Sticky Space Coverage

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Abstract

Space can be covered with point-like objects. Space covered by a countable set of point-like objects behaves differently from space that is covered by an uncountable set of point-like objects. The document treats the consequences of the change in behavior.

Number systems and coordinate systems

This document applies coordinate markers to navigate between point-like objects that exist in otherwise empty space. The markers use identifiers that are borrowed from a number system. The location of a marker point need not coincide with the virtual location of the corresponding number. However, in the maiden state of the coordinate system, borrowing the identification means that the location of the coordinate marker is identical to the virtual location of the corresponding number.

We apply Cartesian coordinates and, in some cases, spherical coordinates because especially in multidimensional situations the events in local and global coordinates are easier comprehended by humans than local and global events in functions that apply borderless parameter spaces, which stretch over multiple dimensions. In the maiden state of the coordinate system, the coordinate markers locate at the same locations as the corresponding numbers. The relation between the number system and the coordinate system corresponds to the relation between a parameter space and the function that applies the parameter space.

Vector space

We start with completely empty space. Completely empty space is synonym for the ultimate nothingness. This start involves the insertion of two point-like objects in a completely empty space. The first added point is the base point of a vector. The second point is the pointer of the vector. The vector has a length and a direction. The direction defines a direction line. The direction line contains at least two separate points. The integrity of the vector is conserved when it is shifted in parallel as one unit to a different location. This turns empty space into a vector space. Vectors can reach any available location in the vector space. We shall see that a sticky medium is synonymous with a completely covered space. We apply the vector space to generate number systems. Number systems define virtual locations. Coordinate markers turn these virtual locations into the locations of the point-like objects. In this way the markers help navigating in the set of point-like objects.

The real number system

Counting and addition

We start with generating a simple number system. One possibility is that the vector is shifted along its direction line such that its base point takes the location of the pointer location of the original vector. This action creates a new vector that consists of the base point of the first vector and the pointer location of the second vector. The length of the third vector is twice the length of the first vector. All contributing points find a position at the same direction line. The contributing points act as counts and the shift installs the addition procedure.
Repeating the shift and addition procedures generates the set of the natural numbers. The procedure of addition can be reversed into subtraction until the base point of the first vector is passed. This is reason to identify this point as the condition in which space is back to being completely empty; For that reason, this point is called zero. If reverse addition is taken further, then this action introduces negative integer numbers. Together with zero and the natural numbers, this constitutes the set of the integer numbers.

Multiplication, division, and fractions

The following step is the introduction of multiplication by combining multiple additions of the same integer number. Multiplication with integer numbers does not introduce new numbers, but the reverse operation that we will call division can introduce new numbers that we call fractions or ratios. In this way, the number system is extended to the set of the rational numbers. All rational numbers except zero can be applied as a divisor. Scientists have shown that all rational numbers can be labelled with a natural number. This means that the set of rational numbers is still countable. This also indicates that all rational numbers are still surrounded by empty space.

Superseding countability

Up to so far, all rational numbers take a location on the same direction line. The square of a rational number is a multiplication of that number with itself. The result is a rational number. The reverse operation is called square root and this operation does not always result in a ratio. However, a converging series of rational numbers can approach the result arbitrarily close. Many numbers exist that are not rational numbers and can be approached arbitrarily close by converging series of rational numbers. We call these numbers irrational numbers. The set of irrational numbers is not countable. If the set of the rational numbers is merged with the set of the irrational numbers, then the set of real numbers results. The set of all real numbers completely covers the same direction line. If the set covers all irrational numbers, then around the real numbers no space is left. This fact drastically changes the behavior of the covering set of point-like objects.

Spatial dimensions

Different arithmetic

The direction line that is covered by all real numbers leaves no space to add extra numbers. If we want to add all square roots of negative real numbers, then we must use one or three new direction lines that are independent of the direction line that is occupied by the real numbers. The independent direction lines cross at point zero. The arithmetic on these new direction lines differs from the arithmetic of the real number direction line. We call the new direction lines spatial direction lines. The spatial arithmetic will automatically add a third independent direction line when a second spatial direction line is added. The real number direction line together with one spatial direction line forms the set of the complex numbers. The real number direction line together with three spatial direction lines form the set of the quaternions. Multiplying spatial numbers with real numbers is straightforward. In handling the arithmetic of multidimensional number systems, it is wise to treat the combined number as a sum of a real number and a spatial number.

