

# On some Ramanujan equations: mathematical connections with $\phi$ , $\zeta(2)$ , Monstrous Moonshine and various parameters of Particle Physics. II

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## Abstract

*In this paper we have described and analyzed some Ramanujan equations. Furthermore, we have obtained several mathematical connections with  $\phi$ ,  $\zeta(2)$ , Monstrous Moonshine and various parameters of Particle Physics.*

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[https://link.springer.com/chapter/10.1007/978-81-322-0767-2\\_12](https://link.springer.com/chapter/10.1007/978-81-322-0767-2_12)

<https://www.sciencephoto.com/media/228058/view/indian-mathematician-srinivasa-ramanujan>

**We want to highlight that the development of the various equations was carried out according to our possible logical and original interpretation**

From:

**The legacy of Ramanujan's mock theta functions: Harmonic Maass forms in number theory** - *Ken Ono* - Emory University

## First few minimal polynomials

$n$	$x^{h_n} - (24n - 1)p(n)x^{h_n-1} + \dots$
1	$x^3 - 23 \cdot 1x^2 + \frac{3592}{23}x - 419$
2	$x^5 - 47 \cdot 2x^4 + \frac{169659}{47}x^3 - 65838x^2 + \frac{1092873176}{47^2}x + \frac{1454023}{47}$
3	$x^7 - 71 \cdot 3x^6 + \frac{1312544}{71}x^5 - 723721x^4 + \frac{44648582886}{71^2}x^3$ $+ \frac{9188934683}{71}x^2 + \frac{166629520876208}{71^3}x + \frac{2791651635293}{71^2}$
4	$x^8 - 95 \cdot 5x^7 + \frac{9032603}{95}x^6 - 9455070x^5 + \frac{3949512899743}{95^2}x^4$ $- \frac{97215753021}{19}x^3 + \frac{9776785708507683}{95^3}x^2$ $- \frac{53144327916296}{19^2}x - \frac{134884469547631}{5^4 \cdot 19}$

Now, we analyze the following equation:

$$x^{h_n} - (24n - 1)p(n)x^{h_n-1} + \dots$$

For  $n = 1$

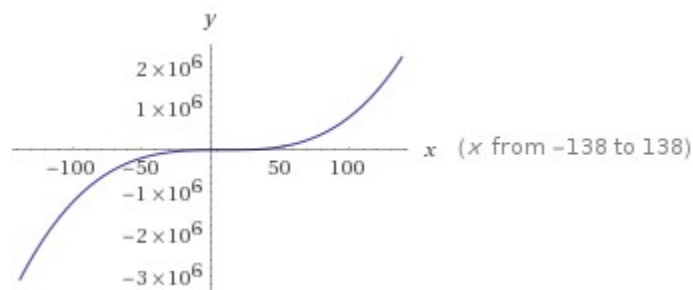
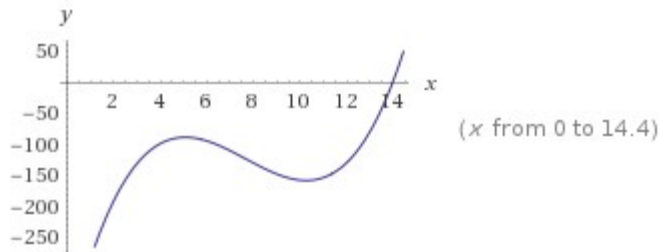
$$x^3 - 23 \cdot 1x^2 + \frac{3592}{23}x - 419$$

**Input:**

$$x^3 - 23 \times 1x^2 + \frac{3592}{23}x - 419$$

**Result:**

$$x^3 - 23x^2 + \frac{3592x}{23} - 419$$

**Plots:****Alternate forms:**

$$\frac{1}{23} (23x^3 - 529x^2 + 3592x - 9637)$$

$$x \left( (x - 23)x + \frac{3592}{23} \right) - 419$$

$$\left( x - \frac{23}{3} \right)^3 - \frac{1391}{69} \left( x - \frac{23}{3} \right) - \frac{3319}{27}$$

**Real root:**

$$x \approx 13.965$$

13.965

**Complex roots:**

$$x \approx 4.5173 - 3.0979i$$

$$x \approx 4.5173 + 3.0979i$$

**Polynomial discriminant:**

$$\Delta = -\frac{4565299489}{12167}$$

## Properties as a real function:

### Domain

$\mathbb{R}$  (all real numbers)

### Range

$\mathbb{R}$  (all real numbers)

### Surjectivity

surjective onto  $\mathbb{R}$

$\mathbb{R}$  is the set of real numbers

### Derivative:

$$\frac{d}{dx}\left(x^3 - 23x^2 + \frac{3592x}{23} - 419\right) = 3x^2 - 46x + \frac{3592}{23}$$

### Indefinite integral:

$$\int\left(-419 + \frac{3592x}{23} - 23x^2 + x^3\right)dx = \frac{x^4}{4} - \frac{23x^3}{3} + \frac{1796x^2}{23} - 419x + \text{constant}$$

### Local maximum:

$$\max\left\{x^3 - 23x^2 + \frac{3592x}{23} - 419\right\} = \frac{2782\sqrt{\frac{1391}{23}}}{621} - \frac{3319}{27} \text{ at } x = \frac{23}{3} - \frac{\sqrt{\frac{1391}{23}}}{3}$$

### Local minimum:

$$\min\left\{x^3 - 23x^2 + \frac{3592x}{23} - 419\right\} = -\frac{3319}{27} - \frac{2782\sqrt{\frac{1391}{23}}}{621} \text{ at } x = \frac{23}{3} + \frac{\sqrt{\frac{1391}{23}}}{3}$$

For  $n = 2$ :

$$x^5 - 47 \cdot 2x^4 + (169659/47)x^3 - 65838x^2 + (1092873176/47^2)x + (1454023/47)$$

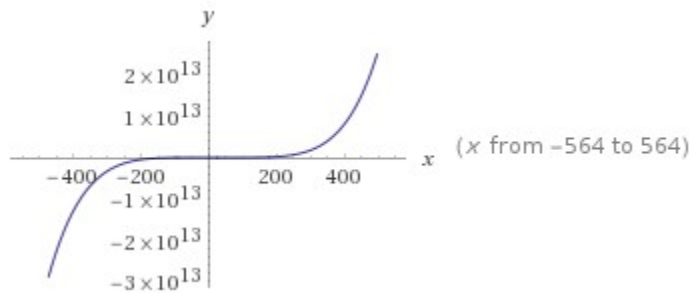
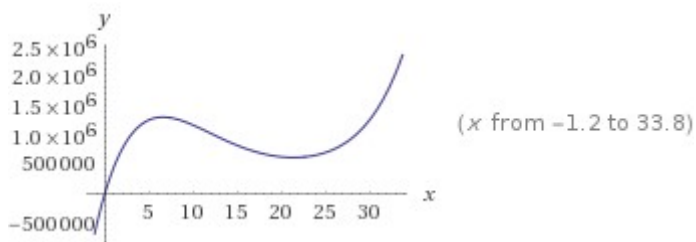
### Input:

$$x^5 - 47 \times 2x^4 + \frac{169659}{47}x^3 - 65838x^2 + \frac{1092873176}{47^2}x + \frac{1454023}{47}$$

### Result:

$$x^5 - 94x^4 + \frac{169659x^3}{47} - 65838x^2 + \frac{1092873176x}{2209} + \frac{1454023}{47}$$

## Plots:



## Alternate forms:

$$\frac{2209 x^5 - 207646 x^4 + 7973973 x^3 - 145436142 x^2 + 1092873176 x + 68339081}{2209}$$

$$x \left( x \left( x \left( (x - 94)x + \frac{169659}{47} \right) - 65838 \right) + \frac{1092873176}{2209} \right) + \frac{1454023}{47}$$

## Real root:

$$x \approx -0.062018$$

**-0.062018**

## Complex roots:

$$x \approx 20.6403 - 12.6922 i$$

$$x \approx 20.6403 + 12.6922 i$$

$$x \approx 26.3907 - 12.3761 i$$

$$x \approx 26.3907 + 12.3761 i$$

## Polynomial discriminant:

$$\Delta = \frac{2528238138127819617117455890143164552778601}{52599132235830049}$$

## Properties as a real function:

### Domain

$\mathbb{R}$  (all real numbers)

### Range

$\mathbb{R}$  (all real numbers)

### Surjectivity

surjective onto  $\mathbb{R}$

$\mathbb{R}$  is the set of real numbers

### Derivative:

$$\frac{d}{dx} \left( x^5 - 47 \times 2 x^4 + \frac{169\,659 x^3}{47} - 65\,838 x^2 + \frac{1\,092\,873\,176 x}{47^2} + \frac{1\,454\,023}{47} \right) =$$
$$5x^4 - 376x^3 + \frac{508\,977 x^2}{47} - 131\,676 x + \frac{1\,092\,873\,176}{2209}$$

### Indefinite integral:

$$\int \left( \frac{1\,454\,023}{47} + \frac{1\,092\,873\,176 x}{2209} - 65\,838 x^2 + \frac{169\,659 x^3}{47} - 94 x^4 + x^5 \right) dx =$$
$$\frac{x^6}{6} - \frac{94 x^5}{5} + \frac{169\,659 x^4}{188} - 21\,946 x^3 + \frac{546\,436\,588 x^2}{2209} + \frac{1\,454\,023 x}{47} + \text{constant}$$

### Local maximum:

$$\max \left\{ x^5 - 47 \times 2 x^4 + \frac{169\,659 x^3}{47} - 65\,838 x^2 + \frac{1\,092\,873\,176 x}{47^2} + \frac{1\,454\,023}{47} \right\} =$$

1300272 at  $x \approx 6.5625$

### Local minimum:

$$\min \left\{ x^5 - 47 \times 2 x^4 + \frac{169\,659 x^3}{47} - 65\,838 x^2 + \frac{1\,092\,873\,176 x}{47^2} + \frac{1\,454\,023}{47} \right\} =$$

617871 at  $x \approx 21.403$

For n = 3:

$$x^7 - 71 \cdot 3x^6 + (1312544/71)x^5 - 723721x^4 + (44648582886/71^2)x^3 + (9188934683/71)x^2 + (166629520876208/71^3)x + (2791651635293/71^2)$$

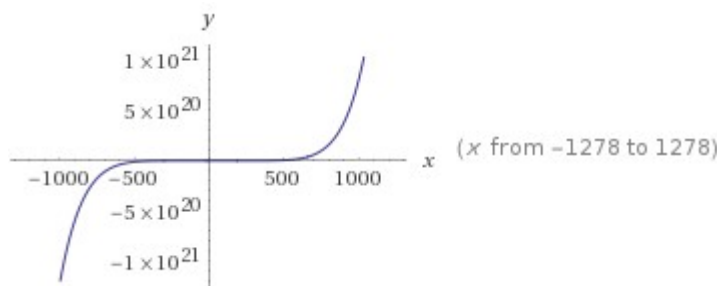
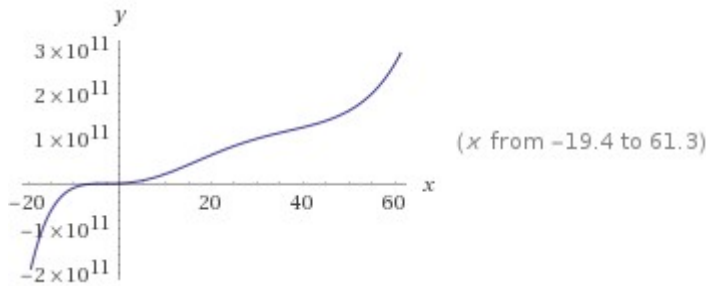
**Input:**

$$x^7 - 71 \times 3x^6 + \frac{1312544}{71}x^5 - 723721x^4 + \frac{44648582886}{71^2}x^3 + \frac{9188934683}{71}x^2 + \frac{166629520876208}{71^3}x + \frac{2791651635293}{71^2}$$

**Result:**

$$x^7 - 213x^6 + \frac{1312544x^5}{71} - 723721x^4 + \frac{44648582886x^3}{5041} + \frac{9188934683x^2}{71} + \frac{166629520876208x}{357911} + \frac{2791651635293}{5041}$$

**Plots:**



**Alternate forms:**

$$\frac{1}{357911} (357911x^7 - 76235043x^6 + 6616534304x^5 - 259027706831x^4 + 3170049384906x^3 + 46321419737003x^2 + 166629520876208x + 198207266105803)$$



$$x \left( x \left( x \left( x \left( x \left( (x - 213)x + \frac{1312544}{71} \right) - 723721 \right) + \frac{44648582886}{5041} \right) + \frac{9188934683}{71} \right) + \frac{166629520876208}{357911} \right) + \frac{2791651635293}{5041}$$

### Real root:

$$x \approx -3.82803$$

**-3.82803**

### Complex roots:

$$x \approx -2.74264 - 1.06423 i$$

$$x \approx -2.74264 + 1.06423 i$$

$$x \approx 53.2947 - 31.6846 i$$

$$x \approx 53.2947 + 31.6846 i$$

$$x \approx 57.8619 - 31.6254 i$$

### Polynomial discriminant:

$\Delta =$

$$-(122\,306\,003\,717\,526\,555\,105\,694\,803\,361\,386\,003\,457\,879\,331\,530\,415\,917\,038 \cdot 237\,161\,108\,279\,333\,230\,415\,809\,828\,983\,538\,342\,898\,752\,489 / 752\,359\,350\,923\,790\,893\,319\,063\,566\,949\,457\,370\,471)$$

