The Grand Structure of Physical Reality

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Summary

This document concerns the discovery of the grand structure of physical reality in terms of mathematical constructs and mechanisms. The grand structure emerges from simple foundations and leads via spaces that show an increasing complexity to a system of Hilbert spaces that will be called Hilbert repository. This structure acts as a powerful storage medium that physical reality applies to store and retrieve prepared data. The repository can easily capture the dynamic geometric data of all objects that will ever exist in our universe. It can also archive the universe as a dynamic manifold and all other physical fields that play in the lifetime of the universe. This enables physical reality to prepare data in a creation episode in which no running time exists and play the story of all archived objects during a running episode in which a flowing time indicates the progression of that life story as an ongoing embedding process.

The investigation concerns a hierarchy of spaces that show increasing degrees of complexity. Finding more complicated spaces is not a big problem. This turns out to be quite simple. The move to a more complicated platform appears always accompanied by several significant restrictions and understanding where these limitations come from is a much bigger problem. Today's mathematics is only just able to explain the restrictions of the Hilbert space. This no longer applies to the system of Hilbert spaces, which in this document is called the Hilbert repository. Current math cannot explain the restrictions and the extra features of the Hilbert repository. This fact will particularly

interest those who are curious about the structure and behavior of physical reality. The approach is quite different from the usual path and provides other insights. The usual way tries to deduce new insights from what we know about classical physics. The story makes it clear that only mathematics cannot provide a complete picture of reality, while experiments alone also cannot expose physical reality. The combination of mathematics and experimentation produces the best results.

The behavior of fields plays an important role in most theoeries. Basic physical fields are dynamic fields like our universe and the fields that are raised by electric charges. These fields are dynamic continuums. Most physical theories treat these fields by applying gravitational theories or by Maxwell equations. Mathematically these fields can be represented by quaternionic fields. Dedicated normal operators in quaternionic non-separable Hilbert spaces can represent these quaternionic fields in their continuum eigenspaces. Quaternionic functions can describe these fields. Quaternionic differential and integral calculus can describe the behavior of these fields and the interaction of these fields with countable sets of quaternions. All quaternionic fields obey the same quaternionic differential equations. The basic fields differ in their start and boundary conditions.

The paper introduces the concept of the Hilbert repository. It is part of a hierarchy of structures that mark increasingly complicated realizations of a purely mathematical model that describes and explains most features of observable physical reality. That model is the Hilbert Book Model.

The paper treats the mathematical and experimental underpinning of the Hilbert Book Model.

1 Introduction

The fact that physical objects can be represented and modeled by mathematical constructs is applied in many physical theories.

Quite often function theory is applied and less frequently the representation is embedded in a topological space, such as a Hilbert space. The Hilbert space has the advantage that it can act as a repository for dynamic geometrical data and for dynamic fields. If a system of Hilbert spaces is applied, then a very powerful and flexible modeling platform results that easily can cope with the diversity and the dynamics of objects that are encountered in the universe. This system of Hilbert spaces wiil be called Hilbert repository and forms the base model of the Hilbert Book Model. Mathematics severely restricts the possibilities of this platform. This appears an advantage rather than a discredit because it limits the extension of the model in arbitrary directions. Consequently, the extension of the platform can only develop in a direction that leads to the structure and behavior of the type of physical reality that we observe. This results in an astonishing correspondence between the Hilbert repository and what experimental physicists call the Standard Model of particle physics.

2 A set of interrelated structures

The Hilbert repository is a concept that is a member of a range of concepts that mark increasing levels of complexity of a purely mathematical model of physical reality. This range covers

Vector spaces

Number systems

The Hilbert lattice and the Hilbert space

The Hilbert repository

The Hilbert Book Model

2.1 Vector space

What is striking is that physical reality applies many kinds of space. Mathematicians and physicists have given the simplest form of space the name vector space. With combinations of simple numbers and vectors, you can use simple calculation procedures to get from a chosen position to reach all positions in the vector space. The vectors have a starting position and an end position. That gives them a direction and a length. The length is characterized with a number. Multiplying vectors by numbers shortens or elongates the vector. Multiplying with a negative scalar reverses the direction of the vector. Two vectors are added by shifting one of the vectors parallel so that its endpoint and the starting point of the other vector coincide. The starting point of the first vector and the endpoint of the second vector are now the sum vector.

2.2 Number systems

There are many different number systems. The simplest number systems arrange their elements along a straight line. The numbers we have learned to count with are called natural numbers. If we add the number zero and let negative numbers appear, then the integers are created. If we add all the fractions, the rational numbers are created. With those numbers, we can approach all points on a straight line randomly close. However, that does not mean that we can end up on that line at all locations. The root of two is a location that cannot be represented with a fraction. For example, there are infinite many numbers that do not belong to the rational numbers. We call those numbers irrational. All rational numbers can be enumerated with a different natural number. That does not apply to the irrational numbers. That set of numbers is not countable. Merging the rational numbers and the irrational numbers provides the set of real numbers. The amalgamation results in a continuum. The continuum has different properties than the countable set. Continuums have more consistency.

This is particularly important in the more complicated number systems that do not fit on a straight line.

2.2.1 Complex numbers

The roots of the negative numbers do not fit on the same straight line. We place the roots of the negative numbers on an independent line that crosses at the position of the number zero with the line of the real numbers. The flat plane that spans the two lines now includes all the combinations of the real numbers and the roots of the negative real numbers. The combined numbers are called complex numbers. The axis of the roots of negative real numbers is called the imaginary axis. This naming is unfortunately confusing. The word imaginary also has other meanings.

The two lines represent independent vectors. The image of the first line on the second line has zero length. We can also say that the lines are perpendicular to each other, but that is a dangerous step, because both the human brain and physical reality do not allow more than three lines or vectors to be perpendicular to each other. However, vector spaces can exist in which more than three vectors are independent among themselves. The maximum number of independent vectors is called the dimension of the vector space. The vector space taken up by the complex numbers is thus two-dimensional. Complex numbers can be considered as the aggregation of two real numbers, each of which has one of the axes as domicile.

2.2.2 Quaternions

Similarly, two complex numbers can be merged into one four-dimensional number. In this way, new numbers emerge, with which we can also calculate and which the discoverer about two centuries ago has called quaternions. Quaternions consist of a real part and an imaginary part. The real part covers one dimension. The imaginary part covers the remaining three dimensions. The square of the imaginary part delivers a negative real number just as with the complex numbers. The product of two different imaginary quaternions delivers a new imaginary quaternion and a real number that together produce a new quaternion. The imaginary subspace of the quaternions acts as a three-dimensional vector space. The imaginary parts are three-dimensional vectors.

The product c= a b of two quaternions a and b consists thereby of five terms.

$c = c + \mathbf{c} = a\, b \equiv (a_r + \mathbf{a})\,(b_r + \mathbf{b}) = a_r\, b_r - \langle \mathbf{a},\mathbf{b}\rangle + \mathbf{a}\, b_r + a_r\, \mathbf{b} \pm \mathbf{a}\times\mathbf{b}$

$c_r = a_r\, b_r - \langle \mathbf{a},\mathbf{b}\rangle$

$\mathbf{c} = \mathbf{a}\, b_r + a_r\, \mathbf{b} \pm \mathbf{a}\times\mathbf{b}$

$\mathbf{a}\,\mathbf{b} = -\langle \mathbf{a},\mathbf{b}\rangle \pm \mathbf{a}\times\mathbf{b}$

$c^* = c_r - \mathbf{c}$ is the conjugated of c

$\|c\|$ is the norm of c.

$c\, c^* = \|c\|^2$

In these equations, the real part a_r of quaternion a is indicated by a suffix $_r$

Here, the imaginary and thus vectorial part **a** of this quaternion is represented with a bold font.

$\langle \mathbf{a},\mathbf{b}\rangle$ is the inner vector product of **a** and **b.** It is a real number.

$\mathbf{a}\times\mathbf{b}$ is the outer vector product of **a** and **b**. It is a vector that is perpendicular to **a** and perpendicular to **b**.

In the quaternionic number system, the scalar real part can simply be added to the vectorial and therefore imaginary part.

The product of two quaternions is not always commutative. a b is not always equal to b a. This is because $\mathbf{a}\times\mathbf{b} = -\mathbf{b}\times\mathbf{a}$

The ± sign in the formula indicates that there are right-handed quaternions and left-handed quaternions. In calculations these types cannot be mixed. The user must make a choice in advance.

2.2.3 Octonions and higher dimensions

Two quaternions can form one octonion. Thus, the number systems of the octonions cover an eight-dimensional vector space that divides into a one-dimensional real part and a seven-dimensional imaginary part. It seems that physical reality does use quaternions but no octonions. That is because the product of octonions is not always associative. For octonions, (a b) c is not always equal to a (b c).

2.2.4 Coordinates

Apart from in different dimensions, number systems exist in a large number of versions and these versions differ in the way in which a Cartesian and a polar coordinate system specify the order of the elements. This choice determines the geometric symmetry of the version.

2.2.5 Inner products

Inner products play a major role in vector spaces that have more than one dimension. Inner vector products of independent vectors are always zero. The number of independent vectors determines the dimension of the vector space. Various sets of independent vectors can span the same vector space. If all participating vectors have length 1, then we call such a system an orthonormal base. By depicting a chosen vector with length 1 on all members of such a base and determining the length and direction of the image, a row of numbers is created that precisely captures the direction of the chosen vector. In some forms of vector spaces, the image of the chosen vector on the base vector is characterized by a more general number than a simple length size, Instead, a complex number or a quaternion characterizes the scaling number.

The inner product of two vectors α and β is indicated as

$\langle\alpha, \beta\rangle = \langle\beta, \alpha\rangle^* = \sum \alpha_i \beta_i^*$

Here are α_i and β_i the image numbers that belong to base component e_i. The superfix* marks the conjugate of the number.

3 Hilbert spaces

Hilbert spaces emerge from orthomodular lattices because the set of closed subspaces of a separable Hilbert space is a Hilbert lattice, which is lattice isomorphic with an orthomodular lattice [3][4]. Only a subtle difference exists between a Hilbert space and its underlying vector space. A separable Hilbert space is a complete vector space that features an inner product. The value of the inner product must be a member of an associative division ring [5]. Only three suitable number systems exist that are associative division rings. These are the real number system, the complex number system, and the quaternionic number system. Octonions and biquaternions do not fit. Depending on their dimension these number systems exist in many versions that distinguish in the Cartesian and polar coordinate systems that sequence the members of the version. Each Hilbert space manages the selected version of the number system in the eigenspace of a dedicated normal operator that the author calls the ***reference operator***. This eigenspace acts as the ***private parameter space*** of the Hilbert space. A category of normal operators exists for which the members share the eigenvectors of the reference operator and apply a selected function and the parameter value that belongs to the eigenvector to generate a new eigenvalue by taking the target value of the function as the new eigenvalue. In this way, the eigenspace of the new operator becomes a sampled field.

Hilbert spaces are also known as function spaces [3]. This paper does not employ this possibility. Instead, it uses the approach of Paul Dirac to couple eigenvectors directly to eigenvalues, as is shown above for the reference operator and for the operators that share the eigenvectors of the reference operator to define their eigenspaces via selected functions. Later we define the footprint operator that archives the production of the private stochastic process, which generates the ongoing hopping path of an elementary particle. The physical fields are also expressed in this way.

3.1 Bra's and ket's

Paul Dirac introduced a handy formulation for the inner product that applies a bra and a ket [3].

Mainstream physics tends to exploit complex number-based Hilbert spaces and most theories apply only one Hilbert space. In contrast, this paper applies a system of quaternionic Hilbert spaces. If members of a number systems are mentioned in relation to a Hilbert space, then a selected version of that number system is meant. A Cartesian and polar coordinate system determine that version.

The bra $\langle \vec{f} |$ is a covariant vector, and the ket $|\vec{g}\rangle$ is a contravariant vector. The inner product $\langle \vec{f} | \vec{g} \rangle$ acts as a metric. It has a quaternionic value. Since the product of quaternions is not commutative, care must be taken with the format of the formulas.

For bra vectors hold

$$\langle \vec{f} | + \langle \vec{g} | = \langle \vec{g} | + \langle \vec{f} | = \langle \vec{f} + \vec{g} | \tag{3.1.1}$$

$$(\langle \vec{f} + \vec{g} |) + \langle \vec{h} | = \langle \vec{f} | + (\langle \vec{g} + \vec{h} |) = \langle \vec{f} + \vec{g} + \vec{h} | \tag{3.1.2}$$

For ket vectors hold

$$\left|\vec{f}\right\rangle + |\vec{g}\rangle = |\vec{g}\rangle + \left|\vec{f}\right\rangle = \left|\vec{f} + \vec{g}\right\rangle \tag{3.1.3}$$

$$\left(\left|\vec{f} + \vec{g}\right\rangle\right) + \left|\vec{h}\right\rangle = \left|\vec{f}\right\rangle + \left(\left|\vec{g} + \vec{h}\right\rangle\right) = \left|\vec{f} + \vec{g} + \vec{h}\right\rangle \tag{3.1.4}$$

For the inner product holds

$$\left\langle \vec{f} \mid \vec{g}\right\rangle = \left\langle \vec{g} \mid \vec{f}\right\rangle^* \tag{3.1.5}$$

For quaternionic numbers α and β hold

$$\left\langle \alpha \vec{f} \mid \vec{g}\right\rangle = \left\langle \vec{g} \mid \alpha \vec{f}\right\rangle^* = \left(\left\langle \vec{g} \mid \vec{f}\right\rangle \alpha\right)^* = \alpha^* \left\langle \vec{f} \mid \vec{g}\right\rangle \tag{3.1.6}$$

$$\left\langle \vec{f} \mid \beta\vec{g}\right\rangle = \left\langle \vec{f} \mid \vec{g}\right\rangle \beta \tag{3.1.7}$$

$$\begin{aligned}\left\langle (\alpha + \beta) \vec{f} \mid \vec{g}\right\rangle &= \alpha^* \left\langle \vec{f} \mid \vec{g}\right\rangle + \beta^* \left\langle \vec{f} \mid \vec{g}\right\rangle \\ &= (\alpha + \beta)^* \left\langle \vec{f} \mid \vec{g}\right\rangle\end{aligned} \tag{3.1.8}$$

Thus

$$\alpha \left|\vec{f}\right\rangle \tag{3.1.9}$$

$$\left\langle \alpha \vec{f}\right| = \alpha^* \left\langle \vec{f}\right| \tag{3.1.10}$$

$$|\alpha\vec{g}\rangle = |\vec{g}\rangle \alpha \tag{3.1.11}$$

We made a choice. Another possibility would be $\left\langle \alpha \vec{f}\right| = \alpha \left\langle \vec{f}\right|$ and $|\alpha\vec{g}\rangle = \alpha^* |\vec{g}\rangle$

In mathematics a topological space is called separable if it contains a countable dense subset; that is, there exists a sequence $\left\{\left|\vec{f}_i\right\rangle\right\}_{i=\infty}^{i=0}$ of elements of the space such that every nonempty open subset of the space contains at least one element of the sequence.

