How to obtain a mass of a graviton, Starting with a first integral and then comparing it with an equivalence between Planck length and a De Broglie wavelength. To obtain early universe entropy. How we can tie this discussion into the influence of the 5th dimension in Pre Planckian space-time ? As well as the formation of voids ?

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Abstract

Using the Klauder enhanced quantization as a way to specify the cosmological constant as a baseline for the mass of a graviton, we eventually come up and then we will go to the relationship of a Planck Length to a De Broglie length in order to link how we construct a massive graviton mass, with cosmological constant and to interface that with entropy in the early universe. We then close with a reference to the possible quantum origins of e folding and inflation. This objective once achieved is connected with a possible mechanism for the creation of voids, in the later universe, using a construction of shock fronts from J. P Onstriker, 1991 and followed up afterwards with Mukhanov's physical foundations to Cosmology book section as to indicate how variable input into self reproduction of the Universe structures may lead to void formation in the present era. A connection with Wesson's 5 dimensional cosmology is brought up in terms of a generalized uncertainty principle which may lead to variations of varying energy input into self reproducing cosmological structures which could enable non uniform structure formation and hence voids. One of the stunning results is that the figure of number of gravitons, about 10^58 , early on, is commensurate with a need for negative pressure, (middle of manuscript) which is a stunning result, partly based on Volovik and weakly interacting Bose gas model for pressure, which is completely unexpected. Note that in quantum physics, the idea statistically is that at large quantum numbers, we have an approach to classical physics results. We will do the same as to our cosmological work. This requires $n^2_{quant-number}$ would be >> 1, in our last set of equations, which as we indicate has the surprise condition that for Pre – Planckian space-time that a very large value for initial Pre Planckian dimensions $d_{\dim}$ for inputs into the Pre Planckian state, prior to emergence into Planckian cosmology conditions

1. Start with the General Relativity First integral.

We use the Padmanabhan 1st integral [1], of the form , with the third entry of Eq. (1) having a Ricci scalar defined via [9] and usually the curvature $\aleph$ set as extremely small, with the general relativity[2], [3]

$$\begin{aligned} & S_1 = \frac{1}{2\kappa}\int\sqrt{-g}\cdot d^4x\cdot(\Re-2\Lambda) \\ & \& -g = -\det g_{uv} \\ & \& \Re = 6\cdot\left(\frac{\ddot{a}}{a}+\left(\frac{\dot{a}}{a}\right)^2+\frac{\aleph}{a^2}\right) \end{aligned} \tag{1}$$

Also, the variation of $\delta g_{tt} \approx a^2_{\min}\phi$ as given by [4,5] will have an inflaton, ϕ given by[6] .Leading to the inflaton which is combined into other procedures for a solution to the cosmological constant problem. Here, $a_{\min}$ is a minimum value of the scale factor and is not zero, but close to it.

1a. Next for the idea from Klauder

We are going to go to page 78 by Klauder [3] of what he calls on page 78 a restricted Quantum action principle which he writes as: S_2 where we write a 1-1 equivalence as in [1] , which is also seen in [2]

$$S_2 = \int_0^T dt \cdot \left[p(t)\dot{q}(t) - H_N\left(p(t), q(t)\right)\right] \approx S_1 = \frac{1}{2\kappa}\int \sqrt{-g}\cdot d^4x \cdot \left(\Re - 2\Lambda\right) \qquad (2)$$

Our assumption is that Λ is a constant, hence we assume then the following approximation , from[2] which is the precursor of activity as given in [3,4,5, 6] we have

$$\begin{aligned}
&\frac{p_0^{\ 2}}{2} = \frac{p_0^{\ 2}(N)}{2} + N; \quad for \quad 0 < N \leq \infty, and \quad q = q_0 \pm p_0 t \\
&V_N(x) = 0; \quad for \quad 0 < x < 1 \\
&V_N(x) = N; \quad otherwise \\
&H_N\left(p(t), q(t)\right) = \frac{p_0^2}{2} + \frac{(\hbar\cdot\pi)^2}{2} + N; \quad for \quad 0 < N \leq \infty
\end{aligned} \qquad (3)$$

Our innovation is to then equate $q = q_0 \pm p_0 t \sim \phi$ and to assume small time step values. Then as in [6]

$$\Lambda \approx \frac{-\left[\frac{V_0}{3\gamma - 1} + 2N + \frac{\gamma\cdot(3\gamma-1)}{8\pi G\cdot\tilde{t}^2}\right]}{\frac{1}{\kappa}\int\sqrt{-g}\cdot d^3x} + \left.\left(6\cdot\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right)\right)\right|_{t=\tilde{t}} \qquad (4)$$

These are terms within the bubble of space-time given in [1] using the same inflaton potential. The scale factor is presumed here to obey the value of the scale factor in [7]

2. Why this is linked to gravity/massive gravitons, and possibly early universe entropy

Klauder's program[3] is to embed via Eq.(3) as a quantum mechanical well for a Pre Planckian-system for inflaton physics as given by Eq. (3). as given in Klauder's treatment of the action integral as of page 87 of [3] where Klauder talks of the weak correspondence principle, where an enhanced classical Hamiltonian, is given 1-1 correspondence with quantum effects, in a non-vanishing fashion. If so, by Novello [8] and Eq. (3) we have then for early universe conditions, that we will be leading up to using an algorithm for massive gravitons, as in [6] , and [8]

$$m_g^{\ 2} = \left(\frac{\hbar\cdot\sqrt{\Lambda}}{c}\right)^2 \approx \frac{\hbar^2}{c^2}\left[\frac{-\left[\frac{V_0}{3\gamma - 1} + 2N + \frac{\gamma\cdot(3\gamma-1)}{8\pi G\cdot\tilde{t}^2}\right]}{\frac{1}{\kappa}\int\sqrt{-g}\cdot d^3x} + \left.\left(6\cdot\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right)\right)\right|_{t=\tilde{t}}\right]_{\tilde{t}=t(Planck)} \qquad (5)$$

The long and short of it is, to tie this value of the cosmological constant, and the production of gravitons due to early universe conditions, to a relationship between De Broglie wavelength, Planck length, and if the velocity v gets to a partial value close to the speed of light, that, we have, say by using [11] as given by Diosi, in Dice (2018) for quantum systems, if we have instead of a velocity much smaller than the speed of light, a situation where the particle moves very quickly (a fraction of the speed of light) that instead of the slow massive particle postulated in [11]

$$\lambda_{De-Broglie} \approx \frac{2\pi\hbar}{m_g v} \cdot \sqrt{1-\frac{v^2}{c^2}} \cong \ell_{Planck} \approx \sqrt{\frac{\hbar G}{c^3}}$$

$$\begin{aligned} &\Rightarrow if \quad v(particle) \to c - \xi^{+}; then \\ &\varepsilon(energy - particle) \approx E_{Planck}(Planck - energy) \end{aligned} \quad (6)$$