### Properties as a real function:

#### Domain

$\mathbb{R}$  (all real numbers)

#### Range

$\mathbb{R}$  (all real numbers)

#### Bijectivity

bijjective from its domain to  $\mathbb{R}$

$\mathbb{R}$  is the set of real numbers

**Derivative:**

$$\frac{d}{dx} \left( x^7 - 71 \times 3 x^6 + \frac{1312544 x^5}{71} - 723721 x^4 + \frac{44648582886 x^3}{71^2} + \frac{9188934683 x^2}{71} + \frac{166629520876208 x}{71^3} + \frac{2791651635293}{71^2} \right) =$$

$$7x^6 - 1278x^5 + \frac{6562720x^4}{71} - 2894884x^3 + \frac{133945748658x^2}{5041} + \frac{18377869366x}{71} + \frac{166629520876208}{357911}$$

**Indefinite integral:**

$$\int \left( \frac{2791651635293}{5041} + \frac{166629520876208x}{357911} + \frac{9188934683x^2}{71} + \frac{44648582886x^3}{5041} - 723721x^4 + \frac{1312544x^5}{71} - 213x^6 + x^7 \right) dx =$$

$$\frac{x^8}{8} - \frac{213x^7}{7} + \frac{656272x^6}{213} - \frac{723721x^5}{5} + \frac{22324291443x^4}{10082} + \frac{9188934683x^3}{213} + \frac{83314760438104x^2}{357911} + \frac{10082}{5041}x + \text{constant}$$

For n = 4:

$$x^8 - 95 \times 5x^7 + (9032603/95)x^6 - 9455070x^5 + (3949512899743/95^2)x^4 - (97215753021/19)x^3 + (9776785708507683/95^3)x^2 - (53144327916296/19^2)x - ((134884469547631)/(5^4 \times 19))$$

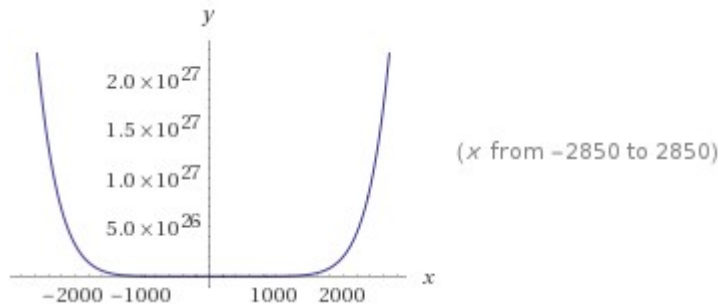
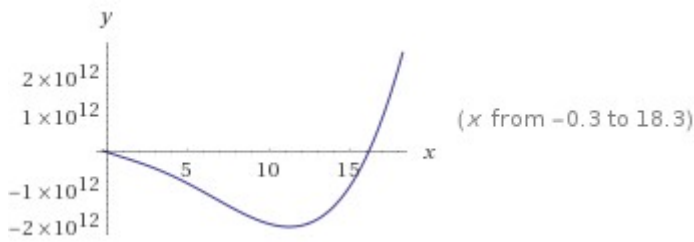
**Input:**

$$x^8 - 95 \times 5x^7 + \frac{9032603}{95}x^6 - 9455070x^5 + \frac{3949512899743}{95^2}x^4 - \frac{97215753021}{19}x^3 + \frac{9776785708507683}{95^3}x^2 - \frac{53144327916296}{19^2}x - \frac{134884469547631}{5^4 \times 19}$$

**Result:**

$$x^8 - 475x^7 + \frac{9032603x^6}{95} - 9455070x^5 + \frac{3949512899743x^4}{9025} - \frac{97215753021x^3}{19} + \frac{9776785708507683x^2}{857375} - \frac{53144327916296x}{361} - \frac{134884469547631}{11875}$$

**Plots:**



**Alternate forms:**

$$\frac{1}{4286875} (4286875 x^8 - 2036265625 x^7 + 407596210375 x^6 - 40532703206250 x^5 + 1876018627377925 x^4 - 21934304275363125 x^3 + 48883928542538415 x^2 - 631088894006015000 x - 48693293506694791)$$

$$x \left( x \left( x \left( x \left( x \left( (x - 475)x + \frac{9032603}{95} \right) - 9455070 \right) + \frac{3949512899743}{9025} \right) - \frac{97215753021}{19} \right) + \frac{9776785708507683}{857375} \right) - \frac{53144327916296}{361} - \frac{134884469547631}{11875}$$

**Real roots:**

$$x \approx -0.0766863$$

$$x \approx 16.1807$$

16.1807

**Complex roots:**

$$x \approx -0.123902 - 5.45402 i$$

$$x \approx -0.123902 + 5.45402 i$$

$$x \approx 112.822 - 66.0681 i$$

$$x \approx 112.822 + 66.0681 i$$

$$x \approx 116.75 - 66.0535 i$$

## Polynomial discriminant:

$\Delta =$

$$\begin{aligned} &-(16\,303\,706\,823\,851\,487\,469\,124\,604\,901\,546\,922\,325\,379\,204\,956\,360\,511\,639) \cdot \\ &365\,811\,312\,022\,416\,725\,852\,746\,265\,661\,511\,558\,134\,436\,491\,724\,732\,456) \cdot \\ &917\,039\,804\,476\,277\,042\,644\,939\,842\,908\,733\,761 / \\ &8876466339898690179501496341752910614013671875) \end{aligned}$$

## Properties as a real function:

### Domain

$\mathbb{R}$  (all real numbers)

### Range

$\{y \in \mathbb{R} : y \geq -2.02135 \times 10^{12}\}$

$\mathbb{R}$  is the set of real numbers

## Derivative:

$$\begin{aligned} \frac{d}{dx} &\left( x^8 - 95 \times 5 x^7 + \frac{9\,032\,603 x^6}{95} - 9\,455\,070 x^5 + \right. \\ &\frac{3\,949\,512\,899\,743 x^4}{95^2} - \frac{97\,215\,753\,021 x^3}{19} + \frac{9\,776\,785\,708\,507\,683 x^2}{95^3} - \\ &\left. \frac{53\,144\,327\,916\,296 x}{19^2} - \frac{134\,884\,469\,547\,631}{5^4 \times 19} \right) = \\ &8 x^7 - 3325 x^6 + \frac{54\,195\,618 x^5}{95} - 47\,275\,350 x^4 + \frac{15\,798\,051\,598\,972 x^3}{9025} - \\ &\frac{291\,647\,259\,063 x^2}{19} + \frac{19\,553\,571\,417\,015\,366 x}{857\,375} - \frac{53\,144\,327\,916\,296}{361} \end{aligned}$$

## Indefinite integral:

$$\begin{aligned} \int &\left( -\frac{134\,884\,469\,547\,631}{11\,875} - \frac{53\,144\,327\,916\,296 x}{361} + \right. \\ &\frac{9\,776\,785\,708\,507\,683 x^2}{857\,375} - \frac{97\,215\,753\,021 x^3}{19} + \\ &\left. \frac{3\,949\,512\,899\,743 x^4}{9025} - 9\,455\,070 x^5 + \frac{9\,032\,603 x^6}{95} - 475 x^7 + x^8 \right) dx = \\ &\frac{x^9}{9} - \frac{475 x^8}{8} + \frac{9\,032\,603 x^7}{665} - 1575\,845 x^6 + \frac{3\,949\,512\,899\,743 x^5}{45\,125} - \\ &\frac{97\,215\,753\,021 x^4}{76} + \frac{3\,258\,928\,569\,502\,561 x^3}{857\,375} - \\ &\frac{26\,572\,163\,958\,148 x^2}{361} - \frac{134\,884\,469\,547\,631 x}{11\,875} + \text{constant} \end{aligned}$$

Thence, we have the following results:

16.1807; -3.82803; -0.062018; 13.965 from the sum, we obtain:

$$(16.1807 - 3.82803 - 0.062018 + 13.965)$$

**Input interpretation:**

$$16.1807 - 3.82803 - 0.062018 + 13.965$$

**Result:**

26.255652

**26.255652**

From which:

$$1 + \left( \left[ \frac{1}{16.1807 - 3.82803 - 0.062018 + 13.965} \right]^{1/7} \right)$$

**Input interpretation:**

$$1 + \sqrt[7]{\frac{1}{16.1807 - 3.82803 - 0.062018 + 13.965}}$$

**Result:**

1.626980284590375628290948641634877987296665923756974396736...

**1.6269802845...**

Multiplying the results, we obtain:

$$(16.1807 * -3.82803 * -0.062018 * 13.965)$$

**Input interpretation:**

$$16.1807 \times (-3.82803) \times (-0.062018) \times 13.965$$

**Result:**

53.64525762266855877

**Repeating decimal:**

53.64525762266855877

53.6452576....

From which:

$$89/(16.1807 * -3.82803 * -0.062018 * 13.965)$$

**Input interpretation:**

$$\frac{89}{16.1807 \times (-3.82803) \times (-0.062018) \times 13.965}$$

**Result:**

1.659046930597492314988205672127604954674092871643273450190...

1.65904693059..... result very near to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164.2696$  i.e. 1.65578...

From the sum, we obtain also:

$$1/16(16.1807 - 3.82803 - 0.062018 + 13.965)$$

**Input interpretation:**

$$\frac{1}{16} (16.1807 - 3.82803 - 0.062018 + 13.965)$$

**Result:**

1.64097825

1.64097825

or:

-3.82803; -0.062018; 13.965; -0.0766863 from the sum, we obtain:

$$(-3.82803 - 0.062018 + 13.965 - 0.0766863)$$

**Input interpretation:**

$$-3.82803 - 0.062018 + 13.965 - 0.0766863$$

**Result:**

9.9982657

9.9982657 result very near to the black hole entropy 9.9340, that is equal to  $\ln(20619)$

From which:

$$1 + (1 / (-3.82803 - 0.062018 + 13.965 - 0.0766863))^{1/5}$$

**Input interpretation:**

$$1 + \sqrt[5]{\frac{1}{-3.82803 - 0.062018 + 13.965 - 0.0766863}}$$

**Result:**

1.630979232144283110759090634444211355669807299929612036364...

1.630979232144...

Multiplying the results, we obtain:

$$(-3.82803 * 0.062018 * 13.965 * -0.0766863)$$

**Input interpretation:**

$$-3.82803 \times 0.062018 \times 13.965 \times (-0.0766863)$$

**Result:**

0.25424464452274919493

**Repeating decimal:**

0.254244644522749194930

0.254244644.....

From which:

$$((\exp(((((-3.82803 * 0.062018 * 13.965 * -0.0766863))))))^{2}$$

**Input interpretation:**

$$\exp^2(-3.82803 \times 0.062018 \times 13.965 \times (-0.0766863))$$

**Result:**

1.662777320642077929001686837618729289795597115909326532978...

1.6627773206... result very near to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164.2696$  i.e. 1.65578...

or:

$$1 + 1 / (((2\pi * (((-3.82803 * 0.062018 * 13.965 * -0.0766863))))))$$

**Input interpretation:**

$$1 + \frac{1}{2\pi(-3.82803 \times 0.062018 \times 13.965 \times (-0.0766863))}$$

**Result:**

1.625991329692116826418453430998378379543606203552977599840...

1.625991329...

**Alternative representations:**

$$1 + \frac{1}{(2\pi)0.062018(-3.82803 \times 13.965(-0.0766863))} = 1 + \frac{1}{91.5281^\circ}$$

$$1 + \frac{1}{(2\pi)0.062018(-3.82803 \times 13.965(-0.0766863))} = 1 + \frac{1}{0.508489 i \log(-1)}$$

$$1 + \frac{1}{(2\pi)0.062018(-3.82803 \times 13.965(-0.0766863))} = 1 + \frac{1}{0.508489 \cos^{-1}(-1)}$$

**Series representations:**

$$1 + \frac{1}{(2\pi)0.062018(-3.82803 \times 13.965(-0.0766863))} = 1 + \frac{0.491652}{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

$$1 + \frac{1}{(2\pi)0.062018(-3.82803 \times 13.965(-0.0766863))} = 1 + \frac{0.983305}{-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}}$$





Now, we have:

Example for  $E$  :  $y^2 = x^3 + 10x^2 - 20x + 8$ .

