On the development of various equations concerning the Ramanujan Manuscript Book 1. New possible mathematical connections with several topics regarding the Dark Matter and the Supersymmetry Breaking.

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Abstract

In this research thesis, we analyze further equations concerning the Ramanujan Manuscript Book 1. We obtain new possible mathematical connections with several topics regarding the Dark Matter and the Supersymmetry Breaking.

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From:

On the Zeros of the Davenport Heilbronn Function
S. A. Gritsenko - Received May 15, 2016 - ISSN 0081-5438, Proceedings of the Steklov Institute of Mathematics, 2017, Vol. 296, pp. 65–87.

A. A. Karatsuba, "On the zeros of arithmetic Dirichlet series without Euler product," Izv. Ross. Akad. Nauk, Ser. Mat. 57 (5), 3–14 (1993)

We have:

Let

$$\varkappa = \frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}$$

and χ_1 be a character modulo 5 such that $\chi_1(2) = i$.

The Davenport–Heilbronn function $f(s)$ is defined by the equality

$$f(s) = \frac{1-i\varkappa}{2}L(s,\chi_1) + \frac{1+i\varkappa}{2}L(s,\overline{\chi}_1), \qquad \text{where} \quad L(s,\chi) = \sum_{n=1}^{\infty}\frac{\chi(n)}{n^s}.$$

The function $f(s)$ satisfies the Riemann-type functional equation

$$g(s) = g(1-s), \qquad \text{where} \quad g(s) = \left(\frac{\pi}{5}\right)^{-s/2}\Gamma\left(\frac{s+1}{2}\right)f(s),$$

but there is no Euler product for it.

$$(\sqrt{10-2\sqrt{5}}-2)/(\sqrt{5}-1) = \kappa$$

Input:

$$\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}$$

Decimal approximation:

0.2840790438404122960282918323931261690910880884457375827591626661

…

0.28407904384…. = κ

The study of this function provides the following representations:

Alternate forms:

$$\frac{1}{4}\left(\sqrt{10-2\sqrt{5}}-2\sqrt{5}+\sqrt{5\left(10-2\sqrt{5}\right)}-2\right)$$

$$\frac{1}{4}\left(1+\sqrt{5}\right)\left(\sqrt{10-2\sqrt{5}}-2\right)$$

$$\frac{1}{2}\left(-1-\sqrt{5}+\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

Minimal polynomial:

$$x^4+2x^3-6x^2-2x+1$$

Expanded forms:

$$\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}-1}-\frac{2}{\sqrt{5}-1}$$

$$\frac{1}{4}\sqrt{10-2\sqrt{5}}+\frac{1}{4}\sqrt{5\left(10-2\sqrt{5}\right)}+\frac{1}{2}\left(-1-\sqrt{5}\right)$$

For (((($\sqrt{}$(10-2$\sqrt{}$5) -2))/(($\sqrt{}$5-1)))) = 8πG; G = 0.011303146014

Indeed:

(((($\sqrt{}$(10-2$\sqrt{}$5) -2))/(($\sqrt{}$5-1))))/(8π)

Input:

$$\frac{\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}}{8\pi}$$

Result:

$$\frac{\sqrt{10-2\sqrt{5}}-2}{8\left(\sqrt{5}-1\right)\pi}$$

Decimal approximation:

0.011303146014005214797375012944203574468576031392001780859490966 7...

0.01130314…. = g (gravitational coupling constant)

The study of this function provides the following representations:

Property:

$$\frac{-2+\sqrt{10-2\sqrt{5}}}{8\left(-1+\sqrt{5}\right)\pi}$$ is a transcendental number

Alternate forms:

$$\frac{\sqrt{10-2\sqrt{5}}-2\sqrt{5}+\sqrt{5\left(10-2\sqrt{5}\right)}-2}{32\pi}$$

$$-\frac{1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}}{16\pi}$$

$$\frac{-1-\sqrt{5}+\sqrt{2\left(5+\sqrt{5}\right)}}{16\pi}$$

Expanded forms:

$$-\frac{1}{16\pi}-\frac{\sqrt{5}}{16\pi}+\frac{\sqrt{10-2\sqrt{5}}}{32\pi}+\frac{\sqrt{5\left(10-2\sqrt{5}\right)}}{32\pi}$$

$$\frac{\sqrt{10-2\sqrt{5}}}{8\left(\sqrt{5}-1\right)\pi}-\frac{1}{4\left(\sqrt{5}-1\right)\pi}$$

Series representations:

$$\frac{\sqrt{10-2\sqrt{5}}-2}{(8\pi)\left(\sqrt{5}-1\right)}=\frac{-2+\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}}{8\pi\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)}$$

$$\frac{\sqrt{10-2\sqrt{5}}-2}{(8\pi)\left(\sqrt{5}-1\right)} = \frac{-2+\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(9-2\sqrt{5}\right)^{-k}}{k!}}{8\pi\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)}$$

$$\frac{\sqrt{10-2\sqrt{5}}-2}{(8\pi)\left(\sqrt{5}-1\right)} = \frac{-2+\sqrt{z_0}\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}}{8\pi\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}\right)}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

We note that:

(((√(10-2√5) -2))/((√5-1)))*((2 i (sqrt(5) - 1) t + sqrt(5) - 1)/(2 (sqrt(2 (5 - sqrt(5))) - 2)))

Input:

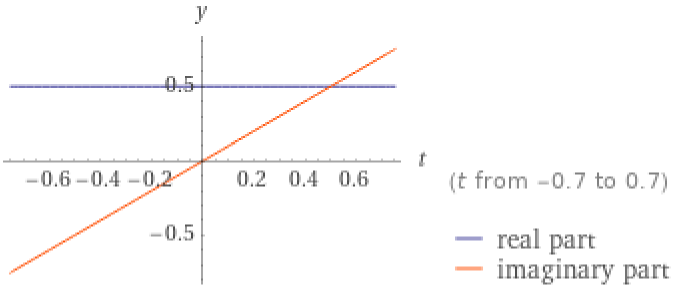
$$\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1} \times \frac{2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}$$

i is the imaginary unit

Exact result:

$$\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\left(2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1\right)}{2\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}$$

Plot:



The study of this function provides the following representations:

Alternate form assuming t>0:

$$\frac{i\sqrt{10-2\sqrt{5}}\,t}{\sqrt{2\left(5-\sqrt{5}\right)}-2}-\frac{2\,i\,t}{\sqrt{2\left(5-\sqrt{5}\right)}-2}+\frac{\sqrt{5\left(10-2\sqrt{5}\right)}}{2\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}-\frac{\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}-\frac{\sqrt{5}}{\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}+\frac{1}{\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}$$

Alternate forms:

$$\frac{1}{8}\left(1+\sqrt{5}\right)\left(2\,i\sqrt{2\left(3-\sqrt{5}\right)}\,t+\sqrt{5}-1\right)$$

$$\frac{1}{2}\left(1+2\,i\,t\right)$$

$$\frac{1}{2} + i\,t$$

1/2+it = real part of every nontrivial zero of the Riemann zeta function

Derivative:

$$\frac{d}{dt}\left(\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\left(2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1\right)}{\left(\sqrt{5}-1\right)\left(2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)\right)}\right) = i$$

Indefinite integral:

$$\int \frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\left(2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1\right)}{\left(\sqrt{5}-1\right)\left(2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)\right)}\,dt = \frac{t}{2} + \frac{i\,t^2}{2} + \text{constant}$$

And again:

(((√(10-2√5) -2))/((2x)))*((2 i (sqrt(5) - 1) t + sqrt(5) - 1)/(2 (sqrt(2 (5 - sqrt(5))) - 2))) = (1/2+it)

Input:

$$\frac{\sqrt{10-2\sqrt{5}}-2}{2\,x} \times \frac{2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)} = \frac{1}{2} + i\,t$$

i is the imaginary unit

Exact result:

$$\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\left(2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1\right)}{4\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}=\frac{1}{2}+i\,t$$

The study of this function provides the following representations:

Alternate form assuming t and x are real:

$$\frac{\sqrt{5}-1}{x}=2$$

Alternate form:

$$\frac{\left(\sqrt{5}-1\right)(1+2\,i\,t)}{4\,x}=\frac{1}{2}+i\,t$$

Alternate form assuming t and x are positive:

$$2\,x+1=\sqrt{5}$$

Expanded forms:

$$\frac{i\sqrt{5\left(10-2\sqrt{5}\right)}\;t}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}-\frac{i\sqrt{10-2\sqrt{5}}\;t}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}-\frac{i\sqrt{5}\;t}{\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}+\frac{i\,t}{\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}+\frac{\sqrt{5\left(10-2\sqrt{5}\right)}}{4\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}-\frac{\sqrt{10-2\sqrt{5}}}{4\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}-\frac{\sqrt{5}}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}+\frac{1}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}=\frac{1}{2}+i\,t$$

$$\frac{i\sqrt{5}\,t}{2\,x} - \frac{i\,t}{2\,x} + \frac{\sqrt{5}}{4\,x} - \frac{1}{4\,x} = \frac{1}{2} + i\,t$$

Solutions:

$$t = \frac{i}{2}, \quad x \neq 0$$

$$x = \frac{\sqrt{5}}{2} - \frac{1}{2}$$

Input:

$$\frac{\sqrt{5}}{2} - \frac{1}{2}$$

Decimal approximation:

0.6180339887498948482045868343656381177203091798057628621354486227

…

$0.6180339887\ldots. = \frac{1}{\phi}$

Solution for the variable x:

$$x = \frac{-2\,i\sqrt{5}\,t + 2\,i\,t - \sqrt{5} + 1}{-2 - 4\,i\,t}$$

Implicit derivatives:

$$\frac{\partial x(t)}{\partial t} = \frac{2\left(-1 + \sqrt{5} - 2\,x\right)x}{\left(-1 + \sqrt{5}\right)(-i + 2\,t)}$$

$$\frac{\partial t(x)}{\partial x} = \frac{\left(-1+\sqrt{5}\right)(-i+2t)}{2\left(-1+\sqrt{5}-2x\right)x}$$

Dark Energy and Dark Matter

We consider $\Lambda = 1.1056 * 10^{-52}$ that is the value of the Cosmological Constant and 299792458 , that is the value of the speed of light in *m/s* , the terms of the Einstein equation $E = mc^2$ ($E = 1.1056 * 10^{-52}$ (Λ = Dark Energy)). Thence, we can to obtain the mass *m*:

(1.1056*10^-52)/(299792458)^2

Input interpretation

$$\frac{1.1056\times 10^{-52}}{299\,792\,458^2}$$

Result

$1.2301459019728805386116738651359598467212878595555184012098\ldots \times 10^{-69}$

$1.2301459019\ldots*10^{-69}$

From (**Ramanujan Manuscript Book 1** – *Srinivasa Ramanujan*):

From:

$$\frac{d}{d\beta}\left(\frac{1+\left(\frac{1}{2}\right)^2(1-\beta)+\left(\frac{3}{2\times 4}\right)^2(1-\beta)^2}{1+\left(\frac{1}{2}\right)^2\beta+\left(\frac{3}{2\times 4}\right)^2\beta^2}\right) = \frac{450\left(\beta^2-\beta-8\right)}{\left(9\beta^2+16\beta+64\right)^2}$$

we obtain, for $\beta = 8.7$ and 37.6 :

(450 (-8 - 8.7 + 8.7^2))/(64 + 16 8.7 + 9 8.7^2)^2

Input

$$\frac{450\left(-8-8.7+8.7^2\right)}{\left(64+16\times 8.7+9\times 8.7^2\right)^2}$$

Result

0.033937794919068686650271704531215936015466340471155683067489039 5

…

0.033937794919….

(450 (-8 - 37.6 + 37.6^2))/(64 + 16 37.6 + 9 37.6^2)^2

Input

$$\frac{450\left(-8-37.6+37.6^2\right)}{\left(64+16\times 37.6+9\times 37.6^2\right)^2}$$

Result

0.003434192029732870982019368439007252196204192746407368070004878 1

…

0.00343419202….

From the two previous analyzed expressions, we obtain:

((((450 (-8 - 8.7 + 8.7^2))/(64 + 16 8.7 + 9 8.7^2)^2 + (450 (-8 - 37.6 + 37.6^2))/(64 + 16 37.6 + 9 37.6^2)^2)))^48

Input

$$\left(\frac{450\left(-8-8.7+8.7^2\right)}{\left(64+16\times 8.7+9\times 8.7^2\right)^2}+\frac{450\left(-8-37.6+37.6^2\right)}{\left(64+16\times 37.6+9\times 37.6^2\right)^2}\right)^{48}$$

Result

$3.035407818663256821636610760596676935071201181338030158007 1\ldots \times 10^{-69}$

3.035407818…*10^-69

and:

1/((([2.6 -25/9 ln(2.6(9*2.6+16)+64)+1/(144*sqrt2)*(((625*tan^-1((9*2.6+8)/(16sqrt2)))))]+[17.6 -25/9 ln(17.6(9*17.6+16)+64)+1/(144*sqrt2)*(((625*tan^-1((9*17.6+8)/(16sqrt2)))))]))^72

Input

$$1\Bigg/\left(\left(2.6-\frac{25}{9}\log(2.6\,(9\times 2.6+16)+64)+\frac{1}{144\sqrt{2}}\left(625\tan^{-1}\left(\frac{9\times 2.6+8}{16\sqrt{2}}\right)\right)\right)+\left(17.6-\frac{25}{9}\log(17.6\,(9\times 17.6+16)+64)+\frac{1}{144\sqrt{2}}\left(625\tan^{-1}\left(\frac{9\times 17.6+8}{16\sqrt{2}}\right)\right)\right)\right)^{72}$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

Result

$1.24433\ldots \times 10^{-69}$

(result in radians)

1.24433…*10^-69

We obtain, after some calculations:

(16 sqrt(2/3) log^(3/2)(2))/(e^(5/2) log^2(3)) ((1/(1+(zeta(2))^2)(3.035407*10^-69 + 1.24433× 10^-69)+1.24433× 10^-69))

where

$$\frac{16\sqrt{\frac{2}{3}}\log^{3/2}(2)}{e^{5/2}\log^2(3)}$$

= 0.5127284721865…..

Input interpretation

$$\frac{16\sqrt{\frac{2}{3}}\log^{3/2}(2)}{e^{5/2}\log^2(3)}\left(\frac{1}{1+\zeta(2)^2}\left(3.035407\times 10^{-69}+1.24433\times 10^{-69}\right)+1.24433\times 10^{-69}\right)$$

$\log(x)$ is the natural logarithm

$\zeta(s)$ is the Riemann zeta function

Result

$1.23014\ldots \times 10^{-69}$

1.23014…*10^{-69}

From:

Majorana Fermion Dark Matter in Minimally Extended Left-Right Symmetric Model - *M. J. Neves, Nobuchika Okada and Satomi Okada* - arXiv:2103.08873v1 [hep-ph] 16 Mar 2021

We have:

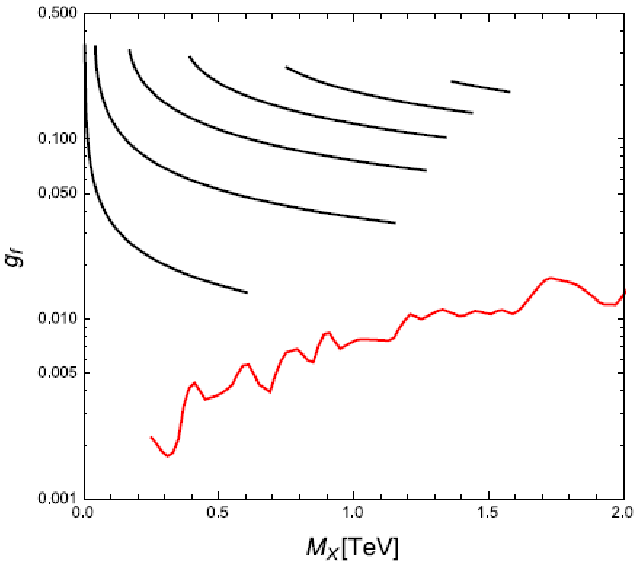


FIG. 6. The plot of M_X versus g_f for the annihilation process (ii) $\bar{\zeta}_\ell \zeta_\ell \to X\,X$. The black solid curves from left to right depict the results for $a = 0.3$, 1, 2, 3, 4 and 5, respectively, from left to right, along which $\Omega_{DM} h^2 = 0.12$ is satisfied. The diagonal red line shows the upper bound on g_f as a function of $M_X \geq 0.25$ TeV from the LHC Run-2 results. No allowed region exists for $M_X \geq 0.25$ TeV, which can simultaneously satisfy the cosmological and LHC constraints.

Combining the results with the perturbative condition and the LHC Run-2 constraints, we have found the allowed parameter region to be very narrow. As for the process (ii), we have found that the parameter region (g_f as a function of M_X) satisfying the cosmological constraint appears far above the upper bound on g_f (for $M_X \geq 0.25$ TeV), and no allowed parameter region exists.

Now, we consider the possible mass of a DM particle equal to 250 GeV. We obtain:

250 GeV = Kg

Input interpretation

convert 250 GeV/c^2 to kilograms

Result

4.457×10^{-25} kg (kilograms)

$4.457 * 10^{-25}$ kg

Now we consider the following value

$\omega/\omega_3 \mid 5+3 \mid m_{u/d} = 240 - 345 \mid 0.937 - 1.000$

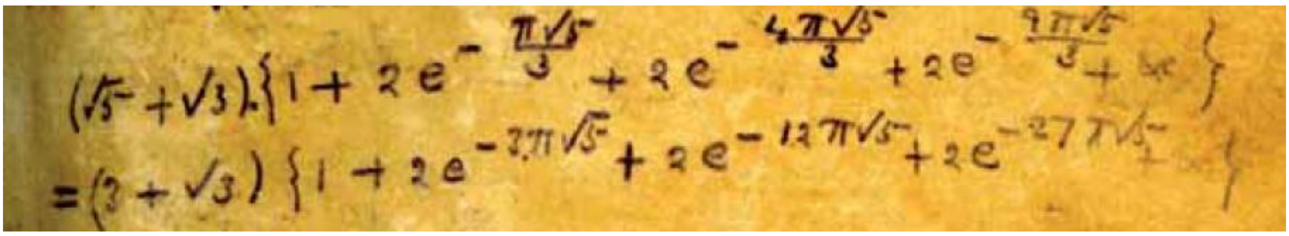
i.e. 0.937 and we perform the following mean:

0.937 + 0.9568666373 / 2 = 0.94693331865

where 0.9568666373 is the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1} = 1 - \cfrac{e^{-\pi}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-3\pi}}{1+\cfrac{e^{-4\pi}}{1+\dots}}}} \approx 0.9568666373$$

From the following expressions



$$(\sqrt{5}+\sqrt{3}).\left\{1+2e^{-\frac{\pi\sqrt{5}}{3}}+2e^{-\frac{4\pi\sqrt{5}}{3}}+2e^{-\frac{9\pi\sqrt{5}}{3}}+ \&c\right\}$$
$$=(3+\sqrt{3})\left\{1+2e^{-2\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-27\pi\sqrt{5}}+ \&c\right\}$$

$$(3+\sqrt{3})\left(1+2e^{-2\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-27\pi\sqrt{5}}\right)$$

$$(\sqrt{5}+\sqrt{3})\left(1+2e^{-(\pi\sqrt{5})/3}+2e^{-1/3\times 4(\pi\sqrt{5})}+2e^{-1/3\times 9(\pi\sqrt{5})}\right)$$

after some calculations, we obtain:

((((3+sqrt3) [1+2e^(-2Pi*sqrt5)+2e^(-12Pi*sqrt5)+2e^(-27Pi*sqrt5)] + (sqrt5+sqrt3) [1+2e^(-1/3*Pi*sqrt5)+2e^(-1/3*4Pi*sqrt5)+2e^(-1/3*9Pi*sqrt5)])))*2*0.94693

Input

$$\left(\left(3+\sqrt{3}\right)\left(1+2\,e^{-2\pi\sqrt{5}}+2\,e^{-12\pi\sqrt{5}}+2\,e^{-27\pi\sqrt{5}}\right)+\left(\sqrt{5}+\sqrt{3}\right)\left(1+2\,e^{-(\pi\sqrt{5})/3}+2\,e^{-1/3\times4(\pi\sqrt{5})}+2\,e^{-1/3\times9(\pi\sqrt{5})}\right)\right)\times2\times0.94693$$

Result

17.9237...

17.9237

The study of this function provides the following representations:

Series representations

$$\left(\left(3+\sqrt{3}\right)\left(1+2\,e^{-2\pi\sqrt{5}}+2\,e^{-12\pi\sqrt{5}}+2\,e^{-27\pi\sqrt{5}}\right)+\left(\sqrt{5}+\sqrt{3}\right)\left(1+2\,e^{1/3(\pi\sqrt{5})(-1)}+2\,e^{1/3(4(\pi\sqrt{5}))(-1)}+2\,e^{1/3(9(\pi\sqrt{5}))(-1)}\right)\right)$$

$$2\times0.94693=1.89386\,e^{-27\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}\left(6+6\,e^{15\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}+6\,e^{25\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}+3\,e^{27\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}+\right.$$
$$2\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}+2\,e^{15\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}+$$
$$2\,e^{25\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}+e^{27\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}\sqrt{2}$$
$$\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}+2\,e^{24\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\left(2^k\sqrt{2}+\sqrt{4}\right)+$$
$$2\,e^{77/3\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\left(2^k\sqrt{2}+\sqrt{4}\right)+$$
$$2\,e^{80/3\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\left(2^k\sqrt{2}+\sqrt{4}\right)+$$
$$\left.e^{27\pi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\left(2^k\sqrt{2}+\sqrt{4}\right)\right)$$

$$\left((3+\sqrt{3})\left(1+2e^{-2\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-27\pi\sqrt{5}}\right)+\right.$$
$$\left.(\sqrt{5}+\sqrt{3})\left(1+2e^{1/3(\pi\sqrt{5})(-1)}+2e^{1/3(4(\pi\sqrt{5}))(-1)}+2e^{1/3(9(\pi\sqrt{5}))(-1)}\right)\right)$$

$$2\times 0.94693 = 1.89386\exp\left(-27\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\left(6+6\exp\left(15\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)+6\exp\left(25\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)+\right.$$
$$3\exp\left(27\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)+2\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+$$
$$2\exp\left(15\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+$$
$$2\exp\left(25\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+$$
$$\exp\left(27\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+$$
$$2\exp\left(24\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)_k\left(\left(-\frac{1}{2}\right)^k\sqrt{2}+\left(-\frac{1}{4}\right)^k\sqrt{4}\right)}{k!}+$$
$$2\exp\left(\frac{77}{3}\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)_k\left(\left(-\frac{1}{2}\right)^k\sqrt{2}+\left(-\frac{1}{4}\right)^k\sqrt{4}\right)}{k!}+$$
$$2\exp\left(\frac{80}{3}\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)_k\left(\left(-\frac{1}{2}\right)^k\sqrt{2}+\left(-\frac{1}{4}\right)^k\sqrt{4}\right)}{k!}+$$
$$\left.\exp\left(27\pi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)_k\left(\left(-\frac{1}{2}\right)^k\sqrt{2}+\left(-\frac{1}{4}\right)^k\sqrt{4}\right)}{k!}\right)$$

$$\left(\left(3+\sqrt{3}\right)\left(1+2\,e^{-2\pi\sqrt{5}}+2\,e^{-12\pi\sqrt{5}}+2\,e^{-27\pi\sqrt{5}}\right)+\left(\sqrt{5}+\sqrt{3}\right)\left(1+2\,e^{1/3\left(\pi\sqrt{5}\right)(-1)}+2\,e^{1/3\left(4\left(\pi\sqrt{5}\right)\right)(-1)}+2\,e^{1/3\left(9\left(\pi\sqrt{5}\right)\right)(-1)}\right)\right)$$

$$2\times 0.94693 = 1.89386\,e^{-27\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}$$

$$\Bigg(6+6\,e^{15\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}+6\,e^{25\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}+$$

$$3\,e^{27\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}+2\,e^{24\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left((3-z_0)^k+(5-z_0)^k\right)z_0^{-k}}{k!}+$$

$$2\,e^{\frac{77}{3}\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left((3-z_0)^k+(5-z_0)^k\right)z_0^{-k}}{k!}+$$

$$2\,e^{\frac{80}{3}\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left((3-z_0)^k+(5-z_0)^k\right)z_0^{-k}}{k!}+$$

$$e^{27\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left((3-z_0)^k+(5-z_0)^k\right)z_0^{-k}}{k!}+$$

$$2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}+$$

$$2\,e^{15\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}+$$

$$2\,e^{25\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}+$$

$$e^{27\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\Bigg)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

From:

1/((1.23041*10^-69)*3.264346*10^94) (((((((3+sqrt3) [1+2e^(-2Pi*sqrt5)+2e^(-12Pi*sqrt5)+2e^(-27Pi*sqrt5)]+(sqrt5+sqrt3)[1+2e^(-1/3*Pi*sqrt5)+2e^(-1/3*4Pi*sqrt5)+2e^(-1/3*9Pi*sqrt5)])))*2*0.94693)))

where $1.23041 * 10^{-69}$ is the result that we have obtained previously and $3.264346 * 10^{94}$ is the value of the Planck Density. Multiplying the two values and the above expression, we obtain:

Input interpretation

$$\frac{1}{1.23041\times 10^{-69}\times 3.264346\times 10^{94}} \left(\left(\left(3+\sqrt{3}\right)\left(1+2e^{-2\pi\sqrt{5}}+2e^{-12\pi\sqrt{5}}+2e^{-27\pi\sqrt{5}}\right)+\left(\sqrt{5}+\sqrt{3}\right)\left(1+2e^{-(\pi\sqrt{5})/3}+2e^{-1/3\times 4\pi\sqrt{5}}+2e^{-1/3\times 9\pi\sqrt{5}}\right)\right)\times 2\times 0.94693\right)$$

Result

$4.46253\ldots \times 10^{-25}$

$4.46253\ldots * 10^{-25}$ result that is very near to the value of the possible mass of a DM particle equal to 250 GeV = 4.457×10^{-25} kg

Or, from the previous expression

$$\left(\sqrt{5}+\sqrt{3}\right)\left(1+2e^{-(\pi\sqrt{5})/3}+2e^{-1/3\times 4(\pi\sqrt{5})}+2e^{-1/3\times 9(\pi\sqrt{5})}\right)$$

and the cosmological constant, after some calculations:

