Mathematical connections beetween some Ramanujan equations, ϕ and various parameters of Quantum Geometry, String Theory and Particle Physics. IV

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Abstract

In this paper we have described and analyzed some Ramanujan expressions. We have obtained several mathematical connections with ϕ and various parameters of Quantum Geometry, String Theory and Particle Physics.

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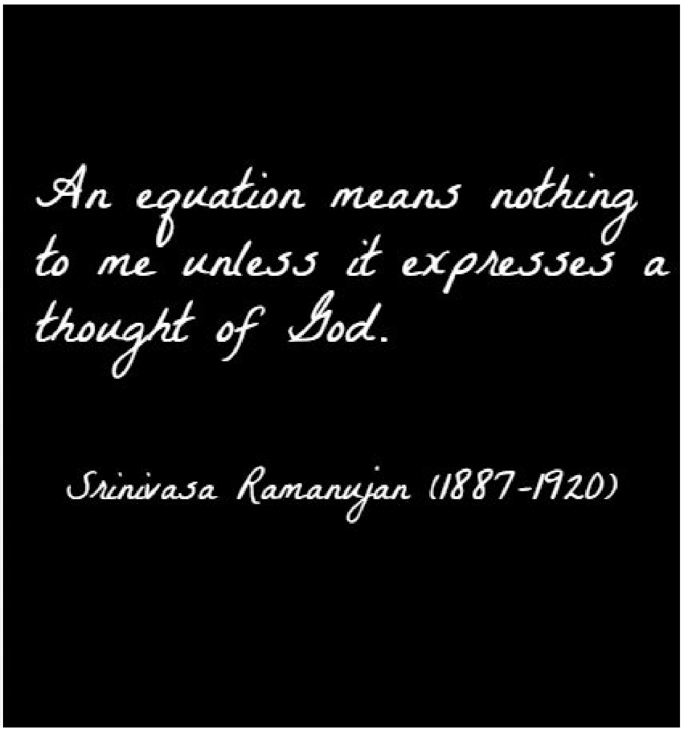


https://mobygeek.com/features/indian-mathematician-srinivasa-ramanujan-quotes-11012

We want to highlight that the development of the various equations was carried out according an our possible logical and original interpretation

From:

Chiral Asymmetry in Four-Dimensional Open-String Vacua
C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Ya.S. Stanev - arXiv:hep-th/9606169v3 11 Jul 1996

We have that:

The corresponding type IIB partition function is

$$T = |V_8 - S_8|^2 \left(\frac{\sum q^{\frac{\alpha'}{4} p_{La} G^{ab} p_{Lb}} \bar{q}^{\frac{\alpha'}{4} p_{Ra} G^{ab} p_{Rb}}}{\eta^2(q)\eta^2(\bar{q})} \right)^3 , \qquad (2.2)$$

where V_8 and S_8 are level-one $SO(8)$ characters, $(p_a)_{L,R} = m_a + \frac{1}{2} G_{ab} n^b$ and $q = e^{2\pi i \tau}$. As anticipated, we begin with a vanishing NS-NS tensor B in order to obtain a CP group of maximum size.

From

$$T = |V_8 - S_8|^2 \left(\frac{\sum q^{\frac{\alpha'}{4} p_{La} G^{ab} p_{Lb}} \bar{q}^{\frac{\alpha'}{4} p_{Ra} G^{ab} p_{Rb}}}{\eta^2(q)\eta^2(\bar{q})} \right)^3$$

We know that: (from Wikipedia)

For any complex number τ with $\mathrm{Im}(\tau) > 0$, let $q = e^{2\pi i\tau}$, then the eta function is defined by,

$$\eta(\tau) = e^{\frac{\pi i\tau}{12}} \prod_{n=1}^{\infty}(1 - e^{2n\pi i\tau}) = q^{\frac{1}{24}} \prod_{n=1}^{\infty}(1 - q^n).$$

The notation $q \equiv e^{2\pi i\tau}$ is now standard in number theory, though many older books use q for the nome $e^{\pi i\tau}$. Raising the eta equation to the 24th power and multiplying by $(2\pi)^{12}$ gives

$$\Delta(\tau) = (2\pi)^{12}\eta^{24}(\tau)$$

where Δ is the modular discriminant. The presence of 24 can be understood by connection with other occurrences, such as in the 24-dimensional Leech lattice.

From:

DEDEKIND'S η-FUNCTION AND ROGERS–RAMANUJAN IDENTITIES
S. OLE WARNAAR AND WADIM ZUDILIN - arXiv:1001.1571v2 [math.CO] 22 Jan 2010

Now, we have:

Conjecture 1.1. *For k, n positive integers, $N = 2n$ and $p \in \{1, k\}$,*

$$(1.4)\quad \sum \frac{q^{\frac{1}{2}\sum_{a,b=1}^{N-1}\sum_{i=1}^{k-1} C_{ab} M_i^{(a)} M_i^{(b)} + \sum_{a=1}^{N-1}\sum_{i=p}^{k-1}(-1)^a M_i^{(a)}}}{\prod_{a=1}^{N-1}\prod_{i=1}^{k-1}(q)_{m_i^{(a)}}} = \frac{1}{(q)_\infty^{2n^2-n}} \sum \xi(\boldsymbol{v}/\boldsymbol{\rho})(-1)^{|\boldsymbol{v}|-|\boldsymbol{\rho}+k-p|} q^{\frac{\|\boldsymbol{v}\|^2 - \|\boldsymbol{\rho}+k-p\|^2}{2(2k+2n-1)}},$$

where the sum on the left is over $m_i^{(a)} \in \mathbb{N}$ (for all $1 \le a \le N-1$ and $1 \le i \le k-1$) and the sum on the right is over $\boldsymbol{v} \in (\mathbb{Z}/2)^n$ such that $v_i \equiv \rho_i + k - p \pmod{2k+2n-1}$. The integers $M_i^{(a)}$ are defined as $M_i^{(a)} = m_i^{(a)} + \cdots + m_{k-1}^{(a)}$, i.e., $m_i^{(a)} = M_i^{(a)} - M_{i+1}^{(a)}$ for $1 \le i \le k-2$ and $m_{k-1}^{(a)} = M_{k-1}^{(a)}$.

Theorem 1.2 (Generalised Rogers–Ramanujan identities). *Conjecture 1.1 is true for $k = 2$. That is, for n a positive integer and $N = 2n$,*

$$(1.5)\quad \sum_{\boldsymbol{m} \in \mathbb{N}^{N-1}} \frac{q^{\frac{1}{2}\boldsymbol{m}C\boldsymbol{m}^t}}{(q)_{\boldsymbol{m}}} = \frac{1}{(q)_\infty^{2n^2-n}} \sum \xi(\boldsymbol{v}/\boldsymbol{\rho})(-1)^{|\boldsymbol{v}|-|\boldsymbol{\rho}|} q^{\frac{\|\boldsymbol{v}\|^2 - \|\boldsymbol{\rho}\|^2}{2(2n+3)}},$$

where the sum on the right is over $\boldsymbol{v} \in (\mathbb{Z}/2)^n$ such that $v_i \equiv \rho_i \pmod{2n+3}$, and

$$\sum_{\boldsymbol{m} \in \mathbb{N}^{N-1}} \frac{q^{\frac{1}{2}\boldsymbol{m}C\boldsymbol{m}^t + |\boldsymbol{m}|_-}}{(q)_{\boldsymbol{m}}} = \frac{1}{(q)_\infty^{2n^2-n}} \sum \xi(\boldsymbol{v}/\boldsymbol{\rho})(-1)^{|\boldsymbol{v}|-|\boldsymbol{\rho}+1|} q^{\frac{\|\boldsymbol{v}\|^2 - \|\boldsymbol{\rho}+1\|^2}{2(2n+3)}},$$

where the sum on the right is over $\boldsymbol{v} \in (\mathbb{Z}/2)^n$ such that $v_i \equiv \rho_i + 1 \pmod{2n+3}$. In the above $(q)_{\boldsymbol{m}} = (q)_{m_1} \dots (q)_{m_{N-1}}$, C is the Cartan matrix of A_{N-1}, i.e.,

$$\frac{1}{2}\boldsymbol{m}C\boldsymbol{m}^t = \sum_{i=1}^{N-1} m_i^2 - \sum_{i=1}^{N-2} m_i m_{i+1},$$

and, for $\boldsymbol{m} \in \mathbb{N}^{N-1}$,

$$|\boldsymbol{m}|_- = \sum_{i=1}^{N-1} (-1)^{i-1} m_i.$$

Now, we have:

To provide further support for Conjecture 1.1, we show below that a standard asymptotic analysis applied to (1.4) implies an identity for the Rogers dilogarithm due to Kirillov.

If we denote the summand on the left by $S(K, N; a, i)$ then

$$S(2k, N; a, i) = S(2k, N; a, 2k-i-1) \quad \text{for } 1 \leq i \leq k-1$$

and $S(2k, N; a, 2k-1) = \mathrm{L}(1)$. Hence

(3.2)
$$\frac{1}{\mathrm{L}(1)} \sum_{a=1}^{N-1} \sum_{i=1}^{k-1} \mathrm{L}\left(\frac{\sin\left(\frac{a\pi}{2k+N-1}\right) \sin\left(\frac{(N-a)\pi}{2k+N-1}\right)}{\sin\left(\frac{(i+a)\pi}{2k+N-1}\right) \sin\left(\frac{(i+N-a)\pi}{2k+N-1}\right)} \right) = \frac{N(N-1)(k-1)}{2k+N-1}.$$

Lemma 3.1. *Let B be a $d \times d$ symmetric, positive definite, rational matrix and let*

$$\sum_{i=0}^{\infty} a_i q^i = \sum_{m \in \mathbb{N}^d} \frac{q^{\frac{1}{2} m B m^t}}{(q)_m}.$$

Then

$$\lim_{m \to \infty} \frac{\log^2 a_m}{4m} = \sum_{i=1}^{d} \mathrm{L}(x_i),$$

where the x_i for $1 \leq i \leq d$ are the solutions of

$$x_i = \prod_{j=1}^{d} (1 - x_j)^{B_{ij}}$$

such that $x_i \in (0, 1)$ for all i.

Hence, denoting the q-series on either side of (1.6) by $\sum_{i\geq 0} a_i q^i$, we find that

$$\frac{1}{\mathrm{L}(1)} \lim_{m\to\infty} \frac{\log^2 a_m}{4m} = \mathrm{LHS}(3.2).$$

The right-hand side of (1.6) is a specialised standard module of $\mathrm{A}_{2n}^{(2)}$ [13, 15]. Exploiting its modular properties [16] we obtain

$$\frac{1}{\mathrm{L}(1)} \lim_{m\to\infty} \frac{\log^2 a_m}{4m} = \mathrm{RHS}(3.2)$$

(recall that $N = 2n$), leading to (3.2).

$\mathrm{L}(1) = \pi^2/6.$

For k, N positive integers and $n = \lfloor N/2 \rfloor$,

For N = 3, k = 5, a = 1, i = 2, from

$$\frac{1}{\mathrm{L}(1)} \sum_{a=1}^{N-1} \sum_{i=1}^{k-1} \mathrm{L}\left(\frac{\sin\left(\frac{a\pi}{2k+N-1}\right) \sin\left(\frac{(N-a)\pi}{2k+N-1}\right)}{\sin\left(\frac{(i+a)\pi}{2k+N-1}\right) \sin\left(\frac{(i+N-a)\pi}{2k+N-1}\right)} \right) = \frac{N(N-1)(k-1)}{2k+N-1}.$$

we obtain:

6/(Pi^2) [(sin(Pi/(2*5+3-1)) sin(2Pi/(2*5+3-1))) / (sin(3Pi/(2*5+3-1)) sin(4Pi/(2*5+3-1)))]

Input:

$$\frac{6}{\pi^2} \times \frac{\sin\left(\frac{\pi}{2\times 5+3-1}\right) \sin\left(2 \times \frac{\pi}{2\times 5+3-1}\right)}{\sin\left(3 \times \frac{\pi}{2\times 5+3-1}\right) \sin\left(4 \times \frac{\pi}{2\times 5+3-1}\right)}$$

Exact result:

$$\frac{\sqrt{3}\left(\sqrt{3}-1\right)}{\pi^2}$$

Decimal approximation:

0.128470112975467657208234930926204801824769601854170797897...

0.12847011…

Property:

$\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}$ is a transcendental number

Alternate forms:

$$\frac{3-\sqrt{3}}{\pi^2}$$

$$\frac{3}{\pi^2}-\frac{\sqrt{3}}{\pi^2}$$

Alternative representations:

$$\frac{\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)6}{\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)\pi^2}=\frac{6}{\frac{\csc\left(\frac{\pi}{12}\right)\csc\left(\frac{2\pi}{12}\right)\pi^2}{\csc\left(\frac{3\pi}{12}\right)\csc\left(\frac{4\pi}{12}\right)}}$$

$$\frac{\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)6}{\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)\pi^2}=\frac{6\cos\left(\frac{\pi}{2}+\frac{\pi}{12}\right)\cos\left(\frac{\pi}{2}+\frac{2\pi}{12}\right)}{\left(\cos\left(\frac{\pi}{2}+\frac{3\pi}{12}\right)\cos\left(\frac{\pi}{2}+\frac{4\pi}{12}\right)\right)\pi^2}$$

$$\frac{\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)6}{\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)\pi^2}=\frac{6\cos\left(\frac{\pi}{2}-\frac{2\pi}{12}\right)\cos\left(\frac{\pi}{2}-\frac{\pi}{12}\right)}{\left(\cos\left(\frac{\pi}{2}-\frac{4\pi}{12}\right)\cos\left(\frac{\pi}{2}-\frac{3\pi}{12}\right)\right)\pi^2}$$

Series representations:

$$\frac{\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)6}{\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)\pi^2}=$$
$$\frac{6\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\,2^{-3-2k_1-4k_2}\times 3^{-2-2k_1-2k_2}\,\pi^{2+2k_1+2k_2}}{(1+2k_1)!\,(1+2k_2)!}}{\pi^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,3^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\,4^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}}$$

$$\frac{\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)6}{\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)\pi^2}=\frac{6\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{\left(-\frac{25}{16}\right)^{k_1}(-1)^{k_2}\,9^{-k_1-k_2}\,(-\pi)^{2k_1+2k_2}}{(2k_1)!\,(2k_2)!}}{\pi^2\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{16}\right)^k(-\pi)^{2k}}{(2k)!}\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{36}\right)^k(-\pi)^{2k}}{(2k)!}}$$

$$\frac{\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)6}{\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)\pi^2}=\frac{6\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}(-1)^{k_1+k_2}\,J_{1+2k_1}\left(\frac{\pi}{12}\right)J_{1+2k_2}\left(\frac{\pi}{6}\right)}{\pi^2\left(\sum_{k=0}^{\infty}(-1)^k\,J_{1+2k}\left(\frac{\pi}{4}\right)\right)\sum_{k=0}^{\infty}(-1)^k\,J_{1+2k}\left(\frac{\pi}{3}\right)}$$

