the zero-point energy of the vacuum not cause a large cosmological constant? What cancels it out?
Because that energy runs infinite time cycles, infinite multiverses. Ln(2) is the cosmological constant and its huge enough to bind infinite multiverses and run the show for infinite time cycles.

Dark matter: What is the identity of dark matter? Is it a particle? Is it the lightest superpartner (LSP)? Or, do the phenomena attributed to dark matter point not to some form of matter but actually to an extension of gravity?
Failed combinations like Quaterquarks, Pentaquarks, or moderately successful Hexaquarks, Dodecaquarks and so on all acting like Bose Einstein condensates but not getting amplified enough to get detected by photons, sounds fit as dark matter candidates..

Dark energy: What is the cause of the observed accelerated expansion (de Sitter phase) of the universe? Why is the energy density of the dark energy component of the same magnitude as the density of matter at present when the two evolve quite differently over time; could it be simply that we are observing at exactly the right time? Is dark energy a pure cosmological constant or are models of quintessence such as phantom energy applicable?
Observed accelerated expansion (de Sitter phase) of the universe is due to non linear stages or phases or epochs in cycle of changes. The energy density of the dark energy component remain always of the same magnitude as of the density of matter through cycle of changes. There is nothing called right time as time itself does not exist. Dark energy apart from being a cosmological constant is also a mathematical constant having value of natural logarithm of 2. There can be infinite cycles, so the universal cosmological constant may also be infinitely small as we keep on dividing spaces into infinite partitions.

Dark flow: Is a non-spherically symmetric gravitational pull from outside the observable universe responsible for some of the observed motion of large objects such as galactic clusters in the universe?
No, The universe has got zero gaussian curvature meaning at large scale space-time is flat but at smaller scales it can have both positive or negative curvature. Due to some local non zero curvature observable universe may feel an outside pull.

Axis of evil: Some large features of the microwave sky at distances of over 13 billion light years appear to be aligned with both the motion and orientation of the solar system. Is this due to systematic errors in processing, contamination of results by local effects, or an unexplained violation of the Copernican principle?
No, this is not an error, actually this is so. The first cyclic number is 4.33 billion. Three cycles of that comes around 13 billion. So beyond 13 billion light years its more or less same cycle so the same synchronised motion, same orientation.

Shape of the universe: What is the 3-manifold of comoving space, i.e. of a comoving spatial section of the universe, informally called the "shape" of the universe? Neither the curvature nor the topology is presently known, though the curvature is known to be "close" to zero on observable scales. The cosmic inflation hypothesis suggests that the shape of the universe may be unmeasurable, but, since 2003, Jean-Pierre Luminet, et al., and other groups have suggested that the shape of the universe may be the Poincaré dodecahedral space. Is the shape unmeasurable; the Poincaré space; or another 3-manifold?
3-manifold of comoving space is spherical. As we take snapshots over different time we cannot follow the time cycles so it appears flat like a straight line. If we project it further to infinities it will give us zero curvature meaning a true flat space time over infinite time.

The largest structures in the universe are larger than expected. Current cosmological models say there should be very little structure on scales larger than a few hundred million light years across, due to the expansion of the universe trumping the effect of gravity. But the Sloan Great Wall is 1.38 billion light-years in length. And the largest structure currently known, the Hercules–Corona Borealis Great Wall, is up to 10 billion light-years in length. Are these actual structures or random density fluctuations? If they are real structures, they contradict the 'End of Greatness' hypothesis which asserts that at a scale of 300 million light-years structures seen in smaller surveys are randomized to the extent that the smooth distribution of the

universe is visually apparent.
Again this is due to scale gap. Why we are settling with one single bang. Series of bangs across the universe have bounded the smaller structures into a greater structure. Random density fluctuations are needed to explain the first bang, thereafter it does not remain random, there is a frequent pattern among the random numbers such as primes. Due to that pattern small series of bangs keep on happening every now and then. There is no end of greatness so the hypothesis is wrong. From an infinitely small patch of sky to the whole omniverse everything is connected with every other thing through cycles and zeros. That's why at large scale it looks smooth every direction, within such large scale there are randomness over small scales.