On spatial direction lines, the square of the spatial numbers results in a negative real number. Spatial numbers can be natural, rational, and irrational. Also, in spatial dimensions, the addition of all irrational numbers will supersede countability. The main difference between real numbers and spatial numbers lays in the value of the square of the numbers. In real numbers, the square is always a positive real number. In spatial numbers, the square is always a negative real number. The product of two arbitrary spatial numbers is the sum of a real scalar and a new spatial number that is perpendicular to both factors. The real scalar equals the inner product of the two spatial factors. The new spatial number equals the outer product of the two spatial factors.

Multidimensional arithmetic

The real number arithmetic and the spatial number arithmetic can be mixed. Spatial numbers that reside on different spatial direction lines can be added and multiplied. This will make the spatial number space of the quaternions isotropic. The coordinate markers will capture the geometric symmetry and the location of the geometric center. Real numbers can be added and multiplied by spatial numbers.

For multidimensional numbers, we will use boldface to indicate the spatial part and we will indicate the real part with suffix $_r$.

Thus, the number a will be represented by the sum $a=a_r+\mathbf{a}$. This means that the product $c=a\,b$ of two numbers a and b will split into several terms

$$c = c_r+\mathbf{c} = a\,b = (a_r+\mathbf{a})\,(b_r+\mathbf{b}) = a_r\,b_r + a_r\,\mathbf{b} + \mathbf{a}\,b_r + \mathbf{a}\,\mathbf{b}$$

The product d of two spatial numbers $\mathbf{a}$ and $\mathbf{b}$ results in a real scalar part d_r and a new spatial part $\mathbf{d}$

$$d = d_r+\mathbf{d} = \mathbf{a}\,\mathbf{b}$$

$d_r = -\langle\mathbf{a},\mathbf{b}\rangle$ is the inner product of $\mathbf{a}$ and $\mathbf{b}$

$\mathbf{d} = \mathbf{a} \times \mathbf{b}$ is the outer product of $\mathbf{a}$ and $\mathbf{b}$

The spatial vector $\mathbf{d}$ is independent of $\mathbf{a}$ and independent of $\mathbf{b}$. This means that $\langle\mathbf{a},\mathbf{d}\rangle=0$, and $\langle\mathbf{b},\mathbf{d}\rangle=0$

For the inner product and the norm, $\|\mathbf{a}\|$ holds $\langle\mathbf{a},\mathbf{a}\rangle = \|\mathbf{a}\|^2$

$$\langle\mathbf{a},\mathbf{b}\rangle=\|\mathbf{a}\|\,\|\mathbf{b}\|\cos(\alpha)$$

In mathematics this formula defines the geometric scalar vector product. It is also called the dot product of two vectors.

$$\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\|\,\|\mathbf{b}\|\sin(\alpha)$$

Only three mutually independent spatial numbers can be involved in the outer product.

These formulas still do not determine the sign of the outer product. Apart from that sign, the outer product is fixed.

The product of multidimensional numbers will split into five terms.

$$c = c_r + \mathbf{c} = a\,b \equiv (a_r + \mathbf{a})\,(b_r + \mathbf{b}) = a_r\,b_r - \langle\mathbf{a},\mathbf{b}\rangle + \mathbf{a}\,b_r + a_r\,\mathbf{b} \pm \mathbf{a}\times\mathbf{b}$$

Before these formulas are used, the sign of the outer product must be selected.

Symmetry

The number of mutually independent direction lines in a number system is called the dimension of the number system. The sequencing on a direction line can be done in one direction or in the reverse direction. The direction of the first direction line is arbitrary. Also, the location of point zero is arbitrary. The coordinate system captures these choices at its maiden state. Thus, the same number system exists in many versions that are distinguished by the selected coordinate system. The coordinate system reflects the geometric symmetry and the geometric center of the number system.