Integral representations:

$$\frac{\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)6}{\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)\pi^2} = \int_0^1\int_0^1 \cos\left(\frac{\pi t_1}{12}\right)\cos\left(\frac{\pi t_2}{6}\right)dt_2\,dt_1$$

$$\frac{\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)6}{\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)\pi^2} = \frac{\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\pi^2/(144 s)+s}}{s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\pi^2/(576 s)+s}}{s^{3/2}}\,ds}{\pi^2\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\pi^2/(36 s)+s}}{s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\pi^2/(64 s)+s}}{s^{3/2}}\,ds} \quad \text{for } \gamma > 0$$

$$\frac{\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)6}{\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)\pi^2} = \frac{6\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{12^{-1+2s}\pi^{1-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{24^{-1+2s}\pi^{1-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds}{\pi^2\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{6^{-1+2s}\pi^{1-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{8^{-1+2s}\pi^{1-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds} \quad \text{for } 0 < \gamma < 1$$

Multiple-argument formulas:

$$\frac{\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)6}{\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)\pi^2} = \frac{6\,U_{-\frac{11}{12}}(\cos(\pi))\,U_{-\frac{5}{6}}(\cos(\pi))}{\pi^2\,U_{-\frac{3}{4}}(\cos(\pi))\,U_{-\frac{2}{3}}(\cos(\pi))}$$

$$\frac{\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)6}{\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)\pi^2} = \frac{6\cos\left(\frac{\pi}{24}\right)\cos\left(\frac{\pi}{12}\right)\sin\left(\frac{\pi}{24}\right)\sin\left(\frac{\pi}{12}\right)}{\pi^2\cos\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{\pi}{6}\right)}$$

$$\frac{\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)6}{\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)\pi^2} = \frac{6\left(3\sin\left(\frac{\pi}{36}\right)-4\sin^3\left(\frac{\pi}{36}\right)\right)\left(3\sin\left(\frac{\pi}{18}\right)-4\sin^3\left(\frac{\pi}{18}\right)\right)}{\pi^2\left(3\sin\left(\frac{\pi}{12}\right)-4\sin^3\left(\frac{\pi}{12}\right)\right)\left(3\sin\left(\frac{\pi}{9}\right)-4\sin^3\left(\frac{\pi}{9}\right)\right)}$$

and:

6/(Pi^2) [(sin(Pi/(2*5+3-1)) sin(2Pi/(2*5+3-1))) / (sin(3Pi/(2*5+3-1)) sin(4Pi/(2*5+3-1)))]x = (3*2*4) / (2*5+3-1)

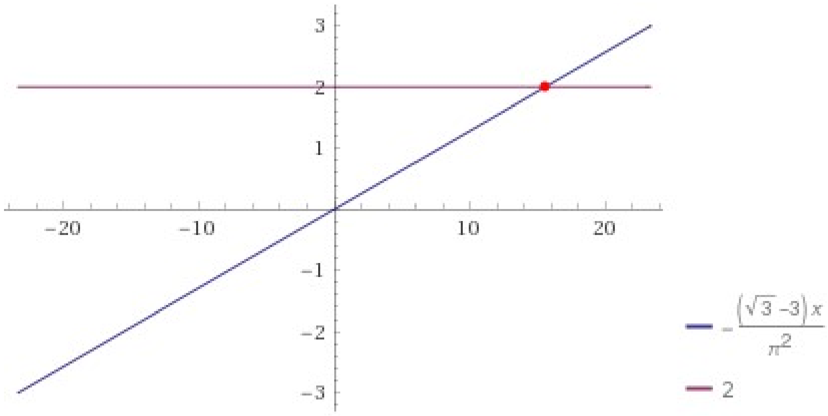
Input:

$$\frac{6}{\pi^2} \times \frac{\sin\left(\frac{\pi}{2\times5+3-1}\right)\sin\left(2\times\frac{\pi}{2\times5+3-1}\right)}{\sin\left(3\times\frac{\pi}{2\times5+3-1}\right)\sin\left(4\times\frac{\pi}{2\times5+3-1}\right)}\,x = \frac{3\times2\times4}{2\times5+3-1}$$

Exact result:

$$\frac{\sqrt{3}\,(\sqrt{3}-1)\,x}{\pi^2} = 2$$

Plot:



Alternate forms:

$$-\frac{(\sqrt{3}-3)\,x}{\pi^2} = 2$$

$$-\frac{\sqrt{3}\,x}{\pi^2} + \frac{3\,x}{\pi^2} - 2 = 0$$

Expanded form:

$$\frac{3\,x}{\pi^2} - \frac{\sqrt{3}\,x}{\pi^2} = 2$$

Solution:

$$x = -\frac{2\,\pi^2}{\sqrt{3}-3}$$

$$x \approx 15.568$$

L = 15.568

Thence, from

$$T = |V_8 - S_8|^2 \left(\frac{\sum \; q^{\frac{\alpha'}{4} p_{La} G^{ab} p_{Lb}} \; \bar{q}^{\frac{\alpha'}{4} p_{Ra} G^{ab} p_{Rb}}}{\eta^2(q)\eta^2(\bar{q})} \right)^3$$

and

$$\frac{1}{\mathrm{L}(1)} \sum_{a=1}^{N-1} \sum_{i=1}^{k-1} \mathrm{L} \left(\frac{\sin\left(\frac{a\pi}{2k+N-1}\right) \sin\left(\frac{(N-a)\pi}{2k+N-1}\right)}{\sin\left(\frac{(i+a)\pi}{2k+N-1}\right) \sin\left(\frac{(i+N-a)\pi}{2k+N-1}\right)} \right)$$

We put $\mathrm{L} = |V_8 - S_8|^2$ and

$$\left[\frac{1}{\mathrm{L}(1)} \sum_{a=1}^{N-1} \sum_{i=1}^{k-1} \left(\frac{\sin\left(\frac{a\pi}{2k+N-1}\right) \sin\left(\frac{(N-a)\pi}{2k+N-1}\right)}{\sin\left(\frac{(i+a)\pi}{2k+N-1}\right) \sin\left(\frac{(i+N-a)\pi}{2k+N-1}\right)} \right) \right]^3 =$$

$$= \left[\left(\frac{\sum \; q^{\frac{\alpha'}{4} p_{La} G^{ab} p_{Lb}} \; \bar{q}^{\frac{\alpha'}{4} p_{Ra} G^{ab} p_{Rb}}}{\eta^2(q)\eta^2(\bar{q})} \right)^3 \right]$$

we obtain:

(((2 *1/ (((((6/(Pi^2) [(sin(Pi/(2*5+3-1)) sin(2Pi/(2*5+3-1))) / (sin(3Pi/(2*5+3-1)) sin(4Pi/(2*5+3-1)))])))))))^2 * ((((sqrt(3) (-1 + sqrt(3)))/π^2)))^3

Input:

$$\left(2\times\frac{1}{\frac{6}{\pi^2}\times\frac{\sin\left(\frac{\pi}{2\times5+3-1}\right)\sin\left(2\times\frac{\pi}{2\times5+3-1}\right)}{\sin\left(3\times\frac{\pi}{2\times5+3-1}\right)\sin\left(4\times\frac{\pi}{2\times5+3-1}\right)}}\right)^2\left(\frac{\sqrt{3}\,(-1+\sqrt{3})}{\pi^2}\right)^3$$

Exact result:

$$\frac{4\sqrt{3}\,(\sqrt{3}-1)}{\pi^2}$$

Decimal approximation:

0.513880451901870628832939723704819207299078407416683191588...

0.5138804519… . result very near to the value of the following Ramanujan continued fraction:

$$2\int_0^\infty\frac{t^2\,dt}{e^{\sqrt{3}t}\sinh t}=\cfrac{1}{1+\cfrac{1^3}{1+\cfrac{1^3}{3+\cfrac{2^3}{1+\cfrac{2^3}{5+\cfrac{3^3}{1+\cfrac{3^3}{7+\ldots}}}}}}}\approx 0.5269391135$$

= 0.5269391135

Property:

$\frac{4\sqrt{3}\,(-1+\sqrt{3})}{\pi^2}$ is a transcendental number

Alternate forms:

$$-\frac{4(\sqrt{3}-3)}{\pi^2}$$

$$\frac{12-4\sqrt{3}}{\pi^2}$$

$$\frac{12}{\pi^2}-\frac{4\sqrt{3}}{\pi^2}$$

Alternative representations:

$$\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3 = \left(\frac{2}{\frac{6}{\frac{\csc\left(\frac{\pi}{12}\right)\csc\left(\frac{2\pi}{12}\right)\pi^2}{\csc\left(\frac{3\pi}{12}\right)\csc\left(\frac{4\pi}{12}\right)}}}\right)^2\left(\frac{\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)^3$$

$$\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3 = \left(\frac{2}{\frac{6\cos\left(\frac{\pi}{2}+\frac{\pi}{12}\right)\cos\left(\frac{\pi}{2}+\frac{2\pi}{12}\right)}{\left(\cos\left(\frac{\pi}{2}+\frac{3\pi}{12}\right)\cos\left(\frac{\pi}{2}+\frac{4\pi}{12}\right)\right)\pi^2}}\right)^2\left(\frac{\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)^3$$

$$\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3 = \left(\frac{2}{\frac{6\cos\left(\frac{\pi}{2}-\frac{2\pi}{12}\right)\cos\left(\frac{\pi}{2}-\frac{\pi}{12}\right)}{\left(\cos\left(\frac{\pi}{2}-\frac{4\pi}{12}\right)\cos\left(\frac{\pi}{2}-\frac{3\pi}{12}\right)\right)\pi^2}}\right)^2\left(\frac{\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)^3$$

Series representations:

$$\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3 = \left(\sqrt{2}^3\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{4}\right)\right)^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{3}\right)\right)^2\left(\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)^3\left(-1+\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)^3\right)\Big/\left(9\pi^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{12}\right)\right)^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{6}\right)\right)^2\right)$$

$$\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2 \left(\frac{\sqrt{3}\,(-1+\sqrt{3})}{\pi^2}\right)^3 = \left(\exp^3\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,4^{-2k}\,(-\pi)^{2k}}{(2k)!}\right)^2 \left(\sum_{k=0}^{\infty}\frac{(-1)^k\,6^{-2k}\,(-\pi)^{2k}}{(2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3 \left(-1+\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3\right) \Big/ \left(9\pi^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,3^{-2k}\,(-\pi)^{2k}}{(2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,5^{2k}\times 12^{-2k}\,(-\pi)^{2k}}{(2k)!}\right)^2\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2 \left(\frac{\sqrt{3}\,(-1+\sqrt{3})}{\pi^2}\right)^3 = \left(\exp^3\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,3^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2 \left(\sum_{k=0}^{\infty}\frac{(-1)^k\,4^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3 \left(-1+\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3\right) \Big/ \left(9\pi^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,6^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,12^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

Multiple-argument formulas:

$$\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2 \left(\frac{\sqrt{3}\,(-1+\sqrt{3})}{\pi^2}\right)^3 = \frac{U_{-\frac{3}{4}}(\cos(\pi))^2\,U_{-\frac{2}{3}}(\cos(\pi))^2\,(-1+\sqrt{3})^3\,\sqrt{3}^3}{9\pi^2\,U_{-\frac{11}{12}}(\cos(\pi))^2\,U_{-\frac{5}{6}}(\cos(\pi))^2}$$

$$\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times5+3-1}\right)\sin\left(\frac{2\pi}{2\times5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times5+3-1}\right)\sin\left(\frac{4\pi}{2\times5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3 =$$

$$\frac{\cos^2\left(\frac{\pi}{8}\right)\cos^2\left(\frac{\pi}{6}\right)\sin^2\left(\frac{\pi}{8}\right)\sin^2\left(\frac{\pi}{6}\right)\left(-1+\sqrt{3}\right)^3\sqrt{3}^3}{9\pi^2\cos^2\left(\frac{\pi}{24}\right)\cos^2\left(\frac{\pi}{12}\right)\sin^2\left(\frac{\pi}{24}\right)\sin^2\left(\frac{\pi}{12}\right)}$$

$$\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times5+3-1}\right)\sin\left(\frac{2\pi}{2\times5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times5+3-1}\right)\sin\left(\frac{4\pi}{2\times5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3 =$$

$$\frac{\left(3\sin\left(\frac{\pi}{12}\right)-4\sin^3\left(\frac{\pi}{12}\right)\right)^2\left(3\sin\left(\frac{\pi}{9}\right)-4\sin^3\left(\frac{\pi}{9}\right)\right)^2\left(-1+\sqrt{3}\right)^3\sqrt{3}^3}{9\pi^2\left(3\sin\left(\frac{\pi}{36}\right)-4\sin^3\left(\frac{\pi}{36}\right)\right)^2\left(3\sin\left(\frac{\pi}{18}\right)-4\sin^3\left(\frac{\pi}{18}\right)\right)^2}$$

From which:

Pi*(((2 *1/ (((((6/(Pi^2) [(sin(Pi/(2*5+3-1)) sin(2Pi/(2*5+3-1))) / (sin(3Pi/(2*5+3-1)) sin(4Pi/(2*5+3-1)))]))))))^2 * ((((sqrt(3) (-1 + sqrt(3)))/π^2)))^3

Input:

$$\pi\left(2\times\frac{1}{\frac{6}{\pi^2}\times\frac{\sin\left(\frac{\pi}{2\times5+3-1}\right)\sin\left(2\times\frac{\pi}{2\times5+3-1}\right)}{\sin\left(3\times\frac{\pi}{2\times5+3-1}\right)\sin\left(4\times\frac{\pi}{2\times5+3-1}\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3$$

Exact result:

$$\frac{4\sqrt{3}\left(\sqrt{3}-1\right)}{\pi}$$

Decimal approximation:

1.614403052518319860407904327688466923237485576304392158746...