How to quantize GR
Here goes the second quantisation of GR from Dirac equation

$$i\hbar\gamma^{\mu}\partial_{\mu}\psi - mc\psi = 0 \implies -\frac{i\hbar\gamma^{\mu}\partial_{\mu}\psi}{mc\psi} = -1$$

Name	Dimension	Expression	Value (SI unit)
Planck length	Length (L)	$l_{\mathrm{R}} = \left(\sqrt{\frac{\hbar G}{c^3}}\right)^{-2}$	$2.612280807 \times 10^{70} m$
Planck mass	Mass (M)	$m_{\mathrm{R}} = \left(\sqrt{\frac{\hbar c}{G}}\right)^{-2}$	$4.736869309 \times 10^{16} kg$
Planck time	Time (T)	$t_{\mathrm{R}} = \left(\frac{l_{\mathrm{R}}}{c}\right)^{-2} = \left(\frac{\hbar}{m_{\mathrm{R}}c^2}\right)^{-2} = \left(\sqrt{\frac{\hbar G}{c^5}}\right)^{-2}$	$2.906554422 \times 10^{89} s$
Planck charge	Electric charge (Q)	$q_{\mathrm{R}} = \left(\sqrt{4\pi\varepsilon_0\hbar c}\right)^{-2} = \left(\frac{e}{\sqrt{\alpha}}\right)^{-2}$	$3.517672633 \times 10^{36} C$
Planck temperature	Temperature (Θ)	$T_{\mathrm{R}} = \left(\frac{m_{\mathrm{R}}c^2}{k_{\mathrm{B}}}\right)^{-2} = \left(\sqrt{\frac{\hbar c^5}{Gk_{\mathrm{B}}^2}}\right)^{-2}$	$2.007279736 \times 10^{-64} K$

Table 1: Tabulated value of squared Planck Units

Is there a third
Here goes the third quantisation from the Schrödinger equation $\hat{\mathrm{H}}\,|\Psi\rangle - E|\Psi\rangle = 0$.

Where is the first one
The first one is the original one we are using now.

But how the physics equations will balance, it cannot be one sided change, what should the relativistic version of it
Here it goes.

The next one should be one fourth
No, its one third, remember zeta zeros lies on pi by 3.

Name	Dimension	Expression	Value (SI unit)
Planck length	Length (L)	$l_{\mathrm{R}} = \left(\sqrt{\frac{\hbar G}{c^3}}\right)^{-4}$	$6.824007974 \times 10^{140} m$
Planck mass	Mass (M)	$m_{\mathrm{R}} = \left(\sqrt{\frac{\hbar c}{G}}\right)^{-4}$	$2.243793085 \times 10^{33} kg$
Planck time	Time (T)	$t_{\mathrm{R}} = \left(\frac{l_{\mathrm{R}}}{c}\right)^{-4} = \left(\frac{\hbar}{m_{\mathrm{R}} c^2}\right)^{-4} = \left(\sqrt{\frac{\hbar G}{c^5}}\right)^{-4}$	$8.448058605 \times 10^{178} s$
Planck charge	Electric charge (Q)	$q_{\mathrm{R}} = \left(\sqrt{4\pi\varepsilon_0 \hbar c}\right)^{-4} = \left(\frac{e}{\sqrt{\alpha}}\right)^{-4}$	$1.237402075 \times 10^{73} C$
Planck temperature	Temperature (Θ)	$T_{\mathrm{R}} = \left(\frac{m_{\mathrm{R}} c^2}{k_{\mathrm{B}}}\right)^{-4} = \left(\sqrt{\frac{\hbar c^5}{G k_{\mathrm{B}}^2}}\right)^{-4}$	$4.029171939 \times 10^{-128} K$