$\Delta$	$c_g^+(-\Delta)$	$L'(E(\Delta), 1)$
-3	1.0267149116 ...	1.4792994920 ...
-4	1.2205364009 ...	1.8129978972 ...
$\vdots$	$\vdots$	$\vdots$
-136	-4.8392675993 ...	5.7382407649 ...
-139	<b>-6</b>	<b>0</b>
-151	-0.8313568817 ...	6.6975085515 ...
$\vdots$	$\vdots$	$\vdots$
-815	121.1944103120 ...	4.7492583693 ...
-823	<b>312</b>	<b>0</b>

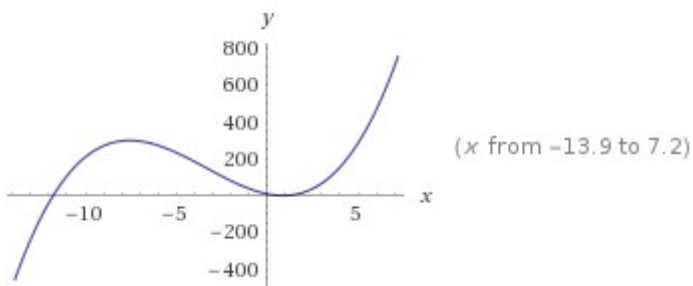
From the above equation, we obtain:

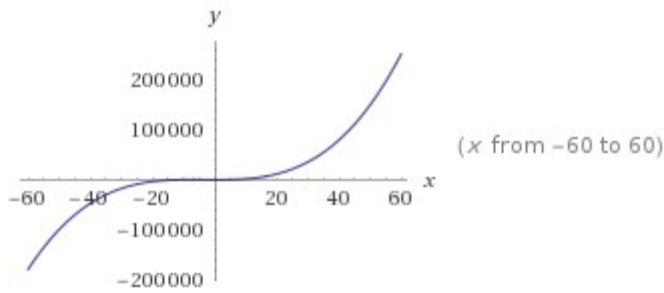
$$x^3 + 10x^2 - 20x + 8$$

**Input:**

$$x^3 + 10x^2 - 20x + 8$$

**Plots:**





**Alternate forms:**

$$x(x(x+10)-20)+8$$

$$\left(x + \frac{10}{3}\right)^3 - \frac{160}{3}\left(x + \frac{10}{3}\right) + \frac{4016}{27}$$

**Roots:**

$$x \approx -11.759$$

$$x \approx 0.57452$$

$$x \approx 1.1842$$

**Polynomial discriminant:**

$$\Delta = 9472$$

**Properties as a real function:**

**Domain**

$\mathbb{R}$  (all real numbers)

**Range**

$\mathbb{R}$  (all real numbers)

**Surjectivity**

surjective onto  $\mathbb{R}$

$\mathbb{R}$  is the set of real numbers

**Derivative:**

$$\frac{d}{dx}(x^3 + 10x^2 - 20x + 8) = 3x^2 + 20x - 20$$

**Indefinite integral:**

$$\int (8 - 20x + 10x^2 + x^3) dx = \frac{x^4}{4} + \frac{10x^3}{3} - 10x^2 + 8x + \text{constant}$$

**Local maximum:**

$$\max\{x^3 + 10x^2 - 20x + 8\} = \frac{16}{27} (251 + 80\sqrt{10}) \text{ at } x = -\frac{10}{3} - \frac{4\sqrt{10}}{3}$$

**Local minimum:**

$$\min\{x^3 + 10x^2 - 20x + 8\} = -\frac{16}{27} (80\sqrt{10} - 251) \text{ at } x = \frac{4\sqrt{10}}{3} - \frac{10}{3}$$

**Definite integral area below the axis between the smallest and largest real roots:**

$$\int_{\substack{\text{root of } 8-20x+10x^2+x^3 \text{ near } x=1.18421 \\ \text{root of } 8-20x+10x^2+x^3 \text{ near } x=-11.7587}} (8 - 20x + 10x^2 + x^3) \theta(-8 + 20x - 10x^2 - x^3) dx = \left(-\frac{4}{3} + \frac{4i}{3}\right) \sqrt{\left(\frac{1}{3}\left(2524480i - \frac{199156227200i2^{2/3}}{\sqrt[3]{-125691391948343369 + 13968975010143i\sqrt{111}}}\right) - \frac{\sqrt[3]{-125691391948343369 + 13968975010143i\sqrt{111}}}{199156227200 \times 2^{2/3} \sqrt{3}}\right) - i\sqrt[3]{2(-125691391948343369 + 13968975010143i\sqrt{111})} - \sqrt{3}\sqrt[3]{2(-125691391948343369 + 13968975010143i\sqrt{111})}\right)} \approx -0.477379$$

$\theta(x)$  is the Heaviside step function

**Definite integral area above the axis between the smallest and largest real roots:**

$$\int_{\substack{\text{root of } 8-20x+10x^2+x^3 \text{ near } x=1.18421 \\ \text{root of } 8-20x+10x^2+x^3 \text{ near } x=-11.7587}} (8 - 20x + 10x^2 + x^3) \theta(8 - 20x + 10x^2 + x^3) dx = \frac{8}{3} \sqrt{\left(\frac{1}{3}\left(1262240 + \frac{199156227200 \times 2^{2/3}}{\sqrt[3]{-125691391948343369 + 13968975010143i\sqrt{111}}}\right) + \frac{\sqrt[3]{-125691391948343369 + 13968975010143i\sqrt{111}}}{199156227200 \times 2^{2/3} \sqrt{3}}\right) + i\sqrt[3]{2(-125691391948343369 + 13968975010143i\sqrt{111})} + \sqrt{3}\sqrt[3]{2(-125691391948343369 + 13968975010143i\sqrt{111})}\right)} \approx 2118.72$$

2118.72 result very near to the rest mass of strange D meson 2112.3

From which, inserting in the expression the value of polynomial discriminant  $\Delta = 9274$ , we obtain:

$$2 * 9274 / ((((-4.8392675993^3 + 10 * 4.8392675993^2 - 20 * (-4.8392675993) + 8) / 2 + (-5.7382407649^3 + 10 * 5.7382407649^2 - 20 * (5.7382407649) + 8) / 2))) - 7$$

**Input interpretation:**

$$2 \times 9274 / \left( \frac{1}{2} (-4.8392675993^3 + 10 \times 4.8392675993^2 - 20 \times (-4.8392675993) + 8) + \frac{1}{2} (-5.7382407649^3 + 10 \times 5.7382407649^2 + 20 \times (-5.7382407649) + 8) \right) - 7$$

**Result:**

136.1140341536590944541587315790150237885671917986795986875...

136.114034153...

and:

$$2 * 9274 / ((((-6^3 + 10 * 6^2 - 20 * (-6) + 8) / 2))) + (8 * 1/2) - 1$$

**Input:**

$$2 \times \frac{9274}{\frac{1}{2} (-6^3 + 10 \times 6^2 - 20 \times (-6) + 8)} + 8 \times \frac{1}{2} - 1$$

**Exact result:**

$$\frac{4739}{34}$$

**Decimal approximation:**

139.3823529411764705882352941176470588235294117647058823529...

139.38235294...

$$2 * 9274 / ((((-6^3 + 10 * 6^2 - 20 * (-6) + 8) / 2))) + (8 * 1/2) + 11$$

**Input:**

$$2 \times \frac{9274}{\frac{1}{2} (-6^3 + 10 \times 6^2 - 20 \times (-6) + 8)} + 8 \times \frac{1}{2} + 11$$

**Exact result:**

$$\frac{5147}{34}$$

**Decimal approximation:**

151.3823529411764705882352941176470588235294117647058823529...

151.3823529411...

$$4 \cdot \frac{1}{\left( \frac{9274}{\left( \left( \left( 121.1944^3 + 10 \cdot 121.1944^2 - 20 \cdot (121.1944) + 8 \right) + \left( 4.74925^3 + 10 \cdot 4.74925^2 - 20 \cdot (4.74925) + 8 \right) \right) \right) \right)} - 15$$

**Input interpretation:**

$$4 \times \frac{1}{\frac{9274}{(121.1944^3 + 10 \times 121.1944^2 + 20 \times (-121.1944) + 8) + (4.74925^3 + 10 \times 4.74925^2 + 20 \times (-4.74925) + 8)}} - 15$$

**Result:**

815.2022793172031861656243260728919560060383868880741858960...

815.202279317...

$$\frac{1}{2} \left( \frac{9274 \cdot \left( \frac{312^3 + 10 \cdot 312^2 - 20 \cdot (312) + 8}{2} + 8 \cdot \frac{1}{2} \right)}{9274^2} - 21 \right)$$

**Input:**

$$\frac{1}{2} \left( 9274 \left( \frac{1}{2} (312^3 + 10 \times 312^2 - 20 \times 312 + 8) + 8 \times \frac{1}{2} \right) \right) \times \frac{1}{9274^2} - 21$$

**Exact result:**

$$\frac{3819941}{4637}$$

**Decimal approximation:**

823.7957731291783480698727625620012939400474444684062971748...

823.795773129...

Now, we want to analyze again the previous equation:

$$x^{h_n} - (24n - 1)p(n)x^{h_n-1} + \dots$$

For  $n = 1$ , and putting the equation equal to 1728, we obtain:

$$x^5 - 47 \times 2x^4 + (169659/47)x^3 - 65838x^2 + (1092873176/47^2)x + (1454023/47) = 1728$$

**Input:**

$$x^5 - 47 \times 2x^4 + \frac{169659}{47}x^3 - 65838x^2 + \frac{1092873176}{47^2}x + \frac{1454023}{47} = 1728$$

**Exact result:**

$$x^5 - 94x^4 + \frac{169659x^3}{47} - 65838x^2 + \frac{1092873176x}{2209} + \frac{1454023}{47} = 1728$$

**Alternate forms:**

$$2209x^5 - 207646x^4 + 7973973x^3 - 145436142x^2 + 1092873176x = -64521929$$

$$x^5 - 94x^4 + \frac{169659x^3}{47} - 65838x^2 + \frac{1092873176x}{2209} + \frac{1372807}{47} = 0$$

$$x^5 + \frac{169659x^3}{47} + \frac{1092873176x}{2209} + \frac{1372807}{47} = 94x^4 + 65838x^2$$

**Real solution:**

$$x \approx -0.0585807$$

$$\mathbf{-0.0585807}$$

Indeed, from

$$x^{h_n} - (24n - 1)p(n)x^{h_n-1} + \dots$$

for  $x = -0.0585807$ , we obtain:

$$(-0.0585807)^5 - (24 \times 2 - 1) \times 2(-0.0585807)^4 + (169659/47)(-0.0585807)^3 - 65838(-0.0585807)^2 + (1092873176/47^2)(-0.0585807) + (1454023/47)$$

**Input interpretation:**

$$(-0.0585807)^5 - (24 \times 2 - 1) \times 2 (-0.0585807)^4 + \frac{169\,659}{47} (-0.0585807)^3 - 65\,838 (-0.0585807)^2 + \frac{1\,092\,873\,176}{47^2} \times (-0.0585807) + \frac{1\,454\,023}{47}$$

**Result:**

1727.979581746689191675210379176792483079841557265731100045...  
 1727.9795817...  $\approx$  1728

Furthermore, we obtain also:

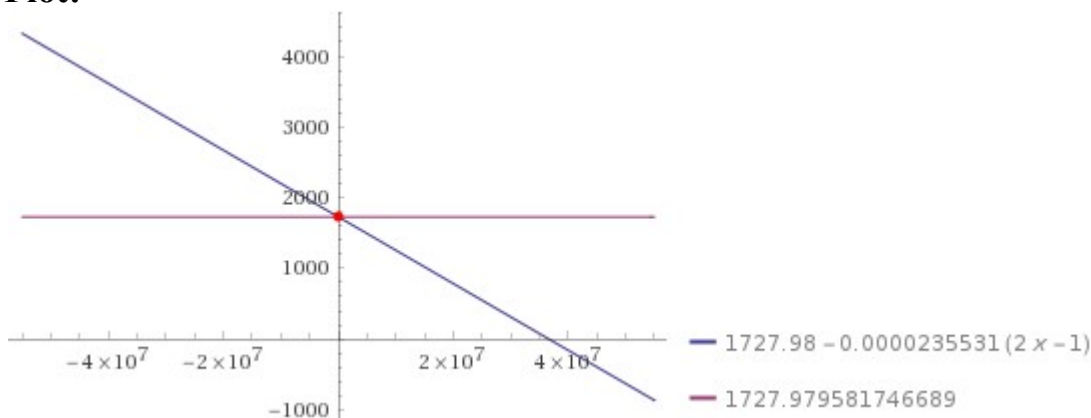
$$(-0.0585807)^5 - (x \times 2 - 1) \times 2 (-0.0585807)^4 + \frac{169\,659}{47} (-0.0585807)^3 - 65\,838 (-0.0585807)^2 + \frac{1\,092\,873\,176}{47^2} \times (-0.0585807) + \frac{1\,454\,023}{47} = 1727.979581746689$$

**Input interpretation:**

$$(-0.0585807)^5 - (x \times 2 - 1) \times 2 (-0.0585807)^4 + \frac{169\,659}{47} (-0.0585807)^3 - 65\,838 (-0.0585807)^2 + \frac{1\,092\,873\,176}{47^2} \times (-0.0585807) + \frac{1\,454\,023}{47} = 1727.979581746689$$

**Result:**

$$1727.98 - 0.0000235531 (2x - 1) = 1727.979581746689$$

**Plot:****Alternate forms:**

$$0.00113055 - 0.0000471062 x = 0$$

$$1727.98 - 0.0000471062 x = 1727.979581746689$$

$$-0.0000471062 (x - 3.66826 \times 10^7) = 1727.979581746689$$



**Solution:**

$$x \approx 24.$$

24 value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

And again:

$$\left( \left( \left( (-0.0585807)^5 - (24 \times 2 - 1) \times 2 (-0.0585807)^4 + \frac{169659}{47} (-0.0585807)^3 - 65838 (-0.0585807)^2 + \frac{1092873176}{47^2} (-0.0585807) + \frac{1454023}{47} \right) \right)^{1/15} \right)$$

**Input interpretation:**

$$\left( (-0.0585807)^5 - (24 \times 2 - 1) \times 2 (-0.0585807)^4 + \frac{169659}{47} (-0.0585807)^3 - 65838 (-0.0585807)^2 + \frac{1092873176}{47^2} \times (-0.0585807) + \frac{1454023}{47} \right)^{(1/15)}$$

**Result:**

1.643750534658957753630587245387375839596126563752760485016...

