Quantum description of Newtonian gravity

Farid Abrari
1400 Woodeden Dr, Mississauga, Ont, L5H 2T9, Canada
farid.abrari@gmail.com

July 1, 2021

Abstract

The smooth Newtonian model of gravity is quantized using the results obtained from the combined theory of Special Relativity (SR) and Quantum Mechanics (QM). The resulting quantum model of gravity, unlike the classical Newtonian model, predicts that there exists an upper limit to the distance between a given pair of masses, called the *action distance*, beyond which they become gravitationally unbound. Equivalently, at any given radial distance from a large gravitating body of mass, there exists a minimum mass below which the particle would not gravitationally bind to the gravitating body. The *attractable mass limit* of a gravitating body is determined by equating action distance with the surface radius of the body. Moreover, the quantum model of gravity indicates that the *escape velocity* from a large gravitating body is a function of the mass of the escaping particle as well. This quantum effect of gravity become significant if the mass of the escaping particles, such as the gas molecules from the exosphere of a planet, are comparable to the attractable mass limit of the planet. The significant discrepancy observed in the escape rates of CH_4 and N_2 species from Pluto's exosphere is used to constrain the *reference mass* of the combined SR-QM theory to $\bar{m} = 3.2E - 45$ (kg). The latter is thought to be the physical cut-off limit for massless particles. An Earth bound experiment is also proposed to test the predictions of the combined SR-QM theory and determine the reference mass with a higher accuracy.

Keywords — Quantum Gravity, Escape Velocity, Reference Mass, Pluto, Free Fall

1 Introduction

In the classical Newtonian model of gravity, the gravitational field surrounding a large body of mass extends *unboundedly* in space. Furthermore, the *local acceleration* of a particle falling freely in such a gravitational field is a *continuous* function of the shrinking distance between them. However, the combined theory of Special Relativity (SR) and Quantum Mechanics (QM) indicates that the local acceleration of the physical particles do **not** vary *continuously* but rather under a fixed *steps*. The *quanta of acceleration* of a particle is found to be inversely proportional to the square of its rest mass m, therefore, *the higher the rest mass the smaller the acceleration quanta.* This makes the accelerated motion of massive particles smoother than those of lighter particles. Having obtained the quantum of acceleration as a function of rest mass, then the classical Newtonian model of gravity was subsequently quantized by invoking the equivalency principle between gravity and local acceleration.

2 Background

We begin with a very brief description of the combined theory of SR-QM whose details can be found in [1]. According to the theory, the inherent uncertainties in the position ϵ, time $\delta t'$, momentum $\delta p'$ and energy $\delta E'$ of a particle are all functions of its rest mass m. Moreover, the theory necessitates the existence of a reference particle with rest mass $\bar{m}$ which the physical particles are all scaled against. *The reference particle represents a particle whose rest mass $\bar{m}$ is the smallest none-zero mass physically possible.* Therefore, any particle with rest mass $m < \bar{m}$ is physically considered as a massless particle, i.e. $\{m = 0 \mid m < \bar{m}\}$. As will be discussed later, one of the main applications of the quantized model of gravity is the estimation of the reference particle mass.

The notion of *rest frame* in the combined SR-QM theory is revised into *an inertial frame of reference in which a quantum particle has a minimum inherent uncertainty in its position.* The rest frame in SR-QM theory is called the *nearest* frame, or alternatively *near-rest* frame, as the particle is at the nearest state to being stationary in that frame. From the theory, we learn that the *uncertainty* ϵ_n in the spatial position of a quantum particle, as viewed from an inertial frame of reference which is in motion relative to the particle's nearest frame, is given by [1]:

$$\epsilon_n = A\sqrt{\frac{\bar{m}}{m}n} \qquad n = 1, 2, ... \tag{1}$$

where *quantum index* n is a dimensionless positive integer. The *constant* A is a fixed integer multiple of Planck length l_p. A is also the Compton wavelength of the reference particle $A = h/\bar{m}c$ [1]. Quantized equation of the particle velocity v'_n, under the non-relativistic condition $v'_n << c$, is given by:

$$v'_n = c\sqrt{\frac{\bar{m}}{m}n} \qquad n = 1, 2, ... \tag{2}$$

where c is the speed of light. From Eqn's 1 and 2, it is evident that the *higher the velocity of a particle relative to an inertial frame, the higher is the spatial uncertainty* in that frame. It then follows that the lowest quantum index, $n = 1$, corresponds to a condition in which the particle is at the nearest state to being stationary.

3 Local acceleration

The *local acceleration* of a particle, a, is defined as the magnitude of the acceleration of the particle relative to an inertial frame of reference which the accelerating particle is found to be *instantaneously* at near-rest with [2]. The derivative of the quantum velocity from Eqn 2, with respect to the *coordinate time*, gives the *quantized* acceleration of the particle as follows:

$$a_{\dot{n}} = \dot{n}c\sqrt{\frac{\bar{m}}{m}} \qquad \dot{n} = 0, \pm 1, \pm 2, ... \tag{3}$$

Unlike the quantum index n which is always a positive number, the *quantum rate index* $\dot{n}$ could be either a negative or positive integer, or zero. The quantum rate index $\dot{n} < 0$ is for a *decelerating* particle where quantum index n reduces in coordinate time, $\dot{n} > 0$ is for an *accelerating* particle where quantum index n increases in time and $\dot{n} = 0$ is for a stationary particle or a particle with a non-varying momentum (constant velocity) where the quantum index n is constant in coordinate time. The unit of quantum rate index $\dot{n}$ is sec^{-1}.

