Exploring reality

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Abstract

This examination of reality is running like an exciting detective novel. The research uses a hierarchy of spaces that show varying degrees of complexity. Finding more complicated spaces is not a big problem. This turns out to be quite simple. The move to a more complicated form is always accompanied by several significant limitations and understanding where these limitations come from is a much bigger problem. Today's mathematics is only just able to explain the limitations of the Hilbert space. This no longer applies to the system of Hilbert spaces, which in this document is called the Hilbert repository. Current math cannot explain the restrictions and the extra features of the Hilbert repository. This fact will particularly interest those who are curious about the structure and behavior of physical reality. The approach is quite different from the usual path and provides other insights. The usual way tries to deduce new insights from what we know about classical physics. The story makes it clear that only mathematics cannot provide a complete picture of reality, while experiments alone also cannot expose physical reality. The combination of mathematics and experimentation produces the best results.

1 The research

If you have plenty of time to think, you often come to areas that intrigue you greatly. The coronavirus makes us lock ourselves up and causes that we have a lot of time to think deeply about all sorts of things. What intrigues me greatly is the way our living environment has come about and how it works in the way that we can perceive. It is why

I went to study physics. Let us name what we live in physical reality. Physical reality apparently possesses all kinds of structures and mechanisms from which we can experience the effects through our senses and through tools. People, especially mathematicians and physicists, have also devised such structures and mechanisms.

2 Vector space

What is striking is that physical reality applies many kinds of space. Mathematicians and physicists have given the simplest form of space the name vector space. With combinations of simple numbers and vectors, you can use simple calculation procedures to get from a chosen position to reach all positions in the vector space. The vectors have a starting position and an end position. That gives them a direction and a length. The length is characterized with a number. Multiplying vectors by numbers shortens or elongates the vector. Multiplying with a negative scalar reverses the direction of the vector. Two vectors are added by shifting one of the vectors parallel so that its endpoint and the starting point of the other vector coincide. The starting point of the first vector and the endpoint of the second vector are now the sum vector.

There are many different number systems. The simplest number systems arrange their elements along a straight line. The numbers we have learned to count with are called natural numbers. If we add the number zero and let negative numbers appear, then the integers are created. If we add all the fractions, the rational numbers are created. With those numbers, we can approach all points on a straight line randomly close. However, that does not mean that we can end up on that line at all locations. The root of two is a location that cannot be represented with a fraction. For example, there are infinite many numbers that do not belong to the rational numbers. We call those numbers irrational. All rational numbers can be enumerated with a different natural number. That does not apply to the irrational numbers. That set of numbers is not countable. Merging the rational numbers and the irrational numbers provides the set of real numbers. The amalgamation results in a continuum. The continuum has different properties than the countable set. Continuums have more consistency. This is particularly important in the more complicated number systems that do not fit on a straight line.

The roots of the negative numbers do not fit on the same straight line. We place the roots of the negative numbers on an independent line that crosses at the position of the number zero with the line of the real numbers. The flat plane that spans the two lines now includes all the combinations of the real numbers and the roots of the negative real numbers. The combined numbers are called complex numbers. The axis of the roots of negative real numbers is called the imaginary axis. This naming is unfortunately confusing. The word imaginary also has other meanings.

The two lines represent independent vectors. The image of the first line on the second line has zero length. We can also say that the lines are perpendicular to each other, but that is a dangerous step, because both the human brain and physical reality do not allow more than three lines or vectors to be perpendicular to each other. However, vector spaces can exist in which more than three vectors are independent among themselves. The maximum number of independent vectors is called the dimension of the vector space. The vector space taken up by the complex numbers is thus two-dimensional. Complex numbers can be considered as the aggregation of two real numbers, each of which has one of the axes as domicile. Similarly, two complex numbers can be merged into one four-dimensional number. In this way, new numbers emerge, with which we can also calculate and which the discoverer more than two centuries ago has called quaternions. Quaternions consist of a real part and an imaginary part. The real part covers one dimension. The imaginary part covers the remaining three dimensions. The square of the imaginary part delivers a negative real number just as with the complex numbers. The product of two different imaginary quaternions delivers a new imaginary quaternion and a real number that together produce a new quaternion. The imaginary subspace of the quaternions acts as a three-dimensional vector space. The imaginary parts are three-dimensional vectors.

The product c= a b of two quaternions a and b consists thereby of five terms.

