On the detailed analysis of a fundamental Ramanujan identity. New possible mathematical connections with the Cosmological Constant value in quantum space-time and some parameters of Theoretical Physics.

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Abstract

In this paper, we analyze a fundamental Ramanujan identity and obtain new possible mathematical connections with the Cosmological Constant value in quantum space-time and some parameters of Theoretical Physics.

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With regard the Dark Energy and Cosmological constant, we know that fundamental are the following results: 2.846 * 10^{-122} and 0.3516 * 10^{122}

$$\Lambda = 3H^2 = \lambda_P (H / h_P)^2 = \lambda_P (l_P / L_H)^2 = (2.846 \pm 0.076)\ 10^{-122}\ m_P^2$$

$$\Lambda_Q = 3H_Q^2 = \lambda_P (h_P / H)^2 = \lambda_P (L_H / l_P)^2 = (0.3516 \pm 0.094)\ 10^{122} m_P^2$$

(**New Quantum Structure of the Space-Time -** *Norma G. SANCHEZ* - arXiv:1910.13382v1 [physics.gen-ph] 28 Oct 2019)

From:

The man who new infinity: a life of the genius Ramanujan - *Robert Kanigel* - Copyright © 1991

We have that:

This second paper went back to two striking identities he had discovered sometime before 1913 and later shown to Hardy. (An identity is an equation true for all values of the variable. So that whereas $x-2=3$ is an ordinary equation, true only for $x=5$, $(x-2)(x+2) = x^2-4$ is true for every value of x.) One of them was

$$1 + \frac{q}{1-q} + \frac{q^4}{(1-q)(1-q^2)} + \frac{q^9}{(1-q)(1-q^2)(1-q^3)} + \dots$$
$$= \frac{1}{(1-q)(1-q^4)(1-q^6)(1-q^9)(1-q^{11})(1-q^{14})(1-q^{16})(1-q^{19}) \dots}$$

In this paper, we study in more detail the above Ramanujan's identity.

For q = 8 , we obtain, from the left-hand side:

1+8/(1-8)+(8^4)/((1-8)(1-8^2))+(8^9)/((1-8)(1-8^2)(1-8^3))

Input

$$1 + \frac{8}{1-8} + \frac{8^4}{(1-8)(1-8^2)} + \frac{8^9}{(1-8)(1-8^2)(1-8^3)}$$

Exact result

$$-\frac{132156865}{225351}$$

Decimal approximation

−586.4489840293586449583094816530656620117061827992775714330089504...

-586.448984029…

And, from the right-hand side:

1/((1-8) (1-8^4) (1-8^6) (1-8^9) (1-8^11) (1-8^14) (1-8^16) (1-8^19))

Input

$$\frac{1}{(1-8)(1-8^4)(1-8^6)(1-8^9)(1-8^{11})(1-8^{14})(1-8^{16})(1-8^{19})}$$

Exact result

1/
1545607834683264031320606556246611906812684783871043615763742 ⋱
996985311225

Decimal approximation

$6.4699464997531309563419487504443142418531834611884160211845\ldots \times 10^{-73}$

$6.469946499753\ldots.*10^{-73}$

We have that:

[1+8/(1-8)+(8^4)/((1-8)(1-8^2))+(8^9)/((1-8)(1-8^2)(1-8^3))] x = [1/((1-8) (1-8^4) (1-8^6) (1-8^9) (1-8^11) (1-8^14) (1-8^16) (1-8^19))]

Input

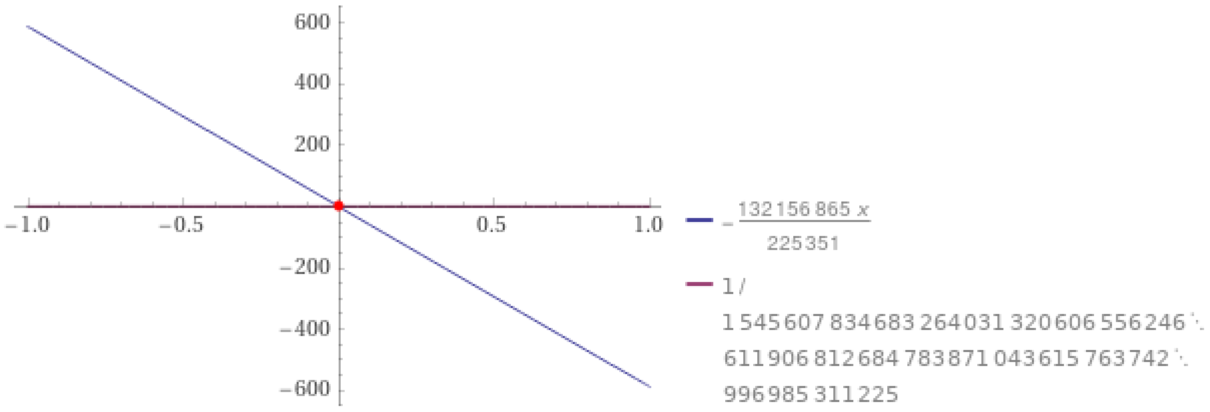
$$\left(1+\frac{8}{1-8}+\frac{8^4}{(1-8)\left(1-8^2\right)}+\frac{8^9}{(1-8)\left(1-8^2\right)\left(1-8^3\right)}\right)x = \frac{1}{(1-8)\left(1-8^4\right)\left(1-8^6\right)\left(1-8^9\right)\left(1-8^{11}\right)\left(1-8^{14}\right)\left(1-8^{16}\right)\left(1-8^{19}\right)}$$

Exact result

$$-\frac{132\,156\,865\,x}{225\,351} = 1/1\,545\,607\,834\,683\,264\,031\,320\,606\,556\,246\,611\,906\,812\,684\,783\,871\,043\,615\,763\,742\,996\,985\,311\,225$$

<u>The study of this function provides the following representations</u>:

Plot



Alternate form

$$-\frac{132\,156\,865\,x}{225\,351} - 1/1\,545\,607\,834\,683\,264\,031\,320\,606\,556\,246\,611\,906\,812\,684\,783\,871\,043\,615\,763\,742\,996\,985\,311\,225 = 0$$

Solution

$$x = -(1/906\,420\,144\,357\,817\,104\,634\,961\,337\,522\,346\,900\,950\,235\,691\,297\,574\,399\,659\,202\,112\,026\,497\,253\,375)$$

$$-(1/906\,420\,144\,357\,817\,104\,634\,961\,337\,522\,346\,900\,950\,235\,691\,297\,574\,399\,659\,202\,112\,026\,497\,253\,375)$$ (irreducible)

Decimal approximation

$-1.1032411473031444973688006997301098650576016207423433801 8\ldots \times 10^{-75}$

$-1.1032411473\ldots.*10^{-75}$

Indeed, we obtain:

-1.1032411473031444973688*10^-75 [1+8/(1-8)+(8^4)/((1-8)(1-8^2))+(8^9)/((1-8)(1-8^2)(1-8^3))]

Input interpretation

$$-1.10324114730314449736888 \times 10^{-75} \left(1+\frac{8}{1-8}+\frac{8^4}{(1-8)(1-8^2)}+\frac{8^9}{(1-8)(1-8^2)(1-8^3)}\right)$$

Result

$6.46994649975313095634194464688419399070782911990627953725552... \times 10^{-73}$

6.469946499…*10^{-73}

We have also:

[1+8/(1-8)+(8^4)/((1-8)(1-8^2))+(8^9)/((1-8)(1-8^2)(1-8^3))] = [1/((1-8) (1-8^4) (1-8^6) (1-8^9) (1-8^11) (1-8^14) (1-8^16) (1-8^19))] x

Input

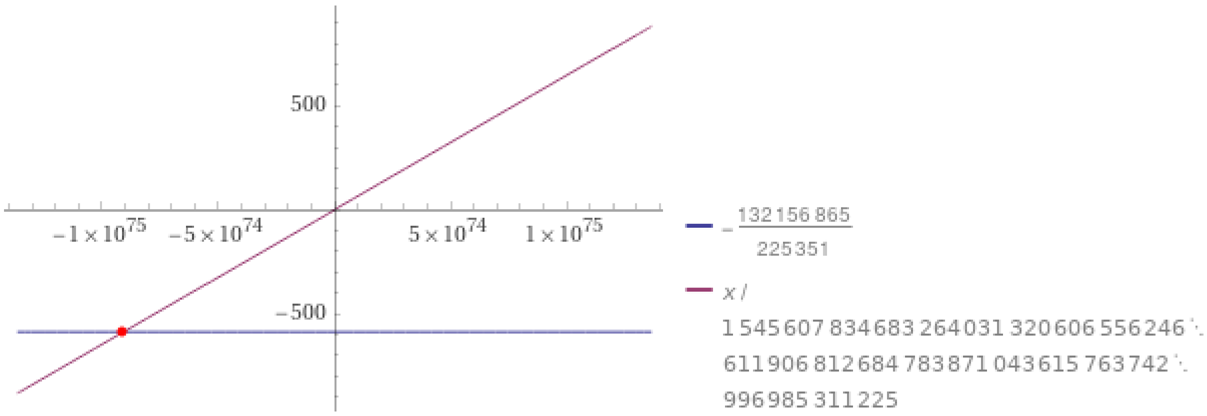
$$1+\frac{8}{1-8}+\frac{8^4}{(1-8)(1-8^2)}+\frac{8^9}{(1-8)(1-8^2)(1-8^3)} = \frac{1}{(1-8)(1-8^4)(1-8^6)(1-8^9)(1-8^{11})(1-8^{14})(1-8^{16})(1-8^{19})}x$$

Result

$$-\frac{132156865}{225351} = x / 1545607834683264031320606556246611906812684783871043615763742996985311225$$

The study of this function provides the following representations:

Plot



Alternate form

$$-(x/1\,545\,607\,834\,683\,264\,031\,320\,606\,556\,246\,611\,906\,812\,684\,783\,871\,043\,615\,763\,742\,996\,985\,311\,225) - \frac{132\,156\,865}{225\,351} = 0$$

Solution

$$x = -906\,420\,144\,357\,817\,104\,634\,961\,337\,522\,346\,900\,950\,235\,691\,297\,574\,399\,659\,202\,112\,026\,497\,253\,375$$

Result

$$-9.06420144357817104634961337522346900950235691297574399659202112026497253375 \times 10^{74}$$

-9.06420144357…*10^74

Dividing the two previous expressions, we obtain:

-9.064201443578171046349613375223469009502356912975743996592021120264 97253375×10^74 ((-1.1032411473031444973688*10^-75 [1+8/(1-8)+(8^4)/((1-8)(1-8^2))+(8^9)/((1-8)(1-8^2)(1-8^3))]))

Input interpretation

$$-9.06420144357817104634961337522346900950235691297574399659202112027\cdot\\ 6497253375\times 10^{74}\left(-1.1032411473031444973688\times 10^{-75}\right.\\ \left(1+\frac{8}{1-8}+\frac{8^4}{(1-8)\left(1-8^2\right)}+\frac{8^9}{(1-8)\left(1-8^2\right)\left(1-8^3\right)}\right)\Bigg)$$

Result

$-586.44898402935864495830910969811000410925726572781904864578939 57$

...

-586.448984029…..

For q = 0.8 , we obtain:

1+0.8/(1-0.8)+(0.8^4)/((1-0.8)(1-0.8^2))+(0.8^9)/((1-0.8)(1-0.8^2)(1-0.8^3))

Input

$$1+\frac{0.8}{1-0.8}+\frac{0.8^4}{(1-0.8)\left(1-0.8^2\right)}+\frac{0.8^9}{(1-0.8)\left(1-0.8^2\right)\left(1-0.8^3\right)}$$

Result

14.5088378870673952641165755919854280510018214936247723132969034 60

...

14.508837887….

And:

1/((1-0.8) (1-0.8^4) (1-0.8^6) (1-0.8^9) (1-0.8^11) (1-0.8^14) (1-0.8^16) (1-0.8^19))

Input

$$\frac{1}{(1-0.8)\left(1-0.8^{4}\right)\left(1-0.8^{6}\right)\left(1-0.8^{9}\right)\left(1-0.8^{11}\right)\left(1-0.8^{14}\right)\left(1-0.8^{16}\right)\left(1-0.8^{19}\right)}$$

Result

15.837498650895593983158865862378517502176377643354122928331114622
...

15.83749865…..

We have:

((1+0.8/(1-0.8)+(0.8^4)/((1-0.8)(1-0.8^2))+(0.8^9)/((1-0.8)(1-0.8^2)(1-0.8^3))))*x = ((1/((1-0.8) (1-0.8^4) (1-0.8^6) (1-0.8^9) (1-0.8^11) (1-0.8^14) (1-0.8^16) (1-0.8^19))))

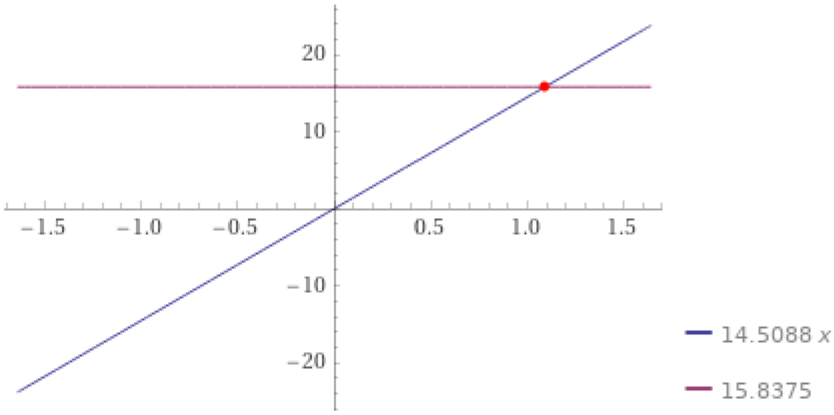
Input

$$\left(1+\frac{0.8}{1-0.8}+\frac{0.8^{4}}{(1-0.8)\left(1-0.8^{2}\right)}+\frac{0.8^{9}}{(1-0.8)\left(1-0.8^{2}\right)\left(1-0.8^{3}\right)}\right)x = \frac{1}{(1-0.8)\left(1-0.8^{4}\right)\left(1-0.8^{6}\right)\left(1-0.8^{9}\right)\left(1-0.8^{11}\right)\left(1-0.8^{14}\right)\left(1-0.8^{16}\right)\left(1-0.8^{19}\right)}$$

Result

$14.5088\,x = 15.8375$

Plot



Alternate form

$14.5088\,x - 15.8375 = 0$

Alternate form assuming x is real

$14.5088\, x + 0 = 15.8375$

Solution

$x \approx 1.09158$

1.09158

Indeed:

((1+0.8/(1-0.8)+(0.8^4)/((1-0.8)(1-0.8^2))+(0.8^9)/((1-0.8)(1-0.8^2)(1-0.8^3))))*(π! - 2 - 3/π - π)

where (π! - 2 - 3/π - π) ≈ 1.09158

Input

$$\left(1+\frac{0.8}{1-0.8}+\frac{0.8^4}{(1-0.8)\left(1-0.8^2\right)}+\frac{0.8^9}{(1-0.8)\left(1-0.8^2\right)\left(1-0.8^3\right)}\right)\left(\pi!-2-\frac{3}{\pi}-\pi\right)$$

$n!$ is the factorial function

Result

15.8373…

15.8373….

The study of this function provides the following representations:

Alternative representations

$$\left(1+\frac{0.8}{1-0.8}+\frac{0.8^4}{(1-0.8)\left(1-0.8^2\right)}+\frac{0.8^9}{(1-0.8)\left(1-0.8^2\right)\left(1-0.8^3\right)}\right) \left(\pi!-2-\frac{3}{\pi}-\pi\right)= \left(-2-\pi+\Gamma(1+\pi)-\frac{3}{\pi}\right)\left(1+\frac{0.8}{0.2}+\frac{0.8^4}{0.2\left(1-0.8^2\right)}+\frac{0.8^9}{0.2\left(1-0.8^2\right)\left(1-0.8^3\right)}\right)$$

$$\left(1+\frac{0.8}{1-0.8}+\frac{0.8^4}{(1-0.8)\left(1-0.8^2\right)}+\frac{0.8^9}{(1-0.8)\left(1-0.8^2\right)\left(1-0.8^3\right)}\right) \left(\pi!-2-\frac{3}{\pi}-\pi\right)= \left(-2-\pi+(1)_\pi-\frac{3}{\pi}\right)\left(1+\frac{0.8}{0.2}+\frac{0.8^4}{0.2\left(1-0.8^2\right)}+\frac{0.8^9}{0.2\left(1-0.8^2\right)\left(1-0.8^3\right)}\right)$$

$$\left(1+\frac{0.8}{1-0.8}+\frac{0.8^4}{(1-0.8)\left(1-0.8^2\right)}+\frac{0.8^9}{(1-0.8)\left(1-0.8^2\right)\left(1-0.8^3\right)}\right) \left(\pi!-2-\frac{3}{\pi}-\pi\right)= \left(-2-\pi+\Gamma(1+\pi,0)-\frac{3}{\pi}\right)\left(1+\frac{0.8}{0.2}+\frac{0.8^4}{0.2\left(1-0.8^2\right)}+\frac{0.8^9}{0.2\left(1-0.8^2\right)\left(1-0.8^3\right)}\right)$$

Series representations

$$\left(1+\frac{0.8}{1-0.8}+\frac{0.8^4}{(1-0.8)\left(1-0.8^2\right)}+\frac{0.8^9}{(1-0.8)\left(1-0.8^2\right)\left(1-0.8^3\right)}\right) \left(\pi!-2-\frac{3}{\pi}-\pi\right)\propto -29.0177-\frac{43.5265}{\pi}-14.5088\,\pi+ 14.5088\,e^{-\pi}\,\pi^{1/2+\pi}\exp\left(\sum_{k=0}^{\infty}\frac{\pi^{-1-2k}\,B_{2+2k}}{2+6k+4k^2}\right)\sqrt{2\pi} \quad \text{for } \infty\to\pi$$

$$\left(1+\frac{0.8}{1-0.8}+\frac{0.8^4}{(1-0.8)\left(1-0.8^2\right)}+\frac{0.8^9}{(1-0.8)\left(1-0.8^2\right)\left(1-0.8^3\right)}\right)\left(\pi!-2-\frac{3}{\pi}-\pi\right)=$$
$$-29.0177-\frac{43.5265}{\pi}-14.5088\,\pi+14.5088\sum_{k=0}^{\infty}\frac{(\pi-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}$$

for $((n_0 \notin \mathbb{Z}$ or $n_0 \geq 0)$ and $n_0 \to \pi)$

$$\left(1+\frac{0.8}{1-0.8}+\frac{0.8^4}{(1-0.8)\left(1-0.8^2\right)}+\frac{0.8^9}{(1-0.8)\left(1-0.8^2\right)\left(1-0.8^3\right)}\right)\left(\pi!-2-\frac{3}{\pi}-\pi\right)\propto$$
$$-29.0177-\frac{43.5265}{\pi}-14.5088\,\pi+14.5088\,e^{-\pi}\,\pi^{1/2+\pi}\sqrt{2\pi}+$$
$$14.5088\,e^{-\pi}\,\pi^{1/2+\pi}\sqrt{2\pi}\sum_{k=1}^{\infty}\sum_{j=1}^{2k}\frac{(-1)^j\,2^{-j-k}\,\pi^{-k}\,\mathcal{D}_{2(j+k),j}}{(j+k)!}$$

for $\left((\infty \to \pi \text{ and } \mathcal{D}_{m,j}=(-1+m)\left((-2+m)\,\mathcal{D}_{-3+m,-1+j}+\mathcal{D}_{-1+m,j}\right) \text{ and } \mathcal{D}_{0,0}=1 \text{ and } \mathcal{D}_{m,1}=(-1+m)! \text{ and } \mathcal{D}_{m,j}=0\right) \text{ for } m \leq -1+3j\right)$

B_n is the n^{th} Bernoulli number

$\mathbb{Z}$ is the set of integers

Integral representations

$$\left(1+\frac{0.8}{1-0.8}+\frac{0.8^4}{(1-0.8)\left(1-0.8^2\right)}+\frac{0.8^9}{(1-0.8)\left(1-0.8^2\right)\left(1-0.8^3\right)}\right)\left(\pi!-2-\frac{3}{\pi}-\pi\right)=-29.0177-\frac{43.5265}{\pi}-14.5088\,\pi+14.5088\int_0^1\log^{\pi}\left(\frac{1}{t}\right)dt$$

$$\left(1+\frac{0.8}{1-0.8}+\frac{0.8^4}{(1-0.8)\left(1-0.8^2\right)}+\frac{0.8^9}{(1-0.8)\left(1-0.8^2\right)\left(1-0.8^3\right)}\right)\left(\pi!-2-\frac{3}{\pi}-\pi\right)=-29.0177-\frac{43.5265}{\pi}-14.5088\,\pi+$$
$$14.5088\int_1^{\infty}e^{-t}\,t^{\pi}\,dt+14.5088\sum_{k=0}^{\infty}\frac{(-1)^k}{(1+k+\pi)\,k!}$$

For q = 0.6 , we obtain:

1+0.6/(1-0.6)+(0.6^4)/((1-0.6)(1-0.6^2))+(0.6^9)/((1-0.6)(1-0.6^2)(1-0.6^3))

Input

$$1+\frac{0.6}{1-0.6}+\frac{0.6^4}{(1-0.6)\left(1-0.6^2\right)}+\frac{0.6^9}{(1-0.6)\left(1-0.6^2\right)\left(1-0.6^3\right)}$$

Result

3.0564617346938775510204081632653061224489795918367346938775510204
...

3.0564617346…..

And:

1/((1-0.6) (1-0.6^4) (1-0.6^6) (1-0.6^9) (1-0.6^11) (1-0.6^14) (1-0.6^16) (1-0.6^19))

Input

$$\frac{1}{(1-0.6)\left(1-0.6^4\right)\left(1-0.6^6\right)\left(1-0.6^9\right)\left(1-0.6^{11}\right)\left(1-0.6^{14}\right)\left(1-0.6^{16}\right)\left(1-0.6^{19}\right)}$$

Result

3.0580058932413726370246885719568042623998197586475526448637182406
...

3.058005893….

We have:

((1+0.6/(1-0.6)+(0.6^4)/((1-0.6)(1-0.6^2))+(0.6^9)/((1-0.6)(1-0.6^2)(1-0.6^3))))*x = ((1/((1-0.6) (1-0.6^4) (1-0.6^6) (1-0.6^9) (1-0.6^11) (1-0.6^14) (1-0.6^16) (1-0.6^19))))

Input

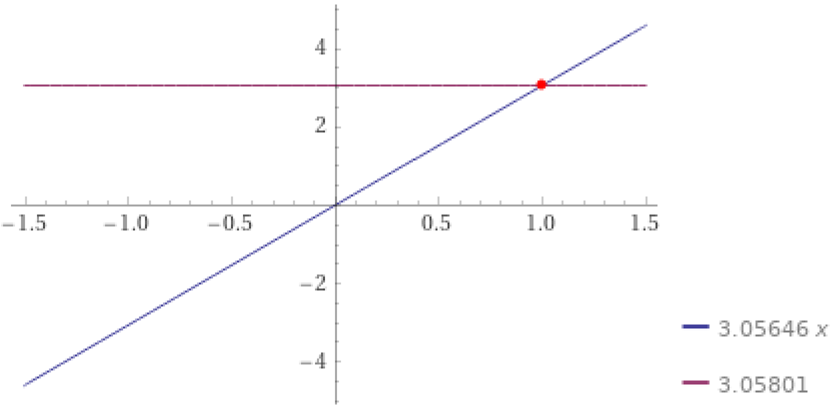
$$\left(1+\frac{0.6}{1-0.6}+\frac{0.6^4}{(1-0.6)\left(1-0.6^2\right)}+\frac{0.6^9}{(1-0.6)\left(1-0.6^2\right)\left(1-0.6^3\right)}\right)x = \frac{1}{(1-0.6)\left(1-0.6^4\right)\left(1-0.6^6\right)\left(1-0.6^9\right)\left(1-0.6^{11}\right)\left(1-0.6^{14}\right)\left(1-0.6^{16}\right)\left(1-0.6^{19}\right)}$$

Result

$3.05646\,x = 3.05801$

<u>The study of this function provides the following representations</u>:

Plot



Alternate form

$3.05646\,x - 3.05801 = 0$

Alternate form assuming x is real

$3.05646\,x + 0 = 3.05801$

Solution

$x \approx 1.00051$

1.00051

Indeed:

((1+0.6/(1-0.6)+(0.6^4)/((1-0.6)(1-0.6^2))+(0.6^9)/((1-0.6)(1-0.6^2)(1-0.6^3))))*(e^π-10π+7log(π)+tan^(-1)(π)) where (e^π-10π+7log(π)+tan^(-1)(π)) ≈ 1.00051

Input

$$\left(1+\frac{0.6}{1-0.6}+\frac{0.6^4}{(1-0.6)\left(1-0.6^2\right)}+\frac{0.6^9}{(1-0.6)\left(1-0.6^2\right)\left(1-0.6^3\right)}\right)\left(e^\pi-10\pi+7\log(\pi)+\tan^{-1}(\pi)\right)$$

log(x) is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

Result

3.05800…

(result in radians)

3.058….