1.614403052518…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Property:

$\frac{4\sqrt{3}\,(-1+\sqrt{3})}{\pi}$ is a transcendental number

Alternate forms:

$$-\frac{4(\sqrt{3}-3)}{\pi}$$

$$\frac{12-4\sqrt{3}}{\pi}$$

$$\frac{12}{\pi}-\frac{4\sqrt{3}}{\pi}$$

Alternative representations:

$$\pi\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\,(-1+\sqrt{3})}{\pi^2}\right)^3 = \pi\left(\frac{2}{\frac{6}{\frac{\csc\left(\frac{\pi}{12}\right)\csc\left(\frac{2\pi}{12}\right)\pi^2}{\csc\left(\frac{3\pi}{12}\right)\csc\left(\frac{4\pi}{12}\right)}}}\right)^2\left(\frac{(-1+\sqrt{3})\sqrt{3}}{\pi^2}\right)^3$$

$$\pi\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\,(-1+\sqrt{3})}{\pi^2}\right)^3 = \pi\left(\frac{2}{\frac{6\cos\left(\frac{\pi}{2}+\frac{\pi}{12}\right)\cos\left(\frac{\pi}{2}+\frac{2\pi}{12}\right)}{\left(\cos\left(\frac{\pi}{2}+\frac{3\pi}{12}\right)\cos\left(\frac{\pi}{2}+\frac{4\pi}{12}\right)\right)\pi^2}}\right)^2\left(\frac{(-1+\sqrt{3})\sqrt{3}}{\pi^2}\right)^3$$

$$\pi\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times5+3-1}\right)\sin\left(\frac{2\pi}{2\times5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times5+3-1}\right)\sin\left(\frac{4\pi}{2\times5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3 = \pi\left(\frac{2}{\frac{6\cos\left(\frac{\pi}{2}-\frac{2\pi}{12}\right)\cos\left(\frac{\pi}{2}-\frac{\pi}{12}\right)}{\left(\cos\left(\frac{\pi}{2}-\frac{4\pi}{12}\right)\cos\left(\frac{\pi}{2}-\frac{3\pi}{12}\right)\right)\pi^2}}\right)^2\left(\frac{\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)^3$$

Series representations:

$$\pi\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times5+3-1}\right)\sin\left(\frac{2\pi}{2\times5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times5+3-1}\right)\sin\left(\frac{4\pi}{2\times5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3 = \left(\sqrt{2}^3\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{4}\right)\right)^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{3}\right)\right)^2\left(\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)^3\left(-1+\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)^3\right)\Bigg/\left(9\pi\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{12}\right)\right)^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{6}\right)\right)^2\right)$$

$$\pi\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times5+3-1}\right)\sin\left(\frac{2\pi}{2\times5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times5+3-1}\right)\sin\left(\frac{4\pi}{2\times5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3 = \left(\exp^3\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k 4^{-2k}(-\pi)^{2k}}{(2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k 6^{-2k}(-\pi)^{2k}}{(2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3\left(-1+\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3\right)\Bigg/\left(9\pi\left(\sum_{k=0}^{\infty}\frac{(-1)^k 3^{-2k}(-\pi)^{2k}}{(2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k 5^{2k}\times12^{-2k}(-\pi)^{2k}}{(2k)!}\right)^2\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\pi \left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times5+3-1}\right)\sin\left(\frac{2\pi}{2\times5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times5+3-1}\right)\sin\left(\frac{4\pi}{2\times5+3-1}\right)\right)}} \right)^2 \left(\frac{\sqrt{3}\,(-1+\sqrt{3})}{\pi^2} \right)^3 = \left(\exp^3\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor \right) \sqrt{x}^3 \left(\sum_{k=0}^{\infty} \frac{(-1)^k\, 3^{-1-2k}\, \pi^{1+2k}}{(1+2k)!} \right)^2 \left(\sum_{k=0}^{\infty} \frac{(-1)^k\, 4^{-1-2k}\, \pi^{1+2k}}{(1+2k)!} \right)^2 \left(\sum_{k=0}^{\infty} \frac{(-1)^k\, (3-x)^k\, x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)^3 \left(-1 + \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k\, (3-x)^k\, x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)^3 \right) \Bigg/ \left(9\pi \left(\sum_{k=0}^{\infty} \frac{(-1)^k\, 6^{-1-2k}\, \pi^{1+2k}}{(1+2k)!} \right)^2 \left(\sum_{k=0}^{\infty} \frac{(-1)^k\, 12^{-1-2k}\, \pi^{1+2k}}{(1+2k)!} \right)^2 \right) \quad \text{for } (x \in \mathbb{R} \text{ and } x < 0)$$

Multiple-argument formulas:

$$\pi \left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times5+3-1}\right)\sin\left(\frac{2\pi}{2\times5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times5+3-1}\right)\sin\left(\frac{4\pi}{2\times5+3-1}\right)\right)}} \right)^2 \left(\frac{\sqrt{3}\,(-1+\sqrt{3})}{\pi^2} \right)^3 = \frac{U_{-\frac{3}{4}}(\cos(\pi))^2\, U_{-\frac{2}{3}}(\cos(\pi))^2\, (-1+\sqrt{3})^3\, \sqrt{3}^3}{9\pi\, U_{-\frac{11}{12}}(\cos(\pi))^2\, U_{-\frac{5}{6}}(\cos(\pi))^2}$$

$$\pi \left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times5+3-1}\right)\sin\left(\frac{2\pi}{2\times5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times5+3-1}\right)\sin\left(\frac{4\pi}{2\times5+3-1}\right)\right)}} \right)^2 \left(\frac{\sqrt{3}\,(-1+\sqrt{3})}{\pi^2} \right)^3 = \frac{\cos^2\left(\frac{\pi}{8}\right)\cos^2\left(\frac{\pi}{6}\right)\sin^2\left(\frac{\pi}{8}\right)\sin^2\left(\frac{\pi}{6}\right)(-1+\sqrt{3})^3\,\sqrt{3}^3}{9\pi\cos^2\left(\frac{\pi}{24}\right)\cos^2\left(\frac{\pi}{12}\right)\sin^2\left(\frac{\pi}{24}\right)\sin^2\left(\frac{\pi}{12}\right)}$$

$$\pi \left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times5+3-1}\right)\sin\left(\frac{2\pi}{2\times5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times5+3-1}\right)\sin\left(\frac{4\pi}{2\times5+3-1}\right)\right)}} \right)^2 \left(\frac{\sqrt{3}\,(-1+\sqrt{3})}{\pi^2} \right)^3 = \frac{\left(3\sin\left(\frac{\pi}{12}\right)-4\sin^3\left(\frac{\pi}{12}\right)\right)^2\left(3\sin\left(\frac{\pi}{9}\right)-4\sin^3\left(\frac{\pi}{9}\right)\right)^2(-1+\sqrt{3})^3\,\sqrt{3}^3}{9\pi\left(3\sin\left(\frac{\pi}{36}\right)-4\sin^3\left(\frac{\pi}{36}\right)\right)^2\left(3\sin\left(\frac{\pi}{18}\right)-4\sin^3\left(\frac{\pi}{18}\right)\right)^2}$$

From

$$T = |V_8 - S_8|^2 \left(\frac{\sum q^{\frac{\alpha'}{4} p_{La} G^{ab} p_{Lb}} \; \bar{q}^{\frac{\alpha'}{4} p_{Ra} G^{ab} p_{Rb}}}{\eta^2(q)\eta^2(\bar{q})} \right)^3$$

and

(((2 *1/ ((((6/(Pi^2) [(sin(Pi/(2*5+3-1)) sin(2Pi/(2*5+3-1))) / (sin(3Pi/(2*5+3-1)) sin(4Pi/(2*5+3-1)))]))))))^2 * ((((sqrt(3) (-1 + sqrt(3)))/π^2)))^3

$$\left(2 \times \frac{1}{\frac{6}{\pi^2} \times \frac{\sin\left(\frac{\pi}{2\times5+3-1}\right)\sin\left(2\times\frac{\pi}{2\times5+3-1}\right)}{\sin\left(3\times\frac{\pi}{2\times5+3-1}\right)\sin\left(4\times\frac{\pi}{2\times5+3-1}\right)}}\right)^2 \left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3$$

$$\frac{4\sqrt{3}\left(\sqrt{3}-1\right)}{\pi^2}$$

0.513880451901870628832939723704819207299078407416683191588...

0.5138804519…

we obtain:

(377+21)/ (((((((2 *1/ ((((6/(Pi^2) [(sin(Pi/(2*5+3-1)) sin(2Pi/(2*5+3-1))) / (sin(3Pi/(2*5+3-1)) sin(4Pi/(2*5+3-1)))]))))))^2 * ((((sqrt(3) (-1 + sqrt(3)))/π^2)))^3))))

Input:

$$\frac{377+21}{\left(2 \times \frac{1}{\frac{6}{\pi^2} \times \frac{\sin\left(\frac{\pi}{2\times5+3-1}\right)\sin\left(2\times\frac{\pi}{2\times5+3-1}\right)}{\sin\left(3\times\frac{\pi}{2\times5+3-1}\right)\sin\left(4\times\frac{\pi}{2\times5+3-1}\right)}}\right)^2 \left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3}$$

Exact result:

$$\frac{199\,\pi^2}{2\sqrt{3}\left(\sqrt{3}-1\right)}$$

Decimal approximation:

774.4992021529573957731702270299423614701416528567726777110...

774.4992021529… result very near to the rest mass of Charged rho meson 775.11

Property:

$\frac{199\pi^2}{2\sqrt{3}\left(-1+\sqrt{3}\right)}$ is a transcendental number

Alternate forms:

$$\frac{199}{12}\left(3+\sqrt{3}\right)\pi^2$$

$$\frac{1}{12}\left(597+199\sqrt{3}\right)\pi^2$$

$$\frac{199\pi^2}{6-2\sqrt{3}}$$

Alternative representations:

$$\frac{377+21}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3}=\frac{398}{\left(\frac{\frac{2}{6}}{\frac{\csc\left(\frac{\pi}{12}\right)\csc\left(\frac{2\pi}{12}\right)\pi^2}{\csc\left(\frac{3\pi}{12}\right)\csc\left(\frac{4\pi}{12}\right)}}\right)^2\left(\frac{\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)^3}$$

$$\frac{377+21}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3}=\frac{398}{\left(\frac{2}{\frac{6\cos\left(\frac{\pi}{2}+\frac{\pi}{12}\right)\cos\left(\frac{\pi}{2}+\frac{2\pi}{12}\right)}{\left(\cos\left(\frac{\pi}{2}+\frac{3\pi}{12}\right)\cos\left(\frac{\pi}{2}+\frac{4\pi}{12}\right)\right)\pi^2}}\right)^2\left(\frac{\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)^3}$$

$$\frac{377+21}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3}=\frac{398}{\left(\frac{2}{\frac{6\cos\left(\frac{\pi}{2}-\frac{2\pi}{12}\right)\cos\left(\frac{\pi}{2}-\frac{\pi}{12}\right)}{\left(\cos\left(\frac{\pi}{2}-\frac{4\pi}{12}\right)\cos\left(\frac{\pi}{2}-\frac{3\pi}{12}\right)\right)\pi^2}}\right)^2\left(\frac{\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)^3}$$

Series representations:

$$\frac{377+21}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times5+3-1}\right)\sin\left(\frac{2\pi}{2\times5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times5+3-1}\right)\sin\left(\frac{4\pi}{2\times5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} = \left(3582\,\pi^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{12}\right)\right)^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{6}\right)\right)^2\right) \Big/ \left(\sqrt{2}^3\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{4}\right)\right)^2 \left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{3}\right)\right)^2\left(\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)^3\left(-1+\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)^3\right)$$

$$\frac{377+21}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times5+3-1}\right)\sin\left(\frac{2\pi}{2\times5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times5+3-1}\right)\sin\left(\frac{4\pi}{2\times5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} = \left(3582\,\pi^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k 3^{-2k}(-\pi)^{2k}}{(2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k 5^{2k}\times 12^{-2k}(-\pi)^{2k}}{(2k)!}\right)^2\right) \Big/ \left(\exp^3\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k 4^{-2k}(-\pi)^{2k}}{(2k)!}\right)^2 \left(\sum_{k=0}^{\infty}\frac{(-1)^k 6^{-2k}(-\pi)^{2k}}{(2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3 \left(-1+\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{377+21}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} =$$

$$\left(3582\,\pi^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,6^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,12^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\right)\Bigg/$$
$$\left(\exp^3\left(i\,\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,3^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\right.$$
$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,4^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3$$
$$\left.\left(-1+\exp\left(i\,\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3\right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

Multiple-argument formulas:

$$\frac{377+21}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} =$$
$$\frac{3582\,\pi^2\,U_{-\frac{11}{12}}(\cos(\pi))^2\,U_{-\frac{5}{6}}(\cos(\pi))^2}{U_{-\frac{3}{4}}(\cos(\pi))^2\,U_{-\frac{2}{3}}(\cos(\pi))^2\left(-1+\sqrt{3}\right)^3\sqrt{3}^3}$$

$$\frac{377+21}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} =$$
$$\frac{3582\,\pi^2\cos^2\left(\frac{\pi}{24}\right)\cos^2\left(\frac{\pi}{12}\right)\sin^2\left(\frac{\pi}{24}\right)\sin^2\left(\frac{\pi}{12}\right)}{\cos^2\left(\frac{\pi}{8}\right)\cos^2\left(\frac{\pi}{6}\right)\sin^2\left(\frac{\pi}{8}\right)\sin^2\left(\frac{\pi}{6}\right)\left(-1+\sqrt{3}\right)^3\sqrt{3}^3}$$

$$\frac{377+21}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} =$$

$$\frac{3582\,\pi^2\left(3\sin\left(\frac{\pi}{36}\right)-4\sin^3\left(\frac{\pi}{36}\right)\right)^2\left(3\sin\left(\frac{\pi}{18}\right)-4\sin^3\left(\frac{\pi}{18}\right)\right)^2}{\left(3\sin\left(\frac{\pi}{12}\right)-4\sin^3\left(\frac{\pi}{12}\right)\right)^2\left(3\sin\left(\frac{\pi}{9}\right)-4\sin^3\left(\frac{\pi}{9}\right)\right)^2\left(-1+\sqrt{3}\right)^3\sqrt{3}^3}$$

and:

(233+21)/ (((((((2 *1/ ((((6/(Pi^2) [(sin(Pi/(2*5+3-1)) sin(2Pi/(2*5+3-1))) / (sin(3Pi/(2*5+3-1)) sin(4Pi/(2*5+3-1)))]))))))))^2 * ((((sqrt(3) (-1 + sqrt(3)))/π^2)))^3))))

Input:

$$\frac{233+21}{\left(2\times\frac{1}{\frac{6}{\pi^2}\times\frac{\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(2\times\frac{\pi}{2\times 5+3-1}\right)}{\sin\left(3\times\frac{\pi}{2\times 5+3-1}\right)\sin\left(4\times\frac{\pi}{2\times 5+3-1}\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3}$$

Exact result:

$$\frac{127\,\pi^2}{2\sqrt{3}\left(\sqrt{3}-1\right)}$$

Decimal approximation:

494.2783852935959259456915519236315573201406528281916083884...