$c = c + \mathbf{c} = a\, b \equiv (a_r + \mathbf{a})\,(b_r + \mathbf{b}) = a_r\, b_r - \langle \mathbf{a},\mathbf{b} \rangle + \mathbf{a}\, b_r + a_r\, \mathbf{b} \pm \mathbf{a}\times\mathbf{b}$

$c_r = a_r\, b_r - \langle \mathbf{a},\mathbf{b} \rangle$

$\mathbf{c} = \mathbf{a}\, b_r + a_r\, \mathbf{b} \pm \mathbf{a}\times\mathbf{b}$

$\mathbf{a}\,\mathbf{b} = -\langle \mathbf{a},\mathbf{b} \rangle \pm \mathbf{a}\times\mathbf{b}$

$c^* = c_r - \mathbf{c}$ is the conjugated of c

$\|c\|$ is the norm of c.

$c\, c^* = \|c\|^2$

In these equations, the real part a_r of quaternion a is indicated by a suffix $_r$

The imaginary and thus vectorial part **a** of this quaternion is represented with a bold font.

$\langle \mathbf{a},\mathbf{b} \rangle$ is the inner vector product of **a** and **b.** It is a real number.

$\mathbf{a}\times\mathbf{b}$ is the outer vector product of **a** and **b**. It is a vector that is perpendicular to **a** and perpendicular to **b**.

In the quaternionic number system, the scalar real part can simply be added to the vectorial and therefore imaginary part.

The product of two quaternions is not always commutative. a b is not always equal to b a. This is because $\mathbf{a}\times\mathbf{b} = -\mathbf{b}\times\mathbf{a}$

The formula indicates that there are right-handed quaternions and left-handed quaternions. The user must make a choice in advance.

Two quaternions can form one octonion. Thus, the number systems of the octonions cover an eight-dimensional vector space that divides into a one-dimensional real part and a seven-dimensional imaginary part. It seems that physical reality does use quaternions but no octonions. That is because the product of octonions is not always associative. For octonions, (a b) c is not always equal to a (b c).

Apart from in different dimensions, number systems exist in a large number of versions and these versions differ in the way in which a Cartesian and a polar coordinate system specify the order of the elements. This choice determines the geometric symmetry of the version.

2.1 Inner products

Inner products play a major role in vector spaces that have more than one dimension. Inner vector products of independent vectors are always zero. The number of independent vectors determines the dimension of the vector space. Various sets of independent vectors can span the same vector space. If all participating vectors have length 1, then we call such a system a base. By depicting a chosen vector with length 1 on all members of such a base and determining the length and direction of the image, a row of numbers is created that precisely captures the direction of the chosen vector. In some forms of vector spaces, the image of the chosen vector on the base vector is characterized by a more general number than a simple length size, Instead, a complex number or a quaternion characterizes the scaling number.

The inner product of two vectors α and β is indicated as

$\langle \alpha, \beta \rangle = \langle \beta, \alpha \rangle^* = \sum \alpha_i \beta_i^*$

Here are α_i and β_i the image numbers that belong to base component e_i. The superfix* marks the conjugate of the number.

3 Other spaces

3.1 The Hilbert spaces

The possibilities of physical reality do not stop at the number systems described above. The vector space can also be applied in other ways. It is possible to map vector spaces onto themselves. In this way some vectors can be mapped on their own direction line. A scale factor then indicates what the image does to the vector. After correcting for the length of the original vector, the inner product of the original vector and the map provides the scale factor. The vectors in which this happens are called eigenvectors. The scale factors are designated to be eigenvalues of the map. In a simple vector space, this scale factor is a rational number. A Hilbert space provides each vector pair with an inner product. The number value of the inner product can be a real number, or it can be a complex number, or it can be a quaternion. The eigenvalues of an image together form the eigenspace of the map. Because the map manages the eigenspace, the name operator is also used for the map. David Hilbert and others discovered this curious behavior of this type of vector space. John von Neumann, David Hilbert's assistant, ensured that this vector space was given the name Hilbert space. The fact that the Hilbert space can only use a small number of number systems for the number value of the inner product was assumed early on but was only supported by hard mathematical evidence many decades later.