1.64375053465.....

$$\left( \left( \left( (-0.0585807)^5 - (24 \times 2 - 1) \times 2 (-0.0585807)^4 + \frac{169659}{47} (-0.0585807)^3 - 65838 (-0.0585807)^2 + \frac{1092873176}{47^2} (-0.0585807) + \frac{1454023}{47} \right) \right)^{1/15} - (21+5) \frac{1}{10^3} \right)$$

**Input interpretation:**

$$\left( (-0.0585807)^5 - (24 \times 2 - 1) \times 2 (-0.0585807)^4 + \frac{169659}{47} (-0.0585807)^3 - 65838 (-0.0585807)^2 + \frac{1092873176}{47^2} \times (-0.0585807) + \frac{1454023}{47} \right)^{(1/15)} - (21+5) \times \frac{1}{10^3}$$

**Result:**

1.617750534658957753630587245387375839596126563752760485016...

1.6177505346589....

From:

**Can't you just feel the moonshine?**

Ken Ono (Emory University) -

<http://people.oregonstate.edu/~petschec/ONTD/Talk2.pdf> - March 30, 2017

We have:

Using the K3 surface elliptic genus, there is a mock modular form

$$H(\tau) = q^{-\frac{1}{8}} (-2 + 45q + 231q^2 + 770q^3 + 2277q^4 + 5796q^5 + \dots)$$

If  $q = e^{2\pi i\tau}$ , for  $i\tau = i(1+i)$ , we obtain from the above expression:

$$\exp(2\pi i(1+i))$$

**Input:**

$$\exp(2\pi i(1+i))$$

**Exact result:**

$$e^{-2\pi}$$

**Decimal approximation:**

0.001867442731707988814430212934827030393422805002475317199...

0.0018674427...

Thence, inserting this value in the expression, we obtain:

$$H(\tau) = q^{-\frac{1}{8}} (-2 + 45q + 231q^2 + 770q^3 + 2277q^4 + 5796q^5 + \dots)$$

$$(0.0018674427)^{-1/8} (-2 + 45 \cdot 0.0018674427 + 231 \cdot 0.0018674427^2 + 770 \cdot 0.0018674427^3 + 2277 \cdot 0.0018674427^4 + 5796 \cdot 0.0018674427^5)$$

**Input interpretation:**

$$0.0018674427^{-1/8} (-2 + 45 \times 0.0018674427 + 231 \times 0.0018674427^2 + 770 \times 0.0018674427^3 + 2277 \times 0.0018674427^4 + 5796 \times 0.0018674427^5)$$

**Result:**

-4.20047008...

**-4.20047008...**

We obtain also:

$$\left( \left( \left( -0.0018674427 \right)^{-1/8} \left( -2 + 45 * 0.0018674427 + 231 * 0.0018674427^2 + 770 * 0.0018674427^3 + 2277 * 0.0018674427^4 + 5796 * 0.0018674427^5 \right) \right) \right)^{1/3}$$

**Input interpretation:**

$$\left( -0.0018674427^{-1/8} \left( -2 + 45 \times 0.0018674427 + 231 \times 0.0018674427^2 + 770 \times 0.0018674427^3 + 2277 \times 0.0018674427^4 + 5796 \times 0.0018674427^5 \right) \right)^{(1/3)}$$

**Result:**

1.61348884...

**1.61348884...**

From:

**Applications of Harmonic Maass Forms**

By *Michael John Griffin* - UMI 3708974 - Published by ProQuest LLC (2015).

Copyright in the Dissertation held by the Author.

We have:



**Input:**

$$-\left( \frac{\exp\left(\pi \sqrt{\frac{16}{6} - \frac{1}{144}}\right)}{2 \sqrt{16 - \frac{1}{24}}} + \frac{\exp\left(\frac{\pi}{2} \sqrt{\frac{\pi}{6} - \frac{1}{144}}\right)}{\sqrt{16 - \frac{1}{24}}} \right) \quad (\text{A})$$

**Exact result:**

$$-\sqrt{\frac{6}{383}} e^{(\sqrt{383} \pi)/12} - 2 \sqrt{\frac{6}{383}} e^{1/2 \sqrt{\pi/6 - 1/144} \pi}$$

**Decimal approximation:**

-21.7921604566254747127459424621662443480967531405723267207...  
[-21.79216....](#)

Now, from

$$\sum_{e \in \mathcal{W}_a} \sum_{\substack{\rho = \frac{\alpha}{\gamma} \in \mathcal{S}_{Nh} \\ (\frac{\gamma}{\gamma, h}, \frac{N}{h}) = \frac{N}{eh}}} \epsilon_g(L_\rho) \frac{1-i}{\sqrt{2}} 2\pi \left| \frac{-(h, \gamma)^2}{eh^2} + \frac{1}{24} \right|^{\frac{1}{4}} \times$$

$$\sum_{\substack{c > 0 \\ (c, Nh) = (\gamma, Nh)}} \frac{K_c(\frac{1}{2}, L, \nu_\eta, -1, n)}{c} \cdot I_{\frac{1}{2}} \left( \frac{4\pi}{c} \sqrt{\left| \frac{-(h, \gamma)^2}{eh^2} + \frac{1}{24} \right| \left| n + \frac{1}{24} \right|} \right)$$

we obtain:

$$-\left( \left( \frac{1535}{437 \pi} - \frac{85 \pi}{437} \right) \frac{2\pi}{\sqrt{2}} \right)^{0.25} \left[ \left( \frac{16^2}{6^2} + \frac{1}{24} \right) / \left( 16 + \frac{1}{24} \right) \right]^{0.25} * \frac{1}{8} * \left( \exp\left( \frac{2\pi}{8} \left( \frac{1}{5}(-2+3) + 2 \cdot \frac{5+8}{13} \right) \right) \right) * \left( \frac{4\pi}{8} \sqrt{\left( \frac{16^2}{6^2} + \frac{1}{24} \right) \left( 16 + \frac{1}{24} \right)} \right)$$

**Input:**

$$-\left( \left( \frac{1535}{437 \pi} - \frac{85 \pi}{437} \right) \times \frac{2\pi}{\sqrt{2}} \right) \left( \left( \frac{16^2}{6^2} + \frac{1}{24} \right) \right)^{0.25} \times \frac{1}{8}$$

$$\exp\left( 2\pi \left( \left( 1 \left( \frac{1}{5}(-2+3) \right) + 2 \times \frac{5+8}{13} \right) \times \frac{1}{8} \right) \right) \times \frac{4\pi}{8} \sqrt{\left( \frac{16^2}{6^2} + \frac{1}{24} \right) \left( 16 + \frac{1}{24} \right)}$$

**Result:**

-21.79216...

-21.79216... result equal to the value of the previous Ramanujan formula

**Series representations:**

$$\begin{aligned}
& -\frac{1}{(8 \times 8) \sqrt{2}} \\
& \left( \left( \frac{16^2 + \frac{1}{24}}{16 + \frac{1}{24}} \right)^{0.25} \exp\left(\frac{2}{8} \pi \left( \frac{1}{5} (-2 + 3) + \frac{2(5+8)}{13} \right)\right) \left( (4\pi) \sqrt{\left( \frac{16^2}{6^2} + \frac{1}{24} \right) \left( 16 + \frac{1}{24} \right)} \right) \right) \\
& \left( \frac{1535}{437\pi} - \frac{85\pi}{437} \right) (2\pi) = \\
& \left( 0.019868 \pi \left[ -18.0588 \exp\left(\frac{11\pi}{20}\right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{198275}{1728} - z_0\right)^k z_0^{-k}}{k!} + \right. \right. \\
& \quad \left. \left. \pi^2 \exp\left(\frac{11\pi}{20}\right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{198275}{1728} - z_0\right)^k z_0^{-k}}{k!} \right] \right) / \\
& \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2 - z_0)^k z_0^{-k}}{k!} \right) \text{ for (not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{(8 \times 8) \sqrt{2}} \\
& \left( \left( \frac{16^2 + \frac{1}{24}}{16 + \frac{1}{24}} \right)^{0.25} \exp\left(\frac{2}{8} \pi \left( \frac{1}{5} (-2 + 3) + \frac{2(5+8)}{13} \right)\right) \left( (4\pi) \sqrt{\left( \frac{16^2}{6^2} + \frac{1}{24} \right) \left( 16 + \frac{1}{24} \right)} \right) \right) \\
& \left( \frac{1535}{437\pi} - \frac{85\pi}{437} \right) (2\pi) = \\
& \left( 0.019868 \pi \left[ -18.0588 \exp\left(\frac{11\pi}{20}\right) \exp\left(i\pi \left\lfloor \frac{\arg\left(\frac{198275}{1728} - x\right)}{2\pi} \right\rfloor\right) \right. \right. \\
& \quad \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{198275}{1728} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \pi^2 \exp\left(\frac{11\pi}{20}\right) \\
& \quad \left. \left. \exp\left(i\pi \left\lfloor \frac{\arg\left(\frac{198275}{1728} - x\right)}{2\pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{198275}{1728} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right] \right) / \\
& \left( \exp\left(i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{(8 \times 8) \sqrt{2}} \\
& \left( \left( \frac{16^2 + \frac{1}{24}}{16 + \frac{1}{24}} \right)^{0.25} \exp\left(\frac{2}{8} \pi \left( \frac{1}{5} (-2 + 3) + \frac{2(5+8)}{13} \right)\right) \left( (4\pi) \sqrt{\left( \frac{16^2 + \frac{1}{24}}{6^2 + \frac{1}{24}} \right) \left( 16 + \frac{1}{24} \right)} \right) \right) \\
& \left( \frac{1535}{437\pi} - \frac{85\pi}{437} \right) (2\pi) = \\
& \left( 0.019868 \left( \frac{1}{z_0} \right)^{-1/2 [\text{arg}(2-z_0)/(2\pi)] + 1/2 [\text{arg}(\frac{198275}{1728} - z_0)/(2\pi)]} z_0^{-1/2 [\text{arg}(2-z_0)/(2\pi)]} \right. \\
& \left. \left( -18.0588 \pi \exp\left(\frac{11\pi}{20}\right) z_0^{1/2 [\text{arg}(\frac{198275}{1728} - z_0)/(2\pi)]} \right. \right. \\
& \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{198275}{1728} - z_0\right)^k z_0^{-k}}{k!} + \pi^3 \exp\left(\frac{11\pi}{20}\right) z_0^{1/2 [\text{arg}(\frac{198275}{1728} - z_0)/(2\pi)]} \right) \right) \\
& \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{198275}{1728} - z_0\right)^k z_0^{-k}}{k!} \right) \right) / \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right)
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{(8 \times 8) \sqrt{2}} \\
& \left( \left( \frac{16^2 + \frac{1}{24}}{16 + \frac{1}{24}} \right)^{0.25} \exp\left(\frac{2}{8} \pi \left( \frac{1}{5} (-2 + 3) + \frac{2(5+8)}{13} \right)\right) \left( (4\pi) \sqrt{\left( \frac{16^2 + \frac{1}{24}}{6^2 + \frac{1}{24}} \right) \left( 16 + \frac{1}{24} \right)} \right) \right) \\
& \left( \frac{1535}{437\pi} - \frac{85\pi}{437} \right) (2\pi) = \\
& \left( 0.019868 \left( \frac{1}{z_0} \right)^{-1/2 [\text{arg}(2-z_0)/(2\pi)] + 1/2 [\text{arg}(\frac{198275}{1728} - z_0)/(2\pi)]} z_0^{-1/2 [\text{arg}(2-z_0)/(2\pi)]} \right. \\
& \left. \left( -18.0588 \pi \exp\left(\frac{11\pi}{20}\right) z_0^{1/2 [\text{arg}(\frac{198275}{1728} - z_0)/(2\pi)]} \right) \right. \\
& \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{198275}{1728} - z_0\right)^k z_0^{-k}}{k!} + \pi^3 \exp\left(\frac{11\pi}{20}\right) z_0^{1/2 [\text{arg}(\frac{198275}{1728} - z_0)/(2\pi)]} \right) \right) \\
& \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{198275}{1728} - z_0\right)^k z_0^{-k}}{k!} \right) \right) / \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right)
\end{aligned}$$

or:

$$-((0.5070258573)(2\pi)/\sqrt{2}) * [(((16^2)/(6^2)+1/24)) / (16+1/24)]^{0.25} * 1/8 * (((\exp(((2\pi)*((1*(-2+3)/5 + 2*(5+8)/13)*1/8)))))) * (4\pi)/8 * \sqrt{(((16^2)/(6^2)+1/24))*(16+1/24))}$$

**Input interpretation:**

$$-\left(0.5070258573 \times \frac{2\pi}{\sqrt{2}}\right) \left(\left(\frac{\frac{16^2}{6^2} + \frac{1}{24}}{16 + \frac{1}{24}}\right)^{0.25} \times \frac{1}{8}\right) \exp\left(2\pi \left(\left(1 \left(\frac{1}{5}(-2+3)\right) + 2 \times \frac{5+8}{13}\right) \times \frac{1}{8}\right)\right) \times \frac{4\pi}{8} \sqrt{\left(\frac{16^2}{6^2} + \frac{1}{24}\right)\left(16 + \frac{1}{24}\right)}$$

**Result:**

-21.79216...