4 Quantized Newtonian gravity

Now let us consider a special case where a small particle of mass m is in the state of *free fall* towards a larger body of mass M, with this condition that $M >> m$. The latter is to emphasize that only the smaller mass m is in the state of free fall and the bigger mass M is stationary. Hence, in the equations to follow the masses m and M are *not interchangeable*. From the equivalence principle of General Relativity [3], the state of an accelerating particle under the local acceleration a is *physically identical to the state of its free fall in a uniform gravitational field* g, if, and only if, $g = a$. Accordingly, the quantum acceleration given in Eqn 3, can also be used to describe the gravitational acceleration of the particle m when falling freely in the gravitational field of a much larger gravitating body M as follows:

$$\frac{GM}{R_{\dot{n}}^2} = g_{\dot{n}} = \dot{n}c\sqrt{\frac{\bar{m}}{m}} \qquad \dot{n} = 1, 2, ... \tag{4}$$

Note that in this equation, only the positive integers of $\dot{n}$ are taken into consideration to further emphasize that if a particle is in a gravitational field, and it is also in the state of free fall, it then must be in acceleration (i.e $\dot{n} > 0$). Now recall that in the classical Newtonian theory of gravity, the distance R in the equation $g(R) = \frac{GM}{R^2}$ can be increased indefinitely; and subsequently, the gravitational force of M, acting from a distance R on the particle m, can be reduced indefinitely. Therefore, without an upper limit on the distance R, as $R \to \infty$ then $g(R) \to 0$; i.e. the gravitational acceleration of m drops further and further without any physical limit on its *non-zero* minimum. In the quantum description of gravity, however, *there exists an upper limit to the distance of gravitational influence* beyond which the gravitational acceleration of the free falling particle m drops below its *acceleration quanta*. The latter is obtained by setting $\dot{n} = 1$ in Eqn 4 as follows:

$$g_1 = c\sqrt{\frac{\bar{m}}{m}} \tag{5}$$

According to Eqn 5, *the higher the mass m of a particle the smaller is its acceleration quanta*. Therefore, from quantum gravity perspective, *free fall of more massive particles occur in a finer quantum steps than those of lighter particles, (i.e. smoother than lighter particles)*. More specifically, it follows that the ratio of quantum accelerations of a pair of particles is equal to the square of the inverse of their mass ratio. This is simply obtained by writing Eqn 5 for a pair of particles with masses m_1 and m_2 and then finding the ratio of their quantum accelerations as follows:

$$\frac{g_{11}}{g_{12}} = \sqrt{\frac{m_2}{m_1}} \tag{6}$$

where g_{11} is the acceleration quanta of m_1 and g_{12} is the acceleration quanta of m_2.

5 The action distance

The *maximum* distance between a large gravitating body M and a particle of small mass m beyond which they gravitationally become unbounded is called the *action distance*. The latter can be determined by setting the quantum rate index $\dot{n} = 1$ in Eqn 4 and then solving for the distance:

$$R_1 = \sqrt[4]{\frac{m}{\bar{m}}}\sqrt{\frac{GM}{c}} \tag{7}$$

Again, note that the masses m and M are not interchangeable in these set of equations, as discussed before. From Eqn 7, it is clear that the higher the masses M and m the higher the action distance between the pair. In other words, the gravitational influence of M stretches to higher distances in the case of its interaction with a more massive particle. When the distance R between the masses increases beyond the action distance R_1, the quantum rate index drops from $\dot{n} = 1$ at R_1 to $\dot{n} = 0$ at distances $R > R_1$, indicating that the gravitational force at any distance greater than the action distance is zero, i.e. the particle m is not gravitationally bound to the body M any more. Note that

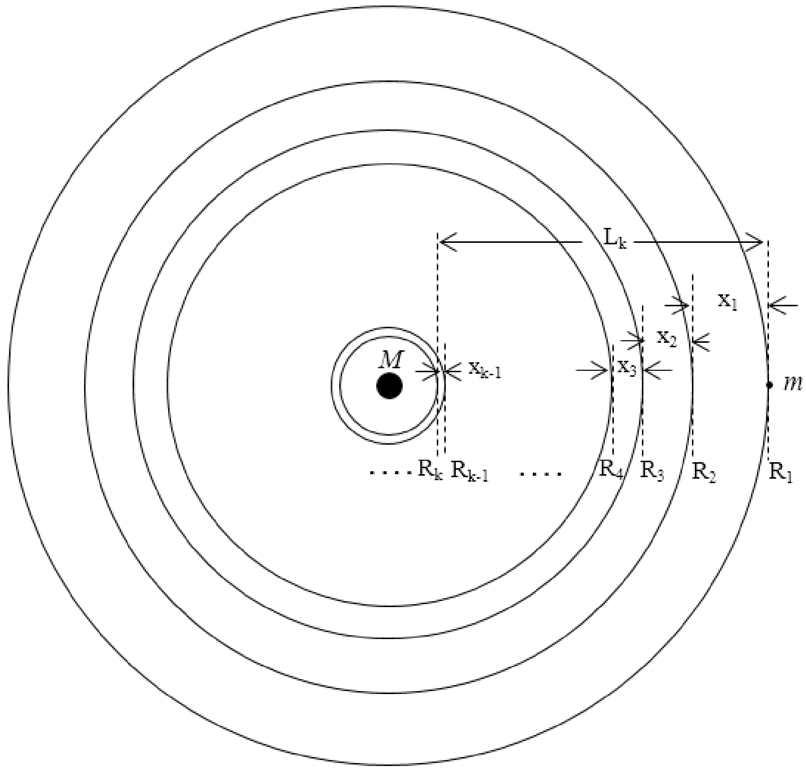

Figure 1: The rings of constant gravity progressively narrow down in width as altitude drops

$\dot{n} = 0$ means velocity of the particle remains invariant in time (i.e. particle is not gravitationally being accelerated over time). The *quanta* of the external force that can accelerate a particle of rest mass m with a single acceleration quanta g_1 is then given by:

$$f_1 = mg_1 = c\sqrt{m\bar{m}} \tag{8}$$

In general, for a given pair of masses M and m, each quantum rate index $\dot{n}$ is related to a *fixed* distance between the masses obtained from Eqn 4 by:

$$R_{\dot{n}} = \sqrt[4]{\frac{m}{\bar{m}}}\sqrt{\frac{GM}{\dot{n} \cdot c}} \tag{9}$$