If so then, we will be looking at using Ng version of entropy via use of infinite quantum Statistics, [12] we have for a clearly specified value of mass of the graviton, say $m_g \approx 10^{-62}\, grams$ as in [13], then we have for the negative components

We are specifying here,

$$\begin{aligned} &If \quad c \equiv 1, m_g \approx 10^{-62} grams, \\ &E_{Planck}(Planck - energy) \cong 2.18 \times 10^{-5} grams \\ &\cong \left(m_g \approx 10^{-62} grams\right) \times N(entropy - number) \\ &\Rightarrow N(entropy - number) \cong 10^{58} \equiv 10^{\mathbb{N}}, and \therefore \mathbb{N} \cong 58 \end{aligned} \quad (7)$$

3. Can this tie in with early universe e folds ? i.e. from [14] e folds are between 55 to 60

E folds in cosmology are a way of delineating if we have enough expansion of the universe is in line with inflation.in order to solve the most important cosmological problems. As seen in [11] we can have

$$\mathbb{N}(e - fold, \cos mol) \approx -\int dt \cdot H\left(\cos mol\right) \quad (8)$$

Where $H\left(\cos mol\right)$ is a value of the Friedman equation, and if we use [13] be defined via that the potential energy, V, of initial inflation is initially over shadowed by the contributions of the Friedman equation, H, at the onset of inflation. Then

$$\mathbb{N}(e - fold, \cos mol) \approx 55 - 60 \quad (9)$$

What we wish to explore will be if Eq. (9) above is consistent with

$$N(entropy - number) \cong 10^{58} \equiv 10^{\mathbb{N}}, and \therefore \mathbb{N} \cong 58 \quad (10)$$

Doing so may involve use of the Corda article, as given in [12]

4. Now for foundational treatment as to if we may have an influence of the 5th dimension in our problem.

Wesson, [13] has a procedure as far as a five dimensional uncertainty principle which is written as, if $n = L / l$

Where L is for 4th dimensions, and l is a five dimensional representation where $l = h / mc$ and we have

$$dS_5^2 = \left(\frac{L}{l}\right)^2 ds_4^2 - \left(\frac{L}{l}\right)^4 dl^2 \quad (11)$$

Such that if $n = L/l$, i..e. a ratio of L and small l, as given by saying that the above is

$$dS_5^2 = n^2 ds_4^2 - n^4 dl^2 \qquad (11a)$$

$$\begin{aligned}
&\left|dp_\alpha dx^\alpha\right| = \frac{h}{c} \cdot \frac{dn^2}{n} \\
&\xrightarrow[\alpha \to 0, \Pr e-Planckian]{} \\
&\left|dp_0 dx^0\right| = \frac{h}{c} \cdot \frac{dn^2}{n} = \left|\frac{dE}{c} \cdot cdt\right| \\
&\Rightarrow \left|dE \cdot dt\right| \approx \int \frac{h}{c} \cdot \frac{dn^2}{n} \\
&\xrightarrow[h=c=1, \Pr e-Planckian, n=L/l]{} \\
&\left|\Delta E \cdot \Delta t\right| \approx n \cdot \left(\ln n - 1\right)
\end{aligned} \qquad (12)$$

Using a n expansion of the form from CRC tables[14]

$$\ln_e n = (n-1) - \frac{1}{2} \cdot \left(n-1\right)^2 + \frac{1}{3} \cdot \left(n-1\right)^3 + \ldots \qquad (13)$$

Up to cubic roots we obtain one real root and 2 conjugate complex roots of, if we use minimum uncertainty of $\Delta E \Delta t = \hbar \to 1$, and set c=1, we have then

$$\begin{aligned}
&n_1 \approx 1.54715 \\
&n_2 \approx .42643 + 1.2242i \\
&n_3 \approx .42643 - 1.2242i
\end{aligned} \qquad (14)$$

If so for the real case, of n, we have about the Planckian regime we look at

$$l = \frac{l(Planck) = l_p}{1.54715} \qquad (15)$$

We will then look at the consequences of the real root, first, in terms of variation of minimum time step before going to other cases, but for the record, we have then the weird case of , for real root n in eq. (14) that

$$\Delta t \approx \frac{-.845184}{\Delta E} \quad real \quad iff \quad \Delta E < 0 \qquad (16)$$

5. Under what conditions would ΔE < 0? How would negative energy tie into negative Pressure which is normally expected in the onset of inflation?

First of all, look at conditions for rapid acceleration of the Universe, i.e. to have this according to the GR theory we have by [15] if a(t) is a scale factor, then the Friedman equations read as

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}\sum_j \left(\rho_j + 3p_j\right) + \frac{\Lambda_b}{3}$$
$$If \quad a(t) = a_{\min} t^{\alpha} \quad then \quad if \quad j = 1(gravitons) \tag{17}$$
$$3\alpha^2 - 3\alpha + 4\pi G_N(\rho + 3p) = \Lambda_b$$

Now, look at a concept of pressure. Here. If the first expression is tabulated about Planck time (or just before)

$$\Lambda \approx \frac{-\left[p(\tilde{t})\dot{q}(\tilde{t}) - H_N\left(p(\tilde{t}), q(\tilde{t})\right)\right]}{\frac{1}{\kappa}\int\sqrt{-g}\cdot d^3x} + \frac{\frac{1}{2\kappa}\int\sqrt{-g}\cdot d^3x\cdot\left(\Re = 6\cdot\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{\aleph}{a^2}\right)\right)\Bigg|_{t=\tilde{t}}}{\frac{1}{2\kappa}\int\sqrt{-g}\cdot d^3x} = 3\alpha^2 - 3\alpha + 4\pi G_N(\rho + 3p) = \Lambda_b \tag{18}$$

We can then make the identification that we have negative pressure, we then have if we have both pressure and energy negative then we can make the following pairing of terms, i.e. first for the negative terms in Eq.(18)

$$\begin{aligned} &If \quad a(t) = a_{\min} t^{\alpha} \quad then \quad if \quad j = 1(gravitons) \\ &3\alpha^2 - 3\alpha + 4\pi G_N(\rho + 3p) = \Lambda_b \\ &and \Lambda_b = \Lambda \quad from \quad Eq.(4) \\ &\Rightarrow -\frac{6p_{momentum}(t)\dot{q}(t)\cdot\kappa}{2\cdot\int\sqrt{-g}d^3x} = -3\alpha - 12\pi G_N\cdot\left|P_{pressure}\right| \end{aligned} \tag{19}$$

Momentum and the time derivative of "space" are in the last line of Eq(19) specified as of the interior up to the boundary of a space-time bubble defined by $a_{\min}$ for the left hand side of Eq(19) , last line.