The study of this function provides the following representations:

Alternative representations

$$\left(1+\frac{0.6}{1-0.6}+\frac{0.6^4}{(1-0.6)\left(1-0.6^2\right)}+\frac{0.6^9}{(1-0.6)\left(1-0.6^2\right)\left(1-0.6^3\right)}\right)\left(e^\pi-10\pi+7\log(\pi)+\tan^{-1}(\pi)\right)=\left(-10\pi+\tan^{-1}(1,\pi)+7\log(\pi)+e^\pi\right)\left(1+\frac{0.6}{0.4}+\frac{0.6^4}{0.4\left(1-0.6^2\right)}+\frac{0.6^9}{0.4\left(1-0.6^2\right)\left(1-0.6^3\right)}\right)$$

$$\left(1+\frac{0.6}{1-0.6}+\frac{0.6^4}{(1-0.6)\left(1-0.6^2\right)}+\frac{0.6^9}{(1-0.6)\left(1-0.6^2\right)\left(1-0.6^3\right)}\right)\left(e^\pi-10\pi+7\log(\pi)+\tan^{-1}(\pi)\right)=\left(-10\pi+\tan^{-1}(\pi)+7\log_e(\pi)+e^\pi\right)\left(1+\frac{0.6}{0.4}+\frac{0.6^4}{0.4\left(1-0.6^2\right)}+\frac{0.6^9}{0.4\left(1-0.6^2\right)\left(1-0.6^3\right)}\right)$$

$$\left(1+\frac{0.6}{1-0.6}+\frac{0.6^4}{(1-0.6)\left(1-0.6^2\right)}+\frac{0.6^9}{(1-0.6)\left(1-0.6^2\right)\left(1-0.6^3\right)}\right)\left(e^\pi-10\,\pi+7\log(\pi)+\tan^{-1}(\pi)\right)=\left(-10\,\pi+\tan^{-1}(\pi)+7\log(a)\log_a(\pi)+e^\pi\right)\left(1+\frac{0.6}{0.4}+\frac{0.6^4}{0.4\left(1-0.6^2\right)}+\frac{0.6^9}{0.4\left(1-0.6^2\right)\left(1-0.6^3\right)}\right)$$

Series representations

$$\left(1+\frac{0.6}{1-0.6}+\frac{0.6^4}{(1-0.6)\left(1-0.6^2\right)}+\frac{0.6^9}{(1-0.6)\left(1-0.6^2\right)\left(1-0.6^3\right)}\right)\left(e^\pi-10\,\pi+7\log(\pi)+\tan^{-1}(\pi)\right)=3.05646\,e^\pi-30.5646\,\pi+3.05646\tan^{-1}(\pi)+21.3952\log(-1+\pi)-21.3952\sum_{k=1}^{\infty}\frac{(-1)^k(-1+\pi)^{-k}}{k}$$

$$\left(1+\frac{0.6}{1-0.6}+\frac{0.6^4}{(1-0.6)\left(1-0.6^2\right)}+\frac{0.6^9}{(1-0.6)\left(1-0.6^2\right)\left(1-0.6^3\right)}\right)\left(e^\pi-10\,\pi+7\log(\pi)+\tan^{-1}(\pi)\right)=3.05646\,e^\pi-30.5646\,\pi+21.3952\log(\pi)+\frac{1.52823\,\pi^2}{\sqrt{\pi^2}}-3.05646\sum_{k=0}^{\infty}\frac{(-1)^k\,\pi^{-1-2k}}{1+2k}$$

$$\left(1+\frac{0.6}{1-0.6}+\frac{0.6^4}{(1-0.6)\left(1-0.6^2\right)}+\frac{0.6^9}{(1-0.6)\left(1-0.6^2\right)\left(1-0.6^3\right)}\right)\left(e^\pi-10\,\pi+7\log(\pi)+\tan^{-1}(\pi)\right)=3.05646\,e^\pi-30.5646\,\pi+21.3952\log(\pi)+3.05646\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{5}\right)^k 2^{1+2k}\,F_{1+2k}\left(\frac{\pi}{1+\sqrt{1+\frac{4\pi^2}{5}}}\right)^{1+2k}}{1+2k}$$

F_n is the n^{th} Fibonacci number

Integral representations

$$\left(1+\frac{0.6}{1-0.6}+\frac{0.6^4}{(1-0.6)\left(1-0.6^2\right)}+\frac{0.6^9}{(1-0.6)\left(1-0.6^2\right)\left(1-0.6^3\right)}\right)\left(e^\pi-10\pi+7\log(\pi)+\tan^{-1}(\pi)\right)=$$
$$3.05646\,e^\pi-30.5646\,\pi+3.05646\,\pi\int_0^1\frac{1}{1+\pi^2t^2}\,dt+21.3952\log(\pi)$$

$$\left(1+\frac{0.6}{1-0.6}+\frac{0.6^4}{(1-0.6)\left(1-0.6^2\right)}+\frac{0.6^9}{(1-0.6)\left(1-0.6^2\right)\left(1-0.6^3\right)}\right)\left(e^\pi-10\pi+7\log(\pi)+\tan^{-1}(\pi)\right)=$$
$$3.05646\,e^\pi-30.5646\,\pi+\int_1^\pi\left(\frac{21.3952}{t}+\frac{3.05646\,(-1+\pi)\,\pi}{1-2\pi+\pi^2\left(2-2t+t^2\right)}\right)dt$$

$$\left(1+\frac{0.6}{1-0.6}+\frac{0.6^4}{(1-0.6)\left(1-0.6^2\right)}+\frac{0.6^9}{(1-0.6)\left(1-0.6^2\right)\left(1-0.6^3\right)}\right)\left(e^\pi-10\pi+7\log(\pi)+\tan^{-1}(\pi)\right)=3.05646\,e^\pi-30.5646\,\pi-$$
$$\frac{0.764115\,i}{\sqrt{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(1+\pi^2\right)^{-s}\,\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds+$$
$$21.3952\log(\pi)\quad\text{for } 0<\gamma<\frac{1}{2}$$

Continued fraction representations

$$\left(1+\frac{0.6}{1-0.6}+\frac{0.6^4}{(1-0.6)\left(1-0.6^2\right)}+\frac{0.6^9}{(1-0.6)\left(1-0.6^2\right)\left(1-0.6^3\right)}\right)$$

$$\left(e^\pi-10\pi+7\log(\pi)+\tan^{-1}(\pi)\right)=$$

$$3.05646\left(e^\pi+7\log(\pi)+\pi\left(-10+\frac{1}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{k^2\pi^2}{1+2k}}\right)\right)=$$

$$3.05646\left(e^\pi+7\log(\pi)+\pi\left(-10+\cfrac{1}{1+\cfrac{\pi^2}{3+\cfrac{4\pi^2}{5+\cfrac{9\pi^2}{7+\cfrac{16\pi^2}{9+\dots}}}}}\right)\right)$$

$$\left(1+\frac{0.6}{1-0.6}+\frac{0.6^4}{(1-0.6)\left(1-0.6^2\right)}+\frac{0.6^9}{(1-0.6)\left(1-0.6^2\right)\left(1-0.6^3\right)}\right)$$

$$\left(e^\pi-10\pi+7\log(\pi)+\tan^{-1}(\pi)\right)=$$

$$3.05646\left(e^\pi-9\pi+7\log(\pi)-\frac{\pi^3}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\left(1+(-1)^{1+k}+k\right)^2\pi^2}{3+2k}}\right)=$$

$$3.05646\left(e^\pi-9\pi+7\log(\pi)-\cfrac{\pi^3}{3+\cfrac{9\pi^2}{5+\cfrac{4\pi^2}{7+\cfrac{25\pi^2}{9+\cfrac{16\pi^2}{11+\dots}}}}}\right)$$

$$\left(1+\frac{0.6}{1-0.6}+\frac{0.6^4}{(1-0.6)(1-0.6^2)}+\frac{0.6^9}{(1-0.6)(1-0.6^2)(1-0.6^3)}\right)\left(e^\pi-10\pi+7\log(\pi)+\tan^{-1}(\pi)\right)=$$

$$3.05646\left(e^\pi-10\pi+\frac{\pi}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{k^2\pi^2}{1+2k}}+\frac{7(-1+\pi)}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{(-1+\pi)\left\lfloor\frac{1+k}{2}\right\rfloor^2}{1+k}}\right)=$$

$$3.05646\left(e^\pi-10\pi+\frac{\pi}{1+\frac{\pi^2}{3+\frac{4\pi^2}{5+\frac{9\pi^2}{7+\frac{16\pi^2}{9+\dots}}}}}+\frac{7(-1+\pi)}{1+\frac{-1+\pi}{2+\frac{-1+\pi}{3+\frac{4(-1+\pi)}{4+\frac{4(-1+\pi)}{5+\dots}}}}}\right)$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

For q = 0.5 , we obtain:

1+0.5/(1-0.5)+(0.5^4)/((1-0.5)(1-0.5^2))+(0.5^9)/((1-0.5)(1-0.5^2)(1-0.5^3))

Input

$$1+\frac{0.5}{1-0.5}+\frac{0.5^4}{(1-0.5)(1-0.5^2)}+\frac{0.5^9}{(1-0.5)(1-0.5^2)(1-0.5^3)}$$

Result

2.172619047619047619047619047619047619047619047619047619047619047619047619047619047 6…

2.172619047619….

And:

1/((1-0.5) (1-0.5^4) (1-0.5^6) (1-0.5^9) (1-0.5^11) (1-0.5^14) (1-0.5^16) (1-0.5^19))

Input

$$\frac{1}{(1-0.5)\left(1-0.5^{4}\right)\left(1-0.5^{6}\right)\left(1-0.5^{9}\right)\left(1-0.5^{11}\right)\left(1-0.5^{14}\right)\left(1-0.5^{16}\right)\left(1-0.5^{19}\right)}$$

Result

2.172667547742295869006321667636102731816287150646429822785037772 4
...

2.172667547742….

We have:

1+0.5/(1-0.5)+(0.5^4)/((1-0.5)(1-0.5^2))+(0.5^9)/((1-0.5)(1-0.5^2)(1-0.5^3)) *x = 1/((1-0.5) (1-0.5^4) (1-0.5^6) (1-0.5^9) (1-0.5^11) (1-0.5^14) (1-0.5^16) (1-0.5^19))

Input

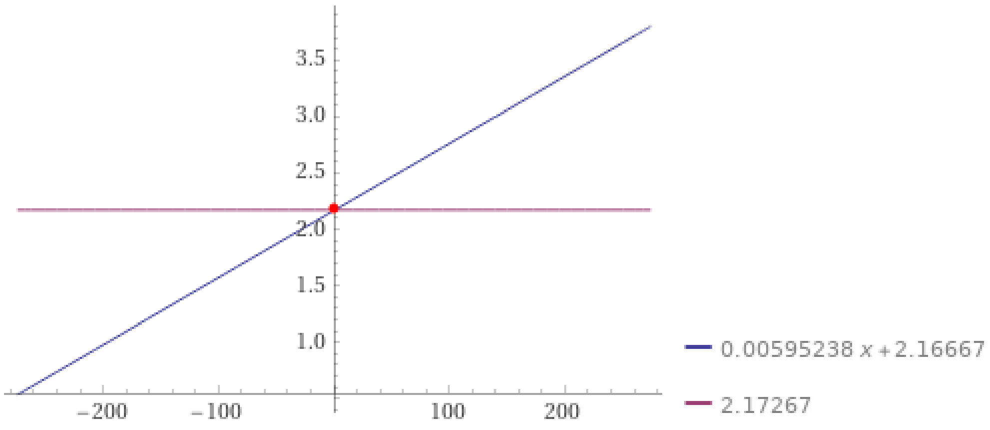
$$1+\frac{0.5}{1-0.5}+\frac{0.5^{4}}{(1-0.5)\left(1-0.5^{2}\right)}+\frac{0.5^{9}}{(1-0.5)\left(1-0.5^{2}\right)\left(1-0.5^{3}\right)}\,x=\frac{1}{(1-0.5)\left(1-0.5^{4}\right)\left(1-0.5^{6}\right)\left(1-0.5^{9}\right)\left(1-0.5^{11}\right)\left(1-0.5^{14}\right)\left(1-0.5^{16}\right)\left(1-0.5^{19}\right)}$$

Result

$0.00595238\,x+2.16667=2.17267$

The study of this function provides the following representations:

Plot



Alternate forms

$0.00595238\,x - 0.00600088 = 0$

$0.00595238\,(x + 364) = 2.17267$

Solution

$x \approx 1.00815$

1.00815

Indeed:

1+0.5/(1-0.5)+(0.5^4)/((1-0.5)(1-0.5^2))+(0.5^9)/((1-0.5)(1-0.5^2)(1-0.5^3)) * ((log(81/4) - 2))

where ((log(81/4) - 2)) ≈ 1.00815

Input

$$1+\frac{0.5}{1-0.5}+\frac{0.5^4}{(1-0.5)\left(1-0.5^2\right)}+\frac{0.5^9}{(1-0.5)\left(1-0.5^2\right)\left(1-0.5^3\right)}\left(\log\left(\frac{81}{4}\right)-2\right)$$

$\log(x)$ is the natural logarithm

Result

2.17267…

2.17267….

<u>The study of this function provides the following representations</u>:

Alternative representations

$$1+\frac{0.5}{1-0.5}+\frac{0.5^4}{(1-0.5)\left(1-0.5^2\right)}+\frac{\left(\log\left(\frac{81}{4}\right)-2\right)0.5^9}{(1-0.5)\left(1-0.5^2\right)\left(1-0.5^3\right)}= 1+\frac{0.5}{0.5}+\frac{0.5^4}{0.5\left(1-0.5^2\right)}+\frac{\left(-2+\log_e\left(\frac{81}{4}\right)\right)0.5^9}{0.5\left(1-0.5^2\right)\left(1-0.5^3\right)}$$

$$1+\frac{0.5}{1-0.5}+\frac{0.5^4}{(1-0.5)\left(1-0.5^2\right)}+\frac{\left(\log\left(\frac{81}{4}\right)-2\right)0.5^9}{(1-0.5)\left(1-0.5^2\right)\left(1-0.5^3\right)}= 1+\frac{0.5}{0.5}+\frac{0.5^4}{0.5\left(1-0.5^2\right)}+\frac{\left(-2+\log(a)\log_a\left(\frac{81}{4}\right)\right)0.5^9}{0.5\left(1-0.5^2\right)\left(1-0.5^3\right)}$$

$$1+\frac{0.5}{1-0.5}+\frac{0.5^4}{(1-0.5)\left(1-0.5^2\right)}+\frac{\left(\log\left(\frac{81}{4}\right)-2\right)0.5^9}{(1-0.5)\left(1-0.5^2\right)\left(1-0.5^3\right)}= 1+\frac{0.5}{0.5}+\frac{0.5^4}{0.5\left(1-0.5^2\right)}+\frac{\left(-2-\text{Li}_1\left(1-\frac{81}{4}\right)\right)0.5^9}{0.5\left(1-0.5^2\right)\left(1-0.5^3\right)}$$

Series representations

$$1+\frac{0.5}{1-0.5}+\frac{0.5^4}{(1-0.5)\left(1-0.5^2\right)}+\frac{\left(\log\left(\frac{81}{4}\right)-2\right)0.5^9}{(1-0.5)\left(1-0.5^2\right)\left(1-0.5^3\right)}=$$
$$2.15476+0.00595238\log\left(\frac{77}{4}\right)-0.00595238\sum_{k=1}^{\infty}\frac{\left(-\frac{4}{77}\right)^k}{k}$$

$$1+\frac{0.5}{1-0.5}+\frac{0.5^4}{(1-0.5)\left(1-0.5^2\right)}+\frac{\left(\log\left(\frac{81}{4}\right)-2\right)0.5^9}{(1-0.5)\left(1-0.5^2\right)\left(1-0.5^3\right)}=$$
$$2.15476+0.0119048\,i\,\pi\left\lfloor\frac{\arg\left(\frac{81}{4}-x\right)}{2\pi}\right\rfloor+0.00595238\log(x)-$$
$$0.00595238\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{81}{4}-x\right)^k x^{-k}}{k}\quad\text{for } x<0$$

$$1+\frac{0.5}{1-0.5}+\frac{0.5^4}{(1-0.5)\left(1-0.5^2\right)}+\frac{\left(\log\left(\frac{81}{4}\right)-2\right)0.5^9}{(1-0.5)\left(1-0.5^2\right)\left(1-0.5^3\right)}=$$
$$2.15476+0.00595238\left\lfloor\frac{\arg\left(\frac{81}{4}-z_0\right)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+0.00595238\log(z_0)+$$
$$0.00595238\left\lfloor\frac{\arg\left(\frac{81}{4}-z_0\right)}{2\pi}\right\rfloor\log(z_0)-0.00595238\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{81}{4}-z_0\right)^k z_0^{-k}}{k}$$

Integral representations

$$1+\frac{0.5}{1-0.5}+\frac{0.5^4}{(1-0.5)\left(1-0.5^2\right)}+\frac{\left(\log\left(\frac{81}{4}\right)-2\right)0.5^9}{(1-0.5)\left(1-0.5^2\right)\left(1-0.5^3\right)}=$$
$$2.15476+0.00595238\int_1^{\frac{81}{4}}\frac{1}{t}\,dt$$

$$1+\frac{0.5}{1-0.5}+\frac{0.5^4}{(1-0.5)\left(1-0.5^2\right)}+\frac{\left(\log\left(\frac{81}{4}\right)-2\right)0.5^9}{(1-0.5)\left(1-0.5^2\right)\left(1-0.5^3\right)}=$$

$$2.15476+\frac{0.00297619}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\left(\frac{4}{77}\right)^s\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \text{ for } -1<\gamma<0$$

Hence, the identity is verified the more q tends to 0 (q = 8, q = 0.8, q = 0.6, q = 0.5, ..., and so on)

Now, we analyzing in more detail the below identity:

1+q/(1-q)+(q^4)/((1-q)(1-q^2))+(q^9)/((1-q)(1-q^2)(1-q^3)) = 1/((1-q) (1-q^4) (1-q^6) (1-q^9) (1-q^11) (1-q^14) (1-q^16) (1-q^19))

Input

$$1+\frac{q}{1-q}+\frac{q^4}{(1-q)\left(1-q^2\right)}+\frac{q^9}{(1-q)\left(1-q^2\right)\left(1-q^3\right)}=$$
$$\frac{1}{(1-q)\left(1-q^4\right)\left(1-q^6\right)\left(1-q^9\right)\left(1-q^{11}\right)\left(1-q^{14}\right)\left(1-q^{16}\right)\left(1-q^{19}\right)}$$

Real solutions

$q=0$

$q\approx-1.15616$

Complex solutions

$q \approx -1.02475 - 0.02369\, i$

$q \approx -1.02475 + 0.02369\, i$

$q \approx -0.97872 - 0.16104\, i$

$q \approx -0.97872 + 0.16104\, i$

$q \approx -0.96193 - 0.29400\, i$

Integer solution

$q = 0$

Numerical solution

$q \approx -1.15615784536085\ldots$

Subtracting the right-hand side to the left-hand side, we obtain the following expression:

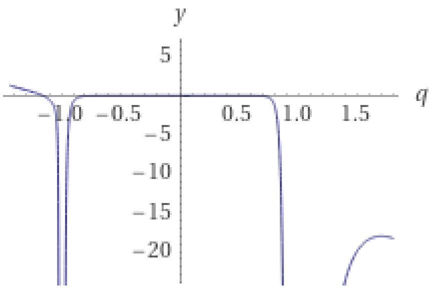
1+q/(1-q)+(q^4)/((1-q)(1-q^2))+(q^9)/((1-q)(1-q^2)(1-q^3)) - ((1/((1-q) (1-q^4) (1-q^6) (1-q^9) (1-q^11) (1-q^14) (1-q^16) (1-q^19))))

Input

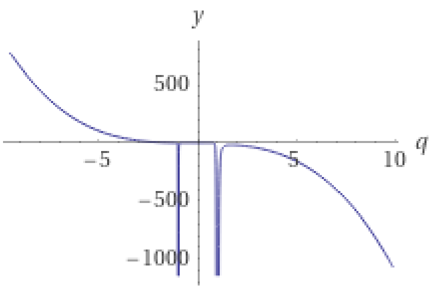
$$1+\frac{q}{1-q}+\frac{q^4}{(1-q)(1-q^2)}+\frac{q^9}{(1-q)(1-q^2)(1-q^3)}-\frac{1}{(1-q)(1-q^4)(1-q^6)(1-q^9)(1-q^{11})(1-q^{14})(1-q^{16})(1-q^{19})}$$

The study of this function provides the following representations:

Plots **(figures that can be related to the open strings)**



(q from -1.4 to 1.8)



(q from -9.5 to 9.8)

Alternate forms

$$\frac{q^4}{(q-1)^2(q+1)}-\frac{q^9}{(q-1)^3(q+1)(q^2+q+1)}-\frac{1}{(q-1)(q^4-1)(q^6-1)(q^9-1)(q^{11}-1)(q^{14}-1)(q^{16}-1)(q^{19}-1)}+\frac{q}{1-q}+1$$

$$-\left(\left(q^{16}\left(q^{67}+q^{64}+q^{62}-q^{58}-q^{56}-q^{55}-2q^{53}-2q^{51}-q^{50}-3q^{48}+q^{47}-q^{46}-q^{45}+q^{44}-q^{43}+2q^{42}+q^{41}+q^{40}+3q^{39}+4q^{37}+q^{36}+q^{35}+3q^{34}+3q^{32}+2q^{29}-2q^{28}+q^{27}-q^{26}-2q^{25}-3q^{23}-q^{22}-2q^{21}-2q^{20}-q^{19}-3q^{18}-q^{16}-q^{15}-q^{13}+q^{12}+q^{9}+q^{7}+q^{6}+q^{4}+q^{3}+q^{2}+1\right)\right)/\left((q-1)^8(q+1)^4(q^2+1)^2(q^2-q+1)(q^2+q+1)^2(q^4+1)(q^6+q^3+1)(q^6-q^5+q^4-q^3+q^2-q+1)(q^6+q^5+q^4+q^3+q^2+q+1)(q^8+1)(q^{10}+q^9+q^8+q^7+q^6+q^5+q^4+q^3+q^2+q+1)(q^{18}+q^{17}+q^{16}+q^{15}+q^{14}+q^{13}+q^{12}+q^{11}+q^{10}+q^9+q^8+q^7+q^6+q^5+q^4+q^3+q^2+q+1)\right)\right)$$

$$-q^3-q^2-\frac{q}{486(q^2+q+1)^2}+\frac{-5q-19}{256(q^2+1)}-\frac{1}{36(q^2-q+1)}+\frac{7(24q+79)}{2916(q^2+q+1)}-\frac{1}{256(q^2+1)^2}-\frac{q^2}{32(q^4+1)}+\frac{-q^6-q^2}{8(q^8+1)}+\frac{-q^5-q^2}{9(q^6+q^3+1)}+\frac{-2q^5+q^3-1}{14(q^6-q^5+q^4-q^3+q^2-q+1)}+\frac{2q^5+q^3-2q^2-2q+1}{98(q^6+q^5+q^4+q^3+q^2+q+1)}+\frac{-q^9-q^7-2q^6-q^5-q^4-q^3-q^2-2q-1}{11(q^{10}+q^9+q^8+q^7+q^6+q^5+q^4+q^3+q^2+q+1)}-q-\frac{14\,629\,339\,271}{5\,096\,577\,024\,(q-1)}+\frac{739}{14\,336\,(q+1)}-\frac{26\,662\,281\,473}{30\,579\,462\,144\,(q-1)^2}-\frac{1949}{258\,048\,(q+1)^2}-\frac{585\,437}{4\,257\,792\,(q-1)^3}-\frac{19}{43\,008\,(q+1)^3}-\frac{2\,508\,965}{485\,388\,288\,(q-1)^4}-\frac{1}{86\,016\,(q+1)^4}+\frac{40\,811}{60\,673\,536\,(q-1)^5}-\frac{533}{8\,667\,648\,(q-1)^6}+\frac{1}{280\,896\,(q-1)^7}-\frac{1}{10\,112\,256\,(q-1)^8}+\left(2q^{17}+2q^{16}-2q^{14}-4q^{13}-2q^{12}-q^{11}-5q^{10}-4q^9-2q^8-2q^7-2q^6-4q^5-5q^4-q^3-2q^2-4q-2\right)/\left(19\left(q^{18}+q^{17}+q^{16}+q^{15}+q^{14}+q^{13}+q^{12}+q^{11}+q^{10}+q^9+q^8+q^7+q^6+q^5+q^4+q^3+q^2+q+1\right)\right)-2$$

Derivative

$$\frac{d}{dq}\left(1+\frac{q}{1-q}+\frac{q^4}{(1-q)(1-q^2)}+\frac{q^9}{(1-q)(1-q^2)(1-q^3)}-\frac{1}{(1-q)(1-q^4)(1-q^6)(1-q^9)(1-q^{11})(1-q^{14})(1-q^{16})(1-q^{19})}\right)=$$

$$-\frac{19q^{18}}{(1-q)(1-q^4)(1-q^6)(1-q^9)(1-q^{11})(1-q^{14})(1-q^{16})(1-q^{19})^2}-\frac{16q^{15}}{(1-q)(1-q^4)(1-q^6)(1-q^9)(1-q^{11})(1-q^{14})(1-q^{16})^2(1-q^{19})}-\frac{14q^{13}}{(1-q)(1-q^4)(1-q^6)(1-q^9)(1-q^{11})(1-q^{14})^2(1-q^{16})(1-q^{19})}+\frac{3q^{11}}{(1-q)(1-q^2)(1-q^3)^2}+\frac{2q^{10}}{(1-q)(1-q^2)^2(1-q^3)}-\frac{11q^{10}}{(1-q)(1-q^4)(1-q^6)(1-q^9)(1-q^{11})^2(1-q^{14})(1-q^{16})(1-q^{19})}+\frac{q^9}{(1-q)^2(1-q^2)(1-q^3)}+\frac{9q^8}{(1-q)(1-q^2)(1-q^3)}-\frac{9q^8}{(1-q)(1-q^4)(1-q^6)(1-q^9)^2(1-q^{11})(1-q^{14})(1-q^{16})(1-q^{19})}-\frac{6q^5}{(1-q)(1-q^4)(1-q^6)^2(1-q^9)(1-q^{11})(1-q^{14})(1-q^{16})(1-q^{19})}+\frac{2q^5}{(1-q)(1-q^2)^2}+\frac{q^4}{(1-q)^2(1-q^2)}+\frac{4q^3}{(1-q)(1-q^2)}-\frac{4q^3}{(1-q)(1-q^4)^2(1-q^6)(1-q^9)(1-q^{11})(1-q^{14})(1-q^{16})(1-q^{19})}+\frac{q}{(1-q)^2}+\frac{1}{1-q}-\frac{1}{(1-q)^2(1-q^4)(1-q^6)(1-q^9)(1-q^{11})(1-q^{14})(1-q^{16})(1-q^{19})}$$

For q = 0.5 , from:

$$-q^3 - q^2 - \frac{q}{486(q^2+q+1)^2} + \frac{-5q-19}{256(q^2+1)} - \frac{1}{36(q^2-q+1)} + \frac{7(24q+79)}{2916(q^2+q+1)} - \frac{1}{256(q^2+1)^2} - \frac{q^2}{32(q^4+1)} + \frac{-q^6-q^2}{8(q^8+1)} + \frac{-q^5-q^2}{9(q^6+q^3+1)} + \frac{-2q^5+q^3-1}{14(q^6-q^5+q^4-q^3+q^2-q+1)} + \frac{2q^5+q^3-2q^2-2q+1}{98(q^6+q^5+q^4+q^3+q^2+q+1)} + \frac{-q^9-q^7-2q^6-q^5-q^4-q^3-q^2-2q-1}{11(q^{10}+q^9+q^8+q^7+q^6+q^5+q^4+q^3+q^2+q+1)} - q - \frac{14\,629\,339\,271}{5\,096\,577\,024\,(q-1)} + \frac{739}{14\,336(q+1)} - \frac{26\,662\,281\,473}{30\,579\,462\,144\,(q-1)^2} - \frac{1949}{258\,048\,(q+1)^2} - \frac{585\,437}{4\,257\,792(q-1)^3} - \frac{19}{43\,008\,(q+1)^3} - \frac{2\,508\,965}{485\,388\,288\,(q-1)^4} - \frac{1}{86\,016(q+1)^4} + \frac{40\,811}{60\,673\,536\,(q-1)^5} - \frac{533}{8\,667\,648\,(q-1)^6} + \frac{1}{280\,896(q-1)^7} - \frac{1}{10\,112\,256\,(q-1)^8} + (2q^{17}+2q^{16}-2q^{14}-4q^{13}-2q^{12}-q^{11}-5q^{10}-4q^9-2q^8-2q^7-2q^6-4q^5-5q^4-q^3-2q^2-4q-2)\big/(19(q^{18}+q^{17}+q^{16}+q^{15}+q^{14}+q^{13}+q^{12}+q^{11}+q^{10}+q^9+q^8+q^7+q^6+q^5+q^4+q^3+q^2+q+1)) - 2$$

i.e.:

-q^3 - q^2 - q/(486 (q^2 + q + 1)^2) + (-5 q - 19)/(256 (q^2 + 1)) - 1/(36 (q^2 - q + 1)) + (7 (24 q + 79))/(2916 (q^2 + q + 1)) - 1/(256 (q^2 + 1)^2) - q^2/(32 (q^4 + 1)) + (-q^6 - q^2)/(8 (q^8 + 1)) + (-q^5 - q^2)/(9 (q^6 + q^3 + 1)) + (-2 q^5 + q^3 - 1)/(14 (q^6 - q^5 + q^4 - q^3 + q^2 - q + 1)) + (2 q^5 + q^3 - 2 q^2 - 2 q + 1)/(98 (q^6 + q^5 + q^4 + q^3 + q^2 + q + 1)) + (-q^9 - q^7 - 2 q^6 - q^5 - q^4 - q^3 - q^2 - 2 q - 1)/(11 (q^10 + q^9 + q^8 + q^7 + q^6 + q^5 + q^4 + q^3 + q^2 + q + 1)) - q - 14629339271/(5096577024 (q - 1)) + 739/(14336 (q + 1)) - 26662281473/(30579462144 (q - 1)^2) - 1949/(258048 (q + 1)^2) - 585437/(4257792 (q - 1)^3) - 19/(43008 (q + 1)^3) - 2508965/(485388288 (q - 1)^4) - 1/(86016 (q + 1)^4) + 40811/(60673536 (q - 1)^5) - 533/(8667648 (q - 1)^6) + 1/(280896 (q - 1)^7) - 1/(10112256 (q - 1)^8) + (2 q^17 + 2 q^16 - 2 q^14 - 4 q^13 - 2 q^12 - q^11 - 5 q^10 - 4 q^9 - 2 q^8 - 2 q^7 - 2 q^6 - 4 q^5 - 5 q^4 - q^3 - 2 q^2 - 4 q - 2)/(19 (q^18 + q^17 + q^16 + q^15 + q^14 + q^13 + q^12 + q^11 + q^10 + q^9 + q^8 + q^7 + q^6 + q^5 + q^4 + q^3 + q^2 + q + 1)) – 2

We divide the above hard expression as follows:

-0.5^3 – 0.5^2 – 0.5/(486 (0.5^2 + 0.5 + 1)^2) + (-5*0.5 - 19)/(256 (0.5^2 + 1)) - 1/(36 (0.5^2 – 0.5 + 1)) + (7 (24*0.5 + 79))/(2916 (0.5^2 + 0.5 + 1))