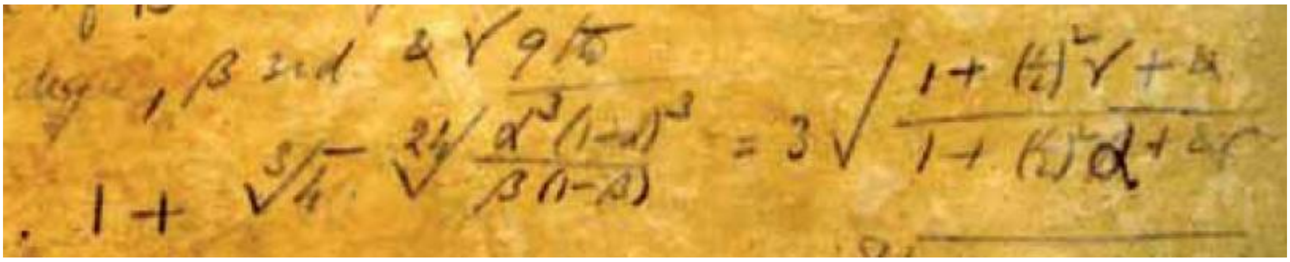
$$\frac{e^{-\frac{2}{3}+\frac{2}{3e}-\frac{2e}{3}+\frac{1}{\pi}+\pi}\,\pi^{2/3-(4e)/3}}{\sin^{5/3}(e\pi)}\times 1.1056\times 10^{-52}\times \frac{1}{\left(\left(\sqrt{5}+\sqrt{3}\right)\left(1+2e^{-(\pi\sqrt{5})/3}+2e^{-1/3\times 4(\pi\sqrt{5})}+2e^{-1/3\times 9(\pi\sqrt{5})}\right)\right)^{24}}$$

Result

$1.23041\ldots \times 10^{-69}$

$1.23041\ldots*10^{-69}$

Now, we have that:



1+4^(1/3) ((α^3(1-α)^3) / (β(1-β)))^(1/24) = 3* sqrt((1+(1/2)^2*γ)/(1+(1/2)^2*α))

From:

1+4^(1/3) ((α^3(1-α)^3) / (β(1-β)))^(1/24)

Input

$$1+\sqrt[3]{4}\sqrt[24]{\frac{\alpha^3(1-\alpha)^3}{\beta(1-\beta)}}$$

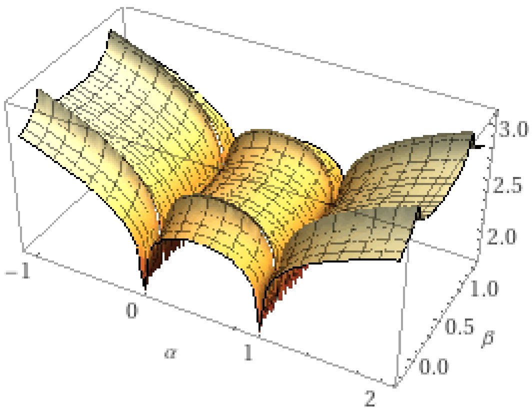
Exact result

$$2^{2/3}\sqrt[24]{\frac{(1-\alpha)^3\alpha^3}{(1-\beta)\beta}}+1$$

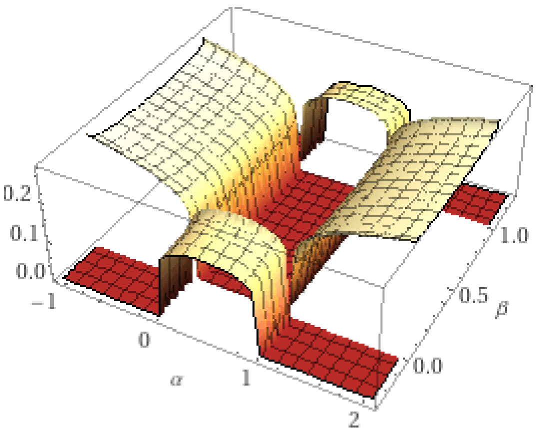
The study of this function provides the following representations:

3D plots

Real part **(figures that can be related to the D-branes)**

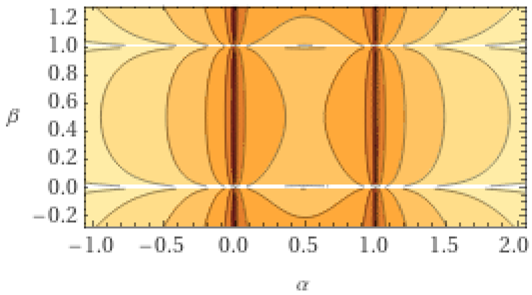


Imaginary part

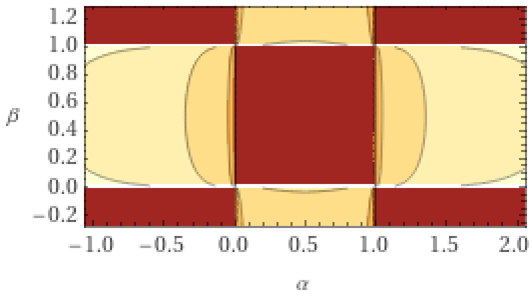


Contour plots

Real part



Imaginary part



Alternate form

$$2^{2/3}\sqrt[24]{\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}}+1$$

Alternate forms assuming α and β are positive

$$\frac{\sqrt[24]{-(\beta-1)\,\beta}+2^{2/3}\sqrt[8]{-(\alpha-1)\,\alpha}\,\exp\left(\frac{1}{12}\,i\,\pi\left\lfloor\frac{-3\arg(1-\alpha)+\arg(1-\beta)+\pi}{2\pi}\right\rfloor\right)}{\sqrt[24]{-(\beta-1)\,\beta}}$$

$$1+\frac{2^{2/3}\sqrt[8]{1-\alpha}\,\sqrt[8]{\alpha}\,\exp\left(\frac{1}{12}\,i\,\pi\left\lfloor-\frac{3\arg(1-\alpha)}{2\pi}+\frac{\arg(1-\beta)}{2\pi}+\frac{1}{2}\right\rfloor\right)}{\sqrt[24]{1-\beta}\,\sqrt[24]{\beta}}$$

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

Roots

(no roots exist)

Series expansion at α=0

$$1+\frac{2^{2/3}\sqrt[8]{\alpha}\sqrt[24]{\frac{\alpha^3}{\beta-\beta^2}}}{\sqrt[8]{\alpha}}-\frac{\alpha^{9/8}\sqrt[24]{\frac{\alpha^3}{\beta-\beta^2}}}{4\left(\sqrt[3]{2}\sqrt[8]{\alpha}\right)}-\frac{7\,\alpha^{17/8}\sqrt[24]{\frac{\alpha^3}{\beta-\beta^2}}}{64\left(\sqrt[3]{2}\sqrt[8]{\alpha}\right)}-\frac{35\,\alpha^{25/8}\sqrt[24]{\frac{\alpha^3}{\beta-\beta^2}}}{512\left(\sqrt[3]{2}\sqrt[8]{\alpha}\right)}+O\left(\alpha^{33/8}\right)$$

(generalized Puiseux series)

Series expansion at α=∞

$$\frac{2^{2/3}\sqrt[4]{\alpha}\sqrt[24]{\frac{\alpha^6}{(\beta-1)\,\beta}}}{\sqrt[4]{\alpha}}+1-\frac{\left(\frac{1}{\alpha}\right)^{3/4}\sqrt[24]{\frac{\alpha^6}{(\beta-1)\,\beta}}}{4\left(\sqrt[3]{2}\sqrt[4]{\alpha}\right)}-\frac{7\left(\frac{1}{\alpha}\right)^{7/4}\sqrt[24]{\frac{\alpha^6}{(\beta-1)\,\beta}}}{64\left(\sqrt[3]{2}\sqrt[4]{\alpha}\right)}-\frac{35\left(\frac{1}{\alpha}\right)^{11/4}\sqrt[24]{\frac{\alpha^6}{(\beta-1)\,\beta}}}{512\left(\sqrt[3]{2}\sqrt[4]{\alpha}\right)}+O\left(\left(\frac{1}{\alpha}\right)^3\right)$$

(generalized Puiseux series)

Derivative

$$\frac{\partial}{\partial\alpha}\left(1+\sqrt[3]{4}\sqrt[24]{\frac{\alpha^3\,(1-\alpha)^3}{\beta\,(1-\beta)}}\right)=\frac{(2\,\alpha-1)\sqrt[24]{\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}}}{4\sqrt[3]{2}\,(\alpha-1)\,\alpha}$$

Indefinite integral

$$\int\left(1+2^{2/3}\sqrt[24]{\frac{(1-\alpha)^3\,\alpha^3}{(1-\beta)\,\beta}}\right)d\alpha=\alpha+\frac{8\times 2^{2/3}\,(\alpha-1)\sqrt[24]{\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}}\;{}_2F_1\left(-\frac{1}{8},\frac{9}{8};\frac{17}{8};1-\alpha\right)}{9\sqrt[8]{\alpha}}+\text{constant}$$

${}_2F_1(a,b;c;x)$ is the hypergeometric function

Limit

$$\lim_{\beta \to \pm\infty} \left(1 + 2^{2/3} \sqrt[24]{\frac{(1-\alpha)^3 \alpha^3}{(1-\beta)\beta}} \right) = 1$$

And from:

3* sqrt((1+(1/2)^2*γ)/(1+(1/2)^2*α))

Input

$$3 \sqrt{\frac{1 + \left(\frac{1}{2}\right)^2 \gamma}{1 + \left(\frac{1}{2}\right)^2 \alpha}}$$

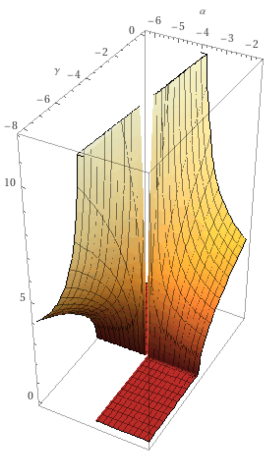
Result

$$3 \sqrt{\frac{\frac{\gamma}{4} + 1}{\frac{\alpha}{4} + 1}}$$

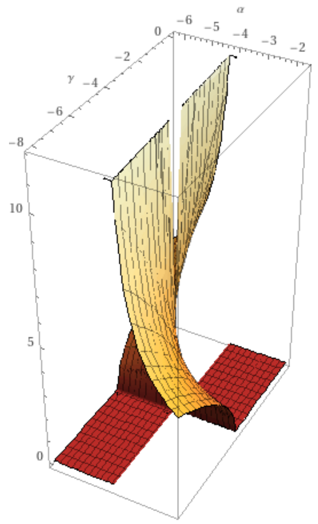
The study of this function provides the following representations:

3D plots

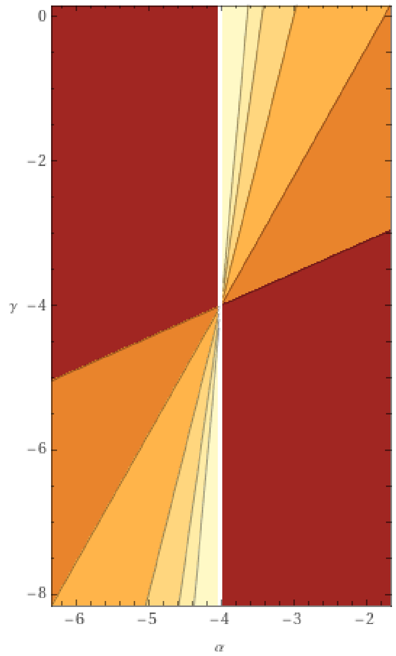
Real part (figures that can be related to the D-branes)



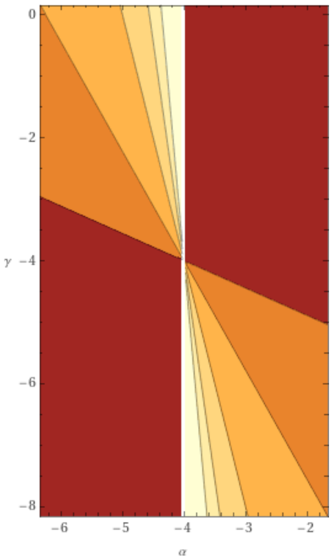
Imaginary part



Contour plots
Real part



Imaginary part



Alternate form

$$3\sqrt{\frac{\gamma+4}{\alpha+4}}$$

Root

$\alpha+4 \neq 0, \quad \gamma=-4$

Series expansion at α=0

$$\frac{3\sqrt{\gamma+4}}{2}-\frac{3}{16}\alpha\sqrt{\gamma+4}+\frac{9}{256}\alpha^2\sqrt{\gamma+4}-\frac{15\alpha^3\sqrt{\gamma+4}}{2048}+\frac{105\alpha^4\sqrt{\gamma+4}}{65536}+O(\alpha^5)$$

(Taylor series)

Series expansion at α=∞

$$3\sqrt{\alpha}\sqrt{\frac{1}{\alpha}}\sqrt{\frac{\gamma+4}{\alpha}}-6\left(\frac{1}{\alpha}\right)^{3/2}\left(\sqrt{\alpha}\sqrt{\frac{\gamma+4}{\alpha}}\right)+18\sqrt{\alpha}\left(\frac{1}{\alpha}\right)^{5/2}\sqrt{\frac{\gamma+4}{\alpha}}-60\left(\frac{1}{\alpha}\right)^{7/2}\left(\sqrt{\alpha}\sqrt{\frac{\gamma+4}{\alpha}}\right)+210\sqrt{\alpha}\left(\frac{1}{\alpha}\right)^{9/2}\sqrt{\frac{\gamma+4}{\alpha}}+O\left(\left(\frac{1}{\alpha}\right)^5\right)$$

(generalized Puiseux series)

Derivative

$$\frac{\partial}{\partial\alpha}\left(3\sqrt{\frac{1+\left(\frac{1}{2}\right)^2\gamma}{1+\left(\frac{1}{2}\right)^2\alpha}}\right) = -\frac{3\sqrt{\frac{\gamma+4}{\alpha+4}}}{2\,\alpha+8}$$

Indefinite integral

$$\int 3\sqrt{\frac{1+\frac{\gamma}{4}}{1+\frac{\alpha}{4}}}\,d\alpha = 6\,(\alpha+4)\sqrt{\frac{\gamma+4}{\alpha+4}} + \text{constant}$$

Global minimum

$$\min\left\{3\sqrt{\frac{1+\left(\frac{1}{2}\right)^2\gamma}{1+\left(\frac{1}{2}\right)^2\alpha}}\right\} = 0 \text{ at } (\alpha,\gamma) = (-8,-4)$$

Limit

$$\lim_{\alpha\to\pm\infty} 3\sqrt{\frac{1+\frac{\gamma}{4}}{1+\frac{\alpha}{4}}} = 0$$

Series representations

$$3\sqrt{\frac{1+\left(\frac{1}{2}\right)^2\gamma}{1+\left(\frac{1}{2}\right)^2\alpha}} = 3\sqrt{\frac{-\alpha+\gamma}{4+\alpha}}\sum_{k=0}^{\infty}\left(\frac{-\alpha+\gamma}{4+\alpha}\right)^{-k}\binom{\frac{1}{2}}{k} \quad \text{for } \left|\frac{\alpha-\gamma}{4+\alpha}\right| > 1$$

$$3\sqrt{\frac{1+\left(\frac{1}{2}\right)^2\gamma}{1+\left(\frac{1}{2}\right)^2\alpha}} = 3\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{-\alpha+\gamma}{4+\alpha}\right)^k\left(-\frac{1}{2}\right)_k}{k!} \quad \text{for } \left|\frac{\alpha-\gamma}{4+\alpha}\right| < 1$$

$$3\sqrt{\frac{1+\left(\frac{1}{2}\right)^2\gamma}{1+\left(\frac{1}{2}\right)^2\alpha}} = 3\sqrt{\frac{-\alpha+\gamma}{4+\alpha}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{-\alpha+\gamma}{4+\alpha}\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!} \quad \text{for } \left|\frac{\alpha-\gamma}{4+\alpha}\right| > 1$$

From the algebraic sum, we obtain:

1+4^(1/3) ((α^3(1-α)^3) / (β(1-β)))^(1/24) - 3* sqrt((1+(1/2)^2*γ)/(1+(1/2)^2*α))

Input

$$1+\sqrt[3]{4}\ \sqrt[24]{\frac{\alpha^3(1-\alpha)^3}{\beta(1-\beta)}}-3\sqrt{\frac{1+\left(\frac{1}{2}\right)^2\gamma}{1+\left(\frac{1}{2}\right)^2\alpha}}$$

Exact result

$$2^{2/3}\sqrt[24]{\frac{(1-\alpha)^3\alpha^3}{(1-\beta)\beta}}-3\sqrt{\frac{\frac{\gamma}{4}+1}{\frac{\alpha}{4}+1}}+1$$

The study of this function provides the following representations:

Alternate form

$$2^{2/3}\sqrt[24]{\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}}-3\sqrt{\frac{\gamma+4}{\alpha+4}}+1$$

Alternate forms assuming α, β, and γ are positive

$$-3\sqrt{\frac{\gamma+4}{\alpha+4}}+\frac{2^{2/3}\sqrt[8]{-(\alpha-1)\alpha}\ \exp\left(\frac{1}{12}\,i\,\pi\left\lfloor\frac{-3\arg(1-\alpha)+\arg(1-\beta)+\pi}{2\pi}\right\rfloor\right)}{\sqrt[24]{-(\beta-1)\beta}}+1$$

$$-3\sqrt{\frac{\frac{\gamma}{4}+1}{\frac{\alpha}{4}+1}}+\frac{2^{2/3}\sqrt[8]{1-\alpha}\ \sqrt[8]{\alpha}\ \exp\left(\frac{1}{12}\,i\,\pi\left\lfloor-\frac{3\arg(1-\alpha)}{2\pi}+\frac{\arg(1-\beta)}{2\pi}+\frac{1}{2}\right\rfloor\right)}{\sqrt[24]{1-\beta}\ \sqrt[24]{\beta}}+1$$

arg(z) is the complex argument

$\lfloor x \rfloor$ is the floor function

Series expansion at α=0

$$\left(1-\frac{3\sqrt{\gamma+4}}{2}\right)+\frac{2^{2/3}\sqrt[8]{\alpha}\sqrt[24]{\frac{\alpha^3}{\beta-\beta^2}}}{\sqrt[8]{\alpha}}+\frac{3}{16}\alpha\sqrt{\gamma+4}+O\left(\alpha^{9/8}\right)$$

(generalized Puiseux series)

Series expansion at α=∞

$$\frac{2^{2/3}\sqrt[4]{\alpha}\sqrt[24]{\frac{\alpha^6}{(\beta-1)\beta}}}{\sqrt[4]{\alpha}}+1-3\sqrt{\frac{1}{\alpha}}\left(\sqrt{\alpha}\sqrt{\frac{\gamma+4}{\alpha}}\right)-\frac{\left(\frac{1}{\alpha}\right)^{3/4}\sqrt[24]{\frac{\alpha^6}{(\beta-1)\beta}}}{4\left(\sqrt[3]{2}\sqrt[4]{\alpha}\right)}+O\left(\left(\frac{1}{\alpha}\right)^{5/4}\right)$$

(generalized Puiseux series)

Derivative

$$\frac{\partial}{\partial\alpha}\left(1+\sqrt[3]{4}\sqrt[24]{\frac{\alpha^3(1-\alpha)^3}{\beta(1-\beta)}}-3\sqrt{\frac{1+\left(\frac{1}{2}\right)^2\gamma}{1+\left(\frac{1}{2}\right)^2\alpha}}\right)=$$

$$\frac{1}{8}\left(\frac{2^{2/3}(2\alpha-1)\sqrt[24]{\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}}}{(\alpha-1)\alpha}+\frac{12\sqrt{\frac{\gamma+4}{\alpha+4}}}{\alpha+4}\right)$$

Indefinite integral

$$\int\left(1+2^{2/3}\sqrt[24]{\frac{(1-\alpha)^3\alpha^3}{(1-\beta)\beta}}-3\sqrt{\frac{1+\frac{\gamma}{4}}{1+\frac{\alpha}{4}}}\right)d\alpha=$$

$$-6(\alpha+4)\sqrt{\frac{\gamma+4}{\alpha+4}}+\alpha+\frac{8\times 2^{2/3}(\alpha-1)\sqrt[24]{\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}}\,{}_2F_1\left(-\frac{1}{8},\frac{9}{8};\frac{17}{8};1-\alpha\right)}{9\sqrt[8]{\alpha}}+\text{constant}$$

${}_2F_1(a,b;c;x)$ is the hypergeometric function

Or, from the alternate forms:

((2^(2/3) (((α - 1)^3 α^3)/((β - 1) β))^(1/24) + 1)) - ((3 sqrt((γ + 4)/(α + 4))))

Input

$$\left(2^{2/3}\sqrt[24]{\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}}+1\right)-3\sqrt{\frac{\gamma+4}{\alpha+4}}$$

Exact result

$$2^{2/3}\sqrt[24]{\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}}-3\sqrt{\frac{\gamma+4}{\alpha+4}}+1$$

The study of this function provides the following representations:

Alternate form assuming α, β, and γ are positive

$$-3\sqrt{\frac{\gamma+4}{\alpha+4}}+\frac{2^{2/3}\sqrt[8]{\alpha-1}\,\sqrt[8]{\alpha}\,\exp\left(\frac{1}{12}\,i\,\pi\left\lfloor\frac{-3\arg(\alpha-1)+\arg(\beta-1)+\pi}{2\pi}\right\rfloor\right)}{\sqrt[24]{\beta-1}\,\sqrt[24]{\beta}}+1$$

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

Property as a function
Parity

even

Series expansion at α=0

$$\left(1-\frac{3\sqrt{\gamma+4}}{2}\right)+\frac{2^{2/3}\sqrt[8]{\alpha}\,\sqrt[24]{\frac{\alpha^3}{\beta-\beta^2}}}{\sqrt[8]{\alpha}}+\frac{3}{16}\,\alpha\sqrt{\gamma+4}+O\left(\alpha^{9/8}\right)$$

(generalized Puiseux series)

Series expansion at α=∞

$$\frac{2^{2/3}\sqrt[4]{\alpha}\sqrt[24]{\frac{\alpha^6}{(\beta-1)\beta}}}{\sqrt[4]{\alpha}}+1-3\sqrt{\frac{1}{\alpha}}\left(\sqrt{\alpha}\sqrt{\frac{\gamma+4}{\alpha}}\right)-\frac{\left(\frac{1}{\alpha}\right)^{3/4}\sqrt[24]{\frac{\alpha^6}{(\beta-1)\beta}}}{4\left(\sqrt[3]{2}\sqrt[4]{\alpha}\right)}+O\left(\left(\frac{1}{\alpha}\right)^{5/4}\right)$$

(generalized Puiseux series)

Derivative

$$\frac{\partial}{\partial\alpha}\left(\left(2^{2/3}\sqrt[24]{\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}}+1\right)-3\sqrt{\frac{\gamma+4}{\alpha+4}}\right)=$$
$$\frac{1}{8}\left(\frac{2^{2/3}(2\alpha-1)\sqrt[24]{\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}}}{(\alpha-1)\alpha}+\frac{12\sqrt{\frac{\gamma+4}{\alpha+4}}}{\alpha+4}\right)$$

Indefinite integral

$$\int\left(1+2^{2/3}\sqrt[24]{\frac{(-1+\alpha)^3\alpha^3}{(-1+\beta)\beta}}-3\sqrt{\frac{4+\gamma}{4+\alpha}}\right)d\alpha=$$
$$-6(\alpha+4)\sqrt{\frac{\gamma+4}{\alpha+4}}+\alpha+\frac{8\times2^{2/3}(\alpha-1)\sqrt[24]{\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}}\,{}_2F_1\left(-\frac{1}{8},\frac{9}{8};\frac{17}{8};1-\alpha\right)}{9\sqrt[8]{\alpha}}+$$
constant

${}_2F_1(a,b;c;x)$ is the hypergeometric function

Performing the derivative:

derivative((1 + 2^(2/3) (((-1 + α)^3 α^3)/((-1 + β) β))^(1/24) - 3 sqrt((4 + γ)/(4 + α))))

Derivative

$$\frac{\partial}{\partial\alpha}\left(1+2^{2/3}\sqrt[24]{\frac{(-1+\alpha)^3\,\alpha^3}{(-1+\beta)\,\beta}}-3\sqrt{\frac{4+\gamma}{4+\alpha}}\right)=$$

$$\frac{\frac{3\,\alpha^3\,(\alpha-1)^2}{(\beta-1)\,\beta}+\frac{3\,\alpha^2\,(\alpha-1)^3}{(\beta-1)\,\beta}}{12\,\sqrt[3]{2}\,\left(\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}\right)^{23/24}}+\frac{3\,(\gamma+4)}{2\,(\alpha+4)^2\,\sqrt{\frac{\gamma+4}{\alpha+4}}}$$

The study of this function provides the following representations:

Alternate forms

$$\frac{1}{8}\left(\frac{2^{2/3}\,(2\,\alpha-1)\,\sqrt[24]{\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}}}{(\alpha-1)\,\alpha}+\frac{12\,\sqrt{\frac{\gamma+4}{\alpha+4}}}{\alpha+4}\right)$$

$$\frac{6\,\sqrt[3]{2}\,(\beta-1)\,\beta\,(\gamma+4)\left(\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}\right)^{23/24}+\alpha^2\left(2\,\alpha^3-5\,\alpha^2+4\,\alpha-1\right)(\alpha+4)^2\,\sqrt{\frac{\gamma+4}{\alpha+4}}}{4\,\sqrt[3]{2}\,(\alpha+4)^2\,(\beta-1)\,\beta\left(\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}\right)^{23/24}\,\sqrt{\frac{\gamma+4}{\alpha+4}}}$$

$$\left(2\times2^{2/3}\,\alpha^7\,\sqrt{\frac{\gamma+4}{\alpha+4}}+11\times2^{2/3}\,\alpha^6\,\sqrt{\frac{\gamma+4}{\alpha+4}}-4\times2^{2/3}\,\alpha^5\,\sqrt{\frac{\gamma+4}{\alpha+4}}-\right.$$
$$49\times2^{2/3}\,\alpha^4\,\sqrt{\frac{\gamma+4}{\alpha+4}}+12\,\beta^2\,\gamma\left(\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}\right)^{23/24}+$$
$$48\,\beta^2\left(\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}\right)^{23/24}-12\,\beta\,\gamma\left(\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}\right)^{23/24}-$$
$$\left.48\,\beta\left(\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}\right)^{23/24}+56\times2^{2/3}\,\alpha^3\,\sqrt{\frac{\gamma+4}{\alpha+4}}-16\times2^{2/3}\,\alpha^2\,\sqrt{\frac{\gamma+4}{\alpha+4}}\right)\Big/$$
$$\left(8\,(\alpha+4)^2\,(\beta-1)\,\beta\left(\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}\right)^{23/24}\,\sqrt{\frac{\gamma+4}{\alpha+4}}\right)$$

Alternate forms assuming α, β, and γ are positive

$$\frac{1}{8}\left(\frac{2^{2/3}(2\alpha-1)\sqrt[24]{\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}}}{(\alpha-1)\alpha}+12\sqrt{\frac{\gamma+4}{(\alpha+4)^3}}\right)$$

$$\frac{1}{8}\left(\frac{12\sqrt{\gamma+4}}{(\alpha+4)^{3/2}}+\frac{2^{2/3}(2\alpha-1)\exp\left(-\frac{23}{12}i\pi\left\lfloor\frac{-3\arg(\alpha-1)+\arg(\beta-1)+\pi}{2\pi}\right\rfloor\right)}{(\alpha-1)^{7/8}\alpha^{7/8}\sqrt[24]{\beta-1}\sqrt[24]{\beta}}\right)$$

$$\frac{3\sqrt{\gamma+4}}{2(\alpha+4)^{3/2}}+\frac{(\beta-1)^{23/24}\beta^{23/24}\left(\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}\right)\exp\left(-\frac{23}{12}i\pi\left\lfloor-\frac{3\arg(\alpha-1)}{2\pi}+\frac{\arg(\beta-1)}{2\pi}+\frac{1}{2}\right\rfloor\right)}{12\sqrt[3]{2}(\alpha-1)^{23/8}\alpha^{23/8}}$$