494.27838529… result very near to the rest mass of Kaon meson 493.677

Property:

$$\frac{127\,\pi^2}{2\sqrt{3}\left(-1+\sqrt{3}\right)}$$ is a transcendental number

Alternate forms:

$$\frac{127}{12}\left(3+\sqrt{3}\right)\pi^2$$

$$\frac{1}{12}\left(381+127\sqrt{3}\right)\pi^2$$

$$\frac{127\pi^2}{6-2\sqrt{3}}$$

Alternative representations:

$$\frac{233+21}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3}=\frac{254}{\left(\frac{2}{\frac{6}{\frac{\csc\left(\frac{\pi}{12}\right)\csc\left(\frac{2\pi}{12}\right)\pi^2}{\csc\left(\frac{3\pi}{12}\right)\csc\left(\frac{4\pi}{12}\right)}}}\right)^2\left(\frac{\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)^3}$$

$$\frac{233+21}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3}=\frac{254}{\left(\frac{2}{\frac{6\cos\left(\frac{\pi}{2}+\frac{\pi}{12}\right)\cos\left(\frac{\pi}{2}+\frac{2\pi}{12}\right)}{\left(\cos\left(\frac{\pi}{2}+\frac{3\pi}{12}\right)\cos\left(\frac{\pi}{2}+\frac{4\pi}{12}\right)\right)\pi^2}}\right)^2\left(\frac{\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)^3}$$

$$\frac{233+21}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3}=\frac{254}{\left(\frac{2}{\frac{6\cos\left(\frac{\pi}{2}-\frac{2\pi}{12}\right)\cos\left(\frac{\pi}{2}-\frac{\pi}{12}\right)}{\left(\cos\left(\frac{\pi}{2}-\frac{4\pi}{12}\right)\cos\left(\frac{\pi}{2}-\frac{3\pi}{12}\right)\right)\pi^2}}\right)^2\left(\frac{\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)^3}$$

Series representations:

$$\frac{233+21}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3}=$$

$$\left(2286\pi^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{12}\right)\right)^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{6}\right)\right)^2\right)\Big/\left(\sqrt{2}^3\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{4}\right)\right)^2\right.$$
$$\left.\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{3}\right)\right)^2\left(\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)^3\left(-1+\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)^3\right)$$

$$\frac{233+21}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} =$$

$$\left(2286\,\pi^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,3^{-2k}\,(-\pi)^{2k}}{(2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,5^{2k}\times 12^{-2k}\,(-\pi)^{2k}}{(2k)!}\right)^2\right)\Big/$$
$$\left(\exp^3\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,4^{-2k}\,(-\pi)^{2k}}{(2k)!}\right)^2\right.$$
$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,6^{-2k}\,(-\pi)^{2k}}{(2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3$$
$$\left.\left(-1+\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3\right)\quad \text{for } (x\in\mathbb{R}\text{ and }x<0)$$

$$\frac{233+21}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} =$$

$$\left(2286\,\pi^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,6^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,12^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\right)\Big/$$
$$\left(\exp^3\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,3^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\right.$$
$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,4^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3$$
$$\left.\left(-1+\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3\right)\quad \text{for } (x\in\mathbb{R}\text{ and }x<0)$$

Multiple-argument formulas:

$$\frac{233+21}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} = \frac{2286\,\pi^2\,U_{-\frac{11}{12}}(\cos(\pi))^2\,U_{-\frac{5}{6}}(\cos(\pi))^2}{U_{-\frac{3}{4}}(\cos(\pi))^2\,U_{-\frac{2}{3}}(\cos(\pi))^2\left(-1+\sqrt{3}\right)^3\sqrt{3}^3}$$

$$\frac{233+21}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} = \frac{2286\,\pi^2\cos^2\left(\frac{\pi}{24}\right)\cos^2\left(\frac{\pi}{12}\right)\sin^2\left(\frac{\pi}{24}\right)\sin^2\left(\frac{\pi}{12}\right)}{\cos^2\left(\frac{\pi}{8}\right)\cos^2\left(\frac{\pi}{6}\right)\sin^2\left(\frac{\pi}{8}\right)\sin^2\left(\frac{\pi}{6}\right)\left(-1+\sqrt{3}\right)^3\sqrt{3}^3}$$

$$\frac{233+21}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} = \frac{2286\,\pi^2\left(3\sin\left(\frac{\pi}{36}\right)-4\sin^3\left(\frac{\pi}{36}\right)\right)^2\left(3\sin\left(\frac{\pi}{18}\right)-4\sin^3\left(\frac{\pi}{18}\right)\right)^2}{\left(3\sin\left(\frac{\pi}{12}\right)-4\sin^3\left(\frac{\pi}{12}\right)\right)^2\left(3\sin\left(\frac{\pi}{9}\right)-4\sin^3\left(\frac{\pi}{9}\right)\right)^2\left(-1+\sqrt{3}\right)^3\sqrt{3}^3}$$

(76-11)/ (((((((2 *1/ ((((6/(Pi^2) [(sin(Pi/(2*5+3-1)) sin(2Pi/(2*5+3-1))) / (sin(3Pi/(2*5+3-1)) sin(4Pi/(2*5+3-1)))]))))))))^2 * ((((sqrt(3) (-1 + sqrt(3)))/π^2)))^3))))

Input:

$$\frac{76-11}{\left(2\times\frac{1}{\frac{6}{\pi^2}\times\frac{\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(2\times\frac{\pi}{2\times 5+3-1}\right)}{\sin\left(3\times\frac{\pi}{2\times 5+3-1}\right)\sin\left(4\times\frac{\pi}{2\times 5+3-1}\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3}$$

Exact result:

$$\frac{65\pi^2}{4\sqrt{3}\,(\sqrt{3}-1)}$$

Decimal approximation:

126.4885631656839967971257908465986268732643402906789549025...

126.4885631… result in the range of the Higgs boson mass 125.18 GeV

Property:

$\frac{65\pi^2}{4\sqrt{3}\,(-1+\sqrt{3})}$ is a transcendental number

Alternate forms:

$$\frac{65}{24}\left(3+\sqrt{3}\right)\pi^2$$

$$\frac{1}{24}\left(195+65\sqrt{3}\right)\pi^2$$

$$\frac{65\pi^2}{12-4\sqrt{3}}$$

Alternative representations:

$$\frac{76-11}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3}=\frac{65}{\left(\frac{2}{\frac{6}{\frac{\csc\left(\frac{\pi}{12}\right)\csc\left(\frac{2\pi}{12}\right)\pi^2}{\csc\left(\frac{3\pi}{12}\right)\csc\left(\frac{4\pi}{12}\right)}}}\right)^2\left(\frac{\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)^3}$$

$$\frac{76-11}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3}=\frac{65}{\left(\frac{2}{\frac{6\cos\left(\frac{\pi}{2}+\frac{\pi}{12}\right)\cos\left(\frac{\pi}{2}+\frac{2\pi}{12}\right)}{\left(\cos\left(\frac{\pi}{2}+\frac{3\pi}{12}\right)\cos\left(\frac{\pi}{2}+\frac{4\pi}{12}\right)\right)\pi^2}}\right)^2\left(\frac{\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)^3}$$

$$\frac{76-11}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} = \frac{65}{\left(\frac{2}{\frac{6\cos\left(\frac{\pi}{2}-\frac{2\pi}{12}\right)\cos\left(\frac{\pi}{2}-\frac{\pi}{12}\right)}{\left(\cos\left(\frac{\pi}{2}-\frac{4\pi}{12}\right)\cos\left(\frac{\pi}{2}-\frac{3\pi}{12}\right)\right)\pi^2}}\right)^2\left(\frac{\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)^3}$$

Series representations:

$$\frac{76-11}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} =$$

$$\left(585\,\pi^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{12}\right)\right)^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{6}\right)\right)^2\right)\Big/\left(\sqrt{2}^3\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{4}\right)\right)^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{3}\right)\right)^2\left(\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)^3\left(-1+\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)^3\right)$$

$$\frac{76-11}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} =$$

$$\left(585\,\pi^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,3^{-2k}\,(-\pi)^{2k}}{(2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,5^{2k}\times 12^{-2k}\,(-\pi)^{2k}}{(2k)!}\right)^2\right)\Big/\left(\exp^3\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,4^{-2k}\,(-\pi)^{2k}}{(2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,6^{-2k}\,(-\pi)^{2k}}{(2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3\left(-1+\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{76-11}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times5+3-1}\right)\sin\left(\frac{2\pi}{2\times5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times5+3-1}\right)\sin\left(\frac{4\pi}{2\times5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} =$$

$$\left(585\,\pi^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,6^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,12^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\right)\Bigg/$$
$$\left(\exp^3\left(i\,\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,3^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\right.$$
$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,4^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3$$
$$\left.\left(-1+\exp\left(i\,\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3\right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

Multiple-argument formulas:

$$\frac{76-11}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times5+3-1}\right)\sin\left(\frac{2\pi}{2\times5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times5+3-1}\right)\sin\left(\frac{4\pi}{2\times5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} =$$
$$\frac{585\,\pi^2\,U_{-\frac{11}{12}}(\cos(\pi))^2\,U_{-\frac{5}{6}}(\cos(\pi))^2}{U_{-\frac{3}{4}}(\cos(\pi))^2\,U_{-\frac{2}{3}}(\cos(\pi))^2\left(-1+\sqrt{3}\right)^3\sqrt{3}^3}$$

$$\frac{76-11}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times5+3-1}\right)\sin\left(\frac{2\pi}{2\times5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times5+3-1}\right)\sin\left(\frac{4\pi}{2\times5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} =$$
$$\frac{585\,\pi^2\cos^2\left(\frac{\pi}{24}\right)\cos^2\left(\frac{\pi}{12}\right)\sin^2\left(\frac{\pi}{24}\right)\sin^2\left(\frac{\pi}{12}\right)}{\cos^2\left(\frac{\pi}{8}\right)\cos^2\left(\frac{\pi}{6}\right)\sin^2\left(\frac{\pi}{8}\right)\sin^2\left(\frac{\pi}{6}\right)\left(-1+\sqrt{3}\right)^3\sqrt{3}^3}$$

$$\frac{76-11}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} = \frac{585\,\pi^2\left(3\sin\left(\frac{\pi}{36}\right)-4\sin^3\left(\frac{\pi}{36}\right)\right)^2\left(3\sin\left(\frac{\pi}{18}\right)-4\sin^3\left(\frac{\pi}{18}\right)\right)^2}{\left(3\sin\left(\frac{\pi}{12}\right)-4\sin^3\left(\frac{\pi}{12}\right)\right)^2\left(3\sin\left(\frac{\pi}{9}\right)-4\sin^3\left(\frac{\pi}{9}\right)\right)^2\left(-1+\sqrt{3}\right)^3\sqrt{3}^3}$$

From which:

27*1/2((([(76-11)/ (((((((2 *1/ ((((6/(Pi^2) [(sin(Pi/(2*5+3-1)) sin(2Pi/(2*5+3-1))) / (sin(3Pi/(2*5+3-1)) sin(4Pi/(2*5+3-1)))])))))))^2 * ((((sqrt(3) (-1 + sqrt(3)))/π^2)))^3))))]+golden ratio)))

Input:

$$27\times\frac{1}{2}\left(\frac{76-11}{\left(2\times\frac{1}{\frac{6}{\pi^2}\times\frac{\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(2\times\frac{\pi}{2\times 5+3-1}\right)}{\sin\left(3\times\frac{\pi}{2\times 5+3-1}\right)\sin\left(4\times\frac{\pi}{2\times 5+3-1}\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3}+\phi\right)$$

ϕ is the golden ratio

Exact result:

$$\frac{27}{2}\left(\phi+\frac{65\,\pi^2}{4\sqrt{3}\left(\sqrt{3}-1\right)}\right)$$

Decimal approximation:

1729.439061584857537211960098693017577378292767851543689823...

1729.43906158…

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

With regard 27 (From Wikipedia):

*"The fundamental group of the complex form, compact real form, or any algebraic version of E_6 is the cyclic group **Z**/3**Z**, and its outer automorphism group is the cyclic group **Z**/2**Z**. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E_6 plays a role in some grand unified theories".*

Property:

$\frac{27}{2}\left(\phi + \frac{65\pi^2}{4\sqrt{3}\,(-1+\sqrt{3})}\right)$ is a transcendental number

Alternate forms:

$$\frac{27}{2}\left(\phi + \frac{65}{24}\left(3+\sqrt{3}\right)\pi^2\right)$$

$$\frac{27}{2}\left(\phi + \frac{65\pi^2}{12-4\sqrt{3}}\right)$$

$$\frac{27}{2}\left(\frac{1}{2}\left(1+\sqrt{5}\right) + \frac{1}{24}\left(195+65\sqrt{3}\right)\pi^2\right)$$

Alternative representations:

$$\frac{27}{2}\left(\frac{76-11}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times5+3-1}\right)\sin\left(\frac{2\pi}{2\times5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times5+3-1}\right)\sin\left(\frac{4\pi}{2\times5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3}+\phi\right)=$$

$$\frac{27}{2}\left(\phi+\frac{65}{\left(\frac{2}{\frac{6}{\frac{\csc\left(\frac{\pi}{12}\right)\csc\left(\frac{2\pi}{12}\right)\pi^2}{\csc\left(\frac{3\pi}{12}\right)\csc\left(\frac{4\pi}{12}\right)}}}\right)^2\left(\frac{\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)^3}\right)$$

$$\frac{27}{2}\left(\frac{76-11}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times5+3-1}\right)\sin\left(\frac{2\pi}{2\times5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times5+3-1}\right)\sin\left(\frac{4\pi}{2\times5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3}+\phi\right)=$$

$$\frac{27}{2}\left(\phi+\frac{65}{\left(\frac{2}{\frac{6\cos\left(\frac{\pi}{2}+\frac{\pi}{12}\right)\cos\left(\frac{\pi}{2}+\frac{2\pi}{12}\right)}{\left(\cos\left(\frac{\pi}{2}+\frac{3\pi}{12}\right)\cos\left(\frac{\pi}{2}+\frac{4\pi}{12}\right)\right)\pi^2}}\right)^2\left(\frac{\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)^3}\right)$$

$$\frac{27}{2}\left(\frac{76-11}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3}+\phi\right)=$$

$$\frac{27}{2}\left(\phi+\frac{65}{\left(\frac{2}{\frac{6\cos\left(\frac{\pi}{2}-\frac{2\pi}{12}\right)\cos\left(\frac{\pi}{2}-\frac{\pi}{12}\right)}{\left(\cos\left(\frac{\pi}{2}-\frac{4\pi}{12}\right)\cos\left(\frac{\pi}{2}-\frac{3\pi}{12}\right)\right)\pi^2}}\right)^2\left(\frac{\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)^3}\right)$$

Series representations:

$$\frac{27}{2}\left(\frac{76-11}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times5+3-1}\right)\sin\left(\frac{2\pi}{2\times5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times5+3-1}\right)\sin\left(\frac{4\pi}{2\times5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3}+\phi\right)=$$

$$\left(27\left(585\pi^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{12}\right)\right)^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{6}\right)\right)^2-\right.\right.$$
$$\phi\sqrt{2}^3\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{4}\right)\right)^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{3}\right)\right)^2\left(\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)^3+$$
$$3\phi\sqrt{2}^4\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{4}\right)\right)^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{3}\right)\right)^2\left(\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)^4-$$
$$3\phi\sqrt{2}^5\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{4}\right)\right)^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{3}\right)\right)^2\left(\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)^5+$$
$$\left.\left.\phi\sqrt{2}^6\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{4}\right)\right)^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{3}\right)\right)^2\left(\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)^6\right)\right)\Big/$$
$$\left(2\sqrt{2}^3\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{4}\right)\right)^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{3}\right)\right)^2\left(\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)^3\right.$$
$$\left.\left(-1+\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)^3\right)$$

$$\frac{27}{2}\left(\frac{76-11}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3}+\phi\right)=$$

$$\begin{aligned}
&\Bigg(27\Bigg(585\,\pi^2\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{25}{144}\right)^k(-\pi)^{2k}}{(2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{9}\right)^k(-\pi)^{2k}}{(2k)!}\right)^2-\\
&\qquad\phi\exp^3\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}^3\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{16}\right)^k(-\pi)^{2k}}{(2k)!}\right)^2\\
&\qquad\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{36}\right)^k(-\pi)^{2k}}{(2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3+\\
&\qquad 3\,\phi\exp^4\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}^4\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{16}\right)^k(-\pi)^{2k}}{(2k)!}\right)^2\\
&\qquad\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{36}\right)^k(-\pi)^{2k}}{(2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4-\\
&\qquad 3\,\phi\exp^5\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}^5\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{16}\right)^k(-\pi)^{2k}}{(2k)!}\right)^2\\
&\qquad\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{36}\right)^k(-\pi)^{2k}}{(2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^5+\\
&\qquad\phi\exp^6\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}^6\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{16}\right)^k(-\pi)^{2k}}{(2k)!}\right)^2\\
&\qquad\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{36}\right)^k(-\pi)^{2k}}{(2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^6\Bigg)\Bigg)\Big/\\
&\Bigg(2\exp^3\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}^3\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{16}\right)^k(-\pi)^{2k}}{(2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{36}\right)^k(-\pi)^{2k}}{(2k)!}\right)^2\\
&\quad\left(\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3\\
&\quad\left(-1+\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3\Bigg)\quad\text{for }(x\in\mathbb{R}\text{ and }x<0)
\end{aligned}$$

$$\frac{27}{2}\left(\frac{76-11}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3}+\phi\right)=$$

$$\left(27\left(585\,\pi^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,6^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,12^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2-\right.\right.$$

$$\phi\exp^3\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,3^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2$$