Table 2: Tabulated value of double squared Planck Units

Name	Dimension	Expression	Value (SI unit)
Planck length	Length (L)	$l_{\mathrm{P}} = \left(\sqrt{\frac{\hbar G}{c^3}}\right)^{\frac{1}{2}}$	$4.020267404 \times 10^{-17} m$
Planck mass	Mass (M)	$m_{\mathrm{P}} = \left(\sqrt{\frac{\hbar c}{G}}\right)^{\frac{1}{2}}$	$1.475274551 \times 10^{-4} kg$
Planck time	Time (T)	$t_{\mathrm{P}} = \left(\frac{l_{\mathrm{P}}}{c}\right)^{\frac{1}{2}} = \left(\frac{\hbar}{m_{\mathrm{P}} c^2}\right)^{\frac{1}{2}} = \left(\sqrt{\frac{\hbar G}{c^5}}\right)^{\frac{1}{2}}$	$2.321905898 \times 10^{-22} s$
Planck charge	Electric charge (Q)	$q_{\mathrm{P}} = \left(\sqrt{4\pi\varepsilon_0 \hbar c}\right)^{\frac{1}{2}} = \left(\frac{e}{\sqrt{\alpha}}\right)^{\frac{1}{2}}$	$1.369505734 \times 10^{-9} C$
Planck temperature	Temperature (Θ)	$T_{\mathrm{P}} = \left(\frac{m_{\mathrm{P}} c^2}{k_{\mathrm{B}}}\right)^{\frac{1}{2}} = \left(\sqrt{\frac{\hbar c^5}{G k_{\mathrm{B}}^2}}\right)^{\frac{1}{2}}$	$1.19028778 \times 10^{16} K$

Table 3: Tabulated value of square rooted Planck Units

Name	Dimension	Expression	Value (SI unit)
Planck length	Length (L)	$l_{\mathrm{P}} = \left(\sqrt{\frac{\hbar G}{c^3}}\right)^{\frac{1}{3}}$	$5.489127187 \times 10^{-11} m$
Planck mass	Mass (M)	$m_{\mathrm{P}} = \left(\sqrt{\frac{\hbar c}{G}}\right)^{\frac{1}{3}}$	$6.061524996 \times 10^{-2} kg$
Planck time	Time (T)	$t_{\mathrm{P}} = \left(\frac{l_{\mathrm{P}}}{c}\right)^{\frac{1}{3}} = \left(\frac{\hbar}{m_{\mathrm{P}} c^2}\right)^{\frac{1}{3}} = \left(\sqrt{\frac{\hbar G}{c^5}}\right)^{\frac{1}{3}}$	$8.201558932 \times 10^{-14} s$
Planck charge	Electric charge (Q)	$q_{\mathrm{P}} = \left(\sqrt{4\pi\varepsilon_0 \hbar c}\right)^{\frac{1}{3}} = \left(\frac{e}{\sqrt{\alpha}}\right)^{\frac{1}{3}}$	$1.233225709 \times 10^{-6} C$
Planck temperature	Temperature (Θ)	$T_{\mathrm{P}} = \left(\frac{m_{\mathrm{P}} c^2}{k_{\mathrm{B}}}\right)^{\frac{1}{3}} = \left(\sqrt{\frac{\hbar c^5}{G k_{\mathrm{B}}^2}}\right)^{\frac{1}{3}}$	$5.253329603 \times 10^{10} K$

Table 4: Tabulated value of cube rooted Planck Units

Can the cyclic numbers be projected further based on 2 values of i found so far, what shall be the rotators

Yes, I iterated the process taking different irrational number as rotators and the same trend line. I could project it up to $e^{33.94}$. *The table can be enlarged up to say* 10^{1000}. *Who is going to take the pain of searching right rotators to build a decimal system based ladder kind of table? I leave it for computer programmers.*