Expanded forms

$$\frac{3\gamma}{2\alpha^2\sqrt{\frac{\gamma}{\alpha+4}+\frac{4}{\alpha+4}}+16\alpha\sqrt{\frac{\gamma}{\alpha+4}+\frac{4}{\alpha+4}}+32\sqrt{\frac{\gamma}{\alpha+4}+\frac{4}{\alpha+4}}}+$$
$$\frac{12}{2\alpha^2\sqrt{\frac{\gamma}{\alpha+4}+\frac{4}{\alpha+4}}+16\alpha\sqrt{\frac{\gamma}{\alpha+4}+\frac{4}{\alpha+4}}+32\sqrt{\frac{\gamma}{\alpha+4}+\frac{4}{\alpha+4}}}+$$
$$\frac{\alpha^5}{4\sqrt[3]{2}(\beta^2-\beta)\left(\frac{\alpha^6}{(\beta-1)\beta}-\frac{3\alpha^5}{(\beta-1)\beta}+\frac{3\alpha^4}{(\beta-1)\beta}-\frac{\alpha^3}{(\beta-1)\beta}\right)^{23/24}}-$$
$$\frac{\alpha^4}{2\sqrt[3]{2}(\beta^2-\beta)\left(\frac{\alpha^6}{(\beta-1)\beta}-\frac{3\alpha^5}{(\beta-1)\beta}+\frac{3\alpha^4}{(\beta-1)\beta}-\frac{\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+$$
$$\frac{\alpha^5}{4\sqrt[3]{2}(\beta-1)\beta\left(\frac{\alpha^6}{(\beta-1)\beta}-\frac{3\alpha^5}{(\beta-1)\beta}+\frac{3\alpha^4}{(\beta-1)\beta}-\frac{\alpha^3}{(\beta-1)\beta}\right)^{23/24}}-$$
$$\frac{3\alpha^4}{4\sqrt[3]{2}(\beta-1)\beta\left(\frac{\alpha^6}{(\beta-1)\beta}-\frac{3\alpha^5}{(\beta-1)\beta}+\frac{3\alpha^4}{(\beta-1)\beta}-\frac{\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+$$
$$\frac{\alpha^3}{\sqrt[3]{2}(\beta-1)\beta\left(\frac{\alpha^6}{(\beta-1)\beta}-\frac{3\alpha^5}{(\beta-1)\beta}+\frac{3\alpha^4}{(\beta-1)\beta}-\frac{\alpha^3}{(\beta-1)\beta}\right)^{23/24}}-$$
$$\frac{\alpha^2}{4\sqrt[3]{2}(\beta-1)\beta\left(\frac{\alpha^6}{(\beta-1)\beta}-\frac{3\alpha^5}{(\beta-1)\beta}+\frac{3\alpha^4}{(\beta-1)\beta}-\frac{\alpha^3}{(\beta-1)\beta}\right)^{23/24}}$$

$$-\frac{5\,\alpha\,\sqrt[24]{\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}}}{4\,\sqrt[3]{2}\,(\alpha-1)^3}+\frac{\sqrt[24]{\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}}}{\sqrt[3]{2}\,(\alpha-1)^3}-\frac{\sqrt[24]{\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}}}{4\,\sqrt[3]{2}\,(\alpha-1)^3\,\alpha}+\frac{\alpha^2\,\sqrt[24]{\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}}}{2\,\sqrt[3]{2}\,(\alpha-1)^3}+\frac{3\,\gamma\,\sqrt{\frac{\gamma+4}{\alpha+4}}}{2\,(\alpha+4)\,(\gamma+4)}+\frac{6\,\sqrt{\frac{\gamma+4}{\alpha+4}}}{(\alpha+4)\,(\gamma+4)}$$

Indefinite integral

$$\int\left(\frac{\frac{3\,(-1+\alpha)^3\,\alpha^2}{(-1+\beta)\,\beta}+\frac{3\,(-1+\alpha)^2\,\alpha^3}{(-1+\beta)\,\beta}}{12\,\sqrt[3]{2}\left(\frac{(-1+\alpha)^3\,\alpha^3}{(-1+\beta)\,\beta}\right)^{23/24}}+\frac{3\,(4+\gamma)}{2\,(4+\alpha)^2\,\sqrt{\frac{4+\gamma}{4+\alpha}}}\right)d\alpha=$$

$$2^{2/3}\,\sqrt[24]{\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}}-3\,\sqrt{\frac{\gamma+4}{\alpha+4}}+\text{constant}$$

And performing the integral, we obtain:

integrate((1 + 2^(2/3) (((-1 + α)^3 α^3)/((-1 + β) β))^(1/24) - 3 sqrt((4 + γ)/(4 + α))))

Indefinite integral

$$\int\left(1+2^{2/3}\,\sqrt[24]{\frac{(-1+\alpha)^3\,\alpha^3}{(-1+\beta)\,\beta}}-3\,\sqrt{\frac{4+\gamma}{4+\alpha}}\right)d\alpha=$$

$$-6\,(\alpha+4)\,\sqrt{\frac{\gamma+4}{\alpha+4}}+\alpha+\frac{8\times 2^{2/3}\,(\alpha-1)\,\sqrt[24]{\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}}\;{}_2F_1\!\left(-\frac{1}{8},\frac{9}{8};\frac{17}{8};1-\alpha\right)}{9\,\sqrt[8]{\alpha}}+\text{constant}$$

${}_2F_1(a,b;c;x)$ is the hypergeometric function

The study of this function provides the following representations:

Alternate form of the integral

$$\frac{9\,\alpha^{9/8} - 54\,(\alpha+4)\,\sqrt[8]{\alpha}\,\sqrt{\frac{\gamma+4}{\alpha+4}} + 8\times 2^{2/3}\,(\alpha-1)\,\sqrt[24]{\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}}\;{}_2F_1\!\left(-\frac{1}{8},\frac{9}{8};\frac{17}{8};1-\alpha\right)}{9\,\sqrt[8]{\alpha}}$$

+ constant

Alternate forms assuming α, β, and γ are positive

$$-6\sqrt{(\alpha+4)(\gamma+4)} + \alpha + \frac{8}{9}\times 2^{2/3}\,(\alpha-1)\,\sqrt[24]{\frac{(\alpha-1)^3}{(\beta-1)\,\beta}}\;{}_2F_1\!\left(-\frac{1}{8},\frac{9}{8};\frac{17}{8};1-\alpha\right) +$$

constant

$$-6\sqrt{(\alpha+4)(\gamma+4)} + \alpha + \frac{8\times 2^{2/3}\,(\alpha-1)^{9/8}\;{}_2F_1\!\left(-\frac{1}{8},\frac{9}{8};\frac{17}{8};1-\alpha\right)\exp\!\left(\frac{1}{12}\,i\,\pi\left\lfloor\frac{-3\arg(\alpha-1)+\arg(\beta-1)+\pi}{2\pi}\right\rfloor\right)}{9\,\sqrt[24]{\beta-1}\,\sqrt[24]{\beta}} +$$

constant

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

Expanded form of the integrals

$$-6\,\alpha\sqrt{\frac{\gamma}{\alpha+4}+\frac{4}{\alpha+4}} - 24\sqrt{\frac{\gamma}{\alpha+4}+\frac{4}{\alpha+4}} + \alpha - \frac{8\times 2^{2/3}\,\sqrt[24]{\frac{\alpha^6}{(\beta-1)\,\beta}-\frac{3\,\alpha^5}{(\beta-1)\,\beta}+\frac{3\,\alpha^4}{(\beta-1)\,\beta}-\frac{\alpha^3}{(\beta-1)\,\beta}}\;{}_2F_1\!\left(-\frac{1}{8},\frac{9}{8};\frac{17}{8};1-\alpha\right)}{9\,\sqrt[8]{\alpha}} + \frac{8}{9}\times 2^{2/3}\,\alpha^{7/8}\sqrt[24]{\frac{\alpha^6}{(\beta-1)\,\beta}-\frac{3\,\alpha^5}{(\beta-1)\,\beta}+\frac{3\,\alpha^4}{(\beta-1)\,\beta}-\frac{\alpha^3}{(\beta-1)\,\beta}}\;{}_2F_1\!\left(-\frac{1}{8},\frac{9}{8};\frac{17}{8};1-\alpha\right) +$$

constant

$$-6\,\alpha\sqrt{\frac{\gamma+4}{\alpha+4}}-24\sqrt{\frac{\gamma+4}{\alpha+4}}+\alpha-\frac{8\times 2^{2/3}\sqrt[24]{\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}}\;{}_2F_1\left(-\frac{1}{8},\frac{9}{8};\frac{17}{8};1-\alpha\right)}{9\sqrt[8]{\alpha}}+\frac{8}{9}\times$$
$$2^{2/3}\,\alpha^{7/8}\sqrt[24]{\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}}\;{}_2F_1\left(-\frac{1}{8},\frac{9}{8};\frac{17}{8};1-\alpha\right)+\text{constant}$$

and deriving again:

$$\frac{\partial}{\partial\alpha}\left(1+2^{2/3}\sqrt[24]{\frac{(-1+\alpha)^3\,\alpha^3}{(-1+\beta)\,\beta}}-3\sqrt{\frac{4+\gamma}{4+\alpha}}\right)=$$
$$\frac{\frac{3\,\alpha^3\,(\alpha-1)^2}{(\beta-1)\,\beta}+\frac{3\,\alpha^2\,(\alpha-1)^3}{(\beta-1)\,\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}\right)^{23/24}}+\frac{3\,(\gamma+4)}{2\,(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}$$

derivative((((3 α^3 (α - 1)^2)/((β - 1) β) + (3 α^2 (α - 1)^3)/((β - 1) β))/(12 2^(1/3) (((α - 1)^3 α^3)/((β - 1) β))^(23/24)) + (3 (γ + 4))/(2 (α + 4)^2 sqrt((γ + 4)/(α + 4)))))

Derivative

$$\frac{\partial}{\partial\gamma}\left(\frac{\frac{3\,\alpha^3\,(\alpha-1)^2}{(\beta-1)\,\beta}+\frac{3\,\alpha^2\,(\alpha-1)^3}{(\beta-1)\,\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}\right)^{23/24}}+\frac{3\,(\gamma+4)}{2\,(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=\frac{3}{4\,(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}$$

The study of this function provides the following representations:

Partial derivatives

$$\frac{\partial}{\partial a \to 1}\left(\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}, a \to 1\right]$$

$$\frac{\partial}{\partial b \to 2}\left(\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}, b \to 2\right]$$

$$\frac{\partial}{\partial c \to 3}\left(\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}, c \to 3\right]$$

$$\frac{\partial}{\partial d \to 4}\left(\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}, d \to 4\right]$$

$$\frac{\partial}{\partial e\to 5}\left(\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}},e\to 5\right]$$

$$\frac{\partial}{\partial f\to 6}\left(\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}},f\to 6\right]$$

$$\frac{\partial}{\partial g\to 7}\left(\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}},g\to 7\right]$$

$$\frac{\partial}{\partial h\to 8}\left(\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}},h\to 8\right]$$

$$\frac{\partial}{\partial i \to 9}\left(\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}},\, i\to 9\right]$$

$$\frac{\partial}{\partial j \to 10}\left(\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}},\, j\to 10\right]$$

$$\frac{\partial}{\partial k \to 11}\left(\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}},\, k\to 11\right]$$

$$\frac{\partial}{\partial l \to 12}\left(\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}},\, l\to 12\right]$$

$$\frac{\partial}{\partial m \to 13}\left(\frac{\frac{3\,\alpha^3\,(\alpha-1)^2}{(\beta-1)\,\beta}+\frac{3\,\alpha^2\,(\alpha-1)^3}{(\beta-1)\,\beta}}{12\,\sqrt[3]{2}\,\left(\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}\right)^{23/24}}+\frac{3\,(\gamma+4)}{2\,(\alpha+4)^2\,\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\,\alpha^3\,(\alpha-1)^2}{(\beta-1)\,\beta}+\frac{3\,\alpha^2\,(\alpha-1)^3}{(\beta-1)\,\beta}}{12\,\sqrt[3]{2}\,\left(\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}\right)^{23/24}}+\frac{3\,(\gamma+4)}{2\,(\alpha+4)^2\,\sqrt{\frac{\gamma+4}{\alpha+4}}},m\to 13\right]$$

$$\frac{\partial}{\partial n \to 14}\left(\frac{\frac{3\,\alpha^3\,(\alpha-1)^2}{(\beta-1)\,\beta}+\frac{3\,\alpha^2\,(\alpha-1)^3}{(\beta-1)\,\beta}}{12\,\sqrt[3]{2}\,\left(\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}\right)^{23/24}}+\frac{3\,(\gamma+4)}{2\,(\alpha+4)^2\,\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\,\alpha^3\,(\alpha-1)^2}{(\beta-1)\,\beta}+\frac{3\,\alpha^2\,(\alpha-1)^3}{(\beta-1)\,\beta}}{12\,\sqrt[3]{2}\,\left(\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}\right)^{23/24}}+\frac{3\,(\gamma+4)}{2\,(\alpha+4)^2\,\sqrt{\frac{\gamma+4}{\alpha+4}}},n\to 14\right]$$

$$\frac{\partial}{\partial o \to 15}\left(\frac{\frac{3\,\alpha^3\,(\alpha-1)^2}{(\beta-1)\,\beta}+\frac{3\,\alpha^2\,(\alpha-1)^3}{(\beta-1)\,\beta}}{12\,\sqrt[3]{2}\,\left(\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}\right)^{23/24}}+\frac{3\,(\gamma+4)}{2\,(\alpha+4)^2\,\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\,\alpha^3\,(\alpha-1)^2}{(\beta-1)\,\beta}+\frac{3\,\alpha^2\,(\alpha-1)^3}{(\beta-1)\,\beta}}{12\,\sqrt[3]{2}\,\left(\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}\right)^{23/24}}+\frac{3\,(\gamma+4)}{2\,(\alpha+4)^2\,\sqrt{\frac{\gamma+4}{\alpha+4}}},o\to 15\right]$$

$$\frac{\partial}{\partial p \to 16}\left(\frac{\frac{3\,\alpha^3\,(\alpha-1)^2}{(\beta-1)\,\beta}+\frac{3\,\alpha^2\,(\alpha-1)^3}{(\beta-1)\,\beta}}{12\,\sqrt[3]{2}\,\left(\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}\right)^{23/24}}+\frac{3\,(\gamma+4)}{2\,(\alpha+4)^2\,\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\,\alpha^3\,(\alpha-1)^2}{(\beta-1)\,\beta}+\frac{3\,\alpha^2\,(\alpha-1)^3}{(\beta-1)\,\beta}}{12\,\sqrt[3]{2}\,\left(\frac{(\alpha-1)^3\,\alpha^3}{(\beta-1)\,\beta}\right)^{23/24}}+\frac{3\,(\gamma+4)}{2\,(\alpha+4)^2\,\sqrt{\frac{\gamma+4}{\alpha+4}}},p\to 16\right]$$

$$\frac{\partial}{\partial q \to 17}\left(\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}, q \to 17\right]$$

$$\frac{\partial}{\partial r \to 18}\left(\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}, r \to 18\right]$$

$$\frac{\partial}{\partial s \to 19}\left(\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}, s \to 19\right]$$

$$\frac{\partial}{\partial t \to 20}\left(\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}, t \to 20\right]$$

$$\frac{\partial}{\partial u \to 21}\left(\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}},u\to 21\right]$$

$$\frac{\partial}{\partial v \to 22}\left(\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}},v\to 22\right]$$

$$\frac{\partial}{\partial w \to 23}\left(\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}},w\to 23\right]$$

$$\frac{\partial}{\partial x \to 24}\left(\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}},x\to 24\right]$$

$$\frac{\partial}{\partial y \to 25}\left(\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}, y \to 25\right]$$

$$\frac{\partial}{\partial z \to 26}\left(\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=$$

$$D\left[\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}, z \to 26\right]$$

Alternate form assuming α and γ are positive

$$\frac{3}{4(\alpha+4)^{3/2}\sqrt{\gamma+4}}$$

Expanded forms

$$\frac{3}{4\alpha^2\sqrt{\frac{\gamma}{\alpha+4}+\frac{4}{\alpha+4}}+32\alpha\sqrt{\frac{\gamma}{\alpha+4}+\frac{4}{\alpha+4}}+64\sqrt{\frac{\gamma}{\alpha+4}+\frac{4}{\alpha+4}}}$$

$$\frac{3\sqrt{\frac{\gamma+4}{\alpha+4}}}{4(\alpha+4)(\gamma+4)}$$

From

$$\frac{3}{4\,\alpha^2 \sqrt{\frac{\gamma}{\alpha+4} + \frac{4}{\alpha+4}} + 32\,\alpha \sqrt{\frac{\gamma}{\alpha+4} + \frac{4}{\alpha+4}} + 64 \sqrt{\frac{\gamma}{\alpha+4} + \frac{4}{\alpha+4}}}$$

For α = γ = 0.5 :

3/(4 0.5^2 sqrt(0.5/(0.5 + 4) + 4/(0.5 + 4)) + 32 0.5 sqrt(0.5/(0.5 + 4) + 4/(0.5 + 4)) + 64 sqrt(0.5/(0.5 + 4) + 4/(0.5 + 4)))

Input

$$\frac{3}{4 \times 0.5^2 \sqrt{\frac{0.5}{0.5+4} + \frac{4}{0.5+4}} + 32 \times 0.5 \sqrt{\frac{0.5}{0.5+4} + \frac{4}{0.5+4}} + 64 \sqrt{\frac{0.5}{0.5+4} + \frac{4}{0.5+4}}}$$

Result

0.037037037037037037037037037037037037037037037037037037037037037037037037037037037037037037037037037037037037037037037037037370...

Repeating decimal

$0.\overline{037}$ (period 3)

0.037

Rational approximation

$$\frac{1}{27}$$

From which:

[1/((3/(4 0.5^2 sqrt(0.5/(0.5 + 4) + 4/(0.5 + 4)) + 32 0.5 sqrt(0.5/(0.5 + 4) + 4/(0.5 + 4)) + 64 sqrt(0.5/(0.5 + 4) + 4/(0.5 + 4)))))]^2

Input

$$\left(\frac{1}{\frac{3}{4\times0.5^2\sqrt{\frac{0.5}{0.5+4}+\frac{4}{0.5+4}}+32\times0.5\sqrt{\frac{0.5}{0.5+4}+\frac{4}{0.5+4}}+64\sqrt{\frac{0.5}{0.5+4}+\frac{4}{0.5+4}}}}\right)^2$$

Result

729

729

10^3+[1/((3/(4 0.5^2 sqrt(0.5/(0.5 + 4) + 4/(0.5 + 4)) + 32 0.5 sqrt(0.5/(0.5 + 4) + 4/(0.5 + 4)) + 64 sqrt(0.5/(0.5 + 4) + 4/(0.5 + 4)))))]^2

Input

$$10^3+\left(\frac{1}{\frac{3}{4\times0.5^2\sqrt{\frac{0.5}{0.5+4}+\frac{4}{0.5+4}}+32\times0.5\sqrt{\frac{0.5}{0.5+4}+\frac{4}{0.5+4}}+64\sqrt{\frac{0.5}{0.5+4}+\frac{4}{0.5+4}}}}\right)^2$$

Result

1729

1729

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

(((10^3+[1/((3/(4 0.5^2 sqrt(0.5/(0.5 + 4) + 4/(0.5 + 4)) + 32 0.5 sqrt(0.5/(0.5 + 4) + 4/(0.5 + 4)) + 64 sqrt(0.5/(0.5 + 4) + 4/(0.5 + 4)))))]^2)))^1/15

Input

$$\sqrt[15]{10^3+\left(\frac{1}{\frac{3}{4\times 0.5^2\sqrt{\frac{0.5}{0.5+4}+\frac{4}{0.5+4}}+32\times 0.5\sqrt{\frac{0.5}{0.5+4}+\frac{4}{0.5+4}}+64\sqrt{\frac{0.5}{0.5+4}+\frac{4}{0.5+4}}}}\right)^2}$$

Result

1.6438152…

1.6438152…. ≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ... (trace of the instanton shape)

(1/27((((10^3+[1/((3/(4 0.5^2 sqrt(0.5/(0.5 + 4) + 4/(0.5 + 4)) + 32 0.5 sqrt(0.5/(0.5 + 4) + 4/(0.5 + 4)) + 64 sqrt(0.5/(0.5 + 4) + 4/(0.5 + 4))))))]^2)))-1))^2

Input

$$\left(\frac{1}{27}\left(\left(\left(10^3+\left(\frac{1}{\frac{3}{4\times 0.5^2\sqrt{\frac{0.5}{0.5+4}+\frac{4}{0.5+4}}+32\times 0.5\sqrt{\frac{0.5}{0.5+4}+\frac{4}{0.5+4}}+64\sqrt{\frac{0.5}{0.5+4}+\frac{4}{0.5+4}}}}\right)^2\right)\right)-1\right)\right)^2$$

Result

4096

$4096 = 64^2$

We have also this other development:

derivative((((3 α^3 (α - 1)^2)/((β - 1) β) + (3 α^2 (α - 1)^3)/((β - 1) β))/(12 2^(1/3) (((α - 1)^3 α^3)/((β - 1) β))^(23/24)) + (3 (γ + 4))/(2 (α + 4)^2 sqrt((γ + 4)/(α + 4)))))

Derivative

$$\frac{\partial}{\partial\alpha}\left(\frac{\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}+\frac{3(\gamma+4)}{2(\alpha+4)^2\sqrt{\frac{\gamma+4}{\alpha+4}}}\right)=-\frac{23\left(\frac{3\alpha^3(\alpha-1)^2}{(\beta-1)\beta}+\frac{3\alpha^2(\alpha-1)^3}{(\beta-1)\beta}\right)^2}{288\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{47/24}}+$$

$$\frac{\frac{6\alpha^3(\alpha-1)}{(\beta-1)\beta}+\frac{18\alpha^2(\alpha-1)^2}{(\beta-1)\beta}+\frac{6\alpha(\alpha-1)^3}{(\beta-1)\beta}}{12\sqrt[3]{2}\left(\frac{(\alpha-1)^3\alpha^3}{(\beta-1)\beta}\right)^{23/24}}-\frac{3(\gamma+4)}{(\alpha+4)^3\sqrt{\frac{\gamma+4}{\alpha+4}}}+\frac{3(\gamma+4)^2}{4(\alpha+4)^4\left(\frac{\gamma+4}{\alpha+4}\right)^{3/2}}$$

For α = 0.5 , β = 0.8 and γ = -2 , we obtain, from the first term of right-hand side:

-(23 ((3 0.5^3 (0.5 - 1)^2)/((0.8 - 1) 0.8) + (3 0.5^2 (0.5 - 1)^3)/((0.8 - 1) 0.8))^2)/(288 2^(1/3) (((0.5 - 1)^3 0.5^3)/((0.8 - 1) 0.8))^(47/24))

Input

$$-\frac{23\left(\frac{3\times0.5^3(0.5-1)^2}{(0.8-1)\times0.8}+\frac{3\times0.5^2(0.5-1)^3}{(0.8-1)\times0.8}\right)^2}{288\sqrt[3]{2}\left(\frac{(0.5-1)^3\times0.5^3}{(0.8-1)\times0.8}\right)^{47/24}}$$

Result

0

From the second term:

((6 0.5^3 (0.5 - 1))/((0.8 - 1) 0.8) + (18 0.5^2 (0.5 - 1)^2)/((0.8 - 1) 0.8) + (6 0.5 (0.5 - 1)^3)/((0.8 - 1) 0.8))/(12 2^(1/3) (((0.5 - 1)^3 0.5^3)/((0.8 - 1) 0.8))^(23/24))

Input

$$\frac{\frac{6\times0.5^3(0.5-1)}{(0.8-1)\times0.8}+\frac{18\times0.5^2(0.5-1)^2}{(0.8-1)\times0.8}+\frac{6\times0.5(0.5-1)^3}{(0.8-1)\times0.8}}{12\sqrt[3]{2}\left(\frac{(0.5-1)^3\times0.5^3}{(0.8-1)\times0.8}\right)^{23/24}}$$

Result

−1.44076…

Adding this result to the last two terms, we obtain in conclusion:

- (3 (γ + 4))/((α + 4)^3 sqrt((γ + 4)/(α + 4))) + (3 (γ + 4)^2)/(4 (α + 4)^4 ((γ + 4)/(α + 4))^(3/2))

-1.44076 - (3 (-2 + 4))/((0.5 + 4)^3 sqrt((-2 + 4)/(0.5 + 4))) + (3 (-2 + 4)^2)/(4 (0.5 + 4)^4 ((-2 + 4)/(0.5 + 4))^(3/2))

Input interpretation

$$-1.44076-\frac{3(-2+4)}{(0.5+4)^3\sqrt{\frac{-2+4}{0.5+4}}}+\frac{3(-2+4)^2}{4(0.5+4)^4\left(\frac{-2+4}{0.5+4}\right)^{3/2}}$$

Result

-1.514834074074074074074074074074074074074074074074074074074074074074...

Repeating decimal

$-1.51483\overline{407}$ (period 3)

-1.51483407

That is:

log(5/(-8 + 3 sqrt(2) + 6 e - 7 e^2 + 4 π + 5 π^2))

Input

$$\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)$$

log(x) is the natural logarithm

Decimal approximation

-1.514834072860029293704249624159999641104997139292888840426419711...

-1.51483407286….