Its values on this countable dense subset determine every continuous function on the separable space $\mathfrak{H}$.

The Hilbert space $\mathfrak{H}$ is separable. That means that a countable row of elements $\left\{\left|\vec{f}_n\right\rangle\right\}$ exists that spans the whole space.

If $\left\langle \vec{f}_m \mid \vec{f}_n \right\rangle = \delta(m,n)$ [1 if n=m; otherwise 0], then $\left\{\left|\vec{f}_n\right\rangle\right\}$ is an orthonormal base of Hilbert space $\mathfrak{H}$.

A ket base $\left\{\left|\vec{k}\right\rangle\right\}$ of $\mathfrak{H}$ is a minimal set of ket vectors $\left|\vec{k}\right\rangle$ that span the full Hilbert space $\mathfrak{H}$.

Any ket vector $\left|\vec{f}\right\rangle$ in $\mathfrak{H}$ can be written as a linear combination of elements of $\left\{\left|\vec{k}\right\rangle\right\}$.

$$\left|\vec{f}\right\rangle = \sum_k \left|\vec{k}\right\rangle\left\langle \vec{k} \mid \vec{f}\right\rangle \qquad (3.1.12)$$

A bra base $\left\{\left\langle\vec{b}\right|\right\}$ of $\mathfrak{H}^\dagger$ is a minimal set of bra vectors $\left\langle\vec{b}\right|$ that span the full Hilbert space $\mathfrak{H}^\dagger$.

Any bra vector $\left\langle\vec{f}\right|$ in $\mathfrak{H}^\dagger$ can be written as a linear combination of elements of $\left\{\left\langle\vec{b}\right|\right\}$.

$$\left\langle\vec{f}\right| = \sum_b \left\langle \vec{f} \mid \vec{b}\right\rangle\left\langle \vec{b}\right| \qquad (3.1.13)$$

Usually, a base selects vectors such that their norm equals 1. Such a base is called an orthonormal base

3.1.1 Operator construction

$\left|\vec{f}\right\rangle\left\langle\vec{g}\right|$ is a constructed operator.

$$\left|\vec{g}\right\rangle\left\langle\vec{f}\right| = \left(\left|\vec{f}\right\rangle\left\langle\vec{g}\right|\right)^\dagger \qquad (3.1.14)$$

The superfix † indictes the adjoint version of the operator.

For the orthonormal base $\{|\vec{q}_i\rangle\}$ consisting of eigenvectors of the reference operator, holds

$$\langle \vec{q}_n | \vec{q}_m \rangle = \delta_{nm} \qquad (3.1.15)$$

The ***reverse bra-ket method*** enables the definition of new operators that are defined by quaternionic functions.

$$\langle \vec{g} | F | \vec{h} \rangle = \sum_{i=1}^{N} \{ \langle \vec{g} | \vec{q}_i \rangle F(q_i) \langle \vec{q}_i | \vec{h} \rangle \} \qquad (3.1.16)$$

The symbol F is used both for the operator F and the quaternionic function $F(q)$. This enables the shorthand

$$F \equiv |\vec{q}_i\rangle F(q_i) \langle \vec{q}_i| \qquad (3.1.17)$$

It is evident that for the adjoint operator

$$F^{\dagger} \equiv |\vec{q}_i\rangle F^{*}(q_i) \langle \vec{q}_i| \qquad (3.1.18)$$

For ***reference operator*** $\mathfrak{R}$ holds

$$\mathfrak{R} = |\vec{q}_i\rangle q_i \langle \vec{q}_i| \qquad (3.1.19)$$

If $\{q_i\}$ consists of all rational values of the version of the quaternionic number system that $\mathfrak{H}$ applies then the eigenspace of $\mathfrak{R}$ represents the private parameter space of the separable Hilbert space $\mathfrak{H}$. It is also the parameter space of the function $F(q)$ that defines the operator F in the formula (3.1.17).

3.1.2 Operator types

I is used to indicate the identity operator.

For normal operator N holds $NN^{\dagger} = NN^{\dagger}$.

For unitary operator U holds $UU^{\dagger} = U^{\dagger}U = I$

For Hermitian operator H holds $H = H^{\dagger}$

For normal operator O holds that $O + O^{\dagger}$ is Hermitian

For anti-Hermitian operator A holds $A = -A^{\dagger}$

For normal operator O holds that $O - O^{\dagger}$ is anti-Hermitian

3.2 Hilbert space facts

Hilbert spaces are vector spaces that support an inner product for each vector pair. In addition, the Hilbert space is complete. The inner product is applied as a metric that closes the subspaces of the Hilbert space. Hilbert spaces can only cope with number systems that are associative division rings. Thus, only real numbers, complex numbers, and quaternions can suit as values of the inner products and as eigenvalues of operators that map Hilbert spaces onto themselves.

3.2.1 Non-separable Hilbert space

Every infinite-dimensional separable Hilbert space $\mathfrak{H}$ owns a unique non-separable companion Hilbert space $\mathcal{H}$ that embeds its separable partner. This is achieved by the closure of the eigenspaces of the reference operator and the defined operators. In this procedure, on many occasions, the notion of the dimension of subspaces loses its sense.

Gelfand triple and ***Rigged Hilbert space*** are other names for the general non-separable Hilbert spaces.

In the non-separable Hilbert space, for operators with continuum eigenspaces, the reverse bra-ket method turns from a summation into an integration.

$$\langle \vec{g} | F | \vec{h} \rangle \equiv \int \iiint \left\{ \langle \vec{g} | \vec{q} \rangle F(q) \langle \vec{q} | \vec{h} \rangle \right\} dV d\tau \qquad (3.2.1)$$

Here we omitted the enumerating subscripts that were used in the countable base of the separable Hilbert space.

The shorthand for the operator F is now

$$F \equiv |\vec{q}\rangle F(q) \langle \vec{q}| \qquad (3.2.2)$$

For eigenvectors $|q\rangle$, the function $F(q)$ defines as

$$F(q) = \langle \vec{q} | F\vec{q} \rangle = \int \iiint \left\{ \langle \vec{q} | \vec{q}' \rangle F(q') \langle \vec{q}' | \vec{q} \rangle \right\} dV' d\tau' \qquad (3.2.3)$$

The reference operator $\mathcal{R}$ that provides the continuum background parameter space as its eigenspace follows from

$$\langle \vec{g} | \mathcal{R}\vec{h} \rangle \equiv \int \iiint \left\{ \langle \vec{g} | \vec{q} \rangle q \langle \vec{q} | \vec{h} \rangle \right\} dV d\tau \qquad (3.2.4)$$

The corresponding shorthand is

$$\mathcal{R} \equiv |\vec{q}\rangle q \langle \vec{q}| \qquad (3.2.5)$$

The reference operator is a special kind of defined operator. Via the quaternionic functions that specify defined operators, it becomes clear that every infinite-dimensional separable Hilbert space owns a unique non-separable companion Hilbert space that can be considered to embed its separable companion.

The reverse bracket method combines Hilbert space operator technology with quaternionic function theory and indirectly with quaternionic differential and integral technology.

3.2.2 Private parameter space

Each Hilbert space manages a private parameter space in the eigenspace of a dedicated reference operator. This parameter space is formed by the version of the number system that the Hilbert space applies to specify its inner products. A Cartesian and a polar ***coordinate system*** determines this version. These coordinate systems sequence the members of the selected version and determine the geometric symmetry of the Hilbert space and its private parameter space. The parameter space determines the ***root geometry*** of the Hilbert space and gives the Hilbert space its ***geometric center*** and its ***geometric symmetry***. This is not the geometric center of the vector space that underlies the Hilbert space. It is the geometric center of the eigenspace of the reference operator.

3.2.3 Fourier transform and momentum space

The normalized eigenvectors of the reference operator form an orthonormal basis of the Hilbert space. We call the subspace spanned by this base the ***configuration space***. Some call it the ***coordinate space*** because it relates to the selected Cartesian coordinate system. We know that in addition to this base, also a spectral base exists that is completely separate from the base of the configuration space. This does not mean that both types of base vectors are mutually independent. It means that any base vector of one type can be written as a linear combination of all base vectors of the other type. A unitary transformation converts one base into the other. An example of such a transformation is the Fourier transformation. A function that is defined in configuration space, will be presented as the Fourier transform of that function in the ***momentum space***. The Fourier transform converts the configuration space into the momentum space.

3.2.3.1 Complex number based Hilbert space

Fourier transforms are commonly described in complex number-based Hilbert spaces. The eigenvectors of the reference operator that belong to eigenvalues that belong to a selected spatial direction span a complex number-based Hilbert space. This enables the conversion of the Fourier transform and the momentum space to the quaternionic Hilbert space.

3.2.4 Archive

Each quaternionic separable Hilbert space acts as an archive for dynamic geometric data of point-like objects in which the eigenvalues of normal operators act as storage bins for a combination of a timestamp and a three-dimensional location.

3.2.5 The Hilbert lattice

The Hilbert lattice is a relational structure that is also known as quantum logic and as an orthomodular lattice. Their discoverers called it quantum logic because the Hilbert lattice shows great similarity with classical logic. However, quantum logic is not a system of logical statements. That is why mathematicians call it an orthomodular lattice. The set of closed subspaces of a Hilbert space is a Hilbert lattice and is lattice isomorphic with the orthomodular lattice. It is completely defined by a set of axioms. Therefore, it can be considered as one of the foundations of the Hilbert Book Model. It needs an underlying vector space and an innerproduct of vector pairs that applies a selected version of a number system to supply its value.

4 A system of separable Hilbert spaces

The definition of a Hilbert space subtly differs from the more basic vector space. The Hilbert space is a vector space that defines an inner vector product for each vector pair. Moreover, a Hilbert space is complete.

4.1 The Hilbert repository

Due to the subtle difference between a Hilbert space and its underlying vector space, and because number systems exist in many versions, a huge number of separable Hilbert spaces can share the same underlying vector space. Sharing the same underlying vector space appears to restrict the choice of the versions of the number system that can be selected. Only versions that have the axes of the Cartesian coordinates parallel to a background separable Hilbert space that is picked from the tolerated collection will be allowed. Only the sequencing of the elements along these axes can be selected freely. This limits the geometric symmetries of the private parameter spaces to a shortlist. In addition the tolerated members must be quaternionic Hilbert spaces.

There are plenty of Hilbert spaces left. One of the Hilbert spaces plays the role of background platform and provides the background parameter space and the background symmetry. The dimension of this Hilbert space is infinite, and its non-separable partner is also part of the background platform. The combined storage capacity is enormous. We call the result a ***Hilbert repository.*** If the Hilbert repository is used to store physical objects in it, then this storage medium can store all the dynamic geometric data of all objects that ever appear in the universe. In addition, the system has storage space for the fields that appear in the universe and that includes the universe field itself.

The requirement that all applied Hilbert spaces use the same underlying vector space introduces some extra restrictions that are known by inspecting the Standard Model but cannot yet be derived from pure mathematical reasons. These restrictions are indicated by the shortlist of electric charges that characterize the Standard Model. The restrictions are that only separable Hilbert spaces are accepted whose Cartesian coordinate systems are parallel to the Cartesian coordinate systems of the background Hilbert spaces.

The restrictions mentioned also lead to additional properties. All participating separable Hilbert spaces glide with their geometric center over the background parameter space. The difference in geometric symmetry between the participating Hilbert spaces and the background symmetry leads to the presence of a source or a sink at the location of the geometric center of the sliding participant. This source or sink corresponds to a symmetry-related charge that generates a symmetry-related field. The value of the charge can be zero.

The difference between the geometric symmetries reduces to the shortlist that also characterizes the list of electric charges and color charges that mark the elementary particles in the Standard Model. This is a remarkable result. See the section on experimental support in this document.

The reason for the shortlist in the Standard Model is the fact that physical reality installs symmetry-related charges that represent sources or sinks at the geometrical centers of the platforms that are formed by separable Hilbert spaces. The geometrical centers of the reference operator eigenspaces of these platforms float with respect to the reference operator eigenspace of a selected background platform.

4.1.1 Elementary particles

These special features do not follow from the mathematics known to us. We can deduce them from measurements made to physical reality. The Hilbert repository has a complicated structure and a special behavior reminiscent of the particle collection described in the Standard Model of the experimental physicists. The sliding platforms behave like elementary particles. In addition, they divide into categories distinguished by their electrical charge. The categories are indicated by type designations. For example, electrons, neutrinos, and quarks form together the elementary fermions. The quarks form a subset and are again divided into types.

4.1.2 Footprint

This is a promising result, but elementary particles have something that in the offered description the sliding platforms do not yet possess. All elementary particles show a footprint that deforms the surrounding field. Let us assume that the sliding platforms support an operator who takes care of the footprint. The elementary particles have a wave function whose square of its modulus results in a detection probability density distribution. The operators of the sliding platform cannot store this continuous detection probability density distribution in its eigenspace. However, it is possible to store a cord of quaternions consisting of combinations of time stamps and hop landing locations that describe an ever-continuing hop path that regularly regenerates a coherent swarm of hop landing locations. If the hopping object is point-shaped, then this swarm is described by a stable location density distribution that corresponds to the mentioned detection probability density distribution.

Mechanisms that produce such coherent swarms with stable location density distributions are common in nature and optical experts often use them. This does not mean that science knows how and why these mechanisms work. I describe these mechanisms as stochastic processes that have a characteristic function. Another description is a process in which a Poisson process combines with a binomial process. The binomial process is implemented by a spatial point spread function. The point spread function plays the role of the location density distribution. The characteristic function of the process equals the Fourier transform of the location density distribution. In optics, this characteristic function is called the Optical Transfer Function.

4.1.3 Elementary particle types

In the Hilbert repository, elementary particles reside on a private floating platform. The Standard Model shows that elementary particles exist in a very short list of types. Especially electric charges occur in a shortlist that covers -3/3, -2/3, -1/3, 0, +1/3, +2/3 and +3/3. Only particles with charge -3/3, 0, +3/3 can produce spherical pulse responses. The other types own a color charge that relates to the dimension in which the geometric symmetry differs in an anisotropic way with the geometric symmetry of the background platform. These particles are called quarks. Quarks must first join into hadrons before they can produce spherical pulse responses that deform the embedding field. This phenomenon is known as ***color confinement***.

