

# On some Ramanujan formulas: mathematical connections with $\phi$ , $\zeta(2)$ and several parameters of String Theory and Particle Physics V.

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## Abstract

*In this paper we have described and analyzed some Ramanujan expressions. We have obtained several mathematical connections with  $\phi$ ,  $\zeta(2)$  and various parameters of String Theory and Particle Physics.*

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*An equation means nothing  
to me unless it expresses a  
thought of God.*

*Srinivasa Ramanujan (1887-1920)*

<https://mobygeek.com/features/indian-mathematician-srinivasa-ramanujan-quotes-11012>

**We want to highlight that the development of the various equations was carried out according to our possible logical and original interpretation**

From:

## Chapter 4 - General Partition Function and Tau Function -

[https://shodhganga.inflibnet.ac.in/bitstream/10603/98249/10/10\\_chapter%204.pdf](https://shodhganga.inflibnet.ac.in/bitstream/10603/98249/10/10_chapter%204.pdf)

$$\begin{aligned}
 \sum_{n=0}^{\infty} p_{24}(5n+4)q^n &= (q^5; q^5)_{\infty}^{24} \left( 4830 \left( \left( 11q + \frac{(q; q)_{\infty}^6}{(q^5; q^5)_{\infty}^6} \right)^2 + 2q^2 \right)^2 - 2q^4 \right) \\
 &\quad - 212520q \left( \left( 11q + \frac{(q; q)_{\infty}^6}{(q^5; q^5)_{\infty}^6} \right)^3 + 3q^2 \left( 11q + \frac{(q; q)_{\infty}^6}{(q^5; q^5)_{\infty}^6} \right) \right) \\
 &\quad + 3487260q^2 \left( \left( 11q + \frac{(q; q)_{\infty}^6}{(q^5; q^5)_{\infty}^6} \right)^2 + 2q^2 \right) \\
 &\quad - 25077360q^3 \left( 11q + \frac{(q; q)_{\infty}^6}{(q^5; q^5)_{\infty}^6} \right) + 14903725q^4 \\
 &= 4830(q; q)_{\infty}^{24} - 48828125q^4(q^5; q^5)_{\infty}^{24} \\
 &= 4830 \sum_{n=0}^{\infty} p_{24}(n)q^n - 48828125 \sum_{n=0}^{\infty} p_{24}(n)q^{5n+4}. \quad (4.3.12)
 \end{aligned}$$

We have that (from OEIS.org)

[A299206](#) **Ramanujan's tau function** (or tau numbers ([A000594](#))) for  $5^n$ .

1, **4830**, -25499225, -359001100500, -488895969711875, 15167983076643206250, 97133231781274332671875, -271470664160664028370625000, -6054036890966043032024015234375, -15985594659896064584391569753906250, 218396847859403327980436336954599609375 ([list](#); [graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))

OFFSET 0,2

LINKS [Table of n, a\(n\) for n=0..10.](#)  
Eric Weisstein's World of Mathematics, [Tau Function](#)

FORMULA G.f.:  $1/(1-4830*x+48828125*x^2)$ .

[A000351](#) Powers of 5:  $a(n) = 5^n$ .  
(Formerly M3937 N1620)

1, 5, 25, 125, 625, 3125, 15625, 78125, 390625, 1953125, 9765625, **48828125**, 244140625, 1220703125, 6103515625, 30517578125, 152587890625, 762939453125, 3814697265625, 19073486328125, 95367431640625, 476837158203125,

2384185791015625, 11920928955078125 ([list](#); [graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))

FORMULA  $a(n) = 5^n.$

We note that:  $5^{11} = 48828125$

A000594 Ramanujan's tau function (or Ramanujan numbers, or tau numbers).  
(Formerly M5153 N2237)

1, -24, **252**, -1472, **4830**, -6048, -16744, 84480, -113643, -115920, 534612, -370944, -577738, 401856, **1217160**, 987136, -6905934, 2727432, 10661420, -7109760, -4219488, -12830688, 18643272, 21288960, -25499225, 13865712, -73279080, 24647168 ([list](#); [graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))

$1217160/252 = 4830$

A002939  $a(n) = 2^n * (2^n - 1).$

0, 2, **12**, 30, 56, 90, 132, 182, **240**, 306, 380, 462, **552**, 650, 756, 870, 992, 1122, 1260, 1406, 1560, 1722, 1892, 2070, 2256, 2450, 2652, 2862, 3080, 3306, 3540, 3782, 4032, 4290, 4556, **4830**, 5112, 5402, 5700, 6006, 6320, 6642, 6972, 7310, 7656, 8010, 8372 ([list](#); [graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))

n a(n)

546 1191372

547 1195742

548 1200120

549 1204506

550 1208900

551 1213302

552 1217712

553 1222130

.....

We have that:

$2 * 35 * (2 * 35 - 1) = 4830$ ;  $2 * 552 * (2 * 552 - 1) = 1217712$ ;  $2 * 12 * (2 * 12 - 1) = 552$

$2 * 8 * (2 * 8 - 1) = 240$ ;  $2 * 2 * (2 * 2 - 1) = 12$

$240 + 12 = 252$ ;  $1217712 - 552 = 1217160$

Thence, we have:

$$\frac{(((2 \times 552 \times (2 \times 552 - 1)) - (2 \times 12 \times (2 \times 12 - 1))))}{(((2 \times 8 \times (2 \times 8 - 1)) + (2 \times 2 \times (2 \times 2 - 1))))}$$

**Input:**

$$\frac{2 \times 552 (2 \times 552 - 1) - 2 \times 12 (2 \times 12 - 1)}{2 \times 8 (2 \times 8 - 1) + 2 \times 2 (2 \times 2 - 1)}$$

**Result:**

4830

4830

From

[A074872](#) Inverse BinomialMean transform of the Fibonacci sequence [A000045](#) (with the initial 0 omitted).

1, 1, 5, 5, 25, 25, 125, 125, 625, 625, 3125, 3125, 15625, 15625, 78125, 78125, 390625, 390625, 1953125, 1953125, 9765625, 9765625, **48828125, 48828125**, 244140625, 244140625, 1220703125, 1220703125, 6103515625, 6103515625 ([list](#); [graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))

We have the following formula:

$$a(n) = (1/(2 \times \sqrt{5})) \times ((1 + \sqrt{5}) \times (\sqrt{5})^n - (1 - \sqrt{5}) \times (-\sqrt{5})^n).$$

For  $n = 22$  and  $n = 23$ , we obtain:

$$(1/(2 \times \sqrt{5})) \times ((1 + \sqrt{5}) \times (\sqrt{5})^{22} - (1 - \sqrt{5}) \times (-\sqrt{5})^{22})$$

**Input:**

$$\frac{1}{2\sqrt{5}} \left( (1 + \sqrt{5}) \sqrt{5}^{22} - (1 - \sqrt{5}) (-\sqrt{5})^{22} \right)$$

**Result:**

48 828 125

48828125

$$(1/(2*\sqrt{5})*((1+\sqrt{5})*(\sqrt{5})^{23} - (1-\sqrt{5})*(-\sqrt{5})^{23}))$$

**Input:**

$$\frac{1}{2\sqrt{5}} \left( (1 + \sqrt{5}) \sqrt{5}^{23} - (1 - \sqrt{5}) (-\sqrt{5})^{23} \right)$$

**Result:**

48 828 125

48828125

Thence, we have developed the following expression:

$$4830 \sum_{n=0}^{\infty} p_{24}(n)q^n - 48828125 \sum_{n=0}^{\infty} p_{24}(n)q^{5n+4}.$$

$$((((4830 \text{ sum}(1*0.5^n), n=0..2)) - ((48828125 \text{ sum}(1*0.5^{(5n+4)}), n=0..2))))))$$

**Input interpretation:**

$$4830 \sum_{n=0}^2 1 \times 0.5^n - 48828125 \sum_{n=0}^2 1 \times 0.5^{5n+4}$$

**Result:**

-3.14165 × 10<sup>6</sup>

-3.14165\*10<sup>6</sup>

Now, we have that:

## Congruence properties of the partition function

Tony Forbes

ADF040c 1.11

Talks for LSBU Mathematics Study Group, 24 Sep, 8 Oct & 19 Nov 2008

### The functions $p_k(n)$

For any  $k$ , define  $p_k(n)$  by

$$\sum_{n=-\infty}^{\infty} p_k(n)x^n = \prod_{n=1}^{\infty} (1 - x^n)^k, \quad |x| < 1,$$

so that  $p_k(n) = 0$  for negative  $n$ . The partition function  $p(n)$  is  $p_{-1}(n)$ . We will be interested in three congruence properties stated and proved by Ramanujan:

$$p(5k + 4) \equiv 0 \pmod{5}, \quad p(7k + 5) \equiv 0 \pmod{7}, \quad p(11k + 6) \equiv 0 \pmod{11}.$$

$n$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$p(n)$	1	1	2	3	5	7	11	15	22	30	42	56	77	101	135	176	231	297	385	490	627

$n$	21	22	23	24	25	26	27	28	29	30	31	32	33
$p(n)$	792	1002	1255	1575	1958	2436	3010	3718	4565	5604	6842	8349	10143

Ramanujan's tau function  $\tau(n)$  is  $p_{24}(n - 1)$ .

$n$	1	2	3	4	5	6	7	8	9	10
$\tau(n)$	1	-24	252	-1472	4830	-6048	-16744	84480	-113643	-115920

Thence, from the previous expression, for  $n = 5$ , we obtain:

$$((((4830 \sum_{n=0}^2 (5 \cdot 0.5^n)) - ((48828125 \sum_{n=0}^2 (5 \cdot 0.5^{5n+4})), n=0..2))))))$$

**Input interpretation:**

$$4830 \sum_{n=0}^2 5 \times 0.5^n - 48828125 \sum_{n=0}^2 5 \times 0.5^{5n+4}$$

**Result:**

$$-1.57083 \times 10^7$$

$$\mathbf{-1.57083 \cdot 10^7}$$

From which:

$$\frac{1}{2}[-(((((((4830 \sum_{n=0}^{\infty} 5 \cdot 0.5^n) - ((48828125 \sum_{n=0}^{\infty} 5 \cdot 0.5^{5n+4})), n=0..2)))))))]^{1/2} - 233 - 21 + \frac{3}{2}$$

**Input interpretation:**

$$\frac{1}{2} \sqrt{-\left(4830 \sum_{n=0}^{\infty} 5 \times 0.5^n - 48828125 \sum_{n=0}^{\infty} 5 \times 0.5^{5n+4}\right)} - 233 - 21 + \frac{3}{2}$$

**Result:**

1729.18

1729.18

This result is very near to the mass of candidate glueball **f<sub>0</sub>(1710) scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

or:

$$(((((((4830 \sum_{n=0}^{\infty} 5 \cdot 0.5^n) - ((48828125 \sum_{n=0}^{\infty} 5^6 \cdot 0.5^{5n+4})), n=0..2)))))))))$$

**Input interpretation:**

$$4830 \sum_{n=0}^{\infty} 5 \times 0.5^n - 48828125 \sum_{n=0}^{\infty} 5^6 \times 0.5^{5n+4}$$

**Result:**

-4.92204 × 10<sup>10</sup>

-4.92204 \* 10<sup>10</sup>

From which:

$$[-(((((((4830 \sum_{n=0}^{\infty} 5 \cdot 0.5^n) - ((48828125 \sum_{n=0}^{\infty} 5^6 \cdot 0.5^{5n+4})), n=0..2)))))))]^{1/4} + 21 + 5$$



**Input interpretation:**

$$\sqrt[4]{-\left(4830 \sum_{n=0}^2 5 \times 0.5^n - 48828125 \sum_{n=0}^2 5^6 \times 0.5^{5n+4}\right) + 21 + 5}$$

**Result:**

497.017

497.017 result practically equal to the rest mass of Kaon meson 497.614

and:

$$1/2[-(((((((4830 \text{ sum}(5*0.5^n), n=0..2)) - ((48828125 \text{ sum}(5^6*0.5^{(5n+4)}), n=0..2)))))))))]^{1/3}-89-13-3/2$$

**Input interpretation:**

$$\frac{1}{2} \sqrt[3]{-\left(4830 \sum_{n=0}^2 5 \times 0.5^n - 48828125 \sum_{n=0}^2 5^6 \times 0.5^{5n+4}\right) - 89 - 13 - \frac{3}{2}}$$

**Result:**

1728.89

1728.89 ≈ 1729

This result is very near to the mass of candidate glueball **f<sub>0</sub>(1710) scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Now, we have:

$$\sum_{t=0}^{\infty} p(5t + 4)x^t = 5 \prod_{n=1}^{\infty} \frac{(1 - x^{5n})^5}{(1 - x^n)^6}$$

5 product (((1-0.5^(5n))^5) / ((1-0.5^n)^6)), n = 1..500

**Input interpretation:**

$$5 \prod_{n=1}^{500} \frac{(1 - 0.5^{5n})^5}{(1 - 0.5^n)^6}$$

**Result:**

7317.5

**7317.5**

5 product (((((1-0.5^(5n))^5) / ((1-0.5^n)^6))), n = 1..1000

**Input interpretation:**

$$5 \prod_{n=1}^{1000} \frac{(1 - 0.5^{5n})^5}{(1 - 0.5^n)^6}$$

**Result:**

7317.5

**7317.5**

Thence:

$$\sum_{n=0}^{\infty} p_{24}(5n+4)q^n = 4830 \sum_{n=0}^{\infty} p_{24}(n)q^n - 48828125 \sum_{n=0}^{\infty} p_{24}(n)q^{5n+4}.$$

we have:

$$-(7317.5)x = (((((4830 \text{ sum}(5*0.5^n), n=0..2)) - ((48828125 \text{ sum}(5*0.5^{(5n+4)}), n=0..2))))))$$