The study of this function provides the following representations:

Alternate forms

$$\log\left(\frac{5}{-8+3\sqrt{2}+(6-7e)e+\pi(4+5\pi)}\right)$$

$$-\log\left(\frac{1}{5}\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)\right)$$

$$\log(5)-\log\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)$$

Alternative representations

$$\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)= \log_e\left(\frac{5}{-8+6e+4\pi-7e^2+5\pi^2+3\sqrt{2}}\right)$$

$$\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)= \log(a)\log_a\left(\frac{5}{-8+6e+4\pi-7e^2+5\pi^2+3\sqrt{2}}\right)$$

$$\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)= -\text{Li}_1\left(1-\frac{5}{-8+6e+4\pi-7e^2+5\pi^2+3\sqrt{2}}\right)$$

Series representations

$$\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right) = -\sum_{k=1}^{\infty} \frac{(-1)^k \left(-1+\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)^k}{k}$$

$$\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right) = 2 i \pi \left\lfloor \frac{\arg\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2} - x\right)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2} - x\right)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right) = 2 i \pi \left\lfloor \frac{\arg\left(5 - \left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right) x\right)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2} - x\right)^k x^{-k}}{k} \quad \text{for } x < 0$$

Integral representation

$$\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right) = \int_1^{\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}} \frac{1}{t}\, dt$$

From which, adding the Ramanujan class invariant

$$\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3$$

we obtain:

(((sqrt(((113+5sqrt(505))/8)) + sqrt(((105+5sqrt(505))/8)))))^3 exp((log(5/(-8 + 3 sqrt(2) + 6 e - 7 e^2 + 4 π + 5 π^2))))

Input

$$\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right)$$

$\log(x)$ is the natural logarithm

Exact result

$$\frac{5\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(105+5\sqrt{505}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(113+5\sqrt{505}\right)}\right)^3}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}$$

Decimal approximation

255.95845465903396631309217880250007006567027836106348450830727785…

255.958454659…. ≈ 256

The study of this function provides the following representations:

Alternate forms

$$\frac{5\left(5\sqrt{5}+\sqrt{101}+\sqrt{10\left(21+\sqrt{505}\right)}\right)^3}{64\left(-8+3\sqrt{2}+(6-7e)e+\pi(4+5\pi)\right)}$$

$$\frac{5\sqrt{338\,881+15\,080\sqrt{505}+4\sqrt{5\left(2\,871\,007\,052+127\,758\,137\sqrt{505}\right)}}}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}$$

$$\frac{5\left(\sqrt{10\left(21+\sqrt{505}\right)}+\sqrt{226+10\sqrt{505}}\right)^3}{64\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)}$$

Expanded form

$$\frac{555\sqrt{\frac{1}{2}\left(105+5\sqrt{505}\right)}}{4\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)}+\frac{25\sqrt{\frac{505}{2}\left(105+5\sqrt{505}\right)}}{4\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)}+\frac{535\sqrt{\frac{1}{2}\left(113+5\sqrt{505}\right)}}{4\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)}+\frac{25\sqrt{\frac{505}{2}\left(113+5\sqrt{505}\right)}}{4\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)}$$

Alternative representations

$$\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right) = \left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7z^2+4\pi+5\pi^2}\right)\right) \text{ for } z=e$$

$$\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right) = \exp\left(\log_e\left(\frac{5}{-8+6e+4\pi-7e^2+5\pi^2+3\sqrt{2}}\right)\right) \left(\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}\right)^3$$

$$\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right) = \left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7w^a+4\pi+5\pi^2}\right)\right) \text{ for } a=\frac{2}{\log(w)}$$

$$\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right) = \exp\left(\log(a)\log_a\left(\frac{5}{-8+6e+4\pi-7e^2+5\pi^2+3\sqrt{2}}\right)\right)\left(\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}\right)^3$$

Integral representation

$$\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right) = \exp\left(\int_1^{\frac{5}{-8+6e-7e^2+4\pi+5\pi^2+3\sqrt{2}}}\frac{1}{t}\,dt\right)\left(\sqrt{\frac{5}{8}\left(21+\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}\right)^3$$

And again, subtracting 5:

(((sqrt(((113+5sqrt(505))/8)) + sqrt(((105+5sqrt(505))/8)))))^3 exp((log(5/(-8 + 3 sqrt(2) + 6 e - 7 e^2 + 4 π + 5 π^2))))-5

Input

$$\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right) - 5$$

$\log(x)$ is the natural logarithm

Exact result

$$\frac{5\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(105+5\sqrt{505}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(113+5\sqrt{505}\right)}\right)^3}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}-5$$

Decimal approximation

250.95845465903396631309217880250007006567027836106348450830727785
...

250.958454659….

The study of this function provides the following representations:

Alternate forms

$$\frac{5\sqrt{338881+15080\sqrt{505}+4\sqrt{5\left(2871007052+127758137\sqrt{505}\right)}}}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}-5$$

$$5\left(\frac{\left(\sqrt{10\left(21+\sqrt{505}\right)}+\sqrt{226+10\sqrt{505}}\right)^3}{64\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)}-1\right)$$

$$-5-\left(5\left(\boxed{\text{root of } x^4-210x^2+12625 \text{ near } x=10.425-1.91847i}+\boxed{\text{root of } x^4-210x^2+12625 \text{ near } x=10.425+1.91847i}+5\sqrt{5}+\sqrt{101}\right)^3\right)\Big/\left(64\left(8-3\sqrt{2}-6e+7e^2-4\pi-5\pi^2\right)\right)$$

Expanded form

$$-5+\frac{555\sqrt{\frac{1}{2}\left(105+5\sqrt{505}\right)}}{4\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)}+\frac{25\sqrt{\frac{505}{2}\left(105+5\sqrt{505}\right)}}{4\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)}+\frac{535\sqrt{\frac{1}{2}\left(113+5\sqrt{505}\right)}}{4\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)}+\frac{25\sqrt{\frac{505}{2}\left(113+5\sqrt{505}\right)}}{4\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)}$$

Alternative representations

$$\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right)-5= \left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7z^2+4\pi+5\pi^2}\right)\right)-5 \text{ for } z=e$$

$$\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right)-5= -5+\exp\left(\log_e\left(\frac{5}{-8+6e+4\pi-7e^2+5\pi^2+3\sqrt{2}}\right)\right) \left(\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}\right)^3$$

$$\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right)-5 = \left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7w^a+4\pi+5\pi^2}\right)\right)-5 \quad \text{for } a=\frac{2}{\log(w)}$$

$$\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right)-5 = -5+\exp\left(\log(a)\log_a\left(\frac{5}{-8+6e+4\pi-7e^2+5\pi^2+3\sqrt{2}}\right)\right)\left(\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}\right)^3$$

Integral representation

$$\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right)-5 = -5+\exp\left(\int_1^{\frac{5}{-8+6e-7e^2+4\pi+5\pi^2+3\sqrt{2}}}\frac{1}{t}\,dt\right)\left(\sqrt{\frac{5}{8}\left(21+\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}\right)^3$$

Thence, considering the result in GeV and converting in Kg, we obtain:

Input interpretation

$$\text{convert}\left(\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right)-5\right)\ \text{GeV}/c^2 \text{ to kilograms}$$

Result

4.474×10^{-25} kg (kilograms)

$4.474*10^{-25}$ result that is very near to the value of the possible mass of a DM particle equal to 250 GeV = 4.457×10^{-25} kg

From the previous expression, we obtain also:

(1/4*(((((sqrt(((113+5sqrt(505))/8)) + sqrt(((105+5sqrt(505))/8)))))^3 exp((log(5/(-8 + 3 sqrt(2) + 6 e - 7 e^2 + 4 π + 5 π^2)))))))^2-Φ+2

Input

$$\left(\frac{1}{4}\left(\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right)\right)\right)^2-\Phi+2$$

$\log(x)$ is the natural logarithm

Φ is the golden ratio conjugate

Exact result

$$-\Phi+2+\frac{25\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(105+5\sqrt{505}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(113+5\sqrt{505}\right)}\right)^6}{16\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)^2}$$

Exact form

$$-\phi + 3 + \frac{25\left(\sqrt{10\left(21+\sqrt{505}\right)} + \sqrt{226 + 10\sqrt{505}}\right)^6}{65\,536\left(-8 + 3\sqrt{2} + 6e - 7e^2 + 4\pi + 5\pi^2\right)^2}$$

ϕ is the golden ratio

Decimal approximation

4096.0526229762967761705502160359403165956235692392397860417921612

...

4096.052622976…. ≈ 4096 = 64^2

The study of this function provides the following representations:

Alternate forms

$$-\Phi + 2 + \frac{8\,472\,025 + 377\,000\sqrt{505} + 8\sqrt{2\,242\,974\,259\,375 + \frac{399\,244\,178\,125\sqrt{505}}{4}}}{16\left(-8 + 3\sqrt{2} + 6e - 7e^2 + 4\pi + 5\pi^2\right)^2}$$

$$\left(25\left(\boxed{\text{root of } x^4 - 210x^2 + 12625 \text{ near } x = 10.425 - 1.91847\,i} + \boxed{\text{root of } x^4 - 210x^2 + 12625 \text{ near } x = 10.425 + 1.91847\,i} + 5\sqrt{5} + \sqrt{101}\right)^6\right) \Big/ \left(65\,536\left(8 - 3\sqrt{2} - 6e + 7e^2 - 4\pi - 5\pi^2\right)^2\right) - \Phi + 2$$

$$-\Phi + 2 + \left(25\left(2711048 + 120640\sqrt{505} + 12251\sqrt{5\left(21+\sqrt{505}\right)\left(113+5\sqrt{505}\right)} + 2725\sqrt{101\left(21+\sqrt{505}\right)\left(113+5\sqrt{505}\right)}\right)\right) \Big/ \left(128\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)^2\right)$$

Expanded form

$$-\Phi + 2 + \frac{8472025}{16\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)^2} + \frac{47125\sqrt{505}}{2\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)^2} + \frac{306275\sqrt{\left(105+5\sqrt{505}\right)\left(113+5\sqrt{505}\right)}}{128\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)^2} + \frac{13625\sqrt{505\left(105+5\sqrt{505}\right)\left(113+5\sqrt{505}\right)}}{128\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)^2}$$

27(1/4*(((((sqrt(((113+5sqrt(505))/8)) + sqrt(((105+5sqrt(505))/8)))))^3 exp((log(5/(-8 + 3 sqrt(2) + 6 e - 7 e^2 + 4 π + 5 π^2)))))))+1+((√(10-2√5) -2))⁄((√5-1))

Input

$$27\left(\frac{1}{4}\left(\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)} + \sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right)\right)\right) + 1 + \frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}$$

$\log(x)$ is the natural logarithm

Exact result

$$1+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}+\frac{135\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(105+5\sqrt{505}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(113+5\sqrt{505}\right)}\right)^3}{4\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)}$$

Decimal approximation

1729.0036479923196849094004987492685991123654670256242580138332881
…

1729.0036479923….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

The study of this function provides the following representations:

Alternate forms

$$\frac{1}{2}\left(1-\sqrt{5}+\sqrt{2\left(5+\sqrt{5}\right)}\right)+\frac{135\sqrt{338881+15080\sqrt{505}+4\sqrt{5\left(2871007052+127758137\sqrt{505}\right)}}}{4\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)}$$

$$\frac{-3+\sqrt{5}+\sqrt{2\left(5-\sqrt{5}\right)}}{\sqrt{5}-1}+$$

$$\Bigg(135\Bigg(555\sqrt{21+\sqrt{505}}-111\sqrt{5\left(21+\sqrt{505}\right)}-25\sqrt{101\left(21+\sqrt{505}\right)}+$$
$$25\sqrt{505\left(21+\sqrt{505}\right)}-107\sqrt{113+5\sqrt{505}}+$$
$$107\sqrt{5\left(113+5\sqrt{505}\right)}+25\sqrt{101\left(113+5\sqrt{505}\right)}-$$
$$5\sqrt{505\left(113+5\sqrt{505}\right)}\Bigg)\Bigg)\Big/$$
$$\left(16\sqrt{2}\left(\sqrt{5}-1\right)\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)\right)$$

$$-\Bigg(\Bigg(-3072+1152\sqrt{2}+1024\sqrt{5}-$$
$$384\sqrt{10}-768\sqrt{5-\sqrt{5}}+1024\sqrt{2\left(5-\sqrt{5}\right)}-$$
$$3375\sqrt{2}\left(21+\sqrt{505}\right)^{3/2}+675\sqrt{10}\left(21+\sqrt{505}\right)^{3/2}-$$
$$228\,825\sqrt{2\left(21+\sqrt{505}\right)}+45\,765\sqrt{10\left(21+\sqrt{505}\right)}+$$
$$10\,125\sqrt{202\left(21+\sqrt{505}\right)}-10\,125\sqrt{1010\left(21+\sqrt{505}\right)}+$$
$$135\sqrt{2}\left(113+5\sqrt{505}\right)^{3/2}-135\sqrt{10}\left(113+5\sqrt{505}\right)^{3/2}+$$
$$42\,525\sqrt{2\left(113+5\sqrt{505}\right)}-42\,525\sqrt{10\left(113+5\sqrt{505}\right)}-$$
$$10\,125\sqrt{202\left(113+5\sqrt{505}\right)}+2025\sqrt{1010\left(113+5\sqrt{505}\right)}+$$
$$2304e-768\sqrt{5}\,e-768\sqrt{2\left(5-\sqrt{5}\right)}\,e-2688e^2+$$
$$896\sqrt{5}\,e^2+896\sqrt{2\left(5-\sqrt{5}\right)}\,e^2+1536\pi-512\sqrt{5}\,\pi-$$
$$512\sqrt{2\left(5-\sqrt{5}\right)}\,\pi+1920\pi^2-640\sqrt{5}\,\pi^2-640\sqrt{2\left(5-\sqrt{5}\right)}\,\pi^2\Bigg)\Big/$$
$$\left(128\left(\sqrt{5}-1\right)\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)\right)\Bigg)$$

Expanded form

$$1-\frac{2}{\sqrt{5}-1}+\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}-1}+\frac{14\,985\sqrt{\frac{1}{2}\left(105+5\sqrt{505}\right)}}{16\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)}+$$
$$\frac{675\sqrt{\frac{505}{2}\left(105+5\sqrt{505}\right)}}{16\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)}+$$
$$\frac{14\,445\sqrt{\frac{1}{2}\left(113+5\sqrt{505}\right)}}{16\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)}+$$
$$\frac{675\sqrt{\frac{505}{2}\left(113+5\sqrt{505}\right)}}{16\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)}$$

Alternative representations

$$\frac{27}{4}\left(\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right)\right)+1+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}=\frac{27}{4}\left(\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7z^2+4\pi+5\pi^2}\right)\right)\right)+1+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1} \quad \text{for } z=e$$

$$\frac{27}{4}\left(\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right)\right)+1+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}=$$

$$1+\frac{27}{4}\exp\left(\log_e\left(\frac{5}{-8+6e+4\pi-7e^2+5\pi^2+3\sqrt{2}}\right)\right)\left(\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}\right)^3+\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}$$

$$\frac{27}{4}\left(\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right)\right)+1+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}=$$

$$1+\frac{27}{4}\exp\left(\log(a)\log_a\left(\frac{5}{-8+6e+4\pi-7e^2+5\pi^2+3\sqrt{2}}\right)\right)\left(\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}\right)^3+\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}$$

$$\frac{27}{4}\left(\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right)\right)+1+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}=\frac{27}{4}\left(\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7w^a+4\pi+5\pi^2}\right)\right)\right)+1+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1} \quad \text{for } a=\frac{2}{\log(w)}$$

Integral representation

$$\frac{27}{4}\left(\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right)\right)+1+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}=$$

$$1+\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}+\frac{27}{4}\exp\left(\int_1^{\frac{5}{-8+6e-7e^2+4\pi+5\pi^2+3\sqrt{2}}}\frac{1}{t}\,dt\right)\left(\sqrt{\frac{5}{8}\left(21+\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}\right)^3$$

(27(1/4*(((((sqrt(((113+5sqrt(505))/8)) + sqrt(((105+5sqrt(505))/8)))))^3 exp((log(5/(-8 + 3 sqrt(2) + 6 e - 7 e^2 + 4 π + 5 π^2)))))))+1+((√(10-2√5) -2))⁄((√5-1)))^1/15

Input

$$\left(27\left(\frac{1}{4}\left(\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right)\right)\right)+1+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)\text{^}(1/15)$$

$\log(x)$ is the natural logarithm

Exact result

$$\left(1+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}+\frac{135\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(105+5\sqrt{505}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(113+5\sqrt{505}\right)}\right)^3}{4\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)}\right)\hat{}\,(1/15)$$

Decimal approximation

1.6438154599659799180239467139113948146360901449111950591292355777

...

1.64381545996…. ≈ $\zeta(2) = \frac{\pi^2}{6}$ = 1.644934 ... (trace of the instanton shape)

The study of this function provides the following representations:

Alternate forms

$$\left(\frac{1}{2}\left(1-\sqrt{5}+\sqrt{2\left(5+\sqrt{5}\right)}\right)+\frac{135\sqrt{338\,881+15\,080\sqrt{505}+4\sqrt{5\left(2\,871\,007\,052+127\,758\,137\sqrt{505}\right)}}}{4\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)}\right)\hat{}\,(1/15)$$

$$1\Bigg/\left(\sqrt[3]{2}\left(\left((\sqrt{5}-1)\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)\right)\Bigg/\left(768-288\sqrt{2}-256\sqrt{5}+96\sqrt{10}+192\sqrt{5-\sqrt{5}}-256\sqrt{2(5-\sqrt{5})}+74925\sqrt{2(21+\sqrt{505})}-14985\sqrt{10(21+\sqrt{505})}-3375\sqrt{202(21+\sqrt{505})}+3375\sqrt{1010(21+\sqrt{505})}-14445\sqrt{2(113+5\sqrt{505})}+14445\sqrt{10(113+5\sqrt{505})}+3375\sqrt{202(113+5\sqrt{505})}-675\sqrt{1010(113+5\sqrt{505})}-576e+192\sqrt{5}\,e+192\sqrt{2(5-\sqrt{5})}\,e+672e^2-224\sqrt{5}\,e^2-224\sqrt{2(5-\sqrt{5})}\,e^2-384\pi+128\sqrt{5}\,\pi+128\sqrt{2(5-\sqrt{5})}\,\pi-480\pi^2+160\sqrt{5}\,\pi^2+160\sqrt{2(5-\sqrt{5})}\,\pi^2\right)\right)\hat{}\,(1/15)\right)$$

All 15th roots of 1 + (sqrt(10 - 2 sqrt(5)) - 2)/(sqrt(5) - 1) + (135 (1/2 sqrt(1/2 (105 + 5 sqrt(505))) + 1/2 sqrt(1/2 (113 + 5 sqrt(505))))^3)/(4 (-8 + 3 sqrt(2) + 6 e - 7 e^2 + 4 π + 5 π^2))

$$\left(1+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}+\frac{135\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(105+5\sqrt{505}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(113+5\sqrt{505}\right)}\right)^3}{4\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)}\right)\hat{}\,(1/15)\,e^0\approx 1.6438$$ (real, principal root)

$$\left(1+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}+\frac{135\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(105+5\sqrt{505}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(113+5\sqrt{505}\right)}\right)^3}{4\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)}\right)^{\wedge}$$

$$(1/15)\,e^{(2i\pi)/15}\approx 1.5017+0.6686\,i$$

$$\left(1+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}+\frac{135\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(105+5\sqrt{505}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(113+5\sqrt{505}\right)}\right)^3}{4\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)}\right)^{\wedge}$$

$$(1/15)\,e^{(4i\pi)/15}\approx 1.0999+1.2216\,i$$

$$\left(1+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}+\frac{135\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(105+5\sqrt{505}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(113+5\sqrt{505}\right)}\right)^3}{4\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)}\right)^{\wedge}$$

$$(1/15)\,e^{(2i\pi)/5}\approx 0.5080+1.5634\,i$$

$$\left(1+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}+\frac{135\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(105+5\sqrt{505}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(113+5\sqrt{505}\right)}\right)^3}{4\left(-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2\right)}\right)^\wedge$$

$$(1/15)\,e^{(8i\pi)/15}\approx -0.17183+1.6348\,i$$

Alternative representations

$$\left(\frac{27}{4}\left(\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right)\right)+1+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)^\wedge(1/15)=$$

$$\left(1+\frac{27}{4}\exp\left(\log_e\left(\frac{5}{-8+6e+4\pi-7e^2+5\pi^2+3\sqrt{2}}\right)\right)\left(\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}\right)^3+\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}\right)^\wedge(1/15)$$

$$\left(\frac{27}{4}\left(\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right)\right)+1+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)\hat{}\,(1/15) =$$

$$\left(1+\frac{27}{4}\exp\left(\log(a)\log_a\left(\frac{5}{-8+6e+4\pi-7e^2+5\pi^2+3\sqrt{2}}\right)\right)\left(\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}\right)^3+\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}\right)\hat{}\,(1/15)$$

$$\left(\frac{27}{4}\left(\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right)\right)+1+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)\hat{}\,(1/15) =$$

$$\left(\frac{27}{4}\left(\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7z^2+4\pi+5\pi^2}\right)\right)\right)+1+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)\hat{}\,(1/15) \quad \text{for } z=e$$

$$\left(\frac{27}{4}\left(\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right)\right)+1+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)\text{^}(1/15)=$$

$$\left(\frac{27}{4}\left(\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7w^a+4\pi+5\pi^2}\right)\right)\right)+1+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)\text{^}(1/15) \quad \text{for } a=\frac{2}{\log(w)}$$

Integral representation

$$\left(\frac{27}{4}\left(\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^3 \exp\left(\log\left(\frac{5}{-8+3\sqrt{2}+6e-7e^2+4\pi+5\pi^2}\right)\right)\right)+1+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)\text{^}(1/15)=$$

$$\left(1+\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}+\frac{27}{4}\exp\left(\int_1^{\frac{5}{-8+6e-7e^2+4\pi+5\pi^2+3\sqrt{2}}}\frac{1}{t}\,dt\right)\left(\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}\right)^3\right)\text{^}(1/15)$$

From:

Cold dark matter protohalo structure around collapse: Lagrangian cosmological perturbation theory versus Vlasov simulations - *Shohei Saga, Atsushi Taruya, and Stéphane Colombi* - arXiv:2111.08836v1 [astro-ph.CO] 16 Nov 2021

We have that:

We explore the structure around shell-crossing time of cold dark matter protohaloes seeded by two or three crossed sine waves of various relative initial amplitudes, by comparing Lagrangian perturbation theory (LPT) up to 10th order to high-resolution cosmological simulations performed with the public Vlasov code `ColDICE`. Accurate analyses of the density, the velocity, and related quantities such as the vorticity are performed by exploiting the fact that `ColDICE` can follow locally the phase-space sheet at the quadratic level. To test LPT predictions beyond shell-crossing, we employ a ballistic approximation, which assumes that the velocity field is frozen just after shell-crossing.

Appendix A: Expressions of the LPT solutions

In this appendix, we present the LPT solutions up to 5th order, which are obtained by solving the recursion relations given in Eqs. (21) and (22). Since higher-order solutions are straightforwardly derived in the same way, we do not explicitly show them here.

From the development of the following equations:

$$\Psi_x^{(1)} = \frac{\epsilon_x}{2\pi} \sin(2\pi\, q_x)\,, \tag{A.2}$$

$$\Psi_x^{(2)} = -\frac{3\epsilon_x^2}{28\pi} \left[\epsilon_1 \cos(2\pi\, q_y) + \epsilon_2 \cos(2\pi\, q_z)\right] \sin(2\pi\, q_x)\,, \tag{A.3}$$

$$\begin{aligned}\Psi_x^{(3)} = \frac{\epsilon_x^3}{2520\pi}\Big[&78\cos(2\pi q_x)(\epsilon_1\cos(2\pi q_y) + \epsilon_2\cos(2\pi q_z))\\ &+ 160\epsilon_1\epsilon_2\cos(2\pi q_y)\cos(2\pi q_z) - 3\epsilon_1^2\cos(4\pi q_y) - 3\epsilon_2^2\cos(4\pi q_z) + 75\left(\epsilon_1^2+\epsilon_2^2\right)\Big]\sin(2\pi\, q_x)\,,\end{aligned} \tag{A.4}$$

$$\begin{aligned}\Psi_x^{(4)} = -\frac{\epsilon_x^4}{7761600\pi}\Big[&\epsilon_2\cos(2\pi q_z)\left(4242\cos(4\pi q_x) + 28550\epsilon_1^2\cos(4\pi q_y) + 208850\epsilon_1^2 + 57015\epsilon_2^2 + 89166\right)\\ &+ 60\cos(2\pi q_x)\left(6010\epsilon_1\epsilon_2\cos(2\pi q_y)\cos(2\pi q_z) + 1274\epsilon_1^2\cos(4\pi q_y) + 1274\epsilon_2^2\cos(4\pi q_z) + 2039\left(\epsilon_1^2+\epsilon_2^2\right)\right)\\ &+ 2\epsilon_1\cos(2\pi q_y)\left(2121\cos(4\pi q_x) - 9303\epsilon_1^2\cos(4\pi q_y) + 14275\epsilon_2^2\cos(4\pi q_z) + 33159\epsilon_1^2 + 104425\epsilon_2^2 + 44583\right)\\ &- 9303\epsilon_2^3\cos(6\pi q_z)\Big]\sin(2\pi\, q_x)\,,\end{aligned} \tag{A.5}$$

$$\begin{aligned}\Psi_x^{(5)} = \frac{\epsilon_x^5}{36793476720000\pi}\Big[&895050\cos(4\pi q_x)\left(181296\epsilon_1\epsilon_2\cos(2\pi q_y)\cos(2\pi q_z) + 41657\left(\epsilon_1^2+\epsilon_2^2\right)\right)\\ &+ 560226137900\epsilon_1^2\epsilon_2\cos(2\pi q_x)\cos(4\pi q_y)\cos(2\pi q_z) + 560226137900\epsilon_1\epsilon_2^2\cos(2\pi q_x)\cos(2\pi q_y)\cos(4\pi q_z)\\ &+ 594423768510\epsilon_1^3\cos(2\pi q_x)\cos(2\pi q_y) - 18642271230\epsilon_1^3\cos(2\pi q_x)\cos(6\pi q_y)\\ &+ 57401651835\epsilon_1^2\cos(4\pi(q_x-q_y)) + 57401651835\epsilon_1^2\cos(4\pi(q_x+q_y)) + 1006179766020\epsilon_1\epsilon_2^2\cos(2\pi q_x)\cos(2\pi q_y)\\ &+ 106526974554\epsilon_1\cos(2\pi q_x)\cos(2\pi q_y) - 6828912090\epsilon_1\cos(6\pi q_x)\cos(2\pi q_y) + 1006179766020\epsilon_1^2\epsilon_2\cos(2\pi q_x)\cos(2\pi q_z)\\ &+ 57401651835\epsilon_2^2\cos(4\pi(q_x-q_z)) + 594423768510\epsilon_2^3\cos(2\pi q_x)\cos(2\pi q_z) - 18642271230\epsilon_2^3\cos(2\pi q_x)\cos(6\pi q_z)\\ &+ 57401651835\epsilon_2^2\cos(4\pi(q_x+q_z)) + 106526974554\epsilon_2\cos(2\pi q_x)\cos(2\pi q_z) - 6828912090\epsilon_2\cos(6\pi q_x)\cos(2\pi q_z)\\ &+ 46659033850\epsilon_1^2\epsilon_2^2\cos(4\pi(q_y-q_z)) + 46659033850\epsilon_1^2\epsilon_2^2\cos(4\pi(q_y+q_z)) + 503168866200\epsilon_1^3\epsilon_2\cos(2\pi q_y)\cos(2\pi q_z)\\ &- 54358176600\epsilon_1^3\epsilon_2\cos(6\pi q_y)\cos(2\pi q_z) + 503168866200\epsilon_1\epsilon_2^3\cos(2\pi q_y)\cos(2\pi q_z) - 54358176600\epsilon_1\epsilon_2^3\cos(2\pi q_y)\cos(6\pi q_z)\\ &+ 877320054000\epsilon_1\epsilon_2\cos(2\pi q_y)\cos(2\pi q_z) + 115709716980\epsilon_1^2\epsilon_2^2\cos(4\pi q_y) - 35064073044\epsilon_1^4\cos(4\pi q_y)\\ &- 1607701095\epsilon_1^4\cos(8\pi q_y) + 241408089450\epsilon_1^2\cos(4\pi q_y) + 115709716980\epsilon_1^2\epsilon_2^2\cos(4\pi q_z) - 35064073044\epsilon_2^4\cos(4\pi q_z)\\ &+ 241408089450\epsilon_2^2\cos(4\pi q_z) - 1607701095\epsilon_2^4\cos(8\pi q_z)\\ &+ 546975\left(\epsilon_1^2\left(940700\epsilon_2^2 + 443058\right) + 171045\epsilon_1^4 + 21\epsilon_2^2\left(8145\epsilon_2^2 + 21098\right)\right)\Big]\sin(2\pi\, q_x)\,.\end{aligned} \tag{A.6}$$

Considering $q_x = q_y = 0.04$ and $q_z = 0$, we obtain:

$$\Psi_x^{(1)} = \frac{\epsilon_x}{2\pi}\sin(2\pi\, q_x)\,,$$
$$\Psi_x^{(2)} = -\frac{3\epsilon_x^2}{28\pi}\left[\epsilon_1\cos(2\pi\, q_y) + \epsilon_2\cos(2\pi\, q_z)\right]\sin(2\pi\, q_x)\,,$$

1/(2Pi) sin(2Pi*0.04)

Input

$$\frac{1}{2\pi}\sin(2\pi\times 0.04)$$

Result

0.0395802248392523361093178871333609356442141070079176777903128940
...