The shortlist forms the motive for the analysis of the geometric symmetries in the section about symmetry flavors. That analysis learns that not symmetry groups form the base for the diversity of elementary particle types. Instead, it appears that geometric symmetry differences between the floating platforms and the background platform deliver the explanation of the existence of the shortlist. It also means that only a very small subset of the versions of the quaternionic number system can be applied in observable floating platforms. This severe restriction means that only versions of the quaternionic number system can be applied in which the Cartesian coordinate systems have their axes parallel to the axes of the Cartesian coordinates in the background platform. Only the direction of the sequencing along these axes may be freely selected. This restriction enables the determination of the geometric symmetry difference by applying enclosure balance equations.

4.2 Two episodes

The footprint operator is already present at the time of the creation of the Hilbert repository and determines the behavior of the elementary particle throughout its existence. This fact is a great mystery. The humanly derived math does not offer an explanation yet. However, the existence of the footprint operator makes it possible to divide the model of physical reality into a preparatory episode in which there is no flowing time and an ongoing episode in which a continuing step-by-step embedding of the hop landing locations mimics the activities of the stochastic processes. The embedding process uses the stored and ordered time stamps to realize the corresponding hop landings. The range of running time is equal to the range of the archived time stamps. At the beginning of the running time, the field that represents our universe is still virginal and corresponds to the background parameter space. After the first footprints completed, the relevant elementary particles can start to form composite objects.

5 The Hilbert Book Model

Without the addition of the stochastic processes, the dynamics of the Hilbert repository is rather dull. The Hilbert Book Model changes this by adding a creation episode and a running episode. In the creation episode, stochastic processes fill the floating part of the Hilbert repository with dynamic representations of the life stories of the elementary particles that during the running episode reside on a floating private separable Hilbert space. The non-separable Hilbert space in the background platform tells the result of the ongoing embedding of the floating part of the Hilbert repository into a dynamic field that the background platform archives in the eigenspace of a dedicated ***universe*** operator. The private stochastic process archives the life story of the elementary particle into the eigenspace of a dedicated footprint operator. The embedding of this eigenspace into the eigenspace that represents the universe mimics the activity of the stochastic process as a function of the archived timestamps. In this way, the eigenspaces of the footprint operators form the motor behind the dynamics of the embedding field that represents the universe.

The Hilbert Book Model is a subject of the Hilbert Book Model Project.

The Hilbert Book Model and related subjects are treated in great detail in "A Self-creating Model of Physical Reality" [2].

This paper focusses on the Hilbert repository, on field theory and on the interaction between discrete objects and fields.

5.1 Fields

Without the Hilbert repository, it is impossible to properly understand the fundamental difference between the dynamic field that represents our universe and the geometric symmetry-related field that is raised by the sources and sinks that reside at the geometric centers of the floating platforms.

5.2 Modularity

A hardly known aspect of the Hilbert Book Model is the fact that elementary particles behave as elementary modules. Together, these elementary modules constitute all other modules that appear in the universe. Some modules constitute modular systems. The configuration of composite modules is controlled by stochastic processes that own a characteristic function. This characteristic function is a dynamic superposition of the characteristic functions of the components of the composite. The superposition coefficients act as internal displacement generators.

5.3 Dark objects

Pulse responses that behave as shock fronts and are field excitations of the universe field, constitute all discrete objects that exist in the universe. Black holes form an exception to this rule. Dark objects can occur in isolation, but they only become noticeable when they combine in huge ensembles. This happens in photons for dark energy objects and in elementary particles for dark matter objects.

6 Modeling platform

Despite the strong restrictions, the system of Hilbert spaces that we call the Hilbert repository, represents a flexible and powerful modeling platform. It acts as a repository for the dynamic geometric data of point-like objects and for dynamic fields. Quaternions can act as storage bins of a scalar timestamp and a three-dimensional location. If the timestamps are sequenced, then the archive tells the life story of the point-like object as an ongoing hopping path. The Hilbert Book Model Project applies this model. In a creation episode, the repository is filled with data. In a subsequent running mode, the archived hop landing locations are embedded as a function of the sequenced archived timestamps in a selected field that we call the universe.

It may seem odd to speak about floating platforms, but on each platform, all relevant eigenspaces appear to be defined relative to the eigenspace of the reference operator at the platform. That is why the author considers the private parameter space as defining the root geometry of the platform.

The free-floating separable Hilbert spaces harbour elementary particles. Elementary particles behave as elementary modules. Together these elementary modules constitute all other massive objects that exist in the model. Discontinuous regions form the exception to this rule. They represent black holes. Discontinuous regions are known as black holes. Some modules constitute modular systems. See the section on the universe field.

The author does not (yet) understand the mathematical reason for the described extra restrictions. The author derives the existence of these extra restrictions from the shortlist of electrical charges and color charges that forms an essential part of the Standard Model. Physical reality applies the restriction to install sources and sinks at the geometrical centers of the floating platforms. These sources and sinks raise geometric symmetry-related fields in the background platform.

6.1 Observers

In the running episode of the model, the timestamps are considered to be sequenced and the embedding process is considered to act as a function of proper time. Observers travel with the instant of proper time that is represented by their archived timestamps. Observers can retrieve archived data that for them have historic timestamps. Since the information is transferred to the observer by the universe, which is a dynamic field, the observers perceive this information in spacetime coordinates. A hyperbolic Lorentz transform converts the archived Euclidean coordinates into the perceived spacetime coordinates. Observers are discrete objects that reside on the floating platforms or they are conglomerates of these objects. Thus, they are elementary modules, composite modules or modular systems. All modules and modular systems are massive objects. Their presence deforms the embedding universe field.

Observers move as one unit. Observers correspond with a reference frame. The reference frame corresponds with the parameter space of the module or modular system. This parameter space conforms to the geometric symmetry of the background platform. However, the geometric center of the reference frame corresponds to the center of mass of the module or modular system that represents the observer. The information about the observed scene is archived in one or more of the floating platforms.

It is possible that floating platforms with other axes orientations exist, but these platforms cannot be perceived by observers.

6.2 Hilbert repository

The author suggests applying the name ***Hilbert repository*** to the sketched system of Hilbert spaces. It is an abstract storage medium for dynamic geometric data of point-like objects. Quaternionic eigenvalues act as storage bins of Euclidean combinations of timestamps and three-dimensional locations. These data combine in eigenspaces of platforms that float over a common background platform. Each platform owns a private parameter space. The system also archives dynamic continuums and mixed dynamic fields in eigenspaces of normal operators in the background platform of the repository.

The repository is filled with data during a creation episode. The sequencing of the archived timestamps turns the archive into a book that tells the life stories of the objects that are contained in the repository. During the running episode, discrete observers can retrieve archived data that for them have historic timestamps. Thus, for the observers, the observed part of the repository acts as a read-only storage medium.

7 The concept of time

7.1 Dynamics

The dynamics of the model is based on the floating of the separable platforms over the background platform and on the ongoing embedding of eigenspaces of the floating platforms that store dynamic geometric data in a selected eigenspace of the background platform. This ongoing embedding occurs as a function of the sequenced archived timestamps. Thus, for observers that travel with the scanning time window, the embedding starts with the lowest ranking timestamp.

7.2 Proper time

The range of proper time corresponds to the range of archived timestamps. In the modeling platform, observers are discrete objects that travel with a scanning proper time window. Observers can only receive information from events that were stored with for them historic timestamps.

The notion of time in the Hilbert book model only means something in relation to the archived timestamps. This means that things could still take place before the value of the first archived proper time instant. This episode includes, among other things, the preparation and archival of the dynamic geometric data of the elementary particles. For each elementary module, a private stochastic process generates the dynamic geometric data that are archived in the eigenspace of the footprint operator. Therefore, the author will call this preceding episode, the creation episode. In the subsequent running episode, the ongoing embedding process will imitate the activity of the private stochastic process.

7.3 Clock rates

Proper time ticks with a minimum step. However, that does not mean that this minimum step is the same in the whole universe. It may depend on the local expansion rate of the universe, and it is possible that the local expansion rate varies with the nearby occurrence of deformation. So, traversing a closed path through a deformed region can result in a difference in time count at the return point between the traveler and the object that stayed at that location because the traveler experienced a different expansion rate of the part of the universe that the traveler traversed. During his trip, the clock of the traveler ran at a different rate than the clock of the staying object. These effects have been measured with accurate clocks.

The metaphor that the Hilbert Book Model steps through the universe with universe-wide progressions steps remains valid, but the page thicknesses in this metaphor can vary from place to place in a fluid way.

7.4 A self-creating model

By restricting the notion of proper time in the described way, it is possible to classify the Hilbert Book Model as a self-creating model. It is now possible to weld a preparatory phase, in which the creation and storage of the dynamic geometrical data of the elementary modules are arranged. Only after this creation episode can observers obtain information. They get this information via the field that embeds them.

7.5 Spacetime

Observers can access information that is retrieved from storage locations that for them have a historic timestamp. That information is transferred to them via the dynamic universe field. This dynamic field embeds both the observer and the observed event. The dynamic geometric data of point-like objects are archived in Euclidean format as a combination of a timestamp and a three-dimensional spatial location. The embedding field affects the format of the transferred information. The observers perceive in spacetime format. A hyperbolic Lorentz transform describes the conversion from the Euclidean storage coordinates to the perceived spacetime coordinates.

7.5.1 Lorentz transform

In dynamic fields, shock fronts move with speed c. In the quaternionic setting, this speed is unity.

$$x^2 + y^2 + z^2 = c^2\tau^2 \quad (6.5.1)$$

In flat dynamic fields, swarms of triggers of spherical pulse responses move with lower speed v.

For the geometric centers of these swarms still holds:

$$x^2 + y^2 + z^2 - c^2\tau^2 = x'^2 + y'^2 + z'^2 - c^2\tau'^2 \quad (6.5.2)$$

If the locations $\{x, y, z\}$ and $\{x', y', z'\}$ move with uniform relative speed v, then

$$ct' = ct\cosh(\omega) - x\sinh(\omega) \quad (6.5.3)$$

$$x' = x\cosh(\omega) - ct\sinh(\omega) \quad (6.5.4)$$

$$\cosh(\omega) = \frac{\exp(\omega) + \exp(-\omega)}{2} = \frac{c}{\sqrt{c^2 - v^2}} \quad (6.5.5)$$

$$\sinh(\omega) = \frac{\exp(\omega) - \exp(-\omega)}{2} = \frac{v}{\sqrt{c^2 - v^2}} \quad (6.5.6)$$

$$\cosh(\omega)^2 - \sinh(\omega)^2 = 1 \quad (6.5.7)$$

This is a hyperbolic transformation that relates two coordinate systems [6].

This transformation can concern two platforms P and P' on which swarms reside and that move with uniform relative speed .

However, it can also concern the storage location P that contains a timestamp t and spatial location $\{x,y,z\}$ and platform P' that has coordinate time t and location $\{x',y',z'\}$.

In this way, the hyperbolic transform relates two individual platforms on which the private swarms of individual elementary particles reside.

It also relates the stored data of an elementary particle and the observed format of these data for the elementary particle that moves with speed relative to the background parameter space.

The Lorentz transform converts a Euclidean coordinate system consisting of a location $\{x,y,z\}$ and proper timestamps τ into the perceived coordinate system that consists of the spacetime coordinates $\{x',y',z',ct'\}$ in which t' plays the role of coordinate time. The uniform velocity v causes time dilation $\Delta t' = \frac{\Delta\tau}{\sqrt{1-\frac{v^2}{c^2}}}$ and length contraction

$\Delta L' = \Delta L\sqrt{1-\frac{v^2}{c^2}}$

7.5.2 Minkowski metric

Spacetime is ruled by the Minkowski metric [7].

In flat field conditions, proper time τ is defined by

$$\tau = \pm\frac{\sqrt{c^2t^2 - x^2 - y^2 - z^2}}{c} \tag{6.5.8}$$

And in deformed fields, still

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 \tag{6.5.9}$$

Here ds is the spacetime interval and $d\tau$ is the proper time interval. dt is the coordinate time interval

7.5.3 Schwarzschild metric

Polar coordinates convert the Minkowski metric to the Schwarzschild metric [8]. The proper time interval $d\tau$ obeys

$$c^2 d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2 d\varphi^2\right) \qquad (6.5.10)$$

Under pure isotropic conditions, the last term on the right side vanishes.

According to mainstream physics, in the environment of a black hole, the symbol r_s stands for the Schwarzschild radius [9].

$$r_s = \frac{2GM}{c^2} \qquad (6.5.11)$$

The variable r equals the distance to the center of mass of the massive object with mass M .

The Hilbert Book model finds a different value for the boundary of a spherical black hole. That radius is a factor of two smaller.

7.6 In the beginning

Before the embedding processes that mimic the activity of the stochastic processes started their action, the content of the universe was empty. It was represented by a flat field that in its spatial part, was equal to the parameter space of the background platform. At the beginning instant, a huge number of these mimicked stochastic processes started their triggering of the dynamic field that represents the universe. The triggers may cause spherical pulse responses that act as spherical shock fronts. These spherical shock fronts temporarily deform the universe field. In that case, they will also persistently expand the universe. Thus, from that moment on, the universe started expanding. This did not happen at a single point. Instead, it happened at a huge number of locations that were distributed all over the spatial part of the parameter space of the quaternionic function that describes the dynamic universe field.

Close to the beginning of time, all distances were equal to the distances in the flat parameter space. Soon, these islands were uplifted with volume that was emitted at nearby locations. This flooding created growing distances between used locations. After some time, all parameter space locations were reached by the generated shock fronts. From that moment on the universe started acting as an everywhere expanded continuum that contained deformations which in advance were very small. Where these deformations grew, the distances grew faster than in the environment. A more uniform expansion appears the rule and local deformations form the exception. Deformations make the information path longer and give the idea that time ticks slower in the deformed and expanded regions. This corresponds with the gravitational redshift of photons.

Composed modules only started to be generated after the presence of enough elementary modules. The generation of photons that reflected the signatures of atoms only started after the presence of these compound modules. However, the spurious one-dimensional shock fronts could be generated from the beginning.

This picture differs considerably from the popular scene of the big bang that started at a single location [10].

The expansion is the fastest in areas where spherical pulse responses are generated. For that reason, it is not surprising that the measured Hubble constant differs from place to place.

8 Basic Fields

Two important dynamic basic fields that exist in the base model couple via the geometric centers of the private parameter spaces of the floating platforms.

8.1 The universe field

The universe is a dynamic field that is represented by a dedicated normal operator in the non-separable Hilbert space, which is part of the background platform. This field exists always and everywhere in the parameter space of the background platform. The field can vibrate, deform and expand as a function of the real part of the parameter space. This real part represents proper time.