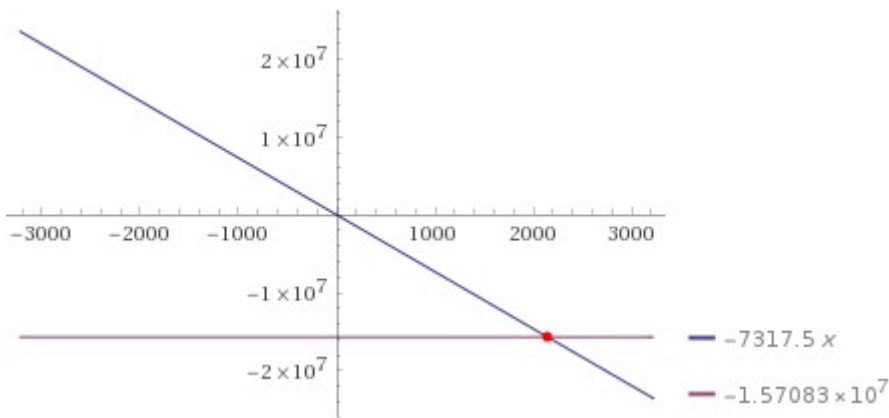
**Input interpretation:**

$$x \times (-7317.5) = 4830 \sum_{n=0}^2 5 \times 0.5^n - 48828125 \sum_{n=0}^2 5 \times 0.5^{5n+4}$$

**Result:**

$$-7317.5 x = -1.57083 \times 10^7$$

**Plot:**



**Alternate form:**

$$1.57083 \times 10^7 - 7317.5 x = 0$$

**Alternate form assuming x is real:**

$$0 - 7317.5 x = -1.57083 \times 10^7$$

**Solution:**

$$x \approx 2146.67$$

**2146.67**

$$-(7317.5)(x+34) = ((((((4830 \sum_{n=0}^2 (5 \cdot 0.5^n)) - ((48828125 \sum_{n=0}^2 (5 \cdot 0.5^{5n+4})), n=0..2))))))$$

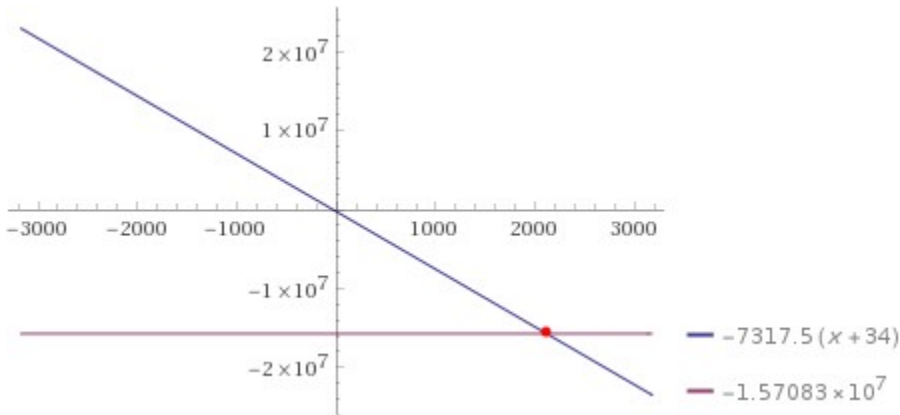
**Input interpretation:**

$$(x + 34) \times (-7317.5) = 4830 \sum_{n=0}^2 5 \times 0.5^n - 48828125 \sum_{n=0}^2 5 \times 0.5^{5n+4}$$

**Result:**

$$-7317.5 (x + 34) = -1.57083 \times 10^7$$

**Plot:**



**Alternate forms:**

$$1.54595 \times 10^7 - 7317.5 x = 0$$

$$-7317.5 (x + 34) = -1.57083 \times 10^7$$

**Expanded form:**

$$-7317.5 x - 248795 = -1.57083 \times 10^7$$

**Solution:**

$$x \approx 2112.67$$

2112.67 result practically equal to the rest mass of Strange D meson 2112.1

and again:

$$-(7317.5)(x+377+34+5+2) = ((((((4830 \sum_{n=0}^2 (5 \cdot 0.5^n), n=0..2)) - ((48828125 \sum_{n=0}^2 (5 \cdot 0.5^{5n+4}), n=0..2))))))$$

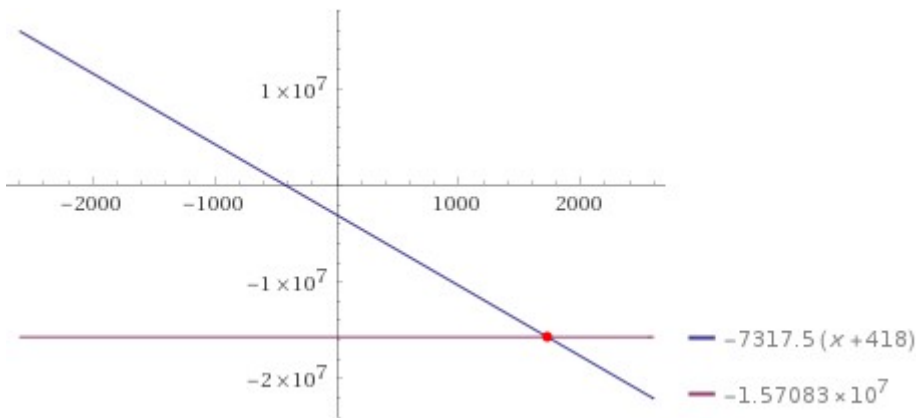
**Input interpretation:**

$$(x + 377 + 34 + 5 + 2) \times (-7317.5) = 4830 \sum_{n=0}^2 5 \times 0.5^n - 48828125 \sum_{n=0}^2 5 \times 0.5^{5n+4}$$

**Result:**

$$-7317.5 (x + 418) = -1.57083 \times 10^7$$

**Plot:**



**Alternate forms:**

$$-7317.5x - 3.05872 \times 10^6 = -1.57083 \times 10^7$$

$$1.26495 \times 10^7 - 7317.5x = 0$$

$$-7317.5(x + 418) = -1.57083 \times 10^7$$

**Solution:**

$$x \approx 1728.67$$

1728.67

This result is very near to the mass of candidate glueball  **$f_0(1710)$  scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

From:

**Efficient implementation of the Hardy-Ramanujan-Rademacher formula**

*Fredrik Johansson* - arXiv:1205.5991v2 [math.NT] 6 Jul 2012

Now, we have that:

$$M(n, N) = \frac{44\pi^2}{225\sqrt{3}}N^{-1/2} + \frac{\pi\sqrt{2}}{75} \left(\frac{N}{n-1}\right)^{1/2} \sinh\left(\frac{\pi}{N}\sqrt{\frac{2n}{3}}\right). \quad (1.8)$$

It is easily shown that  $M(n, cn^{1/2}) \sim n^{-1/4}$  for every positive  $c$ . Rademacher's bound (1.8) therefore implies that  $O(n^{1/2})$  terms in (1.4) suffice to compute  $p(n)$  exactly by forcing  $|R(n, N)| < 1/2$  and rounding to the nearest integer. For example, we can take  $N = \lceil n^{1/2} \rceil$  when  $n \geq 65$ .

For  $n = 256$  and  $N = \sqrt{256}$ , we obtain:

$$M(n, N) = \frac{44\pi^2}{225\sqrt{3}}N^{-1/2} + \frac{\pi\sqrt{2}}{75} \left(\frac{N}{n-1}\right)^{1/2} \sinh\left(\frac{\pi}{N}\sqrt{\frac{2n}{3}}\right).$$

$(44\pi^2)/(225\sqrt{3}) * (16)^{-1/2} + (\pi*\sqrt{2})/75 * (16/255)^{1/2}$   
 $\sinh(((\pi/(16)*\sqrt{2})*\sqrt{(2*256/3)}))$

**Input:**

$$\frac{44\pi^2}{225\sqrt{3}} \times 16^{-1/2} + \left(\frac{1}{75} (\pi\sqrt{2})\right) \sqrt{\frac{16}{255}} \sinh\left(\frac{\pi}{16} \sqrt{\frac{2 \times 256}{3}}\right)$$

$\sinh(x)$  is the hyperbolic sine function

**Exact result:**

$$\frac{11\pi^2}{225\sqrt{3}} + \frac{4}{75} \sqrt{\frac{2}{255}} \pi \sinh\left(\sqrt{\frac{2}{3}} \pi\right)$$

**Decimal approximation:**

0.374474480196453188207662435055045762223802159630964595486...

0.374474480196453...

**Alternate forms:**

$$\frac{\pi \left( 935 \pi + 12 \sqrt{170} \sinh \left( \sqrt{\frac{2}{3}} \pi \right) \right)}{19125 \sqrt{3}}$$

$$\frac{\pi \left( -11 \pi - 12 \sqrt{\frac{2}{85}} \sinh \left( \sqrt{\frac{2}{3}} \pi \right) \right)}{225 \sqrt{3}}$$

$$\frac{935 \sqrt{3} \pi^2 + 12 \sqrt{510} \pi \sinh \left( \sqrt{\frac{2}{3}} \pi \right)}{57375}$$

### Alternative representations:

$$\begin{aligned} & \frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right) (\pi \sqrt{2}) = \\ & \frac{44 \pi^2}{\sqrt{16} (225 \sqrt{3})} + \frac{\pi \sqrt{\frac{16}{255}} \sqrt{2}}{75 \operatorname{csch} \left( \frac{1}{16} \pi \sqrt{\frac{512}{3}} \right)} \end{aligned}$$

$$\begin{aligned} & \frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right) (\pi \sqrt{2}) = \\ & \frac{44 \pi^2}{\sqrt{16} (225 \sqrt{3})} + \frac{1}{75} i \pi \cos \left( \frac{\pi}{2} + \frac{1}{16} i \pi \sqrt{\frac{512}{3}} \right) \sqrt{\frac{16}{255}} \sqrt{2} \end{aligned}$$

$$\begin{aligned} & \frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right) (\pi \sqrt{2}) = \\ & \frac{44 \pi^2}{\sqrt{16} (225 \sqrt{3})} + \frac{\pi \left( -e^{-1/16 \pi \sqrt{512/3}} + e^{1/16 \pi \sqrt{512/3}} \right) \sqrt{\frac{16}{255}} \sqrt{2}}{2 \times 75} \end{aligned}$$

**Series representations:**

$$\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right) (\pi \sqrt{2}) =$$

$$\frac{11 \pi^2}{225 \sqrt{3}} + \frac{8}{75} \sqrt{\frac{2}{255}} \pi \sum_{k=0}^{\infty} I_{1+2k} \left( \sqrt{\frac{2}{3}} \pi \right)$$

$$\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right) (\pi \sqrt{2}) =$$

$$\frac{11 \pi^2}{225 \sqrt{3}} + \frac{4}{75} \sqrt{\frac{2}{255}} \pi \sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}\right)^{-1/2-k} \pi^{1+2k}}{(1+2k)!}$$

$$\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right) (\pi \sqrt{2}) =$$

$$\frac{11 \pi^2}{225 \sqrt{3}} + \frac{4}{75} i \sqrt{\frac{2}{255}} \pi \sum_{k=0}^{\infty} \frac{\left(\frac{1}{6} (-3i + 2\sqrt{6}) \pi\right)^{2k}}{(2k)!}$$

**Integral representations:**

$$\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right) (\pi \sqrt{2}) =$$

$$\frac{11 \pi^2}{225 \sqrt{3}} + \frac{8 \pi^2}{225 \sqrt{85}} \int_0^1 \cosh \left( \sqrt{\frac{2}{3}} \pi t \right) dt$$

$$\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right) (\pi \sqrt{2}) =$$

$$\frac{11 \pi^2}{225 \sqrt{3}} - \frac{2 i \pi^{3/2}}{225 \sqrt{85}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^2/(6s)+s}}{s^{3/2}} ds \text{ for } \gamma > 0$$



From which:

$$1 + \sqrt{\left( \frac{44\pi^2}{225\sqrt{3}} \times 16^{-1/2} + \left( \frac{\pi\sqrt{2}}{75} \right) \sqrt{\frac{16}{255}} \sinh\left( \frac{\pi}{16} \sqrt{\frac{2 \times 256}{3}} \right) \right)}$$

**Input:**

$$1 + \sqrt{\frac{44\pi^2}{225\sqrt{3}} \times 16^{-1/2} + \left( \frac{1}{75} (\pi\sqrt{2}) \right) \sqrt{\frac{16}{255}} \sinh\left( \frac{\pi}{16} \sqrt{\frac{2 \times 256}{3}} \right)}$$

$\sinh(x)$  is the hyperbolic sine function

**Exact result:**

$$1 + \sqrt{\frac{11\pi^2}{225\sqrt{3}} + \frac{4}{75} \sqrt{\frac{2}{255}} \pi \sinh\left( \sqrt{\frac{2}{3}} \pi \right)}$$

**Decimal approximation:**

1.611943200139075969949760296084325418870941463955449598770...

1.611943200139.... result that is a very good approximation to the value of the golden ratio 1.618033988749...