Input

$$-0.5^3 - 0.5^2 - \frac{0.5}{486\left(0.5^2 + 0.5 + 1\right)^2} + \frac{-5 \times 0.5 - 19}{256\left(0.5^2 + 1\right)} - \frac{1}{36\left(0.5^2 - 0.5 + 1\right)} + \frac{7\,(24 \times 0.5 + 79)}{2916\left(0.5^2 + 0.5 + 1\right)}$$

Result

$-0.3547319416449707454998460289465580470871476162481453486744492 03$
…

- 1/(256 (0.5^2 + 1)^2) – 0.5^2/(32 (0.5^4 + 1)) + (-0.5^6 – 0.5^2)/(8 (0.5^8 + 1)) + (-0.5^5 – 0.5^2)/(9 (0.5^6 + 0.5^3 + 1))

Input interpretation

$$-0.35473194164497 - \frac{1}{256\left(0.5^2 + 1\right)^2} - \frac{0.5^2}{32\left(0.5^4 + 1\right)} + \frac{-0.5^6 - 0.5^2}{8\left(0.5^8 + 1\right)} + \frac{-0.5^5 - 0.5^2}{9\left(0.5^6 + 0.5^3 + 1\right)}$$

Result

$-0.4250560730565026851384442695579377745448160608521432132364698 98$
…

-0.4250560730565+(-2*0.5^5 + 0.5^3 - 1)/(14 (0.5^6 – 0.5^5 + 0.5^4 – 0.5^3 + 0.5^2 – 0.5 + 1)) + (2*0.5^5 + 0.5^3 – 2*0.5^2 – 2*0.5 + 1)/(98 (0.5^6 + 0.5^5 + 0.5^4 + 0.5^3 + 0.5^2 + 0.5 + 1))

Input interpretation

$$-0.4250560730565 + \frac{-2\times 0.5^5 + 0.5^3 - 1}{14\left(0.5^6 - 0.5^5 + 0.5^4 - 0.5^3 + 0.5^2 - 0.5 + 1\right)} + \frac{2\times 0.5^5 + 0.5^3 - 2\times 0.5^2 - 2\times 0.5 + 1}{98\left(0.5^6 + 0.5^5 + 0.5^4 + 0.5^3 + 0.5^2 + 0.5 + 1\right)}$$

Result

$-0.52633078913227292041152663226066841312610010127471607577292041\ldots$

-0.5263307891322729204+ (-0.5^9–0.5^7–2*0.5^6–0.5^5–0.5^4–0.5^3–0.5^2–2*0.5-1)/(11(0.5^10+0.5^9+0.5^8+0.5^7+0.5^6+0.5^5+0.5^4+0.5^3+0.5^2+0.5+1))

Input interpretation

$$-0.5263307891322729204 + \frac{-0.5^9 - 0.5^7 - 2\times 0.5^6 - 0.5^5 - 0.5^4 - 0.5^3 - 0.5^2 - 2\times 0.5 - 1}{11\left(0.5^{10} + 0.5^9 + 0.5^8 + 0.5^7 + 0.5^6 + 0.5^5 + 0.5^4 + 0.5^3 + 0.5^2 + 0.5 + 1\right)}$$

Result

$-0.64046677527607538076328107651996269485277790114135986143802460\ldots$

-0.64046677527607538-0.5-14629339271/(5096577024(0.5-1))+739/(14336(0.5+1))-26662281473/(30579462144(0.5-1)^2)-1949/(258048(0.5+1)^2)

Input interpretation

$$-0.64046677527607538 - 0.5 - \frac{14629339271}{5096577024\,(0.5-1)} + \frac{739}{14336\,(0.5+1)} - \frac{26662281473}{30579462144\,(0.5-1)^2} - \frac{1949}{258048\,(0.5+1)^2}$$

Result

$1.1437846514844017842644101569192073229945688870779374817247276172\ldots$

1.14378465148440178-585437/(4257792(0.5-1)^3)-19/(43008(0.5+1)^3)-2508965/(485388288(0.5-1)^4)-1/(86016(0.5+1)^4)+40811/(60673536(0.5- 1)^5)-533/(8667648 (0.5-1)^6)+1/(280896(0.5-1)^7)

Input interpretation

$$1.14378465148440178 - \frac{585437}{4257792\,(0.5-1)^3} - \frac{19}{43008\,(0.5+1)^3} - \frac{2508965}{485388288\,(0.5-1)^4} - \frac{1}{86016\,(0.5+1)^4} + \frac{40811}{60673536\,(0.5-1)^5} - \frac{533}{8667648\,(0.5-1)^6} + \frac{1}{280896\,(0.5-1)^7}$$

Result

2.135014545629135106019875727478066659353209060811399992686542394 1…

2.1350145456291351- 1/(10112256 (0.5 - 1)^8) + (2*0.5^17 + 2 0.5^16 - 2 0.5^14 - 4 0.5^13 - 2 0.5^12 – 0.5^11 - 5 0.5^10 - 4 0.5^9 - 2 0.5^8 - 2 0.5^7 - 2 0.5^6 - 4 0.5^5 - 5 0.5^4 – 0.5^3 - 2 0.5^2 - 4 0.5 - 2)/(19 (0.5^18 + 0.5^17 + 0.5^16 + 0.5^15 + 0.5^14 + 0.5^13 + 0.5^12 + 0.5^11 + 0.5^10 + 0.5^9 + 0.5^8 + 0.5^7 + 0.5^6 + 0.5^5 + 0.5^4 + 0.5^3 + 0.5^2 + 0.5 + 1)) – 2

From the terms highlighted in red, we obtain:

Input

$$19\left(0.5^{18} + 0.5^{17} + 0.5^{16} + 0.5^{15} + 0.5^{14} + 0.5^{13} + 0.5^{12} + 0.5^{11} + 0.5^{10} + 0.5^{9} + 0.5^{8} + 0.5^{7} + 0.5^{6} + 0.5^{5} + 0.5^{4} + 0.5^{3} + 0.5^{2} + 0.5 + 1\right)$$

Result

37.999927520751953125

Thence:

2.1350145456- 1/(10112256 (0.5 - 1)^8)+(2*0.5^17+2 0.5^16-2 0.5^14-4 0.5^13-2 0.5^12-0.5^11-5 0.5^10-4 0.5^9-2 0.5^8-2 0.5^7-2 0.5^6-4 0.5^5-5 0.5^4 -0.5^3-2 0.5^2-4 0.5-2)/ 37.99992752075-2

Input interpretation

$$2.1350145456-\frac{1}{10112256\,(0.5-1)^8}+\frac{1}{37.99992752075}(2\times 0.5^{17}+2\times 0.5^{16}-2\times 0.5^{14}-4\times 0.5^{13}-2\times 0.5^{12}-0.5^{11}-5\times 0.5^{10}-4\times 0.5^{9}-2\times 0.5^{8}-2\times 0.5^{7}-2\times 0.5^{6}-4\times 0.5^{5}-5\times 0.5^{4}-0.5^{3}-2\times 0.5^{2}-4\times 0.5-2)-2$$

Result

−0.000048500152386869515925762251037719116967629259294018015924260
…

-0.000048500152386869515925762251037719116967629259294018015924260

Inverting the above expression, we obtain:

-1/((2.1350145456- 1/(10112256 (0.5 - 1)^8)+(2*0.5^17+2 0.5^16-2 0.5^14-4 0.5^13-2 0.5^12-0.5^11-5 0.5^10-4 0.5^9-2 0.5^8-2 0.5^7-2 0.5^6-4 0.5^5-5 0.5^4 -0.5^3-2 0.5^2-4 0.5-2)/ 37.99992752075-2))

Input interpretation

$$-\left(1\middle/\left(2.1350145456-\frac{1}{10112256\,(0.5-1)^8}+\frac{1}{37.99992752075}(2\times 0.5^{17}+2\times 0.5^{16}-2\times 0.5^{14}-4\times 0.5^{13}-2\times 0.5^{12}-0.5^{11}-5\times 0.5^{10}-4\times 0.5^{9}-2\times 0.5^{8}-2\times 0.5^{7}-2\times 0.5^{6}-4\times 0.5^{5}-5\times 0.5^{4}-0.5^{3}-2\times 0.5^{2}-4\times 0.5-2)-2\right)\right)$$

Result

20618.491917784794007395180476396258996437665586843513151741932271
…

20618.4919177….

From which:

-1/12*1/((2.1350145456-1/(10112256(-0.5)^8)+(2*0.5^17+2 0.5^16-2 0.5^14-4 0.5^13-2 0.5^12-0.5^11-5 0.5^10-4 0.5^9-2 0.5^8-2 0.5^7-2 0.5^6-4 0.5^5-5 0.5^4 -0.5^3-2 0.5^2-4 0.5-2)/37.99992752-2))+11-1/5

Input interpretation

$$-\frac{1}{12}\times 1\Big/\left(2.1350145456-\frac{1}{10112256(-0.5)^8}+\frac{1}{37.99992752}\left(2\times 0.5^{17}+2\times 0.5^{16}-2\times 0.5^{14}-4\times 0.5^{13}-2\times 0.5^{12}-0.5^{11}-5\times 0.5^{10}-4\times 0.5^{9}-2\times 0.5^{8}-2\times 0.5^{7}-2\times 0.5^{6}-4\times 0.5^{5}-5\times 0.5^{4}-0.5^{3}-2\times 0.5^{2}-4\times 0.5-2\right)-2\right)+11-\frac{1}{5}$$

Result

1729.007565394931758279359979762644090293454959568464507734292817O …

1729.0075653949…..

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

(-1/12/((2.1350145456-1/(10112256(-0.5)^8)+(2*0.5^17+2 0.5^16-2 0.5^14-4 0.5^13-2 0.5^12-0.5^11-5 0.5^10-4 0.5^9-2 0.5^8-2 0.5^7-2 0.5^6-4 0.5^5-5 0.5^4 -0.5^3-2 0.5^2-4 0.5-2)/38-2))+11-1/5)^1/15

(we have approximated the value 37.99992752 to 38)

Input interpretation

$$\left(-\left(1\Big/\left(12\left(2.1350145456-\frac{1}{10112256(-0.5)^8}+\frac{1}{38}\left(2\times 0.5^{17}+2\times 0.5^{16}-2\times 0.5^{14}-4\times 0.5^{13}-2\times 0.5^{12}-0.5^{11}-5\times 0.5^{10}-4\times 0.5^{9}-2\times 0.5^{8}-2\times 0.5^{7}-2\times 0.5^{6}-4\times 0.5^{5}-5\times 0.5^{4}-0.5^{3}-2\times 0.5^{2}-4\times 0.5-2\right)-2\right)\right)\right)+11-\frac{1}{5}\right)\hat{}\,(1/15)$$

Result

1.644395706605953018615587609059182859933927684491743919211531346 0...

1.6443957…. ≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ... (trace of the instanton shape)

(1/27(-1/12/((2.13501454-1/(10112256(-0.5)^8)+(2*0.5^17+2 0.5^16-2 0.5^14-4 0.5^13-2 0.5^12-0.5^11-5 0.5^10-4 0.5^9-2 0.5^8-2 0.5^7-2 0.5^6-4 0.5^5-5 0.5^4 - 0.5^3-2 0.5^2-4 0.5-2)/38-2))+11-1/5))^2-47

Input interpretation

$$\left(\frac{1}{27}\left(-\left(1\middle/\left(12\left(2.13501454-\frac{1}{10112256\,(-0.5)^8}+\frac{1}{38}\left(2\times 0.5^{17}+2\times 0.5^{16}-2\times 0.5^{14}-4\times 0.5^{13}-2\times 0.5^{12}-0.5^{11}-5\times 0.5^{10}-4\times 0.5^9-2\times 0.5^8-2\times 0.5^7-2\times 0.5^6-4\times 0.5^5-5\times 0.5^4-0.5^3-2\times 0.5^2-4\times 0.5-2\right)-2\right)\right)\right)+11-\frac{1}{5}\right)\right)^2-47$$

Result

4096.451924319708542517370682848418669035468823678928023013630821 1...

4096.451924319…..≈ 4096 = 64^2

From the same expression, we obtain also:

(-1/12/((2.1350145456-1/(10112256(-0.5)^8)+(2*0.5^17+2 0.5^16-2 0.5^14-4 0.5^13-2 0.5^12-0.5^11-5 0.5^10-4 0.5^9-2 0.5^8-2 0.5^7-2 0.5^6-4 0.5^5-5 0.5^4 - 0.5^3-2 0.5^2-4 0.5-2)/38-2))+11-1/5)^38

Input interpretation

$$\left(-\left(1\Big/\left(12\left(2.1350145456-\frac{1}{10112256(-0.5)^8}+\frac{1}{38}\left(2\times0.5^{17}+2\times0.5^{16}-2\times0.5^{14}-4\times0.5^{13}-2\times0.5^{12}-0.5^{11}-5\times0.5^{10}-4\times0.5^{9}-2\times0.5^{8}-2\times0.5^{7}-2\times0.5^{6}-4\times0.5^{5}-5\times0.5^{4}-0.5^{3}-2\times0.5^{2}-4\times0.5-2\right)-2\right)\right)\right)+11-\frac{1}{5}\right)^{38}$$

Result

$1.3292617063074873247778652207776984523483764460576262018891\ldots\times10^{123}$

$1.329261706307\ldots\times10^{123}$

From this last result, in conclusion, we obtain:

(1.329261706307 × 10^123)*1/((2 e^7 log^8(2))/(log^12(3)))

where

$$\frac{2e^7\log^8(2)}{\log^{12}(3)}\approx37.806080436$$

Input interpretation

$$(1.329261706307\times10^{123})\times\frac{1}{\frac{2e^7\log^8(2)}{\log^{12}(3)}}$$

log(x) is the natural logarithm

Result

$3.516000006059\ldots\times10^{121}$

$0.3516\ldots*10^{122}\approx\Lambda_Q$

The observed value of ρ_Λ or Λ today is precisely the classical dual of its quantum precursor values ρ_Q , Λ_Q in the quantum very early precursor vacuum U_Q as determined by our dual equations

The study of this function provides the following representations:

Alternative representations

$$\frac{1.3292617063070000 \times 10^{123}}{\frac{2 e^7 \log^8(2)}{\log^{12}(3)}} = \frac{1.3292617063070000 \times 10^{123}}{\frac{2 e^7 \log_e^8(2)}{\log_e^{12}(3)}}$$

$$\frac{1.3292617063070000 \times 10^{123}}{\frac{2 e^7 \log^8(2)}{\log^{12}(3)}} = \frac{1.3292617063070000 \times 10^{123}}{\frac{2 e^7 (\log(a) \log_a(2))^8}{(\log(a) \log_a(3))^{12}}}$$

$$\frac{1.3292617063070000 \times 10^{123}}{\frac{2 e^7 \log^8(2)}{\log^{12}(3)}} = \frac{1.3292617063070000 \times 10^{123}}{\frac{2 e^7 (2 \coth^{-1}(3))^8}{(2 \coth^{-1}(2))^{12}}}$$

Series representations

$$\frac{1.3292617063070000 \times 10^{123}}{\frac{2 e^7 \log^8(2)}{\log^{12}(3)}} = \frac{6.6463085315350000 \times 10^{122} \left(2 i \pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k}\right)^{12}}{e^7 \left(2 i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}\right)^8}$$

for $x < 0$

$$\frac{1.3292617063070000\times 10^{123}}{\frac{2 e^7 \log^8(2)}{\log^{12}(3)}} = \left(6.6463085315350000\times 10^{122} \left(\log(z_0) + \left\lfloor \frac{\arg(3-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k}\right)^{12}\right) \Big/ \left(e^7 \left(\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}\right)^8\right)$$

$$\frac{1.3292617063070000\times 10^{123}}{\frac{2 e^7 \log^8(2)}{\log^{12}(3)}} = \left(6.6463085315350000\times 10^{122} \left(2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{3}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k}\right)^{12}\right) \Big/ \left(e^7 \left(2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{2}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}\right)^8\right)$$

Integral representations

$$\frac{1.3292617063070000\times 10^{123}}{\frac{2 e^7 \log^8(2)}{\log^{12}(3)}} = \frac{6.6463085315350000\times 10^{122} \left(\int_1^3 \frac{1}{t}\, dt\right)^{12}}{e^7 \left(\int_1^2 \frac{1}{t}\, dt\right)^8}$$

$$\frac{1.3292617063070000\times 10^{123}}{\frac{2 e^7 \log^8(2)}{\log^{12}(3)}} = \frac{4.1539428322093750\times 10^{121} \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{2^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\, ds\right)^{12}}{e^7\, i^4\, \pi^4 \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\, ds\right)^8} \quad \text{for } -1 < \gamma < 0$$

From the previous result

$$2.1350145456-\frac{1}{10112256\,(0.5-1)^8}+\frac{1}{37.99992752075}\left(2\times 0.5^{17}+2\times 0.5^{16}-2\times 0.5^{14}-4\times 0.5^{13}-2\times 0.5^{12}-0.5^{11}-5\times 0.5^{10}-4\times 0.5^{9}-2\times 0.5^{8}-2\times 0.5^{7}-2\times 0.5^{6}-4\times 0.5^{5}-5\times 0.5^{4}-0.5^{3}-2\times 0.5^{2}-4\times 0.5-2\right)-2$$

integrating, we obtain:

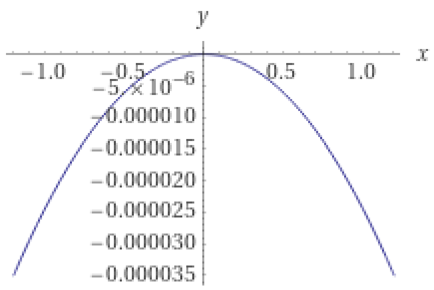
integrate(2.1350145456- 1/(10112256 (0.5 - 1)^8)+(2*0.5^17+2 0.5^16-2 0.5^14-4 0.5^13-2 0.5^12-0.5^11-5 0.5^10-4 0.5^9-2 0.5^8-2 0.5^7-2 0.5^6-4 0.5^5-5 0.5^4 -0.5^3-2 0.5^2-4 0.5-2)/ 37.9999275-2)x

Indefinite integral

$$\int\left(2.1350145456-\frac{1}{10112256\,(0.5-1)^8}+\frac{1}{37.9999275}\left(2\times 0.5^{17}+2\times 0.5^{16}-2\times 0.5^{14}-4\times 0.5^{13}-2\times 0.5^{12}-0.5^{11}-5\times 0.5^{10}-4\times 0.5^{9}-2\times 0.5^{8}-2\times 0.5^{7}-2\times 0.5^{6}-4\times 0.5^{5}-5\times 0.5^{4}-0.5^{3}-2\times 0.5^{2}-4\times 0.5-2\right)-2\right)x\,dx=-0.0000242501\,x^2+\text{constant}$$

<u>The study of this function provides the following representations</u>:

Plot of the integral



(x from −1.2 to 1.2)

Alternate form assuming x is real

$0 - 0.0000242501\,x^2 + \text{constant}$

Definite integral after subtraction of diverging parts

$$\int_0^\infty (-0.0000485002\,x - -0.0000485002\,x)\,dx = 0$$

For x = 1.2 :

-0.0000242501 x^2

(-0.0000242501 * 1.2^2)

Input interpretation

$-0.0000242501 \times 1.2^2$

Result

-0.000034920144

-0.000034920144

From which:

-((1/4 log(4/π))1/(-0.0000242501 * 1.2^2)-2/5

Input interpretation

$$-\left(\frac{1}{4}\log\left(\frac{4}{\pi}\right)\right)\left(-\frac{1}{0.0000242501 \times 1.2^2}\right) - \frac{2}{5}$$

$\log(x)$ is the natural logarithm

Result

1729.0063511772062329628200528274913793576098641317095995347051512
...

1729.0063511772….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

The study of this function provides the following representations :

Alternative representations

$$\frac{-\log\left(\frac{4}{\pi}\right)}{\left(-0.0000242501\times 1.2^2\right)4}-\frac{2}{5}=-\frac{2}{5}-\frac{\log_e\left(\frac{4}{\pi}\right)}{4\left(-0.0000242501\times 1.2^2\right)}$$

$$\frac{-\log\left(\frac{4}{\pi}\right)}{\left(-0.0000242501\times 1.2^2\right)4}-\frac{2}{5}=-\frac{2}{5}-\frac{\log(a)\log_a\left(\frac{4}{\pi}\right)}{4\left(-0.0000242501\times 1.2^2\right)}$$

$$\frac{-\log\left(\frac{4}{\pi}\right)}{\left(-0.0000242501\times 1.2^2\right)4}-\frac{2}{5}=-\frac{2}{5}+\frac{\text{Li}_1\left(1-\frac{4}{\pi}\right)}{4\left(-0.0000242501\times 1.2^2\right)}$$

Series representations

$$\frac{-\log\left(\frac{4}{\pi}\right)}{\left(-0.0000242501\times 1.2^2\right)4}-\frac{2}{5}=-\frac{2}{5}-7159.19\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{4}{\pi}\right)^k}{k}$$

$$\frac{-\log\left(\frac{4}{\pi}\right)}{\left(-0.0000242501\times 1.2^2\right)4}-\frac{2}{5}=-\frac{2}{5}+14318.4\,i\,\pi\left\lfloor\frac{\arg\left(\frac{4}{\pi}-x\right)}{2\pi}\right\rfloor+7159.19\log(x)-7159.19\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{4}{\pi}-x\right)^k x^{-k}}{k}\quad\text{for } x<0$$

$$\frac{-\log\left(\frac{4}{\pi}\right)}{\left(-0.0000242501\times 1.2^2\right)4}-\frac{2}{5}=-\frac{2}{5}+7159.19\left\lfloor\frac{\arg\left(\frac{4}{\pi}-z_0\right)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+7159.19\log(z_0)+7159.19\left\lfloor\frac{\arg\left(\frac{4}{\pi}-z_0\right)}{2\pi}\right\rfloor\log(z_0)-7159.19\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{4}{\pi}-z_0\right)^k z_0^{-k}}{k}$$

Integral representations

$$\frac{-\log\left(\frac{4}{\pi}\right)}{\left(-0.0000242501\times 1.2^2\right)4}-\frac{2}{5}=-\frac{2}{5}+7159.19\int_1^{\frac{4}{\pi}}\frac{1}{t}\,dt$$

$$\frac{-\log\left(\frac{4}{\pi}\right)}{\left(-0.0000242501\times 1.2^2\right)4}-\frac{2}{5}=-\frac{2}{5}+\frac{3579.6}{i\,\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\left(-1+\frac{4}{\pi}\right)^{-s}\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\quad\text{for } -1<\gamma<0$$

And again:

((-((1/4 log(4/π)))1/(-0.0000242501 * 1.2^2)-2/5))^1/15

Input interpretation

$$\sqrt[15]{-\left(\frac{1}{4}\log\left(\frac{4}{\pi}\right)\right)\left(-\frac{1}{0.0000242501\times 1.2^2}\right)-\frac{2}{5}}$$

log(x) is the natural logarithm

Result

1.643816…

$1.643816\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$ (trace of the instanton shape)

Now, performing the following calculations, we obtain also

(2.1350145456- 1/(10112256 (0.5 - 1)^8)+(2*0.5^17+2 0.5^16-2 0.5^14-4 0.5^13-2 0.5^12-0.5^11-5 0.5^10-4 0.5^9-2 0.5^8-2 0.5^7-2 0.5^6-4 0.5^5-5 0.5^4 -0.5^3-2 0.5^2-4 0.5-2)/ 37.9999275-2)dxdy

Input interpretation

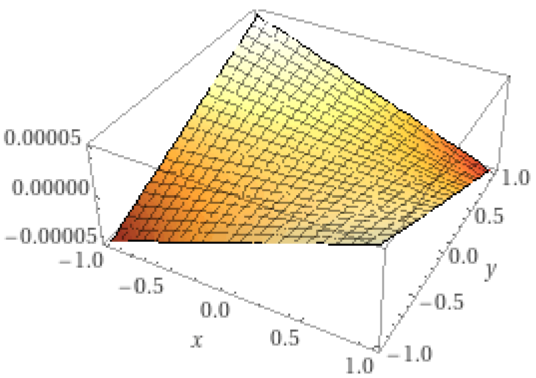
$$\int\int\left(2.1350145456 - \frac{1}{10112256\,(0.5-1)^8} + \frac{1}{37.9999275}\left(2\times 0.5^{17} + 2\times 0.5^{16} - 2\times 0.5^{14} - 4\times 0.5^{13} - 2\times 0.5^{12} - 0.5^{11} - 5\times 0.5^{10} - 4\times 0.5^{9} - 2\times 0.5^{8} - 2\times 0.5^{7} - 2\times 0.5^{6} - 4\times 0.5^{5} - 5\times 0.5^{4} - 0.5^{3} - 2\times 0.5^{2} - 4\times 0.5 - 2\right) - 2\right) dx\, dy$$

Result

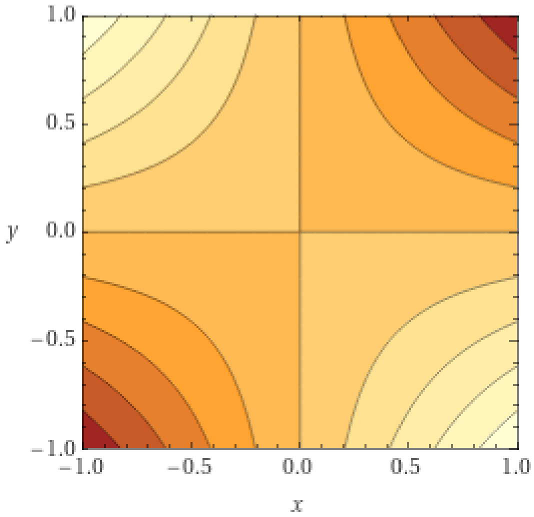
$-0.0000485002\, x\, y$

The study of this function provides the following representations :

3D plot **(figure that can be related to a D-brane/Instanton)**



Contour plot



Indefinite integral assuming all variables are real

$-0.0000242501\, x^2\, y + \text{constant}$

Definite integral over a disk of radius R

$$\iint_{x^2+y^2<R^2} -0.0000485002\, dy\, dx = -0.000152368\, R^2$$

Definite integral over a square of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L} -0.0000485002\, dx\, dy = -0.000194001\, L^2$$

From

$-0.0000485002\, x\, y$

For x = y = 0.5 :

-0.0000485002 (0.5*0.5)

Input interpretation

$-0.0000485002\, (0.5 \times 0.5)$

Result

-0.00001212505

-0.00001212505

From which:

-1/48 1/((-0.0000485002 (0.5*0.5)))+11-1/5

Input interpretation

$$-\frac{1}{48}\left(-\frac{1}{0.0000485002\,(0.5\times 0.5)}\right)+11-\frac{1}{5}$$

Result

1729.0059730333788176818514837739500730581179733966732783232509006 8 …

1729.005973033….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

((-1/48 1/((-0.0000485002 (0.5*0.5)))+11-1/5))^1/15

Input interpretation

$$\sqrt[15]{-\frac{1}{48}\left(-\frac{1}{0.0000485002\,(0.5\times 0.5)}\right)+11-\frac{1}{5}}$$

Result

1.643816…

1.643816…. $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934$ ... (trace of the instanton shape)

From the initial identity, we have also:

1/(1+q/(1-q)+(q^4)/((1-q)(1-q^2))+(q^9)/((1-q)(1-q^2)(1-q^3)))^2 - 1/(((1/((1-q) (1-q^4) (1-q^6) (1-q^9) (1-q^11) (1-q^14) (1-q^16) (1-q^19))))))^2 - √2

Input

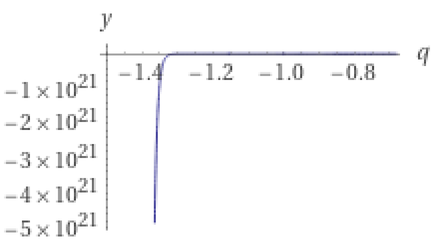
$$\frac{1}{\left(1+\frac{q}{1-q}+\frac{q^4}{(1-q)(1-q^2)}+\frac{q^9}{(1-q)(1-q^2)(1-q^3)}\right)^2}-\frac{1}{\left(\frac{1}{(1-q)(1-q^4)(1-q^6)(1-q^9)(1-q^{11})(1-q^{14})(1-q^{16})(1-q^{19})}\right)^2}-\sqrt{2}$$

Exact result

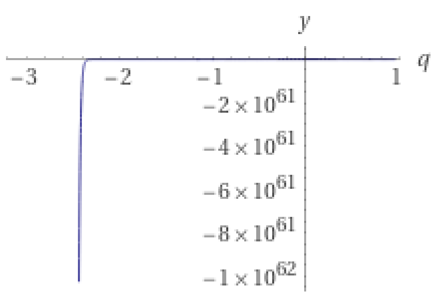
$$\frac{1}{\left(\frac{q^4}{(1-q)(1-q^2)}+\frac{q^9}{(1-q)(1-q^2)(1-q^3)}+\frac{q}{1-q}+1\right)^2}-(1-q)^2(1-q^4)^2(1-q^6)^2(1-q^9)^2(1-q^{11})^2(1-q^{14})^2(1-q^{16})^2(1-q^{19})^2-\sqrt{2}$$

<u>The study of this function provides the following representations</u> :

Plots



(q from -1.5 to -0.7)



(q from -3.1 to 0.9)

For q = 0.9 :

[-(1/(0.9^4/((1-0.9)(1-0.9^2))+0.9^9/((1-0.9)(1-0.9^2) (1-0.9^3))+0.9/(1-0.9)+1)^2-(1-0.9)^2 (1-0.9^4)^2 (1-0.9^6)^2 (1-0.9^9)^2 (1-0.9^11)^2 (1-0.9^14)^2 (1-0.9^16)^2 (1-0.9^19)^2 - √(2))]

Input

$$-\left(\frac{1}{\left(\frac{0.9^4}{(1-0.9)(1-0.9^2)}+\frac{0.9^9}{(1-0.9)(1-0.9^2)(1-0.9^3)}+\frac{0.9}{1-0.9}+1\right)^2}-(1-0.9)^2\left(1-0.9^4\right)^2\left(1-0.9^6\right)^2\left(1-0.9^9\right)^2\left(1-0.9^{11}\right)^2\left(1-0.9^{14}\right)^2\left(1-0.9^{16}\right)^2\left(1-0.9^{19}\right)^2-\sqrt{2}\right)$$

Result

1.414157404268293187005982572349891809517987724490352091911281059 9...