8.1.1 Black hole

We introduce a ***discontinuum*** as the antonym of a continuum. The universe is a mixed field. It can contain a set of enclosed spatial regions that encapsulate a discontinuum. A discontinuum is a dense discrete set. A discontinuum is countable. In physics, the equivalent of a discontinuum is a black hole. The enclosing surface is a continuum with a lower dimension than the enclosed region. No field excitations exist inside the discontinuum. Thus, no field excitations can pass the enclosing surface. Since a discontinuum deforms the surrounding continuum, this enclosed region owns an amount of mass. Together with the spherical shock fronts and the elementary modules, the discontinuums are the only objects in the universe that own mass. The mass of spherical shock fronts is volatile. Only when gathered in coherent and dense ensembles these shock fronts can cause a persistent amount of mass. That happens in the footprint of elementary modules. It also happens in the halos of galaxies.

8.2 Symmetry related fields

Each floating platform features its own private parameter space and with that parameter space, it owns a geometric symmetry. The difference in the geometric symmetry between the floating platform and the background platform defines a geometric symmetry-related charge that is represented by a source or sink that locates at the geometric center of the private parameter space. The sources and sinks mark the locations of the geometrical centers of the corresponding floating platforms in the universe field. In the background platform, these sources and sinks raise a geometric symmetry-related field that corresponds to the geometric symmetry-related charge of the floating platform. The geometric symmetry-related fields can superpose. This superposition is managed in the eigenspace of a dedicated geometric symmetry-related field operator that resides in the non-separable Hilbert space.

9 Field equations

Field equations are quaternionic functions or quaternionic differential and integral equations that describe the behavior of the continuum part of fields.

The differential change can be expressed in terms of a linear combination of partial differentials. Now the total differential change df of field f equals

$$df = \frac{\partial f}{\partial \tau} d\tau + \frac{\partial f}{\partial x} \vec{i} dx + \frac{\partial f}{\partial y} \vec{j} dy + \frac{\partial f}{\partial z} \vec{k} dz \qquad (8.1.1)$$

In this equation, the partial differentials $\frac{\partial f}{\partial \tau}, \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial y}$ behave like quaternionic differential operators.

The quaternionic nabla ∇ assumes the ***special condition*** that partial differentials direct along the axes of the Cartesian coordinate system. Thus,

$$\nabla = \sum_{i=0}^{4} \vec{e}_i \frac{\partial}{\partial x_i} = \frac{\partial}{\partial \tau} + \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z} \tag{8.1.2}$$

However, this way of notation is often considered as abusive. Still, we will apply that notation because the correct notation (8.1.1) leads to the same result. This will be shown in the next section by splitting both the quaternionic nabla and the function in a scalar part and a vector part.

9.1 Quaternionic differential calculus

The first order partial differential equations divide the first-order change of a field in five different parts that each represent a new field. We will represent the field change operator by a quaternionic nabla operator. This operator behaves like a quaternionic multiplier.

A quaternion can store a timestamp in its real part and a three-dimensional spatial location in its imaginary part. The quaternionic nabla ∇ acts as a quaternionic multiplying operator. Quaternionic multiplication obeys the equation

$$\begin{aligned} c = c_r + \vec{c} = ab &= \left(a_r + \vec{a}\right)\left(b_r + \vec{b}\right) \\ &= a_r b_r - \left\langle \vec{a}, \vec{b} \right\rangle + a_r \vec{b} + \vec{a} b_r \pm \vec{a} \times \vec{b} \end{aligned} \tag{8.1.3}$$

The $\pm$ sign indicates the freedom of choice of the handedness of the product rule that exists when selecting a version of the quaternionic number system. The first order partial differential follows from

$$\nabla = \left\{ \frac{\partial}{\partial \tau}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} = \nabla_r + \vec{\nabla} \tag{8.1.4}$$

The spatial nabla $\vec{\nabla}$ is well-known as the del operator and is treated in detail in Wikipedia [11].

$$\phi = \nabla \psi = \left(\frac{\partial}{\partial \tau} + \vec{\nabla} \right)(\psi_r + \vec{\psi}) \\ = \nabla_r \psi_r - \langle \vec{\nabla}, \vec{\psi} \rangle + \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} \quad (8.1.5)$$

In a selected version of the quaternionic number system, only the corresponding version of the quaternionic nabla is active. In the background platform, this version is always and everywhere the same.

The differential $\nabla \psi$ describes the change of field ψ. The five separate terms in the first-order partial differential have a separate physical meaning. All basic fields feature this decomposition. The terms may represent new fields.

$$\phi_r = \nabla_r \psi_r - \langle \vec{\nabla}, \vec{\psi} \rangle \quad (8.1.6)$$

$$\vec{\phi} = \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} = -\vec{E} \pm \vec{B} \quad (8.1.7)$$

$\vec{\nabla} f$ is the gradient of f.

$\langle \vec{\nabla}, \vec{f} \rangle$ is the divergence of $\vec{f}$.

$\vec{\nabla} \times \vec{f}$ is the curl of $\vec{f}$.

The conjugate of the quaternionic nabla operator defines another type of field change.

$$\nabla^* = \nabla_r - \vec{\nabla} \quad (8.1.8)$$

$$\zeta = \nabla^* \phi = \left(\frac{\partial}{\partial \tau} - \vec{\nabla} \right)(\phi_r + \vec{\phi}) \\ = \nabla_r \phi_r + \langle \vec{\nabla}, \vec{\phi} \rangle + \nabla_r \vec{\phi} - \vec{\nabla} \phi_r \mp \vec{\nabla} \times \vec{\phi} \quad (8.1.9)$$

The quaternionic nabla is a normal operator.

$$\nabla^\dagger = \nabla^* = \nabla_r - \vec{\nabla} = \nabla_r + \vec{\nabla}^\dagger = \nabla_r + \vec{\nabla}^* \\ \nabla^\dagger \nabla = \nabla \nabla^\dagger = \nabla^* \nabla = \nabla \nabla^* = \nabla_r \nabla_r + \langle \vec{\nabla}, \vec{\nabla} \rangle \quad (8.1.10)$$

The operators $\nabla_r \nabla_r$ and $\langle \vec{\nabla}, \vec{\nabla} \rangle$ are Hermitian operators. They can also be combined as $\square = \nabla_r \nabla_r - \langle \vec{\nabla}, \vec{\nabla} \rangle$. This is the d'Alembert operator.

9.2 Continuity equations

Continuity equations are partial quaternionic differential equations.

9.2.1 Field excitations

Field excitations are solutions of second-order partial differential equations.

One of the second-order partial differential equations results from combining the two first-order partial differential equations $\phi = \nabla \psi$ and $\zeta = \nabla^* \phi$.

$$\begin{aligned} \zeta &= \nabla^* \varphi = \nabla^* \nabla \psi = \nabla \nabla^* \psi = \left(\nabla_r + \vec{\nabla} \right) \left(\nabla_r - \vec{\nabla} \right) \left(\psi_r + \vec{\psi} \right) \\ &= \left(\nabla_r \nabla_r + \langle \vec{\nabla}, \vec{\nabla} \rangle \right) \psi \end{aligned} \qquad (8.2.1)$$

Integration over the time domain results in the Poisson equation

$$\rho = \langle \vec{\nabla}, \vec{\nabla} \rangle \psi \qquad (8.2.2)$$

Under isotropic conditions, a very special solution of the Poisson equation is the Green's function $\frac{1}{4\pi \left| \vec{q} - \vec{q'} \right|}$ of the affected field. This solution is the spatial Dirac $\delta(\vec{q})$ pulse response of the field under strict isotropic conditions.

$$\nabla \frac{1}{\left| \vec{q} - \vec{q'} \right|} = - \frac{\left(\vec{q} - \vec{q'} \right)}{\left| \vec{q} - \vec{q'} \right|^3} \qquad (8.2.3)$$

$$\left\langle \vec{\nabla},\vec{\nabla} \right\rangle \frac{1}{\left|\vec{q}-\overrightarrow{q'}\right|} \equiv \left\langle \vec{\nabla},\vec{\nabla} \frac{1}{\left|\vec{q}-\overrightarrow{q'}\right|} \right\rangle$$

$$= -\left\langle \vec{\nabla}, \frac{\left(\vec{q}-\overrightarrow{q'}\right)}{\left|\vec{q}-\overrightarrow{q'}\right|^3} \right\rangle = 4\pi\delta\left(\vec{q}-\overrightarrow{q'}\right) \tag{8.2.4}$$

This solution corresponds with an ongoing source or sink that exists in the field.

Change can take place in one dimension or combined in two or three dimensions.

Under isotropic conditions, the dynamic spherical pulse response of the field is a solution of a special form of the equation (8.2.1)

$$\left(\nabla_r \nabla_r + \left\langle \vec{\nabla},\vec{\nabla} \right\rangle\right)\psi = 4\pi\delta\left(\vec{q}-\overrightarrow{q'}\right)\theta\left(\tau \pm \tau'\right) \tag{8.2.5}$$

Here $\theta(\tau)$ is a step function and $\delta(\vec{q})$ is a Dirac pulse response. For the spherical pulse response, the pulse must be isotropic.

After the instant τ', the equation turns into a homogeneous equation. In isotropic conditions it gets the form

$$\left(\frac{\partial^2}{\partial\tau^2} + \frac{\partial^2}{\partial r^2} + 2\frac{\partial}{r\partial r}\right)\psi$$

$$= \left(\frac{\partial^2}{\partial\tau^2} + \frac{\partial^2}{\partial r^2}\right)(\psi r) = 0 \tag{8.2.6}$$

The second line describes the second-order change of ψr in one dimension along the radius r. A solution of this line is

$$\psi r = f\left(r \pm c\tau\vec{n}\right) \tag{8.2.7}$$

The solution of (8.2.6) is described by

$$\psi = \frac{f\left(\left|\vec{q}-\vec{q'}\right| \pm c(\tau-\tau')\vec{n}\right)}{\left|\vec{q}-\vec{q'}\right|} \tag{8.2.8}$$

The normalized vector $\vec{n}$ can be interpreted as the spin of the solution. The spherical pulse response acts either as an expanding or as a contracting spherical shock front. Over time this pulse response integrates into the Green's function. This means that the isotropic pulse injects the volume of the Green's function into the field. Subsequently, the front spreads this volume over the field. The contracting shock front collects the volume of the Green's function and sucks it out of the field. The $\pm$ sign in equation (8.2.5) selects between injection and subtraction.

Apart from the spherical pulse response equation (8.2.5) supports a one-dimensional pulse response that acts as a one-dimensional shock front. This solution is described by

$$\psi = f\left(\left|\vec{q}-\vec{q'}\right| \pm c(\tau-\tau')\vec{n}\right) \tag{8.2.9}$$

Here, the normalized vector $\vec{n}$ can be interpreted as the polarization of the solution. Shock fronts only occur in one and three dimensions. A pulse response can also occur in two dimensions, but in that case, the pulse response is a complicated vibration that looks like the result of a throw of a stone in the middle of a pond.

Equations (8.2.1) and (8.2.2) show that the operators $\frac{\partial^2}{\partial\tau^2}$ and $\langle\vec{\nabla},\vec{\nabla}\rangle$ are valid second-order partial differential operators. These operators combine in the quaternionic equivalent of the wave equation [12].

$$\varphi = \left(\frac{\partial^2}{\partial\tau^2} - \langle\vec{\nabla},\vec{\nabla}\rangle\right)\psi \tag{8.2.10}$$

This equation also offers one-dimensional and three-dimensional shock fronts as its solutions.

$$\psi = \frac{f\left(\left|\vec{q} - \vec{q'}\right| \pm c\left(\tau - \tau'\right)\right)}{\left|\vec{q} - \vec{q'}\right|} \tag{8.2.11}$$

$$\psi = f\left(\left|\vec{q} - \vec{q'}\right| \pm c\left(\tau - \tau'\right)\right) \tag{8.2.12}$$

These pulse responses do not contain the normed vector $\vec{n}$. Apart from pulse responses, the wave equation offers waves as its solutions.

If locally, the field can be split into a time-dependent part $T(\tau)$ and a location-dependent part $A(\vec{q})$, then the homogeneous version of the wave equation can be transformed into the Helmholtz equation [13].

$$\frac{\partial^2 \psi}{\partial \tau^2} = \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi = -\omega^2 \psi \tag{8.2.13}$$

$$\psi(\vec{q}, \tau) = A(\vec{q}) T(\tau) \tag{8.2.14}$$

$$\frac{1}{T} \frac{\partial^2 T}{\partial \tau^2} = \frac{1}{A} \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle A = -\omega^2 \tag{8.2.15}$$

$$\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle A + \omega^2 A \tag{8.2.16}$$

The time-dependent part $T(\tau)$ depends on initial conditions, or it indicates the switch of the oscillation mode. The switch of the oscillation mode means that temporarily the oscillation is stopped and instead an object is emitted or absorbed that compensates the difference in potential energy. The location-dependent part of the field $A(\vec{q})$ describes the possible oscillation modes of the field and depends on boundary conditions. The oscillations have a binding effect. They keep moving objects within a bounded region.

For three-dimensional isotropic spherical conditions, the solutions have the form

$$A(r,\theta,\varphi)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\left\{\left(a_{lm}j_l(kr)\right)+b_{lm}Y_l^m(\theta,\varphi)\right\} \qquad (8.2.17)$$

Here j_l and y_l are the spherical Bessel functions, and Y_l^m are the spherical harmonics [14][15]. These solutions play a role in the spectra of atomic modules.

Planar and spherical waves are the simpler wave solutions of the equation (8.2.13)

$$\psi(\vec{q},\tau)=\exp\left\{\vec{n}\left(\left\langle\vec{k},\vec{q}-\vec{q}_0\right\rangle-\omega\tau+\varphi\right)\right\} \qquad (8.2.18)$$

$$\psi(\vec{q},\tau)=\frac{\exp\left\{\vec{n}\left(\left\langle\vec{k},\vec{q}-\vec{q}_0\right\rangle-\omega\tau+\varphi\right)\right\}}{\left|\vec{q}-\vec{q}_0\right|} \qquad (8.2.19)$$

A more general solution is a superposition of these basic types.

Two quite similar homogeneous second-order partial differential equations exist. They are the homogeneous versions of equations (8.2.5) and (8.2.10). The equation (8.2.5) has spherical shock front solutions with a spin vector that behaves like the spin of elementary particles.

The inhomogeneous pulse activated equations are

$$\left(\nabla_r\nabla_r\pm\left\langle\vec{\nabla},\vec{\nabla}\right\rangle\right)\psi=4\pi\delta\left(\vec{q}-\vec{q'}\right)\theta(\tau\pm\tau') \qquad (8.2.20)$$

9.3 Enclosure balance equations

Enclosure balance equations are quaternionic integral equations that describe the balance between the inside, the border, and the outside of an enclosure.