The real numbers, the complex numbers and the quaternions appear to be the only three division rings that offer an associative multiplication. Hilbert spaces can only cope with associative division rings. Our purpose is to apply Hilbert spaces. So, we do not look for other number systems. Hilbert systems apply a private version of a chosen number system. The private coordinate system selects which version is tolerated. The selected version of the number system is maintained by a dedicated operator that we will call the reference operator. In its eigenspace this operator provides a private parameter space, which settles the private geometric symmetry and the geometric center of the Hilbert space. The private parameter space turns the Hilbert space into a corresponding function space. The eigenvectors of the reference operator form an orthogonal base for the Hilbert space. This allows a special trick that abstracts a complex-number-based Hilbert space from a quaternionic Hilbert space. A complex-number-based Hilbert space can be abstracted from a quaternionic Hilbert space by taking all eigenvectors of its reference operator that belong to the same spatial direction together with the real number eigenvectors and use these as an orthogonal base of the new complex-number-based Hilbert space. This shows that complex-number-based Hilbert spaces can be considered as subspaces of quaternionic Hilbert spaces.

The reverse trick is only possible if in the quaternionic Hilbert space, locally, the conditions are sufficiently isotropic. For moderate deviations, the coordinate markers can help to find a suitable map of the complex-number-based coordinate markers to the quaternionic coordinate markers.

Stickiness

If space is covered with point-like objects that act as markers of a coordinate system, then the behavior of the combination is determined by the cardinal number of the set of point-like objects. If the set is countable, then the set of point-like objects acts as an ensemble of discrete objects. Every member of the set seems to be surrounded by empty space. However, if the set is no longer countable, then the behavior of the combination of space and point-like objects changes from an ensemble of discrete objects to a coherent sticky medium. It looks as if the combination occupies all available space. The combination becomes deformable and mathematically the medium acts as a differentiable continuum. This switch in behavior happens if number systems containing all integer numbers and all rational numbers are suddenly extended by adding all irrational numbers. It means that the coordinates besides concerning integer markers and rational markers also concern irrational markers. The coordinate system puts the numbers in the correct sequence. It means that some coordinate markers merge into the same point. All converging series of markers end in a limit that is also a coordinate marker. A single deformation does not change the sequencing that the coordinate markers indicate. Each dynamic deformation takes response time for the reaction of the sticky medium.

Sticky coordinates

The set of the complex numbers covers two dimensions. For complex numbers, the outer product does not exist. Two extra independent lines can offer a location to other roots of negative numbers. Together the four direction lines constitute the number system of the quaternions. Both the complex numbers and the quaternions contain a one-dimensional subspace that obeys the arithmetic of the real numbers. In the real number system, all squares of numbers deliver a positive scalar. In the spatial dimensions of the number system, all squares of numbers deliver a negative real number. If the real numbers are interpreted as timestamps, then stickiness can be interpreted as a dynamic behavior that covers all spatial dimensions. The stickiness of the medium leads to a particular dynamic behavior of the medium. Any sudden local deformation is quickly spread in all directions over the full medium until the disturbance vanishes at infinity. The spread occurs with a fixed finite speed. Finally, each sudden local deformation expands the medium. The deformations do not touch the number systems. Instead, in the maiden state, the coordinate system reflects the geometric symmetry and the geometric center of the number system. The coordinate markers will be used to follow the deformations and the vibrations of the medium. In the maiden state, the coordinate markers locate at the same locations as the corresponding numbers. The relation between the number system and the coordinate system corresponds to the relation between a parameter space and the function that applies the parameter space.

At the scale of elementary particles, the deformations caused by these particles are recurrently regenerated. This is implemented by the ongoing hopping path of the particle. The hopping path recurrently regenerates a coherent hop landing location swarm that can be described by a stable location density distribution. If the hop landings cause a reaction of the sticky medium, then that reaction blurs the hop landing location distribution. The blur smooths the effect of the hop landing location swarm. Consequently, the deformation can be described by a smooth function, which is a blurred version of the location density distribution that describes the hop landing location swarm.

Humans often have problems comprehending what an infinite set is and are not familiar with uncountable sets. That is why the switch in behavior works counterintuitively.

Functions can describe the deformations and vibrations of the sticky medium. Differential calculus describes the corresponding change of the coordinate markers in fine detail. Mathematicians can interpret the solutions of quaternionic differential equations. Second-order partial differential equations treat the interaction between sticky mediums and point-like actuators.