1.414157404….≈ √2

$\sqrt{2} \approx 1.41421356$

From which:

4+([-(1/(0.9^4/((1-0.9)(1-0.9^2))+0.9^9/((1-0.9)(1-0.9^2) (1-0.9^3))+0.9/(1-0.9)+1)^2-(1-0.9)^2 (1-0.9^4)^2 (1-0.9^6)^2 (1-0.9^9)^2 (1-0.9^11)^2 (1-0.9^14)^2 (1-0.9^16)^2 (1-0.9^19)^2 - √(2))])^24

Input

$$4+\left(\left(-\left(\frac{1}{\left(\frac{0.9^4}{(1-0.9)(1-0.9^2)}+\frac{0.9^9}{(1-0.9)(1-0.9^2)(1-0.9^3)}+\frac{0.9}{1-0.9}+1\right)^2}-(1-0.9)^2(1-0.9^4)^2(1-0.9^6)^2(1-0.9^9)^2(1-0.9^{11})^2(1-0.9^{14})^2(1-0.9^{16})^2(1-0.9^{19})^2-\sqrt{2}\right)\right)\right)^{24}$$

Result

4096.098…

4096.098….≈ 4096 = 64^2

And:

27sqrt(4+([-(1/(0.9^4/((1-0.9)(1-0.9^2))+0.9^9/((1-0.9)(1-0.9^2)(1-0.9^3))+0.9/(1-0.9)+1)^2-(1-0.9)^2(1-0.9^4)^2(1-0.9^6)^2(1-0.9^9)^2(1-0.9^11)^2(1-0.9^14)^2(1-0.9^16)^2(1-0.9^19)^2-√(2))])^24)

Input

$$27\sqrt{\left(4+\left(\left(-\left(\frac{1}{\left(\frac{0.9^4}{(1-0.9)(1-0.9^2)}+\frac{0.9^9}{(1-0.9)(1-0.9^2)(1-0.9^3)}+\frac{0.9}{1-0.9}+1\right)^2}-(1-0.9)^2(1-0.9^4)^2(1-0.9^6)^2(1-0.9^9)^2(1-0.9^{11})^2(1-0.9^{14})^2(1-0.9^{16})^2(1-0.9^{19})^2-\sqrt{2}\right)\right)\right)^{24}\right)}$$

Result

1728.021…

1728.021….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = $8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

(27√(4+([-(1/(0.9^4/((1-0.9)(1-0.9^2))+0.9^9/((1-0.9)(1-0.9^2)(1-0.9^3))+0.9/(1-0.9)+1)^2-(1-0.9)^2(1-0.9^4)^2(1-0.9^6)^2(1-0.9^9)^2(1-0.9^11)^2(1-0.9^14)^2(1-0.9^16)^2(1-0.9^19)^2-√(2))])^24))^1/15

Input

$$\left(27\sqrt{\left(4+\left(-\left(\frac{1}{\left(\frac{0.9^4}{(1-0.9)(1-0.9^2)}+\frac{0.9^9}{(1-0.9)(1-0.9^2)(1-0.9^3)}+\frac{0.9}{1-0.9}+1\right)^2}-(1-0.9)^2(1-0.9^4)^2(1-0.9^6)^2(1-0.9^9)^2(1-0.9^{11})^2(1-0.9^{14})^2(1-0.9^{16})^2(1-0.9^{19})^2-\sqrt{2}\right)\right)^{24}\right)}\right)\verb|^|(1/15)$$

Result

1.6437531424732865011148769780390370501815474588905938137738588345

...

1.6437531424….≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ... (trace of the instanton shape)

We obtain also:

√(6(27√(4+([-(1/(0.9^4/((0.1)(1-0.9^2))+0.9^9/((0.1)(1-0.9^2)(1-0.9^3))+0.9/(0.1)+1)^2-(0.1)^2(1-0.9^4)^2(1-0.9^6)^2(1-0.9^9)^2(1-0.9^11)^2(1-0.9^14)^2(1-0.9^16)^2(1-0.9^19)^2-√(2))])^24))^1/15)

Input

$$\sqrt{\left(6\left(27\sqrt{\left(4+\left(-\left(\frac{1}{\left(\frac{0.9^4}{0.1(1-0.9^2)}+\frac{0.9^9}{0.1(1-0.9^2)(1-0.9^3)}+\frac{0.9}{0.1}+1\right)^2}-0.1^2(1-0.9^4)^2(1-0.9^6)^2(1-0.9^9)^2(1-0.9^{11})^2(1-0.9^{14})^2(1-0.9^{16})^2(1-0.9^{19})^2-\sqrt{2}\right)\right)^{24}\right)}\right)\verb|^|(1/15)\right)}$$

Result

3.14046475...

3.14046475…. ≈ π

And:

2√(6(27√(4+([-(1/(0.9^4/((0.1)(0.19))+0.9^9/((0.1)(0.19)(1-0.9^3))+0.9/(0.1)+1)^2-(0.1)^2(1-0.9^4)^2(1-0.9^6)^2(1-0.9^9)^2(1-0.9^11)^2(1-0.9^14)^2(1-0.9^16)^2(1-0.9^19)^2-√(2))])^24))^1/15)*1/0.99964

Input

$$2\sqrt{\left(6\left(27\sqrt{\left(4+\left(-\left(\frac{1}{\left(\frac{0.9^4}{0.1\times0.19}+\frac{0.9^9}{0.1\times0.19\left(1-0.9^3\right)}+\frac{0.9}{0.1}+1\right)^2}-0.1^2\left(1-0.9^4\right)^2\left(1-0.9^6\right)^2\left(1-0.9^9\right)^2\left(1-0.9^{11}\right)^2\left(1-0.9^{14}\right)^2\left(1-0.9^{16}\right)^2\left(1-0.9^{19}\right)^2-\sqrt{2}\right)\right)^{24}\right)}\right)\right)\text{^}(1/15)}\times\frac{1}{0.99964}$$

Result

6.283191451739775119934885350121592513368138629005334782133771849 4...

6.28319…. = 2πr , where r = 1/0.99964 = 1.00036012964…. ≈ 1.0036

From:

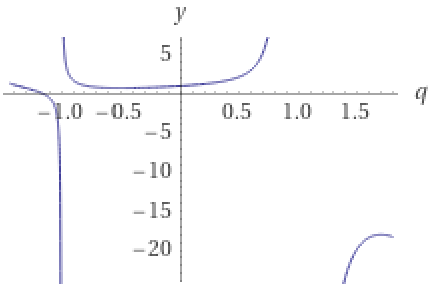
1+q/(1-q)+q^4/((1-q)(1-q^2))+q^9/((1-q)(1-q^2)(1-q^3))

Input

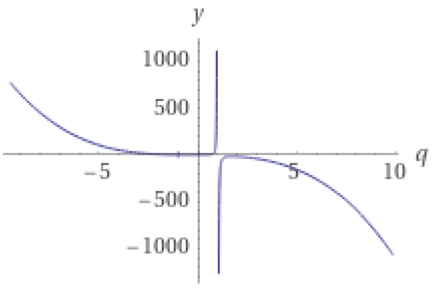
$$1+\frac{q}{1-q}+\frac{q^4}{(1-q)\left(1-q^2\right)}+\frac{q^9}{(1-q)\left(1-q^2\right)\left(1-q^3\right)}$$

<u>The study of this function provides the following representations</u>:

Plots **(figures that can be related to the open strings)**



(q from −1.4 to 1.8)



(q from −9.5 to 9.8)

Alternate forms

$$-\frac{q^9-q^7+q^5+q^4-q^3-q^2+1}{(q-1)^3\,(q+1)\left(q^2+q+1\right)}$$

$$\frac{-q^9+q^7-q^5-q^4+q^3+q^2-1}{(q-1)^3\,(q+1)\left(q^2+q+1\right)}$$

$$-\frac{q^9 - q^7 + q^5 + q^4 - q^3 - q^2 + 1}{(q-1)\left(q^2-1\right)\left(q^3-1\right)}$$

Real root

$q \approx -1.15616$

Complex roots

$q \approx -0.73396 - 0.55718\, i$

$q \approx -0.73396 + 0.55718\, i$

$q \approx -0.38103 - 0.90502\, i$

$q \approx -0.38103 + 0.90502\, i$

$q \approx 0.81070 - 0.76843\, i$

Series expansion at q=0

$1 + q + q^2 + q^3 + 2\, q^4 + O\left(q^5\right)$

(Taylor series)

Series expansion at q=∞

$$-q^3 - q^2 - q - 2 + O\left(\left(\frac{1}{q}\right)^1\right)$$

(Taylor series)

Derivative

$$\frac{d}{dq}\left(1 + \frac{q}{1-q} + \frac{q^4}{(1-q)\left(1-q^2\right)} + \frac{q^9}{(1-q)\left(1-q^2\right)\left(1-q^3\right)}\right) = \frac{-3q^{12} - 2q^{11} + 5q^{10} + 10q^9 + 6q^8 - 4q^7 - 3q^6 + 2q^5 + q^4 + 2q^3 + q^2 + 2q + 1}{(q-1)^4 (q+1)^2 \left(q^2+q+1\right)^2}$$

Indefinite integral

$$\int\left(1 + \frac{q}{1-q} + \frac{q^4}{(1-q)\left(1-q^2\right)} + \frac{q^9}{(1-q)\left(1-q^2\right)\left(1-q^3\right)}\right) dq = \frac{1}{72}\left(-18q^4 - 24q^3 - 36q^2 + 4\log\left(q^2+q+1\right) - 144q + \frac{54}{q-1} + \frac{6}{(q-1)^2} - 233\log(1-q) + 9\log(q+1) + 8\sqrt{3}\tan^{-1}\left(\frac{2q+1}{\sqrt{3}}\right) - 42\right) + \text{constant}$$

(assuming a complex-valued logarithm)

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

Local maximum

$$\max\left\{1 + \frac{q}{1-q} + \frac{q^4}{(1-q)\left(1-q^2\right)} + \frac{q^9}{(1-q)\left(1-q^2\right)\left(1-q^3\right)}\right\} \approx -18.023$$

at $q \approx 1.6989$

Local minimum

$$\min\left\{1 + \frac{q}{1-q} + \frac{q^4}{(1-q)\left(1-q^2\right)} + \frac{q^9}{(1-q)\left(1-q^2\right)\left(1-q^3\right)}\right\} \approx 0.72067$$

at $q \approx -0.49748$

From

$$\frac{d}{dq}\left(1+\frac{q}{1-q}+\frac{q^4}{(1-q)\left(1-q^2\right)}+\frac{q^9}{(1-q)\left(1-q^2\right)\left(1-q^3\right)}\right)=$$
$$\frac{-3q^{12}-2q^{11}+5q^{10}+10q^9+6q^8-4q^7-3q^6+2q^5+q^4+2q^3+q^2+2q+1}{(q-1)^4(q+1)^2\left(q^2+q+1\right)^2}$$

For q = 8 , we obtain

(-3 8^12 - 2 8^11 + 5 8^10 + 10 8^9 + 6 8^8 - 4 8^7 - 3 8^6 + 2 8^5 + 8^4 + 2 8^3 + 8^2 + 2 8 + 1)/((8 - 1)^4 (8 + 1)^2 (8^2 + 8 + 1)^2)

Input

$$\left(-3\times8^{12}-2\times8^{11}+5\times8^{10}+10\times8^9+6\times8^8-4\times8^7-3\times8^6+2\times8^5+8^4+2\times8^3+8^2+2\times8+1\right)/\left((8-1)^4(8+1)^2\left(8^2+8+1\right)^2\right)$$

Exact result

$$-\frac{216535853999}{1036389249}$$

Decimal approximation

$$-208.932941178165386391421356784066755597924964580561757641312622 3\ldots$$

-208.932941178……

From:

$$1+\frac{q}{1-q}+\frac{q^4}{(1-q)\left(1-q^2\right)}+\frac{q^9}{(1-q)\left(1-q^2\right)\left(1-q^3\right)}$$

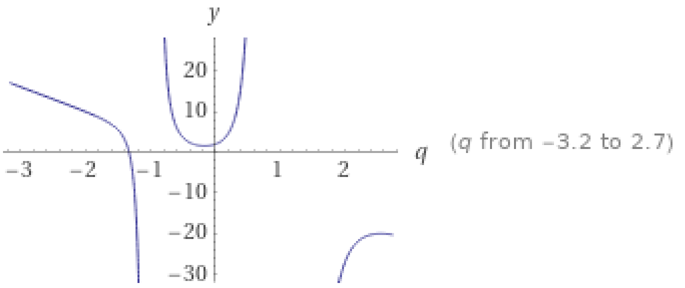
we calculate the second derivative

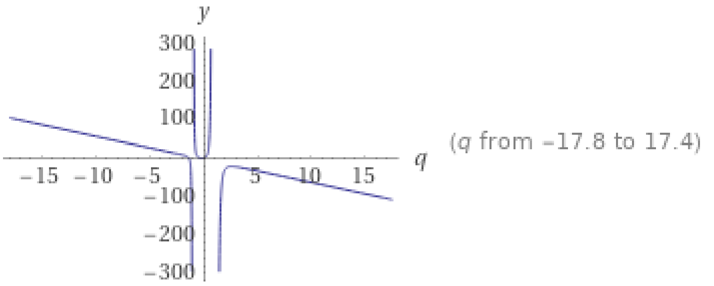
Derivative

$$\frac{d^2}{dq^2}\left(1+\frac{q}{1-q}+\frac{q^4}{(1-q)(1-q^2)}+\frac{q^9}{(1-q)(1-q^2)(1-q^3)}\right)=$$
$$\frac{\left(\frac{8q^2}{(1-q^2)^3}+\frac{2}{(1-q^2)^2}\right)q^4}{1-q}+\frac{4\left(\frac{q^4}{(1-q)^2}+\frac{4q^3}{1-q}\right)q}{(1-q^2)^2}+\frac{\frac{2q^4}{(1-q)^3}+\frac{8q^3}{(1-q)^2}+\frac{12q^2}{1-q}}{1-q^2}+$$
$$2\left(\frac{q^9}{(1-q)^2}+\frac{9q^8}{1-q}\right)\left(\frac{3q^2}{(1-q^2)(1-q^3)^2}+\frac{2q}{(1-q^2)^2(1-q^3)}\right)+$$
$$\frac{\left(\frac{12q^3}{(1-q^2)^2(1-q^3)^2}+\frac{\frac{8q^2}{(1-q^2)^3}+\frac{2}{(1-q^2)^2}}{1-q^3}+\frac{\frac{6q}{(1-q^3)^2}+\frac{18q^4}{(1-q^3)^3}}{1-q^2}\right)q^9}{1-q}+$$
$$\frac{\frac{2q^9}{(1-q)^3}+\frac{18q^8}{(1-q)^2}+\frac{72q^7}{1-q}}{(1-q^2)(1-q^3)}+\frac{2q}{(1-q)^3}+\frac{2}{(1-q)^2}$$

<u>The study of this function provides the following representations</u>:

Plots **(figures that can be related to the open strings)**





Alternate forms

$$\frac{2(q+5)}{9(q^2+q+1)^2} - \frac{2(q+2)}{3(q^2+q+1)^3} - 6q - \frac{233}{36(q-1)^3} + \frac{1}{4(q+1)^3} - \frac{9}{2(q-1)^4} - \frac{2}{(q-1)^5} - 2$$

$$-\left(\left(2\left(3q^{15}+4q^{14}-5q^{13}-17q^{12}-8q^{11}+34q^{10}+70q^9+60q^8+12q^7-9q^6+9q^5+24q^4+21q^3+13q^2+4q+1\right)\right)/\left((q-1)^5(q+1)^3\left(q^2+q+1\right)^3\right)\right)$$

Expanded form

$$\frac{12q^2}{(1-q)(1-q^2)} + \frac{8q^6}{(1-q)(1-q^2)^3} + \frac{4q^5}{(1-q)^2(1-q^2)^2} + \frac{2q^4}{(1-q)^3(1-q^2)} + \frac{18q^4}{(1-q)(1-q^2)^2} + \frac{8q^3}{(1-q)^2(1-q^2)} + \frac{18q^{13}}{(1-q)(1-q^2)(1-q^3)^3} + \frac{12q^{12}}{(1-q)(1-q^2)^2(1-q^3)^2} + \frac{8q^{11}}{(1-q)(1-q^2)^3(1-q^3)} + \frac{6q^{11}}{(1-q)^2(1-q^2)(1-q^3)^2} + \frac{4q^{10}}{(1-q)^2(1-q^2)^2(1-q^3)} + \frac{60q^{10}}{(1-q)(1-q^2)(1-q^3)^2} + \frac{2q^9}{(1-q)^3(1-q^2)(1-q^3)} + \frac{38q^9}{(1-q)(1-q^2)^2(1-q^3)} + \frac{18q^8}{(1-q)^2(1-q^2)(1-q^3)} + \frac{72q^7}{(1-q)(1-q^2)(1-q^3)} + \frac{2q}{(1-q)^3} + \frac{2}{(1-q)^2}$$

Real root

$q \approx -1.33518$

Complex roots

$q \approx -0.86736 - 0.34540\,i$

$q \approx -0.86736 + 0.34540\, i$

$q \approx -0.84597 - 0.87626\, i$

$q \approx -0.84597 + 0.87626\, i$

$q \approx -0.38856 - 1.17497\, i$

Series expansion at q=0

$2 + 6\, q + 24\, q^2 + 40\, q^3 + 90\, q^4 + O(q^5)$

(Taylor series)

Series expansion at q=∞

$-6\, q - 2 + O\left(\left(\frac{1}{q}\right)^3\right)$

(Taylor series)

Indefinite integral

$$\int \left(\frac{2}{(1-q)^2} + \frac{2q}{(1-q)^3} + \frac{\frac{12q^2}{1-q} + \frac{8q^3}{(1-q)^2} + \frac{2q^4}{(1-q)^3}}{1-q^2} + \frac{4q\left(\frac{4q^3}{1-q} + \frac{q^4}{(1-q)^2}\right)}{(1-q^2)^2} + \frac{\frac{72q^7}{1-q} + \frac{18q^8}{(1-q)^2} + \frac{2q^9}{(1-q)^3}}{(1-q^2)(1-q^3)} + \frac{q^4\left(\frac{8q^2}{(1-q^2)^3} + \frac{2}{(1-q^2)^2}\right)}{1-q} + 2\left(\frac{9q^8}{1-q} + \frac{q^9}{(1-q)^2}\right)\left(\frac{3q^2}{(1-q^2)(1-q^3)^2} + \frac{2q}{(1-q^2)^2(1-q^3)}\right) + \frac{q^9\left(\frac{12q^3}{(1-q^2)^2(1-q^3)^2} + \frac{\frac{8q^2}{(1-q^2)^3} + \frac{2}{(1-q^2)^2}}{1-q^3} + \frac{\frac{18q^4}{(1-q^3)^3} + \frac{6q}{(1-q^3)^2}}{1-q^2}\right)}{1-q} \right) dq =$$

$$(-3q^{12} - 2q^{11} + 11q^{10} + 10q^9 - 6q^8 - 16q^7 + 3q^6 + 26q^5 + 7q^4 - 10q^3 - 11q^2 + 2q + 7)/((q-1)^4(q+1)^2(q^2+q+1)^2) + \text{constant}$$

Local maximum

$$\max\left\{ \frac{\left(\frac{8q^2}{(1-q^2)^3} + \frac{2}{(1-q^2)^2}\right)q^4}{1-q} + \frac{4\left(\frac{q^4}{(1-q)^2} + \frac{4q^3}{1-q}\right)q}{(1-q^2)^2} + \frac{\frac{2q^4}{(1-q)^3} + \frac{8q^3}{(1-q)^2} + \frac{12q^2}{1-q}}{1-q^2} + 2\left(\frac{q^9}{(1-q)^2} + \frac{9q^8}{1-q}\right)\left(\frac{3q^2}{(1-q^2)(1-q^3)^2} + \frac{2q}{(1-q^2)^2(1-q^3)}\right) + \frac{\left(\frac{12q^3}{(1-q^2)^2(1-q^3)^2} + \frac{\frac{8q^2}{(1-q^2)^3} + \frac{2}{(1-q^2)^2}}{1-q^3} + \frac{\frac{6q}{(1-q^3)^2} + \frac{18q^4}{(1-q^3)^3}}{1-q^2}\right)q^9}{1-q} + \frac{\frac{2q^9}{(1-q)^3} + \frac{18q^8}{(1-q)^2} + \frac{72q^7}{1-q}}{(1-q^2)(1-q^3)} + \frac{2q}{(1-q)^3} + \frac{2}{(1-q)^2} \right\} \approx -20.022 \text{ at } q \approx 2.5544$$

Local minimum

$$\min\left\{\frac{\left(\frac{8q^2}{(1-q^2)^3}+\frac{2}{(1-q^2)^2}\right)q^4}{1-q}+\frac{4\left(\frac{q^4}{(1-q)^2}+\frac{4q^3}{1-q}\right)q}{(1-q^2)^2}+\frac{\frac{2q^4}{(1-q)^3}+\frac{8q^3}{(1-q)^2}+\frac{12q^2}{1-q}}{1-q^2}+2\left(\frac{q^9}{(1-q)^2}+\frac{9q^8}{1-q}\right)\left(\frac{3q^2}{(1-q^2)(1-q^3)^2}+\frac{2q}{(1-q^2)^2(1-q^3)}\right)+\frac{\left(\frac{12q^3}{(1-q^2)^2(1-q^3)^2}+\frac{\frac{8q^2}{(1-q^2)^3}+\frac{2}{(1-q^2)^2}}{1-q^3}+\frac{\frac{6q}{(1-q^3)^2}+\frac{18q^4}{(1-q^3)^3}}{1-q^2}\right)q^9}{1-q}+\frac{\frac{2q^9}{(1-q)^3}+\frac{18q^8}{(1-q)^2}+\frac{72q^7}{1-q}}{(1-q^2)(1-q^3)}+\frac{2q}{(1-q)^3}+\frac{2}{(1-q)^2}\right\}\approx 1.5385 \text{ at } q\approx -0.16739$$

From the following alternate form:

$$\frac{2(q+5)}{9(q^2+q+1)^2}-\frac{2(q+2)}{3(q^2+q+1)^3}-6q-\frac{233}{36(q-1)^3}+\frac{1}{4(q+1)^3}-\frac{9}{2(q-1)^4}-\frac{2}{(q-1)^5}-2$$

For q = 8 , we obtain:

(2 (8 + 5))/(9 (8^2 + 8 + 1)^2) - (2 (8 + 2))/(3 (8^2 + 8 + 1)^3) - 6 8 - 233/(36 (8 - 1)^3) + 1/(4 (8 + 1)^3) - 9/(2 (8 - 1)^4) - 2/(8 - 1)^5 – 2

Input

$$\frac{2(8+5)}{9(8^2+8+1)^2}-\frac{2(8+2)}{3(8^2+8+1)^3}-6\times 8-\frac{233}{36(8-1)^3}+\frac{1}{4(8+1)^3}-\frac{9}{2(8-1)^4}-\frac{2}{(8-1)^5}-2$$

Exact result

$$-\frac{238413009935042}{4766354156151}$$

Decimal approximation

−50.019994764209665831345399600949595206759665536649494936096797 72

…

-50.0199947642……

Calculate the third derivative of:

$$1+\frac{q}{1-q}+\frac{q^4}{(1-q)(1-q^2)}+\frac{q^9}{(1-q)(1-q^2)(1-q^3)}$$

Derivative

$$\frac{d^3}{dq^3}\left(1+\frac{q}{1-q}+\frac{q^4}{(1-q)(1-q^2)}+\frac{q^9}{(1-q)(1-q^2)(1-q^3)}\right)=$$

$$\frac{\left(\frac{24q}{(1-q^2)^3}+\frac{48q^3}{(1-q^2)^4}\right)q^4}{1-q}+\frac{6\left(\frac{2q^4}{(1-q)^3}+\frac{8q^3}{(1-q)^2}+\frac{12q^2}{1-q}\right)q}{(1-q^2)^2}+$$

$$3\left(\frac{q^4}{(1-q)^2}+\frac{4q^3}{1-q}\right)\left(\frac{8q^2}{(1-q^2)^3}+\frac{2}{(1-q^2)^2}\right)+\frac{\frac{6q^4}{(1-q)^4}+\frac{24q^3}{(1-q)^3}+\frac{36q^2}{(1-q)^2}+\frac{24q}{1-q}}{1-q^2}+$$

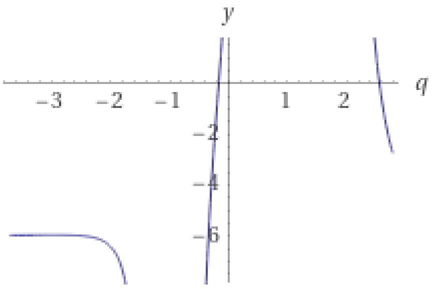
$$3\left(\frac{2q^9}{(1-q)^3}+\frac{18q^8}{(1-q)^2}+\frac{72q^7}{1-q}\right)\left(\frac{3q^2}{(1-q^2)(1-q^3)^2}+\frac{2q}{(1-q^2)^2(1-q^3)}\right)+$$

$$3\left(\frac{q^9}{(1-q)^2}+\frac{9q^8}{1-q}\right)\left(\frac{12q^3}{(1-q^2)^2(1-q^3)^2}+\frac{\frac{8q^2}{(1-q^2)^3}+\frac{2}{(1-q^2)^2}}{1-q^3}+\frac{\frac{6q}{(1-q^3)^2}+\frac{18q^4}{(1-q^3)^3}}{1-q^2}\right)+$$