These integral balance equations base on replacing the del operator $\vec{\nabla}$ by a normed vector $\vec{n}$. Vector $\vec{n}$ is oriented outward and perpendicular to a local part of the closed boundary of the enclosed region.

$$\vec{\nabla}\psi \Leftrightarrow \vec{n}\psi \tag{8.3.1}$$

This approach turns part of the differential continuity equation into a corresponding integral balance equation.

$$\iiint \vec{\nabla}\psi \, dV = \oiint \vec{n}\psi \, dS \tag{8.3.2}$$

$\vec{n}\, dS$ plays the role of a differential surface. $\vec{n}$ is perpendicular to that surface.

This result separates into three parts

$$\begin{aligned}\vec{\nabla}\psi &= -\langle \vec{\nabla}, \vec{\psi} \rangle + \vec{\nabla}\psi_r \pm \vec{\nabla} \times \vec{\psi} \Leftrightarrow \vec{n}\psi \\ &= -\langle \vec{n}, \vec{\psi} \rangle + \vec{n}\psi_r \pm \vec{n} \times \vec{\psi}\end{aligned} \tag{8.3.3}$$

The first part concerns the gradient of the scalar part of the field

$$\vec{\nabla}\psi_r \Leftrightarrow \vec{n}\psi_r \tag{8.3.4}$$

$$\iiint \vec{\nabla}\psi_r \, dV = \oiint \vec{n}\psi_r \, dS \tag{8.3.5}$$

The divergence is treated in an integral balance equation that is known as the Gauss theorem. It is also known as the divergence theorem [16].

$$\langle \vec{\nabla}, \vec{\psi} \rangle \Leftrightarrow \langle \vec{n}, \vec{\psi} \rangle \tag{8.3.6}$$

$$\iiint \langle \vec{\nabla}, \vec{\psi} \rangle \, dV = \oiint \langle \vec{n}, \vec{\psi} \rangle \, dS \tag{8.3.7}$$

The curl is treated in an integrated balance equation

$$\vec{\nabla} \times \vec{\psi} \Leftrightarrow \vec{n} \times \vec{\psi} \tag{8.3.8}$$

$$\iiint \vec{\nabla} \times \vec{\psi} dV = \oiint \vec{n} \times \vec{\psi} dS \tag{8.3.9}$$

Equation (8.3.7) and equation (8.3.9) can be combined in the extended theorem

$$\iiint \vec{\nabla} \vec{\psi} dV = \oiint \vec{n} \ \vec{\psi} dS \tag{8.3.10}$$

The method also applies to other partial differential equations. For example

$$\begin{aligned} \vec{\nabla} \times \left(\vec{\nabla} \times \vec{\psi} \right) = \vec{\nabla} \left\langle \vec{\nabla}, \vec{\psi} \right\rangle - \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \vec{\psi} \Leftrightarrow \vec{\nabla} \times \left(\vec{\nabla} \times \vec{\psi} \right) \\ = \vec{n} \langle \vec{n}, \vec{\psi} \rangle - \langle \vec{n}, \vec{n} \rangle \vec{\psi} \end{aligned} \tag{8.3.11}$$

$$\iiint_V \left\{ \vec{\nabla} \times \left(\vec{\nabla} \times \vec{\psi} \right) \right\} dV = \oiint_S \left\{ \vec{\nabla} \left\langle \vec{\nabla}, \vec{\psi} \right\rangle \right\} dS - \oiint_S \left\{ \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \vec{\psi} \right\} dS \tag{8.3.12}$$

One dimension less, a similar relation exists.

$$\iint_S \left(\left\langle \vec{\nabla} \times \vec{a}, \vec{n} \right\rangle \right) dS = \oint_C \left\langle \vec{a}, d\vec{l} \right\rangle \tag{8.3.13}$$

This is known as the Stokes theorem[17]

The curl can be presented as a line integral

$$\left\langle \vec{\nabla} \times \vec{\psi}, \vec{n} \right\rangle \equiv \lim_{A \to 0} \left(\frac{1}{A} \oint_C \langle \vec{\psi}, d\vec{r} \rangle \right) \tag{8.3.14}$$

9.4 Using volume integrals to determine the symmetry-related charges

In its simplest form in which no discontinuities occur in the integration domain Ω the generalized Stokes theorem runs as

$$\int_\Omega d\omega = \int_{\partial\Omega} \omega = \oint_\Omega \omega \tag{8.4.1}$$

We separate all point-like discontinuities from the domain Ω by encapsulating them in an extra boundary. Symmetry centers represent spherically shaped or cube-shaped closed parameter space regions H_n^x that float on a background parameter space $\mathfrak{R}$. The boundaries ∂H_n^x separate the regions from the domain H_n^x. The regions H_n^x are platforms for local discontinuities in basic fields. These fields are continuous in the domain $\Omega - H$.

$$H = \bigcup_{n=1}^{N} H_n^x \tag{8.4.2}$$

The symmetry centers $\mathfrak{S}_n^x$ are encapsulated in regions H_n^x, and the encapsulating boundary ∂H_n^x is not part of the disconnected boundary, which encapsulates all continuous parts of the quaternionic manifold ω that exists in the quaternionic model.

$$\int_{\Omega - H} d\omega = \int_{\partial\Omega \cup \partial H} \omega = \int_{\partial\Omega} \omega - \sum_{k=1}^{N} \int_{\partial H_n^x} \omega \tag{8.4.3}$$

In fact, it is sufficient that ∂H_n^x surrounds the current location of the elementary module. We will select a boundary, which has the shape of a small cube of which the sides run through a region of the parameter spaces where the manifolds are continuous.

If we take everywhere on the boundary the unit normal to point outward, then this reverses the direction of the normal on ∂H_n^x which negates the integral. Thus, in this formula, the contributions of boundaries $\{\partial H_n^x\}$ are subtracted from the contributions of the boundary $\partial\Omega$. This means that $\partial\Omega$ also surrounds the regions $\{\partial H_n^x\}$

This fact renders the integration sensitive to the ordering of the participating domains.

Domain Ω corresponds to part of the background parameter space $\mathfrak{R}$. As mentioned before the symmetry centers $\mathfrak{S}_n^x$ represent encapsulated regions $\{\partial H_n^x\}$ that float on the background parameter space $\mathfrak{R}$. The

Cartesian axes of $\mathfrak{S}_n^x$ are parallel to the Cartesian axes of background parameter space $\mathfrak{R}$. Only the orderings along these axes may differ.

Further, the geometric center of the symmetry center $\mathfrak{S}_n^x$ is represented by a floating location on parameter space $\mathfrak{R}$.

The symmetry center $\mathfrak{S}_n^x$ is characterized by a private symmetry flavor. That symmetry flavor relates to the Cartesian ordering of this parameter space. With the orientation of the coordinate axes fixed, eight independent Cartesian orderings are possible.

The consequence of the differences in the symmetry flavor on the subtraction can best be comprehended when the encapsulation ∂H_n^x is performed by a ***cubic space form*** that is aligned along the Cartesian axes that act in the background parameter space. Now the six sides of the cube contribute differently to the effects of the encapsulation when the ordering of H_n^x differs from the Cartesian ordering of the reference parameter space $\mathfrak{R}$. Each discrepant axis ordering corresponds to one-third of the surface of the cube. This effect is represented by the ***geometric symmetry-related charge***, which includes the ***color charge*** of the symmetry center. It is easily comprehensible related to the algorithm which below is introduced for the computation of the geometric symmetry-related charge. Also, the relation to the color charge will be clear. ***Thus, this effect couples the ordering of the local parameter spaces to the geometric symmetry-related charge of the encapsulated elementary module.*** The differences with the ordering of the surrounding parameter space determine the value of the geometric symmetry-related charge of the object that resides inside the encapsulation!

9.5 Symmetry flavor

The Cartesian ordering of its private parameter space determines the symmetry flavor of the platform [18]. For that reason, this symmetry is compared with the reference symmetry, which is the symmetry of the background parameter space. Four arrows indicate the symmetry of the platform. The background is represented by:

↑↑↑↑

Now the geometric symmetry-related charge follows in two steps.

1. Count the difference of the spatial part of the geometric symmetry of the platform with the spatial part of the geometric symmetry of the background parameter space.
2. Switch the sign of the result for anti-particles.

Symmetrieversie					
Ordering x y z τ	Sequence	Handedness Right/Left	Color charge	Electric charge * 3	Symmetry type.
↑↑↑↑	⓪	R	N	+0	neutrino
↓↑↑↑	①	L	R	− 1	down quark
↑↓↑↑	②	L	G	− 1	down quark
↓↓↑↑	③	R	B	+2	up quark
↑↑↓↑	④	L	B	−1	down quark
↓↑↓↑	⑤	R	G	+2	up quark
↑↓↓↑	⑥	R	R	+2	up quark
↓↓↓↑	⑦	L	N	− 3	electron
↑↑↑↓	⑧	R	N	+3	positron
↓↑↑↓	⑨	L	R	− 2	anti-up quark
↑↓↑↓	⑩	L	G	− 2	anti-up quark
↓↓↑↓	⑪	R	B	+1	anti-down quark
↑↑↓↓	⑫	L	B	− 2	anti-up quark
↓↑↓↓	⑬	R	G	+1	anti-down quark
↑↓↓↓	⑭	R	R	+1	anti-down quark
↓↓↓↓	⑮	L	N	− 0	anti-neutrino

Probably, the neutrino and the antineutrino own an abnormal handedness.

The suggested particle names that indicate the symmetry type are borrowed from the Standard Model. In the table, compared to the standard model, some differences exist with the selection of the anti-predicate. All considered particles are elementary fermions. The freedom of choice in the polar coordinate system might determine the spin [19]. The azimuth range is 2π radians, and the polar angle range is π radians. Symmetry breaking means a difference between the platform symmetry and the symmetry of the background. Neutrinos do not break the symmetry. Instead, they probably may cause conflicts with the handedness of the multiplication rule.

9.6 Coupling of basic fields

Besides the fact that the geometric center of the elementary particles also form the geometric center of the symmetery related field of this particle the coupling of the symmetry of the particle to the Cartesian coordinate system of the particle couples the basic fields of the particle to the background field that acts as our universe. It tries to keep the Cartesian coordinate systems in parallel. This couples the curl of the particle’s geometric symmetry related field to the curl of the embedding background field. A non-zero curl might even couple to the otherwise undetermined direction of the spin vector in the spherical shock fronts. This couples the direction of spin to a non-zero magnetic field.

9.7 Derivation of physical laws

The quaternionic equivalents of Ampère's law are [20]

$$\vec{J} \equiv \vec{\nabla} \times \vec{B} = \nabla_r \vec{E} \Leftrightarrow \vec{J} \equiv \vec{n} \times \vec{B} = \nabla_r \vec{E} \tag{8.7.1}$$

$$\iint_S \left\langle \vec{\nabla} \times \vec{B}, \vec{n} \right\rangle dS = \oint_C \left\langle \vec{B}, d\vec{l} \right\rangle = \iint_S \left\langle \vec{J} + \nabla_r \vec{E}, \vec{n} \right\rangle dS \tag{8.7.2}$$

The quaternionic equivalents of Faraday's law are [21]:

$$\nabla_r \vec{B} = \vec{\nabla} \times (\nabla_r \vec{\psi}) = -\vec{\nabla} \times \vec{E} \Leftrightarrow \nabla_r \vec{B} = \vec{n} \times (\nabla_r \vec{\psi}) = -\vec{\nabla} \times \vec{E} \qquad (8.7.3)$$

$$\oint_c \left\langle \vec{E}, d\vec{l} \right\rangle = \iint_S \left\langle \vec{\nabla} \times \vec{E}, \vec{n} \right\rangle dS = -\iint_S \left\langle \nabla_r \vec{B}, \vec{n} \right\rangle dS \qquad (8.7.4)$$

$$\vec{J} = \vec{\nabla} \times \left(\vec{B} - \vec{E} \right) = \vec{\nabla} \times \vec{\varphi} - \nabla_r \vec{\varphi} = \vec{v} \rho \qquad (8.7.5)$$

$$\iint_S \left\langle \vec{\nabla} \times \vec{\varphi}, \vec{n} \right\rangle dS = \oint_C \left(\left\langle \vec{\varphi}, d\vec{l} \right\rangle \right) = \iint_S \left\langle \vec{v} \rho + \nabla_r \vec{\varphi}, \vec{n} \right\rangle dS \qquad (8.7.6)$$

The equations (8.7.4) and (8.7.6) enable the derivation of the ***Lorentz force*** [22].

$$\vec{\nabla} \times \vec{E} = -\nabla_r \vec{B} \qquad (8.7.7)$$

$$\frac{d}{d\tau} \iint_S \left\langle \vec{B}, \vec{n} \right\rangle dS = \iint_{S(\tau_0)} \left\langle \dot{\vec{B}}(\tau_0), \vec{n} \right\rangle ds + \frac{d}{d\tau} \iint_{S(\tau)} \left\langle \vec{B}(\tau_0), \vec{n} \right\rangle ds \qquad (8.7.8)$$

The Leibniz integral equation states [23]

$$\begin{aligned} &\frac{d}{dt} \iint_{S(\tau)} \left\langle \vec{X}(\tau_0), \vec{n} \right\rangle dS \\ &= \iint_{S(\tau_0)} \left\langle \dot{\vec{X}}(\tau_0) + \left\langle \vec{\nabla}, \vec{X}(\tau_0) \right\rangle \vec{v}(\tau_0), \vec{n} \right\rangle dS - \oint_{C(\tau_0)} \left\langle \vec{v}(\tau_0) \times \vec{X}(\tau_0), d\vec{l} \right\rangle \end{aligned} \qquad (8.7.9)$$

With $\vec{X} = \vec{B}$ and $\left\langle \vec{\nabla}, \vec{B} \right\rangle = 0$ follows

$$\begin{aligned} &\frac{d\Phi_B}{d\tau} = \\ &\frac{d}{d\tau} \iint_{S(\tau)} \left\langle \dot{\vec{B}}(\tau), \vec{n} \right\rangle dS = \iint_{S(\tau_0)} \left\langle \vec{B}(\tau_0), \vec{n} \right\rangle dS - \oint_{C(\tau_0)} \left\langle \vec{v}(\tau_0) \times \vec{B}(\tau_0), d\vec{l} \right\rangle \\ &= -\oint_{C(\tau_0)} \left\langle E(\tau_0), d\vec{l} \right\rangle - \oint_{C(\tau_0)} \left\langle \vec{v}(\tau_0) \times \vec{B}(\tau_0), d\vec{l} \right\rangle \end{aligned} \qquad (8.7.10)$$

The electromotive force (EMF) ε equals [24]

$$\varepsilon = \oint_{C(\tau_0)} \left\langle \frac{\vec{F}(\tau_0)}{q}, d\vec{l} \right\rangle = -\frac{d\Phi_B}{d\tau}\bigg|_{\tau=\tau_0}$$
$$= \oint_{C(\tau_0)} \left\langle \vec{E}(\tau_0), d\vec{l} \right\rangle + \oint_{C(\tau_0)} \left\langle \vec{v}(\tau_0) \times \vec{B}(\tau_0), d\vec{l} \right\rangle \tag{8.7.11}$$

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \tag{8.7.12}$$

10 Stochastic control

Stochastic processes that own a characteristic function control the coherence and part of the dynamics of most of the discrete objects in the model. A displacement generator that can be considered as part of the characteristic function determines the location of the geometric center of the object.