**Alternate forms:**

$$1 + \frac{\sqrt{\pi \left( 11\pi + 12 \sqrt{\frac{2}{85}} \sinh\left( \sqrt{\frac{2}{3}} \pi \right) \right)}}{15 \sqrt[4]{3}}$$

$$1 + \frac{\sqrt{11\pi^2 + 12 \sqrt{\frac{2}{85}} \pi \sinh\left( \sqrt{\frac{2}{3}} \pi \right)}}{15 \sqrt[4]{3}}$$

$$\frac{3825 + 3^{3/4} \sqrt{85 \pi \left( 935\pi + 12 \sqrt{170} \sinh\left( \sqrt{\frac{2}{3}} \pi \right) \right)}}{3825}$$

**Alternative representations:**

$$1 + \sqrt{\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right)} (\pi \sqrt{2}) =$$

$$1 + \sqrt{\frac{44 \pi^2}{\sqrt{16} (225 \sqrt{3})} + \frac{\pi \sqrt{\frac{16}{255}} \sqrt{2}}{75 \operatorname{csch} \left( \frac{1}{16} \pi \sqrt{\frac{512}{3}} \right)}}$$

$$1 + \sqrt{\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right)} (\pi \sqrt{2}) =$$

$$1 + \sqrt{\frac{44 \pi^2}{\sqrt{16} (225 \sqrt{3})} + \frac{1}{75} i \pi \cos \left( \frac{\pi}{2} + \frac{1}{16} i \pi \sqrt{\frac{512}{3}} \right) \sqrt{\frac{16}{255}} \sqrt{2}}$$

$$1 + \sqrt{\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right)} (\pi \sqrt{2}) =$$

$$1 + \sqrt{\frac{44 \pi^2}{\sqrt{16} (225 \sqrt{3})} + \frac{\pi \left( -e^{-1/16 \pi \sqrt{512/3}} + e^{1/16 \pi \sqrt{512/3}} \right) \sqrt{\frac{16}{255}} \sqrt{2}}{2 \times 75}}$$

**Series representations:**

$$1 + \sqrt{\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right)} (\pi \sqrt{2}) =$$

$$1 + \frac{\sqrt{11 \pi^2 + 24 \sqrt{\frac{2}{85}} \pi \sum_{k=0}^{\infty} I_{1+2k} \left( \sqrt{\frac{2}{3}} \pi \right)}}{15 \sqrt[4]{3}}$$

$$1 + \sqrt{\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right) (\pi \sqrt{2})} =$$

$$1 + \frac{\sqrt{11 \pi^2 + 12 \sqrt{\frac{2}{85}} \pi \sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}\right)^{-1/2-k} \pi^{1+2k}}{(1+2k)!}}{15 \sqrt[4]{3}}$$

$$1 + \sqrt{\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right) (\pi \sqrt{2})} =$$

$$1 + \sqrt{\frac{11 \pi^2}{225 \sqrt{3}} + \frac{4}{75} \sqrt{\frac{2}{255}} \pi \sum_{k=0}^{\infty} \frac{2^{1/2(1+2k)} \times 3^{1/2(-1-2k)} \pi^{1+2k}}{(1+2k)!}}$$

**Integral representations:**

$$1 + \sqrt{\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right) (\pi \sqrt{2})} =$$

$$1 + \sqrt{\frac{11 \pi^2}{225 \sqrt{3}} + \frac{8 \pi^2}{225 \sqrt{85}} \int_0^1 \cosh \left( \sqrt{\frac{2}{3}} \pi t \right) dt}$$

$$1 + \sqrt{\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right) (\pi \sqrt{2})} =$$

$$1 + \sqrt{\frac{11 \pi^2}{225 \sqrt{3}} - \frac{2 i \pi^{3/2}}{225 \sqrt{85}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^2/(6s)+s}}{s^{3/2}} ds \text{ for } \gamma > 0}$$

and:

$$(521+123)/((((44\text{Pi}^2)/(225\text{sqrt}3) * (16)^{-1/2} + (\text{Pi}*\text{sqrt}2)/75 * (16/255)^{1/2} \sinh(((\text{Pi}/(16)*\text{sqrt}((2*256)/3))))))))+7+7/3$$

**Input:**

$$\frac{521 + 123}{\frac{44 \pi^2}{225 \sqrt{3}} \times 16^{-1/2} + \left(\frac{1}{75} (\pi \sqrt{2})\right) \sqrt{\frac{16}{255}} \sinh \left( \frac{\pi}{16} \sqrt{\frac{2 \times 256}{3}} \right)} + 7 + \frac{7}{3}$$

$\sinh(x)$  is the hyperbolic sine function

**Exact result:**

$$\frac{28}{3} + \frac{644}{\frac{11\pi^2}{225\sqrt{3}} + \frac{4}{75} \sqrt{\frac{2}{255}} \pi \sinh\left(\sqrt{\frac{2}{3}} \pi\right)}$$

**Decimal approximation:**

1729.076691177507212558857969458488329143599143835854471508...

1729.0766911775...

This result is very near to the mass of candidate glueball **f<sub>0</sub>(1710) scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

**Alternate forms:**

$$\frac{28}{3} + \frac{12316500\sqrt{3}}{935\pi^2 + 12\sqrt{170}\pi \sinh\left(\sqrt{\frac{2}{3}}\pi\right)}$$

$$\frac{28}{3} - \frac{144900\sqrt{3}}{\pi\left(-11\pi - 12\sqrt{\frac{2}{85}} \sinh\left(\sqrt{\frac{2}{3}}\pi\right)\right)}$$

$$\frac{28}{3} + \frac{644}{\frac{2}{75} \sqrt{\frac{2}{255}} \left(e^{\sqrt{2/3}\pi} - e^{-\sqrt{2/3}\pi}\right) \pi + \frac{11\pi^2}{225\sqrt{3}}}$$

**Alternative representations:**

$$\frac{521 + 123}{\frac{16^{-1/2}(44\pi^2)}{225\sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh\left(\frac{1}{16}\pi \sqrt{\frac{2 \times 256}{3}}\right) \right)} + 7 + \frac{7}{3} =$$

$$\frac{28}{3} + \frac{644}{\frac{44\pi^2}{\sqrt{16}(225\sqrt{3})} + \frac{\pi \sqrt{\frac{16}{255}} \sqrt{2}}{75 \operatorname{csch}\left(\frac{1}{16}\pi \sqrt{\frac{512}{3}}\right)}}$$

$$\frac{521 + 123}{\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right) (\pi \sqrt{2})} + 7 + \frac{7}{3} =$$

$$\frac{28}{3} + \frac{644}{\frac{44 \pi^2}{\sqrt{16} (225 \sqrt{3})} + \frac{1}{75} i \pi \cos \left( \frac{\pi}{2} + \frac{1}{16} i \pi \sqrt{\frac{512}{3}} \right) \sqrt{\frac{16}{255}} \sqrt{2}}$$

$$\frac{521 + 123}{\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right) (\pi \sqrt{2})} + 7 + \frac{7}{3} =$$

$$\frac{28}{3} + \frac{644}{\frac{44 \pi^2}{\sqrt{16} (225 \sqrt{3})} + \frac{1}{75} i \pi \cosh \left( \frac{i \pi}{2} - \frac{1}{16} \pi \sqrt{\frac{512}{3}} \right) \sqrt{\frac{16}{255}} \sqrt{2}}$$

**Series representations:**

$$\frac{521 + 123}{\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right) (\pi \sqrt{2})} + 7 + \frac{7}{3} =$$

$$\frac{28}{3} + \frac{12\,316\,500 \sqrt{3}}{935 \pi^2 + 12 \sqrt{170} \pi \sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}\right)^{-1/2-k} \pi^{1+2k}}{(1+2k)!}}$$

$$\frac{521 + 123}{\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right) (\pi \sqrt{2})} + 7 + \frac{7}{3} =$$

$$\frac{28}{3} + \frac{12\,316\,500 \sqrt{3}}{\pi \left( 935 \pi + 12 i \sqrt{170} \sum_{k=0}^{\infty} \frac{\left(\frac{1}{6} (-3i+2\sqrt{6}) \pi\right)^{2k}}{(2k)!} \right)}$$

$$\frac{521 + 123}{\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right) (\pi \sqrt{2})} + 7 + \frac{7}{3} =$$

$$\frac{28}{3} + \frac{644}{\frac{11 \pi^2}{225 \sqrt{3}} + \frac{4}{75} \sqrt{\frac{2}{255}} \pi \sum_{k=0}^{\infty} \frac{2^{1/2(1+2k)} \times 3^{1/2(-1-2k)} \pi^{1+2k}}{(1+2k)!}}$$

**Integral representations:**

$$\frac{521 + 123}{\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right) (\pi \sqrt{2})} + 7 + \frac{7}{3} =$$

$$\frac{28}{3} + \frac{644}{\frac{11 \pi^2}{225 \sqrt{3}} + \frac{8 \pi^2}{225 \sqrt{85}} \int_0^1 \cosh \left( \sqrt{\frac{2}{3}} \pi t \right) dt}$$

$$\frac{521 + 123}{\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right) (\pi \sqrt{2})} + 7 + \frac{7}{3} =$$

$$\frac{28}{3} + \frac{644}{\frac{11 \pi^2}{225 \sqrt{3}} - \frac{2 i \pi^{3/2}}{225 \sqrt{85}} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{\pi^2 / (6 s) + s}}{s^{3/2}} ds} \text{ for } \gamma > 0$$

$$47 / \left( \left( \left( \left( \left( \left( \left( \frac{44 \pi^2}{225 \sqrt{3}} \right) \times 16^{-1/2} \right) + \left( \frac{\pi \sqrt{2}}{75} \right) \times \left( \frac{16}{255} \right)^{1/2} \right) \right) \right) \right) \sinh \left( \left( \frac{\pi}{16} \right) \times \sqrt{\left( \frac{2 \times 256}{3} \right)} \right) \right) \right) \right) \right) \right)$$

**Input:**

$$\frac{47}{\frac{44 \pi^2}{225 \sqrt{3}} \times 16^{-1/2} + \left( \frac{1}{75} (\pi \sqrt{2}) \right) \sqrt{\frac{16}{255}} \sinh \left( \frac{\pi}{16} \sqrt{\frac{2 \times 256}{3}} \right)}$$

$\sinh(x)$  is the hyperbolic sine function

**Exact result:**

$$\frac{47}{\frac{11 \pi^2}{225 \sqrt{3}} + \frac{4}{75} \sqrt{\frac{2}{255}} \pi \sinh \left( \sqrt{\frac{2}{3}} \pi \right)}$$

**Decimal approximation:**

125.5092202153356713099373569842892621165877221950597725066...

125.50922021... result very near to the Higgs boson mass 125.18 GeV

**Alternate forms:**

$$\frac{898875 \sqrt{3}}{\pi \left( 935 \pi + 12 \sqrt{170} \sinh \left( \sqrt{\frac{2}{3}} \pi \right) \right)}$$

$$\frac{898875 \sqrt{3}}{935 \pi^2 + 12 \sqrt{170} \pi \sinh\left(\sqrt{\frac{2}{3}} \pi\right)}$$

$$- \frac{10575 \sqrt{3}}{\pi \left(-11 \pi - 12 \sqrt{\frac{2}{85}} \sinh\left(\sqrt{\frac{2}{3}} \pi\right)\right)}$$

### Alternative representations:

$$\frac{47}{\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh\left(\frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}}\right) \right) (\pi \sqrt{2})} =$$

$$\frac{\frac{44 \pi^2}{\sqrt{16} (225 \sqrt{3})} + \frac{\pi \sqrt{\frac{16}{255}} \sqrt{2}}{75 \operatorname{csch}\left(\frac{1}{16} \pi \sqrt{\frac{512}{3}}\right)}}{47}$$

$$\frac{47}{\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh\left(\frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}}\right) \right) (\pi \sqrt{2})} =$$

$$\frac{\frac{44 \pi^2}{\sqrt{16} (225 \sqrt{3})} + \frac{1}{75} i \pi \cos\left(\frac{\pi}{2} + \frac{1}{16} i \pi \sqrt{\frac{512}{3}}\right) \sqrt{\frac{16}{255}} \sqrt{2}}{47}$$

$$\frac{47}{\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh\left(\frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}}\right) \right) (\pi \sqrt{2})} =$$

$$\frac{\frac{44 \pi^2}{\sqrt{16} (225 \sqrt{3})} + \frac{1}{75} i \pi \cosh\left(\frac{i \pi}{2} - \frac{1}{16} \pi \sqrt{\frac{512}{3}}\right) \sqrt{\frac{16}{255}} \sqrt{2}}{47}$$

### Series representations:

$$\frac{47}{\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh\left(\frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}}\right) \right) (\pi \sqrt{2})} =$$

$$\frac{898875 \sqrt{3}}{935 \pi^2 + 24 \sqrt{170} \pi \sum_{k=0}^{\infty} I_{1+2k}\left(\sqrt{\frac{2}{3}} \pi\right)}$$

$$\frac{47}{\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right) (\pi \sqrt{2})} = \frac{935 \pi^2 + 12 \sqrt{170} \pi \sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}\right)^{-1/2-k} \pi^{1+2k}}{(1+2k)!}}$$

$$\frac{47}{\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right) (\pi \sqrt{2})} = \frac{\pi \left( 935 \pi + 12 i \sqrt{170} \sum_{k=0}^{\infty} \frac{\left(\frac{1}{6} (-3 i + 2 \sqrt{6}) \pi\right)^{2k}}{(2k)!} \right)}{898875 \sqrt{3}}$$

### Integral representation:

$$\frac{47}{\frac{16^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh \left( \frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}} \right) \right) (\pi \sqrt{2})} = \frac{2696625}{\pi^{3/2} \left( 935 \sqrt{3} \pi - 6 i \sqrt{85} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{\pi^2/(6s)+s}}{s^{3/2}} ds \right)} \quad \text{for } \gamma > 0$$

$$55 / \left( \left( \left( \left( \left( \left( \frac{44 \pi^2}{225 \sqrt{3}} \right) \right) \right) \right) \right) \left( \frac{16}{255} \right)^{-1/2} + \left( \frac{\pi \sqrt{2}}{75} \right) \left( \frac{16}{255} \right)^{1/2} \sinh \left( \left( \frac{\pi}{16} \right) \sqrt{\left( \frac{2 \times 256}{3} \right)} \right) \right) - 8 + \frac{1}{2}$$

### Input:

$$\frac{55}{\frac{44 \pi^2}{225 \sqrt{3}} \times 16^{-1/2} + \left( \frac{1}{75} (\pi \sqrt{2}) \right) \sqrt{\frac{16}{255}} \sinh \left( \frac{\pi}{16} \sqrt{\frac{2 \times 256}{3}} \right)} - 8 + \frac{1}{2}$$

sinh(x) is the hyperbolic sine function

### Exact result:

$$\frac{55}{\frac{11 \pi^2}{225 \sqrt{3}} + \frac{4}{75} \sqrt{\frac{2}{255}} \pi \sinh \left( \sqrt{\frac{2}{3}} \pi \right)} - \frac{15}{2}$$

### Decimal approximation:

139.3724917413502536605649922156576471577090366112401593162...