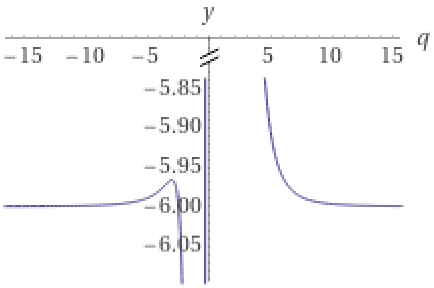
$$\frac{1}{1-q}\left(\frac{9\left(\frac{8q^2}{(1-q^2)^3}+\frac{2}{(1-q^2)^2}\right)q^2}{(1-q^3)^2}+\frac{\frac{24q}{(1-q^2)^3}+\frac{48q^3}{(1-q^2)^4}}{1-q^3}+\frac{\frac{108q^3}{(1-q^3)^3}+\frac{6}{(1-q^3)^2}+\frac{162q^6}{(1-q^3)^4}}{1-q^2}+\frac{6\left(\frac{6q}{(1-q^3)^2}+\frac{18q^4}{(1-q^3)^3}\right)q}{(1-q^2)^2}\right)q^9+$$

$$\frac{\frac{6q^9}{(1-q)^4}+\frac{54q^8}{(1-q)^3}+\frac{216q^7}{(1-q)^2}+\frac{504q^6}{1-q}}{(1-q^2)(1-q^3)}+\frac{6q}{(1-q)^4}+\frac{6}{(1-q)^3}$$

Plots **(figures that can be related to the open strings)**



(q from −3.7 to 2.8)



(q from −16.6 to 15.7)

Alternate forms

$$\frac{4-2q(q(q(q+8)+6)-4)}{3(q^2+q+1)^4}+\frac{233}{12(q-1)^4}-\frac{3}{4(q+1)^4}+\frac{18}{(q-1)^5}+\frac{10}{(q-1)^6}-6$$

$$\frac{6q}{(q^2+q+1)^4}-\frac{2}{3(q^2+q+1)^2}-\frac{2(2q-1)}{(q^2+q+1)^3}+\frac{233}{12(q-1)^4}-\frac{3}{4(q+1)^4}+\frac{18}{(q-1)^5}+\frac{10}{(q-1)^6}-6$$

$$\left(6\left(-q^{18}-2q^{17}+q^{16}+8q^{15}+12q^{14}+16q^{13}+56q^{12}+164q^{11}+294q^{10}+380q^9+372q^8+276q^7+174q^6+148q^5+132q^4+84q^3+35q^2+10q+1\right)\right)/\left((q-1)^6(q+1)^4\left(q^2+q+1\right)^4\right)$$

Expanded form

$$\frac{162 q^{15}}{(1-q)(1-q^2)(1-q^3)^4} + \frac{108 q^{14}}{(1-q)(1-q^2)^2(1-q^3)^3} + \frac{72 q^{13}}{(1-q)(1-q^2)^3(1-q^3)^2} + \frac{54 q^{13}}{(1-q)^2(1-q^2)(1-q^3)^3} + \frac{48 q^{12}}{(1-q)(1-q^2)^4(1-q^3)} + \frac{36 q^{12}}{(1-q)^2(1-q^2)^2(1-q^3)^2} + \frac{594 q^{12}}{(1-q)(1-q^2)(1-q^3)^3} + \frac{24 q^{11}}{(1-q)^2(1-q^2)^3(1-q^3)} + \frac{18 q^{11}}{(1-q)^3(1-q^2)(1-q^3)^2} + \frac{378 q^{11}}{(1-q)(1-q^2)^2(1-q^3)^2} + \frac{12 q^{10}}{(1-q)^3(1-q^2)^2(1-q^3)} + \frac{240 q^{10}}{(1-q)(1-q^2)^3(1-q^3)} + \frac{180 q^{10}}{(1-q)^2(1-q^2)(1-q^3)^2} + \frac{6 q^9}{(1-q)^4(1-q^2)(1-q^3)} + \frac{114 q^9}{(1-q)^2(1-q^2)^2(1-q^3)} + \frac{816 q^9}{(1-q)(1-q^2)(1-q^3)^2} + \frac{54 q^8}{(1-q)^3(1-q^2)(1-q^3)} + \frac{486 q^8}{(1-q)(1-q^2)^2(1-q^3)} + \frac{216 q^7}{(1-q)^2(1-q^2)(1-q^3)} + \frac{48 q^7}{(1-q)(1-q^2)^4} + \frac{504 q^6}{(1-q)(1-q^2)(1-q^3)} + \frac{24 q^6}{(1-q)^2(1-q^2)^3} + \frac{12 q^5}{(1-q)^3(1-q^2)^2} + \frac{120 q^5}{(1-q)(1-q^2)^3} + \frac{6 q^4}{(1-q)^4(1-q^2)} + \frac{54 q^4}{(1-q)^2(1-q^2)^2} + \frac{24 q^3}{(1-q)^3(1-q^2)} + \frac{96 q^3}{(1-q)(1-q^2)^2} + \frac{36 q^2}{(1-q)^2(1-q^2)} + \frac{24 q}{(1-q)(1-q^2)} + \frac{6 q}{(1-q)^4} + \frac{6}{(1-q)^3}$$

Real roots

$q \approx -0.167388$

$q \approx 2.55444$

Complex roots

$q \approx -1.4460 - 0.4247\, i$

$q \approx -1.4460 + 0.4247\, i$

$q \approx -0.94237 - 0.62521\, i$

$q \approx -0.94237 + 0.62521\, i$

$q \approx -0.7864 - 1.3517\, i$

Series expansion at q=0

$$6 + 48\, q + 120\, q^2 + 360\, q^3 + 630\, q^4 + O\left(q^5\right)$$

(Taylor series)

Series expansion at q=∞

$$-6 + \frac{18}{q^4} + \frac{96}{q^5} + O\left(\left(\frac{1}{q}\right)^6\right)$$

(Laurent series)

Indefinite integral

$$\int \Bigg(\frac{6}{(1-q)^3} + \frac{6q}{(1-q)^4} + \frac{\frac{24q}{1-q} + \frac{36q^2}{(1-q)^2} + \frac{24q^3}{(1-q)^3} + \frac{6q^4}{(1-q)^4}}{1-q^2} +$$

$$\frac{6q\left(\frac{12q^2}{1-q} + \frac{8q^3}{(1-q)^2} + \frac{2q^4}{(1-q)^3}\right)}{(1-q^2)^2} + \frac{\frac{504q^6}{1-q} + \frac{216q^7}{(1-q)^2} + \frac{54q^8}{(1-q)^3} + \frac{6q^9}{(1-q)^4}}{(1-q^2)(1-q^3)} +$$

$$\frac{q^4\left(\frac{48q^3}{(1-q^2)^4} + \frac{24q}{(1-q^2)^3}\right)}{1-q} + 3\left(\frac{4q^3}{1-q} + \frac{q^4}{(1-q)^2}\right)\left(\frac{8q^2}{(1-q^2)^3} + \frac{2}{(1-q^2)^2}\right) +$$

$$3\left(\frac{72q^7}{1-q} + \frac{18q^8}{(1-q)^2} + \frac{2q^9}{(1-q)^3}\right)\left(\frac{3q^2}{(1-q^2)(1-q^3)^2} + \frac{2q}{(1-q^2)^2(1-q^3)}\right) +$$

$$\frac{1}{1-q} q^9 \Bigg(\frac{\frac{48q^3}{(1-q^2)^4} + \frac{24q}{(1-q^2)^3}}{1-q^3} + \frac{9q^2\left(\frac{8q^2}{(1-q^2)^3} + \frac{2}{(1-q^2)^2}\right)}{(1-q^3)^2} +$$

$$\frac{\frac{162q^6}{(1-q^3)^4} + \frac{108q^3}{(1-q^3)^3} + \frac{6}{(1-q^3)^2}}{1-q^2} + \frac{6q\left(\frac{18q^4}{(1-q^3)^3} + \frac{6q}{(1-q^3)^2}\right)}{(1-q^2)^2} \Bigg) +$$

$$3\left(\frac{9q^8}{1-q} + \frac{q^9}{(1-q)^2}\right) \Bigg(\frac{12q^3}{(1-q^2)^2(1-q^3)^2} + \frac{\frac{8q^2}{(1-q^2)^3} + \frac{2}{(1-q^2)^2}}{1-q^3} +$$

$$\frac{\frac{18q^4}{(1-q^3)^3} + \frac{6q}{(1-q^3)^2}}{1-q^2} \Bigg) \Bigg) dq =$$

$$\frac{1}{36}\left(\frac{8(q+5)}{(q^2+q+1)^2} - \frac{24(q+2)}{(q^2+q+1)^3} - 216q - \frac{233}{(q-1)^3} + \frac{9}{(q+1)^3} - \frac{162}{(q-1)^4} - \frac{72}{(q-1)^5}\right) + \text{constant}$$

Local maximum

$$
\begin{aligned}
\max\Bigg\{ & \frac{\left(\frac{24q}{(1-q^2)^3} + \frac{48q^3}{(1-q^2)^4}\right)q^4}{1-q} + \\
& \frac{6\left(\frac{2q^4}{(1-q)^3} + \frac{8q^3}{(1-q)^2} + \frac{12q^2}{1-q}\right)q}{(1-q^2)^2} + \frac{\frac{6q^4}{(1-q)^4} + \frac{24q^3}{(1-q)^3} + \frac{36q^2}{(1-q)^2} + \frac{24q}{1-q}}{1-q^2} + \\
& 3\left(\frac{q^4}{(1-q)^2} + \frac{4q^3}{1-q}\right)\left(\frac{8q^2}{(1-q^2)^3} + \frac{2}{(1-q^2)^2}\right) + \\
& 3\left(\frac{2q^9}{(1-q)^3} + \frac{18q^8}{(1-q)^2} + \frac{72q^7}{1-q}\right)\left(\frac{3q^2}{(1-q^2)(1-q^3)^2} + \frac{2q}{(1-q^2)^2(1-q^3)}\right) + \\
& 3\left(\frac{q^9}{(1-q)^2} + \frac{9q^8}{1-q}\right) \\
& \left(\frac{12q^3}{(1-q^2)^2(1-q^3)^2} + \frac{\frac{8q^2}{(1-q^2)^3} + \frac{2}{(1-q^2)^2}}{1-q^3} + \frac{\frac{6q}{(1-q^3)^2} + \frac{18q^4}{(1-q^3)^3}}{1-q^2}\right) + \\
& \frac{1}{1-q}\Bigg(\frac{9\left(\frac{8q^2}{(1-q^2)^3} + \frac{2}{(1-q^2)^2}\right)q^2}{(1-q^3)^2} + \frac{\frac{24q}{(1-q^2)^3} + \frac{48q^3}{(1-q^2)^4}}{1-q^3} + \\
& \frac{\frac{108q^3}{(1-q^3)^3} + \frac{6}{(1-q^3)^2} + \frac{162q^6}{(1-q^3)^4}}{1-q^2} + \frac{6\left(\frac{6q}{(1-q^3)^2} + \frac{18q^4}{(1-q^3)^3}\right)q}{(1-q^2)^2}\Bigg)q^9 + \\
& \frac{\frac{6q^9}{(1-q)^4} + \frac{54q^8}{(1-q)^3} + \frac{216q^7}{(1-q)^2} + \frac{504q^6}{1-q}}{(1-q^2)(1-q^3)} + \frac{6q}{(1-q)^4} + \frac{6}{(1-q)^3}\Bigg\} \approx \\
& -5.9664 \text{ at } q \approx -2.9866
\end{aligned}
$$

Limit

$$\lim_{q\to\pm\infty}\left(\frac{6}{(1-q)^3}+\frac{6q}{(1-q)^4}+\frac{\frac{24q}{1-q}+\frac{36q^2}{(1-q)^2}+\frac{24q^3}{(1-q)^3}+\frac{6q^4}{(1-q)^4}}{1-q^2}+\frac{6q\left(\frac{12q^2}{1-q}+\frac{8q^3}{(1-q)^2}+\frac{2q^4}{(1-q)^3}\right)}{(1-q^2)^2}+\frac{\frac{504q^6}{1-q}+\frac{216q^7}{(1-q)^2}+\frac{54q^8}{(1-q)^3}+\frac{6q^9}{(1-q)^4}}{(1-q^2)(1-q^3)}+\frac{q^4\left(\frac{48q^3}{(1-q^2)^4}+\frac{24q}{(1-q^2)^3}\right)}{1-q}+3\left(\frac{4q^3}{1-q}+\frac{q^4}{(1-q)^2}\right)\left(\frac{8q^2}{(1-q^2)^3}+\frac{2}{(1-q^2)^2}\right)+3\left(\frac{72q^7}{1-q}+\frac{18q^8}{(1-q)^2}+\frac{2q^9}{(1-q)^3}\right)\left(\frac{3q^2}{(1-q^2)(1-q^3)^2}+\frac{2q}{(1-q^2)^2(1-q^3)}\right)+\frac{1}{1-q}q^9\left(\frac{\frac{48q^3}{(1-q^2)^4}+\frac{24q}{(1-q^2)^3}}{1-q^3}+\frac{9q^2\left(\frac{8q^2}{(1-q^2)^3}+\frac{2}{(1-q^2)^2}\right)}{(1-q^3)^2}+\frac{\frac{162q^6}{(1-q^3)^4}+\frac{108q^3}{(1-q^3)^3}+\frac{6}{(1-q^3)^2}}{1-q^2}+\frac{6q\left(\frac{18q^4}{(1-q^3)^3}+\frac{6q}{(1-q^3)^2}\right)}{(1-q^2)^2}\right)+3\left(\frac{9q^8}{1-q}+\frac{q^9}{(1-q)^2}\right)\left(\frac{12q^3}{(1-q^2)^2(1-q^3)^2}+\frac{\frac{8q^2}{(1-q^2)^3}+\frac{2}{(1-q^2)^2}}{1-q^3}+\frac{\frac{18q^4}{(1-q^3)^3}+\frac{6q}{(1-q^3)^2}}{1-q^2}\right)\right)=-6$$

From the following alternate form, for q = 8

$$\frac{4-2q(q(q(q+8)+6)-4)}{3(q^2+q+1)^4}+\frac{233}{12(q-1)^4}-\frac{3}{4(q+1)^4}+\frac{18}{(q-1)^5}+\frac{10}{(q-1)^6}-6$$

we obtain:

(4 - 2 8 (8 (8 (8 + 8) + 6) - 4))/(3 (8^2 + 8 + 1)^4) + 233/(12 (8 - 1)^4) - 3/(4 (8 + 1)^4) + 18/(8 - 1)^5 + 10/(8 - 1)^6 – 6

Input

$$\frac{4-2\times 8(8(8(8+8)+6)-4)}{3\left(8^2+8+1\right)^4}+\frac{233}{12(8-1)^4}-\frac{3}{4(8+1)^4}+\frac{18}{(8-1)^5}+\frac{10}{(8-1)^6}-6$$

Exact result

$$-\frac{43775689876970974}{7306820921379483}$$

Decimal approximation

$-5.991071951535715492953350851334978756746082877819817069426204753\ldots$

-5.9910719515357…..

In conclusion, we calculate the fourth derivative of

$$1+\frac{q}{1-q}+\frac{q^4}{(1-q)\left(1-q^2\right)}+\frac{q^9}{(1-q)\left(1-q^2\right)\left(1-q^3\right)}$$

Derivative

$$\frac{d^4}{dq^4}\left(1+\frac{q}{1-q}+\frac{q^4}{(1-q)\left(1-q^2\right)}+\frac{q^9}{(1-q)\left(1-q^2\right)\left(1-q^3\right)}\right)=$$

$$\frac{1}{1-q}\left(\frac{12\left(\frac{48\,q^3}{(1-q^2)^4}+\frac{24\,q}{(1-q^2)^3}\right)q^2}{\left(1-q^3\right)^2}+\frac{8\left(\frac{162\,q^6}{(1-q^3)^4}+\frac{108\,q^3}{(1-q^3)^3}+\frac{6}{(1-q^3)^2}\right)q}{\left(1-q^2\right)^2}+\right.$$

$$\frac{\frac{384\,q^4}{(1-q^2)^5}+\frac{288\,q^2}{(1-q^2)^4}+\frac{24}{(1-q^2)^3}}{1-q^3}+\frac{\frac{1944\,q^8}{(1-q^3)^5}+\frac{1944\,q^5}{(1-q^3)^4}+\frac{360\,q^2}{(1-q^3)^3}}{1-q^2}+$$

$$\left.6\left(\frac{8\,q^2}{\left(1-q^2\right)^3}+\frac{2}{\left(1-q^2\right)^2}\right)\left(\frac{18\,q^4}{\left(1-q^3\right)^3}+\frac{6\,q}{\left(1-q^3\right)^2}\right)\right)q^9+$$

$$\frac{\left(\frac{384\,q^4}{(1-q^2)^5}+\frac{288\,q^2}{(1-q^2)^4}+\frac{24}{(1-q^2)^3}\right)q^4}{1-q}+\frac{8\left(\frac{6\,q^4}{(1-q)^4}+\frac{24\,q^3}{(1-q)^3}+\frac{36\,q^2}{(1-q)^2}+\frac{24\,q}{1-q}\right)q}{\left(1-q^2\right)^2}+$$

$$\frac{24\,q}{(1-q)^5}+\frac{\frac{24\,q^4}{(1-q)^5}+\frac{96\,q^3}{(1-q)^4}+\frac{144\,q^2}{(1-q)^3}+\frac{96\,q}{(1-q)^2}+\frac{24}{1-q}}{1-q^2}+$$

$$\frac{\frac{24\,q^9}{(1-q)^5}+\frac{216\,q^8}{(1-q)^4}+\frac{864\,q^7}{(1-q)^3}+\frac{2016\,q^6}{(1-q)^2}+\frac{3024\,q^5}{1-q}}{\left(1-q^2\right)\left(1-q^3\right)}+$$

$$4\left(\frac{q^4}{(1-q)^2}+\frac{4\,q^3}{1-q}\right)\left(\frac{48\,q^3}{\left(1-q^2\right)^4}+\frac{24\,q}{\left(1-q^2\right)^3}\right)+$$

$$6\left(\frac{2\,q^4}{(1-q)^3}+\frac{8\,q^3}{(1-q)^2}+\frac{12\,q^2}{1-q}\right)\left(\frac{8\,q^2}{\left(1-q^2\right)^3}+\frac{2}{\left(1-q^2\right)^2}\right)+$$

$$4\left(\frac{6\,q^9}{(1-q)^4}+\frac{54\,q^8}{(1-q)^3}+\frac{216\,q^7}{(1-q)^2}+\frac{504\,q^6}{1-q}\right)$$

$$\left(\frac{3\,q^2}{\left(1-q^2\right)\left(1-q^3\right)^2}+\frac{2\,q}{\left(1-q^2\right)^2\left(1-q^3\right)}\right)+4\left(\frac{q^9}{(1-q)^2}+\frac{9\,q^8}{1-q}\right)$$

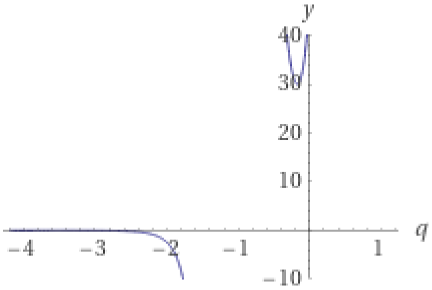
$$\left(\frac{9\left(\frac{8\,q^2}{(1-q^2)^3}+\frac{2}{(1-q^2)^2}\right)q^2}{\left(1-q^3\right)^2}+\frac{6\left(\frac{18\,q^4}{(1-q^3)^3}+\frac{6\,q}{(1-q^3)^2}\right)q}{\left(1-q^2\right)^2}+\frac{\frac{48\,q^3}{(1-q^2)^4}+\frac{24\,q}{(1-q^2)^3}}{1-q^3}+\right.$$

$$\left.\frac{\frac{162\,q^6}{(1-q^3)^4}+\frac{108\,q^3}{(1-q^3)^3}+\frac{6}{(1-q^3)^2}}{1-q^2}\right)+6\left(\frac{2\,q^9}{(1-q)^3}+\frac{18\,q^8}{(1-q)^2}+\frac{72\,q^7}{1-q}\right)$$

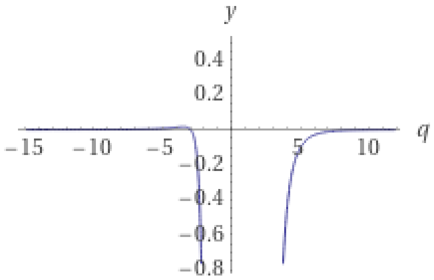
$$\left(\frac{12\,q^3}{\left(1-q^2\right)^2\left(1-q^3\right)^2}+\frac{\frac{8\,q^2}{(1-q^2)^3}+\frac{2}{(1-q^2)^2}}{1-q^3}+\frac{\frac{18\,q^4}{(1-q^3)^3}+\frac{6\,q}{(1-q^3)^2}}{1-q^2}\right)+\frac{24}{(1-q)^4}$$

<u>The study of this function provides the following representations</u>:

Plots **(figures that can be related to the open strings)**



(q from -4.2 to 1.2)



(q from -14.8 to 11.8)

Alternate form

$$-((24(3q^{17}+29q^{16}+138q^{15}+425q^{14}+935q^{13}+1652q^{12}+2426q^{11}+2922q^{10}+2915q^{9}+2605q^{8}+2132q^{7}+1516q^{6}+917q^{5}+500q^{4}+230q^{3}+77q^{2}+16q+2))/((q-1)^{7}(q+1)^{5}(q^{2}+q+1)^{5}))$$

Partial fraction expansion

$$\frac{24(q+2)}{(q^2+q+1)^5}+\frac{8(q+8)}{3(q^2+q+1)^3}-\frac{24(q+3)}{(q^2+q+1)^4}-\frac{233}{3(q-1)^5}+\frac{3}{(q+1)^5}-\frac{90}{(q-1)^6}-\frac{60}{(q-1)^7}$$

Expanded form

$$\frac{1944\,q^{17}}{(1-q)\left(1-q^2\right)\left(1-q^3\right)^5}+\frac{1296\,q^{16}}{(1-q)\left(1-q^2\right)^2\left(1-q^3\right)^4}+\frac{864\,q^{15}}{(1-q)\left(1-q^2\right)^3\left(1-q^3\right)^3}+$$
$$\frac{648\,q^{15}}{(1-q)^2\left(1-q^2\right)\left(1-q^3\right)^4}+\frac{576\,q^{14}}{(1-q)\left(1-q^2\right)^4\left(1-q^3\right)^2}+$$
$$\frac{432\,q^{14}}{(1-q)^2\left(1-q^2\right)^2\left(1-q^3\right)^3}+\frac{7776\,q^{14}}{(1-q)\left(1-q^2\right)\left(1-q^3\right)^4}+\frac{384\,q^{13}}{(1-q)\left(1-q^2\right)^5\left(1-q^3\right)}+$$
$$\frac{288\,q^{13}}{(1-q)^2\left(1-q^2\right)^3\left(1-q^3\right)^2}+\frac{216\,q^{13}}{(1-q)^3\left(1-q^2\right)\left(1-q^3\right)^3}+$$
$$\frac{4968\,q^{13}}{(1-q)\left(1-q^2\right)^2\left(1-q^3\right)^3}+\frac{192\,q^{12}}{(1-q)^2\left(1-q^2\right)^4\left(1-q^3\right)}+$$
$$\frac{144\,q^{12}}{(1-q)^3\left(1-q^2\right)^2\left(1-q^3\right)^2}+\frac{3168\,q^{12}}{(1-q)\left(1-q^2\right)^3\left(1-q^3\right)^2}+$$
$$\frac{2376\,q^{12}}{(1-q)^2\left(1-q^2\right)\left(1-q^3\right)^3}+\frac{96\,q^{11}}{(1-q)^3\left(1-q^2\right)^3\left(1-q^3\right)}+\frac{2016\,q^{11}}{(1-q)\left(1-q^2\right)^4\left(1-q^3\right)}+$$
$$\frac{72\,q^{11}}{(1-q)^4\left(1-q^2\right)\left(1-q^3\right)^2}+\frac{1512\,q^{11}}{(1-q)^2\left(1-q^2\right)^2\left(1-q^3\right)^2}+$$
$$\frac{12\,024\,q^{11}}{(1-q)\left(1-q^2\right)\left(1-q^3\right)^3}+\frac{48\,q^{10}}{(1-q)^4\left(1-q^2\right)^2\left(1-q^3\right)}+\frac{960\,q^{10}}{(1-q)^2\left(1-q^2\right)^3\left(1-q^3\right)}+$$
$$\frac{720\,q^{10}}{(1-q)^3\left(1-q^2\right)\left(1-q^3\right)^2}+\frac{7248\,q^{10}}{(1-q)\left(1-q^2\right)^2\left(1-q^3\right)^2}+$$
$$\frac{24\,q^{9}}{(1-q)^5\left(1-q^2\right)\left(1-q^3\right)}+\frac{456\,q^{9}}{(1-q)^3\left(1-q^2\right)^2\left(1-q^3\right)}+\frac{4344\,q^{9}}{(1-q)\left(1-q^2\right)^3\left(1-q^3\right)}+$$
$$\frac{3264\,q^{9}}{(1-q)^2\left(1-q^2\right)\left(1-q^3\right)^2}+\frac{216\,q^{8}}{(1-q)^4\left(1-q^2\right)\left(1-q^3\right)}+\frac{1944\,q^{8}}{(1-q)^2\left(1-q^2\right)^2\left(1-q^3\right)}+$$
$$\frac{8856\,q^{8}}{(1-q)\left(1-q^2\right)\left(1-q^3\right)^2}+\frac{384\,q^{8}}{(1-q)\left(1-q^2\right)^5}+\frac{864\,q^{7}}{(1-q)^3\left(1-q^2\right)\left(1-q^3\right)}+$$
$$\frac{4896\,q^{7}}{(1-q)\left(1-q^2\right)^2\left(1-q^3\right)}+\frac{192\,q^{7}}{(1-q)^2\left(1-q^2\right)^4}+\frac{2016\,q^{6}}{(1-q)^2\left(1-q^2\right)\left(1-q^3\right)}+$$
$$\frac{96\,q^{6}}{(1-q)^3\left(1-q^2\right)^3}+\frac{1056\,q^{6}}{(1-q)\left(1-q^2\right)^4}+\frac{3024\,q^{5}}{(1-q)\left(1-q^2\right)\left(1-q^3\right)}+$$
$$\frac{48\,q^{5}}{(1-q)^4\left(1-q^2\right)^2}+\frac{480\,q^{5}}{(1-q)^2\left(1-q^2\right)^3}+\frac{24\,q^{4}}{(1-q)^5\left(1-q^2\right)}+\frac{216\,q^{4}}{(1-q)^3\left(1-q^2\right)^2}+$$
$$\frac{984\,q^{4}}{(1-q)\left(1-q^2\right)^3}+\frac{96\,q^{3}}{(1-q)^4\left(1-q^2\right)}+\frac{384\,q^{3}}{(1-q)^2\left(1-q^2\right)^2}+\frac{144\,q^{2}}{(1-q)^3\left(1-q^2\right)}+$$
$$\frac{336\,q^{2}}{(1-q)\left(1-q^2\right)^2}+\frac{96\,q}{(1-q)^2\left(1-q^2\right)}+\frac{24\,q}{(1-q)^5}+\frac{24}{(1-q)\left(1-q^2\right)}+\frac{24}{(1-q)^4}$$

Real root

$q \approx -2.98658$

Complex roots

$q \approx -1.5721 - 2.1641\, i$

$q \approx -1.5721 + 2.1641\, i$

$q \approx -1.13293 - 0.66444\, i$

$q \approx -1.13293 + 0.66444\, i$

$q \approx -0.79254 - 0.46536\, i$

Series expansion at q=0

$$48 + 240\, q + 1080\, q^2 + 2520\, q^3 + 6720\, q^4 + O(q^5)$$

(Taylor series)

Series expansion at q=∞

$$-\frac{72}{q^5} - \frac{480}{q^6} - \frac{1800}{q^7} - \frac{5040}{q^8} + O\left(\left(\frac{1}{q}\right)^9\right)$$

(Laurent series)