Therefore, as shown in Fig 1, due to the characteristics of such quantization, the gravitational free fall of mass m occurs in a series of *constant* accelerations, each corresponding to a distinct quantum rate index $\dot{n}$. The free fall distance of each quantum rate index $\dot{n}$ (and hence acceleration) is given by:

$$x_{\dot{n}} = \sqrt[4]{\frac{m}{\bar{m}}}(\sqrt{\frac{GM}{\dot{n}c}} - \sqrt{\frac{GM}{(\dot{n}+1)c}}) \tag{10}$$

It then follows that the distance traveled from the rate index $\dot{n} = 1$ to $\dot{n} = k$ is given by:

$$L_k = \sum_{j=1}^{k-1} x_j = \sqrt[4]{\frac{m}{\bar{m}}}(\sqrt{\frac{GM}{c}} - \sqrt{\frac{GM}{kc}}) \qquad k > 1 \tag{11}$$

Finally, from the last equation, the quantum rate index k corresponding to a given free fall distance L_k, measured from R_1, as shown in Fig 1, is given by the following:

$$k = \frac{GM}{(\sqrt{\frac{GM}{c}} - L_k\sqrt[4]{\frac{\bar{m}}{m}})^2 c} \tag{12}$$

It is worth to note that as the mass m continues to fall to lower and lower altitudes in the gravitational field of M, the quantum rate index $\dot{n}$ continues to progressively increase, and the distance $x_{\dot{n}}$ between two successive $\dot{n}$'s continues to progressively reduce in length, resulting in an increasingly smooth variation in the acceleration of the mass m as it gets closer and closer to the gravitating body M. Before concluding this section, let us consider two masses m_1 and m_2 with such mass ratios that permits $R_{\dot{n}_1} = R_{\dot{n}_2}$. Using Eqn 9, for such situation we will have the following relation between the quantum rates indices and the masses:

$$\frac{\dot{n}_1}{\dot{n}_2} = \sqrt{\frac{m_1}{m_2}} \tag{13}$$

Note that since quantum rate indices are variables that always assume integer values, the condition $R_{\dot{n}_1} = R_{\dot{n}_2}$ of two particles will only be met, if, and only if $\sqrt{\frac{m_1}{m_2}}$ itself is an integer number (or inverse of an integer number). Accordingly, this means that in the quantum formulation of gravity, particles of such mass ratios will have exactly the same gravitational acceleration at those coinciding distances from a gravitating body. To realize this, we replace for $\dot{n}_1$ in Eqn 4 from Eqn 13 as follows:

$$g_{m_1} = \dot{n}_1 \cdot c\sqrt{\frac{\bar{m}}{m_1}} = \dot{n}_2\sqrt{\frac{m_1}{m_2}} \cdot c\sqrt{\frac{\bar{m}}{m_1}} = \dot{n}_2 \cdot c\sqrt{\frac{\bar{m}}{m_2}} = g_{m_2} \tag{14}$$

For any other radial distances between two successive coinciding $R_{\dot{n}_1} = R_{\dot{n}_2}$, as shown in Fig 2, the acceleration of the heavier particle will be higher by some *integer multiples* of the heavier particle's quantum acceleration, before they become equal again in the next coinciding distance. The situation illustrated in Fig 2 is for a mass ratio of 9. From Eqn 6, the ratio of acceleration quanta in this case will be $\frac{g_{11}}{g_{12}} = 3$; and hence, there is precisely three quantum acceleration upticks of the heavier mass per one uptick of the lighter mass. Hence, *in average*, gravitational acceleration of the massive particle will be $0.5g_{11}$ higher per quantum acceleration uptick of the lighter particle. Therefore, unlike General Relativity or Newtonian model of gravity, the quantum model of gravity indicates that the free fall of particles with different mass occur under slightly different gravitational accelerations. Therefore, comparing a pair of particles of large and small mass - both fall in vacuum from equal heights and initial velocities - the accumulation of tiny effects of quantum gravity would result in the massive particle to attain a higher velocity and fall ahead of the lighter particle. Inversely, in free ascending scenarios, starting from equal heights and initial velocities, the massive particle would gain a higher altitude compared to the lighter particle.

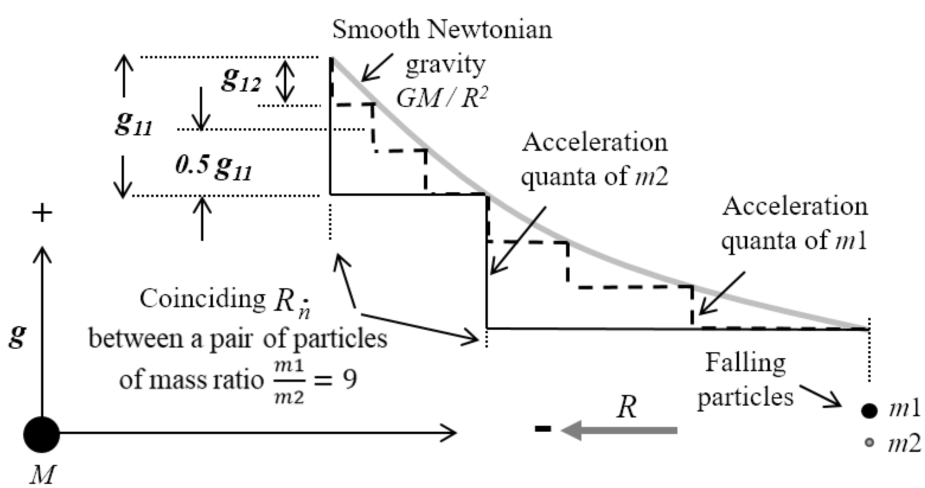

Figure 2: Quantum free fall accelerations of two particles with mass ratio of 9

Finally, let us remark here that unlike the smooth Newtonian gravity, where the stable orbital paths of particles around a large gravitating body are always circular, in the situations where quantum effects of gravity are apparent the stable orbital paths of particles are *polygons*. The polygons are fit within the rings of constant gravity, as shown in Fig 1. The Newtonian circular motion, then corresponds to a special case where the width of the rings are reduced to zero (i.e. the case of classical mechanics) and therefore the polygons have infinite sides. One of the manifestations of the phenomenon, is believed to be the Saturn's *hexagon*. These two topics are introduced in two separate papers that will be submitted following the current paper.