We will after this is described go to the positive terms in Eq.(18). We get then

$$\begin{aligned} &If \quad a(t) = a_{\min} t^{\alpha} \quad then \quad if \quad j = 1(gravitons) \\ &3\alpha^2 - 3\alpha + 4\pi G_N(\rho + 3p) = \Lambda_b \\ &and \Lambda_b = \Lambda \quad from \quad Eq.(4) \\ &\Rightarrow \frac{\left|H_{Potential-well-Hamiltonian}(p_{momentum}(t)q(t))\right|\cdot\kappa}{2\cdot\int\sqrt{-g}d^3x} + 6(\alpha^2 + \frac{\aleph}{a_{\min}^2 t^{2\alpha}})\Bigg|_{t=t(Planck)} \\ &= 6\alpha + 4\pi G_N\left|\rho_{early-universe-density}\right| \end{aligned} \tag{20}$$

We will then be looking at how we can then equate out a negative energy and a negative pressure for this Pre Planckian to Planckian physics transition.

6. **Explicit calculation for a negative pressure in this Pre Planckian to Planckian physics transition**

We will transition to Reference {15} by Volovik, 2003 which has the following expression for pressure in a vacuum state of weakly interacting Bose Gases. i.e.

$$\begin{aligned}
P_{Bose-fluid} &= \frac{1}{2\hbar^3}\sqrt{-g}\cdot\left(E^3_{Planck2}E^1_{Planck1} - \frac{16}{15\pi^2}E^4_{Planck1}\right) \\
\& E^1_{Planck1} &= mc^2, \\
E^1_{Planck2} &= \hbar c/\Theta, \\
\Theta &\sim \sqrt[3]{n(particle-density)} \sim n^{1/3}
\end{aligned} \tag{21}$$

For our problem if we configure the initial contents of the "well" we assume for having a near singularity, for space-time expansion start we can have n = N/V , with N as the number of would be "gravitons, , and V being the "Volume of space-time for our evaluation". Whereas, m = N times the mass of a graviton. If so a simple calculation for this problem would have, then a negative value for pressure if we have the following, namely

$$\begin{aligned}
&E^3_{Planck2} < \left(\frac{16}{15\pi^2}\right)\times E^3_{Planck1} \\
&\Rightarrow \hbar c/\left(N_{gravitons}m_{graviton}/Vol\right)^{1/3} < \left(\frac{16}{15\pi^2}\right)^{1/3}\times N_{gravitons}m_{graviton}c^2
\end{aligned} \tag{22}$$

Here, use $m_{graviton} \le 10^{-62} - 10^{-65}\, grams$ which is from [16] , and Planck Mass $m_{Planck} \approx 2.176\times 10^{-5}\, grams$ [17]

Use, here that Vol = (Planck length, cubed) times 1/(1.54715), cubed = Planck length, cubed times 0.27002422918

Therefore if we use Planck length set equal to 1 and h bar = 1 and Planck mass = 1, we have Eq. (22) re written as

$$\left(\frac{15\pi^2}{16}\right)^{1/3}\times .27002422918 \le (.5\times 10^{-58}m_{Planck})^{4/3}N^{4/3}_{graviton} \tag{23}$$

Or roughly

$$\left(\frac{15\pi^2}{16}\right)^{1/3}\times .27002422918 \le 10^{-77}N^{4/3}_{graviton} \Rightarrow 10^{77} \le N^{4/3}_{graviton} \tag{24}$$

Or an upper bound of say for graviton mass of 10^-62 grams, we have that we have negative pressure in our system for the number of gravitons being less than 10^58 , in a volume about .27 times the cube of Planck length. This is stunning because in Eq.(7) we have an entropy number of 10^57 to 10^58, which is amazing because it suggests that the entropy generation we pick is tied in explicitly for the generation of negative pressure which is essential for inflation.

7. **Now for how we could consider having ΔE drop as negative energy, in our problem of Pre-Planckian physics right before the onset of inflation. With a flip over to ultra high temperature- energy conditions.**

From [18] we have the following relationship, i.e. see referenced [18] have in its Eq.(8) the following value

$$
\begin{aligned}
&E=\frac{d}{2}PV \\
&d=\dim, \\
&P=\Pr essure \\
&V=Volume
\end{aligned}
\tag{25}
$$

The discussion as to implementation of Eq. (25) has that if the conditions in section 6 above are obtained for negative pressure, that in the Pre Planckian state we have at a chance, a quadratic dispersion relationship. In addition, Reference [18] claims that this is a result of a derivation from the Virial theorem as given in [19], so then that we may look at

$$
\begin{aligned}
&(Heisenberg)\frac{dP}{dt}=\frac{i}{h}\left[H,P\right]\xrightarrow[{[P,X]=\frac{\hbar}{i}I}]{}\frac{dP}{dt}=-V'(X) \\
&\Leftrightarrow (Schrodinger-Ehrenfest\quad Theorem)\frac{d\left\langle P\right\rangle}{dt}=-\left\langle V'(X)\right\rangle \\
&\&(Schrodinger)\frac{d\left\langle P\right\rangle}{dt}=-\left\langle V'(X)\right\rangle\ for\quad classical\quad \frac{dP}{dt}=-V'(X) \\
&\Leftrightarrow (Heisenberg)\frac{dP}{dt}=\frac{i}{h}\left[H,P\right]\xrightarrow[{[P,X]=\frac{\hbar}{i}I}]{}\frac{dP}{dt}=-V'(X)
\end{aligned}
\tag{26}
$$

This is in a way of referring to [18] and [19] a way to ascertain the correctness of using Eq. (25) in the Pre-Planckian to Planckian transition in space-time

Having said that. We will then state that what we believe is that V as volume , as given in Eq. (25) would be roughly about .27 times the cube of Planck length, as a starting point, for investigation and that we would then have a transition up to the Planck length. Prior to nucleation of space-time

