Advanced Hilbert space technology

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Abstract

Hilbert spaces are relevant because these extensions of vector spaces are capable of archiving sets of numbers in a structured way such that these data can be retrieved in a well-organized way.

Textbooks about Hilbert spaces do not contain all subjects that make Hilbert spaces interesting and relevant. This paper adds some valuable aspects of Hilbert spaces that you will not easily find in regular textbooks about Hilbert spaces.

The section on conclusions shows that advanced Hilbert space technology leads to other results than mainstream physics does.

1 Sets

This paper considers the universe as a special kind of set.

The universe is often described as all of space and time or spacetime and its contents. In short, it is a special kind of set. Humans cannot think about subjects without giving these subjects an identifier and a sufficient description. Physical reality must operate without these linguistic tools. It probably acts via a trial-and-error approach. It does that very successfully. Humans use their senses and their brain to create an impression of their living environment. This is done such that the incoming information is filtered before it is accepted such that incorrect or noisy information does not cause a psychotic reaction of the brain. Look at “How the brain works” in https://vixra.org/author/j_a_j_van_leunen

The consequence is that humans do not observe their environment. They interpret the information that is retrieved from their living environment. For humans their living environment is an association of information that is gathered or created about their environment. Thus, it is accepted that the brain creates part of the applied information. This interpretation is even more important when instruments are applied that empower our senses.

Mathematics is created by humans. This is the reason that a mathematical treatment of the considered set introduces number systems into the set. The number system will be used to give locations in space an identity. Since the coverage of plain space with number systems can occur in many ways, a special structure brings order into the coverage. This special structure appears to be a system of Hilbert spaces. In other words, this paper concerns a special type of set. A system of Hilbert spaces that all share the same underlying vector space describes this set.

Set theory does not always establish beforehand the issues of what acts as the container of the set and what the restrictions for the members of the set are. This paper considers a set that is contained in plain space and the members of the set are point-like objects that act like locations. In this way, the switch from countable content to uncountable content can be interpreted as a fundamental change in the behavior of the content of the set. According to set theory, the switch changes this kind of set into a continuum. This does not exclude the possibility that encapsulated subregions inside the continuum contain discrete sets. It also allows that the container incorporates discrete subjects that interact with the continuum. All other content of the container acts as a tool that helps navigate the members of the set and is not supposed to influence the behavior of the set.

The system of Hilbert spaces provides a regular place for all contents of the container. Each Hilbert space relates uniquely the identifiers to the corresponding locations.

2 Vector space

In this paper, a plain space is a container that has the capability to harbor sets of point-like objects that represent locations. Empty space contains nothing that can be referred to. It has no size, no boundaries, and no center.

A vector is a combination of two point-like objects that are connected by a line. This line defines the direction of the vector. One of the points is the base of the vector and the other point is its pointer. The vector has a length that is represented by a scalar. Shifting the vector along its direction line does not change the integrity of the vector. Also shifting the vector parallel to its direction does not change its integrity. Adding a vector to an empty space turns that space into a vector space. Vectors obey vector arithmetic. Via that arithmetic, vectors can reach all locations of point-like objects that are contained in the vector space.

For example, by recurrently repeating the described shift along the direction line, the set of natural numbers can be constructed such that each new vector pointer location is identified by a corresponding natural number. This enables humans to think about these vector pointer locations.

2.1 Vector arithmetic

In this section vectors that reside in a vector space will be indicated with boldface and scalars will be indicated with italics.

The addition of vectors is commutative. It can be done by shifting one of the vectors in parallel until it coincides with the alternative point of the other vector. Now the two resulting points represent the vector sum. Vector addition is commutative. Addition creates new vectors.

$$\mathbf{v} + \mathbf{w} = \mathbf{w} + \mathbf{v} \tag{1.1}$$

Vector addition is also associative.

$$(\mathbf{u}+\mathbf{v})+\mathbf{w}=\mathbf{u}+(\mathbf{v}+\mathbf{w}) \tag{1.2}$$

Multiplication with a scalar is commutative. This multiplication may change the length and thus the integrity of the vector. It may create a new vector.

$$\mathbf{w}=a\mathbf{v}=\mathbf{v}a \tag{1.3}$$

Multiplication with scalars is distributive for scalars and vectors.

$$\begin{aligned}(a+b)\mathbf{v}&=a\mathbf{v}+b\mathbf{v}\\ a(\mathbf{v}+\mathbf{w})&=a\mathbf{v}+a\mathbf{w}\end{aligned} \tag{1.4}$$

Multiplication with negative scalars reverses the direction of the vector. In particular

$$(-1)\mathbf{v}=-\mathbf{v} \tag{1.5}$$

This paper does not define multiplication of vectors for a simple vector space. Later will be clarified that this restriction avoids interpretation problems with the concept of inner product spaces.