6 Numerical simulation of quantum free fall

As discussed in the previous section, motion of any particle of sufficiently small mass in a gravitational free fall is consisted of a numerous quantum free falls, each under a constant local acceleration. In this section, a very simple recursive numerical algorithm is described in order to demonstrate the quantum effects of gravity as predicted by this theory. Starting from a given initial height, a falling particle of mass m and a gravitating body of mass M, first the quantum rate index j corresponding to the initial radial position R_j of the particle is found using:

$$j = \frac{GM}{{R_j}^2 c}\sqrt{\frac{m}{\bar{m}}} \tag{15}$$

Then the local acceleration g_j and the quantum distance x_j corresponding to the quantum rate index j are found using Eqns 4 and 10, respectively. Next, the quantum free fall duration t_j, corresponding to the distance x_j, is found using the following:

$$t_j = \sqrt{(\frac{v'_{j-1}}{g_j})^2 + \frac{2x_j}{g_j}} - \frac{v'_{j-1}}{g_j} \tag{16}$$

where v'_{j-1} is the velocity of the falling mass m at the *end* of the previous free fall of quantum rate $j-1$. If free fall starts from a stationary condition, v'_{j-1} is then set to zero at the very beginning of the simulation. Finally, the free fall velocity at the end of the current quantum rate index j is calculated using:

$$v'_j = v'_{j-1} + g_j t_j \tag{17}$$

The recursive process is then repeated for the next quantum rate index $j+1$; obtaining distances, elapsed times, velocities and accelerations as free fall continues by continuous increase of the index j, one unit at a time.

7 Attractable mass limit

As discussed earlier, in both the Newtonian model of gravity and General Relativity, there is no physical upper limit to the distance between a pair of gravitationally bounded masses M and m, beyond which the force of gravity drops below a physical minimum required to accelerate them. In the quantum model of gravity, however, we find that there exists a distance of gravitational influence R_1, associated with the quantum rate index $\dot{n} = 1$, beyond which the gravitational force of the gravitating body M on particle m drops below the quanta $c\sqrt{m\bar{m}}$; and therefore, by not being able to accelerate the particle it becomes physically inconsequential. Similarly, in this section we want to show that, unlike the existing continuous theories of gravity, the quantum model of gravity offers a minimum particle mass m_r below which the gravitational force between the masses m and M drops below the quanta. It is obvious that the binding mass limit is a function of radial distance r from the gravitating mass M. When the binding distance is taken to be the *surface radius* R_s of the body M, the corresponding minimum gravitationally binding mass of the body M is denoted by m_s and is called the Attractable Mass Limit (AML). To find the latter, we begin with the action distance of Eqn 7 and ask the following question: for a given gravitating body of mass M and radius R_s, what is the minimum mass m_s whose gravitationally binding distance R_1 is equal to the radius of the body R_s? Therefore, by setting $R_1 = R_s$ in Eqn 7 we arrive at:

$$m_s = \xi_s \, \bar{m} \tag{18}$$

The constant of proportionality ξ_s, called the *constant of gravitational attraction* of body M, is a dimensionless parameter given by:

$$\xi_s = (\frac{c}{g_s})^2 \tag{19}$$

where g_s is gravitational strength at the *surface* radius R_s of the body M. According to the theory, for a large gravitating body of mass like a planet, the gaseous species with molecular mass $m < m_s$ will not be gravitationally bound to the planet's surface. The minimum mass limit m_r at any other radial distance $R_r > R_s$ is given by $m_r = \xi_r \, \bar{m}$. The constant of gravitational attraction ξ_r is then given by $\xi_r = (\frac{c}{g_r})^2$, where g_r is gravitational strength at the radial distance R_r. Therefore, as expected, we always have $m_s < m_r$, i.e. the attractable mass limit of body M increases with increasing distance from it.

We conclude this section by noting since a particle with mass $m < m_s$ is not gravitationally bound to the gravitating body, the escape velocity of classical mechanics does not apply to it. Therefore, unlike that of classical mechanics, the escape velocity from quantum gravity is expected to be a function of the mass of the escaping particle, such that at the minimum mass limit m_s the escape velocity of the particle is expected to be the particle's velocity quanta v_1. The latter is the smallest non-zero velocity that a particle of mass m could have [1].

8 Escape velocity of quantum gravity

The *classical escape velocity* v_{ec} of a particle of mass m from a gravitating body of mass M and radius R_s is obtained by equating the total energy of the escaping particle at the *surface* of the body and that of *infinitely far away*, where both the gravitational potential and kinetic energy (KE) of the particle are assumed to asymptotically reach zero:

$$\frac{1}{2}mv_{ec}^2 - \frac{GMm}{R_s} = 0 - 0 = 0 \tag{20}$$

Accordingly, the classical escape velocity v_{ec} is determined to be:

$$v_{ec} = \sqrt{\frac{2GM}{R_s}} \tag{21}$$

From Eqn 21, the classical escape velocity is found to be independent of the mass m of the escaping particle, an aspect that is shown here not to be true in the case of particles of very small mass; such as the case of gas molecules escaping the exosphere of planets. The effect of quantum gravity on the escape velocity could be determined by simply accounting for the potential and KE energies of the particle at the *action distance* R_1, instead of infinitely far away which is not physically appealing. Therefore:

$$\frac{1}{2}mv_{eq}^2 - \frac{GMm}{R_s} = \frac{1}{2}mv_1^2 - \frac{GMm}{R_1} \tag{22}$$