The study of this function provides the following representations:

Alternative representations

$$\frac{\sin(2\pi\, 0.04)}{2\pi} = \frac{\cos(0.42\,\pi)}{2\pi}$$

$$\frac{\sin(2\pi\, 0.04)}{2\pi} = -\frac{\cos(0.58\,\pi)}{2\pi}$$

$$\frac{\sin(2\pi\, 0.04)}{2\pi} = \frac{\cosh(0.42\,i\,\pi)}{2\pi}$$

Series representations

$$\frac{\sin(2\pi\, 0.04)}{2\pi} = \frac{\sum_{k=0}^{\infty} \frac{(-1)^k\, 0.08^{1+2k}\, \pi^{1+2k}}{(1+2k)!}}{2\pi}$$

$$\frac{\sin(2\pi\, 0.04)}{2\pi} = \frac{\sum_{k=0}^{\infty} (-1)^k\, J_{1+2k}(0.08\,\pi)}{\pi}$$

$$\frac{\sin(2\pi\, 0.04)}{2\pi} = \sum_{k=0}^{\infty} \frac{(-1)^k\, (0.04 + k)\, ((0.08)_k)^3}{(k!)^3}$$

Integral representations

$$\frac{\sin(2\pi\, 0.04)}{2\pi} = 0.04 \int_0^1 \cos(0.08\,\pi\, t)\, dt$$

$$\frac{\sin(2\pi\, 0.04)}{2\pi} = \frac{0.01\sqrt{\pi}}{i\,\pi} \int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{e^{-(0.0016\,\pi^2)/s+s}}{s^{3/2}}\, ds \quad \text{for } \gamma > 0$$

$$\frac{\sin(2\pi\, 0.04)}{2\pi} = \frac{0.01\sqrt{\pi}}{i\,\pi^2} \int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{e^{6.43775\, s}\, \pi^{1-2s}\, \Gamma(s)}{\Gamma\left(\frac{3}{2} - s\right)}\, ds \quad \text{for } 0 < \gamma < 1$$

Multiple-argument formulas

$$\frac{\sin(2\pi\, 0.04)}{2\pi} = \frac{\cos(0.04\,\pi)\sin(0.04\,\pi)}{\pi}$$

$$\frac{\sin(2\pi\, 0.04)}{2\pi} = \frac{3\sin(0.0266667\pi) - 4\sin^3(0.0266667\pi)}{2\pi}$$

$$\frac{\sin(2\pi\, 0.04)}{2\pi} = \frac{U_{-0.92}(\cos(\pi))\sin(\pi)}{2\pi}$$

Inverting:

1/((1/(2Pi) sin(2Pi*0.04)))

Input

$$\frac{1}{\frac{1}{2\pi}\sin(2\pi\times 0.04)}$$

Result

25.2651…

The study of this function provides the following representations:

Alternative representations

$$\frac{1}{\frac{\sin(2\pi\, 0.04)}{2\pi}} = \frac{1}{\frac{\cos(0.42\pi)}{2\pi}}$$

$$\frac{1}{\frac{\sin(2\pi\, 0.04)}{2\pi}} = -\frac{1}{\frac{\cos(0.58\pi)}{2\pi}}$$

$$\frac{1}{\frac{\sin(2\pi\, 0.04)}{2\pi}} = \frac{1}{\frac{\cosh(0.42\, i\, \pi)}{2\pi}}$$

Series representations

$$\frac{1}{\frac{\sin(2\pi\, 0.04)}{2\pi}} = \frac{\pi}{\sum_{k=0}^{\infty} (-1)^k J_{1+2k}(0.08\,\pi)}$$

$$\frac{1}{\frac{\sin(2\pi\, 0.04)}{2\pi}} = \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k (0.04+k)\left((0.08)_k\right)^3}{(k!)^3}}$$

$$\frac{1}{\frac{\sin(2\pi\, 0.04)}{2\pi}} = \frac{2\pi}{\sum_{k=0}^{\infty} \frac{(-1)^k\, 0.08^{1+2k}\, \pi^{1+2k}}{(1+2k)!}}$$

Integral representations

$$\frac{1}{\frac{\sin(2\pi\, 0.04)}{2\pi}} = \frac{25}{\int_0^1 \cos(0.08\,\pi\, t)\, dt}$$

$$\frac{1}{\frac{\sin(2\pi\, 0.04)}{2\pi}} = \frac{100\, i\, \pi}{\sqrt{\pi}\, \int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{e^{-(0.0016\,\pi^2)/s+s}}{s^{3/2}}\, ds} \quad \text{for } \gamma > 0$$

$$\frac{1}{\frac{\sin(2\pi\, 0.04)}{2\pi}} = \frac{100.\, i\, \pi^2}{\sqrt{\pi}\, \int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{e^{6.43775\, s}\, \pi^{1-2s}\, \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\, ds} \quad \text{for } 0 < \gamma < 1$$

Multiple-argument formulas

$$\frac{1}{\frac{\sin(2\pi\, 0.04)}{2\pi}} = \frac{\pi}{\cos(0.04\,\pi)\sin(0.04\,\pi)}$$

$$\frac{1}{\frac{\sin(2\pi\,0.04)}{2\pi}} = \frac{2\pi}{3\sin(0.0266667\,\pi) - 4\sin^3(0.0266667\,\pi)}$$

$$\frac{1}{\frac{\sin(2\pi\,0.04)}{2\pi}} = \frac{2\pi}{U_{-0.92}(\cos(\pi))\sin(\pi)}$$

-3/(28Pi) [cos(2Pi*0.04)+cos(2Pi*0)] sin(2Pi*0.04)

Input

$$-\frac{3}{28\pi}(\cos(2\pi\times 0.04) + \cos(2\pi\times 0))\sin(2\pi\times 0.04)$$

Result

$$-0.016696492314050856284027366878697971543119633474674909992624718\ldots$$

The study of this function provides the following representations:

Alternative representations

$$\frac{((\cos(2\pi\,0.04) + \cos(2\pi\,0))\sin(2\pi\,0.04))(-3)}{28\pi} = -\frac{3(\cosh(0) + \cosh(0.08\,i\,\pi))\cos(0.42\,\pi)}{28\pi}$$

$$\frac{((\cos(2\pi\,0.04)+\cos(2\pi\,0))\sin(2\pi\,0.04))(-3)}{28\pi} = -\frac{3\cos(0.42\pi)\left(1+\frac{1}{2}\left(e^{-0.08\,i\,\pi}+e^{0.08\,i\,\pi}\right)\right)}{28\pi}$$

$$\frac{((\cos(2\pi\,0.04)+\cos(2\pi\,0))\sin(2\pi\,0.04))(-3)}{28\pi} = \frac{3\cos(0.58\pi)\left(1+\frac{1}{2}\left(e^{-0.08\,i\,\pi}+e^{0.08\,i\,\pi}\right)\right)}{28\pi}$$

Series representations

$$\frac{((\cos(2\pi\,0.04)+\cos(2\pi\,0))\sin(2\pi\,0.04))(-3)}{28\pi} = -\frac{1}{14\pi}\,3\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}(-1)^{k_1}\,J_{1+2k_1}(0.08\pi)\left(\frac{(-1)^{k_2}\,0.42^{1+2k_2}\,(-\pi)^{2k_2}\,\pi}{(1+2k_2)!}+\frac{(-1)^{k_2}\,2^{-1-2k_2}\,(-\pi)^{2k_2}\,\pi}{(1+2k_2)!}\right)$$

$$\frac{((\cos(2\pi\,0.04)+\cos(2\pi\,0))\sin(2\pi\,0.04))(-3)}{28\pi} = -\frac{3\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_2}\,0.08^{1+2k_2}\,\pi^{1+2k_2}\left(\frac{\cos\left(\frac{\pi k_1}{2}+z_0\right)(0.08\pi-z_0)^{k_1}}{k_1!}+\frac{\cos\left(\frac{\pi k_1}{2}+z_0\right)(-z_0)^{k_1}}{k_1!}\right)}{(1+2k_2)!}}{28\pi}$$

$$\frac{((\cos(2\pi\,0.04)+\cos(2\pi\,0))\sin(2\pi\,0.04))(-3)}{28\pi} = -\frac{1}{14\pi}\,3\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}(-1)^{k_1}\,J_{1+2k_1}(0.08\pi)\left(\frac{\cos\left(\frac{\pi k_2}{2}+z_0\right)(0.08\pi-z_0)^{k_2}}{k_2!}+\frac{\cos\left(\frac{\pi k_2}{2}+z_0\right)(-z_0)^{k_2}}{k_2!}\right)$$

$J_n(z)$ is the Bessel function of the first kind

Integral representations

$$\frac{((\cos(2\pi\, 0.04) + \cos(2\pi\, 0)) \sin(2\pi\, 0.04)) (-3)}{28\pi} = 0.000685714 \left(\int_0^1 \cos(0.08\pi\, t)\, dt\right)\left(-25. + \pi \int_0^1 \sin(0.08\pi\, t)\, dt\right)$$

$$\frac{((\cos(2\pi\, 0.04) + \cos(2\pi\, 0)) \sin(2\pi\, 0.04)) (-3)}{28\pi} = -\frac{0.00428571 \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^s \left(1+e^{-(0.0016\pi^2)/s}\right)}{\sqrt{s}}\, ds\right)\left(\int_0^1 \cos(0.08\pi\, t)\, dt\right)\sqrt{\pi}}{i\pi} \quad \text{for } \gamma > 0$$

$$\frac{((\cos(2\pi\, 0.04) + \cos(2\pi\, 0)) \sin(2\pi\, 0.04)) (-3)}{28\pi} = 0.00857143 \left(\int_0^1 \cos(0.08\pi\, t)\, dt\right)\left(\int_{\frac{\pi}{2}}^0 \sin(t)\, dt + \int_{\frac{\pi}{2}}^{0.08\pi} \sin(t)\, dt\right)$$

Multiple-argument formulas

$$\frac{((\cos(2\pi\, 0.04) + \cos(2\pi\, 0)) \sin(2\pi\, 0.04)) (-3)}{28\pi} = \frac{3\cos(0.04\pi)\sin(0.04\pi)\left(-1 + \sin^2(0) + \sin^2(0.04\pi)\right)}{7\pi}$$

$$\frac{((\cos(2\pi\, 0.04) + \cos(2\pi\, 0)) \sin(2\pi\, 0.04)) (-3)}{28\pi} = -\frac{3\cos(0.04\pi)\left(-1 + \cos^2(0) + \cos^2(0.04\pi)\right)\sin(0.04\pi)}{7\pi}$$

$$\frac{((\cos(2\pi\, 0.04) + \cos(2\pi\, 0)) \sin(2\pi\, 0.04)) (-3)}{28\pi} = \frac{3\left(-1 + \cos^2(0) + \cos^2(0.04\pi)\right)\sin(0.0266667\pi)\left(-3 + 4\sin^2(0.0266667\pi)\right)}{14\pi}$$

Inverting:

1/(((-3/(28Pi) [cos(2Pi*0.04)+cos(2Pi*0)] sin(2Pi*0.04))))

Input

$$\frac{1}{-\frac{3}{28\pi}(\cos(2\pi\times 0.04)+\cos(2\pi\times 0))\sin(2\pi\times 0.04)}$$

Result

$-59.8928\ldots$

The study of this function provides the following representations:

Alternative representations

$$\frac{1}{\frac{((\cos(2\pi\,0.04)+\cos(2\pi\,0))\sin(2\pi\,0.04))(-3)}{28\pi}} = -\frac{1}{\frac{3(\cosh(0)+\cosh(0.08\,i\,\pi))\cos(0.42\,\pi)}{28\pi}}$$

$$\frac{1}{\frac{((\cos(2\pi\,0.04)+\cos(2\pi\,0))\sin(2\pi\,0.04))(-3)}{28\pi}} = -\frac{1}{\frac{3(\cosh(0)+\cosh((-0.08\,i)\,\pi))\cos(0.42\,\pi)}{28\pi}}$$

$$\frac{1}{\frac{((\cos(2\pi\,0.04)+\cos(2\pi\,0))\sin(2\pi\,0.04))(-3)}{28\pi}} = \frac{1}{\frac{3(\cosh(0)+\cosh((-0.08\,i)\,\pi))\cos(0.58\,\pi)}{28\pi}}$$

Series representations

$$\frac{1}{\frac{((\cos(2\pi\,0.04)+\cos(2\pi\,0))\sin(2\pi\,0.04))(-3)}{28\pi}} = -\frac{14\pi}{3\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}(0.08\,\pi)\right)\sum_{k=0}^{\infty}\frac{0.42\left(-\frac{1}{4}\right)^k\left(1.19048+e^{-0.348707\,k}\right)(-\pi)^{2k}\,\pi}{(1+2k)!}}$$

$$\frac{1}{\frac{((\cos(2\pi\,0.04)+\cos(2\pi\,0)) \sin(2\pi\,0.04))\,(-3)}{28\pi}} = -\frac{14\pi}{3\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}(0.08\pi)\right)\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)\left((0.08\pi-z_0)^k+(-z_0)^k\right)}{k!}}$$

$$\frac{1}{\frac{((\cos(2\pi\,0.04)+\cos(2\pi\,0)) \sin(2\pi\,0.04))\,(-3)}{28\pi}} = -\frac{28\pi}{3\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,0.08^{1+2k}\,\pi^{1+2k}}{(1+2k)!}\right)\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)\left((0.08\pi-z_0)^k+(-z_0)^k\right)}{k!}}$$

$J_n(z)$ is the Bessel function of the first kind

Integral representations

$$\frac{1}{\frac{((\cos(2\pi\,0.04)+\cos(2\pi\,0)) \sin(2\pi\,0.04))\,(-3)}{28\pi}} = \frac{116.667}{\left(\int_0^1 \cos(0.08\pi t)\,dt\right)\int_{\frac{\pi}{2}}^{0}(0.84\sin(0.08\pi+0.84t)+\sin(t))\,dt}$$

$$\frac{1}{\frac{((\cos(2\pi\,0.04)+\cos(2\pi\,0)) \sin(2\pi\,0.04))\,(-3)}{28\pi}} = -\frac{233.333\,i\,\pi}{\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{s}\left(1+e^{-(0.0016\pi^2)/s}\right)}{\sqrt{s}}\,ds\right)\left(\int_0^1\cos(0.08\pi t)\,dt\right)\sqrt{\pi}} \quad \text{for } \gamma > 0$$

$$\frac{1}{\frac{((\cos(2\pi\,0.04)+\cos(2\pi\,0)) \sin(2\pi\,0.04))\,(-3)}{28\pi}} = \frac{5833.33\,i\,\pi}{\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-(0.0016\pi^2)/s+s}}{s^{3/2}}\,ds\right)\left(-25+\pi\int_0^1\sin(0.08\pi t)\,dt\right)\sqrt{\pi}} \quad \text{for } \gamma > 0$$

Multiple-argument formulas

$$\frac{1}{\frac{((\cos(2\pi\,0.04)+\cos(2\pi\,0))\sin(2\pi\,0.04))(-3)}{28\pi}} = \frac{7\pi}{3\cos(0.04\,\pi)\sin(0.04\,\pi)\left(-1+\sin^2(0)+\sin^2(0.04\,\pi)\right)}$$

$$\frac{1}{\frac{((\cos(2\pi\,0.04)+\cos(2\pi\,0))\sin(2\pi\,0.04))(-3)}{28\pi}} = -\frac{7\pi}{3\cos(0.04\,\pi)\left(-1+\cos^2(0)+\cos^2(0.04\,\pi)\right)\sin(0.04\,\pi)}$$

$$\frac{1}{\frac{((\cos(2\pi\,0.04)+\cos(2\pi\,0))\sin(2\pi\,0.04))(-3)}{28\pi}} = \frac{14\pi}{3\left(-1+\cos^2(0)+\cos^2(0.04\,\pi)\right)\sin(0.0266667\,\pi)\left(-3+4\sin^2(0.0266667\,\pi)\right)}$$

From:

$$\Psi_x^{(3)} = \frac{\epsilon_x^3}{2520\pi}\Big[78\cos(2\pi q_x)(\epsilon_1\cos(2\pi q_y)+\epsilon_2\cos(2\pi q_z)) + 160\epsilon_1\epsilon_2\cos(2\pi q_y)\cos(2\pi q_z) - 3\epsilon_1^2\cos(4\pi q_y) - 3\epsilon_2^2\cos(4\pi q_z) + 75\left(\epsilon_1^2+\epsilon_2^2\right)\Big]\sin(2\pi\,q_x)\,,$$

1/(2520Pi) ((((78 cos(2Pi*0.04)(cos(2Pi*0.04)+1*cos(2Pi*0))+160 cos(2Pi*0.04) cos(2Pi*0)-3 cos(4Pi*0.04) – 3 cos(4Pi*0) + 75(1+1)))))*sin(2Pi*0.04)

Input

$$\frac{1}{2520\pi}(78\cos(2\pi\times 0.04)(\cos(2\pi\times 0.04)+1\cos(2\pi\times 0)) + 160\cos(2\pi\times 0.04)\cos(2\pi\times 0) - 3\cos(4\pi\times 0.04) - 3\cos(4\pi\times 0) + 75(1+1))\sin(2\pi\times 0.04)$$

Result

0.0140751624818295702740279573547741308366570105750668907084172267

...

The study of this function provides the following representations:

Alternative representations

$$\frac{1}{2520\pi}(78\cos(2\pi\,0.04)(\cos(2\pi\,0.04)+1\cos(2\pi\,0))+160\cos(2\pi\,0.04)\cos(2\pi\,0)-3\cos(4\pi\,0.04)-3\cos(4\pi\,0)+75(1+1))\sin(2\pi\,0.04)=\frac{1}{2520\pi}(150-3\cosh(0)+160\cosh(0)\cosh(0.08\,i\,\pi)+78\cosh(0.08\,i\,\pi)(\cosh(0)+\cosh(0.08\,i\,\pi))-3\cosh(0.16\,i\,\pi))\cos(0.42\,\pi)$$

$$\frac{1}{2520\pi}(78\cos(2\pi\,0.04)(\cos(2\pi\,0.04)+1\cos(2\pi\,0))+160\cos(2\pi\,0.04)\cos(2\pi\,0)-3\cos(4\pi\,0.04)-3\cos(4\pi\,0)+75(1+1))\sin(2\pi\,0.04)=\frac{1}{2520\pi}(150-3\cosh(0)+160\cosh(0)\cosh((-0.08\,i)\pi)+78\cosh((-0.08\,i)\pi)(\cosh(0)+\cosh((-0.08\,i)\pi))-3\cosh((-0.16\,i)\pi))\cos(0.42\,\pi)$$

$$\frac{1}{2520\pi}(78\cos(2\pi\,0.04)(\cos(2\pi\,0.04)+1\cos(2\pi\,0))+160\cos(2\pi\,0.04)\cos(2\pi\,0)-3\cos(4\pi\,0.04)-3\cos(4\pi\,0)+75(1+1))\sin(2\pi\,0.04)=-\frac{1}{2520\pi}(150-3\cosh(0)+160\cosh(0)\cosh((-0.08\,i)\pi)+78\cosh((-0.08\,i)\pi)(\cosh(0)+\cosh((-0.08\,i)\pi))-3\cosh((-0.16\,i)\pi))\cos(0.58\,\pi)$$

Series representations

$$\frac{1}{2520\pi}(78\cos(2\pi\,0.04)(\cos(2\pi\,0.04)+1\cos(2\pi\,0))+160\cos(2\pi\,0.04)\cos(2\pi\,0)-3\cos(4\pi\,0.04)-3\cos(4\pi\,0)+75(1+1))\sin(2\pi\,0.04)=-\frac{1}{1260\pi}\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}(0.08\pi)\right)\left(-150-78\left(\sum_{k=0}^{\infty}-\frac{0.42(-1)^k e^{-1.735k}(-\pi)^{2k}\pi}{(1+2k)!}\right)^2+3\sum_{k=0}^{\infty}\frac{0.34\left(-\frac{1}{4}\right)^k\left(1.47059+e^{-0.771325k}\right)(-\pi)^{2k}\pi}{(1+2k)!}-238\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\,0.42^{1+2k_1}\times 2^{-1-2k_2}(-\pi)^{2+2k_1+2k_2}}{(1+2k_1)!(1+2k_2)!}\right)$$

$$\frac{1}{2520\pi}(78\cos(2\pi\,0.04)(\cos(2\pi\,0.04)+1\cos(2\pi\,0))+160\cos(2\pi\,0.04)\cos(2\pi\,0)-3\cos(4\pi\,0.04)-3\cos(4\pi\,0)+75(1+1))\sin(2\pi\,0.04)=-\frac{1}{2520\pi}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,0.08^{1+2k}\pi^{1+2k}}{(1+2k)!}\right)\left(-150+3\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)\left((0.16\pi-z_0)^k+(-z_0)^k\right)}{k!}-78\left(\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)(0.08\pi-z_0)^k}{k!}\right)^2-238\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{\cos\left(\frac{\pi k_1}{2}+z_0\right)\cos\left(\frac{\pi k_2}{2}+z_0\right)(0.08\pi-z_0)^{k_1}(-z_0)^{k_2}}{k_1!\,k_2!}\right)$$

$$\frac{1}{2520\,\pi}(78\cos(2\,\pi\,0.04)\,(\cos(2\,\pi\,0.04)+1\cos(2\,\pi\,0))+160\cos(2\,\pi\,0.04)\cos(2\,\pi\,0)-3\cos(4\,\pi\,0.04)-3\cos(4\,\pi\,0)+75\,(1+1))\sin(2\,\pi\,0.04)=-\frac{1}{1260\,\pi}\left(\sum_{k=0}^{\infty}(-1)^k\,J_{1+2\,k}(0.08\,\pi)\right)\left(-150+3\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\,\pi}{2}+z_0\right)\left((0.16\,\pi-z_0)^k+(-z_0)^k\right)}{k!}-78\left(\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\,\pi}{2}+z_0\right)(0.08\,\pi-z_0)^k}{k!}\right)^2-238\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{\cos\left(\frac{\pi\,k_1}{2}+z_0\right)\cos\left(\frac{\pi\,k_2}{2}+z_0\right)(0.08\,\pi-z_0)^{k_1}\,(-z_0)^{k_2}}{k_1!\,k_2!}\right)$$

$J_n(z)$ is the Bessel function of the first kind

Integral representations

$$\frac{1}{2520\,\pi}(78\cos(2\,\pi\,0.04)\,(\cos(2\,\pi\,0.04)+1\cos(2\,\pi\,0))+160\cos(2\,\pi\,0.04)\cos(2\,\pi\,0)-3\cos(4\,\pi\,0.04)-3\cos(4\,\pi\,0)+75\,(1+1))\sin(2\,\pi\,0.04)=0.0000158476\left(\int_0^1\cos(0.08\,\pi\,t)\,dt\right)\left(921.474-63.141\,\pi\int_0^1\sin(0.08\,\pi\,t)\,dt+\pi^2\left(\int_0^1\sin(0.08\,\pi\,t)\,dt\right)^2+0.961538\,\pi\int_0^1\sin(0.16\,\pi\,t)\,dt\right)$$

$$\frac{1}{2520\,\pi}(78\cos(2\,\pi\,0.04)\,(\cos(2\,\pi\,0.04)+1\cos(2\,\pi\,0))+160\cos(2\,\pi\,0.04)\cos(2\,\pi\,0)-3\cos(4\,\pi\,0.04)-3\cos(4\,\pi\,0)+75\,(1+1))\sin(2\,\pi\,0.04)=0.00755556\left(\int_0^1\cos(0.08\,\pi\,t)\,dt\right)\left(0.630252+0.012605\int_{\frac{\pi}{2}}^{0}\sin(t)\,dt+0.327731\left(\int_{\frac{\pi}{2}}^{0.08\,\pi}\sin(t)\,dt\right)^2+0.012605\int_{\frac{\pi}{2}}^{0.16\,\pi}\sin(t)\,dt+\int_0^1\int_0^1\sin\left(\frac{1}{2}\,(\pi-\pi\,t_1)\right)\sin(\pi\,(0.5-0.42\,t_2))\,dt_2\,dt_1\right)$$

$$\frac{1}{2520\pi}(78\cos(2\pi\,0.04)(\cos(2\pi\,0.04)+1\cos(2\pi\,0))+160\cos(2\pi\,0.04)\cos(2\pi\,0)-3\cos(4\pi\,0.04)-3\cos(4\pi\,0)+75(1+1))\sin(2\pi\,0.04)=$$
$$\frac{1}{i\pi}3.9619\times10^{-6}\left(921.474\sqrt{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-(0.0016\pi^2)/s+s}}{s^{3/2}}\,ds+\pi^2\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-(0.0016\pi^2)/s+s}}{s^{3/2}}\,ds\right)\left(\int_0^1\sin(0.08\pi t)\,dt\right)^2\sqrt{\pi}+\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-(0.0016\pi^2)/s+s}}{s^{3/2}}\,ds\right)\left(\int_0^1(-63.141\pi\sin(0.08\pi t)+0.961538\pi\sin(0.16\pi t))\,dt\right)\sqrt{\pi}\right)\text{ for }\gamma>0$$