$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,4^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3+$$

$$3\,\phi\exp^4\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}^4\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,3^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2$$

$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,4^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4-$$

$$3\,\phi\exp^5\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}^5\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,3^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2$$

$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,4^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^5+$$

$$\phi\exp^6\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}^6\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,3^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2$$

$$\left.\left.\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,4^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^6\right)\right)\Big/$$

$$\left(2\exp^3\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,3^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\right.$$

$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,4^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3$$

$$\left.\left(-1+\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3\right)\quad\text{for }(x\in\mathbb{R}\text{ and }x<0)$$

Multiple-argument formulas:

$$\frac{27}{2}\left(\frac{76-11}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3}+\phi\right)=$$

$$\frac{27}{2}\left(\phi+\frac{585\,\pi^2\,U_{-\frac{11}{12}}(\cos(\pi))^2\,U_{-\frac{5}{6}}(\cos(\pi))^2}{U_{-\frac{3}{4}}(\cos(\pi))^2\,U_{-\frac{2}{3}}(\cos(\pi))^2\left(-1+\sqrt{3}\right)^3\sqrt{3}^3}\right)$$

$$\frac{27}{2}\left(\frac{76-11}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3}+\phi\right)=$$

$$\frac{27}{2}\left(\phi+\frac{585\,\pi^2\cos^2\left(\frac{\pi}{24}\right)\cos^2\left(\frac{\pi}{12}\right)\sin^2\left(\frac{\pi}{24}\right)\sin^2\left(\frac{\pi}{12}\right)}{\cos^2\left(\frac{\pi}{8}\right)\cos^2\left(\frac{\pi}{6}\right)\sin^2\left(\frac{\pi}{8}\right)\sin^2\left(\frac{\pi}{6}\right)\left(-1+\sqrt{3}\right)^3\sqrt{3}^3}\right)$$

$$\frac{27}{2}\left(\frac{76-11}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3}+\phi\right)=$$

$$\frac{27}{2}\left(\phi+\frac{585\,\pi^2\left(3\sin\left(\frac{\pi}{36}\right)-4\sin^3\left(\frac{\pi}{36}\right)\right)^2\left(3\sin\left(\frac{\pi}{18}\right)-4\sin^3\left(\frac{\pi}{18}\right)\right)^2}{\left(3\sin\left(\frac{\pi}{12}\right)-4\sin^3\left(\frac{\pi}{12}\right)\right)^2\left(3\sin\left(\frac{\pi}{9}\right)-4\sin^3\left(\frac{\pi}{9}\right)\right)^2\left(-1+\sqrt{3}\right)^3\sqrt{3}^3}\right)$$

and:

(76-4)/ (((((((2 *1/ ((((6/(Pi^2) [(sin(Pi/(2*5+3-1)) sin(2Pi/(2*5+3-1))) / (sin(3Pi/(2*5+3-1)) sin(4Pi/(2*5+3-1)))])))))))^2 * ((((sqrt(3) (-1 + sqrt(3)))/π^2)))^3))))

Input:

$$\frac{76-4}{\left(2\times\frac{1}{\frac{6}{\pi^2}\times\frac{\sin\left(\frac{\pi}{2\times5+3-1}\right)\sin\left(2\times\frac{\pi}{2\times5+3-1}\right)}{\sin\left(3\times\frac{\pi}{2\times5+3-1}\right)\sin\left(4\times\frac{\pi}{2\times5+3-1}\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3}$$

Exact result:

$$\frac{6\sqrt{3}\,\pi^2}{\sqrt{3}-1}$$

Decimal approximation:

140.11040842968073491373933755315540207500050001429053466 12...

140.1104084… result very near to the rest mass of Pion meson 139.57 MeV

Property:

$$\frac{6\sqrt{3}\,\pi^2}{-1+\sqrt{3}}$$ is a transcendental number

Alternate forms:

$$3\left(3+\sqrt{3}\right)\pi^2$$

$$\left(9+3\sqrt{3}\right)\pi^2$$

Alternative representations:

$$\frac{76-4}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times5+3-1}\right)\sin\left(\frac{2\pi}{2\times5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times5+3-1}\right)\sin\left(\frac{4\pi}{2\times5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3}=\frac{72}{\left(\frac{2}{\frac{6}{\frac{\csc\left(\frac{\pi}{12}\right)\csc\left(\frac{2\pi}{12}\right)\pi^2}{\csc\left(\frac{3\pi}{12}\right)\csc\left(\frac{4\pi}{12}\right)}}}\right)^2\left(\frac{\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)^3}$$

$$\frac{76-4}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} = \frac{72}{\left(\frac{2}{\frac{6\cos\left(\frac{\pi}{2}+\frac{\pi}{12}\right)\cos\left(\frac{\pi}{2}+\frac{2\pi}{12}\right)}{\left(\cos\left(\frac{\pi}{2}+\frac{3\pi}{12}\right)\cos\left(\frac{\pi}{2}+\frac{4\pi}{12}\right)\right)\pi^2}}\right)^2\left(\frac{\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)^3}$$

$$\frac{76-4}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} = \frac{72}{\left(\frac{2}{\frac{6\cos\left(\frac{\pi}{2}-\frac{2\pi}{12}\right)\cos\left(\frac{\pi}{2}-\frac{\pi}{12}\right)}{\left(\cos\left(\frac{\pi}{2}-\frac{4\pi}{12}\right)\cos\left(\frac{\pi}{2}-\frac{3\pi}{12}\right)\right)\pi^2}}\right)^2\left(\frac{\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)^3}$$

Series representations:

$$\frac{76-4}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} =$$

$$\left(648\pi^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{12}\right)\right)^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{6}\right)\right)^2\right)\Big/\left(\sqrt{2}^3\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{4}\right)\right)^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{3}\right)\right)^2\left(\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)^3\left(-1+\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)^3\right)$$

$$\frac{76-4}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} =$$

$$\left(648\pi^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k 3^{-2k}(-\pi)^{2k}}{(2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k 5^{2k}\times 12^{-2k}(-\pi)^{2k}}{(2k)!}\right)^2\right)\Big/\left(\exp^3\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k 4^{-2k}(-\pi)^{2k}}{(2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k 6^{-2k}(-\pi)^{2k}}{(2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3\left(-1+\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{76-4}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} =$$

$$\left(648\,\pi^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,6^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,12^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\right)\Big/$$

$$\left(\exp^3\left(i\,\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,3^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\right.$$

$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,4^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3$$

$$\left.\left(-1+\exp\left(i\,\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3\right)\quad \text{for } (x\in\mathbb{R}\text{ and }x<0)$$

Multiple-argument formulas:

$$\frac{76-4}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} = \frac{648\,\pi^2\,U_{-\frac{11}{12}}(\cos(\pi))^2\,U_{-\frac{5}{6}}(\cos(\pi))^2}{U_{-\frac{3}{4}}(\cos(\pi))^2\,U_{-\frac{2}{3}}(\cos(\pi))^2\left(-1+\sqrt{3}\right)^3\sqrt{3}^3}$$

$$\frac{76-4}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} = \frac{648\,\pi^2\cos^2\left(\frac{\pi}{24}\right)\cos^2\left(\frac{\pi}{12}\right)\sin^2\left(\frac{\pi}{24}\right)\sin^2\left(\frac{\pi}{12}\right)}{\cos^2\left(\frac{\pi}{8}\right)\cos^2\left(\frac{\pi}{6}\right)\sin^2\left(\frac{\pi}{8}\right)\sin^2\left(\frac{\pi}{6}\right)\left(-1+\sqrt{3}\right)^3\sqrt{3}^3}$$

$$\frac{76-4}{\left(\frac{2}{\frac{6\left(\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(\frac{2\pi}{2\times 5+3-1}\right)\right)}{\pi^2\left(\sin\left(\frac{3\pi}{2\times 5+3-1}\right)\sin\left(\frac{4\pi}{2\times 5+3-1}\right)\right)}}\right)^2\left(\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\right)^3} =$$

$$\frac{648\,\pi^2\left(3\sin\left(\frac{\pi}{36}\right)-4\sin^3\left(\frac{\pi}{36}\right)\right)^2\left(3\sin\left(\frac{\pi}{18}\right)-4\sin^3\left(\frac{\pi}{18}\right)\right)^2}{\left(3\sin\left(\frac{\pi}{12}\right)-4\sin^3\left(\frac{\pi}{12}\right)\right)^2\left(3\sin\left(\frac{\pi}{9}\right)-4\sin^3\left(\frac{\pi}{9}\right)\right)^2\left(-1+\sqrt{3}\right)^3\sqrt{3}^3}$$

Now, we have that:

$$V_8 - S_8 = A_0\chi_0 + A_+\chi_- + A_-\chi_+ \quad . \tag{2.5}$$

A_0, A_+ and A_- may be expressed in terms of the four level-one $SO(2)$ characters and of the 12 characters ξ_m $(m = -5, \dots, 6)$ of the $N = 2$ superconformal model with $c = 1$ (equivalent to the rational torus at radius $R = \sqrt{12}$), of conformal weight $h_m = \frac{m^2}{24}$, as

$$\begin{aligned} A_0 &= V_2\xi_0 + O_2\xi_6 - S_2\xi_{-3} - C_2\xi_3 \\ A_+ &= V_2\xi_4 + O_2\xi_{-2} - S_2\xi_1 - C_2\xi_{-5} \\ A_- &= V_2\xi_{-4} + O_2\xi_2 - S_2\xi_5 - C_2\xi_{-1} \quad . \end{aligned} \tag{2.6}$$

The spectrum of the "parent" type IIB string on the Z orbifold can thus be extracted from the torus amplitude

$$\begin{aligned} T = {} & \frac{1}{3}\,\Xi_{0,0}(q)\,\Xi_{0,0}(\bar{q})\sum q^{\frac{\alpha'}{4}p_{La}G^{ab}p_{Lb}}\bar{q}^{\frac{\alpha'}{4}p_{Ra}G^{ab}p_{Rb}} + \frac{1}{3}\sum_{\epsilon=\pm 1}\Xi_{0,\epsilon}(q)\,\Xi_{0,\epsilon}(\bar{q}) \\ & + \frac{1}{3}\sum_{\eta=\pm 1}\sum_{\epsilon=0,\pm 1}\Xi_{\eta,\epsilon}(q)\,\Xi_{-\eta,-\epsilon}(\bar{q}) \quad , \end{aligned} \tag{2.7}$$

where

$$\begin{aligned} \Xi_{0,\epsilon}(q) &= \left(\frac{A_0\chi_0 + \omega^\epsilon A_+\chi_- + \bar{\omega}^\epsilon A_-\chi_+}{{H_{0,\epsilon}}^3}\right)(q) \\ \Xi_{+,\epsilon}(q) &= \left(\frac{A_0\chi_+ + \omega^\epsilon A_+\chi_0 + \bar{\omega}^\epsilon A_-\chi_-}{{H_{+,\epsilon}}^3}\right)(q) \\ \Xi_{-,\epsilon}(q) &= \left(\frac{A_0\chi_- + \omega^\epsilon A_-\chi_0 + \bar{\omega}^\epsilon A_+\chi_+}{{H_{-,\epsilon}}^3}\right)(q) \quad . \end{aligned} \tag{2.8}$$

For:

(((2 / ((((6/(Pi^2) [(sin(Pi/(2*5+3-1)) sin(2Pi/(2*5+3-1))) / (sin(3Pi/(2*5+3-1)) sin(4Pi/(2*5+3-1)))])))))))

Input:

$$\frac{2}{\frac{6}{\pi^2} \times \frac{\sin\left(\frac{\pi}{2\times 5+3-1}\right)\sin\left(2\times\frac{\pi}{2\times 5+3-1}\right)}{\sin\left(3\times\frac{\pi}{2\times 5+3-1}\right)\sin\left(4\times\frac{\pi}{2\times 5+3-1}\right)}}$$

Exact result:

$$\frac{2\pi^2}{\sqrt{3}\,(\sqrt{3}-1)}$$

Decimal approximation:

15.56782315885341499041548195035060023055561111269894829569...

15.5678231588…

Property:

$\frac{2\pi^2}{\sqrt{3}\,(-1+\sqrt{3})}$ is a transcendental number

From which:

11+2+2.567823158853414 99

Input interpretation:

11 + 2 + 2.567823158853414 99

Result:

15.567823158853414 99

15.567823158853414 99

Thence:

$$V_8 - S_8 = A_0\chi_0 + A_+\chi_- + A_-\chi_+$$

$A_0\chi_0 = 11$; $A_+\chi_- = 2$ and $A_-\chi_+ = 2.567823158853414 99$
From:

$$\begin{aligned}
\Xi_{0,\epsilon}(q) &= \left(\frac{A_0\chi_0 + \omega^\epsilon A_+\chi_- + \bar{\omega}^\epsilon A_-\chi_+}{{H_{0,\epsilon}}^3}\right)(q) \\
\Xi_{+,\epsilon}(q) &= \left(\frac{A_0\chi_+ + \omega^\epsilon A_+\chi_0 + \bar{\omega}^\epsilon A_-\chi_-}{{H_{+,\epsilon}}^3}\right)(q) \\
\Xi_{-,\epsilon}(q) &= \left(\frac{A_0\chi_- + \omega^\epsilon A_-\chi_0 + \bar{\omega}^\epsilon A_+\chi_+}{{H_{-,\epsilon}}^3}\right)(q) \\
\omega &= e^{\frac{2i\pi}{3}}
\end{aligned}$$

For q = exp(2Pi) and $H_{0,\varepsilon} = 2$; $H_{+,\varepsilon} = 4$ and $H_{-,\varepsilon} = 8$, we obtain:

1/8(((11+ exp((2Pi*i)/3)*2 + exp((2Pi*i)/3) * 2.56782315885341499))) * exp(2Pi)

Input interpretation:

$$\left(\frac{1}{8}\left(11+\exp\left(\frac{1}{3}(2\pi i)\right)\times 2+\exp\left(\frac{1}{3}(2\pi i)\right)\times 2.56782315885341499\right)\right)\exp(2\pi)$$

i is the imaginary unit

Result:

583.4240772541280370... +
264.7906431341982238... i

Polar coordinates:

$r = 640.700974411037615$ (radius), $\theta = 24.4112178167003351°$ (angle)