Combining influences

The sticky medium transfers information between events and observers of that event. Observers can perceive the event via interaction with the sticky medium. The transfer of the information occurs with finite speed. This fact affects the perceived information. If the speed of information transfer is fixed, then a hyperbolic transformation can mathematically describe the involved coordinate transformation. The observer will perceive in spacetime coordinates. Provided that nothing deforms the information transfer path, a hyperbolic Lorentz transform describes the conversion from Cartesian coordinates to spacetime coordinates. Coordinates can describe the dynamic deformations but do not represent coordinate transforms that account for the effects of information transfer through the sticky medium.

Tensors can combine the coordinate transforms and the influence of deformations. The gravitation field describes the sticky medium. Tensors do not work correctly when multiple fields affect the observer. This occurs when both the gravitation field and electric fields affect the observer. First, the origin of gravitation and the origin of electric charge must be cleared. Another disadvantage of tensors is that the tool is so complicated that it obscures more than it elucidates. In many cases, the coordinate transformation can be ignored, and the application of untransformed coordinates suffices to describe what the observer perceives.

Hilbert repository

In a structure that supports both countable sets of point-like objects and uncountable sets of point-like objects, the interaction between discrete points and the sticky medium can be investigated.

All Hilbert spaces own a natural parameter space, which presents the maiden state of the selected coordinate system. This coordinate system determines the geometric symmetry and the geometric center of the Hilbert space.

The Hilbert repository is a system of Hilbert spaces that all share the same underlying vector space. One of them acts as a background platform. Most members of the system are separable quaternionic Hilbert spaces that float with their geometric center over the natural parameter space of the other Hilbert spaces. The background platform owns a non-separable companion Hilbert space that embeds its separable partner. The sharing of the same underlying vector space restricts the type of Hilbert spaces that can join the system. At their maiden states, all direction lines of the Cartesian coordinate systems must be parallel to the direction lines of the background platform. Only the order along these axes can be selected freely.

A separable Hilbert space can only support operators that manage countable eigenspaces. A non-separable Hilbert space also offers operators that manage eigenspaces for which the eigenvalues are no longer countable. These eigenspaces contain a sticky medium. The Hilbert repository contains only one non-separable Hilbert space. The Hilbert repository shows many features that it shares with the Standard Model of the elementary fermions.

The Hilbert repository is explained in
Preprint The Standard Model of Particle Physics and the Hilbert Repository

The Hilbert repository uses the same space in different ways. Each number system can fill the space in its own way. Coordinate systems fixate the symmetry and the center location. Each Hilbert space uses the space in its own way as a storage medium for numbers. The Hilbert space selects the coordinate system that determines the symmetry and the center location of the chosen number system. The Hilbert repository allows the Hilbert spaces to work together to generate a dynamic system in which parts of Hilbert spaces are depicted on a background. The observers are part of the image. We can recognize aspects of our living environment. Since the Hilbert repository uses only one non-separable Hilbert space, all represented fields reside in the same sticky medium.

Quaternionic differential calculus

Fields that reside in the sticky medium are maintained in eigenspaces of dedicated operators in the background platform of the Hilbert repository. Quaternionic differential calculus describes the dynamic behavior of these fields. The calculus applies to all such fields.

Quaternionic differential calculus applies the quaternionic nabla operator ∇. This calculus uses proper time τ.

None of these equations use physical dimensions.

We use boldface to indicate vectors.

$\nabla \equiv \nabla_r + \boldsymbol{\nabla}$

$\boldsymbol{\nabla} \equiv \{\partial/\partial x, \partial/\partial y, \partial/\partial z\}$

This is the well-known del operator.

$\nabla_r \equiv \partial/\partial\tau$

In the quaternionic differential calculus, differentiation is a multiplier operation.