-21.79216... as above

**Series representations:**

$$\frac{1}{(8 \times 8) \sqrt{2}} \left(\left(\frac{\frac{16^2}{6^2} + \frac{1}{24}}{16 + \frac{1}{24}}\right)^{0.25} \exp\left(\frac{2}{8} \pi \left(\frac{1}{5}(-2+3) + \frac{2(5+8)}{13}\right)\right)\right) \left(4\pi \sqrt{\left(\frac{16^2}{6^2} + \frac{1}{24}\right)\left(16 + \frac{1}{24}\right)}\right) \\ 0.507026 (2\pi) = - \frac{0.0517901 \pi^2 \exp\left(\frac{11\pi}{20}\right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{198275}{1728} - z_0\right)^k z_0^{-k}}{k!}}{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}$$

for (not ( $z_0 \in \mathbb{R}$  and  $-\infty < z_0 \leq 0$ ))

$$\frac{1}{(8 \times 8) \sqrt{2}} \left(\left(\frac{\frac{16^2}{6^2} + \frac{1}{24}}{16 + \frac{1}{24}}\right)^{0.25} \exp\left(\frac{2}{8} \pi \left(\frac{1}{5}(-2+3) + \frac{2(5+8)}{13}\right)\right)\right) \left(4\pi \sqrt{\left(\frac{16^2}{6^2} + \frac{1}{24}\right)\left(16 + \frac{1}{24}\right)}\right) \\ 0.507026 (2\pi) = \frac{0.0517901 \pi^2 \exp\left(\frac{11\pi}{20}\right) \exp\left(i\pi \left\lfloor \frac{\arg\left(\frac{198275}{1728} - x\right)}{2\pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{198275}{1728} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}{\exp\left(i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}$$

for ( $x \in \mathbb{R}$  and  $x < 0$ )



$$\begin{aligned}
& -\frac{1}{(8 \times 8) \sqrt{2}} \\
& \left( \left( \frac{16^2}{6^2} + \frac{1}{24} \right)^{0.25} \exp\left(\frac{2}{8} \pi \left( \frac{1}{5} (-2+3) + \frac{2(5+8)}{13} \right)\right) \left( 4\pi \sqrt{\left( \frac{16^2}{6^2} + \frac{1}{24} \right) \left( 16 + \frac{1}{24} \right)} \right) \right) \\
& 0.507026 (2\pi) = \\
& -\left( \left( 0.0517901 \pi^2 \exp\left(\frac{11\pi}{20}\right) \left(\frac{1}{z_0}\right)^{-1/2 [\operatorname{arg}(2-z_0)/(2\pi)]+1/2 [\operatorname{arg}\left(\frac{198275}{1728}-z_0\right)/(2\pi)]} \right. \right. \\
& \left. \left. \frac{z_0^{-1/2 [\operatorname{arg}(2-z_0)/(2\pi)]+1/2 [\operatorname{arg}\left(\frac{198275}{1728}-z_0\right)/(2\pi)]}}{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{198275}{1728} - z_0\right)^k z_0^{-k}}{k!}} \right) \right) / \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right) \right)
\end{aligned}$$

and again:

$$\begin{aligned}
& -(1/((\pi \cdot 0.6277981322))) (2\pi) / \sqrt{2} * [(((16^2)/(6^2)+1/24)) / (16+1/24)]^{0.25} * 1/8 \\
& * (((\exp(((2\pi * ((1 * (-2+3)/5 + 2 * (5+8)/13) * 1/8)))))) * \\
& (4\pi) / 8 * \sqrt{(((16^2)/(6^2)+1/24)) * (16+1/24))}
\end{aligned}$$

**Input interpretation:**

$$\begin{aligned}
& -\frac{1}{\pi \times 0.6277981322} \times \frac{2\pi}{\sqrt{2}} \left( \left( \frac{16^2}{6^2} + \frac{1}{24} \right)^{0.25} \times \frac{1}{8} \right. \\
& \left. \exp\left(2\pi \left( \left( 1 \left( \frac{1}{5} (-2+3) \right) + 2 \times \frac{5+8}{13} \right) \times \frac{1}{8} \right)\right) \times \frac{4\pi}{8} \sqrt{\left( \frac{16^2}{6^2} + \frac{1}{24} \right) \left( 16 + \frac{1}{24} \right)} \right)
\end{aligned}$$

**Result:**

-21.79216...

-21.79216...

**Series representations:**

$$\frac{\left(\left(\frac{16^2 + \frac{1}{24}}{6^2 + \frac{1}{24}}\right)^{0.25} \exp\left(\frac{2}{8} \pi \left(\frac{1}{5} (-2 + 3) + \frac{2(5+8)}{13}\right)\right) (4\pi) \sqrt{\left(\frac{16^2}{6^2} + \frac{1}{24}\right) \left(16 + \frac{1}{24}\right)}\right) (2\pi)}{((\pi 0.627798) (8 \times 8)) \sqrt{2}} =$$

$$\frac{0.162703 \pi \exp\left(\frac{11\pi}{20}\right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{198275}{1728} - z_0\right)^k z_0^{-k}}{k!}}{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}$$

for (not ( $z_0 \in \mathbb{R}$  and  $-\infty < z_0 \leq 0$ ))

$$\frac{\left(\left(\frac{16^2 + \frac{1}{24}}{6^2 + \frac{1}{24}}\right)^{0.25} \exp\left(\frac{2}{8} \pi \left(\frac{1}{5} (-2 + 3) + \frac{2(5+8)}{13}\right)\right) (4\pi) \sqrt{\left(\frac{16^2}{6^2} + \frac{1}{24}\right) \left(16 + \frac{1}{24}\right)}\right) (2\pi)}{((\pi 0.627798) (8 \times 8)) \sqrt{2}} =$$

$$\frac{0.162703 \pi \exp\left(\frac{11\pi}{20}\right) \exp\left(i \pi \left\lfloor \frac{\operatorname{arg}\left(\frac{198275}{1728} - x\right)}{2\pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{198275}{1728} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}{\exp\left(i \pi \left\lfloor \frac{\operatorname{arg}(2-x)}{2\pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}$$

for ( $x \in \mathbb{R}$  and  $x < 0$ )

$$\frac{\left(\left(\frac{16^2 + \frac{1}{24}}{6^2 + \frac{1}{24}}\right)^{0.25} \exp\left(\frac{2}{8} \pi \left(\frac{1}{5} (-2 + 3) + \frac{2(5+8)}{13}\right)\right) (4\pi) \sqrt{\left(\frac{16^2}{6^2} + \frac{1}{24}\right) \left(16 + \frac{1}{24}\right)}\right) (2\pi)}{((\pi 0.627798) (8 \times 8)) \sqrt{2}} =$$

$$\frac{\left(0.162703 \pi \exp\left(\frac{11\pi}{20}\right) \left(\frac{1}{z_0}\right)^{-1/2 \lfloor \operatorname{arg}(2-z_0)/(2\pi) \rfloor + 1/2 \lfloor \operatorname{arg}\left(\frac{198275}{1728} - z_0\right)/(2\pi) \rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{198275}{1728} - z_0\right)^k z_0^{-k}}{k!} \right) / \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right)}{\left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{198275}{1728} - z_0\right)^k z_0^{-k}}{k!} \right) / \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right)}$$

From which:

$$-(1/((\pi * 0.6277981322))) * ((2\pi) / \sqrt{2}) * [(((16^2) / (6^2 + 1/x)) / (16 + 1/x))^0.25 * 1/8 * ((\exp(((2\pi) * ((1 * (-2 + 3) / 5 + 2 * (5 + 8) / 13) * 1/8)))))) * (4\pi) / 8 * \sqrt{(((16^2) / (6^2 + 1/x)) * (16 + 1/x))} = -21.79216$$

**Input interpretation:**

$$-\frac{1}{\pi \times 0.6277981322} \times \frac{2\pi}{\sqrt{2}}$$

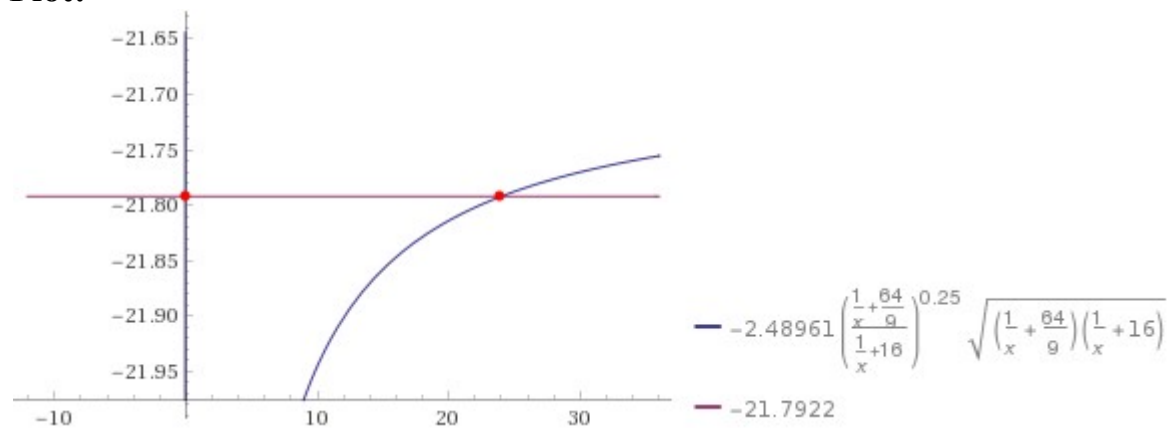
$$\left( \left( \frac{16^2}{6^2} + \frac{1}{x} \right)^{0.25} \times \frac{1}{8} \exp \left( 2\pi \left( \left( 1 \left( \frac{1}{5} (-2+3) \right) + 2 \times \frac{5+8}{13} \right) \times \frac{1}{8} \right) \right) \times \frac{4\pi}{8} \right)$$

$$\sqrt{\left( \frac{16^2}{6^2} + \frac{1}{x} \right) \left( 16 + \frac{1}{x} \right)} = -21.79216$$

**Result:**

$$-2.48961 \left( \frac{1 + \frac{64}{9}}{\frac{1}{x} + 16} \right)^{0.25} \sqrt{\left( \frac{1}{x} + \frac{64}{9} \right) \left( \frac{1}{x} + 16 \right)} = -21.7922$$

**Plot:**



**Alternate form assuming x is real:**

$$\sqrt{\frac{(16x+1)(64x+9)}{x^2}} \left( \frac{64x+9}{144x+9} \right)^{0.25} = 26.2597$$

**Alternate forms:**

$$- \frac{1.43738 \sqrt{\left( \frac{1}{x} + \frac{64}{9} \right) \left( \frac{1}{x} + 16 \right)} (-64x-9)^{0.25}}{(-16x-1)^{0.25}} = -21.7922$$

$$\sqrt[4]{\frac{64x+9}{16x+1}} \sqrt{\frac{1024x^2+208x+9}{x^2}} = 45.4831$$

$$-0.829871 \left( \frac{1 + \frac{64}{9}}{\frac{1}{x} + 16} \right)^{0.25} \sqrt{\frac{1024x^2+208x+9}{x^2}} = -21.7922$$

**Alternate form assuming x is positive:**

$$(16x+1)^{0.25} (64x+9)^{0.75} = 45.4831x$$

**Alternate forms assuming  $x > 0$ :**

$$-2.48961 \left(\frac{1}{x} + \frac{64}{9}\right)^{0.75} \left(\frac{1}{x} + 16\right)^{0.25} = -21.7922$$

$$(-2.48961 + 0i) \left(\frac{1}{x} + \frac{64}{9}\right)^{0.75} \left(\frac{1}{x} + 16\right)^{0.25} = -21.7922$$

**Solutions:**

$$x \approx -0.0519004$$

$$x \approx 24.0001$$

24.0001  $\approx$  24 value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

and:

$$\left[ \left( \frac{1}{\pi \times 0.6277981322} \right) \left( \frac{2\pi}{\sqrt{2}} \right) \left[ \left( \frac{16^2}{6^2} + \frac{1}{24} \right) / \left( 16 + \frac{1}{24} \right) \right]^{0.25} \right]^{1/8} \times \left[ \exp \left( \frac{2\pi i}{5} \left( 1 \times (-2+3) + 2 \times \frac{5+8}{13} \times \frac{1}{8} \right) \right) \right]^{4\pi/8} \sqrt[6]{ \left( \frac{16^2}{6^2} + \frac{1}{24} \right) \left( 16 + \frac{1}{24} \right) } \right]^{1/6}$$

**Input interpretation:**

$$\left( \left( \frac{1}{\pi \times 0.6277981322} \times \frac{2\pi}{\sqrt{2}} \left( \frac{16^2}{6^2} + \frac{1}{24} \right)^{0.25} \right) \times \frac{1}{8} \right) \times \left[ \exp \left( 2\pi i \left( \left( 1 \times \left( \frac{1}{5} (-2+3) \right) + 2 \times \frac{5+8}{13} \times \frac{1}{8} \right) \right) \right) \right]^{4\pi/8} \sqrt[6]{ \left( \frac{16^2}{6^2} + \frac{1}{24} \right) \left( 16 + \frac{1}{24} \right) } \right]^{1/6}$$

**Result:**

1.671283199351855910864385526644339544073633144643303077899...

1.67128319935....