Indefinite integral

$$\int\Bigg(\frac{24}{(1-q)^4}+\frac{24\,q}{(1-q)^5}+\frac{\frac{24}{1-q}+\frac{96\,q}{(1-q)^2}+\frac{144\,q^2}{(1-q)^3}+\frac{96\,q^3}{(1-q)^4}+\frac{24\,q^4}{(1-q)^5}}{1-q^2}+$$

$$\frac{8\,q\left(\frac{24\,q}{1-q}+\frac{36\,q^2}{(1-q)^2}+\frac{24\,q^3}{(1-q)^3}+\frac{6\,q^4}{(1-q)^4}\right)}{\left(1-q^2\right)^2}+$$

$$\frac{\frac{3024\,q^5}{1-q}+\frac{2016\,q^6}{(1-q)^2}+\frac{864\,q^7}{(1-q)^3}+\frac{216\,q^8}{(1-q)^4}+\frac{24\,q^9}{(1-q)^5}}{\left(1-q^2\right)\left(1-q^3\right)}+$$

$$\frac{q^4\left(\frac{384\,q^4}{(1-q^2)^5}+\frac{288\,q^2}{(1-q^2)^4}+\frac{24}{(1-q^2)^3}\right)}{1-q}+$$

$$4\left(\frac{4\,q^3}{1-q}+\frac{q^4}{(1-q)^2}\right)\left(\frac{48\,q^3}{\left(1-q^2\right)^4}+\frac{24\,q}{\left(1-q^2\right)^3}\right)+$$

$$6\left(\frac{12\,q^2}{1-q}+\frac{8\,q^3}{(1-q)^2}+\frac{2\,q^4}{(1-q)^3}\right)\left(\frac{8\,q^2}{\left(1-q^2\right)^3}+\frac{2}{\left(1-q^2\right)^2}\right)+$$

$$4\left(\frac{504\,q^6}{1-q}+\frac{216\,q^7}{(1-q)^2}+\frac{54\,q^8}{(1-q)^3}+\frac{6\,q^9}{(1-q)^4}\right)\left(\frac{3\,q^2}{\left(1-q^2\right)\left(1-q^3\right)^2}+\frac{2\,q}{\left(1-q^2\right)^2\left(1-q^3\right)}\right)+$$

$$4\left(\frac{9\,q^8}{1-q}+\frac{q^9}{(1-q)^2}\right)\Bigg(\frac{\frac{48\,q^3}{(1-q^2)^4}+\frac{24\,q}{(1-q^2)^3}}{1-q^3}+\frac{9\,q^2\left(\frac{8\,q^2}{(1-q^2)^3}+\frac{2}{(1-q^2)^2}\right)}{\left(1-q^3\right)^2}+$$

$$\frac{\frac{162\,q^6}{(1-q^3)^4}+\frac{108\,q^3}{(1-q^3)^3}+\frac{6}{(1-q^3)^2}}{1-q^2}+\frac{6\,q\left(\frac{18\,q^4}{(1-q^3)^3}+\frac{6\,q}{(1-q^3)^2}\right)}{\left(1-q^2\right)^2}\Bigg)+$$

$$6\left(\frac{72\,q^7}{1-q}+\frac{18\,q^8}{(1-q)^2}+\frac{2\,q^9}{(1-q)^3}\right)\left(\frac{12\,q^3}{\left(1-q^2\right)^2\left(1-q^3\right)^2}+\frac{\frac{8\,q^2}{(1-q^2)^3}+\frac{2}{(1-q^2)^2}}{1-q^3}+\frac{\frac{18\,q^4}{(1-q^3)^3}+\frac{6\,q}{(1-q^3)^2}}{1-q^2}\right)+$$

$$\frac{1}{1-q}q^9\Bigg(\frac{\frac{384\,q^4}{(1-q^2)^5}+\frac{288\,q^2}{(1-q^2)^4}+\frac{24}{(1-q^2)^3}}{1-q^3}+\frac{12\,q^2\left(\frac{48\,q^3}{(1-q^2)^4}+\frac{24\,q}{(1-q^2)^3}\right)}{\left(1-q^3\right)^2}+$$

$$\frac{\frac{1944\,q^8}{(1-q^3)^5}+\frac{1944\,q^5}{(1-q^3)^4}+\frac{360\,q^2}{(1-q^3)^3}}{1-q^2}+\frac{8\,q\left(\frac{162\,q^6}{(1-q^3)^4}+\frac{108\,q^3}{(1-q^3)^3}+\frac{6}{(1-q^3)^2}\right)}{\left(1-q^2\right)^2}+$$

$$6\left(\frac{8\,q^2}{\left(1-q^2\right)^3}+\frac{2}{\left(1-q^2\right)^2}\right)\left(\frac{18\,q^4}{\left(1-q^3\right)^3}+\frac{6\,q}{\left(1-q^3\right)^2}\right)\Bigg)\Bigg)dq=$$

$$\frac{2-4\,q}{\left(q^2+q+1\right)^3}-\frac{2}{3\left(q^2+q+1\right)^2}+\frac{6\,q}{\left(q^2+q+1\right)^4}+\frac{233}{12\,(q-1)^4}-\frac{3}{4\,(q+1)^4}+\frac{18}{(q-1)^5}+\frac{10}{(q-1)^6}+\text{constant}$$

From the following alternate form

$$-\left(\left(24\left(3 q^{17}+29 q^{16}+138 q^{15}+425 q^{14}+935 q^{13}+1652 q^{12}+2426 q^{11}+2922 q^{10}+2915 q^{9}+2605 q^{8}+2132 q^{7}+1516 q^{6}+917 q^{5}+500 q^{4}+230 q^{3}+77 q^{2}+16 q+2\right)\right)/\left((q-1)^{7}(q+1)^{5}\left(q^{2}+q+1\right)^{5}\right)\right)$$

For q = 8 , we obtain:

-(24(3 8^17+29 8^16+138 8^15+425 8^14+935 8^13+1652 8^12+2426 8^11+2922 8^10+2915 8^9+2605 8^8+2132 8^7+1516 8^6+917 8^5+500 8^4+230 8^3+77 8^2+16 8+2))/((8-1)^7 (8+1)^5 (8^2+8 +1)^5)

Input

$$-\left(\left(24\left(3\times 8^{17}+29\times 8^{16}+138\times 8^{15}+425\times 8^{14}+935\times 8^{13}+1652\times 8^{12}+2426\times 8^{11}+2922\times 8^{10}+2915\times 8^{9}+2605\times 8^{8}+2132\times 8^{7}+1516\times 8^{6}+917\times 8^{5}+500\times 8^{4}+230\times 8^{3}+77\times 8^{2}+16\times 8+2\right)\right)/\left((8-1)^{7}(8+1)^{5}\left(8^{2}+8+1\right)^{5}\right)\right)$$

Exact result

$$-\frac{178357997181664784}{33604069417424242317}$$

Decimal approximation

$$-0.0053076308992857673684499701438789523366963936864536015137262 96\ldots$$

-0.00530763089……

From the algebraic sum of the four results highlighted in red, after some calculations, we obtain:

-1/4*[(-208.932941178-50.0199947642+5.9910719515357+0.00530763089)-Pi]

Input interpretation

$$-\frac{1}{4}((-208.932941178 - 50.0199947642 + 5.9910719515357 + 0.00530763089) - \pi)$$

Result

64.0245372533…

64.0245372533…. ≈ 64 = 8^2

(-1/4*[(-208.932941178-50.0199947642+5.9910719515357+0.00530763089)-Pi])^2-Pi

Input interpretation

$$\left(-\frac{1}{4}((-208.932941178 - 50.0199947642 + 5.9910719515357 + 0.00530763089) - \pi)\right)^2 - \pi$$

Result

4095.99977785…

4095.99977785…. ≈ 4096 = 64^2

The study of this function provides the following representations:

Alternative representations

$$\left(\frac{1}{4}((-208.9329411780000 - 50.01999476420000 + 5.99107195153570000 + 0.00530763) - \pi)(-1)\right)^2 - \pi = -180\,° + \left(-\frac{1}{4}(-252.957 - 180\,°)\right)^2$$

$$\left(\frac{1}{4}\left((-208.9329411780000-50.01999476420000+5.99107195153570000+0.00530763)-\pi\right)(-1)\right)^2-\pi=i\log(-1)+\left(-\frac{1}{4}\left(-252.957+i\log(-1)\right)\right)^2$$

$$\left(\frac{1}{4}\left((-208.9329411780000-50.01999476420000+5.99107195153570000+0.00530763)-\pi\right)(-1)\right)^2-\pi=-\cos^{-1}(-1)+\left(-\frac{1}{4}\left(-252.957-\cos^{-1}(-1)\right)\right)^2$$

Series representations

$$\left(\frac{1}{4}\left((-208.9329411780000-50.01999476420000+5.99107195153570000+0.00530763)-\pi\right)(-1)\right)^2-\pi=3999.19+122.478\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}+\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^2$$

$$\left(\frac{1}{4}\left((-208.9329411780000-50.01999476420000+5.99107195153570000+0.00530763)-\pi\right)(-1)\right)^2-\pi=0.25\left(15752.8+242.957\sum_{k=1}^{\infty}\frac{2^k}{\binom{2k}{k}}+\left(\sum_{k=1}^{\infty}\frac{2^k}{\binom{2k}{k}}\right)^2\right)$$

$$\left(\frac{1}{4}\left(\left(-208.9329411780000-50.01999476420000+5.99107195153570000+0.00530763\right)-\pi\right)(-1)\right)^2-\pi=0.0625\left(63987.+489.913\sum_{k=0}^{\infty}\frac{2^{-k}\,(-6+50\,k)}{\binom{3\,k}{k}}+\left(\sum_{k=0}^{\infty}\frac{2^{-k}\,(-6+50\,k)}{\binom{3\,k}{k}}\right)^2\right)$$

Integral representations

$$\left(\frac{1}{4}\left(\left(-208.9329411780000-50.01999476420000+5.99107195153570000+0.00530763\right)-\pi\right)(-1)\right)^2-\pi=0.25\left(15996.8+244.957\int_0^{\infty}\frac{1}{1+t^2}\,dt+\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^2\right)$$

$$\left(\frac{1}{4}\left(\left(-208.9329411780000-50.01999476420000+5.99107195153570000+0.00530763\right)-\pi\right)(-1)\right)^2-\pi=0.25\left(15996.8+244.957\int_0^{\infty}\frac{\sin(t)}{t}\,dt+\left(\int_0^{\infty}\frac{\sin(t)}{t}\,dt\right)^2\right)$$

$$\left(\frac{1}{4}\left(\left(-208.9329411780000-50.01999476420000+5.99107195153570000+0.00530763\right)-\pi\right)(-1)\right)^2-\pi=3999.19+122.478\int_0^{1}\sqrt{1-t^2}\,dt+\left(\int_0^{1}\sqrt{1-t^2}\,dt\right)^2$$

27*(-1/4*[(-208.932941178-50.0199947642+5.9910719515357+0.00530763089)-Pi])+1/3

Input interpretation

$$27\left(-\frac{1}{4}((-208.932941178-50.0199947642+5.9910719515357+0.00530763089)-\pi)\right)+\frac{1}{3}$$

Result

1728.99583917…

1728.99583917…. ≈ 1729

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

The study of this function provides the following representations:

Alternative representations

$$\frac{27}{4}(-1)((-208.9329411780000-50.01999476420000+5.99107195153570000+0.00530763)-\pi)+\frac{1}{3}=\frac{1}{3}-\frac{27}{4}(-252.957-180\,°)$$

$$\frac{27}{4}(-1)((-208.9329411780000-50.01999476420000+5.99107195153570000+0.00530763)-\pi)+\frac{1}{3}=\frac{1}{3}-\frac{27}{4}(-252.957+i\log(-1))$$

$$\frac{27}{4}(-1)((-208.9329411780000 - 50.01999476420000 + 5.99107195153570000 + 0.00530763) - \pi) + \frac{1}{3} = \frac{1}{3} - \frac{27}{4}\left(-252.957 - \cos^{-1}(-1)\right)$$

Series representations

$$\frac{27}{4}(-1)((-208.9329411780000 - 50.01999476420000 + 5.99107195153570000 + 0.00530763) - \pi) + \frac{1}{3} = 1707.79 + 27\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\frac{27}{4}(-1)((-208.9329411780000 - 50.01999476420000 + 5.99107195153570000 + 0.00530763) - \pi) + \frac{1}{3} = 1694.29 + 13.5\sum_{k=1}^{\infty}\frac{2^k}{\binom{2k}{k}}$$

$$\frac{27}{4}(-1)((-208.9329411780000 - 50.01999476420000 + 5.99107195153570000 + 0.00530763) - \pi) + \frac{1}{3} = 1707.79 + 6.75\sum_{k=0}^{\infty}\frac{2^{-k}(-6+50k)}{\binom{3k}{k}}$$

Integral representations

$$\frac{27}{4}(-1)((-208.9329411780000 - 50.01999476420000 + 5.99107195153570000 + 0.00530763) - \pi) + \frac{1}{3} = 1707.79 + 13.5\int_0^{\infty}\frac{1}{1+t^2}\,dt$$

$$\frac{27}{4}(-1)((-208.9329411780000 - 50.01999476420000 + 5.99107195153570000 + 0.00530763) - \pi) + \frac{1}{3} = 1707.79 + 27\int_0^{1}\sqrt{1-t^2}\,dt$$

$$\frac{27}{4}(-1)((-208.9329411780000 - 50.01999476420000 + 5.99107195153570000 + 0.00530763) - \pi) + \frac{1}{3} = 1707.79 + 13.5 \int_0^\infty \frac{\sin(t)}{t}\, dt$$

(27*(-1/4*[(-208.932941178-50.0199947642+5.9910719515357+0.00530763089)-Pi])+1/3)^1/15

Input interpretation

$$\left(27\left(-\frac{1}{4}((-208.932941178 - 50.0199947642 + 5.9910719515357 + 0.00530763089) - \pi)\right) + \frac{1}{3}\right)\hat{}\,(1/15)$$

Result

1.643814965026...

1.643814965026..... ≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ... (trace of the instanton shape)

Now, from

$$1 + \frac{q}{1-q} + \frac{q^4}{(1-q)(1-q^2)} + \frac{q^9}{(1-q)(1-q^2)(1-q^3)}$$

we perform the following integrations:

integrate((1+q/(1-q)+q^4/((1-q)(1-q^2))+q^9/((1-q)(1-q^2)(1-q^3))))

Indefinite integral

$$\int\left(1+\frac{q}{1-q}+\frac{q^4}{(1-q)(1-q^2)}+\frac{q^9}{(1-q)(1-q^2)(1-q^3)}\right)dq =$$
$$\frac{1}{72}\left(-18q^4-24q^3-36q^2+4\log(q^2+q+1)-144q+\frac{54}{q-1}+\frac{6}{(q-1)^2}-233\log(1-q)+9\log(q+1)+8\sqrt{3}\tan^{-1}\left(\frac{2q+1}{\sqrt{3}}\right)-42\right)+\text{constant}$$

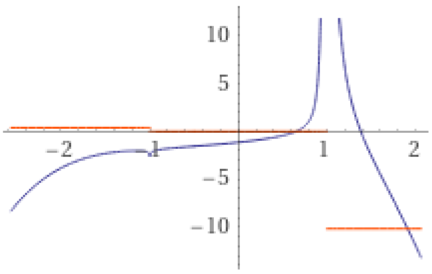
(assuming a complex-valued logarithm)

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

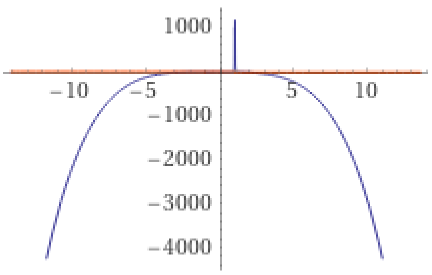
The study of this function provides the following representations:

Plots of the integral **(figures that can be related to the open strings)**



(q from −2.6 to 2.1)

real part
imaginary part



(q from −14.1 to 13.6)

real part
imaginary part

Alternate forms of the integral

$$\frac{1}{72}\left(4\log(q^2+q+1)-\frac{6\left(\left(q\left(3q^3-2q^2+q+16\right)-35\right)q^2+q+15\right)}{(q-1)^2}-233\log(1-q)+9\log(q+1)+8\sqrt{3}\tan^{-1}\left(\frac{2q+1}{\sqrt{3}}\right)\right)+\text{constant}$$

$$-\frac{q^4}{4}-\frac{q^3}{3}-\frac{q^2}{2}+\frac{1}{72}\left(4\log(q^2+q+1)-233\log(1-q)+9\log(q+1)+8\sqrt{3}\tan^{-1}\left(\frac{2q+1}{\sqrt{3}}\right)-42\right)-2q+\frac{3}{4(q-1)}+\frac{1}{12(q-1)^2}+\text{constant}$$

$$-\frac{q^4}{4}-\frac{q^3}{3}-\frac{q^2}{2}+\frac{1}{18}\log(q^2+q+1)-2q+\frac{3}{4(q-1)}+\frac{1}{12(q-1)^2}-\frac{233}{72}\log(1-q)+\frac{1}{8}\log(q+1)+\frac{i\log\left(1-\frac{i(2q+1)}{\sqrt{3}}\right)}{6\sqrt{3}}-\frac{i\log\left(1+\frac{i(2q+1)}{\sqrt{3}}\right)}{6\sqrt{3}}-\frac{7}{12}+\text{constant}$$

Expanded form of the integral

$$-\frac{q^4}{4}-\frac{q^3}{3}-\frac{q^2}{2}+\frac{1}{18}\log(q^2+q+1)-2q+\frac{3}{4(q-1)}+\frac{1}{12(q-1)^2}-\frac{233}{72}\log(1-q)+\frac{1}{8}\log(q+1)+\frac{\tan^{-1}\left(\frac{2q+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{7}{12}+\text{constant}$$

Series expansion of the integral at q=-1

$$\frac{1}{432}\left(54\log(q+1)-8\sqrt{3}\,\pi+279-1398\log(2)\right)+\frac{9(q+1)}{16}+O\left((q+1)^2\right)$$

(Puiseux series)

Series expansion of the integral at q=0

$$\left(\frac{\pi}{18\sqrt{3}} - \frac{5}{4}\right) + q + \frac{q^2}{2} + \frac{q^3}{3} + \frac{q^4}{4} + O(q^5)$$

(Taylor series)

Series expansion of the integral at q=1

$$\frac{1}{12(q-1)^2} + \frac{3}{4(q-1)} + \frac{1}{216}\left(-699\log(1-q) + 8\sqrt{3}\,\pi - 792 + 3\log(41\,472)\right) - \frac{695(q-1)}{144} + O\left((q-1)^2\right)$$

(generalized Puiseux series)

Series expansion of the integral at q=-(-1)$^{1/3}$

$$-\frac{\pi\left\lfloor\frac{\frac{3\pi}{2}-\arg\left(q+\sqrt[3]{-1}\right)}{2\pi}\right\rfloor}{3\sqrt{3}} + \left(\frac{1}{72\left(1+\sqrt[3]{-1}\right)^2}\left(4(-1)^{2/3}\log\left(-i\sqrt{3}\left(q+\sqrt[3]{-1}\right)\right) + 8\sqrt[3]{-1}\log\left(-i\sqrt{3}\left(q+\sqrt[3]{-1}\right)\right) + 4\log\left(-i\sqrt{3}\left(q+\sqrt[3]{-1}\right)\right) + 8(-1)^{5/6}\sqrt{3}\log\left(q+e^{(i\pi)/3}\right) - 4\sqrt[6]{-1}\sqrt{3}\log\left(q+e^{(i\pi)/3}\right) + 4i\sqrt{3}\log\left(q+e^{(i\pi)/3}\right) - 233(-1)^{2/3}\log\left(3+i\sqrt{3}\right) - 466\sqrt[3]{-1}\log\left(3+i\sqrt{3}\right) - 233\log\left(3+i\sqrt{3}\right) - 4(-1)^{5/6}\sqrt{3}\log(3) + 2\sqrt[6]{-1}\sqrt{3}\log(3) - 2i\sqrt{3}\log(3) + 233(-1)^{2/3}\log(2) + 466\sqrt[3]{-1}\log(2) + 233\log(2) + 2(-1)^{2/3}\sqrt{3}\,\pi + 4\sqrt[3]{-1}\sqrt{3}\,\pi + 2\sqrt{3}\,\pi - 6(-1)^{5/6}\pi + 3\sqrt[6]{-1}\pi - 3i\pi + 222(-1)^{2/3} + 12\sqrt[3]{-1} - 204\right) + \left(i\left(21i - 78\sqrt[6]{-1} + 105(-1)^{5/6} + 7\sqrt{3} - 31\sqrt[3]{-1}\sqrt{3} - 40(-1)^{2/3}\sqrt{3}\right)\left(q+\sqrt[3]{-1}\right)\right)\Big/\left(6\left(1+\sqrt[3]{-1}\right)^3\left(i+\sqrt{3}\right)\left(-3i+\sqrt{3}\right)\right) + O\left(\left(q+\sqrt[3]{-1}\right)^2\right)\right)$$

We calculate the double integrate of

$$1+\frac{q}{1-q}+\frac{q^4}{(1-q)(1-q^2)}+\frac{q^9}{(1-q)(1-q^2)(1-q^3)}$$

Input

$$\int\left(\int\left(1+\frac{q}{1-q}+\frac{q^4}{(1-q)(1-q^2)}+\frac{q^9}{(1-q)(1-q^2)(1-q^3)}\right)dq\right)dq$$

Exact result

$$\frac{1}{72}\left(-\frac{18q^5}{5}-6q^4-12q^3-72q^2+4q\log(q^2+q+1)+2\log(q^2+q+1)-12\left(\frac{1}{2}\log(q^2+q+1)-\frac{\tan^{-1}\left(\frac{2q+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+174q-\frac{6}{q-1}+9q\log(q+1)+233(1-q)\log(1-q)+54\log(q-1)+9\log(q+1)+8\sqrt{3}\,q\tan^{-1}\left(\frac{2q+1}{\sqrt{3}}\right)+4\sqrt{3}\tan^{-1}\left(\frac{2q+1}{\sqrt{3}}\right)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

The study of this function provides the following representations:

Indefinite integral

$$\int\int\left(1+\frac{q}{1-q}+\frac{q^4}{(1-q)(1-q^2)}+\frac{q^9}{(1-q)(1-q^2)(1-q^3)}\right)dq\,dq=$$
$$\frac{1}{360}\left(360c_1q-18q^5-30q^4-60q^3-360q^2+20q\log(q^2+q+1)-20\log(q^2+q+1)+870q-\frac{30}{q-1}+45q\log(q+1)-5(233q-287)\log(1-q)+45\log(q+1)+40\sqrt{3}\,(q+1)\tan^{-1}\left(\frac{2q+1}{\sqrt{3}}\right)\right)+c_2$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

Series expansion of the integral at q=-1

$$\begin{cases} \left(\frac{1}{720} + \frac{q+1}{1440} + O\left((q+1)^2\right)\right)\left(\left(270 - 270\,(q+1) + O\left((q+1)^2\right)\right)\right. \\ \quad \left((i\,\pi + \log(2)) - \frac{q+1}{2} + O\left((q+1)^2\right)\right)^* + \\ \quad 270\left((i\,\pi + \log(2)) - \frac{q+1}{2} + O\left((q+1)^2\right)\right)^* + \\ \quad \left(2\,(-1167 + 2330\log(2)) - \frac{2}{3}\left(-135\log(q+1) + 6990\log(2) + \right.\right. \\ \quad\quad \left.\left.\left. 20\sqrt{3}\,\pi - 2718\right)(q+1) + O\left((q+1)^2\right)\right)\right) & \operatorname{Im}(q) < 0 \\ \frac{1}{360}\,i\,(1167\,i + 270\,\pi - 2600\,i\log(2)) + \frac{1}{432}\,(q+1) \\ \quad \left(54\log(q+1) - 8\sqrt{3}\,\pi + 225 - 1398\log(2)\right) + O\left((q+1)^2\right) & \text{(otherwise)} \end{cases}$$

Series expansion of the integral at q=1

$$-\frac{1}{12\,(q-1)} + \left(\frac{3}{4}\log(q-1) + \frac{2\,\pi}{9\sqrt{3}} + \frac{67}{60} + \frac{\log(2)}{4}\right) + \frac{1}{216}\,(q-1)\left(-699\log(1-q) + 8\sqrt{3}\,\pi - 93 + 3\log(41\,472)\right) + O\left((q-1)^2\right)$$

(generalized Puiseux series)

Series expansion of the integral at q=-(-1)$^{1/3}$

$$\left\lfloor \frac{\frac{3\pi}{2} - \arg\left(q + \sqrt[3]{-1}\right)}{2\pi} \right\rfloor \left(\frac{\left(-1 + \sqrt[3]{-1}\right)\pi}{3\sqrt{3}} - \frac{\pi\left(q + \sqrt[3]{-1}\right)}{3\sqrt{3}} + O\left(\left(q + \sqrt[3]{-1}\right)^2\right) \right) +$$

$$\left(\frac{1}{360\left(1 + \sqrt[3]{-1}\right)} \Big(-20(-1)^{2/3}\log\left(-i\sqrt{3}\left(q + \sqrt[3]{-1}\right)\right) - 40\sqrt[3]{-1}\log\left(-i\sqrt{3}\left(q + \sqrt[3]{-1}\right)\right) - 20\log\left(-i\sqrt{3}\left(q + \sqrt[3]{-1}\right)\right) + 20\sqrt[6]{-1}\sqrt{3}\log\left(q + e^{(i\pi)/3}\right) + 20i\sqrt{3}\log\left(q + e^{(i\pi)/3}\right) + 1165(-1)^{2/3}\log\left(3 + i\sqrt{3}\right) + 2330\sqrt[3]{-1}\log\left(3 + i\sqrt{3}\right) + 1165\log\left(3 + i\sqrt{3}\right) + 270\sqrt[3]{-1}\log\left(-3 - i\sqrt{3}\right) + 270\log\left(-3 - i\sqrt{3}\right) - 10\sqrt[6]{-1}\sqrt{3}\log(3) - 10i\sqrt{3}\log(3) - 1165(-1)^{2/3}\log(2) - 2600\sqrt[3]{-1}\log(2) - 1435\log(2) - 10(-1)^{2/3}\sqrt{3}\pi + 10\sqrt{3}\pi - 15\sqrt[6]{-1}\pi - 15i\pi - 1218(-1)^{2/3} - 900\sqrt[3]{-1} + 348 \Big) + \right.$$

$$\Big(\Big(-8(-1)^{5/6}\sqrt{3}\log\left(-i\sqrt{3}\left(q + \sqrt[3]{-1}\right)\right) + 4\sqrt[6]{-1}\sqrt{3}\log\left(-i\sqrt{3}\left(q + \sqrt[3]{-1}\right)\right) - 4i\sqrt{3}\log\left(-i\sqrt{3}\left(q + \sqrt[3]{-1}\right)\right) + 12(-1)^{2/3}\log\left(-i\sqrt{3}\left(q + \sqrt[3]{-1}\right)\right) + 24\sqrt[3]{-1}\log\left(-i\sqrt{3}\left(q + \sqrt[3]{-1}\right)\right) + 12\log\left(-i\sqrt{3}\left(q + \sqrt[3]{-1}\right)\right) + 24(-1)^{5/6}\sqrt{3}\log\left(q + e^{(i\pi)/3}\right) - 12\sqrt[6]{-1}\sqrt{3}\log\left(q + e^{(i\pi)/3}\right) + 12i\sqrt{3}\log\left(q + e^{(i\pi)/3}\right) + 12(-1)^{2/3}\log\left(q + e^{(i\pi)/3}\right) + 24\sqrt[3]{-1}\log\left(q + e^{(i\pi)/3}\right) + 12\log\left(q + e^{(i\pi)/3}\right) + 466(-1)^{5/6}\sqrt{3}\log\left(3 + i\sqrt{3}\right) - 233\sqrt[6]{-1}\sqrt{3}\log\left(3 + i\sqrt{3}\right) + 233i\sqrt{3}\log\left(3 + i\sqrt{3}\right) - 699(-1)^{2/3}\log\left(3 + i\sqrt{3}\right) - 1398\sqrt[3]{-1}\log\left(3 + i\sqrt{3}\right) - 699\log\left(3 + i\sqrt{3}\right) - 12(-1)^{5/6}\sqrt{3}\log(3) + 6\sqrt[6]{-1}\sqrt{3}\log(3) - 6i\sqrt{3}\log(3) - 6(-1)^{2/3}\log(3) - 12\sqrt[3]{-1}\log(3) - 6\log(3) - 466(-1)^{5/6}\sqrt{3}\log(2) + 233\sqrt[6]{-1}\sqrt{3}\log(2) - 233i\sqrt{3}\log(2) + 699(-1)^{2/3}\log(2) + 1398\sqrt[3]{-1}\log(2) + 699\log(2) + 3(-1)^{2/3}\sqrt{3}\pi + 6\sqrt[3]{-1}\sqrt{3}\pi + 3\sqrt{3}\pi - 30(-1)^{5/6}\pi + 15\sqrt[6]{-1}\pi - 15i\pi + 302(-1)^{5/6}\sqrt{3} - 298\sqrt[6]{-1}\sqrt{3} - 2i\sqrt{3} + 510(-1)^{2/3} + 714\sqrt[3]{-1} + 222 \Big) \left(q + \sqrt[3]{-1}\right) \Big) \Big/$$

$$\left. \left(36\left(1 + \sqrt[3]{-1}\right)^2 \left(i + \sqrt{3}\right)\left(-3i + \sqrt{3}\right)\right) + O\left(\left(q + \sqrt[3]{-1}\right)^2\right) \right)$$

arg(z) is the complex argument

$\lfloor x \rfloor$ is the floor function

Now, we perform the various integrations of :

$$1+\frac{q}{1-q}+\frac{q^4}{(1-q)\left(1-q^2\right)}+\frac{q^9}{(1-q)\left(1-q^2\right)\left(1-q^3\right)}$$