2.2 Number systems

We intend to apply the vector space to cover the empty space with one or more associated number systems that become the base of a coordinate system that helps us to navigate in the vector space. The coordinate markers will be point-like objects that will be identified with the corresponding element of the number system. After finishing the construction of the number system, the point-like coordinate markers will be detached from the numbers, but these markers will keep their identification with the number. In this way, the life story of the coordinate marker can be followed via this identifier. For multidimensional numbers, the real part of the number may act as a progress indicator for the dynamic behavior. The detached number system acts like a parameter space for a function that acts like the detached coordinate markers. The detached coordinate markers represent the target values of the function.

The arithmetic of vectors enables the construction of number systems. Several types of number systems exist. Hilbert spaces can only cope with number systems that are associative division rings. This reduces the choice to the real numbers, the complex numbers and the quaternions. Later will become clear that the complex numbers and the quaternions are mixtures of real numbers and spatial numbers. Another name for spatial number is imaginary number. This paper avoids that name because it may generate confusion.

3 Real numbers

The locations inside the vector space form a set. The set theory describes what happens in the set. If the locations are confined to the same direction line, then the set may expose the arithmetic of the real numbers. The square of a real number is zero or it equals a positive real number. We start with a simple subset. The set of natural numbers can be constructed by subsequently shifting a vector along the direction line of the vector. The shift replaces the base point of the vector such that it coincides with the former location of the pointer of the vector.

The shifts introduce the procedures of counting and addition. An ongoing shift generates the set of natural numbers. This set is countable. We shall call a set countable when every member of the set can be labeled with a natural number. The resulting point-like objects are members of an ordered set. Reversing the shift introduces the subtraction procedure. If counting is reversed, then the point will be reached that the space is empty again. This is the reason to give the first base point a special identifier. It will be called point zero. If subtraction proceeds past point zero, then the negative integer numbers are introduced. Together with zero and the natural numbers, the negative numbers form the integer numbers.

We can add multiple shifts in one action or shifts that involve longer vectors. This does not introduce new integer numbers. It introduces a procedure that in arithmetic is called multiplication. The reverse procedure is called division. Division can introduce new numbers that can be interpreted as ratios. Numbers that can be interpreted as ratios are called rational numbers. Rational numbers constitute classes of numbers that in addition and multiplication feature the same value. Each class corresponds with a location in vector space to which a vector points. Scientists have proven that every rational number class can be labeled with a natural number. Thus, the set of rational number classes is countable. Without the addition of other numbers, all rational numbers appear to be surrounded by empty volume. On the applied direction line still, abundant empty volume is left to insert other numbers.

Rational numbers can be squared. The result is again a rational number that results when the number is multiplied by itself. The reverse procedure is called square root. The square root need not result in a rational number. However, a converging sequence of rational numbers can approach the result arbitrarily close. If the converging series does not result in an existing rational number, then the result is called an irrational number. For many situations, a converging series of rational numbers does not result in a rational number. The missing number belongs to the irrational numbers. The set of irrational numbers cannot be counted. Each completely connected set of irrational numbers is uncountable.

Adding to the rational numbers in one lump all limits of converging series that are not rational numbers makes the total set uncountable and turns the connected set into a continuum. This defines the set of real numbers. In this continuum, none of the members is surrounded by empty volume. Empty volume does not contain point-like objects that act as locations. In addition to the rational numbers, the real numbers contain all irrational numbers for which the square is a positive real number. Irrational numbers are limits of converging series of numbers where the limit is not a rational number. In the set of real numbers, all converging series of numbers result in a limit which is also a real number. Like the rational numbers, the real numbers form classes in which the members feature the same value. Featuring the same value means occupying the same location. The location is a point-like object that is identified by a real number class. With the help of the real number classes, we can navigate along the locations on the real number direction line. It is important to notice that that switching from a countable set to an uncountable set cannot be achieved in a step-by-step fashion. Also, countable infinity cannot be reached in a step-by-step fashion.

3.1 Real arithmetic

We will indicate the real numbers with suffix $_r$.