From which:

$$\left[ \left( \frac{1}{\pi \times 0.6277981322} \right) \left( \frac{2\pi}{\sqrt{2}} \left( \frac{16^2}{6^2} + \frac{1}{24} \right)^{0.25} \right) \right] \times \frac{1}{8} \left[ \exp \left( \frac{2\pi}{5} (-2+3) + 2 \times \frac{5+8}{13} \times \frac{1}{8} \right) \right] \left( \frac{4\pi}{8} \sqrt{\left( \frac{16^2}{6^2} + \frac{1}{24} \right) \left( 16 + \frac{1}{24} \right)} \right)^{\frac{1}{6} - \frac{2^2+7^2}{10^3}}$$

**Input interpretation:**

$$\left( \left( \frac{1}{\pi \times 0.6277981322} \times \frac{2\pi}{\sqrt{2}} \left( \frac{16^2}{6^2} + \frac{1}{24} \right)^{0.25} \right) \times \frac{1}{8} \right) \left( \exp \left( 2\pi \left( \left( \frac{1}{5} (-2+3) \right) + 2 \times \frac{5+8}{13} \times \frac{1}{8} \right) \right) \times \frac{4\pi}{8} \sqrt{\left( \frac{16^2}{6^2} + \frac{1}{24} \right) \left( 16 + \frac{1}{24} \right)} \right)^{\frac{1}{6} - \frac{2^2+7^2}{10^3}}$$

**Result:**

1.618283199351855910864385526644339544073633144643303077899...

[1.61828319935.....](#)

**Series representations:**

$$\sqrt[6]{ \frac{\left( \exp \left( \frac{2}{8} \pi \left( \frac{1}{5} (-2+3) + \frac{2(5+8)}{13} \right) \right) (4\pi) \sqrt{\left( \frac{16^2}{6^2} + \frac{1}{24} \right) \left( 16 + \frac{1}{24} \right)} \right) \left( \frac{2\pi}{8} \left( \frac{16^2}{6^2} + \frac{1}{24} \right)^{0.25} \right)}{(8 \times 8) (\pi \cdot 0.627798) \sqrt{2}} } - \frac{2^2+7^2}{10^3} = 0.738867 \left( -0.0717315 + \sqrt[6]{ \frac{\pi \exp \left( \frac{11\pi}{20} \right) \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{1}{2} \right)_k \left( \frac{198275}{1728} - z_0 \right)^k z_0^{-k}}{k!}}{\sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{1}{2} \right)_k (2-z_0)^k z_0^{-k}}{k!}} } \right)$$

for (not  $(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)$ )

$$\sqrt[6]{\frac{\left(\exp\left(\frac{2}{8}\pi\left(\frac{1}{5}(-2+3)+\frac{2(5+8)}{13}\right)\right)(4\pi)\sqrt{\left(\frac{16^2}{6^2}+\frac{1}{24}\right)\left(16+\frac{1}{24}\right)}\right)\left(2\pi\left(\frac{16^2+\frac{1}{24}}{16+\frac{1}{24}}\right)^{0.25}\right)}{(8\times 8)(\pi 0.627798)\sqrt{2}} - \frac{2^2+7^2}{10^3} = 0.738867 \left[ -0.0717315 + \frac{\pi \exp\left(\frac{11\pi}{20}\right) \exp\left(i\pi\left[\frac{\operatorname{arg}\left(\frac{198275}{1728}-x\right)}{2\pi}\right]\right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{198275}{1728}-x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}{\exp\left(i\pi\left[\frac{\operatorname{arg}(2-x)}{2\pi}\right]\right) \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}} \right] \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\sqrt[6]{\frac{\left(\exp\left(\frac{2}{8}\pi\left(\frac{1}{5}(-2+3)+\frac{2(5+8)}{13}\right)\right)(4\pi)\sqrt{\left(\frac{16^2}{6^2}+\frac{1}{24}\right)\left(16+\frac{1}{24}\right)}\right)\left(2\pi\left(\frac{16^2+\frac{1}{24}}{16+\frac{1}{24}}\right)^{0.25}\right)}{(8\times 8)(\pi 0.627798)\sqrt{2}} - \frac{2^2+7^2}{10^3} = 0.738867 \left[ -0.0717315 + \left( \left( \pi \exp\left(\frac{11\pi}{20}\right) \left(\frac{1}{z_0}\right)^{-1/2 [\operatorname{arg}(2-z_0)/(2\pi)]+1/2 [\operatorname{arg}\left(\frac{198275}{1728}-z_0\right)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{198275}{1728}-z_0\right)^k z_0^{-k}}{k!} \right) / \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right) \right]^{\wedge (1/6)}$$

Now, we have:

In physical terms the monster Lie algebra  $\mathfrak{m}$  is (“about half” of) the space of “physical states” of a bosonic string moving in the quotient

$$(\mathbb{R}^{24}/\Lambda \oplus \mathbb{R}^{1,1}/II_{1,1}) / (\mathbb{Z}/2\mathbb{Z}) \quad (7.4)$$

of the 26-dimensional torus  $II_{25,1} \otimes_{\mathbb{Z}} \mathbb{R}/II_{25,1} \simeq \mathbb{R}^{24}/\Lambda \oplus \mathbb{R}^{1,1}/II_{1,1}$  by the Kummer involution  $x \mapsto -x$ . The monster Lie algebra  $\mathfrak{m}$  is constructed in a functorial way from  $V^{\natural}$  (cf. [66]), inherits an action by the monster from  $V^{\natural}$ , and admits a monster-invariant grading by  $II_{1,1}$ .

The denominator identity for a Kac–Moody algebra  $\mathfrak{g}$  equates a product indexed by the positive roots of  $\mathfrak{g}$  with a sum indexed by the Weyl group of  $\mathfrak{g}$ . A BKM algebra also admits a denominator identity, which for the case of the monster Lie algebra  $\mathfrak{m}$  is the beautiful *Koike–Norton–Zagier formula*

$$p^{-1} \prod_{\substack{m,n \in \mathbb{Z} \\ m > 0}} (1 - p^m q^n)^{c(mn)} = J(\sigma) - J(\tau), \quad (7.5)$$

where  $\sigma \in \mathbb{III}$  and  $p = e^{2\pi i \sigma}$  (and  $c(n)$  is the coefficient of  $q^n$  in  $J(\tau)$ , cf. (1.33)). Since the right hand side of (7.5) implies that the left hand side has no terms  $p^m q^n$  with  $mn \neq 0$ , this identity imposes many non-trivial polynomial relations upon the coefficients of  $J(\tau)$ . Among these is

$$c(4n + 2) = c(2n + 2) + \sum_{k=1}^n c(k)c(2n - k + 1), \quad (7.6)$$

which was first found by Mahler [221] by a different method, along with similar expressions for  $c(4n)$ ,  $c(4n + 1)$ , and  $c(4n + 3)$ , which are also entailed in (7.5). Taken together these relations allow us to compute the coefficients of  $J(\tau)$  recursively, given just the values

$$\begin{aligned} c(1) &= 196884, \\ c(2) &= 21493760, \\ c(3) &= 864299970, \\ c(5) &= 333202640600. \end{aligned} \quad (7.7)$$



Rademacher “perfected” the circle method introduced by Hardy–Ramanujan, and he obtained an exact convergent series expression for the combinatorial partition function  $p(n)$ . In 1938 he generalized this work [255] and obtained such exact formulas for the Fourier coefficients of general modular functions. For the elliptic modular invariant  $J(\tau) = \sum_n c(n)q^n$  (cf. (1.33)), Rademacher’s formula (which was obtained earlier by Petersson [251], via a different method) may be written as

$$c(n) = 4\pi^2 \sum_{c>0} \sum_{\substack{0<a<c \\ (a,c)=1}} \frac{e^{-2\pi i \frac{a}{c}} e^{2\pi i n \frac{d}{c}}}{c^2} \sum_{k \geq 0} \frac{(4\pi^2)^k}{c^{2k}} \frac{1}{(k+1)!} \frac{n^k}{k!}, \quad (7.15)$$

where  $d$ , in each summand, is a multiplicative inverse for  $a$  modulo  $c$ , and  $(a, c)$  is the greatest common divisor of  $a$  and  $c$ . Having established the formula (7.15), Rademacher sought to reverse the process, and use it to derive the modular invariance of  $J(\tau)$ . That is, he set out to prove directly that  $J_0(\tau + 1) = J_0(-1/\tau) = J_0(\tau)$ , when  $J_0(\tau)$  is defined by setting  $J_0(\tau) = q^{-1} + \sum_{n>0} c(n)q^n$ , with  $c(n)$  defined by (7.15).

Now, from (7.15):

$$c(n) = 4\pi^2 \sum_{c>0} \sum_{\substack{0<a<c \\ (a,c)=1}} \frac{e^{-2\pi i \frac{a}{c}} e^{2\pi i n \frac{d}{c}}}{c^2} \sum_{k \geq 0} \frac{(4\pi^2)^k}{c^{2k}} \frac{1}{(k+1)!} \frac{n^k}{k!},$$

for  $c(1) = 196884$ , and  $a, c, d, n$  and  $k = 1$ , we obtain:

$$(4\pi^2)^2 * (e^{-2\pi i}) * (e^{2\pi i}) * (4\pi^2)^2 * 1/2! * 1/1!$$

**Input:**

$$(4\pi^2)^2 e^{-2\pi i} e^{2\pi i} (4\pi^2)^2 \times \frac{1}{2!} \times \frac{1}{1!}$$

$n!$  is the factorial function

$i$  is the imaginary unit

**Exact result:**

$$8\pi^4$$



### Decimal approximation:

779.2727282720194978915226615096408899978206853814833735317...

(using the principal branch of the logarithm for complex exponentiation)

779.2727282... result very near to the rest mass of Omega meson 782.65

### Property:

$8\pi^4$  is a transcendental number

### Alternative representations:

$$\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)}{2! \times 1!} = \frac{16 e^{-2i\pi} e^{2i\pi} (\pi^2)^2}{\Gamma(2)\Gamma(3)}$$

$$\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)}{2! \times 1!} = \frac{16 e^{-2i\pi} e^{2i\pi} (\pi^2)^2}{(0!! \times 1!!)(1!! \times 2!!)}$$

$$\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)}{2! \times 1!} = \frac{16 e^{-2i\pi} e^{2i\pi} (\pi^2)^2}{\Gamma(2, 0)\Gamma(3, 0)}$$

### Series representations:

$$\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)}{2! \times 1!} = 720 \sum_{k=1}^{\infty} \frac{1}{k^4}$$

$$\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)}{2! \times 1!} = 768 \sum_{k=0}^{\infty} \frac{1}{(1+2k)^4}$$

$$\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)}{2! \times 1!} = 2048 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^4$$

### Integral representations:

$$\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)}{2! \times 1!} = 2048 \left( \int_0^1 \sqrt{1-t^2} dt \right)^4$$

$$\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)}{2! \times 1!} = 128 \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^4$$

$$\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)}{2! \times 1!} = 128 \left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^4$$

From

$$\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)}{2! \times 1!} = 720 \sum_{k=1}^{\infty} \frac{1}{k^4}$$

we obtain:

$$(((720 \sum_{k=1}^{\infty} 1/k^4))) * 3/\pi^4$$

**Input interpretation:**

$$\left( 720 \sum_{k=1}^{\infty} \frac{1}{k^4} \right) \times \frac{3}{\pi^4}$$

**Result:**

24

24 value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

and:

$$((((e^{(-2\pi i)} 4\pi^2 e^{(2\pi i)}) (4\pi^2))/(2! 1!)))) 3/\pi^4$$

**Input:**

$$\frac{(e^{-2\pi i} \times 4\pi^2 e^{2\pi i})(4\pi^2)}{2! \times 1!} \times \frac{3}{\pi^4}$$

$n!$  is the factorial function

$i$  is the imaginary unit

**Result:**

24

24 as above

### Alternative representations:

$$\frac{3((e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2))}{\pi^4 (2! \times 1!)} = \frac{48 e^{-2i\pi} e^{2i\pi} (\pi^2)^2}{(\Gamma(2) \Gamma(3)) \pi^4}$$

$$\frac{3((e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2))}{\pi^4 (2! \times 1!)} = \frac{48 e^{-2i\pi} e^{2i\pi} (\pi^2)^2}{(0!! (1!!)^2 2!!) \pi^4}$$

$$\frac{3((e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2))}{\pi^4 (2! \times 1!)} = \frac{48 e^{-2i\pi} e^{2i\pi} (\pi^2)^2}{(\Gamma(2, 0) \Gamma(3, 0)) \pi^4}$$

### Series representation:

$$\frac{3((e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2))}{\pi^4 (2! \times 1!)} = \frac{48}{\left( \sum_{k=0}^{\infty} \frac{(1-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right) \sum_{k=0}^{\infty} \frac{(2-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}}$$

for  $(n_0 \geq 0 \text{ or } n_0 \notin \mathbb{Z})$  and  $n_0 \rightarrow 1$  and  $n_0 \rightarrow 2$

### Integral representations:

$$\frac{3((e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2))}{\pi^4 (2! \times 1!)} = \frac{48}{\left( \int_0^1 \log\left(\frac{1}{t}\right) dt \right) \int_0^1 \log^2\left(\frac{1}{t}\right) dt}$$

$$\frac{3((e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2))}{\pi^4 (2! \times 1!)} = \frac{48}{\left( \int_0^{\infty} e^{-t} t dt \right) \int_0^{\infty} e^{-t} t^2 dt}$$

$$\frac{3((e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2))}{\pi^4 (2! \times 1!)} = \frac{48}{\left( \int_1^{\infty} e^{-t} t dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{(2+k)k!} \right) \left( \int_1^{\infty} e^{-t} t^2 dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{(3+k)k!} \right)}$$

We know that the **Coxeter number**  $h$  is the order of a **Coxeter element** of an irreducible Coxeter group.  $E_8$  has Coxeter number 30. Thence, from

$$(4\pi^2) e^{-2\pi i} e^{2\pi i} (4\pi^2) \times \frac{1}{2!} \times \frac{1}{1!}$$

we obtain

$$1/30((((4\pi^2) * (e^{(-2\pi i)}) * (e^{(2\pi i)}) * (4\pi^2) * 1/2! * 1/1!)))$$

**Input:**

$$\frac{1}{30} \left( (4\pi^2) e^{-2\pi i} e^{2\pi i} (4\pi^2) \times \frac{1}{2!} \times \frac{1}{1!} \right)$$

$n!$  is the factorial function

$i$  is the imaginary unit

**Exact result:**

$$\frac{4\pi^4}{15}$$

**Decimal approximation:**

25.97575760906731659638408871698802966659402284604944578439...