10.1 Elementary modules

Elementary modules are controlled by the first type of stochastic process. These processes are inhomogeneous spatial Poisson point processes. They can be considered as a combination of a genuine Poisson process and a binomial process that is implemented by a spatial point spread function. The process generates an ongoing hopping path that recurrently regenerates a coherent hop landing location swarm. A location density distribution describes this swarm and equals the Fourier transform of the characteristic function of the process. Further, it equals the square of the modulus of what physicists would call the wavefunction of the elementary module.

Each elementary particle behaves like an elementary module. Together, the elementary modules constitute all massive objects that exist in the universe. Discontinuous regions form the exception to this rule. Some modules constitute modular systems.

10.2 Composite modules

Composite modules are controlled by the second type of stochastic process. The characteristic function of these stochastic processes is a dynamic superposition of the characteristic functions of their components. The superposition coefficients act as displacement generators. This means that the composition of composite modules is defined in Fourier space (also called momentum space). In that environment, the location in the configuration space has no significance. Thus, components of a composite can locate far from each other in configuration space. This is the reason that entanglement exists. Entanglement becomes noticeable when components obey exclusion principles.

10.3 Atoms

Compound modules are composite modules for which the geometric centers of the platforms of the components coincide. The charges of the platforms of the elementary modules establish the binding of the corresponding platforms. Physicists and chemists call these compound modules atoms or atomic ions.

In free compound modules, the geometric symmetry-related charges do not take part in the oscillations. The targets of the private stochastic processes of the elementary modules oscillate. This means that the hopping path of the elementary module folds around the oscillation path and the hop landing location swarm gets smeared along the oscillation path. The oscillation path is a solution of the Helmholtz equation. Each fermion must use a different oscillation mode. A change of the oscillation mode goes together with the emission or the absorption of a photon. The center of emission coincides with the geometrical center of the compound module. During the emission or absorption, the oscillation mode and the hopping path halt, such that the emitted photon does not lose its integrity. Since all photons share the same emission duration, that duration must coincide with the regeneration cycle of the hop landing location swarm. Absorption cannot be interpreted so easily. In fact, it can only be comprehended as a time-reversed emission act. Otherwise, the absorption would require an incredible aiming precision for the photon.

The type of stochastic process that controls the binding of components appears to be responsible for the absorption and emission of photons and the change of oscillation modes. If photons arrive with too low energy, then the energy is spent on the kinetic energy of the common platform. If photons arrive with too high energy, then the energy is distributed over the available oscillation modes, and the rest is spent on the kinetic energy of the common platform, or it escapes into free space. The process must somehow archive the modes of the components. It can apply the private platform of the components for that purpose. Most probably, the current value of the dynamic superposition coefficient is stored in the eigenspace of a special superposition operator.

10.4 Acceleration at a distance

Far from the geometric center of the platform on which a composite module resides the potential of the field that couples to this geometrical center takes the shape of the Green's function of the field. For example, the gravitation potential takes the shape

$$\phi(r) \approx \frac{GM}{r} \qquad (9.4.1)$$

The geometric symmetry-related potential takes the shape

$$\varphi(r) \approx \frac{\alpha Q}{r} \qquad (9.4.2)$$

Here we use α to represent the permittivity of free space. In free space, the geometric center of this platform floats with uniform speed over the background platform. Free space means that the embedding field and the geometric symmetry-related field are nearly flat. In free space, the only disturbances are the described potentials.

Physical reality tries to stabilize this situation. This tendency can be represented by an artificial field that tries to keep its change to a minimum. We treat the potentials in a similar way by taking

$$\begin{aligned} GM &\Rightarrow \kappa X \\ \alpha Q &\Rightarrow \kappa X \end{aligned} \qquad (9.4.3)$$

The new artificial field $\xi = \left\{ \frac{\kappa X}{r}, \vec{v} \right\}$ considers a uniformly moving floating platform as a normal situation. It is a combination of the scalar potential $\frac{\kappa X}{r}$ and the uniform speed $\vec{v}$.

If this object accelerates, then the new field $\left\{ \frac{\kappa X}{r}, \vec{v} \right\}$ tries to counteract the change of the field $\dot{\vec{v}}$ by compensating this with an equivalent change of the real part $\frac{\kappa X}{r}$ of the new field.

The first-order change of a field contains five terms. Mathematically, the statement that in first approximation nothing in the field ξ changes indicates that locally, the first-order partial differential $\nabla \xi$ will be equal to zero.

$$\zeta = \nabla \xi = \nabla_r \xi_r - \left\langle \vec{\nabla}, \vec{\xi} \right\rangle + \vec{\nabla} \xi_r + \nabla_r \vec{\xi} \pm \vec{\nabla} \times \vec{\xi} = 0 \qquad (9.4.4)$$

We concentrate on the imaginary terms

$$\vec{\zeta} = \vec{\nabla} \xi_r + \nabla_r \vec{\xi} \pm \vec{\nabla} \times \vec{\xi} = 0 \qquad (9.4.5)$$

For our purpose, the curl $\vec{\nabla} \times \vec{\xi}$ of the vector field $\vec{\xi}$ is expected to be zero. The resulting terms of the equation (9.4.5) are

$$\nabla_r \vec{\xi} + \vec{\nabla} \xi_r = 0 \qquad (9.4.6)$$

According to the equation (11.8.4), this equivalent change is the gradient of the real part of the field.

$$\vec{a} = \dot{\vec{v}} = -\vec{\nabla} \left(\frac{\kappa X}{r} \right) = \frac{\kappa X \, \vec{r}}{|\vec{r}|^3} \qquad (9.4.7)$$

This generated vector field acts on masses or charges that appear in its realm.

Thus, if two uniformly moving objects X_1 and X_2 exist in each other's neighborhood, then any disturbance of the situation will cause the force

$$\vec{F}\left(\vec{r}_1 - \vec{r}_2\right) = X_1\vec{a} = \frac{\kappa X_1 X_2 \left(\vec{r}_1 - \vec{r}_2\right)}{\left|\vec{r}_1 - \vec{r}_2\right|^3} \tag{9.4.8}$$

For the gravitation potential, this is explained in the section on inertia. Inertia involves only one accelerated massive object. It requires the existence of the gradient of the gravitation potential of that massive object. The ongoing expansion of the universe field will establish that gradient.

10.5 Molecules

Molecules are conglomerates of compound modules that each keep their private geometrical center. However, electron oscillations are shared among the compound modules. Together with the geometric symmetry-related charges, this binds the compound modules into the molecule.

10.6 RTOS

The archival of dynamic geometric data that takes place in the creation episode is determining the life story of the elementary particles. The activity of the stochastic processes is mimicked by the ongoing embedding process that implements the dynamic geometric data as an ongoing hopping path that recurrently regenerates a coherent hop landing location swarm which has a stable location density distribution. This location density distribution is the Fourier transform of the characteristic function of the stochastic process that filled the eigenspace of the footprint operator that resides at the private platform of the elementary particle. This activity acts as a Real-Time Operating

System. The recurrent regeneration of the hop landing location swarm implements an effective guard against deadlocks and race conditions.

10.7 Potential energy and kinetic energy

One-dimensional shock fronts are packages of pure energy. They can transfer this energy between an emitter and an absorber. The absorber can convert this energy into kinetic energy or into potential energy. For example, atoms can convert the absorbed energy into the potential energy of internal oscillations of its components. The potential energy of internal oscillations can be converted into pure energy that is emitted by the atom. Energy can also be absorbed by platforms and converted into the kinetic energy of the platform. This works when the platform owns mass. This means that on the platform a source of spherical pulses must be present that deform the embedding field. If the platform contains a resulting geometric symmetry related charge, then the absorbed energy will also be spent to the energy of the geometric symmetry-related field because the geometric symmetry-related charge is located at the geometric center of the platform. On the other hand, the geometric symmetry-related field can influence the kinetic energy of the platform.

All elementary particles own mass. All geometric symmetry-related charges locate at the geometric center of an elementary particle. Together the elementary particles constitute all massive objects that exist in the universe. Discontinuous regions form the exception to this rule.

10.8 Pair production and pair annihilation

Pair production and pair annihilation of elementary particles are exceptional cases of exchanging energy against matter. It is possible to interpret these phenomena as time reversal of a single particle. It turns an elementary particle into its antiparticle.

11 Photons

Photons are objects that still offer significant confusion among physicists. The mainstream interpretation is still that photons are electromagnetic waves. This interpretation conflicts with the known behavior of photons. Photons that are emitted by a nearby star can be detected by a human eye. Since the space between the star and the earth does not contain waveguides, waves cannot do this trick. Electromagnetic fields require the nearby presence of electric charges. Both conditions forbid that photons are implemented by electromagnetic waves.

11.1 Photon structure

Photons are one-dimensional objects that are strings of equidistant energy packages, such that the string obeys the Einstein-Planck relation [25]

$$E = h\nu \qquad (10.1.1)$$

The energy packages are implemented by one-dimensional shock fronts that possess a polarization vector.

Where the light speed c indicates the speed at which shock fronts travel, will Planck's constant indicate the period during which one-dimensional shock fronts will be emitted. We know the frequency of the photon that is emitted at the annihilation of an electron. Thus, we know the rate at which the energy packages that constitute this photon are produced. However, no data are available on the duration D of the photon emission or on the spatial length $L = D / c$ of photons.

$$E = h\nu = N_p E_p \qquad (10.1.2)$$

$$E_p = \frac{h\nu}{N_p} \qquad (10.1.3)$$

$$D = \frac{N_p}{\nu} \tag{10.1.4}$$

$$\nu = \frac{N_p}{D} \tag{10.1.5}$$

$$E = \frac{hN_p}{D} = N_p E_p \tag{10.1.6}$$

$$E_p = \frac{h}{D} \tag{10.1.7}$$

$$h = \frac{E_p}{D} = \frac{cE_p}{L} \tag{10.1.8}$$

Thus, Planck's constant equals the energy E_p of the standard energy packages divided by the emission duration of the photons.

11.2 One-dimensional pulse responses

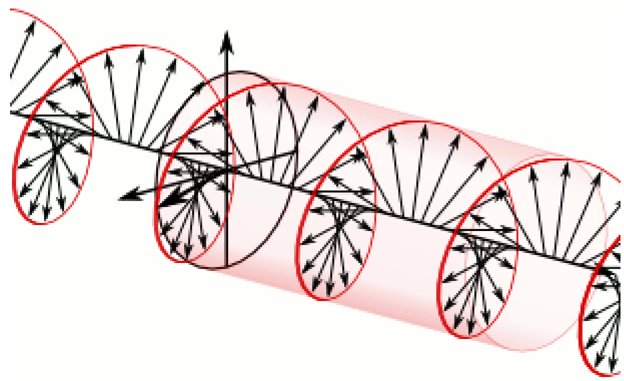
One-dimensional pulse responses that act as one-dimensional shock fronts and possess a polarization vector are solutions of the equation (8.2.5) and are described by the equation (8.2.9).

$$\psi = f\left(\left|\vec{q} - \vec{q}'\right| \pm c\left(\tau - \tau'\right)\vec{n}\right) \tag{10.1.9}$$

During travel, the front $f(\vec{q})$ keeps its shape and its amplitude. So also, during long-range trips, the shock front does not lose its integrity. The one-dimensional pulse response represents an energy package that travels with speed c through its carrier field. The energy of the package has a standard value.

In the animation of this left handed circular polarized photon, the black arrows represent the moving shock fronts. The red line connects the vectors that indicate the amplitudes of the separate shock fronts. Here the picture of a guided wave is borrowed to show the similarity with such EM waves [41]. However,

photons are not EM waves!



11.3 Photon integrity

Except for its speed, the photon emitter determines the properties of the photon. These properties are its frequency, its energy, and its polarization. The energy packages preserve their own integrity. They travel at a constant speed and follow a worldline. Photon emission possesses a fixed duration. It is not an instant process. During emission, the emitter must not move and can only rotate around the direction of travel. Failing these requirements will compromise the integrity of the photon and make it impossible for a distant, tiny absorber to capture the full photon. In that case, the energy packages will spray and fly to multiple locations. Consequently, they will act like dark energy objects.

The absorption of a photon by an atom requires an incredible aiming precision of the emitter. In fact, this absorption can only be comprehended when it is interpreted as the time-reversal of the corresponding emission process. If the absorbing atom cannot cope with the full energy of the photon, then it might absorb only part of the energy packages of the photon. The rest will stay on its route to the next absorber. Absorbing individual energy packages will result in an increase in the kinetic energy of the absorber. Absorbing the full photon or a part of it will result in an increase in the potential energy of the absorber. Usually, this results in a higher oscillation mode of one or more of the components of the absorber.

11.4 Light

Light is a dynamic spatial distribution of photons. Often the location density distribution of photons owns a Fourier transform. In that case, the light may show wave behavior. Photons are one-dimensional particles that feature private frequency and energy. Single photons do not show wave behavior. Photons and light waves will feature different frequencies.

11.5 Optics

Optics is the science of imaging distributions of particles that can be characterized by a location density distribution and a corresponding Fourier transform of that location density distribution. Even though photons have a fixed non-zero spatial length, optics will treat these particles as point-like objects. Also, massive particles, such as elementary particles or their conglomerates can take part in an imaging process. Another name for the location density distribution is point spread function (PSF). Another name for the Fourier transform of the PSF is the optical transfer function (OTF) [26]. Apart from a location density distribution, the swarm of the particles is also characterized by an angular distribution and by an energy distribution. In the case of photons, the energy distribution is also a chromatic distribution.

A linearly operating imaging device can be characterized by its point spread function or alternatively by its OTF. This point spread function is an image of a point-like object. The PSF represents the blur that is introduced by the imaging device. For a homogeneous distribution of particle properties, the OTF of a chain of linearly operating imaging devices equals the product of the OTF's of the separate devices.

The imaging properties of an imaging device may vary as a function of the location and the orientation in the imaging surface.

Without the presence of the traveling particles, the imaging devices keep their OTF. Small apertures and patterns of apertures feature an OTF. That OTF handles single particles similarly as this feature handles distributions of particles.