139.37249174... result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternate forms:**

$$\frac{1051875\sqrt{3}}{935\pi^2 + 12\sqrt{170}\pi \sinh\left(\sqrt{\frac{2}{3}}\pi\right)} - \frac{15}{2}$$

$$-\frac{15}{2} - \frac{12375\sqrt{3}}{\pi\left(-11\pi - 12\sqrt{\frac{2}{85}}\sinh\left(\sqrt{\frac{2}{3}}\pi\right)\right)}$$

$$\frac{55}{\frac{2}{75}\sqrt{\frac{2}{255}}\left(e^{\sqrt{2/3}\pi} - e^{-\sqrt{2/3}\pi}\right)\pi + \frac{11\pi^2}{225\sqrt{3}}} - \frac{15}{2}$$

**Alternative representations:**

$$\frac{55}{\frac{16^{-1/2}(44\pi^2)}{225\sqrt{3}} + \frac{1}{75}\left(\sqrt{\frac{16}{255}}\sinh\left(\frac{1}{16}\pi\sqrt{\frac{2 \times 256}{3}}\right)\right)(\pi\sqrt{2})} - 8 + \frac{1}{2} =$$

$$-\frac{15}{2} + \frac{55}{\frac{44\pi^2}{\sqrt{16}(225\sqrt{3})} + \frac{\pi\sqrt{\frac{16}{255}}\sqrt{2}}{75\operatorname{csch}\left(\frac{1}{16}\pi\sqrt{\frac{512}{3}}\right)}}$$

$$\frac{55}{\frac{16^{-1/2}(44\pi^2)}{225\sqrt{3}} + \frac{1}{75}\left(\sqrt{\frac{16}{255}}\sinh\left(\frac{1}{16}\pi\sqrt{\frac{2 \times 256}{3}}\right)\right)(\pi\sqrt{2})} - 8 + \frac{1}{2} =$$

$$-\frac{15}{2} + \frac{55}{\frac{44\pi^2}{\sqrt{16}(225\sqrt{3})} + \frac{1}{75}i\pi\cos\left(\frac{\pi}{2} + \frac{1}{16}i\pi\sqrt{\frac{512}{3}}\right)\sqrt{\frac{16}{255}}\sqrt{2}}$$

$$\frac{55}{\frac{16^{-1/2}(44\pi^2)}{225\sqrt{3}} + \frac{1}{75}\left(\sqrt{\frac{16}{255}}\sinh\left(\frac{1}{16}\pi\sqrt{\frac{2 \times 256}{3}}\right)\right)(\pi\sqrt{2})} - 8 + \frac{1}{2} =$$

$$-\frac{15}{2} + \frac{55}{\frac{44\pi^2}{\sqrt{16}(225\sqrt{3})} + \frac{1}{75}i\pi\cosh\left(\frac{i\pi}{2} - \frac{1}{16}\pi\sqrt{\frac{512}{3}}\right)\sqrt{\frac{16}{255}}\sqrt{2}}$$

**Series representations:**

$$\frac{55}{\frac{16^{-1/2}(44\pi^2)}{225\sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh\left(\frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}}\right) \right) (\pi \sqrt{2})} - 8 + \frac{1}{2} =$$

$$-\frac{15}{2} + \frac{1051875\sqrt{3}}{935\pi^2 + 12\sqrt{170} \pi \sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}\right)^{-1/2-k} \pi^{1+2k}}{(1+2k)!}}$$

$$\frac{55}{\frac{16^{-1/2}(44\pi^2)}{225\sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh\left(\frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}}\right) \right) (\pi \sqrt{2})} - 8 + \frac{1}{2} =$$

$$-\frac{15}{2} + \frac{1051875\sqrt{3}}{\pi \left( 935\pi + 12i\sqrt{170} \sum_{k=0}^{\infty} \frac{\left(\frac{1}{6}(-3i+2\sqrt{6})\pi\right)^{2k}}{(2k)!} \right)}$$

$$\frac{55}{\frac{16^{-1/2}(44\pi^2)}{225\sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh\left(\frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}}\right) \right) (\pi \sqrt{2})} - 8 + \frac{1}{2} =$$

$$-\frac{15}{2} + \frac{55}{\frac{11\pi^2}{225\sqrt{3}} + \frac{4}{75} \sqrt{\frac{2}{255}} \pi \sum_{k=0}^{\infty} \frac{2^{1/2(1+2k)} \times 3^{1/2(-1-2k)} \pi^{1+2k}}{(1+2k)!}}$$

### Integral representations:

$$\frac{55}{\frac{16^{-1/2}(44\pi^2)}{225\sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh\left(\frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}}\right) \right) (\pi \sqrt{2})} - 8 + \frac{1}{2} =$$

$$-\frac{15}{2} + \frac{55}{\frac{11\pi^2}{225\sqrt{3}} + \frac{8\pi^2}{225\sqrt{85}} \int_0^1 \cosh\left(\sqrt{\frac{2}{3}} \pi t\right) dt}$$

$$\frac{55}{\frac{16^{-1/2}(44\pi^2)}{225\sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{16}{255}} \sinh\left(\frac{1}{16} \pi \sqrt{\frac{2 \times 256}{3}}\right) \right) (\pi \sqrt{2})} - 8 + \frac{1}{2} =$$

$$-\frac{15}{2} + \frac{55}{\frac{11\pi^2}{225\sqrt{3}} - \frac{2i\pi^{3/2}}{225\sqrt{85}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^2/(6s)+s}}{s^{3/2}} ds} \quad \text{for } \gamma > 0$$

For  $n = 4096$  and  $N = \sqrt{4096} = 64$ , we obtain:

$$M(n, N) = \frac{44\pi^2}{225\sqrt{3}} N^{-1/2} + \frac{\pi\sqrt{2}}{75} \left(\frac{N}{n-1}\right)^{1/2} \sinh\left(\frac{\pi}{N}\sqrt{\frac{2n}{3}}\right).$$

$$(44\pi^2)/(225\sqrt{3}) * (64)^{-1/2} + (\pi*\sqrt{2})/75 * (64/4095)^{1/2} \sinh(\frac{(\pi/64)*\sqrt{2*4096}}{3})$$

**Input:**

$$\frac{44\pi^2}{225\sqrt{3}} \times 64^{-1/2} + \left(\frac{1}{75} (\pi\sqrt{2})\right) \sqrt{\frac{64}{4095}} \sinh\left(\frac{\pi}{64} \sqrt{\frac{2 \times 4096}{3}}\right)$$

sinh(x) is the hyperbolic sine function

**Exact result:**

$$\frac{11\pi^2}{450\sqrt{3}} + \frac{8}{225} \sqrt{\frac{2}{455}} \pi \sinh\left(\sqrt{\frac{2}{3}} \pi\right)$$

**Decimal approximation:**

0.187149343693897983020756694260300435028700934433310935425...

0.18714934...

**Alternate forms:**

$$\frac{\pi \left( 5005\sqrt{3} \pi + 48\sqrt{910} \sinh\left(\sqrt{\frac{2}{3}} \pi\right) \right)}{614250}$$

$$\frac{\pi \left( -11\pi - 16\sqrt{\frac{6}{455}} \sinh\left(\sqrt{\frac{2}{3}} \pi\right) \right)}{450\sqrt{3}}$$

$$\frac{5005\sqrt{3} \pi^2 + 48\sqrt{910} \pi \sinh\left(\sqrt{\frac{2}{3}} \pi\right)}{614250}$$

**Alternative representations:**

$$\frac{64^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{64}{4095}} \sinh \left( \frac{1}{64} \pi \sqrt{\frac{2 \times 4096}{3}} \right) \right) (\pi \sqrt{2}) =$$

$$\frac{44 \pi^2}{\sqrt{64} (225 \sqrt{3})} + \frac{\pi \sqrt{\frac{64}{4095}} \sqrt{2}}{75 \operatorname{csch} \left( \frac{1}{64} \pi \sqrt{\frac{8192}{3}} \right)}$$

$$\frac{64^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{64}{4095}} \sinh \left( \frac{1}{64} \pi \sqrt{\frac{2 \times 4096}{3}} \right) \right) (\pi \sqrt{2}) =$$

$$\frac{44 \pi^2}{\sqrt{64} (225 \sqrt{3})} + \frac{1}{75} i \pi \cos \left( \frac{\pi}{2} + \frac{1}{64} i \pi \sqrt{\frac{8192}{3}} \right) \sqrt{\frac{64}{4095}} \sqrt{2}$$

$$\frac{64^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{64}{4095}} \sinh \left( \frac{1}{64} \pi \sqrt{\frac{2 \times 4096}{3}} \right) \right) (\pi \sqrt{2}) =$$

$$\frac{44 \pi^2}{\sqrt{64} (225 \sqrt{3})} + \frac{\pi \left( -e^{-1/64 \pi \sqrt{8192/3}} + e^{1/64 \pi \sqrt{8192/3}} \right) \sqrt{\frac{64}{4095}} \sqrt{2}}{2 \times 75}$$

### Series representations:

$$\frac{64^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{64}{4095}} \sinh \left( \frac{1}{64} \pi \sqrt{\frac{2 \times 4096}{3}} \right) \right) (\pi \sqrt{2}) =$$

$$\frac{11 \pi^2}{450 \sqrt{3}} + \frac{16}{225} \sqrt{\frac{2}{455}} \pi \sum_{k=0}^{\infty} I_{1+2k} \left( \sqrt{\frac{2}{3}} \pi \right)$$

$$\frac{64^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{64}{4095}} \sinh \left( \frac{1}{64} \pi \sqrt{\frac{2 \times 4096}{3}} \right) \right) (\pi \sqrt{2}) =$$

$$\frac{11 \pi^2}{450 \sqrt{3}} + \frac{8}{225} \sqrt{\frac{2}{455}} \pi \sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}\right)^{-1/2-k} \pi^{1+2k}}{(1+2k)!}$$

$$\frac{64^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{64}{4095}} \sinh \left( \frac{1}{64} \pi \sqrt{\frac{2 \times 4096}{3}} \right) \right) (\pi \sqrt{2}) =$$

$$\frac{11 \pi^2}{450 \sqrt{3}} + \frac{8}{225} i \sqrt{\frac{2}{455}} \pi \sum_{k=0}^{\infty} \frac{\left(\frac{1}{6} (-3i + 2\sqrt{6}) \pi\right)^{2k}}{(2k)!}$$

**Integral representations:**

$$\frac{64^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{64}{4095}} \sinh \left( \frac{1}{64} \pi \sqrt{\frac{2 \times 4096}{3}} \right) \right) (\pi \sqrt{2}) =$$

$$\frac{11 \pi^2}{450 \sqrt{3}} + \frac{16 \pi^2}{225 \sqrt{1365}} \int_0^1 \cosh \left( \sqrt{\frac{2}{3}} \pi t \right) dt$$

$$\frac{64^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{64}{4095}} \sinh \left( \frac{1}{64} \pi \sqrt{\frac{2 \times 4096}{3}} \right) \right) (\pi \sqrt{2}) =$$

$$\frac{11 \pi^2}{450 \sqrt{3}} - \frac{4 i \pi^{3/2}}{225 \sqrt{1365}} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{\pi^2 / (6s) + s}}{s^{3/2}} ds \text{ for } \gamma > 0$$

From which:

$$1 + \left( \left( \left( \left( \left( \frac{44 \pi^2}{225 \sqrt{3}} \right) \times 64^{-1/2} + \left( \frac{\pi \sqrt{2}}{75} \right) \sqrt{\frac{64}{4095}} \sinh \left( \frac{\pi}{64} \sqrt{\frac{2 \times 4096}{3}} \right) \right) \right) \right) \right)^{1/4}$$

**Input:**

$$1 + \sqrt[4]{\frac{44 \pi^2}{225 \sqrt{3}} \times 64^{-1/2} + \left( \frac{1}{75} (\pi \sqrt{2}) \right) \sqrt{\frac{64}{4095}} \sinh \left( \frac{\pi}{64} \sqrt{\frac{2 \times 4096}{3}} \right)}$$

sinh(x) is the hyperbolic sine function

**Exact result:**

$$1 + \sqrt[4]{\frac{11 \pi^2}{450 \sqrt{3}} + \frac{8}{225} \sqrt{\frac{2}{455}} \pi \sinh \left( \sqrt{\frac{2}{3}} \pi \right)}$$

**Decimal approximation:**

1.657729130706923031377260260129740986143694759330906384278...

1.6577291307... result very near to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164.2696$  i.e. 1.65578...