SL	Formula	i/j	g_1	g_2	g_3	g_4	g_5	g_6
1	$e^{\frac{i\pi}{g_p}}$	$\ln(2)$	$e^{2.18}$	$e^{1.09}$	$e^{0.73}$	$e^{0.54}$	$e^{0.44}$	$e^{0.36}$
2	$\pi^{\frac{je}{g_p}}$	$\frac{1}{\ln(2)}$	$e^{4.49}$	$e^{2.24}$	$e^{1.5}$	$e^{1.12}$	$e^{0.9}$	$e^{0.75}$
3	$\pi^{\frac{je}{g_p}}$	$\frac{e}{e-2}$	$e^{11.78}$	$e^{5.89}$	$e^{3.93}$	$e^{2.94}$	$e^{2.36}$	$e^{1.96}$
4	$e^{\frac{i\pi}{g_p}}$	$\pi+\phi-1$	$e^{11.81}$	$e^{5.90}$	$e^{3.94}$	$e^{2.95}$	$e^{2.36}$	$e^{1.96}$
5	$e^{\frac{i\pi}{g_p}}$	$\frac{1}{\pi-3}$	$e^{22.19}$	$e^{11.09}$	$e^{7.4}$	$e^{5.55}$	$e^{4.44}$	$e^{3.7}$
6	$\pi^{\frac{je}{g_p}}$	e^2	$e^{22.99}$	$e^{11.50}$	$e^{7.66}$	$e^{5.75}$	$e^{4.60}$	$e^{3.83}$
7	$e^{\frac{i\pi}{g_p}}$	$\frac{4\pi-3}{2\pi(\pi-3)}$	$e^{33.78}$	$e^{16.89}$	$e^{11.26}$	$e^{8.45}$	$e^{6.76}$	$e^{5.63}$
8	$\pi^{\frac{je}{g_p}}$	$e^2+\phi^2$	$e^{33.94}$	$e^{16.97}$	$e^{11.31}$	$e^{8.48}$	$e^{6.78}$	$e^{5.65}$

Continued up to infinity ...

Table 5: Tabulated value of Grand unified scale in ascending order

Can we try this cyclic math on other millenium prize problems.

Yes, these cyclic math techniques are universal and can be applied to every branch of mathematics.

Can we have some hints for Hodge Conjecture dealing with algebraic cycles.

Here is the hint with minimal proof of numerical solution to Hodge Conjecture. When we try to evaluate either $\sum_i c_i Z_i$ *or* $\sum_i c_i[Z_i]$ *we enter into the domain of number theory, more specifically zeta function. We have seen zeta function is simply connected (smooth in calculus terms) whether in integer form or rational number form. Zeta function together with its harmonic counterpart is entirely continuous, bijective, and very much stretchable like topological deformation. We can add, multiply, truncated partial zeta series retaining all it's properties. Even in its minimal state zeta function follows basic laws of algebra very neatly for example* $\zeta(-1)+\zeta(0)=0$ *or* $2\zeta(-1)=1$. *To prove that every Hodge class on X is a linear combination with rational coefficients of the cohomology classes of complex subvarieties we just need compliance with addition laws of algebra and scalar multiplication which zeta function duly complies beyond any doubt. Therefore every Hodge class on X is algebraic. No need to mention that Mumford-Tate group is the full symplectic group.*

Can we have some hints for BSD conjecture dealing elliptic curves.
Here is the hint with minimal proof of numerical solution to BSD conjecture.

$$Z_{E,\mathbf{Q}}(s) = \frac{\zeta(s)\zeta(s-1)}{L(s,E)}$$

$\zeta(s)$ is the usual Riemann zeta function and L(s, E) is called the L-function of E/Q. Kolyvagin showed that a modular elliptic curve E for which L(E, 1) is not zero has rank 0, and a modular elliptic curve E for which L(E, 1) has a first-order zero at s = 1 has rank 1. Hasse–Weil zeta function fails to throw some light on the rank of the abelian group E(K) of points of E at s = 1 as $\zeta(1)$ was known to be undefined . In the light of my proof of Riemann hypothesis and its geralisations we can now evaluate the rank easily. We set Hasse–Weil zeta function in left hand side to -1 and evaluate the right hand side putting $\zeta(1) = 1$ which then give the average rank $\frac{1}{2}$ including zero valued ranks . Similarly we can take harmonic conjugate of Hasse–Weil zeta function as follows:

$$Z^*_{E,\mathbf{Q}}(s) = \frac{\zeta(s).L(s,E)}{\zeta(s-1)}$$

Now setting it to -1 and at s=0 putting $\zeta(-1) = \frac{1}{2}$ and $\zeta(-2) = \frac{-1}{2}$we get the analytic rank of elliptic curves E over Q with order s=1 $L(E,s) > 1$ which equals 1. Following Kolyvagin theorem the Birch and Swinnerton-Dyer conjecture holds for all elliptic curves E over Q with order s=1 $L(E,s) > 1$.Tate-Shafarevich group must be finite for all such elliptic curves.