Substituting for the quanta velocity v_1 from Eqn 2, the action distance R_1 from Eqn 7 and solving for the quantum escape velocity v_{eq} we arrive at:

$$v_{eq} = [2GM(\frac{1}{R_s} - \sqrt{\frac{1.c}{GM}}\sqrt[4]{\frac{\bar{m}}{m}}) + c^2\frac{\bar{m}}{m}]^{1/2} \tag{23}$$

It is evident that for more massive particles as the ratio $\bar{m}/m \to 0$, the quantum effect of gravity diminishes and the escape velocity from Eqn 23 reduces to that of the classical mechanics -

as expected. Moreover, the escape velocity of AML with mass $m = m_s$ and the action distance $R_1 = R_s$ is found to be $v_{eq} = \frac{g_s}{1} = v_1$. Therefore, we learn that the escape velocity of the AML of the gravitating body M is equal to the body's surface gravity -*numerically.* Recall that the unit of numeric 1 in the denominator is $\sec^{-1}$. Therefore, a particle whose escape velocity from Earth is $v_{eq} = 9.81$ (m/sec) is the AML of the planet Earth.

The percent error in the escape velocities from the classical and quantum formulation is, therefore, a function of how close the escaping particle mass is to the attractive mass limit AML of the gravitating body. Such error can be a source of discrepancy between the escape rates of a pair of species, in particular if mass of one of them is too close to m_s of the host planet. This unique characteristic of the quantum model of Newtonian gravity will be used in the next section to constrain the value of reference mass $\bar{m}$.

9 Jeans atmospheric escape mechanism

One the mechanisms of the atmospheric loss corresponds to a condition that the kinetic energy of gas species is adequately high to overcome the gravitational grip of a planet. Such condition is important in particular at the very edge of the atmosphere, called exobase, where the planet's atmosphere interfaces with the vacuum of space. At the edge of the exobase should the gas molecules have speeds higher than the escape velocity they will have a chance to escape into the space instead of remaining confined by their collisions to the surrounding molecules. This molecule by molecule escape condition is not unlike to that of vapor molecules leaving the surface of a boiling liquid when their kinetic energy is adequately high to break them away from the liquid surface. The thermally driven molecule by molecule escape mechanism from planetary atmosphere is known as Jeans escape [5]. The onset of Jeans escape can therefore be described in terms of the *Jeans parameter*, defined as the ratio of gravitational potential energy to the thermal energy of the gas at the exobase as follows:

$$\lambda = -\frac{U(r)}{\kappa T} \tag{24}$$

where $\kappa = 1.38065E-23$ (J/K) is Boltzmann constant, T is the species temperature at the exobase in Kelvin and $r = R_s + H$ is radius of the exobase. The gravitational potential energy $U(r)$ of the escaping molecule is a function of the mass m and radius r as follows:

$$U(r) = -\frac{GMm}{r} \tag{25}$$

The average velocity v_{rms} of gas molecules at temperature T, on the other hand, is given by:

$$\frac{1}{2}mv_{rms}^2 = \frac{3}{2}\kappa T \tag{26}$$

By substituting for the energy terms from Eqn 25 and Eqn 26, we then have the Jeans parameter from Eqn 24 in terms of the gas escape velocity v_{ec} and the gas average molecular velocity v_{rms} as follows:

$$\lambda = \frac{3}{2}(\frac{v_{ec}}{v_{rms}})^2 \tag{27}$$

By examining Eqn 27, it is evident that a large Jeans parameter λ would indicate a high escape velocity v_{ec} and a low gas molecular velocity v_{rms}. This corresponds to the case of a massive planet with a cold exobase for which the atmospheric escape is suppressed due to a large gravity and low molecular velocity. On the other extreme, a small Jeans parameter λ would indicate a low escape velocity v_{ec} and a high gas molecular velocity v_{rms}. This corresponds to the case of a small planet and a hot exobase for which atmospheric escape is more likely.

For a given planet and exobase altitude, the *escape rates* Φ of the species will be a function of their Jeans parameters; if all other influencing variables such as their abundance and temperature were similar. Jeans escape occurs if v_{rms} is greater than some fraction α of the escape velocity v_{ec}. As a rule of thumb [6], if the fraction $\alpha > 1/6$, Jeans escape mechanism is active and as a result Jeans escape rate $\Phi > 0$. Under such condition, the escape rate Φ is inversely proportional to the Jeans parameter λ and directly proportional to the density number N of the species present at the top of the exosphere as follows:

$$\Phi \propto \frac{N}{\lambda} \tag{28}$$

Assuming the constant of proportionality remains invariant between a pair of species, then the ratio of their escape rate at the top of the exobase could be approximated as follows:

$$\frac{\Phi_1}{\Phi_2} \approx \frac{N_1 \lambda_2}{N_2 \lambda_1} \tag{29}$$

where N is the number density in units cm^{-3}, Φ is Jeans escape rate in units sec^{-1} and λ is Jeans Parameter - a dimensionless number.

10 Constraining the reference mass $\bar{m}$

Based on the observational evidence obtained from the *ALICE spectograph* and *New Horizons* mission, the escape rate of Nitrogen N2 molecules from the exosphere of Pluto is found to be 4 orders of magnitudes less than the theoretical values [8] (using escape velocity of classical mechanics). This is rather a large discrepancy in comparison to that of less massive Methane CH4 molecules whose escape rate is found to be inline with the theoretical predictions. The escape rate corresponding to these species are in the range of $\Phi_1 = 4 - 8E25$ CH4 sec^{-1} and $\Phi_2 = 3 - 7E22$ N2 sec^{-1} [9]. The number density corresponding to each specie is given as $N_1 = 2.71E6$ CH4 cm^{-3} and $N_2 = 1.8E6$ N2 cm^{-3}. It is evident that such a large discrepancy in the *average* escape rate ratio $\Phi_1/\Phi_2 = 6000/5$ of CH4 to N2 molecules cannot be explained by their mass ratio of 16/28 and number density ratio 2.71/1.8 alone.