Indefinite integral

$$\int\left(1+\frac{q}{1-q}+\frac{q^4}{(1-q)\left(1-q^2\right)}+\frac{q^9}{(1-q)\left(1-q^2\right)\left(1-q^3\right)}\right)dq =$$
$$\frac{1}{72}\left(-18\,q^4-24\,q^3-36\,q^2+4\log\left(q^2+q+1\right)-144\,q+\frac{54}{q-1}+\frac{6}{(q-1)^2}-233\log(1-q)+9\log(q+1)+8\sqrt{3}\tan^{-1}\left(\frac{2\,q+1}{\sqrt{3}}\right)-42\right)+\text{constant}$$

(assuming a complex-valued logarithm)

1/72 (-18 8^4 - 24 8^3 - 36 8^2 + 4 log(8^2 + 8 + 1) - 144 8 + 54/(8 - 1) + 6/(8 - 1)^2 - 233 log(1 - 8) + 9 log(8 + 1) + 8 sqrt(3) tan^(-1)((2 8 + 1)/sqrt(3)) - 42)

Input

$$\frac{1}{72}\left(-18\times 8^4-24\times 8^3-36\times 8^2+4\log\left(8^2+8+1\right)-144\times 8+\frac{54}{8-1}+\frac{6}{(8-1)^2}-233\log(1-8)+9\log(8+1)+8\sqrt{3}\tan^{-1}\left(\frac{2\times 8+1}{\sqrt{3}}\right)-42\right)$$

Exact Result

$$\frac{1}{72}\left(-\frac{4385802}{49}-233\,(\log(7)+i\,\pi)+9\log(9)+4\log(73)+8\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)$$

(result in radians)

Decimal approximation

$$-1248.642566508517889335731125828818309316881024777567468208311189\ldots - 10.16654289286696978558049872644616905580472875075555078176614 01\ldots\,i$$

(result in radians)

Polar coordinates

$r \approx 1248.7$ (radius), $\theta \approx -3.1335$ (angle)

1248.7

<u>The study of this function provides the following representations</u>:

Polar forms

$$\frac{1}{72}\sqrt{\left(54289\pi^2+\left(-\frac{4385802}{49}-233\log(7)+9\log(9)+4\log(73)+8\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)^2\right)}\left(\cos\left(-\pi-\tan^{-1}\left(\frac{233\pi}{-\frac{4385802}{49}-233\log(7)+9\log(9)+4\log(73)+8\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}\right)\right)+i\sin\left(-\pi-\tan^{-1}\left(\frac{233\pi}{-\frac{4385802}{49}-233\log(7)+9\log(9)+4\log(73)+8\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}\right)\right)\right)$$

Approximate form

$$\frac{1}{72}\sqrt{\left(54289\pi^2+\left(-\frac{4385802}{49}-233\log(7)+9\log(9)+4\log(73)+8\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)^2\right)}\exp\left(i\left(-\pi-\tan^{-1}\left(\frac{233\pi}{-\frac{4385802}{49}-233\log(7)+9\log(9)+4\log(73)+8\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}\right)\right)\right)$$

Alternate forms

$$\frac{1}{3528}\left(-4385802 - 11417\, i\, \pi - 11417\log(7) + 441\log(9) + 196\log(73) + 392\sqrt{3}\,\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)$$

$$\frac{-4385802 - 11417\log(7) + 441\log(9) + 196\log(73) + 392\sqrt{3}\,\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{3528} - \frac{233\, i\, \pi}{72}$$

$$-\frac{1}{3528} i\left(11417\,\pi + i\left(882\log(3) - 11417\log(7) + 2\,(98\log(73) - 2192901) + 392\sqrt{3}\,\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)\right)$$

$$-\frac{730967}{588} + \frac{\log(3)}{4} - \frac{233}{72}(\log(7) + i\,\pi) + \frac{\log(73)}{18} + \frac{\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Expanded form

$$-\frac{730967}{588} - \frac{233\, i\, \pi}{72} - \frac{233\log(7)}{72} + \frac{\log(9)}{8} + \frac{\log(73)}{18} + \frac{\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Alternative representations

$$\frac{1}{72}\left(-18\times 8^4-24\times 8^3-36\times 8^2+4\log\left(8^2+8+1\right)-144\times 8+\frac{54}{8-1}+\frac{6}{(8-1)^2}-233\log(1-8)+9\log(8+1)+8\sqrt{3}\tan^{-1}\left(\frac{2\times 8+1}{\sqrt{3}}\right)-42\right)=\frac{1}{72}\left(-1194-233\log(-7)+9\log(9)+4\log\left(9+8^2\right)+\frac{54}{7}-36\times 8^2-24\times 8^3-18\times 8^4+\frac{6}{7^2}+8\tan^{-1}\left(1,\frac{17}{\sqrt{3}}\right)\sqrt{3}\right)$$

$$\frac{1}{72}\left(-18\times 8^4-24\times 8^3-36\times 8^2+4\log\left(8^2+8+1\right)-144\times 8+\frac{54}{8-1}+\frac{6}{(8-1)^2}-233\log(1-8)+9\log(8+1)+8\sqrt{3}\tan^{-1}\left(\frac{2\times 8+1}{\sqrt{3}}\right)-42\right)=\frac{1}{72}\left(-1194-233\log(-7)+9\log(9)+4\log\left(9+8^2\right)+\frac{54}{7}-36\times 8^2-24\times 8^3-18\times 8^4+\frac{6}{7^2}+4i\left(\log\left(1-\frac{17i}{\sqrt{3}}\right)-\log\left(1+\frac{17i}{\sqrt{3}}\right)\right)\sqrt{3}\right)$$

$$\frac{1}{72}\left(-18\times 8^4-24\times 8^3-36\times 8^2+4\log\left(8^2+8+1\right)-144\times 8+\frac{54}{8-1}+\frac{6}{(8-1)^2}-233\log(1-8)+9\log(8+1)+8\sqrt{3}\tan^{-1}\left(\frac{2\times 8+1}{\sqrt{3}}\right)-42\right)=\frac{1}{72}\left(-1194-233\log(a)\log_a(-7)+9\log(a)\log_a(9)+4\log(a)\log_a\left(9+8^2\right)+\frac{54}{7}-36\times 8^2-24\times 8^3-18\times 8^4+\frac{6}{7^2}+8\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\sqrt{3}\right)$$

Series representations

$$\frac{1}{72}\left(-18\times 8^4-24\times 8^3-36\times 8^2+4\log\left(8^2+8+1\right)-144\times 8+\frac{54}{8-1}+\frac{6}{(8-1)^2}-233\log(1-8)+9\log(8+1)+8\sqrt{3}\tan^{-1}\left(\frac{2\times 8+1}{\sqrt{3}}\right)-42\right)=$$
$$-\frac{730967}{588}-\frac{233\,i\,\pi}{72}+\frac{\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{233\log(6)}{72}+\frac{\log(8)}{8}+\frac{\log(72)}{18}+\sum_{k=1}^{\infty}-\frac{72^{-1-k}\left(4+9^{1+k}-233\times 12^k\right)e^{i\,k\,\pi}}{k}$$

$$\frac{1}{72}\left(-18\times 8^4-24\times 8^3-36\times 8^2+4\log\left(8^2+8+1\right)-144\times 8+\frac{54}{8-1}+\frac{6}{(8-1)^2}-233\log(1-8)+9\log(8+1)+8\sqrt{3}\tan^{-1}\left(\frac{2\times 8+1}{\sqrt{3}}\right)-42\right)=$$
$$-\frac{730967}{588}-\frac{233\,i\,\pi}{72}+\frac{\tan^{-1}(z_0)}{3\sqrt{3}}-\frac{233\log(6)}{72}+\frac{\log(8)}{8}+\frac{\log(72)}{18}+\sum_{k=1}^{\infty}\left(-\frac{233\,(-1)^{-1+k}\,2^{-3-k}\times 3^{-2-k}}{k}+\frac{(-1)^{-1+k}\,8^{-1-k}}{k}+\frac{(-1)^{-1+k}\,2^{-1-3k}\times 9^{-1-k}}{k}+\frac{i\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{17}{\sqrt{3}}-z_0\right)^k}{6\sqrt{3}\,k}\right)$$

for $(i\,z_0 \notin \mathbb{R}$ or $((\text{not } 1 \le i\,z_0 < \infty)$ and $(\text{not } -\infty < i\,z_0 \le -1)))$

$$\frac{1}{72}\left(-18\times 8^4-24\times 8^3-36\times 8^2+4\log\left(8^2+8+1\right)-144\times 8+\frac{54}{8-1}+\frac{6}{(8-1)^2}-233\log(1-8)+9\log(8+1)+8\sqrt{3}\tan^{-1}\left(\frac{2\times 8+1}{\sqrt{3}}\right)-42\right)=$$

$$-\frac{730967}{588}-\frac{233\,i\,\pi}{72}+\frac{\tan^{-1}(z_0)}{3\sqrt{3}}-\frac{233\log(6)}{72}+\frac{\log(8)}{8}+\frac{\log(72)}{18}+\sum_{k=1}^{\infty}\left(\frac{\left(-\frac{1}{8}\right)^{1+k}}{k}+\frac{\left(-\frac{1}{9}\right)^{1+k}2^{-1-3k}}{k}-\frac{233(-1)^{1+k}2^{-3-k}\times 3^{-2-k}}{k}+\frac{i\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{17}{\sqrt{3}}-z_0\right)^k}{6\sqrt{3}\,k}\right)$$

for ($i\,z_0\notin\mathbb{R}$ or ((not $1\leq i\,z_0<\infty$) and (not $-\infty<i\,z_0\leq -1$)))

Integral representations

$$\frac{1}{72}\left(-18\times 8^4-24\times 8^3-36\times 8^2+4\log\left(8^2+8+1\right)-144\times 8+\frac{54}{8-1}+\frac{6}{(8-1)^2}-233\log(1-8)+9\log(8+1)+8\sqrt{3}\tan^{-1}\left(\frac{2\times 8+1}{\sqrt{3}}\right)-42\right)=$$

$$-\frac{730967}{588}-\frac{233\,i\,\pi}{72}+\frac{17}{3}\int_0^1\frac{1}{3+289\,t^2}\,dt-\frac{233\log(7)}{72}+\frac{\log(9)}{8}+\frac{\log(73)}{18}$$

$$\frac{1}{72}\left(-18\times 8^4-24\times 8^3-36\times 8^2+4\log\left(8^2+8+1\right)-144\times 8+\frac{54}{8-1}+\frac{6}{(8-1)^2}-233\log(1-8)+9\log(8+1)+8\sqrt{3}\tan^{-1}\left(\frac{2\times 8+1}{\sqrt{3}}\right)-42\right)=$$

$$-\frac{730967}{588}-\frac{233\,i\,\pi}{72}+\int_1^{73}\frac{1}{216}\left(\frac{17}{3+\frac{289(-1+t)^2}{5184}}+\frac{12}{t}-\frac{5295+672\,t}{88+19\,t+t^2}\right)dt$$

$$\frac{1}{72}\left(-18\times 8^4-24\times 8^3-36\times 8^2+4\log(8^2+8+1)-144\times 8+\frac{54}{8-1}+\frac{6}{(8-1)^2}-233\log(1-8)+9\log(8+1)+8\sqrt{3}\tan^{-1}\left(\frac{2\times 8+1}{\sqrt{3}}\right)-42\right)=$$

$$\frac{1}{3528}\left(-4385802-11417\,i\,\pi+19992\int_0^1\frac{1}{3+289\,t^2}\,dt+3528\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}-\frac{i\,2^{-4-3\,s}\times 9^{-1-s}\left(4+9^{1+s}-233\times 12^s\right)\Gamma(-s)^2\,\Gamma(1+s)}{\pi\,\Gamma(1-s)}\,ds\right)$$

for $-1<\gamma<0$

We perform the double integrate of

$$1+\frac{q}{1-q}+\frac{q^4}{(1-q)\left(1-q^2\right)}+\frac{q^9}{(1-q)\left(1-q^2\right)\left(1-q^3\right)}$$

$$\int\left(\int\left(1+\frac{q}{1-q}+\frac{q^4}{(1-q)\left(1-q^2\right)}+\frac{q^9}{(1-q)\left(1-q^2\right)\left(1-q^3\right)}\right)dq\right)dq$$

we obtain:

$$\frac{1}{72}\left(-\frac{18\,q^5}{5}-6\,q^4-12\,q^3-72\,q^2+4\,q\log\left(q^2+q+1\right)+2\log\left(q^2+q+1\right)-12\left(\frac{1}{2}\log\left(q^2+q+1\right)-\frac{\tan^{-1}\left(\frac{2q+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+174\,q-\frac{6}{q-1}+9\,q\log(q+1)+233\,(1-q)\log(1-q)+54\log(q-1)+9\log(q+1)+8\sqrt{3}\,q\tan^{-1}\left(\frac{2\,q+1}{\sqrt{3}}\right)+4\sqrt{3}\tan^{-1}\left(\frac{2\,q+1}{\sqrt{3}}\right)\right)$$

After the following calculations, considering q = 8 :

(-(18 8^5)/5 - 6 8^4 - 12 8^3 - 72 8^2 + 4 8 log(8^2 + 8 + 1) + 2 log(8^2 + 8 + 1) - 12 (1/2 log(8^2 + 8 + 1) - (tan^(-1)((2 8 + 1)/sqrt(3)))/sqrt(3)) + 174 8

Input

$$-\frac{1}{5}\left(18\times 8^5\right)-6\times 8^4-12\times 8^3-72\times 8^2+4\times 8\log\left(8^2+8+1\right)+2\log\left(8^2+8+1\right)-12\left(\frac{1}{2}\log\left(8^2+8+1\right)-\frac{\tan^{-1}\left(\frac{2\times 8+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+174\times 8$$

Exact Result

$$-\frac{759504}{5}+34\log(73)-12\left(\frac{\log(73)}{2}-\frac{\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

(result in radians)

Decimal approximation

−151770.4877944167936094179133936978823826277901579682525797686088
...

(result in radians)

- 6/(8 - 1) + 9 8 log(8 + 1) + 233 (1 - 8) log(1 - 8) + 54 log(8 - 1) + 9 log(8 + 1) + 8 sqrt(3) 8 tan^(-1)((2 8 + 1)/sqrt(3)) + 4 sqrt(3) tan^(-1)((2 8 + 1)/sqrt(3)))

Input

$$-\frac{6}{8-1}+9\times 8\log(8+1)+233(1-8)\log(1-8)+54\log(8-1)+9\log(8+1)+8\sqrt{3}\times 8\tan^{-1}\left(\frac{2\times 8+1}{\sqrt{3}}\right)+4\sqrt{3}\tan^{-1}\left(\frac{2\times 8+1}{\sqrt{3}}\right)$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result

$$-\frac{6}{7} + 54 \log(7) - 1631 (\log(7) + i \pi) + 81 \log(9) + 68 \sqrt{3} \tan^{-1}\left(\frac{17}{\sqrt{3}}\right)$$

(result in radians)

Decimal approximation

−2718.533456225263706840340199890334895050741330659626884576198 40…
−
5123.937618004952771932571358128869204125583290380797594010134 63… i

(result in radians)

Polar coordinates

$r \approx 5800.4$ (radius), $\theta \approx -2.0586$ (angle)

Thence, in conclusion, we obtain:

1/72(-759504/5 - 12 (-(tan^(-1)(17/sqrt(3)))/sqrt(3) + log(73)/2) + 34 log(73)-6/7 + 68 sqrt(3) tan^(-1)(17/sqrt(3)) + 54 log(7) - 1631 (i π + log(7)) + 81 log(9))

Input

$$\frac{1}{72}\left(-\frac{759504}{5} - 12\left(-\frac{\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log(73)}{2}\right) + 34 \log(73) - \frac{6}{7} + 68 \sqrt{3} \tan^{-1}\left(\frac{17}{\sqrt{3}}\right) + 54 \log(7) - 1631 (i \pi + \log(7)) + 81 \log(9)\right)$$

Exact Result

$$\frac{1}{72}\left(-\frac{5316558}{35}+54\log(7)-1631(\log(7)+i\pi)+81\log(9)+\right.$$
$$\left.34\log(73)+68\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)-12\left(\frac{\log(73)}{2}-\frac{\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)$$

(result in radians)

Decimal approximation

$-2145.680850703361907170253522133169684412201826230942770338812232\ldots$
$-$
$71.165800250068788499063491085123183390633101255288855472362980\,9\ldots\, i$

(result in radians)

Polar coordinates

$r \approx 2146.9$ (radius), $\theta \approx -3.1084$ (angle)

2146.9

The study of this function provides the following representations:

Polar forms

$$\frac{1}{72}\sqrt{\left(2660161\pi^2+\left(-\frac{5316558}{35}-1577\log(7)+81\log(9)+28\log(73)+72\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)^2\right)}\left(\cos\left(-\pi-\tan^{-1}\left((1631\pi)\Big/\left(-\frac{5316558}{35}-1577\log(7)+81\log(9)+34\log(73)+68\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)-12\left(\frac{\log(73)}{2}-\frac{\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)\right)\right)+i\sin\left(-\pi-\tan^{-1}\left((1631\pi)\Big/\left(-\frac{5316558}{35}-1577\log(7)+81\log(9)+34\log(73)+68\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)-12\left(\frac{\log(73)}{2}-\frac{\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)\right)\right)\right)$$

Approximate form

$$\frac{1}{72}\sqrt{\left(2660161\pi^2+\left(-\frac{5316558}{35}-1577\log(7)+81\log(9)+28\log(73)+72\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)^2\right)}\exp\left(i\left(-\pi-\tan^{-1}\left((1631\pi)\Big/\left(-\frac{5316558}{35}-1577\log(7)+81\log(9)+34\log(73)+68\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)-12\left(\frac{\log(73)}{2}-\frac{\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{\sqrt{3}}\right)\right)\right)\right)\right)$$

Alternate forms

$$\frac{1}{2520}\left(-5316558-57085i\pi-55195\log(7)+2835\log(9)+980\log(73)+2520\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)$$

$$\frac{-5316558 - 55195\log(7) + 2835\log(9) + 980\log(73) + 2520\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{2520} - \frac{1631\,i\,\pi}{72}$$

$$-\frac{1}{2520\sqrt{3}}\,i\left(57085\sqrt{3}\,\pi + i\left(\sqrt{3}\,(5670\log(3) - 55195\log(7) + 2\,(490\log(73) - 2658279)) + 7560\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)\right)$$

$$-\frac{886093}{420} + \frac{9\log(3)}{4} + \frac{3\log(7)}{4} - \frac{1631}{72}(\log(7) + i\,\pi) + \frac{17\log(73)}{36} + \frac{17\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{6\sqrt{3}} + \frac{1}{6}\left(\frac{\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(73)}{2}\right)$$

Expanded form

$$-\frac{886093}{420} - \frac{1631\,i\,\pi}{72} - \frac{1577\log(7)}{72} + \frac{9\log(9)}{8} + \frac{7\log(73)}{18} + \sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)$$

Alternative representations

$$\frac{1}{72}\left(-\frac{759504}{5} - 12\left(-\frac{\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log(73)}{2}\right) + 34\log(73) - \frac{6}{7} + 68\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right) + 54\log(7) - 1631\,(i\,\pi + \log(7)) + 81\log(9)\right) =$$
$$\frac{1}{72}\left(54\log(7) - 1631\,(i\,\pi + \log(7)) + 81\log(9) + 34\log(73) - \frac{759504}{5} - \frac{6}{7} - 12\left(\frac{\log(73)}{2} - \frac{\tan^{-1}\left(1, \frac{17}{\sqrt{3}}\right)}{\sqrt{3}}\right) + 68\tan^{-1}\left(1, \frac{17}{\sqrt{3}}\right)\sqrt{3}\right)$$

$$\frac{1}{72}\left(-\frac{759504}{5}-12\left(-\frac{\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log(73)}{2}\right)+34\log(73)-\frac{6}{7}+\right.$$
$$\left.68\sqrt{3}\,\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)+54\log(7)-1631\,(i\,\pi+\log(7))+81\log(9)\right)=$$
$$\frac{1}{72}\left(54\log(a)\log_a(7)-1631\,(i\,\pi+\log(a)\log_a(7))+\right.$$
$$81\log(a)\log_a(9)+34\log(a)\log_a(73)-\frac{759504}{5}-\frac{6}{7}-$$
$$\left.12\left(\frac{1}{2}\log(a)\log_a(73)-\frac{\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{\sqrt{3}}\right)+68\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\sqrt{3}\right)$$

$$\frac{1}{72}\left(-\frac{759504}{5}-12\left(-\frac{\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log(73)}{2}\right)+34\log(73)-\frac{6}{7}+\right.$$
$$\left.68\sqrt{3}\,\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)+54\log(7)-1631\,(i\,\pi+\log(7))+81\log(9)\right)=$$
$$\frac{1}{72}\left(54\log(a)\log_a(7)-1631\,(i\,\pi+\log(a)\log_a(7))+\right.$$
$$81\log(a)\log_a(9)+34\log(a)\log_a(73)-\frac{759504}{5}-\frac{6}{7}-$$
$$\left.12\left(\frac{1}{2}\log(a)\log_a(73)-\frac{\tan^{-1}\left(1,\ \frac{17}{\sqrt{3}}\right)}{\sqrt{3}}\right)+68\tan^{-1}\left(1,\ \frac{17}{\sqrt{3}}\right)\sqrt{3}\right)$$

Series representations

$$\frac{1}{72}\left(-\frac{759504}{5}-12\left(-\frac{\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log(73)}{2}\right)+34\log(73)-\frac{6}{7}+68\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)+54\log(7)-1631(i\pi+\log(7))+81\log(9)\right)=-\frac{886093}{420}-\frac{1631 i\pi}{72}+\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)-\frac{1577\log(6)}{72}+\frac{9\log(8)}{8}+\frac{7\log(72)}{18}+\sum_{k=1}^{\infty}-\frac{72^{-1-k}\left(28+9^{2+k}-1577\times 12^k\right)e^{i k\pi}}{k}$$

$$\frac{1}{72}\left(-\frac{759504}{5}-12\left(-\frac{\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log(73)}{2}\right)+34\log(73)-\frac{6}{7}+68\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)+54\log(7)-1631(i\pi+\log(7))+81\log(9)\right)=-\frac{886093}{420}-\frac{1631 i\pi}{72}+\sqrt{3}\tan^{-1}(z_0)-\frac{1577\log(6)}{72}+\frac{9\log(8)}{8}+\frac{7\log(72)}{18}+\sum_{k=1}^{\infty}\left(-\frac{1577(-1)^{-1+k}2^{-3-k}\times 3^{-2-k}}{k}+\frac{9(-1)^{-1+k}8^{-1-k}}{k}+\frac{7(-1)^{-1+k}2^{-1-3k}\times 9^{-1-k}}{k}+\frac{i\sqrt{3}\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{17}{\sqrt{3}}-z_0\right)^k}{2k}\right)$$

for $(i z_0 \notin \mathbb{R}$ or $((\text{not } 1 \leq i z_0 < \infty)$ and $(\text{not } -\infty < i z_0 \leq -1)))$

$$\frac{1}{72}\left(-\frac{759504}{5}-12\left(-\frac{\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log(73)}{2}\right)+34\log(73)-\frac{6}{7}+68\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)+54\log(7)-1631(i\pi+\log(7))+81\log(9)\right)=$$
$$-\frac{886093}{420}-\frac{1631 i\pi}{72}+\sqrt{3}\tan^{-1}(z_0)-\frac{1577\log(6)}{72}+\frac{9\log(8)}{8}+\frac{7\log(72)}{18}+\sum_{k=1}^{\infty}\left(\frac{9\left(-\frac{1}{8}\right)^{1+k}}{k}+\frac{7\left(-\frac{1}{9}\right)^{1+k}2^{-1-3k}}{k}-\frac{1577(-1)^{1+k}2^{-3-k}\times 3^{-2-k}}{k}+\frac{i\sqrt{3}\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{17}{\sqrt{3}}-z_0\right)^k}{2k}\right)$$

for ($i z_0 \notin \mathbb{R}$ or ((not $1 \leq i z_0 < \infty$) and (not $-\infty < i z_0 \leq -1$)))

Integral representations

$$\frac{1}{72}\left(-\frac{759504}{5}-12\left(-\frac{\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log(73)}{2}\right)+34\log(73)-\frac{6}{7}+68\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)+54\log(7)-1631(i\pi+\log(7))+81\log(9)\right)=$$
$$-\frac{886093}{420}-\frac{1631 i\pi}{72}+\int_1^{73}\frac{1}{72}\left(\frac{51}{3+\frac{289(-1+t)^2}{5184}}+\frac{28}{t}+\frac{81}{8+t}-\frac{1577}{11+t}\right)dt$$

$$\frac{1}{72}\left(-\frac{759504}{5}-12\left(-\frac{\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log(73)}{2}\right)+34\log(73)-\frac{6}{7}+68\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)+54\log(7)-1631(i\pi+\log(7))+81\log(9)\right)=$$
$$-\frac{886093}{420}-\frac{1631 i\pi}{72}+51\int_0^1\frac{1}{3+289t^2}\,dt-\frac{1577\log(7)}{72}+\frac{9\log(9)}{8}+\frac{7\log(73)}{18}$$

$$\frac{1}{72}\left(-\frac{759504}{5}-12\left(-\frac{\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log(73)}{2}\right)+34\log(73)-\frac{6}{7}+68\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)+54\log(7)-1631(i\pi+\log(7))+81\log(9)\right)=$$

$$\frac{1}{2520}\left(-5316558-57085\,i\pi+128520\int_0^1\frac{1}{3+289t^2}\,dt+2520\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i\,2^{-4-3s}\times 9^{-1-s}\left(28+9^{2+s}-1577\times 12^s\right)\Gamma(-s)^2\,\Gamma(1+s)}{\pi\,\Gamma(1-s)}\,ds\right)$$

for $-1<\gamma<0$

$\Gamma(x)$ is the gamma function

Continued fraction representations

$$\frac{1}{72}\left(-\frac{759504}{5}-12\left(-\frac{\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log(73)}{2}\right)+34\log(73)-\frac{6}{7}+68\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)+54\log(7)-1631(i\pi+\log(7))+81\log(9)\right)=$$

$$-\frac{886093}{420}-\frac{1631\,i\pi}{72}-\frac{1577\log(7)}{72}+\frac{9\log(9)}{8}+\frac{7\log(73)}{18}+\frac{17}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{289k^2}{3}}{1+2k}}=$$

$$-\frac{886093}{420}-\frac{1631\,i\pi}{72}-\frac{1577\log(7)}{72}+\frac{9\log(9)}{8}+\frac{7\log(73)}{18}+\frac{17}{1+\frac{289}{3\left(3+\frac{1156}{3\left(5+\frac{867}{7+\frac{4624}{3(9+\ldots)}}\right)}\right)}}$$

$$\frac{1}{72}\left(-\frac{759504}{5}-12\left(-\frac{\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log(73)}{2}\right)+34\log(73)-\frac{6}{7}+68\sqrt{3}\,\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)+54\log(7)-1631\,(i\pi+\log(7))+81\log(9)\right)=$$

$$\left(3\,(-6649358-57085\,i\pi-55195\log(7)+2835\log(9)+980\log(73))+(-5273718-57085\,i\pi-55195\log(7)+2835\log(9)+980\log(73))\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{289}{3}\left(1+(-1)^{1+k}+k\right)^2}{3+2k}\right)\right)\Bigg/\left(2520\left(3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{289}{3}\left(1+(-1)^{1+k}+k\right)^2}{3+2k}\right)\right)=$$

$$\left(3\,(-6649358-57085\,i\pi-55195\log(7)+2835\log(9)+980\log(73))+(-5273718-57085\,i\pi-55195\log(7)+2835\log(9)+980\log(73))\frac{867}{5+\frac{1156}{3\left(7+\frac{7225}{3\left(9+\frac{4624}{3\,(11+\ldots)}\right)}\right)}}\right)\Bigg/\left(2520\left(3+\frac{867}{5+\frac{1156}{3\left(7+\frac{7225}{3\left(9+\frac{4624}{3\,(11+\ldots)}\right)}\right)}}\right)\right)$$

$$\frac{1}{72}\left(-\frac{759504}{5}-12\left(-\frac{\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log(73)}{2}\right)+34\log(73)-\frac{6}{7}+68\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)+54\log(7)-1631(i\pi+\log(7))+81\log(9)\right)=$$

$$-\frac{886093}{420}-\frac{1631 i\pi}{72}-\frac{1577\log(7)}{72}+\frac{9\log(9)}{8}+\frac{7\log(73)}{18}+\frac{17}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{289}{3}(1-2k)^2}{-\frac{4}{3}(-73+143k)}}=-\frac{886093}{420}-\frac{1631 i\pi}{72}-\frac{1577\log(7)}{72}+\frac{9\log(9)}{8}+$$

$$\frac{7\log(73)}{18}+\frac{17}{1+\frac{289}{3\left(-\frac{280}{3}+\frac{867}{-284+\frac{7225}{3\left(-\frac{1424}{3}+\frac{14161}{3\left(-\frac{1996}{3}+\ldots\right)}\right)}}\right)}}$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