Our hypothesis, is that breaching the barrier to full emergence would entail a simultaneous flip from negative (bound energy states) to Positive energy, whereas we would be using a variant of positive energy given as'

$$
E(\inf)\sim\frac{d}{2}\cdot k_B\cdot T_{\inf} \tag{27}
$$

i.e. a release of bound state to unrestrained positive energy would be commenced from the Pre Planckian to Planckian transition.

i.e. eventually , if there is a barrier, of space-time at the surface of a sphere of about .27 times the cube of Plank length, in "volume" that when the barrier was breached, there would be a switch from negative energy, to positive energy, but that the pressure would still be negative, hence "inside" the initial near singularity sphere we would have a negative value of Eq.(27) signifying a BOUND state. Once the barrier collapsed, Eq. (27) would switch to positive, but that in lieu of inflation that the pressure of our system would still follow Eq. (21) and Eq. (22)

All this may be tied into an issue of semi classical reasoning as given below. We include this in to motivate readers to consider how a semi classical set of approximations may lead to bridging the gap between General Relativity and Quantum mechanics . We argue that the challenge in our present problem is to re duplicate the same methodology, but to also find a suitable potential system, instead of just a hierarchy of kinetic energy expressions.

8. Lesson learned, i.e. a way to ascertain if quantum gravity has a chance to be applied Quantum Geometrodynamics and Semi classical approximations, as reference [20] and evolutionary Equations, for quantum states, and its relationships to quantum issues arising in [21]

We wish now to refer to another result which we view as largely in tandem with our quest as to come up with precursors to quantum gravity, i.e. from Kieffer.

Due to how huge this literature is, we will be by necessity restricting ourselves to pages 172 to 177 of [20] as that encompasses Hamiltonian style formalism and also has some connections to the Hamilton Jacobi equation.

We will make this limitation so our methods are not too far removed from the Solvay conference, 1927, i.e. the Hamilton-Jacobi equation makes an appearance, as well as a full stationary Schrodinger equation.

In this discussion, the wave functions are often quantized, or nearly so, albeit usually added gravitational background is semi classical.

To begin our inquiry as to Geometrodynamics, which has some fidelity to the Solvay 1927 conference, we look at the following expansion of the Klein Gordon Equation, without an external potential. i.e.

$$
\begin{aligned}
&\left(\frac{\hbar^2}{c^2}\cdot\frac{\partial^2}{\partial t^2}-\hbar^2\Delta+m^2c^2\right)\Psi_{KG}=0\\
&\&\\
&\Psi_{KG}=\exp(i\cdot S_{example}/\hbar)=c^2S_0+S_1+c^{-2}S_2+\\
&\&\\
&S_0\sim\pm m\cdot t\Rightarrow\left(\Psi_{KG}at\quad c^2\right)\sim\exp(-imc^2t/\hbar)\\
&\&\\
&\left(\Psi_{KG}at\quad c^0\right)\sim\exp(iS_1/\hbar)\Rightarrow i\hbar\Psi_t=\frac{-\hbar^2}{2m}\Delta\Psi\\
&\&\\
&\left(\Psi_{KG}at\quad c^{-2}\right)\sim\exp(iS_2/\hbar)\Rightarrow i\hbar\Psi_t=\frac{-\hbar^2}{2m}\Delta\Psi-\frac{\hbar^4}{8m^3c^2}\Delta\Delta\Psi\\
&\&\\
&\frac{\hbar^4}{8m^3c^2}\Delta\Delta\Psi=\ first-relativistic-correction-term
\end{aligned}
\tag{28}
$$

As a Klein Gordon result, this leads directly to the idea of quantum mechanics, as embedded within a larger theory.

I,e this methodology as brought up by Kieffer, in page 177 of [20] in its own way is fully in sync with some of the investigations of the embedding of quantum mechanics within a larger structure, as has been mentioned in a far more abstract manner by t'Hooft, in [22], although to make further connections, it would be advisable to have a potential term put in, as well as to have more said about relativistic corrections.

As mentioned by [22] , Lammerzahl , C. in [23] has extended this sort of reasoning to quantum optics in a gravitational field. The virtue of this, is that one is NOT using the functional Schrodinger equation, as seen in page 149 of the Wheeler De Witt equations, given in [20]. i.e. the above derivation, within the context of the orders of c, given above, has explicit time dependence put in its evolution equations, and avoids some of the issues of the Wheeler De Witt program. I.e. read page 149 and beyond in [20] as to some of the perils and promises as to this approach.

In addition the c^{0} recovery of the Schrodinger equation, and the c^{-2} recovery of a Schrodinger equation within the context of the Klein Gordon equation is fully in sync with some of the Solvay 1927 deliberations. As given in [21]. And also directly linkable to [22]

What we wish to do is to re duplicate the same sort of power expansion picking off of terms given in Eq. (28) but instead of using the Klein Gordon Equation, without a potential, to use a similar equation with a potential and from there to ascertain an embedding of space time effects largely in sync with t'Hooft as given in [22] at near the Plank regime of space time. Doing so would among other things employ a re do and looking at how our evolution equation so chosen, as mentioned in Eq. (28) may be linked to the issues given in Eq. (3) and Eq. (4) of our manuscript

However, before tying an evolution Equation, from Eq. (28) suitably modified to use parts of Eq. (3) and Eq. (4) we need to consider if we have a Hamiltonian system which is the same as the ENERGY of a system. If we do not have this option, it is a good bet that the system so modeled does NOT conserve energy. Ie. What would that mean for our problem?

9. A major caution to consider, i.e. when we have a Hamiltonian which is not conserved, i.e. when Hamiltonian H no longer is in sync with the ENERGY E of a system

Very simply put, if the Hamiltonian has for any reason a time component to it, so the time derivative of a Hamiltonian is not zero, then the physically modeled system is not conserving energy. i.e. for a Lagrangian L, we have that by [24]

$$\varepsilon(energy) = \dot{q}_{\beta}\frac{\partial L}{\partial \dot{q}_{\beta}} - L \tag{29}$$

Whereas we can write if L has no time dependence , that

$$\frac{d\varepsilon(energy)}{dt} = -\frac{\partial L}{\partial t} = 0, \quad if \quad L \neq L(t) \tag{30}$$

The Lagrangian L = Kinetic energy – Potential energy, hence if we go to look at the Hamiltonian itself we have

H(Hamitonian)=Kinetic energy+ Potential energy) = Total energy E, iff

$$\frac{d\mathrm{H}}{dt} = 0 \Leftrightarrow \mathrm{H} = \varepsilon(energy) \tag{31}$$

Otherwise, we have

$$\frac{d\mathrm{H}}{dt} \neq 0 \Leftrightarrow \mathrm{H} \neq \varepsilon(energy) \tag{32}$$

What we have to decide in terms of the evolution of Eq. (3) and Eq.(4) is do we have a closed or an open physical input into the creation of the Universe. This will profoundly influence how we look at Eq. (20) above, which in turn has a lot to say about how uniformly applicable Eq. (24) actually is. i.e. if we do this, then there is a matter of the self reproduction of the Universe as given by Mukhanov, [25] where we have for a scalar field driving the expansion of the universe, with a scalar field being bigger than the square root of the mass of the universe for domain production as given in [25] , page 353.