To address the discrepancy in the escape rates, we propose to use the quantum escape velocity of Eqn 23 instead of the classical escape velocity $\sqrt{-2U(r)/m}$ traditionally used in the calculations. Taking the extreme values, the ratio of escape rates between two species could be bracketed to the range $571 < \Phi_1/\Phi_2 < 2666$. From Eqn 29, a similar range could be assumed for the ratio $571 < \frac{N_1\lambda_2}{N_2\lambda_1} < 2666$; with the exception that now in the calculation of Jeans parameters of λ_1 (i.e. CH4) and λ_2 (i.e. N2) each specie has its own escape velocity calculated from Eqn 23. The process shows that the reference mass must be constrained to the range $3.1979E-45 < \bar{m} < 3.2039E-45$ in order to address the Jeans escape rate discrepancy between N2 and CH4 species in Pluto. Using the mean value $\bar{m} = 3.2(0)E-45$ (kg), the exobase altitude of 1710 (km) and exobase temperature of $T = 67.78$ (K), the quantum escape velocity of CH4 is found to be only $v_{eq-CH4} = 16.2$ (m/sec) and that of N2 gas molecule is $v_{eq-N2} = 280.5$ (m/sec). In comparison, the classical escape velocity is $v_{ec} = 776.4$ (m/sec) for both species. Evidently, the escape velocity of CH4 being so low promotes its escape rate by 2-3 orders of magnitude higher than that of N2. Table 1 shows the gravitational binding distance of various gas molecules in comparison to the exobase radius $R_r = R_s + H = 1188.3 + 1710 = 2898.3$ (km) of Pluto. The quantum gravity escape velocity of the gaseous species calculated at the exobase altitude and the percent error between the quantum and classical escape velocities are also shown in the table. By taking $\bar{m} = 3.2E-45$ (kg), the H2 and He molecules will not be gravitationally bound to Pluto, as $R_1/R_r < 1$, and therefore, their escape velocity is zero. This makes the percent error between the escape velocities of classical and quantum gravity of the H2 and He molecules infinite. Using the estimated value of the reference mass $\bar{m}$, the resulting quantum escape velocity and action distance of Na atom on the Moon will be determined in the following section.

Gas molecule	m (g/mole)	m (kg)	gravitationally bindind radius R1 (km)	R1/Rr	ve classical gravity (m/sec)	ve quantum gravity (m/sec)	percent error in ve	avg thermal velocity (m/sec)
H2	2.016	3.348E-27	1726	0.596	776.4	0	∞	915.8
He	4.003	6.647E-27	2049	0.707	776.4	0	∞	649.9
CH4	16.040	2.664E-26	2900	**1.000**	776.4	16.2	4678.5	324.6
N2	28.014	4.652E-26	3333	1.150	776.4	280.5	176.8	245.7
O2	32.000	5.314E-26	3446	1.189	776.4	309.6	150.8	229.8
Ar	39.948	6.634E-26	3643	1.257	776.4	351.0	121.2	205.7
CO2	44.010	7.308E-26	3732	1.288	776.4	366.9	111.6	196.0

Table 1: Action distance and escape velocity of various species from Pluto

11 Lunar comet-like sodium tail

According to the quantum model of gravity, Na atoms can escape the lunar surface at lower speeds than the classical escape velocity of $v_{ec} = 2.38$ (km/sec). As shown in table 2, the lunar escape velocity of Na atoms under quantum model of gravity is $v_{eq} = 2.08$ (km/sec), which is about 12% lower than v_{ec}. The lower escape velocity promotes the escape rate of Na atoms by reducing Jeans

Gas molecule	m (g/mole)	m (kg)	gravitationally bindind radius R1 (km)	R1/Rr	ve classical gravity (m/sec)	ve quantum gravity (m/sec)	percent error in ve	avg thermal velocity (m/sec)
He	4.0026	6.647E-27	4855	2.795	2375.9	1904.0	24.8	1367.3
Ne	20.179	3.351E-26	7275	4.188	2375.9	2072.9	14.6	608.9
Na	22.989	3.818E-26	7517	**4.326**	2375.9	2083.3	14.0	570.5
AR	39.348	6.534E-26	8597	4.948	2375.9	2122.3	11.9	436.1
Rn	222.018	3.687E-25	13251	7.627	2375.9	2214.7	7.3	183.6

Table 2: Action distance and escape velocity of various species from Moon

parameter. Therefore, Na atoms of lower ejection speed are able to contribute to the lunar Sodium tail formation. In order to produce an escaping Na atmosphere, the ejection speeds of the Na atoms in the Monte-Carlo simulations [10, 11] can be taken well below 2.0 (km/sec), in particular considering the solar radiation pressure. Moreover, as shown in table 2, the gravitational binding distance R_1 of Sodium atom to Moon under the quantum model of gravity is found to be $4.3R_s$ which is comparable with the size $\approx 5R_s$ of the Sodium atmosphere on the day-side of the moon [10]. A reason for this difference could be that the *actual* average ejection velocity of Sodium atoms are higher than the quantum escape velocity v_{eq}.