**Alternate forms:**

$$1 + \frac{\sqrt[4]{\frac{1}{2} \pi \left( 11 \pi + 16 \sqrt{\frac{6}{455}} \sinh\left(\sqrt{\frac{2}{3}} \pi\right) \right)}}{3^{5/8} \sqrt{5}}$$

$$\frac{2730 + 182^{3/4} \sqrt[4]{15 \pi \left( 5005 \sqrt{3} \pi + 48 \sqrt{910} \sinh\left(\sqrt{\frac{2}{3}} \pi\right) \right)}}{2730}$$

$$\frac{2730 + 182^{3/4} \sqrt[4]{15 \left( 5005 \sqrt{3} \pi^2 + 48 \sqrt{910} \pi \sinh\left(\sqrt{\frac{2}{3}} \pi\right) \right)}}{2730}$$

**Alternative representations:**

$$1 + \sqrt[4]{\frac{64^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{64}{4095}} \sinh\left(\frac{1}{64} \pi \sqrt{\frac{2 \times 4096}{3}}\right) \right)} (\pi \sqrt{2}) =$$

$$1 + \sqrt[4]{\frac{44 \pi^2}{\sqrt{64} (225 \sqrt{3})} + \frac{\pi \sqrt{\frac{64}{4095}} \sqrt{2}}{75 \operatorname{csch}\left(\frac{1}{64} \pi \sqrt{\frac{8192}{3}}\right)}}$$

$$1 + \sqrt[4]{\frac{64^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{64}{4095}} \sinh\left(\frac{1}{64} \pi \sqrt{\frac{2 \times 4096}{3}}\right) \right)} (\pi \sqrt{2}) =$$

$$1 + \sqrt[4]{\frac{44 \pi^2}{\sqrt{64} (225 \sqrt{3})} + \frac{1}{75} i \pi \cos\left(\frac{\pi}{2} + \frac{1}{64} i \pi \sqrt{\frac{8192}{3}}\right) \sqrt{\frac{64}{4095}} \sqrt{2}}$$

$$1 + \sqrt[4]{\frac{64^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{64}{4095}} \sinh\left(\frac{1}{64} \pi \sqrt{\frac{2 \times 4096}{3}}\right) \right)} (\pi \sqrt{2}) =$$

$$1 + \sqrt[4]{\frac{44 \pi^2}{\sqrt{64} (225 \sqrt{3})} + \frac{1}{75} i \pi \cosh\left(\frac{i \pi}{2} - \frac{1}{64} \pi \sqrt{\frac{8192}{3}}\right) \sqrt{\frac{64}{4095}} \sqrt{2}}$$

**Series representations:**

$$1 + 4 \sqrt{\frac{64^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{64}{4095}} \sinh \left( \frac{1}{64} \pi \sqrt{\frac{2 \times 4096}{3}} \right) \right)} (\pi \sqrt{2}) =$$

$$1 + 4 \sqrt{\frac{11 \pi^2}{450 \sqrt{3}} + \frac{16}{225} \sqrt{\frac{2}{455}} \pi \sum_{k=0}^{\infty} I_{1+2k} \left( \sqrt{\frac{2}{3}} \pi \right)}$$

$$1 + 4 \sqrt{\frac{64^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{64}{4095}} \sinh \left( \frac{1}{64} \pi \sqrt{\frac{2 \times 4096}{3}} \right) \right)} (\pi \sqrt{2}) =$$

$$1 + \frac{4 \sqrt{\frac{55 \sqrt{3} \pi^2}{2} + 24 \sqrt{\frac{10}{91}} \pi \sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}\right)^{-1/2-k} \pi^{1+2k}}{(1+2k)!}}{15^{3/4}}$$

$$1 + 4 \sqrt{\frac{64^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{64}{4095}} \sinh \left( \frac{1}{64} \pi \sqrt{\frac{2 \times 4096}{3}} \right) \right)} (\pi \sqrt{2}) =$$

$$1 + 4 \sqrt{\frac{11 \pi^2}{450 \sqrt{3}} + \frac{8}{225} \sqrt{\frac{2}{455}} \pi \sum_{k=0}^{\infty} \frac{2^{1/2(1+2k)} \times 3^{1/2(-1-2k)} \pi^{1+2k}}{(1+2k)!}}$$

**Integral representations:**

$$1 + 4 \sqrt{\frac{64^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{64}{4095}} \sinh \left( \frac{1}{64} \pi \sqrt{\frac{2 \times 4096}{3}} \right) \right)} (\pi \sqrt{2}) =$$

$$1 + 4 \sqrt{\frac{11 \pi^2}{450 \sqrt{3}} + \frac{16 \pi^2}{225 \sqrt{1365}} \int_0^1 \cosh \left( \sqrt{\frac{2}{3}} \pi t \right) dt}$$

$$1 + 4 \sqrt{\frac{64^{-1/2} (44 \pi^2)}{225 \sqrt{3}} + \frac{1}{75} \left( \sqrt{\frac{64}{4095}} \sinh \left( \frac{1}{64} \pi \sqrt{\frac{2 \times 4096}{3}} \right) \right)} (\pi \sqrt{2}) =$$

$$1 + 4 \sqrt{\frac{11 \pi^2}{450 \sqrt{3}} - \frac{4 i \pi^{3/2}}{225 \sqrt{1365}} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{\pi^2/(6s)+s}}{s^{3/2}} ds \text{ for } \gamma > 0}$$

and:

$$322 \times \frac{1}{\left( \frac{44\pi^2}{225\sqrt{3}} \times 64^{-1/2} + \left( \frac{1}{75} (\pi \sqrt{2}) \right) \sqrt{\frac{64}{4095}} \sinh\left( \frac{\pi}{64} \sqrt{\frac{2 \times 4096}{3}} \right) \right)} + 7 + \frac{3}{2}$$

**Input:**

$$322 \times \frac{1}{\frac{44\pi^2}{225\sqrt{3}} \times 64^{-1/2} + \left( \frac{1}{75} (\pi \sqrt{2}) \right) \sqrt{\frac{64}{4095}} \sinh\left( \frac{\pi}{64} \sqrt{\frac{2 \times 4096}{3}} \right)} + 7 + \frac{3}{2}$$

$\sinh(x)$  is the hyperbolic sine function

**Exact result:**

$$\frac{17}{2} + \frac{322}{\frac{11\pi^2}{450\sqrt{3}} + \frac{8}{225} \sqrt{\frac{2}{455}} \pi \sinh\left( \sqrt{\frac{2}{3}} \pi \right)}$$

**Decimal approximation:**

1729.051051072154156463104929194658262224119375844257693054...

[1729.051051072...](#)

This result is very near to the mass of candidate glueball  **$f_0(1710)$  scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

**Alternate forms:**

$$\frac{17}{2} + \frac{197788500}{5005\sqrt{3}\pi^2 + 48\sqrt{910}\pi \sinh\left( \sqrt{\frac{2}{3}} \pi \right)}$$

$$\frac{17}{2} - \frac{144900\sqrt{3}}{\pi \left( -11\pi - 16\sqrt{\frac{6}{455}} \sinh\left( \sqrt{\frac{2}{3}} \pi \right) \right)}$$



$$\frac{17}{2} + \frac{322}{\frac{4}{225} \sqrt{\frac{2}{455}} \left( e^{\sqrt{2/3} \pi} - e^{-\sqrt{2/3} \pi} \right) \pi + \frac{11\pi^2}{450\sqrt{3}}}$$

**Alternative representations:**

$$\frac{\frac{322}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}} \sinh\left(\frac{1}{64}\pi\sqrt{\frac{2 \times 4096}{3}}\right)} + 7 + \frac{3}{2} = \frac{17}{2} + \frac{322}{\frac{44\pi^2}{\sqrt{64}(225\sqrt{3})} + \frac{\pi\sqrt{\frac{64}{4095}}\sqrt{2}}{75 \operatorname{csch}\left(\frac{1}{64}\pi\sqrt{\frac{8192}{3}}\right)}}$$

$$\frac{\frac{322}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}} \sinh\left(\frac{1}{64}\pi\sqrt{\frac{2 \times 4096}{3}}\right)} + 7 + \frac{3}{2} = \frac{17}{2} + \frac{322}{\frac{44\pi^2}{\sqrt{64}(225\sqrt{3})} + \frac{1}{75}i\pi \cos\left(\frac{\pi}{2} + \frac{1}{64}i\pi\sqrt{\frac{8192}{3}}\right)\sqrt{\frac{64}{4095}}\sqrt{2}}$$

$$\frac{\frac{322}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}} \sinh\left(\frac{1}{64}\pi\sqrt{\frac{2 \times 4096}{3}}\right)} + 7 + \frac{3}{2} = \frac{17}{2} + \frac{322}{\frac{44\pi^2}{\sqrt{64}(225\sqrt{3})} + \frac{1}{75}i\pi \cosh\left(\frac{i\pi}{2} - \frac{1}{64}\pi\sqrt{\frac{8192}{3}}\right)\sqrt{\frac{64}{4095}}\sqrt{2}}$$

**Series representations:**

$$\frac{\frac{322}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}} \sinh\left(\frac{1}{64}\pi\sqrt{\frac{2 \times 4096}{3}}\right)} + 7 + \frac{3}{2} = \frac{17}{2} + \frac{322}{197788500 + 5005\sqrt{3}\pi^2 + 48\sqrt{910}\pi \sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}\right)^{-1/2-k} \pi^{1+2k}}{(1+2k)!}}$$

$$\frac{322}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}}\sinh\left(\frac{1}{64}\pi\sqrt{\frac{2\times 4096}{3}}\right)} + 7 + \frac{3}{2} =$$

$$\frac{17}{2} + \frac{197788500}{\pi\left(5005\sqrt{3}\pi + 48i\sqrt{910}\sum_{k=0}^{\infty}\frac{\left(\frac{1}{6}(-3i+2\sqrt{6})\pi\right)^{2k}}{(2k)!}\right)}$$

$$\frac{322}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}}\sinh\left(\frac{1}{64}\pi\sqrt{\frac{2\times 4096}{3}}\right)} + 7 + \frac{3}{2} =$$

$$\frac{17}{2} + \frac{322}{\frac{11\pi^2}{450\sqrt{3}} + \frac{8}{225}\sqrt{\frac{2}{455}}\pi\sum_{k=0}^{\infty}\frac{2^{1/2(1+2k)}\times 3^{1/2(-1-2k)}\pi^{1+2k}}{(1+2k)!}}$$

**Integral representations:**

$$\frac{322}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}}\sinh\left(\frac{1}{64}\pi\sqrt{\frac{2\times 4096}{3}}\right)} + 7 + \frac{3}{2} =$$

$$\frac{17}{2} + \frac{322}{\frac{11\pi^2}{450\sqrt{3}} + \frac{16\pi^2}{225\sqrt{1365}}\int_0^1\cosh\left(\sqrt{\frac{2}{3}}\pi t\right)dt}$$

$$\frac{322}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}}\sinh\left(\frac{1}{64}\pi\sqrt{\frac{2\times 4096}{3}}\right)} + 7 + \frac{3}{2} =$$

$$\frac{17}{2} + \frac{322}{\frac{11\pi^2}{450\sqrt{3}} - \frac{4i\pi^{3/2}}{225\sqrt{1365}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{\pi^2/(6s)+s}}{s^{3/2}}ds} \text{ for } \gamma > 0$$

and:

$$29*1/((((((44Pi^2)/(225sqrt3)) * (64)^{-1/2}) + (Pi*sqrt2)/75 * (64/4095)^{1/2} \sinh((((Pi/64)*sqrt((2*4096)/3))))))))-18+golden\ ratio^2$$

**Input:**

$$29 \times \frac{1}{\frac{44\pi^2}{225\sqrt{3}} \times 64^{-1/2} + \left(\frac{1}{75}(\pi\sqrt{2})\right)\sqrt{\frac{64}{4095}}\sinh\left(\frac{\pi}{64}\sqrt{\frac{2\times 4096}{3}}\right)} - 18 + \phi^2$$

sinh(x) is the hyperbolic sine function

**Exact result:**

$$\phi^2 - 18 + \frac{29}{\frac{11\pi^2}{450\sqrt{3}} + \frac{8}{225} \sqrt{\frac{2}{455}} \pi \sinh\left(\sqrt{\frac{2}{3}} \pi\right)}$$

**Decimal approximation:**

139.5744951101550828526457139978597052124391349545991575782...

139.57449511015... result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternate forms:**

$$\phi^2 - 18 + \frac{17813250}{5005\sqrt{3}\pi^2 + 48\sqrt{910}\pi \sinh\left(\sqrt{\frac{2}{3}}\pi\right)}$$

$$\phi^2 - 18 - \frac{13050\sqrt{3}}{\pi\left(-11\pi - 16\sqrt{\frac{6}{455}} \sinh\left(\sqrt{\frac{2}{3}}\pi\right)\right)}$$

$$-\frac{33}{2} + \frac{\sqrt{5}}{2} + \frac{29}{\frac{11\pi^2}{450\sqrt{3}} + \frac{8}{225} \sqrt{\frac{2}{455}} \pi \sinh\left(\sqrt{\frac{2}{3}} \pi\right)}$$

**Alternative representations:**

$$\frac{29}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}} \sinh\left(\frac{1}{64}\pi\sqrt{\frac{2 \times 4096}{3}}\right)} - 18 + \phi^2 =$$

$$-18 + \phi^2 + \frac{29}{\frac{44\pi^2}{\sqrt{64}(225\sqrt{3})} + \frac{\pi\sqrt{\frac{64}{4095}}\sqrt{2}}{75 \operatorname{csch}\left(\frac{1}{64}\pi\sqrt{\frac{8192}{3}}\right)}}$$

$$\frac{29}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}}\sinh\left(\frac{1}{64}\pi\sqrt{\frac{2\times 4096}{3}}\right)} - 18 + \phi^2 =$$

$$-18 + \phi^2 + \frac{\frac{44\pi^2}{\sqrt{64}(225\sqrt{3})} + \frac{1}{75}i\pi\cos\left(\frac{\pi}{2} + \frac{1}{64}i\pi\sqrt{\frac{8192}{3}}\right)\sqrt{\frac{64}{4095}}\sqrt{2}}{29}$$

$$\frac{29}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}}\sinh\left(\frac{1}{64}\pi\sqrt{\frac{2\times 4096}{3}}\right)} - 18 + \phi^2 =$$

$$-18 + \phi^2 + \frac{\frac{44\pi^2}{\sqrt{64}(225\sqrt{3})} + \frac{1}{75}i\pi\cosh\left(\frac{i\pi}{2} - \frac{1}{64}\pi\sqrt{\frac{8192}{3}}\right)\sqrt{\frac{64}{4095}}\sqrt{2}}{29}$$

**Series representations:**

$$\frac{29}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}}\sinh\left(\frac{1}{64}\pi\sqrt{\frac{2\times 4096}{3}}\right)} - 18 + \phi^2 =$$

$$-\frac{33}{2} + \frac{\sqrt{5}}{2} + \frac{17813250}{5005\sqrt{3}\pi^2 + 96\sqrt{910}\pi\sum_{k=0}^{\infty} I_{1+2k}\left(\sqrt{\frac{2}{3}}\pi\right)}$$

$$\frac{29}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}}\sinh\left(\frac{1}{64}\pi\sqrt{\frac{2\times 4096}{3}}\right)} - 18 + \phi^2 =$$

$$-\frac{33}{2} + \frac{\sqrt{5}}{2} + \frac{17813250}{5005\sqrt{3}\pi^2 + 48\sqrt{910}\pi\sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}\right)^{-1/2-k}\pi^{1+2k}}{(1+2k)!}}$$

$$\frac{29}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}}\sinh\left(\frac{1}{64}\pi\sqrt{\frac{2\times 4096}{3}}\right)} - 18 + \phi^2 =$$

$$-\frac{33}{2} + \frac{\sqrt{5}}{2} + \frac{29}{\frac{11\pi^2}{450\sqrt{3}} + \frac{8}{225}\sqrt{\frac{2}{455}}\pi\sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}\right)^{-1/2-k}\pi^{1+2k}}{(1+2k)!}}$$