Multiple-argument formulas

$$\frac{1}{2520\pi}(78\cos(2\pi\,0.04)(\cos(2\pi\,0.04)+1\cos(2\pi\,0))+160\cos(2\pi\,0.04)\cos(2\pi\,0)-3\cos(4\pi\,0.04)-3\cos(4\pi\,0)+75(1+1))\sin(2\pi\,0.04)=\frac{1}{630\pi}\cos(0.04\pi)\left(236-241\cos^2(0)+\left(-394+476\cos^2(0)\right)\cos^2(0.04\pi)+156\cos^4(0.04\pi)-3\cos^2(0.08\pi)\right)\sin(0.04\pi)$$

$$\frac{1}{2520\pi}(78\cos(2\pi\,0.04)(\cos(2\pi\,0.04)+1\cos(2\pi\,0))+160\cos(2\pi\,0.04)\cos(2\pi\,0)-3\cos(4\pi\,0.04)-3\cos(4\pi\,0)+75(1+1))\sin(2\pi\,0.04)=\frac{1}{630\pi}\cos(0.04\pi)\sin(0.04\pi)\left(230-235\sin^2(0)+\left(-394+476\sin^2(0)\right)\sin^2(0.04\pi)+156\sin^4(0.04\pi)+3\sin^2(0.08\pi)\right)$$

$$\frac{1}{2520\pi}(78\cos(2\pi\,0.04)(\cos(2\pi\,0.04)+1\cos(2\pi\,0))+160\cos(2\pi\,0.04)\cos(2\pi\,0)-3\cos(4\pi\,0.04)-3\cos(4\pi\,0)+75(1+1))\sin(2\pi\,0.04)=-\frac{1}{1260\pi}\left(236-241\cos^2(0)+\left(-394+476\cos^2(0)\right)\cos^2(0.04\pi)+156\cos^4(0.04\pi)-3\cos^2(0.08\pi)\right)\sin(0.0266667\pi)\left(-3+4\sin^2(0.0266667\pi)\right)$$

Inverting:

1/(((1/(2520Pi) ((((78 cos(2Pi*0.04)(cos(2Pi*0.04)+1*cos(2Pi*0))+160 cos(2Pi*0.04) cos(2Pi*0)-3 cos(4Pi*0.04) – 3 cos(4Pi*0) + 75(1+1)))))*sin(2Pi*0.04))))

Input

$$1\Big/\left(\frac{1}{2520\,\pi}\,(78\cos(2\,\pi\times 0.04)\,(\cos(2\,\pi\times 0.04)+1\cos(2\,\pi\times 0))+160\cos(2\,\pi\times 0.04)\cos(2\,\pi\times 0)-3\cos(4\,\pi\times 0.04)-3\cos(4\,\pi\times 0)+75\,(1+1))\sin(2\,\pi\times 0.04)\right)$$

Result

71.047137202924443236604373661613303165969547660528979188627646475
…

The study of this function provides the following representations:

Alternative representations

$$1\Big/\frac{1}{2520\,\pi}\,(78\cos(2\,\pi\,0.04)\,(\cos(2\,\pi\,0.04)+1\cos(2\,\pi\,0))+160\cos(2\,\pi\,0.04)\cos(2\,\pi\,0)-3\cos(4\,\pi\,0.04)-3\cos(4\,\pi\,0)+75\,(1+1))\sin(2\,\pi\,0.04)=1\Big/\frac{1}{2520\,\pi}(150-3\cosh(0)+160\cosh(0)\cosh(0.08\,i\,\pi)+78\cosh(0.08\,i\,\pi)\,(\cosh(0)+\cosh(0.08\,i\,\pi))-3\cosh(0.16\,i\,\pi))\cos(0.42\,\pi)$$

$$1\Big/\frac{1}{2520\,\pi}\,(78\cos(2\,\pi\,0.04)\,(\cos(2\,\pi\,0.04)+1\cos(2\,\pi\,0))+160\cos(2\,\pi\,0.04)\cos(2\,\pi\,0)-3\cos(4\,\pi\,0.04)-3\cos(4\,\pi\,0)+75\,(1+1))\sin(2\,\pi\,0.04)=1\Big/\frac{1}{2520\,\pi}(150-3\cosh(0)+160\cosh(0)\cosh((-0.08\,i)\,\pi)+78\cosh((-0.08\,i)\,\pi)\,(\cosh(0)+\cosh((-0.08\,i)\,\pi))-3\cosh((-0.16\,i)\,\pi))\cos(0.42\,\pi)$$

$$1\Big/\frac{1}{2520\,\pi}\,(78\cos(2\,\pi\,0.04)\,(\cos(2\,\pi\,0.04)+1\cos(2\,\pi\,0))+160\cos(2\,\pi\,0.04)\cos(2\,\pi\,0)-3\cos(4\,\pi\,0.04)-3\cos(4\,\pi\,0)+75\,(1+1))\sin(2\,\pi\,0.04)=-\left(1\Big/\frac{1}{2520\,\pi}(150-3\cosh(0)+160\cosh(0)\cosh((-0.08\,i)\,\pi)+78\cosh((-0.08\,i)\,\pi)\,(\cosh(0)+\cosh((-0.08\,i)\,\pi))-3\cosh((-0.16\,i)\,\pi))\cos(0.58\,\pi)\right)$$

Series representations

$$1\Big/\frac{1}{2520\,\pi}\,(78\cos(2\,\pi\,0.04)\,(\cos(2\,\pi\,0.04)+1\cos(2\,\pi\,0))+160\cos(2\,\pi\,0.04)\cos(2\,\pi\,0)-3\cos(4\,\pi\,0.04)-3\cos(4\,\pi\,0)+75\,(1+1))\sin(2\,\pi\,0.04)=-\left((1260\,\pi)\Big/\left(\left(\sum_{k=0}^{\infty}(-1)^k\,J_{1+2k}(0.08\,\pi)\right)\left(-150+3\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)\left((0.16\,\pi-z_0)^k+(-z_0)^k\right)}{k!}-78\left(\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)(0.08\,\pi-z_0)^k}{k!}\right)^2-238\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{\cos\left(\frac{\pi k_1}{2}+z_0\right)\cos\left(\frac{\pi k_2}{2}+z_0\right)(0.08\,\pi-z_0)^{k_1}\,(-z_0)^{k_2}}{k_1!\,k_2!}\right)\right)\right)$$

$$1\Big/\frac{1}{2520\,\pi}\,(78\cos(2\,\pi\,0.04)\,(\cos(2\,\pi\,0.04)+1\cos(2\,\pi\,0))+160\cos(2\,\pi\,0.04)\cos(2\,\pi\,0)-3\cos(4\,\pi\,0.04)-3\cos(4\,\pi\,0)+75\,(1+1))\sin(2\,\pi\,0.04)=-\left((2520\,\pi)\Big/\left(\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,0.08^{1+2k}\,\pi^{1+2k}}{(1+2k)!}\right)\left(-150+3\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)\left((0.16\,\pi-z_0)^k+(-z_0)^k\right)}{k!}-78\left(\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)(0.08\,\pi-z_0)^k}{k!}\right)^2-238\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{\cos\left(\frac{\pi k_1}{2}+z_0\right)\cos\left(\frac{\pi k_2}{2}+z_0\right)(0.08\,\pi-z_0)^{k_1}\,(-z_0)^{k_2}}{k_1!\,k_2!}\right)\right)\right)$$

$$1\Big/\frac{1}{2520\,\pi}\,(78\cos(2\,\pi\,0.04)\,(\cos(2\,\pi\,0.04)+1\cos(2\,\pi\,0))+160\cos(2\,\pi\,0.04)\cos(2\,\pi\,0)-3\cos(4\,\pi\,0.04)-3\cos(4\,\pi\,0)+75\,(1+1))\sin(2\,\pi\,0.04)=$$

$$(1260\,\pi)\Big/\left(\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}(0.08\,\pi)\right)\left(150-3\,\pi\sum_{k=0}^{\infty}\frac{(-1)^k\,0.34^{1+2k}\,(-\pi)^{2k}}{(1+2k)!}+78\,\pi^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,0.42^{1+2k}\,(-\pi)^{2k}}{(1+2k)!}\right)^2-3\,\pi\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{-1-2k}\,(-\pi)^{2k}}{(1+2k)!}+238\,\pi^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\,0.42^{1+2k_1}\times 2^{-1-2k_2}\,(-\pi)^{2k_1+2k_2}}{(1+2k_1)!\,(1+2k_2)!}\right)\right)$$

$J_n(z)$ is the Bessel function of the first kind

Integral representations

$$1\Big/\frac{1}{2520\,\pi}\,(78\cos(2\,\pi\,0.04)\,(\cos(2\,\pi\,0.04)+1\cos(2\,\pi\,0))+160\cos(2\,\pi\,0.04)\cos(2\,\pi\,0)-3\cos(4\,\pi\,0.04)-3\cos(4\,\pi\,0)+75\,(1+1))\sin(2\,\pi\,0.04)=$$

$$(252404.\,i\,\pi)\Big/\left(\left(\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{-(0.0016\,\pi^2)/s+s}}{s^{3/2}}\,ds\right)\left(921.474+\pi^2\left(\int_0^1\sin(0.08\,\pi\,t)\,dt\right)^2+\int_0^1(-63.141\,\pi\sin(0.08\,\pi\,t)+0.961538\,\pi\sin(0.16\,\pi\,t))\,dt\right)\sqrt{\pi}\right)\quad\text{for }\gamma>0$$

$$1\Big/\frac{1}{2520\,\pi}\,(78\cos(2\,\pi\,0.04)\,(\cos(2\,\pi\,0.04)+1\cos(2\,\pi\,0))+160\cos(2\,\pi\,0.04)\cos(2\,\pi\,0)-3\cos(4\,\pi\,0.04)-3\cos(4\,\pi\,0)+75\,(1+1))\sin(2\,\pi\,0.04)=$$

$$\left(252404.\,i\,\pi^2\right)\Big/\left(\left(\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{6.43775\,s}\,\pi^{1-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds\right)\left(921.474+\pi^2\left(\int_0^1\sin(0.08\,\pi\,t)\,dt\right)^2+\int_0^1(-63.141\,\pi\sin(0.08\,\pi\,t)+0.961538\,\pi\sin(0.16\,\pi\,t))\,dt\right)\sqrt{\pi}\right)\quad\text{for }0<\gamma<1$$

$$1 \Big/ \frac{1}{2520\,\pi}$$
$$(78\cos(2\,\pi\,0.04)\,(\cos(2\,\pi\,0.04) + 1\cos(2\,\pi\,0)) + 160\cos(2\,\pi\,0.04)\cos(2\,\pi\,0) - 3\cos(4\,\pi\,0.04) - 3\cos(4\,\pi\,0) + 75\,(1+1))\sin(2\,\pi\,0.04) =$$
$$(1615.38\,i\,\pi) \Bigg/ \left(\left(\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{e^{-(0.0016\,\pi^2)/s+s}}{s^{3/2}}\,ds\right)\left(1.92308 + 0.0384615\int_{\frac{\pi}{2}}^{0}\sin(t)\,dt + \left(\int_{\frac{\pi}{2}}^{0.08\,\pi}\sin(t)\,dt\right)^2 + 0.0384615\int_{\frac{\pi}{2}}^{0.16\,\pi}\sin(t)\,dt + \int_0^1\int_0^1 \sin\left(\frac{1}{2}\,(\pi - \pi\,t_1)\right)\sin(\pi\,(0.5 - 0.42\,t_2))\,dt_2\,dt_1\right)\sqrt{\pi}\right) \text{ for } \gamma > 0$$

Multiple-argument formulas

$$1 \Big/ \frac{1}{2520\,\pi}$$
$$(78\cos(2\,\pi\,0.04)\,(\cos(2\,\pi\,0.04) + 1\cos(2\,\pi\,0)) + 160\cos(2\,\pi\,0.04)\cos(2\,\pi\,0) - 3\cos(4\,\pi\,0.04) - 3\cos(4\,\pi\,0) + 75\,(1+1))\sin(2\,\pi\,0.04) =$$
$$(630\,\pi) \Big/ \left(\cos(0.04\,\pi)\left(236 - 241\cos^2(0) + \left(-394 + 476\cos^2(0)\right)\cos^2(0.04\,\pi) + 156\cos^4(0.04\,\pi) - 3\cos^2(0.08\,\pi)\right)\sin(0.04\,\pi)\right)$$

$$1 \Big/ \frac{1}{2520\,\pi}$$
$$(78\cos(2\,\pi\,0.04)\,(\cos(2\,\pi\,0.04) + 1\cos(2\,\pi\,0)) + 160\cos(2\,\pi\,0.04)\cos(2\,\pi\,0) - 3\cos(4\,\pi\,0.04) - 3\cos(4\,\pi\,0) + 75\,(1+1))\sin(2\,\pi\,0.04) =$$
$$(630\,\pi) \Big/ \left(\cos(0.04\,\pi)\sin(0.04\,\pi)\left(230 - 235\sin^2(0) + \left(-394 + 476\sin^2(0)\right)\sin^2(0.04\,\pi) + 156\sin^4(0.04\,\pi) + 3\sin^2(0.08\,\pi)\right)\right)$$

$$1 \Big/ \frac{1}{2520\,\pi}$$
$$(78\cos(2\,\pi\,0.04)\,(\cos(2\,\pi\,0.04) + 1\cos(2\,\pi\,0)) + 160\cos(2\,\pi\,0.04)\cos(2\,\pi\,0) - 3\cos(4\,\pi\,0.04) - 3\cos(4\,\pi\,0) + 75\,(1+1))\sin(2\,\pi\,0.04) =$$
$$-\left((1260\,\pi) \Big/ \left(\left(236 - 241\cos^2(0) + \left(-394 + 476\cos^2(0)\right)\cos^2(0.04\,\pi) + 156\cos^4(0.04\,\pi) - 3\cos^2(0.08\,\pi)\right)\sin(0.0266667\,\pi)\left(-3 + 4\sin^2(0.0266667\,\pi)\right)\right)\right)$$

From:

$$\Psi_x^{(4)} = -\frac{\epsilon_x^4}{7761600\pi}\Big[\epsilon_2\cos(2\pi q_z)\left(4242\cos(4\pi q_x) + 28550\epsilon_1^2\cos(4\pi q_y) + 208850\epsilon_1^2 + 57015\epsilon_2^2 + 89166\right)$$
$$+ 60\cos(2\pi q_x)\left(6010\epsilon_1\epsilon_2\cos(2\pi q_y)\cos(2\pi q_z) + 1274\epsilon_1^2\cos(4\pi q_y) + 1274\epsilon_2^2\cos(4\pi q_z) + 2039\left(\epsilon_1^2+\epsilon_2^2\right)\right)$$
$$+ 2\epsilon_1\cos(2\pi q_y)\left(2121\cos(4\pi q_x) - 9303\epsilon_1^2\cos(4\pi q_y) + 14275\epsilon_2^2\cos(4\pi q_z) + 33159\epsilon_1^2 + 104425\epsilon_2^2 + 44583\right)$$
$$- 9303\epsilon_2^3\cos(6\pi q_z)\Big]\sin(2\pi\, q_x)\,,$$

-1/(7761600Pi)
[cos(2Pi*0)((4242cos(4Pi*0.04)+28550cos(4Pi*0.04)+208850+57015+89166))+60cos(2Pi*0.04)((6010cos(2Pi*0.04)cos(2Pi*0)+1274cos(4Pi*0.04)+1274cos(4Pi*0)+2039(1+1))+2cos(2Pi*0.04)((2121cos(4Pi*0.04)-9303cos(4Pi*0.04)+14275cos(4Pi*0)+33159+104425+44583))-9303cos(6Pi*0)]sin(2Pi*0.04)

We dividing the expression and perform the calculations.

-1/(7761600Pi)
[383766.85+60cos(2Pi*0.04)(6010cos(2Pi*0.04)+1274cos(4Pi*0.04)+1274+2039(1+1))+2cos(2Pi*0.04)190148.37-9303]sin(2Pi*0.04)

Input interpretation

$$-\frac{1}{7761600\,\pi}(383766.85 + 60\cos(2\,\pi\times 0.04)\,(6010\cos(2\,\pi\times 0.04) + 1274\cos(4\,\pi\times 0.04) + 1274 + 2039\,(1+1)) + 2\cos(2\,\pi\times 0.04)\times 190148.37 - 9303)\sin(2\,\pi\times 0.04)$$

Result

−0.0148601553018639118906767494634126428957707251753945271461016 14...

The study of this function provides the following representations:

Alternative representations

$$\frac{1}{7\,761\,600\,\pi}\left((383\,767. + 60\cos(2\,\pi\,0.04)\,(6010\cos(2\,\pi\,0.04) + 1274\cos(4\,\pi\,0.04) + 1274 + 2039\,(1+1)) + 2\cos(2\,\pi\,0.04)\,190\,148. - 9303)\sin(2\,\pi\,0.04)\right)(-1) = -\frac{1}{7\,761\,600\,\pi}(374\,464. + 380\,297.\cosh(0.08\,i\,\pi) + 60\cosh(0.08\,i\,\pi)\,(5352 + 6010\cosh(0.08\,i\,\pi) + 1274\cosh(0.16\,i\,\pi)))\cos(0.42\,\pi)$$

$$\frac{1}{7\,761\,600\,\pi}\left((383\,767. + 60\cos(2\,\pi\,0.04)\,(6010\cos(2\,\pi\,0.04) + 1274\cos(4\,\pi\,0.04) + 1274 + 2039\,(1+1)) + 2\cos(2\,\pi\,0.04)\,190\,148. - 9303)\sin(2\,\pi\,0.04)\right)(-1) = \frac{1}{7\,761\,600\,\pi}(374\,464. + 380\,297.\cosh((-0.08\,i)\,\pi) + 60\cosh((-0.08\,i)\,\pi)\,(5352 + 6010\cosh((-0.08\,i)\,\pi) + 1274\cosh((-0.16\,i)\,\pi)))\cos(0.58\,\pi)$$

$$\frac{1}{7\,761\,600\,\pi}\left((383\,767. + 60\cos(2\,\pi\,0.04)\,(6010\cos(2\,\pi\,0.04) + 1274\cos(4\,\pi\,0.04) + 1274 + 2039\,(1+1)) + 2\cos(2\,\pi\,0.04)\,190\,148. - 9303)\sin(2\,\pi\,0.04)\right)(-1) = -\frac{1}{7\,761\,600\,\pi}(374\,464. + 380\,297.\cosh((-0.08\,i)\,\pi) + 60\cosh((-0.08\,i)\,\pi)\,(5352 + 6010\cosh((-0.08\,i)\,\pi) + 1274\cosh((-0.16\,i)\,\pi)))\cos(0.42\,\pi)$$

Series representations

$$\frac{1}{7761600\pi}((383767. + 60\cos(2\pi\,0.04)(6010\cos(2\pi\,0.04) + 1274\cos(4\pi\,0.04) + 1274 + 2039(1+1)) + 2\cos(2\pi\,0.04)\,190148. - 9303)\sin(2\pi\,0.04))(-1) = -\frac{1}{7761600\pi}\left(374464. + 380297.\left(J_0(0.08\pi) + 2\sum_{k=1}^{\infty}(-1)^k J_{2k}(0.08\pi)\right) + 60\left(J_0(0.08\pi) + 2\sum_{k=1}^{\infty}(-1)^k J_{2k}(0.08\pi)\right)\left(5352 + 6010\left(J_0(0.08\pi) + 2\sum_{k=1}^{\infty}(-1)^k J_{2k}(0.08\pi)\right) + 1274\left(J_0(0.16\pi) + 2\sum_{k=1}^{\infty}(-1)^k J_{2k}(0.16\pi)\right)\right)\right)\sum_{k=0}^{\infty}\frac{(-1)^k\,0.08^{1+2k}\,\pi^{1+2k}}{(1+2k)!}$$

$$\frac{1}{7761600\pi}((383767. + 60\cos(2\pi\,0.04)(6010\cos(2\pi\,0.04) + 1274\cos(4\pi\,0.04) + 1274 + 2039(1+1)) + 2\cos(2\pi\,0.04)\,190148. - 9303)\sin(2\pi\,0.04))(-1) = -\frac{1}{7761600\pi}\left(374464. + 380297.\left(J_0(0.08\pi) + 2\sum_{k=1}^{\infty}(-1)^k J_{2k}(0.08\pi)\right) + 60\left(J_0(0.08\pi) + 2\sum_{k=1}^{\infty}(-1)^k J_{2k}(0.08\pi)\right)\left(5352 + 6010\left(J_0(0.08\pi) + 2\sum_{k=1}^{\infty}(-1)^k J_{2k}(0.08\pi)\right) + 1274\left(J_0(0.16\pi) + 2\sum_{k=1}^{\infty}(-1)^k J_{2k}(0.16\pi)\right)\right)\right)\sum_{k=0}^{\infty}\frac{(-1)^k\,e^{-1.735k}\,(-\pi)^{2k}}{(2k)!}$$

$$\frac{1}{7761600\,\pi}\left((383767. + 60\cos(2\pi\, 0.04)\,(6010\cos(2\pi\, 0.04) + 1274\cos(4\pi\, 0.04) + 1274 + 2039\,(1+1)) + 2\cos(2\pi\, 0.04)\,190148. - 9303)\sin(2\pi\, 0.04)\right)(-1) = -\frac{1}{\pi}\,0.092919\left(1.03845\sum_{k=0}^{\infty}(-1)^k J_{1+2k}(0.08\pi) + \left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}(0.08\pi)\right)\left(\sum_{k=0}^{\infty}\frac{(-1)^k e^{-5.05146k}\pi^{2k}}{(2k)!}\right)^2 + 1.94514\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2} e^{-5.05146k_2}\pi^{2k_2} J_{1+2k_1}(0.08\pi)}{(2k_2)!} + 0.21198\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\frac{(-1)^{k_1+k_2+k_3} e^{-5.05146k_2-3.66516k_3}\pi^{2k_2+2k_3} J_{1+2k_1}(0.08\pi)}{(2k_2)!\,(2k_3)!}\right)$$

$J_n(z)$ is the Bessel function of the first kind

Multiple-argument formulas

$$\frac{1}{7761600\,\pi}\left((383767. + 60\cos(2\pi\, 0.04)\,(6010\cos(2\pi\, 0.04) + 1274\cos(4\pi\, 0.04) + 1274 + 2039\,(1+1)) + 2\cos(2\pi\, 0.04)\,190148. - 9303)\sin(2\pi\, 0.04)\right)(-1) = -\frac{1}{\pi}\,0.371676\cos(0.04\pi)\left(0.0763222 + \cos^4(0.04\pi) - 0.10599\cos^2(0.08\pi) + \cos^2(0.04\pi)\left(-0.133421 + 0.21198\cos^2(0.08\pi)\right)\right)\sin(0.04\pi)$$

$$\frac{1}{7761600\,\pi}\left((383767. + 60\cos(2\pi\, 0.04)\,(6010\cos(2\pi\, 0.04) + 1274\cos(4\pi\, 0.04) + 1274 + 2039\,(1+1)) + 2\cos(2\pi\, 0.04)\,190148. - 9303)\sin(2\pi\, 0.04)\right)(-1) = -\frac{1}{\pi}\,0.371676\cos(0.04\pi)\sin(0.04\pi)\left(1.04889 + \sin^4(0.04\pi) - 0.10599\sin^2(0.08\pi) + \sin^2(0.04\pi)\left(-2.07856 + 0.21198\sin^2(0.08\pi)\right)\right)$$

$$\frac{1}{7761600\pi}$$
$$((383767. + 60\cos(2\pi\, 0.04)\,(6010\cos(2\pi\, 0.04) + 1274\cos(4\pi\, 0.04) + 1274 + 2039\,(1+1)) + 2\cos(2\pi\, 0.04)\, 190148. - 9303)\sin(2\pi\, 0.04))$$
$$(-1) = \frac{1}{\pi}\, 0.743352\left(0.0763222 + \cos^4(0.04\pi) - 0.10599\cos^2(0.08\pi) + \cos^2(0.04\pi)\left(-0.133421 + 0.21198\cos^2(0.08\pi)\right)\right)\left(-0.75\sin(0.0266667\pi) + \sin^3(0.0266667\pi)\right)$$

Inverting:

1/((-1/(7761600Pi)
[383766.85+60cos(2Pi*0.04)(6010cos(2Pi*0.04)+1274cos(4Pi*0.04)+1274+2039(1+1))+2cos(2Pi*0.04)190148.37-9303]sin(2Pi*0.04)))

Input interpretation

$$-\left(1\Big/\frac{1}{7761600\pi}(383766.85 + 60\cos(2\pi\times 0.04)\,(6010\cos(2\pi\times 0.04) + 1274\cos(4\pi\times 0.04) + 1274 + 2039\,(1+1)) + 2\cos(2\pi\times 0.04)\times 190148.37 - 9303)\sin(2\pi\times 0.04)\right)$$

Result

$-67.29404771931083548529320823775103317161339438008474603081224064\ldots$

The study of this function provides the following representations:

Alternative representations

$$1\Big/\frac{1}{7761600\pi}((383767. + 60\cos(2\pi\, 0.04)\,(6010\cos(2\pi\, 0.04) + 1274\cos(4\pi\, 0.04) + 1274 + 2039\,(1+1)) + 2\cos(2\pi\, 0.04)\, 190148. - 9303)\sin(2\pi\, 0.04))$$
$$(-1) = -\left(1\Big/\frac{1}{7761600\pi}\,(374464. + 380297.\cosh(0.08\,i\,\pi) + 60\cosh(0.08\,i\,\pi)\,(5352 + 6010\cosh(0.08\,i\,\pi) + 1274\cosh(0.16\,i\,\pi)))\cos(0.42\pi)\right)$$

$$1\Big/\frac{1}{7\,761\,600\,\pi}\,((383\,767. + 60\cos(2\,\pi\,0.04)\,(6010\cos(2\,\pi\,0.04) + 1274\cos(4\,\pi\,0.04) + 1274 + 2039\,(1+1)) + 2\cos(2\,\pi\,0.04)\,190\,148. - 9303)\sin(2\,\pi\,0.04))\,(-1) = 1\Big/\frac{1}{7\,761\,600\,\pi}\,(374\,464. + 380\,297.\cosh((-0.08\,i)\,\pi) + 60\cosh((-0.08\,i)\,\pi)\,(5352 + 6010\cosh((-0.08\,i)\,\pi) + 1274\cosh((-0.16\,i)\,\pi)))\cos(0.58\,\pi)$$

$$1\Big/\frac{1}{7\,761\,600\,\pi}\,((383\,767. + 60\cos(2\,\pi\,0.04)\,(6010\cos(2\,\pi\,0.04) + 1274\cos(4\,\pi\,0.04) + 1274 + 2039\,(1+1)) + 2\cos(2\,\pi\,0.04)\,190\,148. - 9303)\sin(2\,\pi\,0.04))\,(-1) = -\left(1\Big/\frac{1}{7\,761\,600\,\pi}\,(374\,464. + 380\,297.\cosh((-0.08\,i)\,\pi) + 60\cosh((-0.08\,i)\,\pi)\,(5352 + 6010\cosh((-0.08\,i)\,\pi) + 1274\cosh((-0.16\,i)\,\pi)))\cos(0.42\,\pi)\right)$$