The Hilbert space acts as an abstract structured archive for collections of elements of a chosen type of number system. For example, the reference operator manages the chosen version of the number system that the Hilbert space applies to specify the inner products of vector pairs in its eigenspace. This gives the Hilbert space a private parameter space. An entire category of operators shares the reference operator's eigenvectors and replaces the corresponding eigenvalue with the target value of a chosen function corresponding to the parameter. In this way, eigenspaces are created that are described by functions. The parameter space therefore plays the role of the root geometry of the Hilbert space. This root geometry has a fixed Cartesian coordinate system, that provides the geometric center and geometric symmetry of the Hilbert space. This symmetry characterizes all the eigenspaces of the Hilbert space in question.

The normalized eigenvectors of the reference operator form an orthonormal basis of the Hilbert space. We call the subspace spanned by this base the configuration space. We know that in addition to this base, there is also a spectral base that is completely separate from the base of the configuration space. This does not mean that both types of base vectors are independent among themselves. It means that any base vector of one type can be written as a linear combination of all base vectors of the other type. A transformation converts one base into the other. An example of such a transformation is the Fourier transformation. A function that is defined in configuration space, will be presented as the Fourier transform of that function in the momentum space. The Fourier transform converts the configuration space into the momentum space. The inner products of base vectors that belong to different base types define the Fourier transform. Fourier transforms are described in complex number-based Hilbert spaces. The eigenvectors of the reference operator that belong to eigenvalues that belong to a selected spatial direction span a complex number-based Hilbert space. This enables the conversion of the Fourier transform and the momentum space to the quaternionic Hilbert space.

The Hilbert spaces described so far have a countable number of dimensions and therefore the eigenspaces of their operators can also be counted. It can be inferred from the existence of the category of operators who share the reference operator's eigenvectors that each Hilbert space that has countable infinite dimensions owns a unique companion Hilbert space that is non-separable and embeds its separable partner. The non-separable Hilbert space also supports operators who have a continuum eigenspace. Often these eigenspaces can be described by continuous functions.

The collection of the closed subspaces of a Hilbert space forms a relational set that mathematicians call an orthomodular lattice.

3.2 The Hilbert repository

The definition of a Hilbert space subtly differs from the more basic vector space. The Hilbert space is a vector space that defines an inner vector product for each vector pair. Moreover, a Hilbert space is complete.

The difference with the underlying vector space allows many separable Hilbert spaces to share the same underlying vector space. This imposes major restrictions on the Hilbert spaces that can participate in the system of Hilbert spaces. Only Hilbert spaces whose axes of the Cartesian coordinate system run parallel to each other can participate in the system. In addition, it must be quaternionic Hilbert spaces. There are plenty of Hilbert spaces left. One of the Hilbert spaces plays the role of background platform and provides the background parameter space and the background symmetry. The dimension of this Hilbert space is infinite, and its non-separable partner is also part of the background platform. The combined storage capacity is enormous. We call the result a ***Hilbert repository.*** If the Hilbert repository is used to store physical objects in it, then this storage medium can store all the dynamic geometric data of all objects that ever appear in the universe. In addition, the system has storage space for the fields that appear in the universe and that includes the universe field itself. The restrictions mentioned also lead to additional properties. All participating separable Hilbert spaces glide with their geometric center over the background parameter space. The difference in geometric symmetry between the participating Hilbert spaces and the background symmetry leads to the presence of a source or a sink at the location of the geometric center of the sliding participant. This source or sink corresponds to a symmetry-related charge that generates a symmetry-related field. The value of the charge can be zero.

4 Elementary particles

These special features do not follow from the mathematics known to us. We can deduce them from measurements made to physical reality. The Hilbert repository has a complicated structure and a special behavior reminiscent of the particle collection described in the Standard Model of the experimental physicists. The sliding platforms behave like elementary particles. In addition, they divide into categories distinguished by their electrical charge. The categories are indicated by type designations. For example, electrons, neutrinos, and quarks form together the elementary fermions. The quarks form a subset and are again divided into types.

This is a promising result, but elementary particles have something that in the offered description the sliding platforms do not yet possess. All elementary particles show a footprint that deforms the surrounding field. Let us assume that the sliding platforms support an operator who takes care of the footprint. The elementary particles have a wave function whose square of its modulus results in a detection probability density distribution. The operators of the sliding platform cannot store this continuous detection probability density distribution in its eigenspace. However, it is possible to store a cord of quaternions consisting of combinations of time stamps and hop landing locations that describe an ever-continuing hop path that regularly regenerates a coherent swarm of hop landing locations. If the hopping object is point-shaped, then this swarm is described by a stable location density distribution that corresponds to the mentioned detection probability density distribution.