10. What if we wish to consider Mukhanov Self reproduction of the Universe criteria?

First of all we will give pertinent background before we go to the Mukhanov criteria.

Note that from [1,26] we have

$$a(t) = a_{\min} \cdot t^{\gamma} \tag{33}$$

Leading to [1] the inflaton.

$$\phi \approx \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln\left\{\sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot t\right\} \tag{34}$$

And then we can look at the consequences for self reproduction of the universe, given on page 353 of [25] and its figure 8.7 as seen in page 353 , of [25] with a perpetuating continual expansion of the universe, given a mass, m, for which the scalar field of Eq. (34) obeys

$$\phi > m^{1/2} \tag{35}$$

The results of Eq. (35) are accessible in figure 1 below as copied from page 353 of [25]in its Figure 8.7 as seen below.

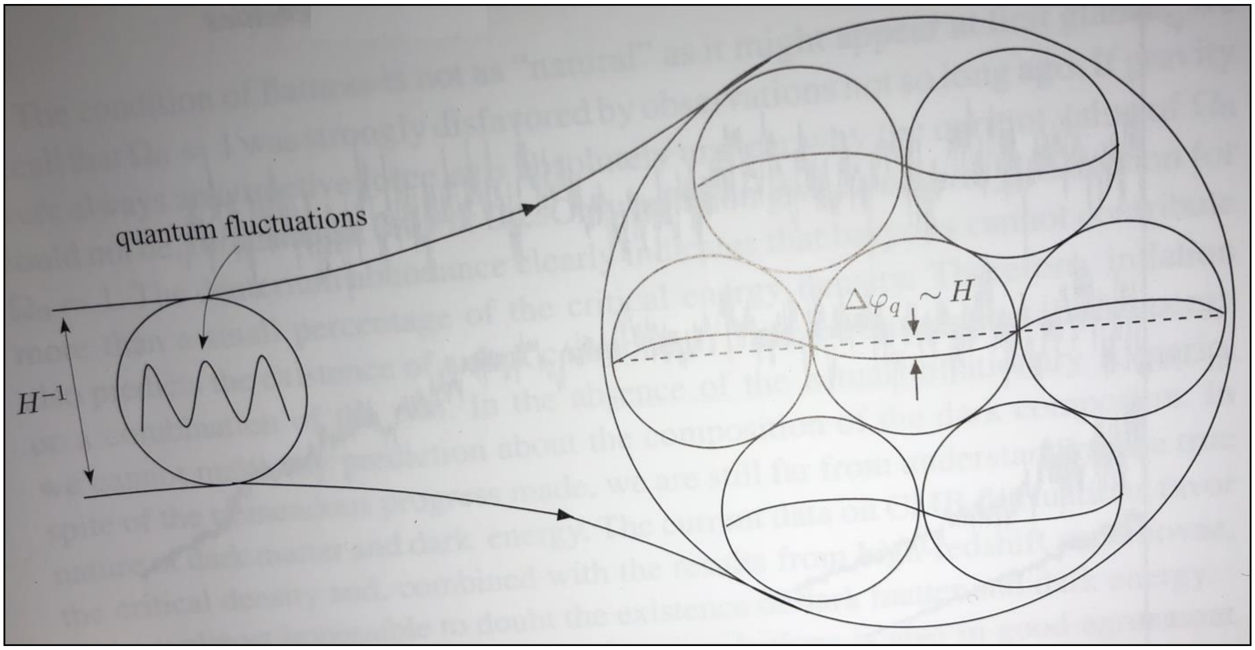

Fig 1. We have in this example a criterion for self reproduction of the universe (based on fig 8.7 of reference [25], i.e. Mukhanov and involving using Eq. (35) above

If we use Clifford Will, as in [16] for velocity of a massive graviton and make the following substitutions, we will have

$$\gamma \to \alpha$$
$$\Delta E = N_g m_g \cdot c^2 \cdot \left(1 - \frac{m_g^2}{E^2 = \hbar^2 \cdot \omega_g^2}\right) \tag{36}$$
$$\Rightarrow \Delta t_{\min} \approx \frac{\hbar}{N_g m_g \cdot c^2 \cdot \left(1 - \frac{m_g^2}{E^2 = \hbar^2 \cdot \omega_g^2}\right)} \propto t$$

Then we have the inequality for self reproduction of the universe as

$$\sqrt{\frac{8\pi G V_0}{\alpha \cdot (3\alpha - 1)}} \frac{\hbar}{N_g m_g \cdot c^2 \cdot \left(1 - \frac{m_g^2}{\hbar^2 \cdot \omega_g^2}\right)} \geq \exp\left(\sqrt{\frac{4\pi G}{\alpha}} \cdot \sqrt{N_g m_g}\right) \tag{37}$$

Also keep in mind the numerical density N, as given above, can be linked to a" particle count" due to Entropy

Then using Kolb and Turner[27] , we would see say,

$$s(entropy-density)=\frac{2\pi^2}{45}g_*\cdot\left(T_{universe}/T_{Planck}\right)^2 \quad (38)$$

And if we have utilization of N(particle count) ~ S (entropy) as given in [28] by Ng, if we solve conclusively for N_g from utilizing Eq. (37) we have that, we can re write Eq. (38) to read as implying

$$\begin{aligned}&\left[N_g/Vol(in-Planck-units)\right]\sim s(entropy-density)=\frac{2\pi^2}{45}g_*\cdot\left(T_{universe}/T_{Planck}\right)^2\\&\Rightarrow\left(T_{universe}/T_{Planck}\right)^2\approx\left(\frac{2\pi^2}{45}g_*\right)^{-1}\cdot\left[N_g/Vol(in-Planck-units)\right]\end{aligned} \quad (39)$$

Should the value of $N_g\propto 10^{58}$ as by earlier arguments in this manuscript, as stated, then if the value of