Series representations

$$1\Big/\frac{1}{7\,761\,600\,\pi}\,((383\,767. + 60\cos(2\,\pi\,0.04)\,(6010\cos(2\,\pi\,0.04) + 1274\cos(4\,\pi\,0.04) + 1274 + 2039\,(1+1)) + 2\cos(2\,\pi\,0.04)\,190\,148. - 9303)\sin(2\,\pi\,0.04))\,(-1) = -\left((10.7621\,\pi)\Big/\left(\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}(0.08\,\pi)\right)\left(1.03845 + 1.94514\sum_{k=0}^{\infty}\frac{(-1)^k e^{-5.05146\,k}\,\pi^{2k}}{(2k)!} + \left(\sum_{k=0}^{\infty}\frac{(-1)^k e^{-5.05146\,k}\,\pi^{2k}}{(2k)!}\right)^2 + 0.21198\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\,e^{-5.05146\,k_1-3.66516\,k_2}\,\pi^{2k_1+2k_2}}{(2k_1)!\,(2k_2)!}\right)\right)\right)$$

$$1\Big/\frac{1}{7761600\,\pi}\left((383767. + 60\cos(2\pi\,0.04)\,(6010\cos(2\pi\,0.04) + 1274\cos(4\pi\,0.04) + 1274 + 2039\,(1+1)) + 2\cos(2\pi\,0.04)\,190148. - 9303)\sin(2\pi\,0.04)\right)$$

$$(-1) = -\left((21.5241\,\pi)\Big/\left(\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,0.08^{1+2k}\,\pi^{1+2k}}{(1+2k)!}\right)\left(1.03845 + 1.94514\sum_{k=0}^{\infty}\frac{(-1)^k\,e^{-5.05146k}\,\pi^{2k}}{(2k)!} + \left(\sum_{k=0}^{\infty}\frac{(-1)^k\,e^{-5.05146k}\,\pi^{2k}}{(2k)!}\right)^2 + 0.21198\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\,e^{-5.05146k_1-3.66516k_2}\,\pi^{2k_1+2k_2}}{(2k_1)!\,(2k_2)!}\right)\right)\right)$$

$$1\Big/\frac{1}{7761600\,\pi}\left((383767. + 60\cos(2\pi\,0.04)\,(6010\cos(2\pi\,0.04) + 1274\cos(4\pi\,0.04) + 1274 + 2039\,(1+1)) + 2\cos(2\pi\,0.04)\,190148. - 9303)\sin(2\pi\,0.04)\right)$$

$$(-1) = -\left((50.7692\,\pi)\Big/\left(\left(\sum_{k=0}^{\infty}(-1)^k\,J_{1+2k}(0.08\,\pi)\right)\left(4.89879 + 9.17604\,\pi\sum_{k=0}^{\infty}\frac{(-1)^k\,0.42^{1+2k}\,(-\pi)^{2k}}{(1+2k)!} + 4.71743\,\pi^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,0.42^{1+2k}\,(-\pi)^{2k}}{(1+2k)!}\right)^2 + \pi^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\,0.34^{1+2k_1}\times 0.42^{1+2k_2}\,(-\pi)^{2k_1+2k_2}}{(1+2k_1)!\,(1+2k_2)!}\right)\right)\right)$$

$J_n(z)$ is the Bessel function of the first kind

Integral representations

$$1\Big/\frac{1}{7761600\pi}\left((383767.+60\cos(2\pi\,0.04)\,(6010\cos(2\pi\,0.04)+1274\cos(4\pi\,0.04)+1274+2039\,(1+1))+2\cos(2\pi\,0.04)\,190148.-9303)\sin(2\pi\,0.04)\right)(-1)=-\left((1076.21\,i\,\pi)\Big/\left(\left(\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{-(0.0016\pi^2)/s+s}}{s^{3/2}}\,ds\right)\left(1.03845-1.94514\int_{\frac{\pi}{2}}^{0.08\pi}\sin(t)\,dt+\left(\int_{\frac{\pi}{2}}^{0.08\pi}\sin(t)\,dt\right)^2+\int_0^1\int_0^1\sin(\pi\,(0.5-0.42\,t_1))\sin(\pi\,(0.5-0.34\,t_2))\,dt_2\,dt_1\right)\sqrt{\pi}\right)\right)\text{ for }\gamma>0$$

$$1\Big/\frac{1}{7761600\pi}\left((383767.+60\cos(2\pi\,0.04)\,(6010\cos(2\pi\,0.04)+1274\cos(4\pi\,0.04)+1274+2039\,(1+1))+2\cos(2\pi\,0.04)\,190148.-9303)\sin(2\pi\,0.04)\right)(-1)=-\left(\left(1076.21\,i\,\pi^2\right)\Big/\left(\left(\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{6.43775\,s}\,\pi^{1-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds\right)\left(1.03845-1.94514\int_{\frac{\pi}{2}}^{0.08\pi}\sin(t)\,dt+\left(\int_{\frac{\pi}{2}}^{0.08\pi}\sin(t)\,dt\right)^2+\int_0^1\int_0^1\sin(\pi\,(0.5-0.42\,t_1))\sin(\pi\,(0.5-0.34\,t_2))\,dt_2\,dt_1\right)\sqrt{\pi}\right)\right)\text{ for }0<\gamma<1$$

$$1\Big/\frac{1}{7761600\,\pi}\,((383767. + 60\cos(2\,\pi\,0.04)\,(6010\cos(2\,\pi\,0.04) + 1274\cos(4\,\pi\,0.04) + 1274 + 2039\,(1+1)) + 2\cos(2\,\pi\,0.04)\,190148. - 9303)\sin(2\,\pi\,0.04))\,(-1) = -\left((168157.\,i\,\pi)\Big/\left(\left(\left(\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{-(0.0016\,\pi^2)/s+s}}{s^{3/2}}\,ds\right)\left(655.557 - 51.964\,\pi\int_0^1\sin(0.08\,\pi\,t)\,dt + \pi^2\left(\int_0^1\sin(0.08\,\pi\,t)\,dt\right)^2 - 5.2995\,\pi\int_0^1\sin(0.16\,\pi\,t)\,dt + \int_0^1\int_0^1\sin(0.08\,\pi\,t_1)\sin(0.16\,\pi\,t_2)\,dt_2\,dt_1\right)\sqrt{\pi}\right)\right)\right) \text{ for } \gamma > 0$$

Multiple-argument formulas

$$1\Big/\frac{1}{7761600\,\pi}\,((383767. + 60\cos(2\,\pi\,0.04)\,(6010\cos(2\,\pi\,0.04) + 1274\cos(4\,\pi\,0.04) + 1274 + 2039\,(1+1)) + 2\cos(2\,\pi\,0.04)\,190148. - 9303)\sin(2\,\pi\,0.04))\,(-1) = -\left((2.69052\,\pi)\Big/\left(\cos(0.04\,\pi)\left(0.0763222 + \cos^4(0.04\,\pi) - 0.10599\cos^2(0.08\,\pi) + \cos^2(0.04\,\pi)\left(-0.133421 + 0.21198\cos^2(0.08\,\pi)\right)\right)\sin(0.04\,\pi)\right)\right)$$

$$1\Big/\frac{1}{7761600\,\pi}\,((383767. + 60\cos(2\,\pi\,0.04)\,(6010\cos(2\,\pi\,0.04) + 1274\cos(4\,\pi\,0.04) + 1274 + 2039\,(1+1)) + 2\cos(2\,\pi\,0.04)\,190148. - 9303)\sin(2\,\pi\,0.04))\,(-1) = -\left((2.69052\,\pi)\Big/\left(\cos(0.04\,\pi)\sin(0.04\,\pi)\left(1.04889 + \sin^4(0.04\,\pi) - 0.10599\sin^2(0.08\,\pi) + \sin^2(0.04\,\pi)\left(-2.07856 + 0.21198\sin^2(0.08\,\pi)\right)\right)\right)\right)$$

$$1\Big/\frac{1}{7761600\,\pi}\,((383767. + 60\cos(2\,\pi\,0.04)\,(6010\cos(2\,\pi\,0.04) + 1274\cos(4\,\pi\,0.04) + 1274 + 2039\,(1+1)) + 2\cos(2\,\pi\,0.04)\,190148. - 9303)\sin(2\,\pi\,0.04))\,(-1) = (1.34526\,\pi)\Big/\left(\left(0.0763222 + \cos^4(0.04\,\pi) - 0.10599\cos^2(0.08\,\pi) + \cos^2(0.04\,\pi)\left(-0.133421 + 0.21198\cos^2(0.08\,\pi)\right)\right)\left(-0.75\sin(0.0266667\,\pi) + \sin^3(0.0266667\,\pi)\right)\right)$$

From the analysis of the following final equation:

$$\Psi_x^{(5)} = \frac{\epsilon_x^5}{36793476720000\pi}\Big[895050\cos(4\pi q_x)\left(181296\epsilon_1\epsilon_2\cos(2\pi q_y)\cos(2\pi q_z) + 41657\left(\epsilon_1^2+\epsilon_2^2\right)\right)$$
$$+ 560226137900\epsilon_1^2\epsilon_2\cos(2\pi q_x)\cos(4\pi q_y)\cos(2\pi q_z) + 560226137900\epsilon_1\epsilon_2^2\cos(2\pi q_x)\cos(2\pi q_y)\cos(4\pi q_z)$$
$$+ 594423768510\epsilon_1^3\cos(2\pi q_x)\cos(2\pi q_y) - 18642271230\epsilon_1^3\cos(2\pi q_x)\cos(6\pi q_y)$$
$$+ 57401651835\epsilon_1^2\cos(4\pi(q_x-q_y)) + 57401651835\epsilon_1^2\cos(4\pi(q_x+q_y)) + 1006179766020\epsilon_1\epsilon_2^2\cos(2\pi q_x)\cos(2\pi q_y)$$
$$+ 106526974554\epsilon_1\cos(2\pi q_x)\cos(2\pi q_y) - 6828912090\epsilon_1\cos(6\pi q_x)\cos(2\pi q_y) + 1006179766020\epsilon_1^2\epsilon_2\cos(2\pi q_x)\cos(2\pi q_z)$$
$$+ 57401651835\epsilon_2^2\cos(4\pi(q_x-q_z)) + 594423768510\epsilon_2^3\cos(2\pi q_x)\cos(2\pi q_z) - 18642271230\epsilon_2^3\cos(2\pi q_x)\cos(6\pi q_z)$$
$$+ 57401651835\epsilon_2^2\cos(4\pi(q_x+q_z)) + 106526974554\epsilon_2\cos(2\pi q_x)\cos(2\pi q_z) - 6828912090\epsilon_2\cos(6\pi q_x)\cos(2\pi q_z)$$
$$+ 46659033850\epsilon_1^2\epsilon_2^2\cos(4\pi(q_y-q_z)) + 46659033850\epsilon_1^2\epsilon_2^2\cos(4\pi(q_y+q_z)) + 503168866200\epsilon_1^3\epsilon_2\cos(2\pi q_y)\cos(2\pi q_z)$$
$$- 54358176600\epsilon_1^3\epsilon_2\cos(6\pi q_y)\cos(2\pi q_z) + 503168866200\epsilon_1\epsilon_2^3\cos(2\pi q_y)\cos(2\pi q_z) - 54358176600\epsilon_1\epsilon_2^3\cos(2\pi q_y)\cos(6\pi q_z)$$
$$+ 877320054000\epsilon_1\epsilon_2\cos(2\pi q_y)\cos(2\pi q_z) + 115709716980\epsilon_1^2\epsilon_2^2\cos(4\pi q_y) - 35064073044\epsilon_1^4\cos(4\pi q_y)$$
$$- 1607701095\epsilon_1^4\cos(8\pi q_y) + 241408089450\epsilon_1^2\cos(4\pi q_y) + 115709716980\epsilon_1^2\epsilon_2^2\cos(4\pi q_z) - 35064073044\epsilon_2^4\cos(4\pi q_z)$$
$$+ 241408089450\epsilon_2^2\cos(4\pi q_z) - 1607701095\epsilon_2^4\cos(8\pi q_z)$$
$$+ 546975\left(\epsilon_1^2\left(940700\epsilon_2^2+443058\right) + 171045\epsilon_1^4 + 21\epsilon_2^2\left(8145\epsilon_2^2+21098\right)\right)\Big]\sin(2\pi q_x)\,. \qquad (A.6)$$

We divide the expression as above and perform the calculations

1/(36793476720000Pi)

Input

$$\frac{1}{36\,793\,476\,720\,000\,\pi}$$

Decimal approximation

$8.6512587164877924463173027032447485454405106699825049176634\ldots \times 10^{-15}$

895050 cos(4Pi*0.04) (181296* cos(2Pi*0.04)+ 41657(1+1))+560226137900cos(2Pi*0.04) cos(4Pi*0.04) + 560226137900cos(2Pi*0.04) cos(2Pi*0.04)

Input

$895\,050\cos(4\pi\times 0.04)(181\,296\cos(2\pi\times 0.04) + 41\,657(1+1)) +$
$560\,226\,137\,900\cos(2\pi\times 0.04)\cos(4\pi\times 0.04) +$
$560\,226\,137\,900\cos(2\pi\times 0.04)\cos(2\pi\times 0.04)$

Result

$1.204161\ldots \times 10^{12}$

594423768510cos(2Pi*0.04) cos(2Pi*0.04)-18642271230cos(2Pi*0.04) cos(6Pi*0.04)+57401651835cos(4Pi(0.04-0.04)) + 57401651835cos(4Pi(0.04+0.04)) + 1006179766020cos(2Pi*0.04) cos(2Pi*0.04)

Input

$$594\,423\,768\,510 \cos(2\pi \times 0.04) \cos(2\pi \times 0.04) - (18\,642\,271\,230 \cos(2\pi \times 0.04)) \cos(6\pi \times 0.04) + 57\,401\,651\,835 \cos(4\pi (0.04 - 0.04)) + 57\,401\,651\,835 \cos(4\pi (0.04 + 0.04)) + 1\,006\,179\,766\,020 \cos(2\pi \times 0.04) \cos(2\pi \times 0.04)$$

Result

$1.576608\ldots \times 10^{12}$

106526974554cos(2Pi*0.04) cos(2Pi*0.04)-6828912090cos(6Pi*0.04) cos(2Pi*0.04) + 1006179766020 cos(2Pi*0.04)

Input

$$106\,526\,974\,554 \cos(2\pi \times 0.04) \cos(2\pi \times 0.04) - (6\,828\,912\,090 \cos(6\pi \times 0.04)) \cos(2\pi \times 0.04) + 1\,006\,179\,766\,020 \cos(2\pi \times 0.04)$$

Result

$1.069686\ldots \times 10^{12}$

57401651835cos(4Pi(0.04-0.04)) + 594423768510 cos(2Pi*0.04)-18642271230 cos(2Pi*0.04)

Input

$$57\,401\,651\,835\cos(4\pi(0.04-0.04)) + 594\,423\,768\,510\cos(2\pi\times 0.04) - 18\,642\,271\,230\cos(2\pi\times 0.04)$$

Result

$6.150939\ldots\times 10^{11}$

57401651835 cos(4Pi(0.04+0.04)) + 106526974554 cos(2Pi*0.04) - 6828912090cos(6Pi*0.04)+46659033850cos(4Pi(0.04-0.04)) + 46659033850 cos(4Pi(0.04+0.04)) + 503168866200cos(2Pi*0.04)

Input

$$57\,401\,651\,835\cos(4\pi(0.04+0.04)) + 106\,526\,974\,554\cos(2\pi\times 0.04) - 6\,828\,912\,090\cos(6\pi\times 0.04) + 46\,659\,033\,850\cos(4\pi(0.04-0.04)) + 46\,659\,033\,850\cos(4\pi(0.04+0.04)) + 503\,168\,866\,200\cos(2\pi\times 0.04)$$

Result

$6.879806\ldots\times 10^{11}$

-54358176600cos(6Pi*0.04) + 503168866200 cos(2Pi*0.04) - 54358176600cos(2Pi*0.04)+ 877320054000 cos(2Pi*0.04) + 115709716980 cos(4Pi*0.04) - 35064073044cos(4Pi*0.04)

Input

$$-54\,358\,176\,600\cos(6\pi\times 0.04) + 503\,168\,866\,200\cos(2\pi\times 0.04) - 54\,358\,176\,600\cos(2\pi\times 0.04) + 877\,320\,054\,000\cos(2\pi\times 0.04) + 115\,709\,716\,980\cos(4\pi\times 0.04) - 35\,064\,073\,044\cos(4\pi\times 0.04)$$

Result

$1.315513\ldots\times 10^{12}$

-1607701095 cos(8Pi*0.04) + 241408089450 cos(4Pi*0.04) + 115709716980 - 35064073044+ 241408089450- 1607701095+ 546975 (940700+ 443058) + 171045 + 21 (8145+ 21098)

Input

$$-1\,607\,701\,095\cos(8\pi\times 0.04)+241\,408\,089\,450\cos(4\pi\times 0.04)+115\,709\,716\,980-35\,064\,073\,044+241\,408\,089\,450-1\,607\,701\,095+546\,975\,(940\,700+443\,058)+171\,045+21\,(8145+21\,098)$$

Result

$1.288014\ldots\times 10^{12}$

Now, from the all results of the various expressions that we have obtained, after some calculations, we obtain, in conclusion:

1/(36793476720000Pi) [1.204161*10^12+1.576608*10^12+1.069686*10^12+6.150939*10^11+6.879806*10^11+1.315513*10^12+1.288014*10^12] (sin(2Pi*0.04))

Input interpretation

$$\frac{1}{36\,793\,476\,720\,000\,\pi}\left(1.204161\times 10^{12}+1.576608\times 10^{12}+1.069686\times 10^{12}+6.150939\times 10^{11}+6.879806\times 10^{11}+1.315513\times 10^{12}+1.288014\times 10^{12}\right)\sin(2\pi\times 0.04)$$

Result

0.016689156216318759938974803518057089587538860130718485359176042 0...

Inverting:

1/((1/(36793476720000Pi) [1.204161*10^12+1.576608*10^12+1.069686*10^12+6.150939*10^11+6.879806*10^11+1.315513*10^12+1.288014*10^12] (sin(2Pi*0.04))))

Input interpretation

$$1/\left(\frac{1}{36\,793\,476\,720\,000\,\pi}\left(1.204161\times 10^{12}+1.576608\times 10^{12}+1.069686\times 10^{12}+6.150939\times 10^{11}+6.879806\times 10^{11}+1.315513\times 10^{12}+1.288014\times 10^{12}\right)\sin(2\,\pi\times 0.04)\right)$$

Result

59.9191…

From the following algebraic sum of the various above results, we obtain:

25.2651 - 59.8928 + 71.0471372 - 67.294047719 + 59.9191 = 29.044489481

and

(25.2651 + 59.8928 + 71.0471372 + 67.294047719 + 59.9191) = 283.418184919

Subtracting the two sums (considering in GeV and converting in Kg), we obtain:

((((25.2651 + 59.8928 + 71.0471372 + 67.294047719 + 59.9191)-(25.2651 - 59.8928 + 71.0471372 - 67.294047719 + 59.9191)))) GeV = kg

Input interpretation

convert ((25.2651 + 59.8928 + 71.0471372 + 67.294047719 + 59.9191) − (25.2651 − 59.8928 + 71.0471372 − 67.294047719 + 59.9191)) GeV/c^2 to kilograms

Result

4.5346×10^{-25} kg (kilograms)

$4.5346*10^{-25}$ kg result that is very near to the value of the possible mass of a DM particle equal to 250 GeV = 4.457×10^{-25} kg

Furthermore, from the first sum, we obtain also:

47/(25.2651 - 59.8928 + 71.0471372 - 67.294047719 + 59.9191)

where 47 is a Lucas number

Input interpretation

$$\frac{47}{25.2651 - 59.8928 + 71.0471372 - 67.294047719 + 59.9191}$$

Result

1.6182071311925084960662731367772599204667700731620237770086629259
...

1.6182071311925….. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Mathematical connections with some sectors of String Theory

Observations

We note that, from the number 8, we obtain as follows:

8^2

64

$8^2 \times 2 \times 8$

1024

$8^4 = 8^2 \times 2^6$

True

$8^4 = 4096$

$8^2 \times 2^6 = 4096$

$2^{13} = 2 \times 8^4$

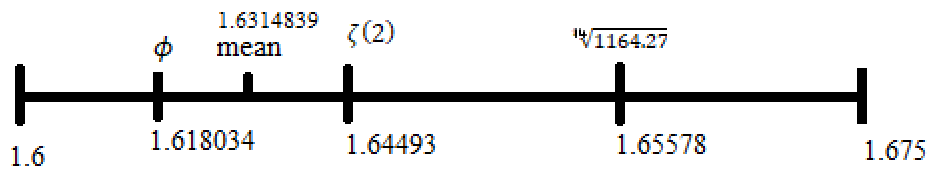
True

$2^{13} = 8192$

$2 \times 8^4 = 8192$

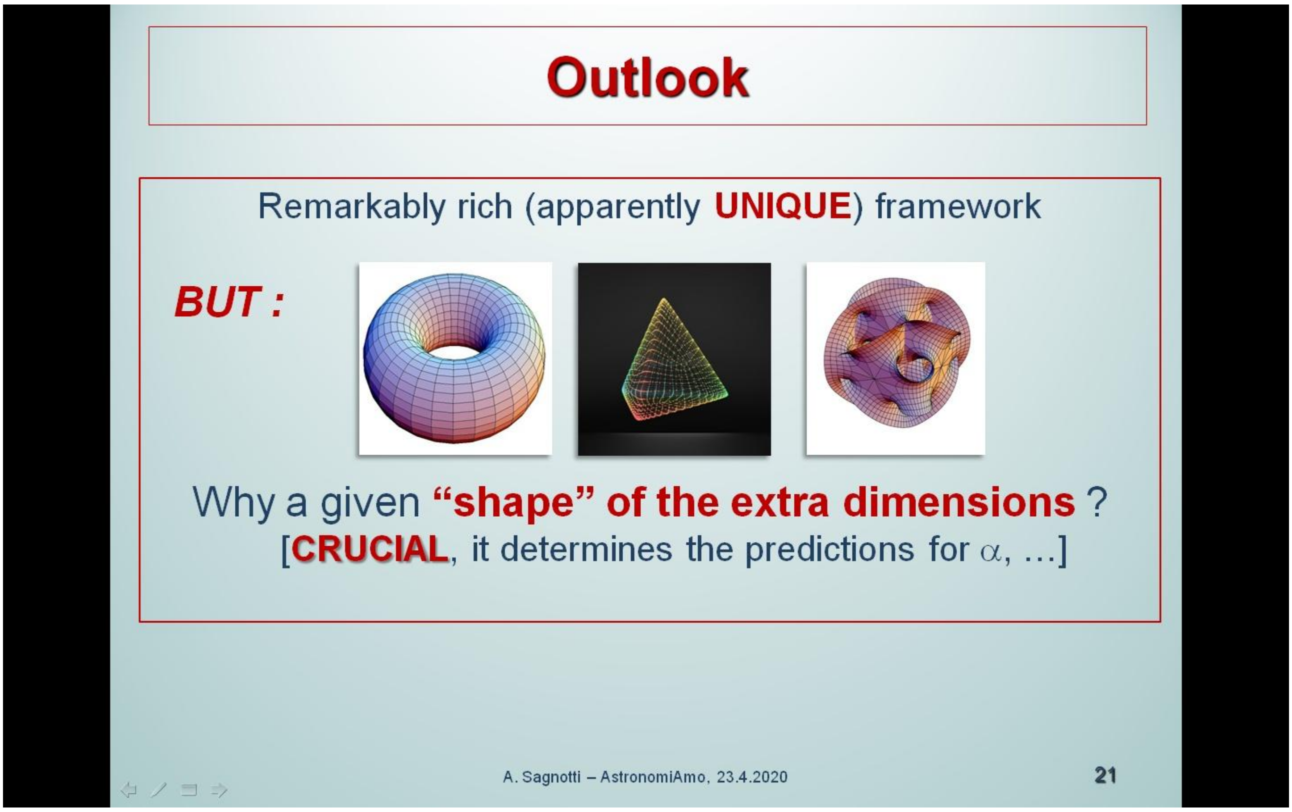
We notice how from the numbers 8 and 2 we get 64, 1024, 4096 and 8192, and that 8 is the fundamental number. In fact $8^2 = 64$, $8^3 = 512$, $8^4 = 4096$. We define it "fundamental number", since 8 is a Fibonacci number, which by rule, divided by the previous one, which is 5, gives 1.6 , a value that tends to the golden ratio, as for all numbers in the Fibonacci sequence

“Golden” Range



Finally we note how $8^2 = 64$, multiplied by 27, to which we add 1, is equal to 1729, the so-called "Hardy-Ramanujan number". Then taking the 15th root of 1729, we obtain a value close to $\zeta(2)$ that 1.6438 ..., which, in turn, is included in the range of what we call "golden numbers". Furthermore for all the results very near to 1728 or 1729, adding $64 = 8^2$, one obtain values about equal to 1792 or 1793. These are values almost equal to the Planck multipole spectrum frequency 1792.35 and to the hypothetical Gluino mass

Appendix



From: *A. Sagnotti – AstronomiAmo, 23.04.2020*

In the above figure, it is said that: “why a given shape of the extra dimensions? Crucial, it determines the predictions for α”.

We propose that whatever shape the compactified dimensions are, their geometry must be based on the values of the golden ratio and ζ(2), (the latter connected to 1728 or 1729, whose fifteenth root provides an excellent approximation to the above mentioned value) which are recurrent as solutions of the equations that we are going to develop. It is important to specify that the initial conditions are **always** values belonging to a fundamental chapter of the work of S. Ramanujan "Modular equations and Appoximations to Pi" (see references). These values are some multiples of 8 (64 and 4096), 276, which added to 4096, is equal to 4372, and finally $e^{\pi\sqrt{22}}$

We have, in certain cases, the following connections:

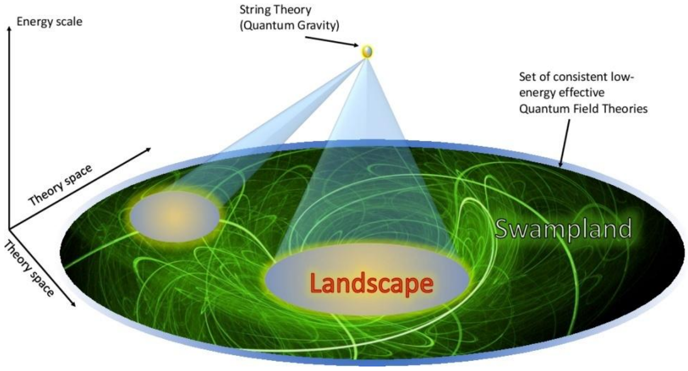


Fig. 1

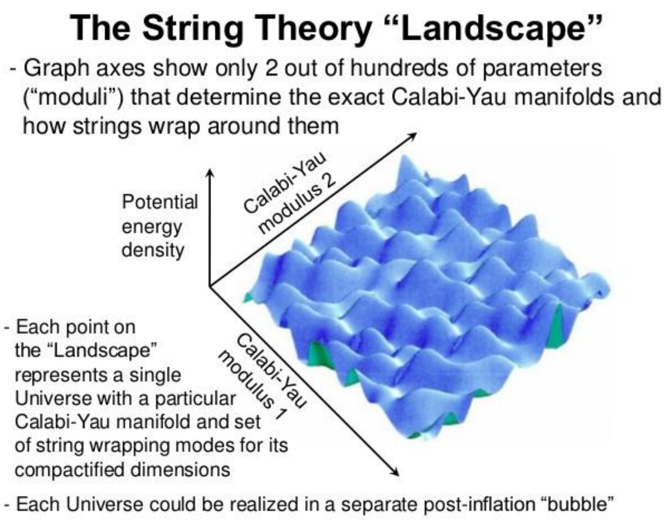


Fig. 2

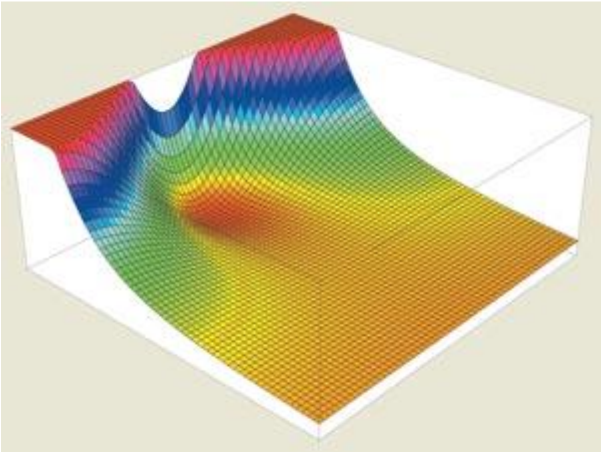


Fig. 3

Stringscape - a small part of the string-theory landscape showing the new de Sitter solution as a local minimum of the energy (vertical axis). The global minimum occurs at the infinite size of the extra dimensions on the extreme right of the figure.