For real numbers, addition and multiplication are commutative and associative.

$$\begin{gathered} b_r + a_r = a_r + b_r \\ (a_r + b_r) + c_r = a_r + (b_r + c_r) \end{gathered} \tag{3.1.1}$$

$$\begin{gathered} b_r a_r = a_r b_r \\ (a_r b_r) c_r = a_r (b_r c_r) \end{gathered} \tag{3.1.2}$$

For real numbers the square is zero or it is positive

$$a_r a_r \geq 0 \tag{3.1.3}$$

3.2 Selection freedom

The arithmetic does not settle the selection freedom that exists in the development of real numbers. For example, inside the vector space, the location of point zero is selected without any restriction. Further, the direction of the direction line can be selected freely. The shift of vectors can be taken upward or downward. The consequence is that real number systems exist in many versions that differ in the choices that are made.

It is possible to add a coordinate marker to each real number class. The coordinate marker links the identity of the real number class to an actual point-like object. Via their values and position, the real number classes form an ordered set. We are interested in the behavior of the set of the point-shaped objects that we call coordinate markers above. The addition of a coordinate system establishes all selections and removes the selection freedom. The coordinate system freezes the version and the geometric symmetry of the number system.

4 Spatial numbers

The square of a non-zero real number is always a positive real number. Numbers exist whose square equals a negative real number. The arithmetic of these numbers differs from the arithmetic of the real numbers. These numbers are spatial numbers. They no longer fit on the direction line of the real numbers. Instead, they reside on independent direction lines that still lay inside the vector space that contains the real number direction line. Spatial numbers may cover one or three independent direction lines. The one-dimensional spatial numbers together with the real numbers constitute the two-dimensional complex numbers. The three-dimensional spatial numbers together with the real numbers constitute the four-dimensional quaternions. If spatial numbers exist beyond the first spatial direction line, then the arithmetic of the spatial numbers enforces the third independent spatial direction line. Some mathematicians tend to call spatial numbers imaginary numbers. This paper avoids that name giving because imaginary has other meanings that might confuse the significance of spatial numbers.

Apart from zero, every spatial number has a direction. We will indicate the spatial numbers with a direction cap. This convention is also followed when the spatial numbers cover only one spatial dimension.

Like the real numbers, the spatial numbers also contain rational numbers and irrational numbers on their direction lines. Like the real numbers, the completely connected set of spatial numbers is uncountable and forms a continuum. This also holds for the complex numbers and the quaternions.

This does not take away that real numbers and spatial numbers can also occur in discrete sets. Discrete sets can be contained in an encapsulated region that is situated within a spatial continuum.

4.1 Spatial arithmetic

For spatial numbers, addition and multiplication are commutative and associative.

$$\vec{b}+\vec{a}=\vec{a}+\vec{b}$$
$$\left(\vec{a}+\vec{b}\right)+\vec{c}=\vec{a}+\left(\vec{b}+\vec{c}\right) \tag{4.1.1}$$

The product d of two spatial numbers $\vec{a}$ and $\vec{b}$ results in a real scalar part d_r and a new spatial part $\vec{d}$

$$d=d_r+\vec{d}=\vec{a}\vec{b} \tag{4.1.2}$$

$d_r=-\left\langle \vec{a},\vec{b}\right\rangle$ is the inner product of $\vec{a}$ and $\vec{b}$

For the inner product and the norm $\|\vec{a}\|$ holds $\langle \vec{a},\vec{a}\rangle=\|\vec{a}\|^2$

$$\left\langle \vec{a},\vec{b}\right\rangle=\|\vec{a}\|\left\|\vec{b}\right\|\cos(\alpha) \tag{4.1.3}$$

The square equals zero or it is a negative real number.

$$\vec{a}\vec{a}=-\langle \vec{a},\vec{a}\rangle\le 0 \tag{4.1.4}$$

$\vec{d}=\vec{a}\times\vec{b}$ is the outer product of $\vec{a}$ and $\vec{b}$

The spatial vector $\vec{d}$ is independent of $\vec{a}$ and independent of $\vec{b}$. This means that $\left\langle \vec{a},\vec{d}\right\rangle=0$ and $\left\langle \vec{b},\vec{d}\right\rangle=0$

$$\left\|\vec{a}\times\vec{b}\right\|=\|\vec{a}\|\left\|\vec{b}\right\||\sin(\alpha)|$$
$$\vec{a}\times\vec{b}=-\vec{b}\times\vec{a} \tag{4.1.5}$$

4.2 Selection freedom

Like real number systems, the spatial number systems exist in many versions that differ in the selections that are made during the development of this number system.