640.700974411037615

(((1/64(((11+ exp((2Pi*i)/3)*2 + exp((2Pi*i)/3) * 2.56782315885341499)))))) * exp(2Pi)

Input interpretation:

$$\left(\frac{1}{64}\left(11+\exp\left(\frac{1}{3}(2\pi i)\right)\times 2+\exp\left(\frac{1}{3}(2\pi i)\right)\times 2.56782315885341499\right)\right)\exp(2\pi)$$

i is the imaginary unit

Result:

72.92800965676600462... +
33.09883039177477797... i

Polar coordinates:

$r = 80.0876218013797019$ (radius), $\theta = 24.4112178167003351°$ (angle)

80.0876218013797019

(((1/512(((11+ exp((2Pi*i)/3)*2 + exp((2Pi*i)/3) * 2.567823158853414 99)))))) * exp(2Pi)

Input interpretation:

$$\left(\frac{1}{512}\left(11+\exp\left(\frac{1}{3}(2\pi i)\right)\times 2+\exp\left(\frac{1}{3}(2\pi i)\right)\times 2.567823158853414 99\right)\right)\exp(2\pi)$$

i is the imaginary unit

Result:

9.116001207095750577... +
4.137353798971847246... i

Polar coordinates:

$r = 10.01095272517246274$ (radius), $\theta = 24.4112178167003351°$ (angle)

10.01095272517246274

From

$$T = |V_8 - S_8|^2 \left(\frac{\sum q^{\frac{\alpha'}{4} p_{La} G^{ab} p_{Lb}} \bar{q}^{\frac{\alpha'}{4} p_{Ra} G^{ab} p_{Rb}}}{\eta^2(q)\eta^2(\bar{q})} \right)^3$$

and:

$$T = \frac{1}{3}\,\Xi_{0,0}(q)\,\Xi_{0,0}(\bar{q}) \sum q^{\frac{\alpha'}{4} p_{La} G^{ab} p_{Lb}} \bar{q}^{\frac{\alpha'}{4} p_{Ra} G^{ab} p_{Rb}} + \frac{1}{3} \sum_{\epsilon=\pm 1} \Xi_{0,\epsilon}(q)\,\Xi_{0,\epsilon}(\bar{q}) + \frac{1}{3} \sum_{\eta=\pm 1} \sum_{\epsilon=0,\pm 1} \Xi_{\eta,\epsilon}(q)\,\Xi_{-\eta,-\epsilon}(\bar{q}) \quad , \qquad (2.7)$$

we obtain:

1/3* 2*640.70097 * ((((sqrt(3) (-1 + sqrt(3)))/π^2))) * (((exp(PI/12) * (1-(exp(2Pi))))))^4 + 1/3* 2*80.08762 + 1/3 *2* 10.01095

Input interpretation:

$$\frac{1}{3}\times 2\times 640.70097\times \frac{\sqrt{3}\,(-1+\sqrt{3})}{\pi^2}\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4 + \frac{1}{3}\times 2\times 80.08762 + \frac{1}{3}\times 2\times 10.01095$$

Result:

$1.2762098\ldots \times 10^{13}$

1.2762098…*10^13

Series representations:

$$\frac{(2\times 640.701)\left(\sqrt{3}\,(-1+\sqrt{3})\right)\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4}{3\pi^2} + \frac{2\times 80.0876}{3} + \frac{2\times 10.011}{3} = 60.0657 + \frac{427.134\exp^4\left(\frac{\pi}{12}\right)(-1+\exp(2\pi))^4\sqrt{2}\left(\sum_{k=0}^{\infty} 2^{-k}\binom{\frac{1}{2}}{k}\right)\left(-1+\sqrt{2}\sum_{k=0}^{\infty} 2^{-k}\binom{\frac{1}{2}}{k}\right)}{\pi^2}$$

$$\frac{(2\times 640.701)\left(\sqrt{3}\,(-1+\sqrt{3})\right)\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4}{3\pi^2} + \frac{2\times 80.0876}{3} + \frac{2\times 10.011}{3} = 60.0657 + \frac{427.134\exp^4\left(\frac{\pi}{12}\right)(-1+\exp(2\pi))^4\sqrt{2}\left(\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\left(-1+\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}{\pi^2}$$

$$\frac{(2\times 640.701)\left(\sqrt{3}\,(-1+\sqrt{3})\right)\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4}{3\pi^2} + \frac{2\times 80.0876}{3} + \frac{2\times 10.011}{3} = 60.0657 - \frac{1}{\pi^2\sqrt{\pi}^2}\,213.567\exp^4\left(\frac{\pi}{12}\right)(-1+\exp(2\pi))^4 \left(\sqrt{\pi} - 0.5\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)$$

From which:

3/sqrt2 ln((((1/3* 2*640.70097 * ((((sqrt(3) (-1 + sqrt(3)))/π^2))) * (((exp(PI/12) * (1-(exp(2Pi))))))^4 + 1/3* 2*80.08762 + 1/3 *2* 10.01095))))

Input interpretation:

$$\frac{3}{\sqrt{2}} \log\left(\frac{1}{3} \times 2 \times 640.70097 \times \frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2} \left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4 + \frac{1}{3} \times 2 \times 80.08762 + \frac{1}{3} \times 2 \times 10.01095\right)$$

$\log(x)$ is the natural logarithm

Result:

64.0161463...

64.0161463… ≈ 64

Alternative representations:

$$\frac{\log\left(\frac{2 \times 640.701\left(\left(\sqrt{3}\left(-1+\sqrt{3}\right)\right)\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4\right)}{3\pi^2} + \frac{2 \times 80.0876}{3} + \frac{2 \times 10.011}{3}\right)3}{\sqrt{2}} = \frac{3\log_e\left(60.0657 + \frac{427.134\left((1-\exp(2\pi))\exp\left(\frac{\pi}{12}\right)\right)^4\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)}{\sqrt{2}}$$

$$\frac{\log\left(\frac{2 \times 640.701\left(\left(\sqrt{3}\left(-1+\sqrt{3}\right)\right)\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4\right)}{3\pi^2} + \frac{2 \times 80.0876}{3} + \frac{2 \times 10.011}{3}\right)3}{\sqrt{2}} = \frac{3\log(a)\log_a\left(60.0657 + \frac{427.134\left((1-\exp(2\pi))\exp\left(\frac{\pi}{12}\right)\right)^4\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)}{\sqrt{2}}$$

Series representations:

$$\frac{\log\left(\frac{2\times 640.701\left(\left(\sqrt{3}\left(-1+\sqrt{3}\right)\right)\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4\right)}{3\pi^2}+\frac{2\times 80.0876}{3}+\frac{2\times 10.011}{3}\right)3}{\sqrt{2}} = \left(3\left(\log\left(59.0657+\frac{427.134\exp^4\left(\frac{\pi}{12}\right)(-1+\exp(2\pi))^4\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(59.0657+\frac{427.134\exp^4\left(\frac{\pi}{12}\right)(-1+\exp(2\pi))^4\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)^{-k}}{k}\right)\right)\Big/\left(\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\log\left(\frac{2\times 640.701\left(\left(\sqrt{3}\left(-1+\sqrt{3}\right)\right)\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4\right)}{3\pi^2}+\frac{2\times 80.0876}{3}+\frac{2\times 10.011}{3}\right)3}{\sqrt{2}} = \left(3\left(\log\left(59.0657+\frac{427.134\exp^4\left(\frac{\pi}{12}\right)(1-\exp(2\pi))^4\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(59.0657+\frac{427.134\exp^4\left(\frac{\pi}{12}\right)(-1+\exp(2\pi))^4\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)^{-k}}{k}\right)\right)\Big/\left(\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\log\left(\frac{2\times 640.701\left(\left(\sqrt{3}\left(-1+\sqrt{3}\right)\right)\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4\right)}{3\pi^2}+\frac{2\times 80.0876}{3}+\frac{2\times 10.011}{3}\right)3}{\sqrt{2}}=$$

$$\left(3\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{-1/2-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\log\left(59.0657+\frac{427.134\exp^4\left(\frac{\pi}{12}\right)(-1+\exp(2\pi))^4\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(59.0657+\frac{427.134\exp^4\left(\frac{\pi}{12}\right)(-1+\exp(2\pi))^4\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)^{-k}}{k}\right)\right)\Bigg/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)$$

Integral representations:

$$\frac{\log\left(\frac{2\times 640.701\left(\left(\sqrt{3}\left(-1+\sqrt{3}\right)\right)\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4\right)}{3\pi^2}+\frac{2\times 80.0876}{3}+\frac{2\times 10.011}{3}\right)3}{\sqrt{2}}=\frac{3}{\sqrt{2}}\int_1^{60.0657+\frac{427.134\exp^4\left(\frac{\pi}{12}\right)(-1+\exp(2\pi))^4\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}}\frac{1}{t}\,dt$$

$$\frac{\log\left(\frac{2\times 640.701\left(\left(\sqrt{3}\left(-1+\sqrt{3}\right)\right)\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4\right)}{3\pi^2}+\frac{2\times 80.0876}{3}+\frac{2\times 10.011}{3}\right)3}{\sqrt{2}}=\frac{3}{2i\pi\sqrt{2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)\left(59.0657+\frac{427.134\exp^4\left(\frac{\pi}{12}\right)(-1+\exp(2\pi))^4\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)^{-s}}{\Gamma(1-s)}\,ds$$

for $-1<\gamma<0$

(((((1/3* 2*640.70097 * ((((sqrt(3) (-1 + sqrt(3)))/π^2))) * (((exp(PI/12) * (1-(exp(2Pi))))))^4 + 1/3* 2*80.08762 + 1/3 *2* 10.01095)))))^1/63

Input interpretation:

$$\left(\frac{1}{3}\times 2\times 640.70097\times \frac{\sqrt{3}\,(-1+\sqrt{3})}{\pi^2}\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\,\pi))\right)^4+\frac{1}{3}\times 2\times 80.08762+\frac{1}{3}\times 2\times 10.01095\right)\text{^}(1/63)$$

Result:

1.614471969...

1.614471969... result that is a good approximation to the value of the golden ratio 1.618033988749...

(((((1/3* 2*640.70097 * ((((sqrt(3) (-1 + sqrt(3)))/π^2))) * (((exp(PI/12) * (1-(exp(2Pi))))))^4 + 1/3* 2*80.08762 + 1/3 *2* 10.01095)))))^1/64

Input interpretation:

$$\left(\frac{1}{3}\times 2\times 640.70097\times \frac{\sqrt{3}\,(-1+\sqrt{3})}{\pi^2}\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\,\pi))\right)^4+\frac{1}{3}\times 2\times 80.08762+\frac{1}{3}\times 2\times 10.01095\right)\text{^}(1/64)$$

Result:

1.602433561...

1.602433561... result, that multiplied by $1/10^{19}$, is practically equal to the value of Elementary Charge

and:

2log base 1.602433561(((((1/3* 2*640.70097 * ((((sqrt(3) (-1 + sqrt(3)))/π^2))) * (((exp(PI/12) * (1-(exp(2Pi))))))^4 + 1/3* 2*80.08762 + 1/3 *2* 10.01095)))))-golden ratio^2

Input interpretation:

$$2\log_{1.602433561}\left(\frac{1}{3}\times 2\times 640.70097\times \frac{\sqrt{3}\,(-1+\sqrt{3})}{\pi^2}\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\,\pi))\right)^4+\frac{1}{3}\times 2\times 80.08762+\frac{1}{3}\times 2\times 10.01095\right)-\phi^2$$

$\log_b(x)$ is the base- b logarithm

ϕ is the golden ratio

Result:

125.381966...

125.381966… result very near to the Higgs boson mass 125.18 GeV

Alternative representation:

$$2 \log_{1.60243}\left(\frac{(2\times 640.701)\left(\sqrt{3}\left(-1+\sqrt{3}\right)\right)\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4}{3\pi^2} + \frac{2\times 80.0876}{3} + \frac{2\times 10.011}{3}\right) - \phi^2 = -\phi^2 + \frac{2\log\left(60.0657 + \frac{427.134\left((1-\exp(2\pi))\exp\left(\frac{\pi}{12}\right)\right)^4\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)}{\log(1.60243)}$$

Series representations:

$$2 \log_{1.60243}\left(\frac{(2\times 640.701)\left(\sqrt{3}\left(-1+\sqrt{3}\right)\right)\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4}{3\pi^2} + \frac{2\times 80.0876}{3} + \frac{2\times 10.011}{3}\right) - \phi^2 = -\phi^2 + 2\log_{1.60243}\left(60.0657 + \frac{1}{\pi^2}\, 427.134 \exp^4\left(\frac{\pi}{12}\right)(-1+\exp(2\pi))^4 \exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\left(\sum_{k=0}^{\infty}\frac{(-1)^k (3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\left(-1+\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$2\log_{1.60243}\left(\frac{(2\times 640.701)\left(\sqrt{3}\left(-1+\sqrt{3}\right)\right)\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4}{3\pi^2}+\frac{2\times 80.0876}{3}+\frac{2\times 10.011}{3}\right)-\phi^2=$$

$$-\phi^2+2\log_{1.60243}\left(60.0657+\frac{1}{\pi^2}\,427.134\exp^4\left(\frac{\pi}{12}\right)(1-\exp(2\pi))^4\right.$$
$$\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$
$$\left.\left(-1+\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

for ($x\in\mathbb{R}$ and $x<0$)

$$2\log_{1.60243}\left(\frac{(2\times 640.701)\left(\sqrt{3}\left(-1+\sqrt{3}\right)\right)\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4}{3\pi^2}+\frac{2\times 80.0876}{3}+\frac{2\times 10.011}{3}\right)-\phi^2=-\phi^2+$$

$$2\log_{1.60243}\left(60.0657+\frac{1}{\pi^2}\,427.134\exp^4\left(\frac{\pi}{12}\right)(1-\exp(2\pi))^4\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor}\right.$$
$$z_0^{1/2(1+\lfloor\arg(3-z_0)/(2\pi)\rfloor)}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)\left(-1+\right.$$
$$\left.\left.\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor\arg(3-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)\right)$$

2log base 1.602433561(((((1/3* 2*640.70097 * ((((sqrt(3) (-1 + sqrt(3)))/π^2))) * (((exp(PI/12) * (1-(exp(2Pi))))))^4 + 1/3* 2*80.08762 + 1/3 *2* 10.01095)))))+11+1/golden ratio

Input interpretation:

$$2\log_{1.602433561}\left(\frac{1}{3}\times 2\times 640.70097\times\frac{\sqrt{3}\left(-1+\sqrt{3}\right)}{\pi^2}\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4+\frac{1}{3}\times 2\times 80.08762+\frac{1}{3}\times 2\times 10.01095\right)+11+\frac{1}{\phi}$$

$\log_b(x)$ is the base- b logarithm

ϕ is the golden ratio

Result:

139.618034...