**Integral representations:**

$$\frac{29}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}}\sinh\left(\frac{1}{64}\pi\sqrt{\frac{2\times 4096}{3}}\right)} - 18 + \phi^2 =$$

$$-\frac{33}{2} + \frac{\sqrt{5}}{2} + \frac{29}{\frac{11\pi^2}{450\sqrt{3}} + \frac{16\pi^2}{225\sqrt{1365}}\int_0^1 \cosh\left(\sqrt{\frac{2}{3}}\pi t\right) dt}$$

$$\frac{29}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}}\sinh\left(\frac{1}{64}\pi\sqrt{\frac{2\times 4096}{3}}\right)} - 18 + \phi^2 =$$

$$-\frac{33}{2} + \frac{\sqrt{5}}{2} + \frac{29}{\frac{11\pi^2}{450\sqrt{3}} - \frac{4i\pi^{3/2}}{225\sqrt{1365}}\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^2/(6s)+s}}{s^{3/2}} ds} \text{ for } \gamma > 0$$

29\*1/((((((44Pi^2)/(225sqrt3) \* (64)^(-1/2) + (Pi\*sqrt2)/75 \* (64/4095)^1/2  
sinh((((Pi/64)\*sqrt((2\*4096)/3))))))))))-29-1/golden ratio

**Input:**

$$29 \times \frac{1}{\frac{44\pi^2}{225\sqrt{3}} \times 64^{-1/2} + \left(\frac{1}{75}(\pi\sqrt{2})\right)\sqrt{\frac{64}{4095}}\sinh\left(\frac{\pi}{64}\sqrt{\frac{2\times 4096}{3}}\right)} - 29 - \frac{1}{\phi}$$

sinh(x) is the hyperbolic sine function

φ is the golden ratio

**Exact result:**

$$-\frac{1}{\phi} - 29 + \frac{29}{\frac{11\pi^2}{450\sqrt{3}} + \frac{8}{225}\sqrt{\frac{2}{455}}\pi\sinh\left(\sqrt{\frac{2}{3}}\pi\right)}$$

**Decimal approximation:**

125.3384271326552931562365403291284289769985165949876318539...

125.33842713265... result very near to the Higgs boson mass 125.18 GeV

**Alternate forms:**

$$-\frac{1}{\phi} - 29 + \frac{17813250}{5005 \sqrt{3} \pi^2 + 48 \sqrt{910} \pi \sinh\left(\sqrt{\frac{2}{3}} \pi\right)}$$

$$-\frac{1}{\phi} - 29 - \frac{13050 \sqrt{3}}{\pi \left(-11 \pi - 16 \sqrt{\frac{6}{455}} \sinh\left(\sqrt{\frac{2}{3}} \pi\right)\right)}$$

$$\frac{1}{2}(-57 - \sqrt{5}) + \frac{29}{\frac{11 \pi^2}{450 \sqrt{3}} + \frac{8}{225} \sqrt{\frac{2}{455}} \pi \sinh\left(\sqrt{\frac{2}{3}} \pi\right)}$$

**Alternative representations:**

$$\frac{29}{\frac{(44 \pi^2) 64^{-1/2}}{225 \sqrt{3}} + \frac{1}{75} (\pi \sqrt{2}) \sqrt{\frac{64}{4095}} \sinh\left(\frac{1}{64} \pi \sqrt{\frac{2 \times 4096}{3}}\right)} - 29 - \frac{1}{\phi} =$$

$$-29 - \frac{1}{\phi} + \frac{29}{\frac{44 \pi^2}{\sqrt{64} (225 \sqrt{3})} + \frac{\pi \sqrt{\frac{64}{4095}} \sqrt{2}}{75 \operatorname{csch}\left(\frac{1}{64} \pi \sqrt{\frac{8192}{3}}\right)}}$$

$$\frac{29}{\frac{(44 \pi^2) 64^{-1/2}}{225 \sqrt{3}} + \frac{1}{75} (\pi \sqrt{2}) \sqrt{\frac{64}{4095}} \sinh\left(\frac{1}{64} \pi \sqrt{\frac{2 \times 4096}{3}}\right)} - 29 - \frac{1}{\phi} =$$

$$-29 - \frac{1}{\phi} + \frac{29}{\frac{44 \pi^2}{\sqrt{64} (225 \sqrt{3})} + \frac{1}{75} i \pi \cos\left(\frac{\pi}{2} + \frac{1}{64} i \pi \sqrt{\frac{8192}{3}}\right) \sqrt{\frac{64}{4095}} \sqrt{2}}$$

$$\frac{29}{\frac{(44 \pi^2) 64^{-1/2}}{225 \sqrt{3}} + \frac{1}{75} (\pi \sqrt{2}) \sqrt{\frac{64}{4095}} \sinh\left(\frac{1}{64} \pi \sqrt{\frac{2 \times 4096}{3}}\right)} - 29 - \frac{1}{\phi} =$$

$$-29 - \frac{1}{\phi} + \frac{29}{\frac{44 \pi^2}{\sqrt{64} (225 \sqrt{3})} + \frac{1}{75} i \pi \cosh\left(\frac{i \pi}{2} - \frac{1}{64} \pi \sqrt{\frac{8192}{3}}\right) \sqrt{\frac{64}{4095}} \sqrt{2}}$$

### Series representations:

$$\frac{29}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}}\sinh\left(\frac{1}{64}\pi\sqrt{\frac{2\times 4096}{3}}\right)} - 29 - \frac{1}{\phi} =$$

$$-29 - \frac{2}{1+\sqrt{5}} + \frac{17813250}{5005\sqrt{3}\pi^2 + 96\sqrt{910}\pi\sum_{k=0}^{\infty} I_{1+2k}\left(\sqrt{\frac{2}{3}}\pi\right)}$$

$$\frac{29}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}}\sinh\left(\frac{1}{64}\pi\sqrt{\frac{2\times 4096}{3}}\right)} - 29 - \frac{1}{\phi} =$$

$$-29 - \frac{2}{1+\sqrt{5}} + \frac{17813250}{5005\sqrt{3}\pi^2 + 48\sqrt{910}\pi\sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}\right)^{-1/2-k}\pi^{1+2k}}{(1+2k)!}}$$

$$\frac{29}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}}\sinh\left(\frac{1}{64}\pi\sqrt{\frac{2\times 4096}{3}}\right)} - 29 - \frac{1}{\phi} =$$

$$-29 - \frac{2}{1+\sqrt{5}} + \frac{29}{\frac{11\pi^2}{450\sqrt{3}} + \frac{16}{225}\sqrt{\frac{2}{455}}\pi\sum_{k=0}^{\infty} I_{1+2k}\left(\sqrt{\frac{2}{3}}\pi\right)}$$

### Integral representations:

$$\frac{29}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}}\sinh\left(\frac{1}{64}\pi\sqrt{\frac{2\times 4096}{3}}\right)} - 29 - \frac{1}{\phi} =$$

$$-29 - \frac{2}{1+\sqrt{5}} + \frac{29}{\frac{11\pi^2}{450\sqrt{3}} + \frac{16\pi^2}{225\sqrt{1365}}\int_0^1 \cosh\left(\sqrt{\frac{2}{3}}\pi t\right) dt}$$

$$\frac{29}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}}\sinh\left(\frac{1}{64}\pi\sqrt{\frac{2\times 4096}{3}}\right)} - 29 - \frac{1}{\phi} =$$

$$-29 - \frac{2}{1+\sqrt{5}} + \frac{29}{\frac{11\pi^2}{450\sqrt{3}} - \frac{4i\pi^{3/2}}{225\sqrt{1365}}\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^2/(6s)+s}}{s^{3/2}} ds} \quad \text{for } \gamma > 0$$

$$\frac{1}{2} \left( \frac{29 \times \frac{1}{\left( \frac{44\pi^2}{225\sqrt{3}} \times 64^{-1/2} + \left( \frac{1}{75} (\pi\sqrt{2}) \sqrt{\frac{64}{4095}} \sinh\left( \frac{\pi}{64} \sqrt{\frac{2 \times 4096}{3}} \right) \right) \right)}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} - 21 - 5 - 0.95686 \right)$$

Where 0.95686 is a value of a following Rogers-Ramanujan continued fraction

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

**Input:**

$$\frac{1}{2} \left( 29 \times \frac{1}{\frac{\frac{44\pi^2}{225\sqrt{3}} \times 64^{-1/2} + \left( \frac{1}{75} (\pi\sqrt{2}) \sqrt{\frac{64}{4095}} \sinh\left( \frac{\pi}{64} \sqrt{\frac{2 \times 4096}{3}} \right) \right)} - 21 - 5 - 0.95686 \right)$$

sinh(x) is the hyperbolic sine function

**Result:**

63.999801...

63.999801...  $\approx$  64

**Alternative representations:**

$$\frac{1}{2} \left( \frac{29}{\frac{\frac{44\pi^2}{225\sqrt{3}} \times 64^{-1/2} + \frac{1}{75} (\pi\sqrt{2}) \sqrt{\frac{64}{4095}} \sinh\left( \frac{1}{64} \pi \sqrt{\frac{2 \times 4096}{3}} \right)} - 21 - 5 - 0.95686} \right) =$$

$$\frac{1}{2} \left( -26.9569 + \frac{29}{\frac{\frac{44\pi^2}{\sqrt{64} (225\sqrt{3})} + \frac{\pi \sqrt{\frac{64}{4095}} \sqrt{2}}{75 \operatorname{csch}\left( \frac{1}{64} \pi \sqrt{\frac{8192}{3}} \right)}} \right)$$



$$\frac{1}{2} \left( \frac{29}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}} \sinh\left(\frac{1}{64}\pi\sqrt{\frac{2 \times 4096}{3}}\right)} - 21 - 5 - 0.95686 \right) =$$

$$\frac{1}{2} \left( -26.9569 + \frac{29}{\frac{44\pi^2}{\sqrt{64}(225\sqrt{3})} + \frac{1}{75}i\pi \cos\left(\frac{\pi}{2} + \frac{1}{64}i\pi\sqrt{\frac{8192}{3}}\right)\sqrt{\frac{64}{4095}}\sqrt{2}} \right)$$

$$\frac{1}{2} \left( \frac{29}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}} \sinh\left(\frac{1}{64}\pi\sqrt{\frac{2 \times 4096}{3}}\right)} - 21 - 5 - 0.95686 \right) =$$

$$\frac{1}{2} \left( -26.9569 + \frac{29}{\frac{44\pi^2}{\sqrt{64}(225\sqrt{3})} + \frac{1}{75}i\pi \cosh\left(\frac{i\pi}{2} - \frac{1}{64}\pi\sqrt{\frac{8192}{3}}\right)\sqrt{\frac{64}{4095}}\sqrt{2}} \right)$$

**Series representations:**

$$\frac{1}{2} \left( \frac{29}{\frac{(44\pi^2)^{64^{-1/2}}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}} \sinh\left(\frac{1}{64}\pi\sqrt{\frac{2 \times 4096}{3}}\right)} - 21 - 5 - 0.95686 \right) =$$

$$-\left( \left( 13.4784 \left( 156.406\pi^2 - \right. \right. \right.$$

$$6883.39 \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} +$$

$$21.3307\pi \exp\left(i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x}^{-2}$$

$$\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_2! k_3!} (-1)^{k_2+k_3} (2-x)^{k_2} (3-x)^{k_3} x^{-k_2-k_3}$$

$$I_{1+2k_1} \left( \frac{1}{64} \pi \sqrt{\frac{8192}{3}} \right) \left( -\frac{1}{2} \right)_{k_2} \left( -\frac{1}{2} \right)_{k_3} \Bigg) /$$

$$\left( 156.406\pi^2 + 21.3307\pi \exp\left(i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \right.$$

$$\left. \sqrt{x}^{-2} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_2! k_3!} (-1)^{k_2+k_3} (2-x)^{k_2} (3-x)^{k_3} x^{-k_2-k_3} \right.$$

$$\left. I_{1+2k_1} \left( \frac{1}{64} \pi \sqrt{\frac{8192}{3}} \right) \left( -\frac{1}{2} \right)_{k_2} \left( -\frac{1}{2} \right)_{k_3} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\begin{aligned}
& \frac{1}{2} \left( \frac{29}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}} \sinh\left(\frac{1}{64}\pi\sqrt{\frac{2 \times 4096}{3}}\right)} - 21 - 5 - 0.95686 \right) = \\
& - \left( \left( 13.4784 \left( \pi^2 - 44.0097 \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \right. \right. \\
& \quad 0.0681901 \pi \exp\left(i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \\
& \quad \left. \sqrt{x}^2 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \left( (-1)^{k_1+k_2} 64^{-1-2k_3} (2-x)^{k_1} \right. \right. \\
& \quad \quad (3-x)^{k_2} x^{-k_1-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} \\
& \quad \quad \left. \left. \left( \pi \sqrt{\frac{8192}{3}} \right)^{1+2k_3} \right) / (k_1! k_2! (1+2k_3)!) \right) \Bigg) / \\
& \left( \pi \left( \pi + 0.0681901 \exp\left(i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x}^2 \right. \right. \\
& \quad \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \left( (-1)^{k_1+k_2} 64^{-1-2k_3} (2-x)^{k_1} (3-x)^{k_2} \right. \right. \\
& \quad \quad \left. \left. x^{-k_1-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} \left( \pi \sqrt{\frac{8192}{3}} \right)^{1+2k_3} \right) / \right. \\
& \quad \left. \left. (k_1! k_2! (1+2k_3)!) \right) \right) \Bigg) \Bigg) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} \left( \frac{29}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}} \sinh\left(\frac{1}{64}\pi\sqrt{\frac{2\times 4096}{3}}\right)} - 21 - 5 - 0.95686 \right) = \\
& - \left( \left( 13.4784 \left( \pi^2 - 44.0097 \exp\left(i\pi\left[\frac{\arg(3-x)}{2\pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right. \right. \right. \\
& \quad \left. \left. \left. + 0.0681901 i \pi \exp\left(i\pi\left[\frac{\arg(2-x)}{2\pi}\right]\right) \exp\left(i\pi\left[\frac{\arg(3-x)}{2\pi}\right]\right) \sqrt{x}^2 \right. \right. \\
& \quad \left. \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_1! k_2! (2k_3)!} (-1)^{k_1+k_2} (2-x)^{k_1} (3-x)^{k_2} \right. \right. \\
& \quad \left. \left. \left. x^{-k_1-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} \left(-\frac{i\pi}{2} + \frac{1}{64}\pi\sqrt{\frac{8192}{3}}\right)^{2k_3} \right) \right) / \\
& \left( \pi \left( \pi + 0.0681901 i \exp\left(i\pi\left[\frac{\arg(2-x)}{2\pi}\right]\right) \exp\left(i\pi\left[\frac{\arg(3-x)}{2\pi}\right]\right) \right. \right. \\
& \quad \left. \left. \sqrt{x}^2 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_1! k_2! (2k_3)!} (-1)^{k_1+k_2} \right. \right. \\
& \quad \left. \left. (2-x)^{k_1} (3-x)^{k_2} x^{-k_1-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} \right. \right. \\
& \quad \left. \left. \left. \left(-\frac{i\pi}{2} + \frac{1}{64}\pi\sqrt{\frac{8192}{3}}\right)^{2k_3} \right) \right) \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