$g_*\sim$ $N_g\propto 10^{58}$ in the case that we have $\left(T_{universe}/T_{Planck}\right)^2\approx 1$, or so, in the early Pre Plankian to Planckian transition, i.e. it means that just prior to the transition to the inflationary regime that we have the following situation As given on page44 of [29]

$$\begin{aligned}&v_{shock-wave}\approx\sqrt{E/m}\\&R_{shock-wave}\approx v_{shock-wave}t\\&m\approx\rho R^3_{shock-wave}\\&\Rightarrow R_{shock-wave}\approx\left(E\cdot t^2/\rho\right)^{1/5}\end{aligned} \quad (40)$$

This shock wave has to be compared with $\Delta t\approx\frac{-.845184}{\Delta E}\quad real\quad iff\quad \Delta E<0$, whereas we were discussing a situation for the diminuation of energy at the start of expansion. As for what I am referring to, see , if we reference variation of change of temperature $\Delta T_{universe}$ against scale factor $a(t)$ as given in page 401 of [30] with $\Delta T_{universe}$ decreasing in value as to expanding scale factor size $a(t)$, hence Eq. (41) below would be negative.

$$\Delta E\approx\frac{d(\dim)}{2}\cdot k_B\cdot\Delta T_{universe} \quad (41)$$

In doing so, we would then in this case see if we use the real root of n, given in Eq. (14) above

$$\Delta t\approx\frac{2\times.845154}{d(\dim)\cdot k_B\cdot\left|\Delta T_{universe}\right|} \quad (42)$$

Then a shock front, right at the starting gate of expansion would look like for the first root of n, in Eq. (14)

$$
\begin{aligned}
&R_{shock-wave} \approx \left(E \cdot t^2 / \rho\right)^{1/5} \\
&\approx \left(2 \times (.845154)^2 / d(\dim) \cdot k_B \cdot \left|\Delta T_{universe}\right|\right)^{1/5} \times \left(1 / N_g \cdot m_g \cdot \left(V_{Volume}\right)\right)^{1/5}
\end{aligned}
\tag{43}
$$

Here the volume, in this case would be .27 times the cube of Planck length, and the mass of a graviton is approximately 10^-62 grams

11. Self reproduction of the universe may entail varying values of Eq. (43) if we look at three roots of n given in Eq. (14), which influences a minimum time step

We state that using the conjugate complex roots of n given in Eq. (14) would lead to different values of the numerator of Eq. (43) which would lead to different values of Eq. (43). We argue that this would induce chaos, and voids in subsequent evolution of space-time. i.e. a matter which we intend to numerically investigate if we have 3 different complementary n values in play used as to Eq. (14) and Eq.(43)

Keep in mind that if we use the values of $m_{Planck} \approx 10^{58} \times m_{graviton} = \hbar = k_B = 1$ due to renormalization, then Eq. (43) becomes if we also assume Planck length scaled to 1 so then we have

$$
\begin{aligned}
&R_{shock-wave} \approx \left(E \cdot t^2 / \rho\right)^{1/5} \\
&\approx \left(2 \times (.845154)^2 / d(\dim) \cdot k_B \cdot \left|\Delta T_{universe}\right|\right)^{1/5} \times \left(1 / N_g \cdot m_g \cdot \left(V_{Volume}\right)\right)^{1/5} \\
&\approx \left(2 \times (.845154)^2 / d(\dim) \cdot \left|\Delta T_{universe}\right|\right)^{1/5} \times \left(1 / .27\right)^{1/5}
\end{aligned}
\tag{44}
$$

This is obviously semi classical, and we will ask readers to consider that what may be used to add more rigor to our analysis would be the process of Bosonification, as seen in [31], page 319-369 of R. Shankar , with the caveat that we would be considering perhaps using advanced field theory , to have relativistic Dirac Fermions obeying Standard Anti Commutation rules by a Boson field theory. The Fermions would be super partners to the spin two gravitons which in SUSY are spin 3/2 gravitinos.

If SUSY is a non starter, and there have been no confirmed data sets for SUSY out of CERN, then we may have to be using gravitons and lump it.

Eq. (44) is for the real root of Eq. (14). Very likely the two complex roots of Eq. (14) would yield different numerator values for the shock wave front formula, and the mixing of all three versions of shock waves, would be itself enough to induce chaos, or at least some of the phenomenology seen in [32]. And if we are lucky in our formulation we may be able to get a potential added to the deliberations of Eq. (28), in terms of hierarchy of embedding space time in terms of a power law development. To do that though would require identifying though a suitable potential added, and we need to find that commensurate potential.

12. More as to a cosmological link to the (Weak) correspondence principle

In physics we have that the correspondence principle is commonly held to be that at large quantum numbers we have an approach to classical results. A request was given to me to quantify that, in terms of mathematics, and the closest which I can come to that is to do the following . I.e. first look at this [33]

Quote

Even if one restricts oneself to Bohr's writings, however, there is still a disagreement among Bohr scholars regarding precisely which of the several relations between classical and quantum mechanics that Bohr discovered should be designated as *the* correspondence principle. There are three primary candidate-definitions in the literature. First, there is the *frequency interpretation*, according to which the correspondence principle is a statistical asymptotic agreement between one component in the Fourier decomposition of the classical frequency and the quantum frequency in the limit of large quantum numbers. Second, there is the *intensity interpretation* according to which it is a statistical agreement in the limit of large quantum numbers between the quantum intensity, understood in terms of the probability of a quantum transition, and the classical intensity, understood as the square of the amplitude of one component of the classical motion. Finally, there is the *selection rule interpretation*, according to which the correspondence principle is the statement that each allowed quantum transition between stationary states corresponds to one harmonic component of the classical motion.

End of quote

In our situation, we most certainly would prefer the first definition, i.e. to look at

Quote

First, there is the *frequency interpretation*, according to which the correspondence principle is a statistical asymptotic agreement between one component in the Fourier decomposition of the classical frequency and the quantum frequency in the limit of large quantum numbers

End of quote

In our situation, a way to illustrate what we are doing is to look at where a large quantum number may play a role as to the approach to classical results.