139.618034… result practically equal to the rest mass of Pion meson 139.57 MeV

Alternative representation:

$$2 \log_{1.60243}\left(\frac{(2\times 640.701)\left(\sqrt{3}\left(-1+\sqrt{3}\right)\right)\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4}{3\pi^2}+\frac{2\times 80.0876}{3}+\frac{2\times 10.011}{3}\right)+11+\frac{1}{\phi}=$$

$$11+\frac{1}{\phi}+\frac{2\log\left(60.0657+\frac{427.134\left((1-\exp(2\pi))\exp\left(\frac{\pi}{12}\right)\right)^4\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)}{\log(1.60243)}$$

Series representations:

$$2 \log_{1.60243}\left(\frac{(2\times 640.701)\left(\sqrt{3}\left(-1+\sqrt{3}\right)\right)\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4}{3\pi^2}+\frac{2\times 80.0876}{3}+\frac{2\times 10.011}{3}\right)+11+\frac{1}{\phi}=$$

$$\frac{1}{\phi}\left(1+11\phi+2\phi\log_{1.60243}\left(60.0657+\frac{1}{\pi^2}\,427.134\exp^4\left(\frac{\pi}{12}\right)(-1+\exp(2\pi))^4\right.\right.$$
$$\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$
$$\left.\left.\left(-1+\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$2\log_{1.60243}\left(\frac{(2\times 640.701)(\sqrt{3}\ (-1+\sqrt{3}))\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4}{3\pi^2}+\frac{2\times 80.0876}{3}+\frac{2\times 10.011}{3}\right)+11+\frac{1}{\phi}=$$

$$\frac{1}{\phi}\left(1+11\phi+2\phi\log_{1.60243}\left(60.0657+\frac{1}{\pi^2}\,427.134\exp^4\left(\frac{\pi}{12}\right)(1-\exp(2\pi))^4\right.\right.$$
$$\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$
$$\left.\left.\left(-1+\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$2\log_{1.60243}\left(\frac{(2\times 640.701)(\sqrt{3}\ (-1+\sqrt{3}))\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4}{3\pi^2}+\frac{2\times 80.0876}{3}+\frac{2\times 10.011}{3}\right)+11+\frac{1}{\phi}=$$

$$\frac{1}{\phi}\left(1+11\phi+2\phi\log_{1.60243}\left(60.0657+\frac{1}{\pi^2}\,427.134\exp^4\left(\frac{\pi}{12}\right)\right.\right.$$
$$(1-\exp(2\pi))^4\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor\arg(3-z_0)/(2\pi)\rfloor)}$$
$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k\,z_0^{-k}}{k!}\right)\left(-1+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor}\right.$$
$$\left.\left.\left.z_0^{1/2(1+\lfloor\arg(3-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k\,z_0^{-k}}{k!}\right)\right)\right)$$

27[log base 1.602433561(((((1/3* 2*640.70097 * ((((sqrt(3) (-1 + sqrt(3)))/π^2))) * (((exp(PI/12) * (1-(exp(2Pi))))))^4 + 1/3* 2*80.08762 + 1/3 *2* 10.01095)))))]+1

Input interpretation:

$$27\log_{1.602433561}\left(\frac{1}{3}\times 2\times 640.70097\times\frac{\sqrt{3}\ (-1+\sqrt{3})}{\pi^2}\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4+\frac{1}{3}\times 2\times 80.08762+\frac{1}{3}\times 2\times 10.01095\right)+1$$

$\log_b(x)$ is the base- b logarithm

Result:

1729.00000...

1729

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternative representation:

$$27 \log_{1.60243}\left(\frac{(2\times 640.701)\left(\sqrt{3}\left(-1+\sqrt{3}\right)\right)\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4}{3\pi^2}+\frac{2\times 80.0876}{3}+\frac{2\times 10.011}{3}\right)+1=1+\frac{27\log\left(60.0657+\frac{427.134\left((1-\exp(2\pi))\exp\left(\frac{\pi}{12}\right)\right)^4\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)}{\log(1.60243)}$$

Series representations:

$$27 \log_{1.60243}\left(\frac{(2\times 640.701)\left(\sqrt{3}\left(-1+\sqrt{3}\right)\right)\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4}{3\pi^2}+\frac{2\times 80.0876}{3}+\frac{2\times 10.011}{3}\right)+1=$$
$$1+27\log_{1.60243}\left(60.0657+\frac{1}{\pi^2}427.134\exp^4\left(\frac{\pi}{12}\right)(-1+\exp(2\pi))^4\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\left(-1+\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$27 \log_{1.60243}\left(\frac{(2\times 640.701)(\sqrt{3}(-1+\sqrt{3}))\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4}{3\pi^2}+\frac{2\times 80.0876}{3}+\frac{2\times 10.011}{3}\right)+1 =$$

$$1+27\log_{1.60243}\left(60.0657+\frac{1}{\pi^2}\,427.134\exp^4\left(\frac{\pi}{12}\right)(1-\exp(2\pi))^4 \exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\left(-1+\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$27 \log_{1.60243}\left(\frac{(2\times 640.701)(\sqrt{3}(-1+\sqrt{3}))\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4}{3\pi^2}+\frac{2\times 80.0876}{3}+\frac{2\times 10.011}{3}\right)+1 = 1+$$

$$27\log_{1.60243}\left(60.0657+\frac{1}{\pi^2}\,427.134\exp^4\left(\frac{\pi}{12}\right)(1-\exp(2\pi))^4\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(3-z_0)/(2\pi)\rfloor)}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)\left(-1+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(3-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)\right)$$

Now, we have that:

$$T = \frac{1}{3}\,\Xi_{0,0}(q)\,\Xi_{0,0}(\bar{q})\sum q^{\frac{\alpha'}{4}p_{La}G^{ab}p_{Lb}}\bar{q}^{\frac{\alpha'}{4}p_{Ra}G^{ab}p_{Rb}}+\frac{1}{3}\sum_{\epsilon=\pm1}\Xi_{0,\epsilon}(q)\,\Xi_{0,\epsilon}(\bar{q}) + \frac{1}{3}\sum_{\eta=\pm1}\sum_{\epsilon=0,\pm1}\Xi_{\eta,\epsilon}(q)\,\Xi_{-\eta,-\epsilon}(\bar{q}) \quad , \tag{2.7}$$

$$\frac{1}{3}\times 2\times 640.70097\times\frac{\sqrt{3}(-1+\sqrt{3})}{\pi^2}\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4+\frac{1}{3}\times 2\times 80.08762+\frac{1}{3}\times 2\times 10.01095$$

$1.2762098\ldots \times 10^{13}$

From which:

$$\frac{3}{\sqrt{2}} \log\left(\frac{1}{3} \times 2 \times 640.70097 \times \frac{\sqrt{3}\,(-1+\sqrt{3})}{\pi^2} \left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4 + \frac{1}{3} \times 2 \times 80.08762 + \frac{1}{3} \times 2 \times 10.01095\right)$$

$64.0161463\ldots$

and:

$$\left(\frac{1}{3} \times 2 \times 640.70097 \times \frac{\sqrt{3}\,(-1+\sqrt{3})}{\pi^2} \left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4 + \frac{1}{3} \times 2 \times 80.08762 + \frac{1}{3} \times 2 \times 10.01095\right)\text{^}(1/64)$$

$1.602433561\ldots$

From which:

[log base 1.602433561((((((1/3* 2*640.70097 * ((((sqrt(3) (-1 + sqrt(3)))/π^2))) * (((exp(PI/12) * (1-(exp(2Pi))))))^4 + 1/3* 2*80.08762 + 1/3 *2* 10.01095)))))]

Input interpretation:

$$\log_{1.602433561}\left(\frac{1}{3} \times 2 \times 640.70097 \times \frac{\sqrt{3}\,(-1+\sqrt{3})}{\pi^2} \left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4 + \frac{1}{3} \times 2 \times 80.08762 + \frac{1}{3} \times 2 \times 10.01095\right)$$

$\log_b(x)$ is the base- b logarithm

Result:

$64.0000000\ldots$

64

Alternative representation:

$$\log_{1.60243}\left(\frac{(2\times 640.701)\left(\sqrt{3}\left(-1+\sqrt{3}\right)\right)\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4}{3\pi^2}+\frac{2\times 80.0876}{3}+\frac{2\times 10.011}{3}\right)=\frac{\log\left(60.0657+\frac{427.134\left((1-\exp(2\pi))\exp\left(\frac{\pi}{12}\right)\right)^4\left(-1+\sqrt{3}\right)\sqrt{3}}{\pi^2}\right)}{\log(1.60243)}$$

Series representations:

$$\log_{1.60243}\left(\frac{(2\times 640.701)\left(\sqrt{3}\left(-1+\sqrt{3}\right)\right)\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4}{3\pi^2}+\frac{2\times 80.0876}{3}+\frac{2\times 10.011}{3}\right)=$$
$$\log_{1.60243}\left(60.0657+\frac{1}{\pi^2}\,427.134\exp^4\left(\frac{\pi}{12}\right)(-1+\exp(2\pi))^4\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\left(-1+\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\quad\text{for } (x\in\mathbb{R}\text{ and } x<0)$$

$$\log_{1.60243}\left(\frac{(2\times 640.701)\left(\sqrt{3}\left(-1+\sqrt{3}\right)\right)\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4}{3\pi^2}+\frac{2\times 80.0876}{3}+\frac{2\times 10.011}{3}\right)=$$
$$\log_{1.60243}\left(60.0657+\frac{1}{\pi^2}\,427.134\exp^4\left(\frac{\pi}{12}\right)(1-\exp(2\pi))^4\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor\arg(3-z_0)/(2\pi)\rfloor)}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)\left(-1+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor\arg(3-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)\right)$$

Integral representations:

$$\log(z)=\int_1^z\frac{1}{t}\,dt$$

$$\log(1+z) = \frac{1}{2\pi i}\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s+1)\,\Gamma(-s)^2}{\Gamma(1-s)\,z^s}\,ds \quad \text{for } (-1<\gamma<0 \text{ and } |\arg(z)|<\pi)$$

From:

Modular equations and approximations to π – *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

Now, from the following Ramanujan equation:

$$G_{65}^2 = \sqrt{\left\{\left(\frac{1+\sqrt{5}}{2}\right)\left(\frac{3+\sqrt{13}}{2}\right)\right\}}\left\{\sqrt{\left(\frac{1+\sqrt{65}}{8}\right)}+\sqrt{\left(\frac{9+\sqrt{65}}{8}\right)}\right\}$$

Sqrt[(((1+sqrt5)/2)((3+sqrt13)/2))] (((sqrt((1+sqrt65)/8)+sqrt((9+sqrt65)/8))))

Input:

$$\sqrt{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\left(\frac{1}{2}\left(3+\sqrt{13}\right)\right)}\left(\sqrt{\frac{1}{8}\left(1+\sqrt{65}\right)}+\sqrt{\frac{1}{8}\left(9+\sqrt{65}\right)}\right)$$

Result:

$$\frac{1}{2}\sqrt{\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)}\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{65}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(9+\sqrt{65}\right)}\right)$$

Decimal approximation:

5.836437260372441913791155982828015314725475269200530282807...

5.83643726…

Alternate forms:

$$\frac{1}{8}\sqrt{\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)}\left(\sqrt{2\left(1+\sqrt{65}\right)}+\sqrt{5}+\sqrt{13}\right)$$

$$\frac{1}{4}\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)}\left(\sqrt{1+\sqrt{65}}+\sqrt{9+\sqrt{65}}\right)$$

$$\frac{1}{8}\sqrt{\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)}\left(\sqrt{2\left(1+\sqrt{65}\right)}+\sqrt{2\left(9+\sqrt{65}\right)}\right)$$

Minimal polynomial:

$$x^8 - 8x^7 + 12x^6 + 8x^5 - 27x^4 + 8x^3 + 12x^2 - 8x + 1$$

From which:

24* sqrt[(((1+sqrt5)/2)((3+sqrt13)/2))] (((sqrt((1+sqrt65)/8)+sqrt((9+sqrt65)/8))))

Input:

$$24\sqrt{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\left(\frac{1}{2}\left(3+\sqrt{13}\right)\right)}\left(\sqrt{\frac{1}{8}\left(1+\sqrt{65}\right)}+\sqrt{\frac{1}{8}\left(9+\sqrt{65}\right)}\right)$$

Result:

$$12\sqrt{\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)}\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{65}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(9+\sqrt{65}\right)}\right)$$

Decimal approximation:

140.07449424893860593098774358787236755341140646081272678 73...

140.07449424…. result very near to the rest mass of Pion meson 139.57 MeV

Alternate forms:

$$3\sqrt{\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)}\left(\sqrt{2\left(1+\sqrt{65}\right)}+\sqrt{5}+\sqrt{13}\right)$$

$$3\sqrt{\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)}\left(\sqrt{1-8i}+\sqrt{1+8i}+\sqrt{5}+\sqrt{13}\right)$$

$$3\sqrt{2\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)}\left(\sqrt{1+\sqrt{65}}+\sqrt{9+\sqrt{65}}\right)$$

Minimal polynomial:

$$x^8 - 192x^7 + 6912x^6 + 110592x^5 - 8957952x^4 + 63700992x^3 + 2293235712x^2 - 36691771392x + 110075314176$$

((((1+golden ratio^2)*(((sqrt[((((1+sqrt5)/2)((3+sqrt13)/2))]
(((sqrt((1+sqrt65)/8)+sqrt((9+sqrt65)/8)))))))^4)))^0.5 -sqrt(1/golden ratio)

Input:

$$\sqrt{(1+\phi^2)\left(\sqrt{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\left(\frac{1}{2}\left(3+\sqrt{13}\right)\right)}\left(\sqrt{\frac{1}{8}\left(1+\sqrt{65}\right)}+\sqrt{\frac{1}{8}\left(9+\sqrt{65}\right)}\right)\right)^4}-\sqrt{\frac{1}{\phi}}$$

ϕ is the golden ratio

Exact result:

$$\frac{1}{4}\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{65}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(9+\sqrt{65}\right)}\right)^2\sqrt{\phi^2+1}-\frac{1}{\sqrt{\phi}}$$

Decimal approximation:

64.00742676327654830540853676673539091977965648126671973548...