12 Earth bound experiment

As discussed earlier, comparing a pair of particles with a distinct mass ratio, the quantum model of Newtonian gravity predicts that the massive particle would attain higher velocity in free fall and eventually fall ahead of the lighter particle. Therefore, this feature of quantum gravity could be used to validate the theory and also determine the reference mass $\bar{m}$ with a higher accuracy. In this section, an experimental set up is proposed in which velocities of a pair of particles are measured after a certain distance of free fall. The required particle mass that such a feable quantum effect could be demonstrated in the Earth's gravitational field is a function of free fall distance and the value of the reference mass - which is not currently known with accuracy. To determine a particle mass that allows to minimize the required height of the device, the free fall velocity of particles of various mass are simulated by taking $\bar{m} = 3.2E-45$ (kg), Earth mass $M = 5.98E24$ (kg) and Earth radius $R_s = 6371$ (km). Figure 3 shows the results, obtained from the numerical simulation described earlier in section 6. The mass of the lightest particle used in the simulations is taken to be $m2 = 1.0E-21$ (kg). The maximum mass ratio used in the simulation is 400, therefore, the mass of the most massive particle used in the simulations of Figure 3 is taken to be $m1 = 400 \times m2 = 4.0E-19$ (kg).

First note that the longer the duration of free fall, the higher the difference the particle velocities. This is as expected - because a longer free fall duration (or distance) allows the small quantum effects of gravity to creep up to a more tangible difference. Moreover, a higher mass ratio between a pair of falling particles results in a higher velocity difference between them. However, as shown in Fig 3, as the mass ratio increases the difference in the velocity of a pair of particles eventually reaches to a plateau. The latter is due to the fact that the quantum effect of gravity diminishes as the mass of the heavier particle increases. Therefore, beyond some large mass ratio, around 400, the quantum free fall velocity of the heavier particle reduces to that of the classical gravity; where the difference in free fall velocities gets plateaued. So for the particle mass $m2 = 1.0E-21$ (kg) and $m1 = 4.0E-19$ (kg), the velocity difference that needs to be measured in the experiment is found to be about 1.898 and 3.147 (mm/sec); for 10 and 15 seconds of free fall, respectively. Taking typical gravitational acceleration $g_s = 9810$ (mm/sec^2), the velocity of the particles at the end of 10 seconds of free fall would be about 98100 (mm/sec). Therefore, the accuracy required in the velocity measurement would be 1.898/98100, i.e. ≈ 20 (ppm). Such a high accuracy indicates that the experiment needs to be done in a vacuum chamber which in turn could pose a practical challenge due to the required height of ≈ 500 (m) for 10 seconds of free fall. It is therefore evident that the

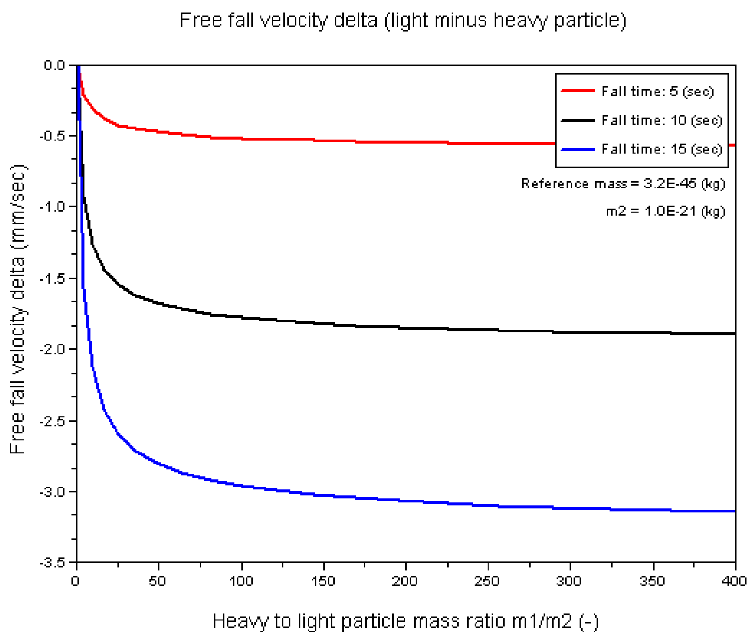

Figure 3: Difference in earth bound free fall velocity of particles with mass ratio m1/m2

higher the accuracy of velocity measurements the lower the height of the vacuum chamber needed in this experiment. Hence, by determining the accuracy achievable in the velocity measurements, the required height of the vacuum chamber (or the duration of free fall) can be determined. Knowing the latter, then numerical simulations similar to Figure 4 can then be used to determine the required particle mass in order to maximize the difference in their free fall velocity. For instance, by taking $m2 = 1.0E-22$ (kg) and $m1 = 4.0E-20$ (kg) the difference in the velocity of the particles would increase to ≈ 4.655 (mm/sec) after 10 seconds of free fall.
Finally, as discussed before, the difference in the velocity of the particles is also a function of the value of the reference mass $\bar{m}$. Figure 5 shows sensitivity in the free fall velocities if the reference mass were 5 times higher or lower than $\bar{m} = 3.2E-45$(kg). Therefore, findings from this experiment could not only validate the theory but also help determine the value of reference mass $\bar{m}$ with a higher certainty.

Using the mass ratio $m1/m2 = 400$, Table 3 summarizes the sensitivity of the particles free fall velocity to the value of the reference mass $\bar{m}$ and fall duration.