Compare:

$c=c_r+\mathbf{c}=a\, b\equiv(a_r+\mathbf{a})(b_r+\mathbf{b})=a_r b_r-\langle\mathbf{a},\mathbf{b}\rangle+\mathbf{a}b_r+a_r\mathbf{b}\pm\mathbf{a}\times\mathbf{b}$

and

$\phi=\phi_r+\boldsymbol{\phi}=\nabla\psi\equiv(\nabla_r+\boldsymbol{\nabla})(\psi_r+\boldsymbol{\psi})=\nabla_r\psi_r-\langle\boldsymbol{\nabla},\boldsymbol{\psi}\rangle+\boldsymbol{\nabla}\psi_r+\nabla_r\boldsymbol{\psi}\pm\boldsymbol{\nabla}\times\boldsymbol{\psi}$

$\nabla\psi$ represents the change of field ψ. This change covers five terms. Part of these terms can compensate for other terms. For example, $\nabla_r\,\boldsymbol{\psi}$ can compensate $\boldsymbol{\nabla}\psi_r$.

$\phi_r=\nabla_r\psi_r-\langle\boldsymbol{\nabla},\ \boldsymbol{\psi}\rangle$

Maxwell equations do not use the real part ϕ_r of ϕ

$\boldsymbol{\phi}=\boldsymbol{\nabla}\psi_r+\nabla_r\,\boldsymbol{\psi}\ \pm\boldsymbol{\nabla}\times\boldsymbol{\psi}$

Double differentiation leads to the quaternionic second order differential equation:

$\zeta=\nabla^*\phi=(\nabla_r-\boldsymbol{\nabla})\ (\nabla_r+\boldsymbol{\nabla})\ (\psi_r+\boldsymbol{\psi}) = \{\nabla_r\nabla_r+\langle\boldsymbol{\nabla},\ \boldsymbol{\nabla}\rangle\}\ (\psi_r+\ \boldsymbol{\psi})\ =\rho_r+\mathbf{J}$

$\zeta=\ \{\nabla_r\nabla_r+\langle\boldsymbol{\nabla},\ \boldsymbol{\nabla}\rangle\}\ \psi$

The equation can be split into two first-order partial differential equations $\phi=\nabla\psi$ and $\zeta=\nabla^*\phi$

This equation offers no waves as its solutions. The quaternionic equivalent of the wave equation is:

$\varphi=\ \{\nabla_r\nabla_r-\langle\boldsymbol{\nabla},\ \boldsymbol{\nabla}\rangle\}\ \psi$

Maxwell-like equations that use special symbols are:

$\mathbf{E}=-\boldsymbol{\nabla}\psi_r-\nabla_r\boldsymbol{\psi}$

$\mathbf{B}=\boldsymbol{\nabla}\times\boldsymbol{\psi}$

$\rho_r=\langle\boldsymbol{\nabla},\mathbf{E}\rangle$

$\mathbf{J}=\boldsymbol{\nabla}\times\mathbf{B}-\nabla_r\mathbf{E}$

$\nabla_r\mathbf{B}=-\boldsymbol{\nabla}\times\mathbf{E}$

The corresponding second-order differential equations are:

$\{\nabla_r\nabla_r\ -\langle\boldsymbol{\nabla},\boldsymbol{\nabla}\rangle\}\ \boldsymbol{\psi}\ =\mathbf{J}$

$\{\nabla_r\nabla_r\ -\langle\boldsymbol{\nabla},\ \boldsymbol{\nabla}\rangle\}\ \psi_r=\ \rho_r$

None of these equations apply compensation for physical units. They are purely mathematical equations.
In relation to the behavior of the sticky medium the solutions of pulse responses are interesting.

Solutions

The two second-order partial differential equations describe the behavior of dark objects.

$\varphi=(\partial^2/\partial\tau^2\pm\ \langle\boldsymbol{\nabla},\ \boldsymbol{\nabla}\rangle)\psi$

A third equation skips the first term

$\varphi=\ \langle\boldsymbol{\nabla},\ \boldsymbol{\nabla}\rangle\psi$

This is the Poisson equation.

Even though the operators are Hermitian operators, these equations are quaternionic differential equations. Thus, φ and ψ are quaternionic functions that own a scalar real part and an imaginary vector part. The solutions are quaternionic functions.