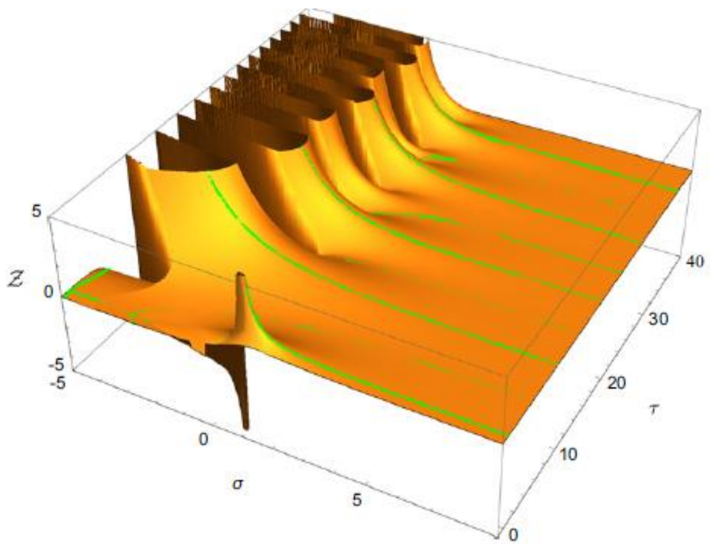


Figure 2. Lines in the complex plane where the Riemann zeta function ζ is real (green) depicted on a relief representing the positive absolute value of ζ for arguments $s \equiv \sigma + i\tau$ where the real part of ζ is positive, and the negative absolute value of ζ where the real part of ζ is negative. This representation brings out most clearly that the lines of constant phase corresponding to phases of integer multiples of 2π run down the hills on the left-hand side, turn around on the right and terminate in the non-trivial zeros. This pattern repeats itself infinitely many times. The points of arrival and departure on the right-hand side of the picture are equally spaced and given by equation (11).

Fig. 4

With regard the Fig. 4 the points of arrival and departure on the right-hand side of the picture are equally spaced and given by the following equation:

$$\tau'_k \equiv k \frac{\pi}{\ln 2},$$
with $k = \ldots, -2, -1, 0, 1, 2$,....

we obtain:

2Pi/(ln(2))

Input:

$$2 \times \frac{\pi}{\log(2)}$$

Exact result:

$$\frac{2\pi}{\log(2)}$$

Decimal approximation:

9.0647202836543876192553658914333336203437229354475911683720330958
...

9.06472028365….

The study of this function provides the following representations:

Alternative representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log_e(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(a)\log_a(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2\coth^{-1}(3)}$$

Series representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor + \log(x) - \sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(z_0) + \left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor + \log(z_0) - \sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}}$$

Integral representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\int_1^2 \frac{1}{t}\,dt}$$

$$\frac{2\pi}{\log(2)} = \frac{4i\pi^2}{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \quad \text{for } -1 < \gamma < 0$$

From which:

(2Pi/(ln(2)))*(1/12 π log(2))

Input:

$$\left(2\times\frac{\pi}{\log(2)}\right)\left(\frac{1}{12}\pi\log(2)\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{\pi^2}{6}$$

Decimal approximation:

1.6449340668482264364724151666460251892189499012067984377355582293
...

1.6449340668…. = ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

From:

Modular equations and approximations to π - *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^6 + (6-\sqrt{37})^6\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

We note that, with regard 4372, we can to obtain the following results:

27((4372)^1/2-2-1/2(((√(10-2√5) -2))⁄((√5-1))))+φ

Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.0526944170905625170637208637148763684189306538457854815447023

...

1729.0526944….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

The study of this function provides the following representations:

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10 - 2\sqrt{5}} - 2\right)}{2\left(\sqrt{5} - 1\right)}$$

Minimal polynomial

$$256 x^8 + 95\,744 x^7 - 3\,248\,750\,080 x^6 - 914\,210\,725\,504 x^5 + 15\,498\,355\,554\,921\,184 x^4 + 2\,911\,478\,392\,539\,914\,656 x^3 - 32\,941\,144\,911\,224\,677\,091\,680 x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456 x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)$$
for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Or:

27((4096+276)^1/2-2-1/2(((√(10-2√5) -2))⁄((√5-1))))+φ

Input

$$27\left(\sqrt{4096+276}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

…

1729.0526944…. as above

The study of this function provides the following representations:

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10 - 2\sqrt{5}} - 2\right)}{2\left(\sqrt{5} - 1\right)}$$

Minimal polynomial

$$256x^8 + 95744x^7 - 3248750080x^6 - 914210725504x^5 + 15498355554921184x^4 + 2911478392539914656x^3 - 32941144911224677091680x^2 - 3092528914069760354714456x + 26320050609744039027169013041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right)\Bigg/\left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4096+276}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=$$

$$\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$

$$108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$

$$\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Bigg/$$

$$\left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4096+276}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=$$

$$\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.$$

$$108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+$$

$$2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-$$

$$\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\Bigg/$$

$$\left(2\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

From which:

(27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ)^1/15

Input

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi}$$

ϕ is the golden ratio

Exact result

$$\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation

1.643818568584986279990230131703681005418575687350518480483418312 4

…

1.64381856858…. ≈ $\zeta(2)=\frac{\pi^2}{6}$ = 1.644934 ...

The study of this function provides the following representations:

Alternate forms

$$\sqrt[15]{\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}}$$

$$\frac{1}{\sqrt[15]{\frac{2\left(\sqrt{5}-1\right)}{166-108\sqrt{5}-108\sqrt{1093}+108\sqrt{5465}-27\sqrt{2\left(5-\sqrt{5}\right)}}}}$$

$$\sqrt[15]{\text{root of } 256 x^8 + 95\,744 x^7 - 3\,248\,750\,080 x^6 - 914\,210\,725\,504 x^5 + 15\,498\,355\,554\,921\,184 x^4 + 2\,911\,478\,392\,539\,914\,656 x^3 - 32\,941\,144\,911\,224\,677\,091\,680 x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456 x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041 \text{ near } x = 1729.05}$$

Minimal polynomial

$$256 x^{120} + 95\,744 x^{105} - 3\,248\,750\,080 x^{90} - 914\,210\,725\,504 x^{75} + 15\,498\,355\,554\,921\,184 x^{60} + 2\,911\,478\,392\,539\,914\,656 x^{45} - 32\,941\,144\,911\,224\,677\,091\,680 x^{30} - 3\,092\,528\,914\,069\,760\,354\,714\,456 x^{15} + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$\sqrt[15]{\frac{1}{2}\left(1+\sqrt{5}\right) + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}} - 2}{2\left(\sqrt{5}-1\right)}\right)}$$

$$\sqrt[15]{-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}}$$

All 15th roots of ϕ + 27 (-2 + 2 sqrt(1093) - (sqrt(10 - 2 sqrt(5)) - 2)/(2 (sqrt(5) - 1)))

$$e^0 \sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}} - 2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.64382$$ (real, principal root)

$$e^{(2 i \pi)/15} \sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}} - 2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.50170 + 0.6686 i$$

$$e^{(4 i \pi)/15} \sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2\left(\sqrt{5} - 1\right)}\right)} \approx 1.0999 + 1.2216\, i$$

$$e^{(2 i \pi)/5} \sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2\left(\sqrt{5} - 1\right)}\right)} \approx 0.5080 + 1.5634\, i$$

$$e^{(8 i \pi)/15} \sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2\left(\sqrt{5} - 1\right)}\right)} \approx -0.17183 + 1.63481\, i$$

Series representations

$$\sqrt[15]{27\left(\sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi} =$$
$$\frac{1}{\sqrt[15]{2}} \left(\left(\left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 2\phi\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(9 - 2\sqrt{5}\right)^{-k}\right) \middle/ \left(-1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)\right)\right) \wedge (1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi} =$$

$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+ 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}- 27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/ \left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\wedge(1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi} =$$

$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+ 108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+ 2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}- 27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/ \left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\wedge(1/15)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Integral representation

$$(1+z)^a = \frac{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0<\gamma<-\mathrm{Re}(a) \text{ and } |\arg(z)|<\pi)$$

From:

An Update on Brane Supersymmetry Breaking
J. Mourad and A. Sagnotti - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E\, \phi} \;=\; -\, \frac{\beta_E^{(p)}\, h^2}{\gamma_E}\; e^{-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

$$16\, k'\, e^{-\,2\,C} \;=\; \frac{h^2 \left(p\,+\,1\,-\,\frac{2\,\beta_E^{(p)}}{\gamma_E}\right) e^{-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}}{(7\,-\,p)}$$

$$(A')^2 \;=\; k\, e^{-\,2\,A} \;+\; \frac{h^2}{16(p\,+\,1)} \left(7\,-\,p\,+\,\frac{2\,\beta_E^{(p)}}{\gamma_E}\right) e^{-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning p, C, β_E and ϕ correspond to the exponents of e (i.e. of exp). Thence we obtain for p = 5 and β_E = 1/2:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18)) we obtain:

Input:

$$\exp\left(-\pi\sqrt{18}\right)$$

Exact result:

$$e^{-3\sqrt{2}\,\pi}$$

Decimal approximation:

$$1.627201622607250929294215673911797954183858113695401 6\ldots \times 10^{-6}$$

$1.6272016\ldots * 10^{-6}$

The study of this function provides the following representations:

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^-}6$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016\ldots * 10\text{^-}6$$

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^-}6$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785…

0.00666501785…

Series representations:

$$\frac{\exp(-\pi\sqrt{18})}{0.000244141} = 4096 \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} 17^{-k} \binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp(-\pi\sqrt{18})}{0.000244141} = 4096 \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{\exp(-\pi\sqrt{18})}{0.000244141} = 4096 \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\, \Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}} \times \frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:

ln(0.00666501784619)

Input interpretation:

log(0.00666501784619)

Result:

–5.010882647757…

-5.010882647757…

Alternative representations:

$$\log(0.006665017846190000) = \log_e(0.006665017846190000)$$

$$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$$

$$\log(0.006665017846190000) = -\text{Li}_1(0.993334982153810000)$$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k (-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2 i \pi \left\lfloor \frac{\arg(0.006665017846190000 - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - z_0)^k z_0^{-k}}{k}$$

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t}\, dt$$

In conclusion:

$$-6C + \phi = -5.010882647757 \ldots$$

and for C = 1, we obtain:

$\phi = -5.010882647757 + 6 = \mathbf{0.989117352243} = \boldsymbol{\phi}$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1} = 1 - \frac{e^{-\pi}}{1+\frac{e^{-2\pi}}{1+\frac{e^{-3\pi}}{1+\frac{e^{-4\pi}}{1+\dots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\dots}}}} \approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value $\mathbf{0.989117352243} = \boldsymbol{\phi}$ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

From

Properties of Nilpotent Supergravity
E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

We have that:

Cosmological inflation with a tiny tensor–to–scalar ratio r, consistently with PLANCK data, may also be described within the present framework, for instance choosing

$$\alpha(\Phi) \;=\; i\,M\left(\Phi \;+\; b\,\Phi\,e^{ik\,\Phi}\right)\;. \tag{4.35}$$

This potential bears some similarities with the Kähler moduli inflation of [32] and with the poly–instanton inflation of [33]. One can verify that $\chi = 0$ solves the field equations, and that the potential along the $\chi = 0$ trajectory is now

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\;. \tag{4.36}$$

We analyzing the following equation:

$$V = \frac{M^2}{3}\left(1 - a\,\phi\,e^{-\gamma\phi}\right)^2 .$$

$$\phi = \varphi - \frac{\sqrt{6}}{k} ,$$

$$a = \frac{b\gamma}{e} < 0 , \qquad \gamma = \frac{k}{\sqrt{6}} < 0 .$$

We have:

(M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

i.e.

V = (M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

For k = 2 and φ = 0.9991104684, that is the value of the scalar field that is equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1 - \cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684$$

we obtain:

V = (M^2)/3*[1-(b/euler number * 2/sqrt6) * (0.9991104684- sqrt6/2) * exp(-(2/sqrt6)(0.9991104684- sqrt6/2))]^2

Input interpretation:

$$V = \frac{M^2}{3}\left(1-\left(\frac{b}{e}\times\frac{2}{\sqrt{6}}\right)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2}{\sqrt{6}}\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\right)\right)^2$$

Result:

$$V = \frac{1}{3}(0.0814845\,b+1)^2\,M^2$$

Solutions:

$$b = \frac{225.913\left(-0.054323\,M^2 \pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} \quad (M \neq 0)$$

Alternate forms:

$$V = 0.00221324\,(b+12.2723)^2\,M^2$$

$$V = 0.00221324\left(b^2\,M^2+24.5445\,b\,M^2+150.609\,M^2\right)$$

$$-0.00221324\,b^2\,M^2-0.054323\,b\,M^2-\frac{M^2}{3}+V=0$$

Expanded form:

$$V = 0.00221324\,b^2\,M^2+0.054323\,b\,M^2+\frac{M^2}{3}$$

Alternate form assuming b, M, and V are positive:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

Alternate form assuming b, M, and V are real:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + 0.333333\,M^2 + 0$$

Derivative:

$$\frac{\partial}{\partial b}\left(\frac{1}{3}\,(0.0814845\,b + 1)^2\,M^2\right) = 0.054323\,(0.0814845\,b + 1)\,M^2$$

Implicit derivatives

$$\frac{\partial b(M, V)}{\partial V} = \frac{154\,317\,775\,011\,120\,075}{36\,961\,748\,(226\,802\,245 + 18\,480\,874\,b)\,M^2}$$

$$\frac{\partial b(M, V)}{\partial M} = -\frac{\frac{226\,802\,245}{18\,480\,874} + b}{M}$$

$$\frac{\partial M(b, V)}{\partial V} = \frac{154\,317\,775\,011\,120\,075}{2\,(226\,802\,245 + 18\,480\,874\,b)^2\,M}$$

$$\frac{\partial M(b, V)}{\partial b} = -\frac{18\,480\,874\,M}{226\,802\,245 + 18\,480\,874\,b}$$

$$\frac{\partial V(b, M)}{\partial M} = \frac{2\,(226\,802\,245 + 18\,480\,874\,b)^2\,M}{154\,317\,775\,011\,120\,075}$$

$$\frac{\partial V(b, M)}{\partial b} = \frac{36\,961\,748\,(226\,802\,245 + 18\,480\,874\, b)\, M^2}{154\,317\,775\,011\,120\,075}$$

Global minimum:

$$\min\left\{\frac{1}{3}\,(0.0814845\, b + 1)^2\, M^2\right\} = 0 \text{ at } (b, M) = (-16, 0)$$

Global minima:

$$\min\left\{\frac{1}{3}\, M^2 \left(1 - \frac{(b\ 2)\left(0.9991104684 - \frac{\sqrt{6}}{2}\right) \exp\left(-\frac{2\left(0.9991104684 - \frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\} = 0$$

for $b = -\frac{226\,802\,245}{18\,480\,874}$

$$\min\left\{\frac{1}{3}\, M^2 \left(1 - \frac{(b\ 2)\left(0.9991104684 - \frac{\sqrt{6}}{2}\right) \exp\left(-\frac{2\left(0.9991104684 - \frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\} = 0$$

for $M = 0$

From:

$$b = \frac{225.913\left(-0.054323\, M^2 \pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} \quad (M \neq 0)$$

we obtain:

(225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2

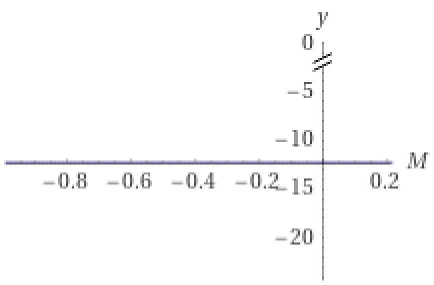
Input interpretation:

$$\frac{225.913\left(-0.054323\,M^2 + 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}$$

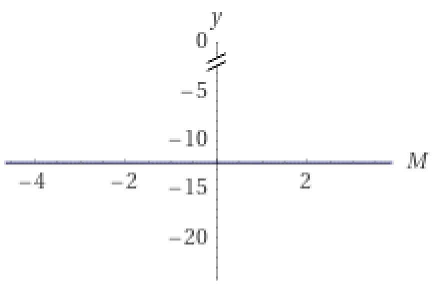
Result:

$$\frac{225.913\left(6.58545\times 10^{-10}\sqrt{M^4} - 0.054323\,M^2\right)}{M^2}$$

Plots:



(M from −1 to 0.2)



(M from −4.6 to 3.9)

Alternate form assuming M is real:

-12.2723

-12.2723 result very near to the black hole entropy value 12.1904 = ln(196884)

The study of this function provides the following representations:

Alternate forms:

$$-\frac{12.2723\left(M^2 - 1.21228\times 10^{-8}\sqrt{M^4}\right)}{M^2}$$

$$\frac{1.48774\times 10^{-7}\sqrt{M^4} - 12.2723\,M^2}{M^2}$$

Expanded form:

$$\frac{1.48774\times 10^{-7}\sqrt{M^4}}{M^2} - 12.2723$$

Property as a function:

Parity

even

Series expansion at M = 0:

$$\left(\frac{1.48774\times 10^{-7}\sqrt{M^4}}{M^2} - 12.2723\right) + O\left(M^6\right)$$

(generalized Puiseux series)

Series expansion at M = ∞:

$$-12.2723$$

Derivative:

$$\frac{d}{dM}\left(\frac{225.913\left(6.58545\times 10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right)=\frac{3.55271\times 10^{-15}}{M}$$

Indefinite integral:

$$\int \frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}\,dM = \frac{1.48774\times 10^{-7}\sqrt{M^4}}{M}-12.2723\,M+\text{constant}$$

Global maximum:

$$\max\left\{\frac{225.913\left(6.58545\times 10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\} = -\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\text{ at } M=-1$$

Global minimum:

$$\min\left\{\frac{225.913\left(6.58545\times 10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\} = -\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\text{ at } M=-1$$

Limit:

$$\lim_{M\to\pm\infty}\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}=-12.2723$$

Definite integral after subtraction of diverging parts:

$$\int_0^\infty \left(\frac{225.913 \left(-0.054323\, M^2 + 6.58545\times 10^{-10} \sqrt{M^4}\right)}{M^2} - -12.2723 \right) dM = 0$$

From b that is equal to

$$\frac{225.913 \left(-0.054323\, M^2 + 6.58545 \times 10^{-10} \sqrt{M^4}\right)}{M^2}$$

From:

$$V = \frac{1}{3} (0.0814845\, b + 1)^2 M^2$$

we obtain:

1/3 (0.0814845 ((225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2) + 1)^2 M^2

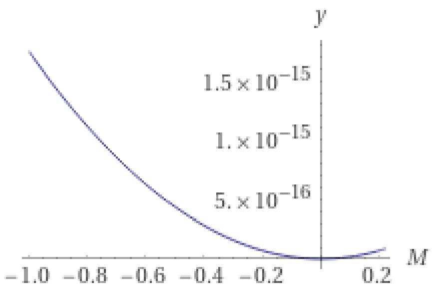
Input interpretation:

$$\frac{1}{3} \left(0.0814845 \times \frac{225.913 \left(-0.054323\, M^2 + 6.58545 \times 10^{-10} \sqrt{M^4}\right)}{M^2} + 1 \right)^2 M^2$$

Result:

0

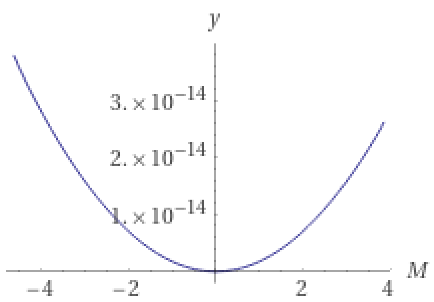
Plots: **(possible mathematical connection with an open string)**



(M from −1 to 0.2)

M = -0.5; M = 0.2

(possible mathematical connection with an open string)



(M from −4.6 to 3.9)

M = 2 ; M = 3

Root:

$M = 0$

Property as a function:

Parity

even

Series expansion at M = 0:

$O(M^{62194})$

(Taylor series)

Series expansion at M = ∞:

$1.75541\times 10^{-15}\, M^2 + O\left(\left(\frac{1}{M}\right)^{62194}\right)$

(Taylor series)

Definite integral after subtraction of diverging parts:

$$\int_0^\infty \left(\frac{1}{3} M^2 \left(1 + \frac{18.4084\left(-0.054323\, M^2 + 6.58545\times 10^{-10} \sqrt{M^4}\right)}{M^2}\right)^2 - 1.75541\times 10^{-15}\, M^2\right) dM = 0$$

For M = - 0.5 , we obtain:

$$\frac{1}{3}\left(0.0814845\times \frac{225.913\left(-0.054323\, M^2 + 6.58545\times 10^{-10} \sqrt{M^4}\right)}{M^2} + 1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 (-0.5)^2 + 6.58545×10^-10 sqrt((-0.5)^4)))/(-0.5)^2) + 1)^2 * (-0.5^2)

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,(-0.5)^2+6.58545\times 10^{-10}\sqrt{(-0.5)^4}\right)}{(-0.5)^2}+1\right)^2$$
$$\left(-0.5^2\right)$$

Result:

$-4.388513449474645453489707833780880208333333333333333333\ldots\times 10^{-16}$

-4.38851344947*10^-16

For M = 0.2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 0.2^2 + 6.58545×10^-10 sqrt(0.2^4)))/0.2^2) + 1)^2 0.2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times 0.2^2+6.58545\times 10^{-10}\sqrt{0.2^4}\right)}{0.2^2}+1\right)^2\times 0.2^2$$

Result:

$7.0216215191594327255835325340494083333333333333333333333333... \times 10^{-17}$

7.021621519159*10^-17

For M = 3:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323\, M^2 + 6.58545 \times 10^{-10}\sqrt{M^4}\right)}{M^2} + 1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 3^2 + 6.58545×10^-10 sqrt(3^4)))/3^2) + 1)^2 3^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323 \times 3^2 + 6.58545 \times 10^{-10}\sqrt{3^4}\right)}{3^2} + 1\right)^2 \times 3^2$$

Result:

$1.579864841810872363256294820161116875 \times 10^{-14}$

1.57986484181*10^-14

For M = 2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 2^2 + 6.58545×10^-10 sqrt(2^4)))/2^2) + 1)^2 2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times2^2+6.58545\times10^{-10}\sqrt{2^4}\right)}{2^2}+1\right)^2\times2^2$$

Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots\times 10^{-15}$

$7.021621519*10^{-15}$

From the four results

7.021621519*10^-15 ; 1.57986484181*10^-14 ; 7.021621519159*10^-17 ;

-4.38851344947*10^-16

we obtain, after some calculations:

sqrt[1/(2Pi)(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 - 4.38851344947*10^-16)]

Input interpretation:

$$\sqrt{\left(\frac{1}{2\pi}\left(7.021621519\times10^{-15}+1.57986484181\times10^{-14}+7.021621519\times10^{-17}-4.38851344947\times10^{-16}\right)\right)}$$

Result:

$5.9776991059\ldots\times10^{-8}$

5.9776991059*10^-8 result very near to the Planck's electric flow 5.975498×10^{-8} that is equal to the following formula:

$$\phi_{\mathrm{P}}^{E}=\mathbf{E}_{\mathrm{P}}l_{\mathrm{P}}^{2}=\phi_{\mathrm{P}}l_{\mathrm{P}}=\sqrt{\frac{\hbar c}{\varepsilon_0}}$$

We note that:

1/55*((([(((1/[(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 -4.38851344947*10^-16)])))^1/7]-((log^(5/8)(2))/(2 2^(1/8) 3^(1/4) e log^(3/2)(3)))))

Input interpretation:

$$\frac{1}{55}\left(\left(1/\left(7.021621519\times10^{-15}+1.57986484181\times10^{-14}+7.021621519\times10^{-17}-4.38851344947\times10^{-16}\right)\right)^\wedge(1/7)-\frac{\log^{5/8}(2)}{2\sqrt[8]{2}\sqrt[4]{3}\,e\log^{3/2}(3)}\right)$$

log(x) is the natural logarithm

Result:

1.6181818182…

1.6181818182… result that is a very good approximation to the value of the golden ratio 1.618033988749...

From the Planck units:

Planck Length

$$l_{\mathrm{P}} = \sqrt{\frac{4\pi\hbar G}{c^3}}$$

5.729475 * 10^{-35} Lorentz-Heaviside value

Planck's Electric field strength

$$\mathbf{E}_{\mathrm{P}} = \frac{F_{\mathrm{P}}}{q_{\mathrm{P}}} = \sqrt{\frac{c^7}{16\pi^2\varepsilon_0\hbar G^2}}$$

1.820306 * 10^{61} V*m Lorentz-Heaviside value

Planck's Electric flux

$$\phi^E_{\mathrm{P}} = \mathbf{E}_{\mathrm{P}} l^2_{\mathrm{P}} = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

5.975498*10^{-8} V*m Lorentz-Heaviside value

Planck's Electric potential

$$\phi_P = V_P = \frac{E_P}{q_P} = \sqrt{\frac{c^4}{4\pi\varepsilon_0 G}}$$

1.042940*10^{27} V Lorentz-Heaviside value

Relationship between Planck's Electric Flux and Planck's Electric Potential

$E_P * l_P$ = $(1.820306 * 10^{61}) * 5.729475 * 10^{-35}$

Input interpretation:

$$\frac{(1.820306\times 10^{61})\times 5.729475}{10^{35}}$$

Result:

1 042 939 771 935 000 000 000 000 000

Scientific notation:

$1.042939771935\times 10^{27}$

$1.042939771935*10^{27} \approx 1.042940*10^{27}$

Or:

$E_P * l_P^2 / l_P$ = $(5.975498*10^{-8})*1/(5.729475 * 10^{-35})$

Input interpretation:

$$5.975498\times 10^{-8}\times \frac{1}{\frac{5.729475}{10^{35}}}$$

Result:

$1.0429398854170757355604134759292954415544181622225422050013\ldots\times 10^{27}$

$1.042939885417*10^{27} \approx 1.042940*10^{27}$

Acknowledgments

We would like to thank Professor **Augusto Sagnotti** theoretical physicist at Scuola Normale Superiore (Pisa – Italy) for his very useful explanations and his availability

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