### Integral representations:

$$\begin{aligned}
& \frac{1}{2} \left( \frac{29}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}} \sinh\left(\frac{1}{64}\pi\sqrt{\frac{2\times 4096}{3}}\right)} - 21 - 5 - 0.95686 \right) = \\
& -13.4784 + \frac{11875500\sqrt{3}}{\pi^2 \left( 20020 + \sqrt{455}\sqrt{2}\sqrt{3}\sqrt{\frac{8192}{3}} \int_0^1 \cosh\left(\frac{1}{64}\pi t \sqrt{\frac{8192}{3}}\right) dt \right)}
\end{aligned}$$

$$\frac{1}{2} \left( \frac{29}{\frac{(44\pi^2)64^{-1/2}}{225\sqrt{3}} + \frac{1}{75}(\pi\sqrt{2})\sqrt{\frac{64}{4095}} \sinh\left(\frac{1}{64}\pi\sqrt{\frac{2 \times 4096}{3}}\right)} - 21 - 5 - 0.95686 \right) =$$

$$-13.4784 + \frac{29}{2 \left( \frac{11\pi^2}{450\sqrt{3}} + \frac{\pi\sqrt{2}\sqrt{\frac{8192}{3}}\sqrt{\pi}}{7200\sqrt{455}i} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{s+\left(\pi^2\sqrt{\frac{8192}{3}}\right)/(16384s)}}{s^{3/2}} ds \right)}$$

for  $\gamma > 0$

From:

**String Theory, Gravity and Particle Physics** – Augusto Sagnotti - 23.04.2020

Fig. 1

**Strings & Gravity**

**"Dilute" gravity on strings** → softer at high energies

• Diluting energy on a scale  $\ell_s$  → **"soft" gravity**

$$\frac{G_N E^2}{\hbar c^5} \Rightarrow \frac{G_N E^2}{\hbar c^5} \times \left[ \frac{\left(\frac{\hbar c}{E}\right)}{\ell_s} \right]^2$$

$$\Delta x \Delta p \geq \hbar$$

A. Sagnotti – AstronomiAmo, 23.4.2020 12

With  $E_{pl} =$  reduced Planck energy  $= 5.51809e+8$  J ,  $h = 1.054571817 \cdot 10^{-34}$  J s ,  
 $c = 299792458$  and  $l_p = 1.61623 \cdot 10^{-35}$  m, we obtain:

From the formula in Fig. 1, we have that:

$$(((1.054571817 \times 10^{-34} * 299792458) / 5.51809 \times 10^8) / (1.61623 \times 10^{-35}))^2$$

**Input interpretation:**

$$\left( \frac{\frac{1.054571817 \times 10^{-34} \times 299792458}{5.51809 \times 10^8}}{1.61623 \times 10^{-35}} \right)^2$$

**Result:**

12.56636994480961896497719363456599895632166078308631432315...

12.566369944809....  $\approx 4\pi$

Or:

$$\left( \frac{E}{E_{Pl}} \right)^2 \times \left( \frac{\Delta x}{\ell_s} \right)^2 \rightarrow \left( \frac{\hbar c}{E_{Pl} \ell_s} \right)^2$$

from which, we obtain:

$$(((1.054571817 \times 10^{-34} * 299792458) / (5.51809 \times 10^8 * 1.61623 \times 10^{-35})))^2$$

**Input interpretation:**

$$\left( \frac{1.054571817 \times 10^{-34} \times 299792458}{5.51809 \times 10^8 \times 1.61623 \times 10^{-35}} \right)^2$$

**Result:**

12.56636994480961896497719363456599895632166078308631432315...

12.5663699448... as above

From which:

$$1/4(((1.054571817 \times 10^{-34} * 299792458) / 5.51809 \times 10^8) / (1.61623 \times 10^{-35}))^2$$

**Input interpretation:**

$$\frac{1}{4} \left( \frac{\frac{1.054571817 \times 10^{-34} \times 299792458}{5.51809 \times 10^8}}{1.61623 \times 10^{-35}} \right)^2$$

**Result:**

3.141592486202404741244298408641499739080415195771578580788...

3.1415924862...  $\approx \pi$

and:

$(((((1.054571817 \times 10^{-34} * 299792458) / 5.51809 \times 10^8) / (1.61623 \times 10^{-35}))^2))^2)^{1/5}$

**Input interpretation:**

$$\sqrt[5]{\left(\frac{\frac{1.054571817 \times 10^{-34} \times 299792458}{5.51809 \times 10^8}}{1.61623 \times 10^{-35}}\right)^2}$$

**Result:**

1.658983123957007944282870649934457594272000531393624844524...

1.658983123957... result very near to the 14th root of the following Ramanujan's

class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164.2696$  i.e. 1.65578...

For

$\frac{G_N E_{Pl}^2}{\hbar c^5}$  multiplied by

$$\left(\frac{\hbar c}{E_{Pl} \ell_s}\right)^2$$

For

$$G_N = M_P^{-2} .$$

$M_{Pl} = 6.13971 \times 10^{-9}$  Kg or  $2.17643524 \times 10^{-8}$  Kg

we obtain:

(a)

$$(6.13971e-9)^{-2} * (((5.51809 * 10^8)^2 / (1.054571817e-34 * 299792458^5)))$$

$$((((1.054571817 * 10^{-34} * 299792458) / 5.51809 * 10^8) / (1.61623 * 10^{-35}))^2))$$

**Input interpretation:**

$$\frac{(5.51809 \times 10^8)^2}{1.054571817 \times 10^{-34} \times 299792458^5} \left( \frac{\frac{1.054571817 \times 10^{-34} \times 299792458}{5.51809 \times 10^8}}{1.61623 \times 10^{-35}} \right)^2$$

$$(6.13971 \times 10^{-9})^2$$

**Result:**

$$3.9747700410469907260377370051302123660015811621834010... \times 10^{26}$$

$$3.97477004... * 10^{26}$$

or:

(b)

$$(2.17643524e-8)^{-2} * (((5.51809 * 10^8)^2 / (1.054571817e-34 * 299792458^5)))$$

$$((((1.054571817 * 10^{-34} * 299792458) / 5.51809 * 10^8) / (1.61623 * 10^{-35}))^2))$$

**Input interpretation:**

$$\frac{(5.51809 \times 10^8)^2}{1.054571817 \times 10^{-34} \times 299792458^5} \left( \frac{\frac{1.054571817 \times 10^{-34} \times 299792458}{5.51809 \times 10^8}}{1.61623 \times 10^{-35}} \right)^2$$

$$(2.17643524 \times 10^{-8})^2$$

**Result:**

$$3.1631240635256766933948719802621169803122624831162112... \times 10^{25}$$

$$3.1631240635... * 10^{25}$$



For the expression (a), we obtain:

$$\ln(\left(\left(\left(6.13971e-9\right)^{-2} * \left(\left(\left(5.51809 * 10^8\right)^2 / \left(1.054571817e-34 * 299792458^5\right)\right)\right)\right) / \left(\left(\left(1.054571817 * 10^{-34} * 299792458\right) / 5.51809 * 10^8\right) / \left(1.61623 * 10^{-35}\right)\right)^2\right)\right)$$

**Input interpretation:**

$$\log \left( \frac{\left( \frac{(5.51809 \times 10^8)^2}{1.054571817 \times 10^{-34} \times 299792458^5} \right) \left( \frac{1.054571817 \times 10^{-34} \times 299792458}{1.61623 \times 10^{-35}} \right)^2}{(6.13971 \times 10^{-9})^2} \right)$$

log(x) is the natural logarithm

**Result:**

61.24718...

61.24718...

From which, adding  $e = 2.71828\dots$  :

$$\ln(\left(\left(\left(6.13971e-9\right)^{-2} * \left(\left(\left(5.51809 * 10^8\right)^2 / \left(1.054571817e-34 * 299792458^5\right)\right)\right)\right) / \left(\left(\left(1.054571817 * 10^{-34} * 299792458\right) / 5.51809 * 10^8\right) / \left(1.61623 * 10^{-35}\right)\right)^2\right)\right) + 2.71828$$

**Input interpretation:**

$$\log \left( \frac{\left( \frac{(5.51809 \times 10^8)^2}{1.054571817 \times 10^{-34} \times 299792458^5} \right) \left( \frac{1.054571817 \times 10^{-34} \times 299792458}{1.61623 \times 10^{-35}} \right)^2}{(6.13971 \times 10^{-9})^2} \right) + 2.71828$$

log(x) is the natural logarithm

**Result:**

63.96546...

63.96546...  $\approx 64$

For

$M_{Pl} = E_{Pl} = 1.22091e+19$  or  $3.44412e+18$ ,  $h = 6.582119569e-16$ ,

$l_{Pl} = 8.19061 * 10^{-29}$ , from the previous expressions, we obtain:

$$\left(\frac{E}{E_{Pl}}\right)^2 \times \left(\frac{\Delta x}{l_s}\right)^2 \rightarrow \left(\frac{\hbar c}{E_{Pl} l_s}\right)^2$$

$$(((6.582119569e-16 * 299792458) / (1.22091e+19 * 8.19061*10^{-29})))^2$$

**Input interpretation:**

$$\left(\frac{6.582119569 \times 10^{-16} \times 299792458}{1.22091 \times 10^{19} \times 8.19061 \times 10^{-29}}\right)^2$$

**Result:**

38937.95547228317937377552550614230009609427199593059462416...

38937.955472283...

and:

$$((((((1.22091e+19)^{-2}) * (((((1.22091e+19)^2 / (6.582119569e-16 * 299792458^5)))))))) * ((((((6.582119569e-16 * 299792458) / 1.22091e+19) / (8.19061*10^{-29})))^2)))$$

**Input interpretation:**

$$\frac{\frac{(1.22091 \times 10^{19})^2}{6.582119569 \times 10^{-16} \times 299792458^5}}{(1.22091 \times 10^{19})^2} \left( \frac{6.582119569 \times 10^{-16} \times 299792458}{1.22091 \times 10^{19}} \right)^2 \frac{1}{8.19061 \times 10^{-29}}$$

**Result:**

2.4428897214020613314723593291522411930647355772341908...  $\times 10^{-23}$

2.4428897214...  $\times 10^{-23}$

From which:

$$\text{colog}\left(\left(\left(\left(\left(\left(1.22091e+19\right)^{-2}\right) * \left(\left(\left(1.22091e+19\right)^2 / \left(6.582119569e-16 * 299792458^5\right)\right)\right)\right)\right)\right) * \left(\left(\left(\left(6.582119569e-16 * 299792458\right) / 1.22091e+19\right) / \left(8.19061 * 10^{-29}\right)\right)^2\right)\right)\right)$$

**Input interpretation:**

$$-\log\left(\frac{\frac{(1.22091 \times 10^{19})^2}{6.582119569 \times 10^{-16} \times 299792458^5}}{(1.22091 \times 10^{19})^2} \left(\frac{6.582119569 \times 10^{-16} \times 299792458}{1.22091 \times 10^{19}}\right)^2\right)$$

log(x) is the natural logarithm

**Result:**

52.0663...

52.0663...

Dividing the result 38937.955472283... of the previous expression by

$$-\log\left(\frac{\frac{(1.22091 \times 10^{19})^2}{6.582119569 \times 10^{-16} \times 299792458^5}}{(1.22091 \times 10^{19})^2} \left(\frac{6.582119569 \times 10^{-16} \times 299792458}{1.22091 \times 10^{19}}\right)^2\right)$$

we obtain:

$$38937.95547228317937 / \text{colog}\left(\left(\left(\left(\left(\left(1.22091e+19\right)^{-2}\right) * \left(\left(\left(1.22091e+19\right)^2 / \left(6.582119569e-16 * 299792458^5\right)\right)\right)\right)\right)\right) * \left(\left(\left(\left(6.582119569e-16 * 299792458\right) / 1.22091e+19\right) / \left(8.19061 * 10^{-29}\right)\right)^2\right)\right)\right)$$

**Input interpretation:**

$$\frac{38937.95547228317937}{-\log\left(\frac{\frac{(1.22091 \times 10^{19})^2}{6.582119569 \times 10^{-16} \times 299792458^5}}{(1.22091 \times 10^{19})^2} \left(\frac{6.582119569 \times 10^{-16} \times 299792458}{1.22091 \times 10^{19}}\right)^2\right)}$$

log(x) is the natural logarithm

**Result:**

747.8536750927920816812719104500108936283666434594662554633...