All solutions of a homogeneous second-order partial differential equation superpose in new solutions of that equation

One of the solutions of the Poisson equation is Green's function

$g(\mathbf{q})=1/|\mathbf{q}-\mathbf{q}_\circ|$

$\nabla g(\mathbf{q})=(\mathbf{q}-\mathbf{q}_\circ)/|\mathbf{q}-\mathbf{q}_\circ|^3$

$\langle\nabla, \nabla\rangle\, g(\mathbf{q})= \langle\nabla, \nabla g(\mathbf{q})\rangle=4\pi\delta(\mathbf{q}-\mathbf{q}_\circ)$

Thus, Green's function is a static pulse response under purely isotropic conditions.

The dark objects behave as shock fronts and operate only as odd-dimensional field excitations. During travel, all shock fronts keep the shape of the front.

The one-dimensional shock fronts also keep their amplitude. Consequently, the one-dimensional shock fronts can travel huge distances without losing their properties. Combined equidistantly in strings, they represent the functionality of photons. This means that the one-dimensional shock fronts are the tiniest possible packages of pure energy.

Depending on the PDE, the solutions can be described by different equations. The solution for the wave equation is

$g(\mathbf{q}, \tau)=f(c\,\tau \pm |\mathbf{q}-\mathbf{q}_\circ|)$

This solution cannot represent polarization.

The solution for the other equation is

$g(\mathbf{q}, \tau)=f(c\,\tau \pm |\mathbf{q}-\mathbf{q}_\circ|\,\mathbf{i})$

The vector $\mathbf{i}$ can indicate the polarization of the shock front.

This solution represents a dark energy object.

A photon is a string of equidistant energy packages that obeys the Einstein-Planck relation

$E = h\,\nu$.

Since photons possess polarization, they use the second solution for their energy packages. Thus, the constituents of photons are not solutions of the wave equation.

The three-dimensional shock fronts require an isotropic trigger. These field excitations integrate over time into the Green's function of the field. That function has some volume, and the pulse response injects this volume into the field. Subsequently, the front spreads the volume over the field. The corresponding solution of the wave equation is

$g(r, \tau) = f(c\,\tau \pm r)/r$

The parameter r is the radius of the spherical front. The formula can also be written as

$g(\mathbf{q}, \tau) = f(c\,\tau \pm |\mathbf{q} - \mathbf{q}_\circ|)/|\mathbf{q} - \mathbf{q}_\circ|$

The solution for the other PED is

$g(\mathbf{q}, \tau) = f(c\,\tau \pm |\mathbf{q} - \mathbf{q}_\circ|\,\mathbf{i})/|\mathbf{q} - \mathbf{q}_\circ|$

In this solution, the vector **i** acts as a spin vector. It is a normed imaginary quaternion.

This solution represents a dark matter object.

This solution tries to remove the deformation in all directions. Consequently, the deformation quickly fades away until it finally vanishes at infinity. Integrated over time the solution equals Green's function. The injected volume stays in the field and finally expands the whole field.

The dark objects describe the typical behavior of the sticky medium. The shock fronts travel with speed c, which is the maximum speed that the sticky medium tolerates.

Coordinates help to navigate through the deformed and expanded sticky medium. Quaternionic differential calculus describes the changes of the coordinates when actuators affect the sticky medium.

Isotropic conditions

The two shock front solutions show an interesting property of the Poisson equation. In isotropic conditions the equation can be rewritten as

$\varphi = \langle \nabla, \nabla \rangle \psi = \{\partial^2/\partial r^2 + (2/r)\,\partial/\partial r\}\,\psi = \{\partial^2/\partial r^2\}\,(r\psi)$

The product $\Phi = r\psi$ is a solution of a one-dimensional equation in which r plays the variable.

The same thing holds for all differential equations that contain the $\langle \nabla, \nabla \rangle$ operator.

So, spherical solutions of the second-order differential equations ξ/r can be obtained from the solutions ξ of one-dimensional second-order differential equations by dividing ξ with the distance r to the center.

It looks as if in isotropic conditions the quaternionic differential calculus can be scaled down to complex-number-based differential calculus. This already works at local scales. If on larger scales the isotropic condition is violated, then the coordinates of the complex-number-based abstraction must be adapted to the possibly deformed Cartesian coordinates of the quaternionic platform. This makes sense in the presence of moderate deformations of the quaternionic field. After adaptation, the map of the complex-number-based coordinate lines become geodesics.

These tricks are possible because complex-number-based Hilbert spaces can be considered as subspaces of quaternionic Hilbert spaces.