64.00742676... ≈ 64

Alternate forms:

$$\frac{-64+(1+\sqrt{5})(3+\sqrt{13})(\sqrt{1-8i}+\sqrt{1+8i}+\sqrt{5}+\sqrt{13})^2\sqrt{\phi(\phi^2+1)}}{64\sqrt{\phi}}$$

$$\frac{1}{4}\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)\sqrt{1+\frac{1}{4}\left(1+\sqrt{5}\right)^2}\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{65}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(9+\sqrt{65}\right)}\right)^2-\sqrt{\frac{2}{1+\sqrt{5}}}$$

$$\sqrt{\text{root of } x^{16}-8402x^{15}+17656355x^{14}-77283330x^{13}+1404766205x^{12}-22747369016x^{11}+135810510772x^{10}-467612304720x^9+982547909920x^8+34705678080x^7-6764648608448x^6+13160819835136x^5+4170418603520x^4-38610595307520x^3-3498995092480x^2+11361752829952x+3023481081856 \text{ near } x=4096.95}$$

Minimal polynomial:

$$x^{32} - 8402\,x^{30} + 17\,656\,355\,x^{28} - 77\,283\,330\,x^{26} + 1\,404\,766\,205\,x^{24} - 22\,747\,369\,016\,x^{22} + 135\,810\,510\,772\,x^{20} - 467\,612\,304\,720\,x^{18} + 982\,547\,909\,920\,x^{16} + 34\,705\,678\,080\,x^{14} - 6\,764\,648\,608\,448\,x^{12} + 13\,160\,819\,835\,136\,x^{10} + 4\,170\,418\,603\,520\,x^{8} - 38\,610\,595\,307\,520\,x^{6} - 3\,498\,995\,092\,480\,x^{4} + 11\,361\,752\,829\,952\,x^{2} + 3\,023\,481\,081\,856$$

Now, we have that:

$$T = \frac{1}{3}\,\Xi_{0,0}(q)\,\Xi_{0,0}(\bar{q})\sum q^{\frac{\alpha'}{4}p_{La}G^{ab}p_{Lb}}\bar{q}^{\frac{\alpha'}{4}p_{Ra}G^{ab}p_{Rb}} + \frac{1}{3}\sum_{\epsilon=\pm1}\Xi_{0,\epsilon}(q)\,\Xi_{0,\epsilon}(\bar{q}) + \frac{1}{3}\sum_{\eta=\pm1}\sum_{\epsilon=0,\pm1}\Xi_{\eta,\epsilon}(q)\,\Xi_{-\eta,-\epsilon}(\bar{q}) \quad , \qquad (2.7)$$

$$\frac{1}{3}\times 2\times 640.70097\times\frac{\sqrt{3}\,(-1+\sqrt{3})}{\pi^2}\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4 + \frac{1}{3}\times 2\times 80.08762 + \frac{1}{3}\times 2\times 10.01095$$

$$= 1.2762098\ldots\times 10^{13}$$

and:

$$\left(\frac{1}{3}\times 2\times 640.70097\times\frac{\sqrt{3}\,(-1+\sqrt{3})}{\pi^2}\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\pi))\right)^4 + \frac{1}{3}\times 2\times 80.08762 + \frac{1}{3}\times 2\times 10.01095\right)\verb|^|\,(1/64)$$

$$1.602433561\ldots$$

From which:

[log base 1.602433561(((((1/3* 2*640.70097 * ((((sqrt(3) (-1 + sqrt(3)))/π^2))) * (((exp(PI/12) * (1-(exp(2Pi))))))^4 + 1/3* 2*80.08762 + 1/3 *2* 10.01095)))))]

$$\log_{1.602433561}\left(\frac{1}{3}\times 2\times 640.70097\times \frac{\sqrt{3}\,(-1+\sqrt{3})}{\pi^2}\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\,\pi))\right)^4 + \frac{1}{3}\times 2\times 80.08762 + \frac{1}{3}\times 2\times 10.01095\right)$$

64.0000000...

Connected with:

$$\frac{1}{4}\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{65}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(9+\sqrt{65}\right)}\right)^2\sqrt{\phi^2+1}-\frac{1}{\sqrt{\phi}}$$

64.00742676327654830540853676673539091977965648126671973548...

Thence the following mathematical connection:

$$\left[\log_{1.602433561}\left(\frac{1}{3}\times 2\times 640.70097\times \frac{\sqrt{3}\,(-1+\sqrt{3})}{\pi^2}\left(\exp\left(\frac{\pi}{12}\right)(1-\exp(2\,\pi))\right)^4 + \frac{1}{3}\times 2\times 80.08762 + \frac{1}{3}\times 2\times 10.01095\right)\right] = 64$$

$$\left[\frac{1}{4}\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{65}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(9+\sqrt{65}\right)}\right)^2\sqrt{\phi^2+1}-\frac{1}{\sqrt{\phi}}\right] = 64.0074\ldots$$

Observations

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125.
In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson:

125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.

Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

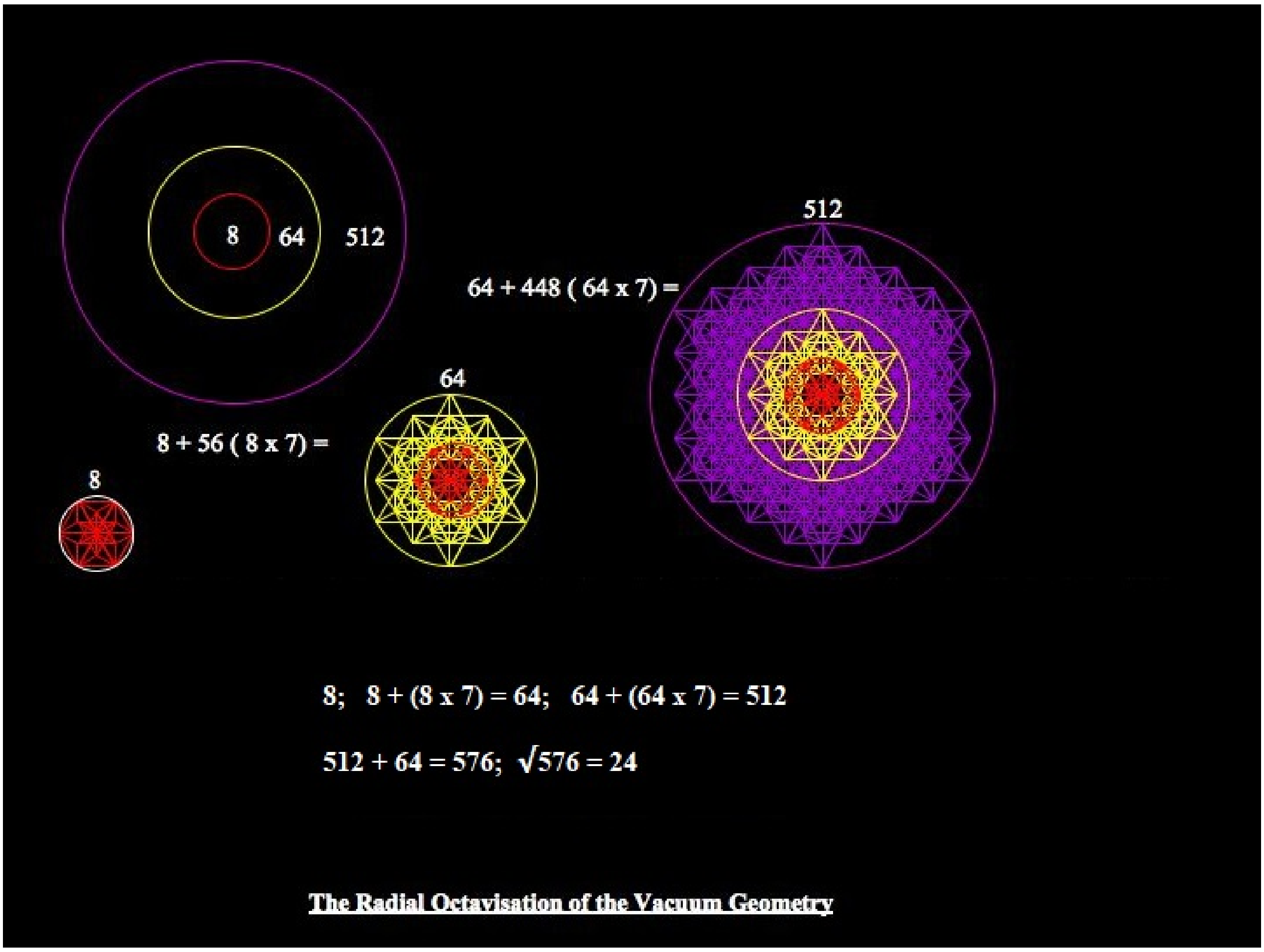
In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

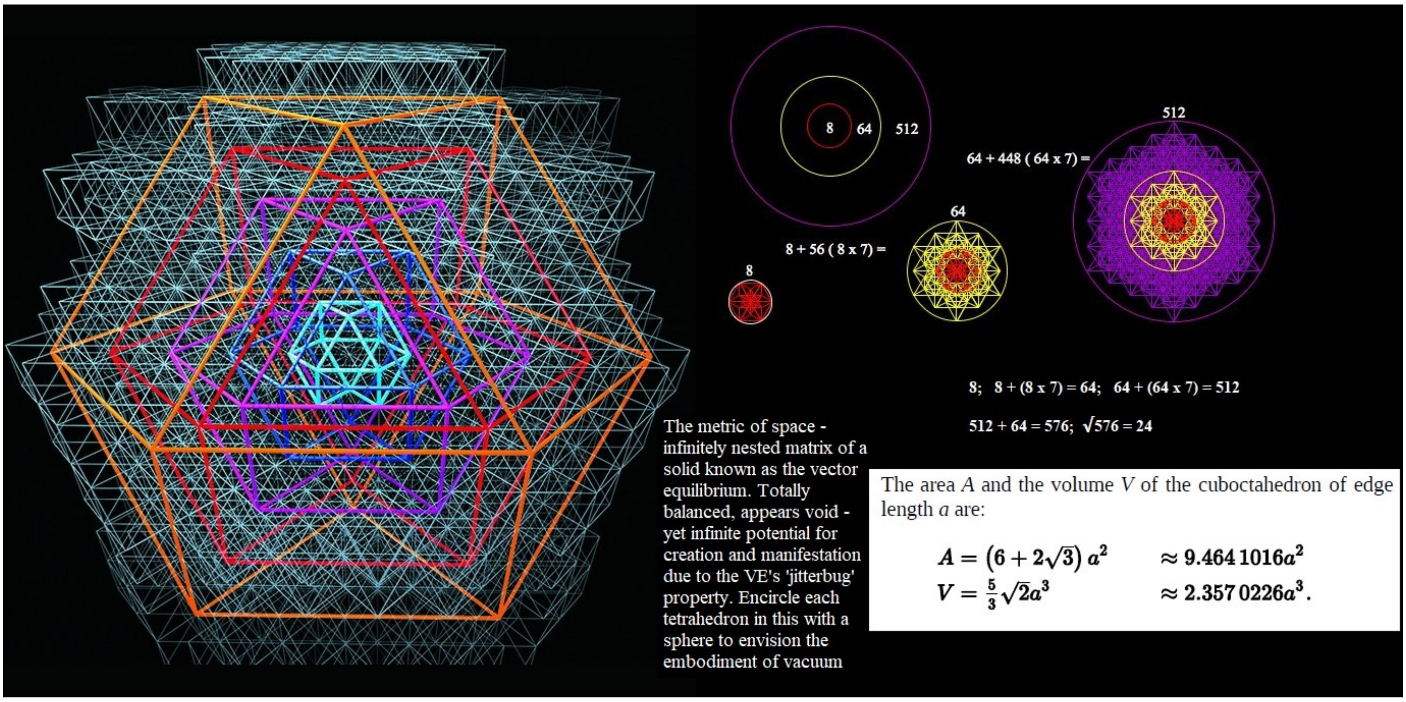
Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749...

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

https://www.pinterest.it/pin/570338740293422619/



https://imgur.com/r/holofractal/6YwLRqH



In geometry, a **cuboctahedron** is a polyhedron with 8 triangular faces and 6 square faces. A cuboctahedron has 12 identical vertices, with 2 triangles and 2 squares meeting at each, and 24 identical edges, each separating a triangle from a square. As such, it is a quasiregular polyhedron, i.e. an Archimedean solid that is not only vertex-transitive but also edge-transitive. It is the only radially equilateral convex polyhedron.

With regard the cuboctahedron, we have the following formulas concerning Area and Volume:

The area A and the volume V of the cuboctahedron of edge length a are:

$$A = \left(6 + 2\sqrt{3}\right) a^2 \qquad \approx 9.464\,1016 a^2$$
$$V = \tfrac{5}{3}\sqrt{2} a^3 \qquad \approx 2.357\,0226 a^3.$$

Now, we observe that the radius of the sphere circumscribed to the cube is equal to:

$$\frac{\sqrt{3}}{2}L$$

while the radius of the sphere inscribed to the octahedron is equal to:

$$\frac{\sqrt{6}}{6}L$$

From the volume formula, for *a* equal to the radius of the sphere circumscribed to the cube, for L = 1.01861677, we obtain:

(5/3*sqrt2)*(((sqrt3)/2)*1.01861677)^3

Input interpretation:

$$\left(\frac{5}{3}\sqrt{2}\right)\left(\frac{\sqrt{3}}{2}\times 1.01861677\right)^3$$

Result:

1.61803573...

1.61803573…

From the area formula, for *a* equal to the radius of the sphere inscribed to the octahedron, for L = 1.01281, we obtain:

(6+2sqrt3)*(((sqrt6)/6)*1.01281)^2

Input interpretation:

$$\left(6+2\sqrt{3}\right)\left(\frac{\sqrt{6}}{6}\times 1.01281\right)^2$$

Result:

1.61802...

1.6180208201…

We note that the two solutions are very good approximations to the value of golden ratio

Furthermore, we have that:

(2/15(sqrt(6)+sqrt(30)))^(1/3)

Input:

$$\sqrt[3]{\frac{2}{15}\left(\sqrt{6}+\sqrt{30}\right)}$$

Decimal approximation:

1.018616404103200144587062791626378585000264771994675875913...

1.01861…

sqrt((3/2 + (3 sqrt(5))/2)/(3 + sqrt(3)))

Input:

$$\sqrt{\frac{\frac{3}{2}+\frac{1}{2}\left(3\sqrt{5}\right)}{3+\sqrt{3}}}$$

Result:

$$\sqrt{\frac{\frac{3}{2}+\frac{3\sqrt{5}}{2}}{3+\sqrt{3}}}$$

Decimal approximation:

1.012814121485659740517350533991522483035419937772355098246...

1.01281…

Thence, in conclusion, we obtain the following equations:

a)

(5/3*sqrt2)*(((sqrt3)/2)*((2/15(sqrt(6)+sqrt(30)))^(1/3))))^3

Input:

$$\left(\frac{5}{3}\sqrt{2}\right)\left(\frac{\sqrt{3}}{2}\sqrt[3]{\frac{2}{15}\left(\sqrt{6}+\sqrt{30}\right)}\right)^3$$

Exact result:

$$\frac{\sqrt{6}+\sqrt{30}}{2\sqrt{6}}$$

Decimal approximation:

1.6180339887498948482045868343656381177203091798057628621354...

1.6180339887…

Alternate forms:

$$\frac{1}{2}\left(1+\sqrt{5}\right)$$

$$\frac{1}{2}+\frac{\sqrt{5}}{2}$$

$$\frac{\sqrt{5}}{2}+\frac{1}{2}$$

Minimal polynomial:

$$x^2-x-1$$

b)

(6+2sqrt3)*(((sqrt6)/6)*(sqrt((3/2+(3sqrt(5))/2)/(3+sqrt(3)))))^2

Input:

$$\left(6+2\sqrt{3}\right)\left(\frac{\sqrt{6}}{6}\sqrt{\frac{\frac{3}{2}+\frac{1}{2}\left(3\sqrt{5}\right)}{3+\sqrt{3}}}\right)^2$$

Result:

$$\frac{(6+2\sqrt{3})\left(\frac{3}{2}+\frac{3\sqrt{5}}{2}\right)}{6(3+\sqrt{3})}$$

Decimal approximation:

1.6180339887498948482045868343656381177203091798057628621354...

1.6180339887…

Alternate forms:

$$\frac{1}{2}\left(1+\sqrt{5}\right)$$

$$\frac{1}{2}+\frac{\sqrt{5}}{2}$$

$$\frac{\sqrt{5}}{2}+\frac{1}{2}$$

Minimal polynomial:

$x^2 - x - 1$

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