Mechanisms that produce such coherent swarms with stable location density distributions are common in nature and optical experts often use them. This does not mean that science knows how and why these mechanisms work. I describe these mechanisms as stochastic processes that have a characteristic function. Another description is a process in which a Poisson process combines with a binomial process. The binomial process is implemented by a spatial point spread function. The point spread function plays the role of the location density distribution. The characteristic function of the process equals the Fourier transform of the location density distribution. In optics, this characteristic function is called the Optical Transfer Function.

4.1 Two episodes

The footprint operator is already present at the time of the creation of the Hilbert repository and determines the behavior of the elementary particle throughout its existence. This fact is a great mystery. The math does not offer an explanation yet. However, the existence of the footprint operator makes it possible to divide the model of physical reality into a preparatory episode in which there is no flowing time and an ongoing episode in which a continuing step-by-step embedding of the hop landing locations mimics the activities of the stochastic processes. The embedding process uses the stored and ordered time stamps to realize the corresponding hop landings. The range of running time is equal to the range of the archived time stamps. At the beginning of the running time, the field that represents our universe is still virginal and corresponds to the background parameter space. After the first footprints completed, the relevant elementary particles can start to form composite objects.

5 Deformation of the embedding field

With the hop path we are not there yet, because the hop landing only causes a deformation of the embedding field if the difference between the symmetry of the sliding platform and the background symmetry is isotropic. We know this from the results of field equations that describe field excitations. In quarks, the difference in symmetry is not isotropic. They must first combine into isotropic hadrons to be able to cause deformation of the embedding field. The hop landings of all other elementary particles can easily produce pulses that produce spherical pulse responses. These spherical pulse responses behave like spherical shock fronts. The front moves at the speed of light away in all directions from the location of the pulse. The amplitude of the pulse decreases proportionally with the distance to the starting location. Integrated over time, the spherical shock front results in the Green's function of the field. The pulse injects the volume of the Green's function into the field and that volume remains in the field. Initially, the pulse deforms the field around the location of the pulse, but the front spreads this deformation throughout the field. This quickly blurs the deformation. To get a permanent deformation, the hop landings in the area must be carried out continuously. The hop path takes care of that. Far from the center at distance r from the footprint, the gravitational potential is equal to V(r)=MG/r. This defines the mass M of the elementary particle.

To be able to say anything about the shape of the gravitational potential near the elementary particle, we need to know the location density distribution. If this is a Gaussian distribution, the gravitational potential equals V(r)=MG ERF(r)/r. For large values of r, this approaches the previously mentioned form. The formula given here is also an approach because part of the deformation fades away early. This new shape is a smooth function and does not show the singularity that the earlier formula shows. The new formula shows how the universe shows near an elementary particle. Far enough away from the particle, the universe is virtually undistorted.

6 Photons

The above story says nothing about photons. Photons are not elementary particles and have a completely different structure. They also move in the universe, but they do not deform that dynamic field. As a result, they do not possess mass. Photons are one-dimensional beaded cords in which each bead is formed by an energy package. These energy packages move at an equal distance from each other. The distance between them determines the color of the photon. The packages are also one-dimensional, and they behave like shock fronts that move at the speed of light. The fronts retain their shape and their amplitude. Photons have no mass and no symmetry-related charge.

Each energy package owns a vector that determines the polarization of the photon.

7 Elementary modules

Elementary particles can behave as elementary modules. Together, the elementary modules form all the modules that occur in the universe. Some modules form modular systems.

Stochastic processes that own a characteristic function, control the definition of composed modules. This characteristic function is the dynamic superposition of the characteristic functions of the components.

In Fourier space where these characteristic functions reside the dynamic superposition coefficients have a different function than in configuration space. They act as displacement generators and determine the internal positions of the components.

In addition to the elementary particles and the modules and modular systems that are assembled, black holes also possess a quantity of mass.

Symmetry-related charges occur only on the geometric centers of elementary particles.

This describes an important part of physical reality.

References

This document is part of the Hilbert Book Model Project;
https://www.researchgate.net/project/The-Hilbert-Book-Model-Project

The topic has been discussed more extensively in
https://www.researchgate.net/publication/339744488_Representing_basic_physical_fields_by_quaternionic_fields