(using the principal branch of the logarithm for complex exponentiation)

25.9757576...  $\approx$  26 result equal to the number of dimensions in Bosonic String Theory. Indeed:

In physical terms the monster Lie algebra  $\mathfrak{m}$  is (“about half” of) the space of “physical states” of a bosonic string moving in the quotient

$$(\mathbb{R}^{24}/\Lambda \oplus \mathbb{R}^{1,1}/II_{1,1}) / (\mathbb{Z}/2\mathbb{Z})$$

of the 26-dimensional torus  $II_{25,1} \otimes_{\mathbb{Z}} \mathbb{R} / II_{25,1} \simeq \mathbb{R}^{24}/\Lambda \oplus \mathbb{R}^{1,1}/II_{1,1}$

**Property:**

$\frac{4\pi^4}{15}$  is a transcendental number

**Alternative representations:**

$$\frac{e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2)}{(2! \times 1!) 30} = \frac{16 e^{-2i\pi} e^{2i\pi} (\pi^2)^2}{30 \Gamma(2) \Gamma(3)}$$

$$\frac{e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2)}{(2! \times 1!) 30} = \frac{16 e^{-2i\pi} e^{2i\pi} (\pi^2)^2}{30 (0!! \times 1!!) (1!! \times 2!!)}$$

$$\frac{e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2)}{(2! \times 1!) 30} = \frac{16 e^{-2i\pi} e^{2i\pi} (\pi^2)^2}{30 \Gamma(2, 0) \Gamma(3, 0)}$$

**Series representations:**

$$\frac{e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2)}{(2! \times 1!) 30} = 24 \sum_{k=1}^{\infty} \frac{1}{k^4}$$

$$\frac{e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2)}{(2! \times 1!) 30} = \frac{128}{5} \sum_{k=0}^{\infty} \frac{1}{(1+2k)^4}$$

$$\frac{e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2)}{(2! \times 1!) 30} = \frac{1024}{15} \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^4$$

**Integral representations:**

$$\frac{e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2)}{(2! \times 1!) 30} = \frac{64}{15} \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^4$$

$$\frac{e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2)}{(2! \times 1!) 30} = \frac{64}{15} \left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^4$$

$$\frac{e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2)}{(2! \times 1!) 30} = \frac{1024}{15} \left( \int_0^1 \sqrt{1-t^2} dt \right)^4$$

We obtain also:

$$((((4\pi^2) * (e^{(-2\pi*i)}) * (e^{(2\pi*i)}) * (4\pi^2) * 1/2! * 1/1!)))-8\pi-11+1$$

**Input:**

$$(4\pi^2) e^{-2\pi i} e^{2\pi i} (4\pi^2) \times \frac{1}{2!} \times \frac{1}{1!} - 8\pi - 11 + 1$$

$n!$  is the factorial function

$i$  is the imaginary unit

**Exact result:**

$$-10 - 8\pi + 8\pi^4$$

**Decimal approximation:**

744.1399870433011519838215144434048669242433301864825269639...

(using the principal branch of the logarithm for complex exponentiation)

744.139987...

**Property:**

$-10 - 8\pi + 8\pi^4$  is a transcendental number

**Alternate form:**

$2(-5 - 4\pi + 4\pi^4)$

**Alternative representations:**

$$\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)}{2! \times 1!} - 8\pi - 11 + 1 = -10 - 8\pi + \frac{16 e^{-2i\pi} e^{2i\pi} (\pi^2)^2}{\Gamma(2)\Gamma(3)}$$

$$\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)}{2! \times 1!} - 8\pi - 11 + 1 = -10 - 8\pi + \frac{16 e^{-2i\pi} e^{2i\pi} (\pi^2)^2}{(0!! \times 1!!)(1!! \times 2!!)}$$

$$\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)}{2! \times 1!} - 8\pi - 11 + 1 = -10 - 8\pi + \frac{16 e^{-2i\pi} e^{2i\pi} (\pi^2)^2}{\Gamma(2, 0)\Gamma(3, 0)}$$

**Series representations:**

$$\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)}{2! \times 1!} - 8\pi - 11 + 1 = -10 - 8\pi + 720 \sum_{k=1}^{\infty} \frac{1}{k^4}$$

$$\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)}{2! \times 1!} - 8\pi - 11 + 1 = -10 - 8\pi + 768 \sum_{k=0}^{\infty} \frac{1}{(1+2k)^4}$$

$$\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)}{2! \times 1!} - 8\pi - 11 + 1 = 2 \left( -5 - 16 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} + 1024 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^4 \right)$$

**Integral representations:**

$$\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)}{2! \times 1!} - 8\pi - 11 + 1 = 2 \left( -5 - 8 \int_0^{\infty} \frac{1}{1+t^2} dt + 64 \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^4 \right)$$

$$\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)}{2! \times 1!} - 8\pi - 11 + 1 = 2 \left( -5 - 16 \int_0^1 \sqrt{1-t^2} dt + 1024 \left( \int_0^1 \sqrt{1-t^2} dt \right)^4 \right)$$

$$\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)}{2! \times 1!} - 8\pi - 11 + 1 = 2 \left( -5 - 8 \int_0^1 \frac{1}{\sqrt{1-t^2}} dt + 64 \left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^4 \right)$$

and:

$$((((4\pi^2) * (e^{(-2\pi*i)}) * (e^{(2\pi*i)}) * (4\pi^2) * 1/2! * 1/1!)))x = 196884$$

**Input:**

$$\left( (4\pi^2) e^{-2\pi i} e^{2\pi i} (4\pi^2) \times \frac{1}{2!} \times \frac{1}{1!} \right) x = 196884$$

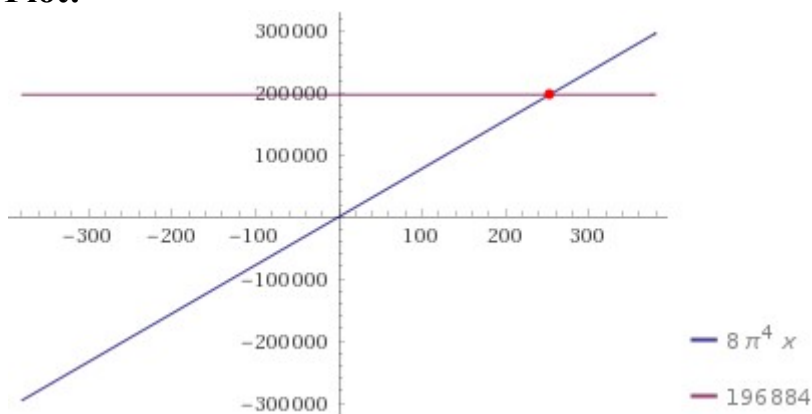
$n!$  is the factorial function

$i$  is the imaginary unit

**Exact result:**

$$8\pi^4 x = 196884$$

**Plot:**



**Alternate form:**

$$8\pi^4 x - 196884 = 0$$

**Solution:**

$$x = \frac{49\,221}{2\pi^4}$$

**Solution:**

$$x \approx 252.65$$

252.65

Thence, we have that:

$$196884 / (((4\pi^2) * (e^{(-2\pi*i)}) * (e^{(2\pi*i)}) * (4\pi^2) * 1/2! * 1/1!))$$

**Input:**

$$\frac{196\,884}{(4\pi^2) e^{-2\pi i} e^{2\pi i} (4\pi^2) \times \frac{1}{2!} \times \frac{1}{1!}}$$

$n!$  is the factorial function

$i$  is the imaginary unit

**Exact result:**

$$\frac{49\,221}{2\pi^4}$$

**Decimal approximation:**

252.6509562789088311726445725954246221774531175189644646254...

(using the principal branch of the logarithm for complex exponentiation)

252.65095627...

**Property:**

$\frac{49\,221}{2\pi^4}$  is a transcendental number

**Alternative representations:**

$$\frac{196\,884}{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)} = \frac{196\,884}{16 e^{-2i\pi} e^{2i\pi} (\pi^2)^2} = \frac{196\,884}{\Gamma(2)\Gamma(3)}$$



$$\frac{196\,884}{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)} = \frac{196\,884}{16 e^{-2i\pi} e^{2i\pi} (\pi^2)^2}$$

$$\frac{196\,884}{2! \times 1!} = \frac{196\,884}{(0!! \times 1!!)(1!! \times 2!!)}$$

$$\frac{196\,884}{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)} = \frac{196\,884}{16 e^{-2i\pi} e^{2i\pi} (\pi^2)^2}$$

$$\frac{196\,884}{2! \times 1!} = \frac{196\,884}{\Gamma(2,0)\Gamma(3,0)}$$

### Series representations:

$$\frac{196\,884}{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)} = \frac{49\,221}{512 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^4}$$

$$\frac{196\,884}{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)} = \frac{49\,221}{512 \left( \sum_{k=0}^{\infty} \frac{(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^4}$$

$$\frac{196\,884}{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)} = \frac{49\,221}{2 \left( \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^4}$$

### Integral representations:

$$\frac{196\,884}{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)} = \frac{49\,221}{512 \left( \int_0^1 \sqrt{1-t^2} dt \right)^4}$$

$$\frac{196\,884}{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)} = \frac{49\,221}{32 \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^4}$$

$$\frac{196\,884}{(e^{-2\pi i} 4\pi^2 e^{2\pi i})(4\pi^2)} = \frac{49\,221}{32 \left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^4}$$

From which, after the following calculations

$$196884/((((4\pi^2) * (e^{(-2\pi*i)}) * (e^{(2\pi*i)}) * (4\pi^2) * 1/2! * 1/1!)))*12*1/4$$

$$(\sqrt{3} - 1) \pi^{-11-4}$$

we obtain:

**Input:**

$$\left( \frac{196884}{(4\pi^2) e^{-2\pi i} e^{2\pi i} (4\pi^2) \times \frac{1}{2!} \times \frac{1}{1!}} \times 12 \times \frac{1}{4} \right) (\sqrt{3} - 1) \pi^{-11-4}$$

$n!$  is the factorial function

$i$  is the imaginary unit

**Exact result:**

$$\frac{147663(\sqrt{3} - 1)}{2\pi^3} - 15$$

**Decimal approximation:**

1728.144130341899991006338316798622839385648944823234292667...

(using the principal branch of the logarithm for complex exponentiation)

1728.144130341...