In conclusion, we perform the triple integral of

$$1+\frac{q}{1-q}+\frac{q^4}{(1-q)(1-q^2)}+\frac{q^9}{(1-q)(1-q^2)(1-q^3)}$$

Input

$$\int\left(\int\left(\int\left(1+\frac{q}{1-q}+\frac{q^4}{(1-q)(1-q^2)}+\frac{q^9}{(1-q)(1-q^2)(1-q^3)}\right)dq\right)dq\right)dq$$

Exact result

$$\frac{1}{360}\left(-3q^6-6q^5-15q^4-120q^3+705q^2+\frac{45}{2}q^2\log(q+1)+10q^2\log(q^2+q+1)-20q\log(q^2+q+1)-\frac{5}{2}(233q^2-466q+353)\log(1-q)-20\log(q^2+q+1)-855q+270q\log(q-1)+45q\log(q+1)+\frac{45}{2}\log(q+1)+20\sqrt{3}(q+2)q\tan^{-1}\left(\frac{2q+1}{\sqrt{3}}\right)\right)$$

<u>The study of this function provides the following representations</u>:

Indefinite integral

$$\int\int\int\left(1+\frac{q}{1-q}+\frac{q^4}{(1-q)(1-q^2)}+\frac{q^9}{(1-q)(1-q^2)(1-q^3)}\right)dq\,dq\,dq=$$
$$\frac{1}{720}\left(30(12c_1+47)q^2+720c_2q+720c_3-6q^6-12q^5-30q^4-240q^3+20(q-2)q\log(q^2+q+1)-40\log(q^2+q+1)-1710q-5(233q-574)q\log(1-q)+45(q+2)q\log(q+1)-1765\log(1-q)+45\log(q+1)+40\sqrt{3}(q+2)q\tan^{-1}\left(\frac{2q+1}{\sqrt{3}}\right)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

Series expansion of the integral at q=0

$$\begin{cases}\frac{1}{720}\left(\left(\left(60+\frac{40\pi}{\sqrt{3}}\right)q+\left(90+\frac{20\pi}{\sqrt{3}}\right)q^2+390q^3+210q^4+147q^5+O(q^6)\right)+(540q+O(q^6))\left(i\pi-q-\frac{q^2}{2}-\frac{q^3}{3}-\frac{q^4}{4}-\frac{q^5}{5}+O(q^6)\right)^*\right) & \text{Im}(q)<0\\ \frac{1}{108}(9+(2\sqrt{3}+81i)\pi)q+\left(\frac{\pi}{36\sqrt{3}}-\frac{5}{8}\right)q^2+\frac{q^3}{6}+\frac{q^4}{24}+\frac{q^5}{60}+O(q^6) & \text{(otherwise)}\end{cases}$$

Series expansion of the integral at q=1

$$\frac{1}{180}\left(-150\log(1-q)+135\log(q-1)+10\sqrt{3}\,\pi-147-15\log(3)+45\log(2)\right)+$$
$$(q-1)\left(\frac{3}{4}\log(q-1)+\frac{2\pi}{9\sqrt{3}}+\frac{11}{30}+\frac{\log(2)}{4}\right)+$$
$$\frac{1}{864}(q-1)^2\left(-1398\log(1-q)+16\sqrt{3}\,\pi+513+24\log(3)+54\log(2)\right)-$$
$$\frac{695}{864}(q-1)^3+O\left((q-1)^4\right)$$

(generalized Puiseux series)

Series expansion of the integral at q=-(-1)$^{1/3}$

$$\left\lfloor \frac{\frac{3\pi}{2} - \arg\left(q + \sqrt[3]{-1}\right)}{2\pi} \right\rfloor \left(-\frac{\sqrt[3]{-1}\left(-2 + \sqrt[3]{-1}\right)\pi}{6\sqrt{3}} + \frac{\left(-1 + \sqrt[3]{-1}\right)\pi\left(q + \sqrt[3]{-1}\right)}{3\sqrt{3}} - \frac{\pi\left(q + \sqrt[3]{-1}\right)^2}{6\sqrt{3}} + O\left(\left(q + \sqrt[3]{-1}\right)^4\right) \right) +$$

$$\left(\frac{1}{720} \left(20\,(-1)^{2/3} \log\left(-i\sqrt{3}\left(q + \sqrt[3]{-1}\right)\right) + 40\sqrt[3]{-1}\log\left(-i\sqrt{3}\left(q + \sqrt[3]{-1}\right)\right) - 40\log\left(-i\sqrt{3}\left(q + \sqrt[3]{-1}\right)\right) - 40\,(-1)^{5/6}\sqrt{3}\log\left(q + e^{(i\pi)/3}\right) - 20\sqrt[6]{-1}\sqrt{3}\log\left(q + e^{(i\pi)/3}\right) - 1165\,(-1)^{2/3}\log\left(3 + i\sqrt{3}\right) - 2330\sqrt[3]{-1}\log\left(3 + i\sqrt{3}\right) - 1765\log\left(3 + i\sqrt{3}\right) - 540\sqrt[3]{-1}\log\left(-3 - i\sqrt{3}\right) + 20\,(-1)^{5/6}\sqrt{3}\log(3) + 10\sqrt[6]{-1}\sqrt{3}\log(3) + 1165\,(-1)^{2/3}\log(2) + 2870\sqrt[3]{-1}\log(2) + 1765\log(2) + 10\,(-1)^{2/3}\sqrt{3}\,\pi - 20\sqrt[3]{-1}\sqrt{3}\,\pi + 30\,(-1)^{5/6}\pi + 15\sqrt[6]{-1}\,\pi - 15\,i\,\pi + 1398\,(-1)^{2/3} + 1740\sqrt[3]{-1} - 246\right) +$$

$$\left(\left(20\,(-1)^{5/6}\sqrt{3}\log\left(-i\sqrt{3}\left(q + \sqrt[3]{-1}\right)\right) + 20\,i\sqrt{3}\log\left(-i\sqrt{3}\left(q + \sqrt[3]{-1}\right)\right) - 60\sqrt[3]{-1}\log\left(-i\sqrt{3}\left(q + \sqrt[3]{-1}\right)\right) - 60\log\left(-i\sqrt{3}\left(q + \sqrt[3]{-1}\right)\right) - 60\,(-1)^{5/6}\sqrt{3}\log\left(q + e^{(i\pi)/3}\right) + 60\,i\sqrt{3}\log\left(q + e^{(i\pi)/3}\right) - 60\sqrt[3]{-1}\log\left(q + e^{(i\pi)/3}\right) + 60\log\left(q + e^{(i\pi)/3}\right) - 1165\,(-1)^{5/6}\sqrt{3}\log\left(3 + i\sqrt{3}\right) - 1165\,i\sqrt{3}\log\left(3 + i\sqrt{3}\right) + 3495\sqrt[3]{-1}\log\left(3 + i\sqrt{3}\right) + 3495\log\left(3 + i\sqrt{3}\right) - 270\,i\sqrt{3}\log\left(-3 - i\sqrt{3}\right) + 810\log\left(-3 - i\sqrt{3}\right) + 30\,(-1)^{5/6}\sqrt{3}\log(3) - 30\,i\sqrt{3}\log(3) + 30\sqrt[3]{-1}\log(3) - 30\log(3) + 1165\,(-1)^{5/6}\sqrt{3}\log(2) + 1435\,i\sqrt{3}\log(2) - 3495\sqrt[3]{-1}\log(2) - 4305\log(2) - 15\sqrt[3]{-1}\sqrt{3}\,\pi + 15\sqrt{3}\,\pi + 75\,(-1)^{5/6}\pi - 75\,i\,\pi - 60\,(-1)^{5/6}\sqrt{3} + 182\sqrt[6]{-1}\sqrt{3} + 35\,i\sqrt{3} - 444\,(-1)^{2/3} - 2880\sqrt[3]{-1} - 1815\right)\left(q + \sqrt[3]{-1}\right)\right) \Big/ \left(180\left(i + \sqrt{3}\right)\left(-3\,i + \sqrt{3}\right)\right) +$$

$$\left(\left(-12\,i\sqrt{3}\log\left(-i\sqrt{3}\left(q + \sqrt[3]{-1}\right)\right) + 12\log\left(-i\sqrt{3}\left(q + \sqrt[3]{-1}\right)\right) + 12\,i\sqrt{3}\log\left(q + e^{(i\pi)/3}\right) + 36\log\left(q + e^{(i\pi)/3}\right) + 699\,i\sqrt{3}\log\left(3 + i\sqrt{3}\right) - 699\log\left(3 + i\sqrt{3}\right) - 6\,i\sqrt{3}\log(3) - 18\log(3) - 699\,i\sqrt{3}\log(2) + 699\log(2) - 3\sqrt{3}\,\pi - 27\,i\,\pi + 202\,(-1)^{5/6}\sqrt{3} + 26\sqrt[6]{-1}\sqrt{3} + 156\,i\sqrt{3} - 242\,(-1)^{2/3} + 86\sqrt[3]{-1} + 688\right)\left(q + \sqrt[3]{-1}\right)^2\right) \Big/ \left(18\left(i + \sqrt{3}\right)^2\left(-3\,i + \sqrt{3}\right)^2\right) +$$

$$\left(4\left(-102 + 624\sqrt[3]{-1} - 237\,(-1)^{2/3} - 48\,i\sqrt{3} - 247\sqrt[6]{-1}\sqrt{3} + 124\,(-1)^{5/6}\sqrt{3}\right)\left(q + \sqrt[3]{-1}\right)^3\right) \Big/ \left(27\left(i + \sqrt{3}\right)^3\left(-3\,i + \sqrt{3}\right)^3\right) + O\left(\left(q + \sqrt[3]{-1}\right)^4\right)\Bigg)$$

$\arg(z)$ is the complex argument

From the above result

$$\frac{1}{360}\left(-3 q^6 - 6 q^5 - 15 q^4 - 120 q^3 + 705 q^2 + \frac{45}{2} q^2 \log(q+1) + 10 q^2 \log\left(q^2+q+1\right) - 20 q \log\left(q^2+q+1\right) - \frac{5}{2}\left(233 q^2 - 466 q + 353\right)\log(1-q) - 20 \log\left(q^2+q+1\right) - 855 q + 270 q \log(q-1) + 45 q \log(q+1) + \frac{45}{2}\log(q+1) + 20\sqrt{3}\,(q+2)\,q \tan^{-1}\left(\frac{2q+1}{\sqrt{3}}\right)\right)$$

for q = 8 , we perform the following calculations:

(-3 8^6 - 6 8^5 - 15 8^4 - 120 8^3 + 705 8^2 + 45/2 8^2 log(8 + 1) + 10 8^2 log(8^2 + 8 + 1) - 20 8 log(8^2 + 8 + 1)

$$-3\times 8^6 - 6\times 8^5 - 15\times 8^4 - 120\times 8^3 + 705\times 8^2 + \frac{45}{2}\times 8^2 \log(8+1) + 10\times 8^2 \log\left(8^2+8+1\right) - 20\times 8 \log\left(8^2+8+1\right)$$

$\log(x)$ is the natural logarithm

Exact result

$$-1\,060\,800 + 1440 \log(9) + 480 \log(73)$$

Decimal approximation

$$-1.0555765760768846163468174814660926255365809458279321146999\ldots \times 10^6$$

- 5/2 (233 8^2 - 466 8 + 353) log(1 - 8) - 20 log(8^2 + 8 + 1) - 855 8 + 270 8 log(8 - 1) + 45 8 log(8 + 1) + 45/2 log(8 + 1) + 20 sqrt(3) (8 + 2) 8 tan^(-1)((2 8 + 1)/sqrt(3)))

$$-\frac{5}{2}\left(233\times 8^2-466\times 8+353\right)\log(1-8)-20\log\left(8^2+8+1\right)-855\times 8+270\times 8\log(8-1)+45\times 8\log(8+1)+\frac{45}{2}\log(8+1)+20\sqrt{3}\,(8+2)\times 8\tan^{-1}\left(\frac{2\times 8+1}{\sqrt{3}}\right)$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result

$$-6840+2160\log(7)-\frac{57\,685}{2}(\log(7)+i\,\pi)+\frac{765\log(9)}{2}-20\log(73)+1600\sqrt{3}\,\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)$$

(result in radians)

From the previous partial results, we obtain in conclusion:

$$\frac{1}{360}\left(-3q^6-6q^5-15q^4-120q^3+705q^2+\frac{45}{2}q^2\log(q+1)+10q^2\log\left(q^2+q+1\right)-20q\log\left(q^2+q+1\right)-\frac{5}{2}\left(233q^2-466q+353\right)\log(1-q)-20\log\left(q^2+q+1\right)-855q+270q\log(q-1)+45q\log(q+1)+\frac{45}{2}\log(q+1)+20\sqrt{3}\,(q+2)\,q\tan^{-1}\left(\frac{2q+1}{\sqrt{3}}\right)\right)$$

1/360 (-1060800 + 1440 log(9) + 480 log(73)-6840 + 2160 log(7) - 57685/2 (log(7) + i π) + (765 log(9))/2 - 20 log(73) + 1600 sqrt(3) tan^(-1)(17/sqrt(3)))

Input

$$\frac{1}{360}\left(-1\,060\,800+1440\log(9)+480\log(73)-6840+2160\log(7)-\frac{57\,685}{2}(\log(7)+i\,\pi)+\frac{1}{2}(765\log(9))-20\log(73)+1600\sqrt{3}\,\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

i is the imaginary unit

Exact Result

$$\frac{1}{360}\left(-1067640+2160\log(7)-\frac{57685}{2}(\log(7)+i\pi)+\frac{3645\log(9)}{2}+460\log(73)+1600\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)$$

(result in radians)

Decimal approximation

$-3081.977660901734172033201423045508158903429219319516889540822 42\ldots - 251.698294753232254112107754950664060937380162226323582337416220\ldots\, i$

(result in radians)

Polar coordinates

$r \approx 3092.2$ (radius), $\theta \approx -3.0601$ (angle)

3092.2

The study of this function provides the following representations:

Polar forms

$$\frac{1}{144}\sqrt{\left(133102369\pi^2+\left(-427056-10673\log(7)+729\log(9)+184\log(73)+640\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)^2\right)}$$

$$\left(\cos\left(-\pi-\tan^{-1}\left((57685\pi)\middle/\left(2\left(-1067640-\frac{53365\log(7)}{2}+\frac{3645\log(9)}{2}+460\log(73)+1600\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)\right)\right)\right)+\right.$$

$$\left.i\sin\left(-\pi-\tan^{-1}\left((57685\pi)\middle/\left(2\left(-1067640-\frac{53365\log(7)}{2}+\frac{3645\log(9)}{2}+460\log(73)+1600\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)\right)\right)\right)\right)$$

Approximate form

$$\frac{1}{144}\sqrt{\left(133\,102\,369\,\pi^2+\left(-427\,056-10\,673\log(7)+729\log(9)+184\log(73)+640\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)^2\right)}\\ \exp\left(i\left(-\pi-\tan^{-1}\left((57\,685\,\pi)\Big/\left(2\left(-1\,067\,640-\frac{53\,365\log(7)}{2}+\frac{3645\log(9)}{2}+460\log(73)+1600\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)\right)\right)\right)\right)$$

Alternate forms

$$\frac{1}{144}\left(-427\,056-11\,537\,i\,\pi-10\,673\log(7)+729\log(9)+184\log(73)+640\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)$$

$$\frac{1}{144}\left(-427\,056-10\,673\log(7)+729\log(9)+184\log(73)+640\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)-\frac{11\,537\,i\,\pi}{144}$$

$$-\frac{1}{144}\,i\left(11\,537\,\pi+i\left(1458\log(3)-10\,673\log(7)+8\,(23\log(73)-53\,382)+640\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)\right)$$

$$-\frac{8897}{3}+\frac{81\log(3)}{8}+6\log(7)-\frac{11\,537}{144}(\log(7)+i\,\pi)+\frac{23\log(73)}{18}+\frac{40\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Expanded form

$$-\frac{8897}{3}-\frac{11\,537\,i\,\pi}{144}-\frac{10\,673\log(7)}{144}+\frac{81\log(9)}{16}+\frac{23\log(73)}{18}+\frac{40\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Alternative representations

$$\frac{1}{360}\left(-1\,060\,800+1440\log(9)+480\log(73)-6840+2160\log(7)-\frac{57\,685}{2}(\log(7)+i\,\pi)+\frac{765\log(9)}{2}-20\log(73)+1600\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)=\frac{1}{360}\left(-1\,067\,640+2160\log(7)-\frac{57\,685}{2}(i\,\pi+\log(7))+\frac{3645\log(9)}{2}+460\log(73)+1600\tan^{-1}\left(1,\frac{17}{\sqrt{3}}\right)\sqrt{3}\right)$$

$$\frac{1}{360}\left(-1\,060\,800+1440\log(9)+480\log(73)-6840+2160\log(7)-\frac{57\,685}{2}(\log(7)+i\,\pi)+\frac{765\log(9)}{2}-20\log(73)+1600\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)=\frac{1}{360}\left(-1\,067\,640+2160\log(7)-\frac{57\,685}{2}(i\,\pi+\log(7))+\frac{3645\log(9)}{2}+460\log(73)+800\,i\left(\log\left(1-\frac{17\,i}{\sqrt{3}}\right)-\log\left(1+\frac{17\,i}{\sqrt{3}}\right)\right)\sqrt{3}\right)$$

$$\frac{1}{360}\left(-1\,060\,800+1440\log(9)+480\log(73)-6840+2160\log(7)-\frac{57\,685}{2}(\log(7)+i\,\pi)+\frac{765\log(9)}{2}-20\log(73)+1600\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)=\frac{1}{360}\left(-1\,067\,640+2160\log(a)\log_a(7)-\frac{57\,685}{2}(i\,\pi+\log(a)\log_a(7))+\frac{3645}{2}\log(a)\log_a(9)+460\log(a)\log_a(73)+1600\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\sqrt{3}\right)$$

Series representations

$$\frac{1}{360}\left(-1060800+1440\log(9)+480\log(73)-6840+2160\log(7)-\frac{57685}{2}(\log(7)+i\pi)+\frac{765\log(9)}{2}-20\log(73)+1600\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)=$$
$$-\frac{8897}{3}-\frac{11537\,i\pi}{144}+\frac{40\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{10673\log(6)}{144}+\frac{81\log(8)}{16}+\frac{23\log(72)}{18}+\sum_{k=1}^{\infty}-\frac{2^{-4-3k}\times 3^{-2-k}\left(-10673(-4)^k+184\left(-\frac{1}{3}\right)^k+(-1)^k 3^{6+k}\right)}{k}$$

$$\frac{1}{360}\left(-1060800+1440\log(9)+480\log(73)-6840+2160\log(7)-\frac{57685}{2}(\log(7)+i\pi)+\frac{765\log(9)}{2}-20\log(73)+1600\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)=$$
$$-\frac{8897}{3}-\frac{11537\,i\pi}{144}+\frac{40\tan^{-1}(z_0)}{3\sqrt{3}}-\frac{10673\log(6)}{144}+\frac{81\log(8)}{16}+\frac{23\log(72)}{18}+\sum_{k=1}^{\infty}\left(\frac{81(-1)^{-1+k}2^{-4-3k}}{k}-\frac{10673(-1)^{-1+k}2^{-4-k}\times 3^{-2-k}}{k}+\frac{23(-1)^{-1+k}2^{-1-3k}\times 9^{-1-k}}{k}+\frac{20\,i\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{17}{\sqrt{3}}-z_0\right)^k}{3\sqrt{3}\,k}\right)$$

for $(i z_0 \notin \mathbb{R}$ or $((\text{not } 1 \leq i z_0 < \infty)$ and $(\text{not } -\infty < i z_0 \leq -1)))$

$$\frac{1}{360}\left(-1\,060\,800+1440\log(9)+480\log(73)-6840+2160\log(7)-\frac{57\,685}{2}(\log(7)+i\pi)+\frac{765\log(9)}{2}-20\log(73)+1600\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)=$$
$$-\frac{8897}{3}-\frac{11\,537\,i\pi}{144}+\frac{40\tan^{-1}(z_0)}{3\sqrt{3}}-\frac{10\,673\log(6)}{144}+\frac{81\log(8)}{16}+\frac{23\log(72)}{18}+\sum_{k=1}^{\infty}\left(\frac{81(-1)^{1+k}\,2^{-4-3k}}{k}+\frac{23\left(-\frac{1}{9}\right)^{1+k}2^{-1-3k}}{k}-\frac{10\,673(-1)^{1+k}\,2^{-4-k}\times 3^{-2-k}}{k}+\frac{20\,i\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{17}{\sqrt{3}}-z_0\right)^k}{3\sqrt{3}\,k}\right)$$

for ($i z_0 \notin \mathbb{R}$ or ((not $1 \leq i z_0 < \infty$) and (not $-\infty < i z_0 \leq -1$)))

Integral representations

$$\frac{1}{360}\left(-1\,060\,800+1440\log(9)+480\log(73)-6840+2160\log(7)-\frac{57\,685}{2}(\log(7)+i\pi)+\frac{765\log(9)}{2}-20\log(73)+1600\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)=$$
$$-\frac{8897}{3}-\frac{11\,537\,i\pi}{144}+\frac{680}{3}\int_0^1\frac{1}{3+289t^2}\,dt-\frac{10\,673\log(7)}{144}+\frac{81\log(9)}{16}+\frac{23\log(73)}{18}$$

$$\frac{1}{360}\left(-1\,060\,800+1440\log(9)+480\log(73)-6840+2160\log(7)-\frac{57\,685}{2}(\log(7)+i\pi)+\frac{765\log(9)}{2}-20\log(73)+1600\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right)=$$
$$-\frac{8897}{3}-\frac{11\,537\,i\pi}{144}+\int_1^{73}\frac{1}{216}\left(\frac{680}{3+\frac{289(-1+t)^2}{5184}}+\frac{276}{t}+\frac{2187}{2(8+t)}-\frac{32\,019}{22+2t}\right)dt$$

$$\frac{1}{360}\left(-1060800 + 1440\log(9) + 480\log(73) - 6840 + 2160\log(7) - \frac{57685}{2}(\log(7) + i\pi) + \frac{765\log(9)}{2} - 20\log(73) + 1600\sqrt{3}\tan^{-1}\left(\frac{17}{\sqrt{3}}\right)\right) =$$

$$\frac{1}{144}\left(-427056 - 11537\,i\pi + 32640\int_0^1 \frac{1}{3+289t^2}\,dt + 144\int_{-i\infty+\gamma}^{i\infty+\gamma} -\frac{i\,2^{-5-3s}\times 9^{-1-s}\left(184 + 9^{3+s} - 10673\times 12^s\right)\Gamma(-s)^2\,\Gamma(1+s)}{\pi\,\Gamma(1-s)}\,ds\right) \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

From the sum of the three obtained results

3092.2 + 1248.7 + 2146.9 = 6487.8

after some calculations, we obtained:

(6487.8)^32 * (((15 e^2)/π^5))

where

$$\frac{15e^2}{\pi^5} \approx 0.36218533314$$

Input interpretation

$$6487.8^{32}\times\frac{15e^2}{\pi^5}$$

Result

$3.51600\ldots\times 10^{121}$

$0.351600\ldots * 10^{122} \approx \Lambda_Q$

The observed value of ρ_Λ or Λ today is precisely the classical dual of its quantum precursor values ρ_Q , Λ_Q in the quantum very early precursor vacuum U_Q as determined by our dual equations

The study of this function provides the following representations:

Alternative representations

$$\frac{6487.8^{32}\left(15\,e^2\right)}{\pi^5} = \frac{6487.8^{32}\left(15\exp^2(z)\right)}{\pi^5} \quad \text{for } z = 1$$

$$\frac{6487.8^{32}\left(15\,e^2\right)}{\pi^5} = \frac{15\times 6487.8^{32}\,e^2}{(180\,°)^5}$$

$$\frac{6487.8^{32}\left(15\,e^2\right)}{\pi^5} = \frac{6487.8^{32}\left(15\exp^2(z)\right)}{(180\,°)^5} \quad \text{for } z = 1$$

Series representations

$$\frac{6487.8^{32}\left(15\,e^2\right)}{\pi^5} = \frac{1.42203\times 10^{120}\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^2}{\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^5}$$

$$\frac{6487.8^{32}\left(15\,e^2\right)}{\pi^5} = \frac{1.42203\times 10^{120}}{\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^5\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\right)^2}$$

$$\frac{6487.8^{32}\left(15\,e^2\right)}{\pi^5} = \frac{1.42203\times 10^{120}\left(\sum_{k=0}^{\infty}\frac{(-1+k)^2}{k!}\right)^2}{\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^5}$$

Integral representations

$$\frac{6487.8^{32}(15 e^2)}{\pi^5} = \frac{4.55051\times 10^{121} e^2}{\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^5}$$

$$\frac{6487.8^{32}(15 e^2)}{\pi^5} = \frac{1.42203\times 10^{120} e^2}{\left(\int_0^1 \sqrt{1-t^2}\,dt\right)^5}$$

$$\frac{6487.8^{32}(15 e^2)}{\pi^5} = \frac{4.55051\times 10^{121} e^2}{\left(\int_0^\infty \frac{\sin(t)}{t}\,dt\right)^5}$$

We obtain also:

sqrt((Pi(6487.8)))*12+16

Input interpretation

$\sqrt{\pi\times 6487.8}\times 12+16$

Result

1729.19…

1729.19…..

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = $8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Series representations

$$\sqrt{\pi\, 6487.8}\ 12 + 16 = 16 + 12\sqrt{-1 + 6487.8\,\pi}\ \sum_{k=0}^{\infty} (-1 + 6487.8\,\pi)^{-k} \binom{\frac{1}{2}}{k}$$

$$\sqrt{\pi\, 6487.8}\ 12 + 16 = 16 + 12\sqrt{-1 + 6487.8\,\pi}\ \sum_{k=0}^{\infty} \frac{(-1)^k (-1 + 6487.8\,\pi)^{-k} \left(-\frac{1}{2}\right)_k}{k!}$$

$$\sqrt{\pi\, 6487.8}\ 12 + 16 = 16 + 12\sqrt{z_0}\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (6487.8\,\pi - z_0)^k\, z_0^{-k}}{k!}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

(1/27(sqrt((Pi(6487.8)))*12+16))^2-5-Φ

Input interpretation

$$\left(\frac{1}{27}\left(\sqrt{\pi \times 6487.8} \times 12 + 16\right)\right)^2 - 5 - \Phi$$

Φ is the golden ratio conjugate

Result

4096.01…

4096.01….≈ 4096 = 64^2

(sqrt((Pi(6487.8)))*12+16)^1/15

Input interpretation

$$\sqrt[15]{\sqrt{\pi \times 6487.8} \times 12 + 16}$$

Result

1.64383…

1.64383….≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ... (trace of the instanton shape)

References

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