Instead of the Point Spread Function optics often uses the Line Spread Function. For the Optical Transfer Function, this means that a cut through the center is taken and the spatial spectrum is reduced to a two-dimensional distribution. The quaternionic representation becomes a complex number-based representation. For the modulus of the Optical Transfer Function, the Modulation Transfer Function this means that the representation becomes symmetric. It is enough to specify one half of the MTF curve. The direction of the cut depends on the selected direction of the line. Often only the cuts with a maximum width and a minimum width are specified to qualify the imaging quality of the imaging device.

11.5.1 Veiling glare

Often the MTF shows a peak near zero spatial frequency. This indicates the presence of less coherent contributions to the Point Spread Function. This phenomenon is called veiling glare. At large scales, the phenomenon corresponds to a halo. For massive objects, this effect generates gravitational lensing. In veiling glare. the image carriers are photons. The dark mass and more persistent massive objects form the halo.

11.5.2 Distributions of particles

Distributions of particles are usually incoherent. That means that the particles are distributed according to multiple properties. For example, light is a distribution of photons. The photons have a location, a color, and an angular direction. Also, their phase can be distributed. If all photons have the same phase, color, and angular direction, then the light beam is called homogeneous. Each photon represents a string of equidistant energy packages. These energy packages are one-dimensional pulse responses.

Moving elementary particles already represent a beam of moving spherical pulse responses. Apart from that, the particles may differ in their location, kinetic energy, their electric charge, their angular distribution, and their mass.

The point spread function only considers the detection location of the elementary particles. This means that the PSF is a function of the other characteristics of the particle beam. The OTF is the Fourier transform of the PSF. Optics must reckon these influences.

In most cases, optical characteristics of imaging devices are only defined for homogeneous particle beams.

12 Gravity

Mainstream physics considers the origin of the deformation of our living space as an unsolved problem. It presents the Higgs mechanism as the explanation of why some elementary particles get their mass. The Hilbert Book Model relates mass to deformation of the field that represents our universe. This deformation causes the mutual attraction of massive objects.

12.1 Difference between the Higgs field and the universe field

The Higgs field corresponds with a Higgs boson [27]. The dynamic field that represents our universe does not own a field generating particle like the Higgs boson that is supposed to generate the Higgs field. The universe field exists always and everywhere. In fact, a private stochastic process generates each elementary particle. The stochastic process produces quaternions that break the symmetry of the background parameter space. Consequently, the embedded quaternion breaks the symmetry of the functions that apply this parameter pace. Thus, the quaternion breaks the symmetry of the field that represents the universe. However, only isotropic symmetry breaks can produce the spherical pulse responses that temporarily deform the universe field. These spherical pulse responses act as spherical shock fronts. The pulse injects volume into the field, and the shock front distributes this volume over the whole field. The volume expands the field persistently, but the initial deformation fades away. The front wipes the deformation away from the location of the pulse.

12.2 Center of mass

In a system of massive objects $p_i, i = 1, 2, 3, \ldots n$, each with static mass m_i at locations r_i, the center of mass $\vec{R}$ follows from [28]

$$\sum_{i=1}^{n} m_i\left(\vec{r}_i - \vec{R}\right) = \vec{0} \tag{11.2.1}$$

Thus

$$\vec{R} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i \tag{11.2.2}$$

Where

$$M = \sum_{i=1}^{n} m_i \tag{11.2.3}$$

In the following, we will consider an ensemble of massive objects that own a center of mass $\vec{R}$ and a fixed combined mass M as a single massive object that locates at $\vec{R}$. $\vec{R}$ can be a dynamic location. In that case, the ensemble must move as one unit. In physical reality, this construct has no point-like equivalent that owns a fixed mass. The problem with the treatise in this paragraph is that in physical reality, point-like objects that possess a static mass do not exist. Only pulse responses that temporarily deform the field exist. Except for black holes, these pulse responses constitute all massive objects that exist in the universe.

12.3 Newton

Newton's laws are nearly correct in nearly flat field conditions. The main formula for Newton's laws is [29]

$$\vec{F} = m\vec{a} \tag{11.3.1}$$

Another law of Newton treats the mutual attraction between massive objects [30].

$$\vec{F}\left(\vec{r}_1 - \vec{r}_2\right) = M_1\vec{a} = \frac{G M_1 M_2 \left(\vec{r}_1 - \vec{r}_2\right)}{\left|\vec{r}_1 - \vec{r}_2\right|^3} \tag{11.3.2}$$

Newton deduced this universal law of gravitation from the results of experiments, but this gravitational attraction can also be derived theoretically from the gravitational potential that is produced by spherical pulse responses.

Massive objects deform the field that embeds these objects. At large distances, a simplified form of the gravitational potential describes properly what occurs.

The following relies heavily on the chapters on quaternionic differential and integral calculus.

12.4 Gauss law

The Gauss law for gravitation is [31]

$$\oiint_{\partial V} \langle \vec{g}, dA \rangle = \iiint_V \langle \vec{\nabla}, \vec{g} \rangle dV = -4\pi G \iiint_V \rho dV = -4\pi GM \qquad (11.4.1)$$

Here $\vec{g}$ is the gravitational field. G is the gravitational constant. M is the encapsulated mass. ρ is the mass density distribution. The differential form of Gauss law is [32]

$$\langle \vec{\nabla}, \vec{g} \rangle = \langle \vec{\nabla}, \vec{\nabla} \rangle \phi = -4\pi G\rho \qquad (11.4.2)$$

$$\vec{g} = -\vec{\nabla}\phi \qquad (11.4.3)$$

ϕ is the gravitational field. Far from the center of mass this gravitation potential equals

$$\phi(r) = \frac{MG}{r} \qquad (11.4.4)$$

12.5 A deforming field excitation

A spherical pulse response is a solution of a homogeneous second-order partial differential equation that was triggered by an isotropic pulse. The corresponding field equation and the corresponding solution are repeated here.

$$\left(\nabla_r\nabla_r+\left\langle\vec{\nabla},\vec{\nabla}\right\rangle\right)\psi=4\pi\delta\left(\vec{q}-\vec{q'}\right)\theta\left(\tau\pm\tau'\right) \tag{11.5.1}$$

Here the $\pm$ sign represents time inversion.

$$\psi=\frac{f\left(\left|\vec{q}-\vec{q'}\right|\pm c\left(\tau-\tau'\right)\vec{n}\right)}{\left|\vec{q}-\vec{q'}\right|} \tag{11.5.2}$$

The spherical pulse response integrates over time into the Green's function of the field. The Green's function is a solution of the Poisson equation.

$$\rho=\left\langle\vec{\nabla},\vec{\nabla}\right\rangle\psi \tag{11.5.3}$$

The Green's function occupies some volume [33].

$$g(\vec{q})=\frac{1}{4\pi\left|\vec{q}-\vec{q'}\right|} \tag{11.5.4}$$

This means that locally the pulse pumps some volume into the field, or it subtracts volume out of the field. The selection between injection and subtraction depends on the sign in the step function in the equation (11.5.1). The dynamics of the spherical pulse response shows that the injected volume quickly spreads over the field. In the case of volume subtraction, the front first collects the volume and finally subtracts it at the trigger location. Gravitation considers the case in which the pulse response injects volume into the field.

Thus, locally and temporarily, the pulse deforms the field, and the injected volume persistently expands the field.

This paper postulates that the spherical pulse response is the only field excitation that temporarily deforms the field, while the injected volume persistently expands the field.

The effect of the spherical pulse response is so tiny and so temporarily that no instrument can ever measure the effect of a single spherical pulse response in isolation. However, when recurrently regenerated in huge numbers in dense and coherent swarms, the pulse responses can cause a significant and persistent deformation that instruments can detect. This is achieved by the stochastic processes that generate the footprint of elementary modules.

The spherical pulse responses are straightforward candidates for what physicists call dark matter objects. A halo of these objects can cause gravitational lensing.

12.6 Gravitational potential

A massive object at a large distance acts as a point-like mass. Far from the center of mass, the gravitational potential of a group of massive particles with combined mass M is

$$\phi(r) \approx \frac{GM}{r} \qquad (11.6.1)$$

At this distance the gravitation potential shows the shape of the Green’s function of the field; however, the amplitude differs. The formula does not indicate that the gravitational potential can cause acceleration for a uniformly moving massive object. However, the gravitational potential is the gravitational potential energy per unit mass. The relation to Newton’s law is shown by the following.

The potential ϕ of a unit mass m at a distance r from a point-mass of mass M can be defined as the work W that needs to be done by an external agent to bring the unit mass in from infinity to that point [34].

$$\phi(\vec{r}) \approx \frac{W}{m} = \frac{1}{m}\int_{\infty}^{\vec{r}} \left\langle \vec{F}, d\vec{r} \right\rangle = \frac{1}{m}\int_{\infty}^{\vec{r}} \left\langle \frac{GmM\,\vec{r}}{|\vec{r}|^3}, d\vec{r} \right\rangle = \frac{GM}{|\vec{r}|} \qquad (11.6.2)$$

12.7 Pulse location density distribution

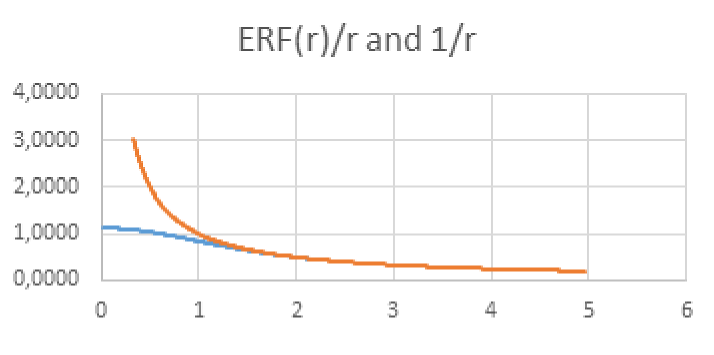
It is false to treat a pulse location density distribution as a set of point-like masses as is done in formulas (11.2.1) and (11.2.2). Instead, the gravitational potential follows from the convolution of the location density distribution and the Green's function. This calculation is still not correct, because the exact result depends on the fact that the deformation that is due to a pulse response quickly fades away and the result also depends on the density of the distribution. If these effects can be ignored, then the resulting gravitational potential of a Gaussian density distribution would be given by [35]

$$g(r) \approx GM \frac{ERF(r)}{r} \tag{11.7.1}$$

Where $ERF(r)$ is the well-known error function. Here the gravitational potential is a perfectly smooth function that at some distance from the center equals the approximated gravitational potential that was described above in equation (11.6.1). As indicated above, the convolution only offers an approximation because this computation does not account for the influence of the density of the swarm and it does not compensate for the fact that the deformation by the individual pulse responses quickly fades away. Thus, the exact result depends on the duration of the recurrence cycle of the swarm.

In the example, we apply a normalized location density distribution, but the actual location density distribution might have a higher amplitude.

This might explain why some elementary module types exist in three generations. These generations appear to have their own mass.



This might also explain why different first-generation elementary particle types show different masses. Due to the convolution, and the coherence of the location density distribution, the blue curve does not show any sign of the singularity that is contained in the red curve, which shows the Green's function.

In physical reality, no point-like static mass object exists. The most important lesson of this investigation is that far from the gravitational center of the distribution the deformation of the field is characterized by the here shown simplified form of the gravitation potential

$$\phi(r) \approx \frac{GM}{r} \quad (11.7.2)$$

Warning: This simplified form shares its shape with the Green's function of the deformed field. This does not mean that the Green's function owns a mass that equals $M_G = \frac{1}{G}$. The functions only share the form of their tail.

12.8 Inertia

The relation between inertia and mass is complicated [36][37]. We apply a field that resists its changing. The condition that for each type of massive object, the gravitational potential is a static function and the condition that in free space, the massive object moves uniformly, establish that inertia rules the dynamics of the situation. These conditions define an artificial quaternionic field that does not change.

The real part of the artificial field is represented by the gravitational potential, and the uniform speed of the massive object represents the imaginary (vector) part of the field.

The change of the quaternionic field can be divided into five separate changes that partly can compensate each other.

The first-order change of a field contains five terms. Mathematically, the statement that in first approximation nothing in the field ξ changes indicates that locally, the first-order partial differential $\nabla\xi$ will be equal to zero.

$$\zeta = \nabla\xi = \nabla_r\xi_r - \langle\vec{\nabla},\vec{\xi}\rangle + \vec{\nabla}\xi_r + \nabla_r\vec{\xi} \pm \vec{\nabla}\times\vec{\xi} = 0 \qquad (11.8.1)$$

Thus

$$\zeta_r = \nabla_r\xi_r - \langle\vec{\nabla},\vec{\xi}\rangle = 0 \qquad (11.8.2)$$

$$\vec{\zeta} = \vec{\nabla}\xi_r + \nabla_r\vec{\xi} \pm \vec{\nabla}\times\vec{\xi} = 0 \qquad (11.8.3)$$

These formulas can be interpreted independently. For example, according to the equation (11.8.2), the variation in time of ξ_r must equal the divergence of $\vec{\xi}$. The terms that are still eligible for change must together be equal to zero. For our purpose, the curl $\vec{\nabla}\times\vec{\xi}$ of the vector field $\vec{\xi}$ is expected to be zero. The resulting terms of the equation (11.8.3) are

$$\nabla_r\vec{\xi} + \vec{\nabla}\xi_r = 0 \qquad (11.8.4)$$

In the following text plays $\vec{\xi}$ the role of the vector field and ξ_r plays the role of the scalar gravitational potential of the considered object. For elementary modules, this special field supports the hop landing location swarm that resides on the floating platform. It reflects the activity of the stochastic process and the uniform movement in the free space of the floating platform over the background platform. It is characterized by a mass value and by the uniform velocity of the platform with respect to the background platform. The real part conforms to the deformation that the stochastic process causes. The imaginary part conforms to the speed of movement of the floating platform. The main characteristic of this field is that it tries to keep its overall change zero. The author calls ξ the ***conservation field***.

At a large distance r, we approximate this potential by using formula

$$\phi(r) \approx \frac{GM}{r} \tag{11.8.5}$$

The new artificial field $\xi = \left\{ \frac{GM}{r}, \vec{v} \right\}$ considers a uniformly moving mass as a normal situation. It is a combination of the scalar potential $\frac{GM}{r}$ and the uniform speed $\vec{v}$.

If this object accelerates, then the new field $\left\{ \frac{GM}{r}, \vec{v} \right\}$ tries to counteract the change of the field $\dot{\vec{v}}$ by compensating this with an equivalent change of the real part $\frac{GM}{r}$ of the new field. According to the equation (11.8.4), this equivalent change is the gradient of the real part of the field.

$$\vec{a} = \dot{\vec{v}} = -\vec{\nabla}\left(\frac{GM}{r} \right) = \frac{GM\,\vec{r}}{|\vec{r}|^3} \tag{11.8.6}$$

This generated vector field acts on masses that appear in its realm.