**Property:**

$$-15 + \frac{147663(-1 + \sqrt{3})}{2\pi^3} \text{ is a transcendental number}$$

**Alternate forms:**

$$\frac{147663\sqrt{3} - 147663}{2\pi^3} - 15$$

$$-15 - \frac{147663}{2\pi^3} + \frac{147663\sqrt{3}}{2\pi^3}$$

$$\frac{3(-49221 + 49221\sqrt{3} - 10\pi^3)}{2\pi^3}$$

**Alternative representations:**

$$\frac{((\sqrt{3} - 1)\pi) 196884 \times 12}{\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2))^4}{2! \times 1!}} - 11 - 4 = -15 + \frac{2362608\pi(-1 + \sqrt{3})}{4(16e^{-2i\pi} e^{2i\pi} (\pi^2)^2)} \frac{\Gamma(2)\Gamma(3)}{\Gamma(2)\Gamma(3)}$$

$$\frac{((\sqrt{3}-1)\pi) 196884 \times 12}{\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2))^4}{2! \times 1!}} - 11 - 4 = -15 + \frac{2362608 \pi (-1 + \sqrt{3})}{4 \frac{(16 e^{-2i\pi} e^{2i\pi} (\pi^2)^2)}{(0!! \times 1!!)(1!! \times 2!!)}}$$

$$\frac{((\sqrt{3}-1)\pi) 196884 \times 12}{\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2))^4}{2! \times 1!}} - 11 - 4 = -15 + \frac{2362608 \pi (-1 + \sqrt{3})}{4 \frac{(16 e^{-2i\pi} e^{2i\pi} (\pi^2)^2)}{\Gamma(2,0)\Gamma(3,0)}}$$

### Series representations:

$$\frac{((\sqrt{3}-1)\pi) 196884 \times 12}{\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2))^4}{2! \times 1!}} - 11 - 4 = -\frac{1}{4\pi^3} 3 \left( 20\pi^3 + 49221 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(1-n_0)^{k_1} (2-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} - 49221 \sqrt{3} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(1-n_0)^{k_1} (2-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} \right)$$

for  $((n_0 \geq 0 \text{ or } n_0 \notin \mathbb{Z}) \text{ and } n_0 \rightarrow 1 \text{ and } n_0 \rightarrow 2)$

$$\frac{((\sqrt{3}-1)\pi) 196884 \times 12}{\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2))^4}{2! \times 1!}} - 11 - 4 = -\frac{1}{4\pi^3} 3 \left( 20\pi^3 + 49221 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(1-n_0)^{k_1} (2-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} - 49221 \sqrt{2} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{2^{-k_1} \binom{\frac{1}{2}}{k_1} (1-n_0)^{k_2} (2-n_0)^{k_3} \Gamma^{(k_2)}(1+n_0) \Gamma^{(k_3)}(1+n_0)}{k_2! k_3!} \right)$$

for  $((n_0 \geq 0 \text{ or } n_0 \notin \mathbb{Z}) \text{ and } n_0 \rightarrow 1 \text{ and } n_0 \rightarrow 2)$

$$\frac{((\sqrt{3}-1)\pi) 196884 \times 12}{\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2))^4}{2! \times 1!}} - 11 - 4 = -\frac{1}{4\pi^3} 3 \left( 20\pi^3 + \right.$$

$$49221 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(1-n_0)^{k_1} (2-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} - 49221 \sqrt{2}$$

$$\left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^{k_1} \left(-\frac{1}{2}\right)_{k_1} (1-n_0)^{k_2} (2-n_0)^{k_3} \Gamma^{(k_2)}(1+n_0) \Gamma^{(k_3)}(1+n_0)}{k_1! k_2! k_3!} \right)$$

for  $(n_0 \geq 0 \text{ or } n_0 \notin \mathbb{Z})$  and  $n_0 \rightarrow 1$  and  $n_0 \rightarrow 2$

### Integral representations:

$$\frac{((\sqrt{3}-1)\pi) 196884 \times 12}{\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2))^4}{2! \times 1!}} - 11 - 4 = -\frac{3 \left( 20\pi^3 + 2 \int_0^1 \int_0^1 \log\left(\frac{1}{t_1}\right) \log^2\left(\frac{1}{t_2}\right) dt_2 dt_1 \right)}{4\pi^3}$$

$$\frac{((\sqrt{3}-1)\pi) 196884 \times 12}{\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2))^4}{2! \times 1!}} - 11 - 4 =$$

$$\frac{3 \left( 20\pi^3 + 49221 \left( \int_0^{\infty} e^{-t} t dt \right) \int_0^{\infty} e^{-t} t^2 dt - 49221 \left( \int_0^{\infty} e^{-t} t dt \right) \left( \int_0^{\infty} e^{-t} t^2 dt \right) \sqrt{3} \right)}{4\pi^3}$$

$$\frac{((\sqrt{3}-1)\pi) 196884 \times 12}{\frac{(e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2))^4}{2! \times 1!}} - 11 - 4 =$$

$$-15 + \frac{147663 (-1 + \sqrt{3}) \left( \int_1^{\infty} e^{-t} t dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{(2+k)k!} \right) \left( \int_1^{\infty} e^{-t} t^2 dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{(3+k)k!} \right)}{4\pi^3}$$

and again:

$$1/11(((196884/(((4\pi^2) * (e^{(-2\pi i)}) * (e^{(2\pi i)}) * (4\pi^2) * 1/2! * 1/1!)))+12)))$$

**Input:**

$$\frac{1}{11} \left( \frac{196884}{(4\pi^2) e^{-2\pi i} e^{2\pi i} (4\pi^2) \times \frac{1}{2!} \times \frac{1}{1!}} + 12 \right)$$

$n!$  is the factorial function

$i$  is the imaginary unit

**Exact result:**

$$\frac{1}{11} \left( 12 + \frac{49\,221}{2\pi^4} \right)$$

**Decimal approximation:**

24.05917784353716647024041569049314747067755613808767860231...

(using the principal branch of the logarithm for complex exponentiation)

24.059177843...  $\approx$  24 value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

**Property:**

$\frac{1}{11} \left( 12 + \frac{49\,221}{2\pi^4} \right)$  is a transcendental number

**Alternate forms:**

$$\frac{12}{11} + \frac{49\,221}{22\pi^4}$$

$$\frac{3}{22} \left( 8 + \frac{16\,407}{\pi^4} \right)$$

$$\frac{3(16\,407 + 8\pi^4)}{22\pi^4}$$

**Alternative representations:**

$$\frac{1}{11} \left( \frac{196\,884}{\frac{e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2)}{2! \times 1!}} + 12 \right) = \frac{1}{11} \left( 12 + \frac{196\,884}{\frac{16 e^{-2i\pi} e^{2i\pi} (\pi^2)^2}{\Gamma(2)\Gamma(3)}} \right)$$

$$\frac{1}{11} \left( \frac{196\,884}{\frac{e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2)}{2! \times 1!}} + 12 \right) = \frac{1}{11} \left( 12 + \frac{196\,884}{\frac{16 e^{-2i\pi} e^{2i\pi} (\pi^2)^2}{(0!! \times 1!!)(1!! \times 2!!)}} \right)$$

$$\frac{1}{11} \left( \frac{196\,884}{\frac{e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2)}{2! \times 1!}} + 12 \right) = \frac{1}{11} \left( 12 + \frac{196\,884}{\frac{16 e^{-2i\pi} e^{2i\pi} (\pi^2)^2}{\Gamma(2,0)\Gamma(3,0)}} \right)$$

**Series representations:**

$$\frac{1}{11} \left( \frac{196884}{\frac{e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2)}{2! \times 1!}} + 12 \right) = \frac{12}{11} + \frac{49221}{5632 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^4}$$

$$\frac{1}{11} \left( \frac{196884}{\frac{e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2)}{2! \times 1!}} + 12 \right) = \frac{12}{11} + \frac{49221}{22 \left( \sum_{k=0}^{\infty} \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^4}$$

$$\frac{1}{11} \left( \frac{196884}{\frac{e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2)}{2! \times 1!}} + 12 \right) = \frac{12}{11} + \frac{49221}{22 \left( \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right) \right)^4}$$

**Integral representations:**

$$\frac{1}{11} \left( \frac{196884}{\frac{e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2)}{2! \times 1!}} + 12 \right) = \frac{12}{11} + \frac{49221}{5632 \left( \int_0^1 \sqrt{1-t^2} dt \right)^4}$$

$$\frac{1}{11} \left( \frac{196884}{\frac{e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2)}{2! \times 1!}} + 12 \right) = \frac{12}{11} + \frac{49221}{352 \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^4}$$

$$\frac{1}{11} \left( \frac{196884}{\frac{e^{-2\pi i} 4\pi^2 e^{2\pi i} (4\pi^2)}{2! \times 1!}} + 12 \right) = \frac{12}{11} + \frac{49221}{352 \left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^4}$$

Now, we have:

$$J(\tau) + 12 = e^{-2\pi i \tau} + \lim_{K \rightarrow \infty} \sum_{\substack{0 < c < K \\ -K^2 < d < K^2 \\ (c,d)=1}} e^{-2\pi i \frac{a\tau+b}{c\tau+d}} - e^{-2\pi i \frac{a}{c}}, \quad (7.17)$$

From which, from the following calculations

$$e^{(-2\pi i)} + e^{(-2\pi i)} - e^{(-2\pi i)} - 12$$

we obtain:

**Input:**

$$e^{-2\pi} + e^{-2\pi} - e^{-2\pi} - 12$$

**Exact result:**

$$e^{-2\pi} - 12$$

**Decimal approximation:**

$$-11.9981325572682920111855697870651729696065771949975246828\dots$$

$$-11.998132557\dots$$

**Property:**

$-12 + e^{-2\pi}$  is a transcendental number

**Alternate form:**

$$-e^{-2\pi} (12 e^{2\pi} - 1)$$

**Alternative representations:**

$$e^{-2\pi} + e^{-2\pi} - e^{-2\pi} - 12 = -12 + e^{-360^\circ}$$

$$e^{-2\pi} + e^{-2\pi} - e^{-2\pi} - 12 = -12 + e^{2i \log(-1)}$$

$$e^{-2\pi} + e^{-2\pi} - e^{-2\pi} - 12 = -12 + e^{-4i \log((1-i)/(1+i))}$$

**Series representations:**

$$e^{-2\pi} + e^{-2\pi} - e^{-2\pi} - 12 = -12 + e^{-8 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

$$e^{-2\pi} + e^{-2\pi} - e^{-2\pi} - 12 = -12 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{-2\pi}$$

$$e^{-2\pi} + e^{-2\pi} - e^{-2\pi} - 12 = -12 + \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{-2\pi}$$

**Integral representations:**

$$e^{-2\pi} + e^{-2\pi} - e^{-2\pi} - 12 = -12 + e^{-8 \int_0^1 \sqrt{1-t^2} dt}$$

$$e^{-2\pi} + e^{-2\pi} - e^{-2\pi} - 12 = -12 + e^{-4} \int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$e^{-2\pi} + e^{-2\pi} - e^{-2\pi} - 12 = -12 + e^{-4} \int_0^{\infty} \frac{1}{(1+t^2)} dt$$

and, multiplying by  $144 = 12^2$ , we obtain:

$$-12^2((e^{-2\pi} + e^{-2\pi} - e^{-2\pi} - 12))$$

**Input:**

$$-12^2 (e^{-2\pi} + e^{-2\pi} - e^{-2\pi} - 12)$$

**Exact result:**

$$-144 (e^{-2\pi} - 12)$$

**Decimal approximation:**

1727.731088246634049610722049337384907623347116079643554323...

$$1727.73108824... \approx 1728$$

**Property:**

$-144(-12 + e^{-2\pi})$  is a transcendental number

**Alternate forms:**

$$1728 - 144 e^{-2\pi}$$

$$144 e^{-2\pi} (12 e^{2\pi} - 1)$$

**Alternative representations:**

$$-12^2 (e^{-2\pi} + e^{-2\pi} - e^{-2\pi} - 12) = -12^2 (-12 + e^{-360^\circ})$$

$$-12^2 (e^{-2\pi} + e^{-2\pi} - e^{-2\pi} - 12) = -12^2 (-12 + e^{2i \log(-1)})$$

$$-12^2 (e^{-2\pi} + e^{-2\pi} - e^{-2\pi} - 12) = -12^2 (-12 + e^{-4i \log((1-i)/(1+i))})$$



### Series representations:

$$-12^2 (e^{-2\pi} + e^{-2\pi} - e^{-2\pi} - 12) = 1728 - 144 e^{-8 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

$$-12^2 (e^{-2\pi} + e^{-2\pi} - e^{-2\pi} - 12) = 1728 - 144 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{-2\pi}$$

$$-12^2 (e^{-2\pi} + e^{-2\pi} - e^{-2\pi} - 12) = 1728 - 144 \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{-2\pi}$$

### Integral representations:

$$-12^2 (e^{-2\pi} + e^{-2\pi} - e^{-2\pi} - 12) = 1728 - 144 e^{-8 \int_0^1 \sqrt{1-t^2} dt}$$

$$-12^2 (e^{-2\pi} + e^{-2\pi} - e^{-2\pi} - 12) = 1728 - 144 e^{-4 \int_0^1 1/\sqrt{1-t^2} dt}$$

$$-12^2 (e^{-2\pi} + e^{-2\pi} - e^{-2\pi} - 12) = 1728 - 144 e^{-4 \int_0^{\infty} 1/(1+t^2) dt}$$

## Observations

*From:*

[https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn\\_RpOSvJIQxWsVLBcJ6KVgd\\_Af\\_hrmDYBNyU8mpSjRs1BDeremA](https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJIQxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA)

*Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that  $p(9) = 30$ ,  $p(9 + 5) = 135$ ,  $p(9 + 10) = 490$ ,  $p(9 + 15) = 1,575$  and so on are all divisible by 5. Note that here the  $n$ 's come at intervals of five units.*

*Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of  $p(n)$  that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.*

*Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of  $n$ 's separated by  $5^3 = 125$  units, saying that the corresponding  $p(n)$ 's should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.*

*From Wikipedia*

*In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field  $\phi$  and a Dirac field  $\psi$ . The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.*

*Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson:*

125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1 + \sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \dots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and  $4096 = 64^2$

(Modular equations and approximations to  $\pi$  - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants  $\pi$ ,  $\phi$ ,  $1/\phi$ , the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

*In mathematics, the Fibonacci numbers, commonly denoted  $F_n$ , form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the  $n$ th Fibonacci number in terms of  $n$  and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as  $n$  increases.*

*Fibonacci numbers are also closely related to Lucas numbers, in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences*

*The beginning of the sequence is thus:*

*0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...*

*The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.*

*The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.<sup>[1]</sup> The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.*

*The sequence of Lucas numbers is:*

*2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803.....*

*All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.*

*A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:*

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

*In geometry, a golden spiral is a logarithmic spiral whose growth factor is  $\varphi$ , the golden ratio.<sup>[1]</sup> That is, a golden spiral gets wider (or further from its origin) by a factor of  $\varphi$  for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies<sup>[3]</sup> - golden spirals are one special case of these logarithmic spirals*

We observe that 1728 and 1729 are results very near to the mass of candidate glueball  **$f_0(1710)$  scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number). In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to  $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$

## References

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**S. Ramanujan to G.H. Hardy** - 12 January 1920

*University of Madras*