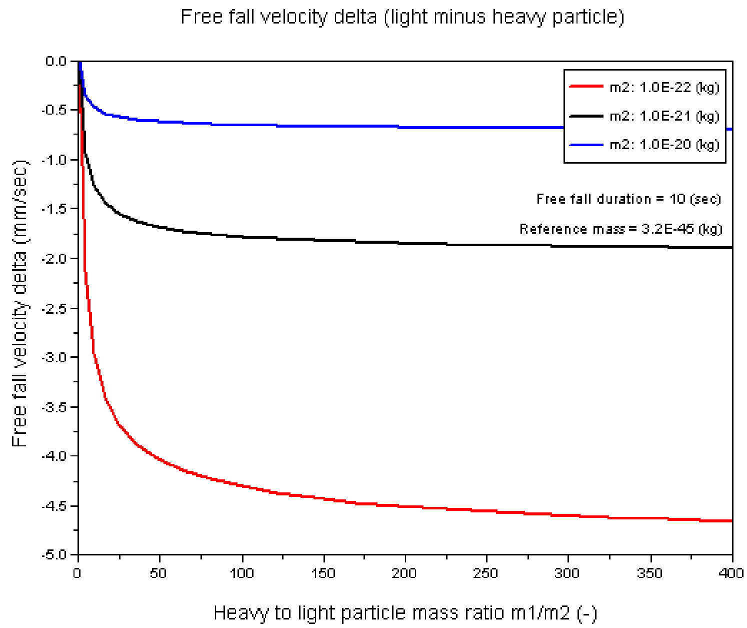

Figure 4: Sensitivity of free fall velocity to selection of particle masses

$\bar{m}$	m2	m1	fall duration	Δv (mm/sec)
3.2E-45	1.0E-21	4.0E-19	5	0.570
3.2E-45	1.0E-21	4.0E-19	10	1.898
3.2E-45	1.0E-21	4.0E-19	15	3.147
3.2E-45	1.0E-20	4.0E-18	10	0.693
3.2E-45	1.0E-22	4.0E-20	10	4.655
1.6E-44	1.0E-21	4.0E-19	10	3.453
6.4E-46	1.0E-21	4.0E-19	10	0.939

Table 3: Velocity difference sensitivity to $\bar{m}$, m1/m2 ratio and fall duration

13 Conclusion

The local acceleration quanta obtained from the combined theory of SR-QM is used to quantize the smooth Newtonian model of gravity surrounding a large gravitating body of mass. In the quantum model of Newtonian gravity, a particle of sufficiently small mass experiences variation of the gravitational field through a series of stepwise quantum accelerations of equal magnitude. Independent of the position of the particle in a field, the magnitude of quantum change in the gravitational acceleration is always fixed. However, the length interval on which a single quantum step in gravity occurs is a function of the distance to the gravitating body. At lower altitudes the quantum steps in gravity are found to be closely packed. By further reduction in altitude, the distance between two successive gravity upticks progressively reduces in length. At higher altitudes, a single quantum step in the gravitational acceleration occurs over a longer radial span; and by further increase in altitude the distance between two successive quantum steps progressively increases in length. The magnitude of the acceleration quanta of a particle is found to be inversely proportional to the square root of its rest mass. This makes the free fall of massive particles be less jerkier than that of lighter particles. Quantum gravity predicts that in a free fall from equal heights and initial velocities, massive particles attain higher velocity than lighter particles. In the combined theory of SR-QM, rest mass of particles are scaled against that of a reference particle. An experiment is proposed to validate the predictions of the quantum model of gravity and also to determine the mass of a reference particle.

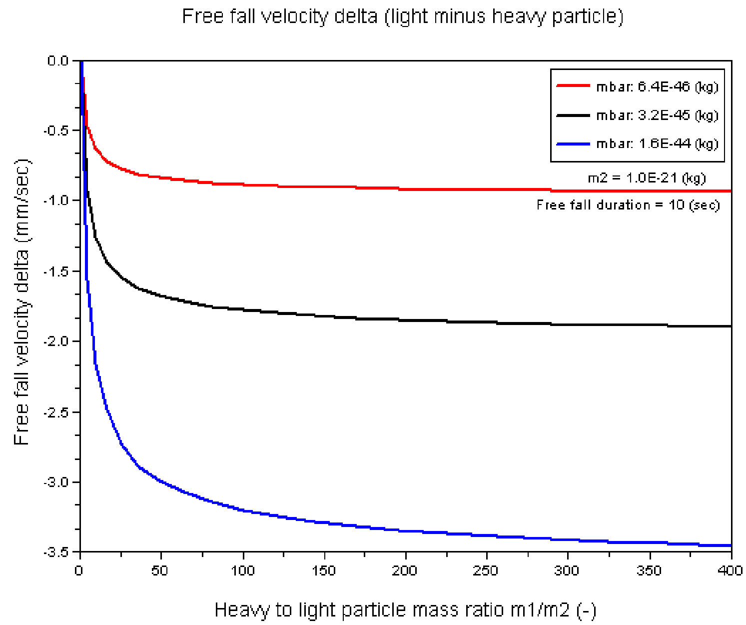

Figure 5: Sensitivity of free fall velocity difference to value of the reference mass $\bar{m}$

References

[1] F. Abrari, 'Combined theory of Special Relativity and Quantum Mechanics', https://viXra.org/abs/2106.0167

[2] F. W. Sears and R. W. Brehme, 'Introduction to the theory of relativity', Addison-Wesley publishing company, 1968, p103

[3] R. C. Tolman, 'Relativity, thermodynamics and cosmology', Oxford University Press, 1934, p174-184

[4] K. Beatty, 'Pluto's atmosphere confounds researchers', March 25 2016, Sky and telescope

[5] J. H. Jeans, 'The dynamical theory of gases', Cambridge University Press, New York, 4th ed. 1925

[6] D.C. Catling, et. al. 'The escape of planetary atmosphere', Scientific American, 2009 - https://geosci.uchicago.edu/~kite/doc/Catling2009.pdf

[7] A. N. Nolkov, et. al. 'Thermally-driven atmosphere escape: transition from hydrodynamic to Jeans escape', arXiv:1009.5110v1[astro-ph.EP]

[8] G. R. Gladstone, 'The atmosphere of Pluto as observed by New Horizons', March 18 2016, Vol 351, issue 6279, Science

[9] L. A. Young, 'Structure and composition of Pluto atmosphere from the New Horizons Solar Ultraviolet Occultation', April 5 2017, arXiv:1704.01511 (astro-ph)

[10] M. Matta, et. al. 'The sodium tail of the moon', Icarus (2009). doi:10.1016/j.icarus.2009.06.017

[11] J. K. Wilson, et. al. 'The outer limits of the lunar sodium exosphere', Geophysical research letters, Vol 30, No 12, 1649, doi:10:1029/2003GL017443, 2003