747.85367509...

From which:

$$13 \times \frac{1}{\left[ \ln 38937.95547228 / \operatorname{colog}\left(\left(\left(\left(\left(1.22091e+19\right)^{-2}\right) * \left(\left(\left(1.22091e+19\right)^2 / \left(6.582119569e-16 * 299792458^5\right)\right)\right)\right)\right) * \left(\left(\left(\left(6.582119569e-16 * 299792458\right) / \left(1.22091e+19\right) / \left(8.19061 * 10^{-29}\right)\right)^2\right)\right)\right)\right]} \right]}$$

**Input interpretation:**

$$13 \times \frac{1}{\left[ \frac{\log(38937.95547228)}{\log\left(\frac{(1.22091 \times 10^{19})^2}{6.582119569 \times 10^{-16} \times 299792458^5} \left(\frac{6.582119569 \times 10^{-16} \times 299792458^2}{1.22091 \times 10^{19}} \frac{1.22091 \times 10^{19}}{8.19061 \times 10^{-29}}\right)\right)} \right]}$$

log(x) is the natural logarithm

**Result:**

64.0378...

64.0378... ≈ 64

or:

$$1.618034 + \frac{1}{12} * 38937.955472 / \operatorname{colog}\left(\left(\left(\left(\left(1.22091e+19\right)^{-2}\right) * \left(\left(\left(1.22091e+19\right)^2 / \left(6.582119569e-16 * 299792458^5\right)\right)\right)\right)\right) * \left(\left(\left(\left(6.582119569e-16 * 299792458\right) / \left(1.22091e+19\right) / \left(8.19061 * 10^{-29}\right)\right)^2\right)\right)\right)\right)$$

**Input interpretation:**

$$1.618034 + \frac{1}{12} \left[ \frac{38937.955472}{\log\left(\frac{(1.22091 \times 10^{19})^2}{6.582119569 \times 10^{-16} \times 299792458^5} \left(\frac{6.582119569 \times 10^{-16} \times 299792458^2}{1.22091 \times 10^{19}} \frac{1.22091 \times 10^{19}}{8.19061 \times 10^{-29}}\right)\right)} \right]$$

log(x) is the natural logarithm

**Result:**

63.9392...

63.9392...  $\approx$  64

We note that from:

[A304834](#)  $a(n) = 36*n^2 - 8*n - 2$  ( $n \geq 1$ ).

26, 126, 298, 542, 858, 1246, 1706, 2238, 2842, 3518, 4266, 5086, 5978, 6942, 7978, 9086, 10266, 11518, 12842, 14238, 15706, 17246, 18858, 20542, 22298, 24126, 26026, 27998, 30042, 32158, 34346, 36606, **38938**, 41342, 43818, 46366, 48986, 51678, 54442, 57278, 60186, 63166, 66218, 69342, 72538 ([list](#); [graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))

where

 $a(n) = 36*n^2 - 8*n - 2$  ( $n \geq 1$ ), for  $n = 33$ , we obtain:

$$36*33^2 - 8*33 - 2$$

**Input:**

$$36 \times 33^2 - 8 \times 33 - 2$$

**Result:**

38938

**38938**

From the formula of coefficients of the '5th order' mock theta function  $\psi_1(q)$ :  
(A053261 OEIS Sequence)

$$\sqrt{\phi} * \exp(\pi * \sqrt{n/15}) / (2 * 5^{1/4} * \sqrt{n}) \text{ for } n = 310$$

$$\sqrt{\phi} * \exp(\pi * \sqrt{310/15}) / (2 * 5^{1/4} * \sqrt{310}) + 377 + 55 - 5$$

where 5, 55 and 377 are Fibonacci's numbers, we obtain:

**Input:**

$$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{310}{15}}\right)}{2 \sqrt[4]{5} \sqrt{310}} + 377 + 55 - 5$$

**Exact result:**

$$\frac{e^{\sqrt{62/3} \pi} \sqrt{\frac{\phi}{62}}}{2 \times 5^{3/4}} + 427$$

**Decimal approximation:**

38937.39874117523387722060265936525674219617003294190764596...

38937

**Property:**

$427 + \frac{e^{\sqrt{62/3} \pi} \sqrt{\frac{\phi}{62}}}{2 \times 5^{3/4}}$  is a transcendental number

**Alternate forms:**

$$427 + \frac{1}{20} \sqrt{\frac{1}{31} (5 + \sqrt{5})} e^{\sqrt{62/3} \pi}$$

$$427 + \frac{\sqrt{\frac{1}{31} (1 + \sqrt{5})} e^{\sqrt{62/3} \pi}}{4 \times 5^{3/4}}$$

$$\frac{1}{620} \left( 264740 + \sqrt[4]{5} \sqrt{31 (1 + \sqrt{5})} e^{\sqrt{62/3} \pi} \right)$$

**Series representations:**

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{310}{15}}\right)}{2 \sqrt[4]{5} \sqrt{310}} + 377 + 55 - 5 = \left( 4270 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (310 - z_0)^k z_0^{-k}}{k!} + \right. \\ \left. 5^{3/4} \exp\left(\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{62}{3} - z_0\right)^k z_0^{-k}}{k!}\right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi - z_0)^k z_0^{-k}}{k!} \right) / \\ \left( 10 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (310 - z_0)^k z_0^{-k}}{k!} \right) \text{ for (not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\begin{aligned}
& \frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{310}{15}}\right)}{2 \sqrt[4]{5} \sqrt{310}} + 377 + 55 - 5 = \\
& \left( 4270 \exp\left(i \pi \left[ \frac{\arg(310-x)}{2\pi} \right] \right) \sum_{k=0}^{\infty} \frac{(-1)^k (310-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \right. \\
& \quad 5^{3/4} \exp\left(i \pi \left[ \frac{\arg(\phi-x)}{2\pi} \right] \right) \exp\left(\pi \exp\left(i \pi \left[ \frac{\arg\left(\frac{62}{3}-x\right)}{2\pi} \right] \right) \sqrt{x} \right. \\
& \quad \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{62}{3}-x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \sum_{k=0}^{\infty} \frac{(-1)^k (\phi-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) / \\
& \left( 10 \exp\left(i \pi \left[ \frac{\arg(310-x)}{2\pi} \right] \right) \sum_{k=0}^{\infty} \frac{(-1)^k (310-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{310}{15}}\right)}{2 \sqrt[4]{5} \sqrt{310}} + 377 + 55 - 5 = \\
& \left( \left( \frac{1}{z_0} \right)^{-1/2 [\arg(310-z_0)/(2\pi)]} z_0^{-1/2 [\arg(310-z_0)/(2\pi)]} \left( 4270 \left( \frac{1}{z_0} \right)^{1/2 [\arg(310-z_0)/(2\pi)]} \right. \right. \\
& \quad \left. \left. z_0^{1/2 [\arg(310-z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (310-z_0)^k z_0^{-k}}{k!} + \right. \right. \\
& \quad 5^{3/4} \exp\left(\pi \left( \frac{1}{z_0} \right)^{1/2 [\arg\left(\frac{62}{3}-z_0\right)/(2\pi)]} z_0^{1/2 (1+[\arg\left(\frac{62}{3}-z_0\right)/(2\pi)])} \right. \\
& \quad \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{62}{3}-z_0\right)^k z_0^{-k}}{k!} \right) \left( \frac{1}{z_0} \right)^{1/2 [\arg(\phi-z_0)/(2\pi)]} z_0^{1/2 [\arg(\phi-z_0)/(2\pi)]} \right. \\
& \quad \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi-z_0)^k z_0^{-k}}{k!} \right) \right) / \left( 10 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (310-z_0)^k z_0^{-k}}{k!} \right)
\end{aligned}$$

Now, we have that:

Fig.2

## Quantum Field Theory

- **Bosons** have **positive** zero-point energy

- **Fermions:**

$$H = \frac{1}{2}(a^*a - aa^*)\hbar\omega = \left( a^*a - \frac{1}{2} \right) \hbar c \sqrt{k^2 + \left( \frac{mc}{\hbar} \right)^2}$$

{ |0>, a^\*|0> }

Pauli principle and **negative** zero-point energy

**Supersymmetry (SUSY)** : zero-point energies cancel for bosons and fermions of same mass

$a^*a = 5.6777661 \times 10^{-18}$  (electron energy)

From:

$$(i) \quad K.E. = \frac{e^2 K}{2a_0}$$

$$K.E. = \frac{(1.6 \times 10^{-19})^2 \times 9 \times 10^9}{2 \times 0.53 \times 10^{-10}}$$

$$K.E. = 21.74 \times 10^{-19} \text{ J}$$

$$K.E. = \frac{21.74 \times 10^{-19}}{1.6 \times 10^{-19}} = 13.59 \text{ eV} \quad \boxed{K.E. = 13.59 \text{ eV}}$$

$$(ii) \quad P.E. = \frac{-e^2 K}{r} = -2K.E.$$

$$P.E. = -2 \times 13.59 \quad \boxed{P.E. = -27.18 \text{ eV}}$$

where K.E. = kinetic energy of an electron and P.E = potential energy of an electron, we obtain putting  $1.60217662 \times 10^{-19}$  instead of  $1.6 \times 10^{-19}$ :

$$(((1.60217662 \times 10^{-19})^2 \times 9 \times 10^9)) / (2 \times 0.53 \times 10^{-10}) * 1 / 1.60217662 \times 10^{-19}$$



**Input interpretation:**

$$\frac{(1.60217662 \times 10^{-19})^2 \times 9 \times 10^9}{2 \times 0.53 \times 10^{-10}} \times \frac{1}{1.60217662 \times 10^{-19}}$$

**Result:**

13.60338639622641509433962264150943396226415094339622641509...

13.6033863... = K. E.

From the formula in Fig. 2

$$(5.6777661 \times 10^{-18} - 1/2) \times 6.582119 \times 10^{-16} \times 299792458 \times \sqrt{((13.60338^2 + ((0.510998950 \times 299792458) / (6.582119 \times 10^{-16})))^2))}$$

where 5.6777661e-18 is the electron energy, we obtain:

**Input interpretation:**

$$\left(5.6777661 \times 10^{-18} - \frac{1}{2}\right) \times 6.582119 \times 10^{-16} \times 299792458 \sqrt{13.60338^2 + \left(\frac{0.510998950 \times 299792458}{6.582119 \times 10^{-16}}\right)^2}$$

**Result:**

-2.296315... × 10<sup>16</sup>  
-2.296315... \* 10<sup>16</sup>

And performing the 9<sup>th</sup> root, we obtain:

$$\left(\left(\left(-\left(5.6777661 \times 10^{-18} - 1/2\right) \times 6.582119 \times 10^{-16} \times 299792458 \times \sqrt{((13.60338^2 + ((0.510998950 \times 299792458) / (6.582119 \times 10^{-16})))^2)}\right)\right)\right)^{1/9} - \phi$$

**Input interpretation:**

$$\left(-\left(5.6777661 \times 10^{-18} - \frac{1}{2}\right) \left(6.582119 \times 10^{-16} \times 299792458 \sqrt{13.60338^2 + \left(\frac{0.510998950 \times 299792458}{6.582119 \times 10^{-16}}\right)^2}\right)\right)^{(1/9) - \phi}$$

φ is the golden ratio

**Result:**

64.131455...

64.131455...  $\approx$  64

**Observations**

*From:*

[https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn\\_RpOSvJIQxWsVLBcJ6KVgd\\_Af\\_hrmDYBNyU8mpSjRs1BDeremA](https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJIQxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA)

*Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that  $p(9) = 30$ ,  $p(9 + 5) = 135$ ,  $p(9 + 10) = 490$ ,  $p(9 + 15) = 1,575$  and so on are all divisible by 5. Note that here the  $n$ 's come at intervals of five units.*

*Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of  $p(n)$  that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.*

*Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of  $n$ 's separated by  $5^3 = 125$  units, saying that the corresponding  $p(n)$ 's should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.*

*From Wikipedia*

*In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field  $\phi$  and a Dirac field  $\psi$ . The Yukawa interaction can be used to describe the nuclear force between nucleons (which*

are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1 + \sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \dots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and  $4096 = 64^2$

*(Modular equations and approximations to  $\pi$  - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)*

*All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants  $\pi$ ,  $\phi$ ,  $1/\phi$ , the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.*

*In mathematics, the Fibonacci numbers, commonly denoted  $F_n$ , form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the  $n$ th Fibonacci number in terms of  $n$  and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as  $n$  increases.*

*Fibonacci numbers are also closely related to Lucas numbers, in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences*

*The beginning of the sequence is thus:*

*0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...*

*The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.*

*The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.<sup>[1]</sup> The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.*

*The sequence of Lucas numbers is:*

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803.....

*All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.*

*A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:*

*2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).*

*In geometry, a golden spiral is a logarithmic spiral whose growth factor is  $\varphi$ , the golden ratio.<sup>[1]</sup> That is, a golden spiral gets wider (or further from its origin) by a factor of  $\varphi$  for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies<sup>[3]</sup> - golden spirals are one special case of these logarithmic spirals*

We observe that 1728 and 1729 are results very near to the mass of candidate glueball  **$f_0(1710)$  scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to  $\zeta(2) =$

$$\frac{\pi^2}{6} = 1.644934 \dots$$

**We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.**

## References

### **Chapter 4 - General Partition Function and Tau Function -**

[https://shodhganga.inflibnet.ac.in/bitstream/10603/98249/10/10\\_chapter%204.pdf](https://shodhganga.inflibnet.ac.in/bitstream/10603/98249/10/10_chapter%204.pdf)

### **Efficient implementation of the Hardy-Ramanujan-Rademacher formula**

*Fredrik Johansson* - arXiv:1205.5991v2 [math.NT] 6 Jul 2012

**String Theory, Gravity and Particle Physics** – *Augusto Sagnotti* - 23.04.2020