Note in Eq. (3) above we have a potential system which we can state the following. Assume that the potential transitions, from Eq. (3) where we look at behavior of a 'potential' inside and outside a boundary of scale factor space time we write as $a_{\min}$

$$\begin{aligned} V_N &= 0, inside - a_{\min} \\ V_N &= N(very), outside - a_{\min} \end{aligned} \tag{45}$$

In physics, the correspondence principle states that the behavior of systems described by the theory of quantum mechanics (or by the old quantum theory) reproduces classical physics in the limit of large quantum numbers

What we are doing is to assume a large quantum number will be generated just about the transition from the interior to the exterior of $a_{\min}$

As it is, I expect that the transition of steps given in Eq. (45) will lead to the following, i.e. as we have the transition from small to large values of a potential given in Eq. (3) as stated in Eq.(45) we would then have the following as given by Pauli, [34] on page 33. We take the spin zero result since it is a BOSON and assume a similar qualitative overlap with spin 2 gravitons.

$$\Lambda \approx \frac{-\left[p(\tilde{t})\dot{q}(\tilde{t}) - H_N\left(p(\tilde{t}), q(\tilde{t})\right)\right]}{\frac{1}{\kappa}\int\sqrt{-g}\cdot d^3x}$$

$$+\frac{\frac{1}{2\kappa}\int\sqrt{-g}\cdot d^3x\cdot\left(\Re = 6\cdot\left(\frac{\ddot{a}}{a}+\left(\frac{\dot{a}}{a}\right)^2+\frac{\aleph}{a^2}\right)\right)\Bigg|_{t=\tilde{t}}}{\frac{1}{2\kappa}\int\sqrt{-g}\cdot d^3x} \tag{46}$$

$$\xrightarrow[quantum-number\to\infty]{} value \propto Vacuum-energy$$

$$Vacuum-energy \propto$$

$$\propto \frac{Energy}{Volume} \approx \left(\frac{1}{2\pi}\right)^3 \int_0^{k(crit)} k^2\sqrt{k^2+m^2}\,dk$$

$$\approx \left(\frac{1}{2\pi}\right)^3 \times \{\ \}$$

$$\{\ \} = \left\{\frac{k^4(crit)}{4} + \frac{m^2\cdot k^2(crit)}{4} - \frac{m^4}{4}\log\left(\frac{2\cdot k(crit)}{m}\right)\right\}$$

So we do not have a complete break down of our results we assume here the following substitutions, that

$$k(crit) \longrightarrow p(momentum) \tag{47}$$

Furthermore, we have then that if we use the speed of a massive graviton as given by [35], i.e. if $\hbar = c = 1, m \longrightarrow m_{graviton} = m_g, \Delta E \Delta t = 1$. Then the vacuum energy would be for Eq. (46) approximately if we used a large quantum number for Eq. (3) for the interior region approaching

$$\begin{aligned}
&p(momentum) \approx \longrightarrow m_g \cdot v(graviton) \approx m_g \cdot \sqrt{1-\left(m_g \Delta t_{\min}\right)^2} \\
&\Lambda \propto vacuum-energy \cdot 8\pi \\
&\approx m_g \cdot \left(1-m_g^2 \Delta t_{\min}^2\right)^{1/2} \cdot \left(m_g^2 \cdot \left(1-m_g^2 \Delta t_{\min}^2\right) + m_g^2\right)^{3/2} \\
&-m_g{}^3 \cdot \left(1-m_g^2 \Delta t_{\min}^2\right)^{1/2} \cdot \left(m_g^2 \cdot \left(1-m_g^2 \Delta t_{\min}^2\right) + m_g^2\right)^{1/2}
\end{aligned} \tag{48}$$

i.e. and this gets into one of the issues brought up by Christian Corda who asked about it. i.e. is there a way to reconcile the value of a cosmological constant as given by Wesson, in 5 dimensional cosmology with that of what is in official data sets. Before going to this issue, we should consider [36]

$$Vacuum-energy = \Lambda \big/ 8\pi G \tag{49}$$

As we have a value of minimum time step from Eq. (36) above, we can then conflate what we are doing with Wesson, i.e. what we did is to assume that there was a projection of space-time from 5 dimensions onto four dimensions i.e. according to this metric as given by Wesson [37], i.e. see its page 44 Eq. (2.42)

$$dS^2 = \left(\frac{l^2}{L^2}\right)\cdot\left[\left(1-\frac{2M}{r}-\frac{r^2}{L^2}\right)dt^2 - \frac{dr^2}{\left(1-\frac{2M}{r}-\frac{r^2}{L^2}\right)} - r^2 d\Omega^2\right] - dl^2 \quad (50)$$

The terms in the brackets refer to a 3 dimensional space, with four dimensional time component, whereas dl^2 is for the 5th dimension. In this context, the cosmological constant, is then according to [37], assuming that L is for four dimensional Space-time

$$\Lambda \equiv 3/L^2 \quad (51)$$

A word of explanation is due here. What I assumed in the calculation of delta t, in terms of time step is to look at a projection and interaction of the fourth and 5th dimensions to come up with the MININUM time step, and then from there to insert it into Eq. (48).

Our working assumption is as follows, i.e. that what we have, as of Eq. (48) should be virtually identical in magnitude to Eq. (51) but it should be understood that L in Eq. (51) is really the present day value of the assumed "radius of the universe". i.e. we are assuming then from Pre Planckian conditions to our present day that the Cosmological constant does not change. In any case, the approximate value of the Cosmological constant in Eq. (51) should be understood to be by observations, approximately as follows, i.e. the true dimension of Λ is a length^{-2}. or 2.888×10^{-122} in Planck units or 4.33×10^{-66} eV^2

We can get some of the observational thinking as to measurements of this constant, via [38], which is from Supernova candle results, so finally as brought up by Christian Corda, there is a matter of connecting the Pre-Planckian with Planckian results, [39], which is the backbone of the delta t term used in Eq. (48)

13. More on a linkage to Pre-Planckian to Planckian physics

One of the striking results in [39] is their treatment of entropy, as given in their Eq. (40), which is brought up to take into consideration the possibility of tunneling. i.e. the variation in entropy, Delta S, is given as

$$i\Delta S = -\frac{ikEt}{\hbar} + \frac{E}{t} \quad (52)$$

My first conclusion is that if there is a tie into the formula 27 of my manuscript that in fact what was done in [39] may be a way to tie in energy, E, with entropy, and make the analogy to Tunneling from the interior to the exterior of a boundary between pre Planckian to Planckian space time more exact.