Thus, if two uniformly moving masses M_1 and M_2 exist in each other's neighborhood, then any disturbance of the situation will cause the gravitational force

$$\vec{F}\left(\vec{r}_1-\vec{r}_2\right)=M_1\vec{a}=\frac{GM_1M_2\left(\vec{r}_1-\vec{r}_2\right)}{\left|\vec{r}_1-\vec{r}_2\right|^3} \qquad (11.8.7)$$

The disturbance by the ongoing expansion of the embedding field suffices to put the gravitational force into action. The description also holds when the field ξ describes a conglomerate of platforms and M_2 represents the mass of the conglomerate.

The artificial field ξ represents the habits of the underlying model that ensures the constancy of the gravitational potential and the uniform floating of the considered massive objects in free space.

Inertia ensures that the third-order differential (the third-order change) of the deformed field is minimized. It does that by varying the speed of the platforms on which the massive objects reside.

Inertia bases mainly on the definition of mass that applies to the region outside the sphere where the gravitational potential behaves like the Green's function of the field. There the formula $\xi_r=\frac{m}{r}$ applies. Further, it bases on the intention of modules to keep the gravitational potential inside the mentioned sphere constant. At least that holds when this potential is averaged over the regeneration period. In that case, the overall change ζ in the conservation field ξ equals zero. Next, the definition of the conservation field supposes that the swarm which causes the deformation moves as one unit. Further, the fact is used that the solutions of the homogeneous second-order partial differential equation can superpose in new solutions of that same equation.

The popular sketch in which the deformation of our living space is presented by smooth dips is obviously false. The story that is represented in this paper shows the deformations as local extensions of the field, which represents the universe. In both sketches, the deformations elongate the information path, but none of the sketches explain why two masses attract each other. The above explanation founds on the habit of the stochastic process to recurrently regenerate the same time average of the gravitational potential, even when that averaged potential moves uniformly. Without the described habit of the stochastic processes, inertia would not exist.

The applied artificial field also explains the gravitational attraction by black holes.

The artificial field that implements mass inertia also plays a role in other fields. Similar tricks can be used to explain the electrical force from the fact that the electrical field is produced by sources and sinks that can be described with the Green's function.

12.9 Elementary particles

For elementary particles, a private stochastic process generates the hop landing locations of the ongoing hopping path that recurrently forms the same hop landing location density distribution. The characteristic function of the stochastic process ensures that the same location density distribution is generated. This does not mean that the same hop landing location swarm is generated! The squared modulus of the wavefunction of the elementary particle equals the generated location density distribution. This explanation means that all elementary particles and all conglomerates of elementary particles are recurrently regenerated.

12.10 Mass

Mass is a property of objects, which has its own significance. Since at large distance, the gravitational potential always has the shape $\phi(r) \approx \frac{GM}{r}$, it does not matter what the massive object is. The formula can be used to determine the mass, even if only is known that the object in question deforms the embedding field. In that case, the formula can still be applied. This is used in the chapter about mixed fields.

In physical reality, no static point-like mass object exists.

12.11 Hop landing generation

The generation of the hopping path is an ongoing process. The generated hop landing location swarm contains a huge number of elements. Each elementary module type is controlled by a corresponding type of stochastic process. For the stochastic process, only the Fourier transform of the location density distribution of the swarm is important. Consequently, for a selected type of elementary module, it does not matter at what instant of the regeneration of the hop landing location swarm the location density distribution is determined. Thus, even when different types are bonded into composed modules, there is no need to synchronize the regeneration cycles of different types. This freedom also means that the number of elements in a hop landing location swarm may differ between elementary module types. This means that the strength of the deformation of the embedding field can differ between elementary module types. The strength of deformation relates to the mass of the elementary modules according to formula (11.6.1).

The requirement for regeneration represents a great mystery. All mass that elementary modules generate appears to dilute away and must be recurrently regenerated. This fact conflicts with the conservation laws of mainstream physics. The deformation work done by the stochastic processes vanishes completely. What results is the ongoing expansion of the field. Thus, these processes must keep generating the particle to which they belong. The stochastic process accurately regenerates the hop landing location swarm, such that its rest mass stays the same.

Only the ongoing embedding of the content that is archived in the floating platform into the embedding field can explain the activity of the stochastic process. This supposes that at the instant of creation, the creator already archived the dynamic geometric data of his creatures into the eigenspaces of the footprint operators. These data consist of a scalar timestamp and a three-dimensional spatial location. The quaternionic eigenvalues act as storage bins.

After the instant of creation, the creator left his creation alone. The set of floating separable Hilbert spaces acts together with the background Hilbert space as a read-only repository. After sequencing the timestamps, the stochastic processes read the storage bins and trigger the embedding of the location into the embedding field in the predetermined sequence.

12.11.1 Freedom

As long as the instant of archival proceeds the passage of the window that scans the Hilbert Book Base Model as a function of progression, then the behavior of the model does not change. This indicates a degree of freedom of the described model.

12.12 Symmetry-related charges

Geometric symmetry-related charges only appear at the geometric center of the private parameter space of the separable Hilbert space that acts as the floating platform for an elementary particle. These charges represent sources or sinks for the corresponding geometric symmetry-related field. Since these phenomena disturb the corresponding geometric symmetry-related field in a static way that can be described by the Green’s function of the field, the same trick that was used to explain inertia can be used here to explain the attraction or the repel of two geometric symmetry-related charges Q_1 and Q_2.

$$\vec{a} = \dot{\vec{v}} = -\vec{\nabla}\left(\frac{Q}{r}\right) = \frac{Q\,\vec{r}}{|\vec{r}|^3} \tag{11.12.1}$$

$$\vec{F}(\vec{r}_1 - \vec{r}_2) = Q_1\vec{a} = \frac{Q_1 Q_2 (\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3} \tag{11.12.2}$$

12.13 Color confinement

Some elementary particle types do not possess an isotropic geometric symmetry. Mainstream physics indicates this fact with a corresponding color charge. Spherical pulse responses require an isotropic pulse. Thus, colored elementary particles cannot generate a gravitational potential. They must first cling together into colorless conglomerates before they can manifest as massive objects. Mesons and baryons are the colorless conglomerates that become noticeable as particles that attract other massive particles.

13 Underpinning

A purely mathematical model of physical reality, such as the Hilbert Book Model requires a solid mathematical foundation and the application of trusted mathematical methods. In addition, at higher levels of complexity, the model must deliver properties and behavior that can be verified by observing physical reality.

13.1 Experimental support

13.1.1 Support by the Standard Model

The Hilbert Book Model applies measuring results on which the Standard Model of mainstream physics is based. In particular, the shortlist of electrical charges and color charges that characterize the elementary particles is used to support the extra restriction of the versions of the quaternionic number system that are applied by the separable Hilbert spaces, which occur in the base model of the Hilbert Book Model. The author does not know the mathematical requirement for this extra restriction. The model uses this restriction to determine the size and location of the sources and sinks that correspond to the geometric symmetry-related charges. The sources and sinks generate the geometric symmetry-related field.

13.1.2 Stochastic control

Unique to the Hilbert Book Model is the application of the control of part of the dynamics of the model by stochastic processes that generate the archived dynamic geometric data of discrete objects.

The stochastic nature of the detection locations of elementary particles has led to the introduction of the wavefunction. During his career in the high-tech industry, the author took part in the development of image intensifier devices and in the establishment of the international standards for the measurement of the Optical Transfer Function and the Detective Quantum Efficiency of imaging equipment. At low dose rates, image intensifying devices make the stochastic nature of the detection of photons and elementary particles visible to the human eye. The concept of the Optical Transfer Function applies the fact that the Point Spread Function, which describes the blur of an imaging device owns a Fourier transform. The measurement of the Detective Quantum Efficiency of an imaging device applies the fact that the underlying stochastic process can be considered as a combination of a Poisson process and a binomial process that is implemented by a spatial point spread function. The measurement of the DQE applies the measurement of the signal to noise ratio of the stream of detected quanta that have passed a small aperture. The relation between the measured signal to noise ratio and the dose rate appears to be typical for the inhomogeneous spatial Poisson point process that generated the detected quanta. Since the generated footprint can be described by a location density distribution that owns a Fourier transform, the stochastic mechanism that generates the dynamic geometric data will own a characteristic function. Consequently, the Hilbert Book Model applies inhomogeneous spatial Poisson point processes as the generators of the footprints of elementary particles. The generated point spread function equals the square of the modulus of the wavefunction of the particle.

The HBM also applies the fact that elementary particles act as elementary modules. Together, they constitute all other massive objects that exist in the universe. Discontinuous regions form the exception to this rule. Some modules constitute modular systems.

The author suggests that the composition of composite modules is defined by the second type of stochastic process. These stochastic processes also own a characteristic function. That characteristic function is a dynamic superposition of the characteristic functions of the components of the composite module. The superposition coefficients act as displacement generators and determine the internal locations of the components.

13.2 Mathematical underpinning

The Hilbert Book Model applies a well-known relational structure as its main foundation. The discoverers of this structure, Garrett Birkhoff and John von Neumann, called it “quantum logic” because the structure shows great similarity with classical logic, which is also known as Boolean logic [38]. Mathematicians called the new relational structure an orthomodular lattice. The Hilbert space itself was discovered shortly before quantum logic was introduced. John von Neumann was an assistant of David Hilbert and gave the Hilbert space its familiar name. Later the group around John Baez gave the name Hilbert lattice to the set of closed subspaces of the Hilbert space. Hilbert lattices are lattice isomorphic to the orthomodular lattice [4].

A Hilbert space is a closed vector space that is equipped with an inner product for each vector pair. Each Hilbert space applies a version of an associative division ring to specify the value of the inner product. Only three suitable associative division rings exist. These are the real numbers, the complex numbers, and the quaternions. Depending on their dimension the number system exists in several versions that distinguish in the way that coordinate systems sequence their members.

The restriction to division rings was already indicated in the paper that introduced quantum logic and got several decades later a hard prove by Maria Pia Solèr.

13.3 Extensions of the notion of a Hilbert space by the author

Several extensions of the notion of a Hilbert space are introduced by the author of this paper.

- The author signaled the possibility to choose a version of the number system for specifying the inner products of the Hilbert space and showed that each separable Hilbert space features a private parameter space that applies the selected version of the number system. This parameter space is managed by a dedicated normal operator that the author calls a reference operator.
- This can be exploited further by reusing the eigenvectors of the reference operator to define a category of normal operators that exchange the eigenvalue of the reference operator by the target value of a selected function. The eigenspaces of these new operators represent sampled fields.
- The author exploited the subtle difference between Hilbert spaces and the underlying vector space by suggesting that a huge number of separable Hilbert spaces can share the same underlying vector space.
- One of these separable Hilbert spaces acts as a background platform. The other separable Hilbert spaces float with the geometrical centers of their private parameter spaces over the parameter space of the background platform.
- This extension of the model appears to introduce a further version choice restriction. Only Cartesian coordinate systems that have their coordinate axes parallel to the axes of a background separable Hilbert space can join this extended model.
- The author does not explain this further version choice restriction. Instead, he indicates that the model applies this restriction to determine the difference in the geometric symmetry between the considered separable Hilbert space and a selected background interface. This possibility helps explain why the floating separable Hilbert spaces feature a geometric symmetry-related charge. The

existence of these charges follows from experiments that support the Standard Model.

- The geometric symmetry-related charges locate at the geometrical centers of the parameter spaces of the floating platforms.
- The Hilbert Book Model postulates that the background separable Hilbert space has infinite dimensions.
- The existence of the private parameter spaces and the fact that eigenspaces of dedicated normal operators represent sampled fields suggest that each infinite-dimensional separable Hilbert space owns a unique non-separable companion Hilbert space that embeds its separable partner. This non-separable Hilbert space also features a private parameter space and a category of normal operators that represent fields. However, in the non-separable companion, the corresponding eigenspaces are continuums.
- The infinite-dimensional separable background Hilbert space and its non-separable companion form the complete background platform.
- The background platform contains a dynamic field that is described by a quaternionic function, which exists always and everywhere in the background parameter space and is continuous except for a series of encapsulated regions. We call this field our universe. A dedicated normal operator manages this field in its eigenspace.
- The background platform contains a dynamic field that reflects the existence of the geometric symmetry-related charges of the floating platforms by corresponding sources and sinks. These sources and sinks correspond to the locations of the geometric centers of the floating platforms. We call this field the geometric symmetry-related field.

- The geometrical centers of the floating platforms couple the two dynamic fields.
- The resulting structure acts as a repository for dynamic geometric data of point-like objects and as a repository for dynamic fields.
- The author suggests that all applied separable Hilbert spaces share a common real number based separable Hilbert space that scans the running part of the model by a proper time window.
- The dynamics of the model results from the floating platforms and from the ongoing embedding of the footprints of the point-like objects into the field that represents the universe.

14 Correspondence with the models that mainstream physics applies

14.1 Features of the Standard Model that the Hilbert Book Model can explain

The experimentally verified results that are part of the SM that the HBM explains are:

- The existence of several types of elementary particles
- The existence of a shortlist of electrical charges that are carried by elementary particle types.
- The existence of color charges.
- The existence of quarks.
- The existence of color confinement.
- The existence of antiparticles.
- The existence of photons

14.2 Features that the Hilbert Book Model cannot explain

The experimentally verified results that are part of the SM that the HBM cannot (yet) explain are:

- The existence of generations of fermions.
- The difference in mass between the lowest generation types of elementary particles.

- The fact that only fermions constitute composite objects.

14.3 Critics of the Hilbert Book Model on the SM

The Hilbert Book Model criticizes the need for the existence of the Higgs mechanism. It offers its own explanation for the gravitational potential of elementary particles. The HBM considers the discovered Higgs particle as a composite of elementary particles and not as an extra elementary particle type that supports a Higgs mechanism.

The Hilbert Book Model criticizes the model that mainstream physics presents for the structure of photons. Mainstream physics states that photons are EM waves. The HBM states that photons are strings of equidistant one-dimensional shock fronts that obey the Einstein-Planck relation $E = h\nu$. One-dimensional shock fronts behave like energy packages that travel with light speed.

The human eye contains cones and rods that can perceive photons. These receptors are not antennas that receive EM waves.

14.4 Dark objects

Mainstream physics suggests the existence of two types of dark objects [39][40]. These are dark energy and dark matter. In contrast to mainstream physics, the Hilbert Book Model presents these two types of dark objects as field excitations that act as shock fronts. Together these special field excitations constitute, except for black holes, all discrete objects that exist in the universe.

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