Can we have some hints for Navier–Stokes dealing turbulence in fluid mechanics.
Here is the hint with minimal proof of numerical solution to Navier–Stokes existence. The continuity equation reads:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_x)}{\partial x} + \frac{\partial(\rho u_y)}{\partial y} + \frac{\partial(\rho u_z)}{\partial z} = 0.$$

When the flow is incompressible, ρ does not change for any fluid particle, and its material derivative vanishes and the continuity equation is reduced to:

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = 0.$$

Now above equation will be true when we can show:

$$\nabla = \left[\frac{\partial}{\partial x}\right]\hat{\mathbf{i}} + \left[\frac{\partial}{\partial y}\right]\hat{\mathbf{j}} + \left[\frac{\partial}{\partial z}\right]\hat{\mathbf{k}} = 0$$

Sum of three unit vectors can be zero only at an angle of $\frac{2\pi}{3}$ *or* $\frac{\pi}{3}$ *which is d-unit circle concepts we have seen in proofs of Riemann Hypothesis. So we can conclude Navier–Stokes solutions always exist in 3D via 4D zeros (considering gravity as the body force) and they are smooth – i.e. they are infinitely differentiable at all points due to analytic number theory. To get a feel about the reason behind requirement of dimensionless numbers such as Reynolds Number, Froude Number, Weber Number, Mach Number, Euler's Number etc.. to find an approximate solutions please refer my work on simplex logarithm, hierarchy of grand unified scale factors etc.*

Can we have some hints for Yang–Mills involving $SU(4)$ group in particle physics.

Here is the hint with minimal proof of numerical solution to Yang–Mills existence. In quantum field theory three to tango combination of electro-streak interactions prevents quarks escaping quark confinement leaving it hard to study gluons isolated. To prove that for any compact simple gauge group G, a non-trivial quantum Yang–Mills theory exists on $\mathbb{R}^4$ *and has a mass gap* $\Delta > 0$ *we need to apply the limit of momenta going to zero similar to critical zeros of Riemann zeta function satisfying all the Wightman axioms. To find the mass gap that unifies QED and QCD into QFT analogous to uniqueness of the vacuum in the way* $\zeta(1) = 1$*, we need a unified coupling constant as strong as, close to* $\frac{\alpha}{2}$ *resulting individual gluons still be massless and the scalar glueballs get its mass borrowed from cyclicity of the vacuum . Such scalar glueballs shall have mass 1000-1500 MEV following the hierarchy of grand unified scale* $10^{\frac{\ln(e^{22.2})}{\ln(e^{7.1})}}$ *following* $SU(4) = \frac{U(1)}{SU(2)\times SU(3)}$ *guage symmetry.*

Can we have some hints for P versus NP problem involving computer science.

I wont give much hint here because of security reasons, but I should tell the answer. For easier problems like RSA prime factorisations it's not even P, it's much less than that. Every given number theoretic problem, if attacked from the right direction it can solved in quadratic time. Really I mean it. Let me warn RSA/SHA users that any kind of prime number based algorithm is not secured at all therefore get rid off numbers as soon as possible. This is last and final call. Disclaimer: If using any of my work, hackers cracks RSA/ SHA encryption tomorrow, and the whole internet security collapses (I know the world is moving to block chain which still uses numbers a lot), I cannot be held responsible for that. Even I cannot be held responsible for, any other kind of losses incurred in whatsoever manner by any person, organization, corporate bodies, countries, economies, communities, races and religions or for any losses caused to humanity at large, our planet earth, the mother nature or the whole existence altogether, and as such I won't be able to compensate a penny (Knowing it very much probable that I may not be *gaining anything at the end, I suffered huge loss of time and money in engaging myself in Riemann hypothesis, even I have gone jobless to finish it, I cannot afford any more losses) for the damages caused, if any. Notwithstanding any contrary provision contained under any law made by human or any advance species (if any), I presume that I am allowed to reveal the results derived from natural laws of mathematics to the mass without knowing the exact consequence. I cannot be questioned, examined, trialled, detained, arrested, prosecuted for an act of mere sharing freely the knowledge I gained without having any ulterior motive. Any unfriendly effort made by anybody in above direction shall be void, therefore not required to be entertained by any appropriate authority or the law keepers. Thanks to everybody for taking me not so seriously.*

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