I would be inclined to take the absolute magnitude of this above entropy expression and to assume the following, i.e. in the aftermath of tunneling right at the nexus of a boundary we would see approximately have for entropy generation, using the absolute magnitude of [39] as well as delta S ~ n(particle counting) by infinite quantum statistics as given by Ng. [10]. An advantage of Eq. (52) if confirmed would be a way to examine the Weak correspondence principle more exactly. We shall comment upon this in our conclusion. Here we take the absolute value of Eq. (52) and we will use that in our conclusion.

$$\left|\Delta S\right| \equiv \sqrt{\left|\frac{kEt}{\hbar}\right|^2 + \left|\frac{E}{t}\right|^2} \approx Particle - count \qquad (53)$$

14. Conclusion : Can we use Eq. (52) and Eq. (53) to quantify a correspondence principle in Cosmology precisely ?

I wish to Thank Christian Corda for bringing this question to my attention. The answer is maybe, but if we do that we can assume that the modeling of E, used in Eq. (53) may be commensurate for the energy levels of a spherical infinite square well, i.e. see this, [40]

Determine the energy levels and normalized wave functions $\psi(\mathbf{r})$ of a particle with zero angular momentum in a spherical "potential well" U(r) = 0, (r < a), U(r) = ∞, (r > a).

Solution:

Concepts:
Three-dimensional "square" potentials

Reasoning:
We are asked to determine the energy levels and normalized wave functions $\psi(\mathbf{r})$ of a particle in a three dimensional square potential.

Details of the calculation:
(a) The Hamiltonian is

$$H = -(h^2/(2m)(1/r)(\partial^2/\partial r^2)r + L^2/(2mr^2) + U(r). \qquad (54)$$

The wave function

$$\psi_{klm}(r,\theta,\varphi) = R_{kl}(r)Y_{lm}(\theta,\varphi) = [u_{kl}(r)/r]Y_{lm}(\theta,\varphi) \qquad (55)$$

is a product of a radial function $R_{kl}(r)$ and the spherical harmonic $Y_{lm}(\theta,\varphi)$.

The differential equation for $u_{kl}(r)$ is

$$[-(\hbar^2/(2m)(\partial^2/\partial r^2) + \hbar^2 l(l+1)/(2mr^2) + U(r)]u_{kl}(r) = E_{kl}u_{kl}(r). \qquad (56)$$

For l = 0 we have

$$[-(\hbar^2/(2m)(\partial^2/\partial r^2) + U(r)]u_{k0}(r) = E_{k0}u_{k0}(r). \qquad (57)$$

With

$U(r) = 0$, $(r < a)$, (58)

$U(r) = \infty$, $(r > a)$

the solutions are
$u_{k0}(r) = A \sin kr$, $ka = n\pi$, $k = n\pi/a$. (59)

We therefore label $u_{k0}(r)$ with the quantum number n as $u_{n0}(r)$.

$E_{n0} = n^2\pi^2\hbar^2/(2ma^2)$ (60)

are the energy levels when $l = 0$.

$\int|\psi_{n00}(r,\theta,\varphi)|^2 d^3r = 1$, (61)

$A^2\int_0^a \sin^2(n\pi r/a)dr = 1$,

$A = (2/a)^{1/2}$.
$\psi_{n00}(r,\theta,\varphi) = (1/(2\pi a)^{1/2})\sin(n\pi r/a)/r$. (62)

My off the top of my head idea is to compare the value of Eq. (60) with the value of Eq. (27) which has an explicit Temperature dependence. Making the approximation that m, in this last set of calculation is the same as the mass of a graviton, and that the term a, as given above is less than or equal to Planck length, if the resulting n, as used in Eq. (60) is large, and ties in with Eq. (27) , with that temperature dependence, we may see the start of classical to quantum correspondence, for large n, and a tie in that way to the Weak correspondence principle. This pre supposes though that Eq. (62) is not completely wrong which will be subsequently investigated.

What we can do is to look also at a relation given by Kerson Huang, in [41], as well as page 481 of the Hubble parameter given in [42]

$$\begin{aligned}
&\frac{8\pi G\rho}{3\tilde{H}^2} - \frac{\kappa_{flatness-meas}}{\tilde{H}^2 a(t)^2} = 1 \\
&\Rightarrow \kappa_{flatness-meas} = -\tilde{H}^2 a(t)^2 + 8\pi G\rho \cdot a(t)^2 \\
&= -\left(1.66 \cdot \sqrt{g_*} T^2_{temp-background}\right)^2 a(t)^2 + 8\pi G \cdot \rho \cdot a(t)^2
\end{aligned} \tag{63}$$

If we use the value of $a(t) = a_{\min} \cdot t^{\gamma}$, and

$$\kappa_{flatness-meas} = -\left(1.66\cdot\sqrt{g_*}T^2_{temp-background}\right)^2 a(t)^2 + 8\pi G\cdot\rho\cdot a(t)^2$$
$$= -\left(1.66\cdot\sqrt{g_*}T^2_{temp-background}\right)^2 a_{\min}{}^2 t^{2\gamma} + 4\pi d_{\dim} k_B T_{temp-background} G\cdot\rho\cdot a_{\min}{}^2 t^{2\gamma} / (V_{Volume-analyzed}) \tag{64}$$

In order for this Eq.(50) to be greater than equal to zero, we would need to have

$$4\pi d_{\dim} k_B T_{temp-background} G > \left(1.66\cdot\sqrt{g_*}T^2_{temp-background}\right)^2 \cdot (V_{Volume-analyzed}) \tag{65}$$

How to tie this into the matter of energy. I.e. use for Pre Planckian to Planckian transitions showing large quantum number values so that the correspondence principle in cosmology would hold would be to have using energy as given in Eq. (60)

$$4\pi d_{\dim} k_B T_{temp-background} G > \left(1.66\cdot\sqrt{g_*}T^2_{temp-background}\right)^2 \cdot (V_{Volume-analyzed})$$
$$\Rightarrow n^2_{quant-number} \le \frac{2m_g \cdot (V_{Volume-analyzed}\cdot d_{\dim}\cdot k_B\cdot G)^{1/3}}{\pi^2\hbar^2\cdot\left(1.66\cdot\sqrt{g_*}\right)^2} \tag{66}$$

For there to be an equality, which would be a necessary condition for having a defacto **correspondence principle in Cosmology, i.e. to have quantum effects for high numbers, i.e.** $n^2_{quant-number}$ >>1, one would likely have, even if we state g_* is a degree of freedom, would be that the stated dimensional values of inputs into a very large value for $d_{\dim}$ for inputs into the Pre Planckian state, prior to emergence into Planckian cosmology conditions would have to be an extremely large number. i.e. we would be looking for conditions in the pre Planckian space time for which $n^2_{quant-number}$ would be >> 1 due to an enormous value for $d_{\dim}$

In saying this, we have to be more precise than we have been wont to be in geometry of pre Planckian space time. And if $n^2_{quant-number}$ approached 1, for whatever the reason, the chances that we could evaluate Eq(53) in terms of the Correspondence principle would evaporate

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