It looks sensible but it is not necessary to take point zero at a common position for the real numbers and the spatial numbers. Point zero of the spatial numbers acts as the geometrical center of the spatial number system. In mixed number systems, the choice for the location of the geometrical center becomes more relevant.

Apart from the fact that the main direction lines must be independent, in a three-dimensional spatial number system the actual spatial directions can be selected freely. The first spatial direction line reduces the angular choice to two π radians. Inside this direction line, the direction of the shift has two choices. In the resulting spatial dimension, the choice of a direction line leaves a range of π radians. The independent direction line can be oriented right-handed or left-handed. Inside that independent direction line, the shift has again two choices. The handedness of the multiplication of spatial numbers is a special kind of symmetry.

4.3 Mixed arithmetic

The addition and multiplication of real numbers with spatial numbers are commutative.

$$\begin{aligned} a_r + \vec{b} &= \vec{b} + a_r \\ a_r \vec{b} &= \vec{b} a_r \end{aligned} \tag{4.2.1}$$

Mixed numbers are indicated without suffix and cap. In the next formula c is a mixed number.

$$c = c_r + \vec{c} \tag{4.2.2}$$

Mathematics often treats spatial numbers as vectors because their behavior corresponds to a large degree to other types of vectors. Also, large differences exist between types of vectors. Mathematics defines the inner product of vectors that represent spatial numbers as the above geometric scalar vector product (4.1.3). It is also called the dot product of two vectors.

Only three mutually independent spatial number parts can be involved in the outer product.

These formulas still do not determine the sign of the outer product. Apart from that sign, the outer product is fixed.

Quaternionic multiplication obeys the equation

$$\begin{aligned} c = c_r + \vec{c} = ab &= \left(a_r + \vec{a}\right)\left(b_r + \vec{b}\right) \\ &= a_r b_r - \left\langle \vec{a}, \vec{b} \right\rangle + a_r \vec{b} + \vec{a} b_r \pm \vec{a} \times \vec{b} \end{aligned} \tag{4.2.3}$$

The $\pm$ sign indicates the freedom of choice of the handedness of the product rule that exists when selecting a version of the quaternionic number system. In this way, the handedness of the product rule is treated as a special kind of symmetry. The version must be selected before it can be used in calculations.

Two quaternions that are each other's inverse can rotate the spatial part of another quaternion.

$$c = ab / a \tag{4.2.4}$$

The construct rotates the spatial part of b that is perpendicular to $\vec{a}$ over an angle that is twice the angular phase θ of $a = \|a\| e^{\vec{i}\theta}$ where $\vec{i} = \vec{a} / \|\vec{a}\|$.

Cartesian quaternionic functions apply a quaternionic parameter space that is sequenced by a Cartesian coordinate system. In the parameter space, the real parts of quaternions are often interpreted as instances of (proper) time, and the spatial parts are often interpreted as spatial locations. With these interpretations, the real parts of quaternionic functions represent dynamic scalar fields. The spatial parts of quaternionic functions represent dynamic vector fields.

4.3.1 Selection freedom

Since the direction lines of real numbers and the direction lines of spatial numbers are independent it is possible to select in the underlying vector space a new location for the geometrical center for each value of the real number system. The vector of the underlying vector space that points to the geometrical center will be called state vector.

5 Hilbert space

5.1 Inner product spaces

In literature, Hilbert spaces are often introduced as inner product spaces. The concept of inner product space is meant to be a way of using the inner product of vectors to apply mixed numbers to pairs of vectors such that these mixed numbers can be interpreted as eigenvalues of operators while the inner product is interpreted as a map of one of the vectors onto the other vector. This idea is sensible for inner products that deliver real numbers but does not make much sense for results that contain a spatial part. To avoid confusion, this paper avoids the application of the notion of inner product space. Instead, this document applies Dirac's bra-ket combination to turn vector spaces into Hilbert spaces. Dirac's bra-ket combination turns a simple vector space into a powerful archive of numbers and continuums.

5.2 Dirac's bra-ket combination

Paul Dirac introduced a handy notation for the relationship that exists between the ingredients of a Hilbert space. The bra-ket combination provides the opportunity to use complex numbers and quaternions as superposition coefficients. The bra-ket combination restricts the applied numbers to members of an associative division ring. This reduces the choice to real numbers, complex numbers, and quaternions. First, we focus on separable Hilbert spaces. In separable Hilbert spaces, the applied sets of numbers are countable. With that restriction, the bra-ket combination turns the underlying vector space into a separable Hilbert space.

By selecting a version of the number system, the symmetry of the number system is fixed. This section treats the case that the Hilbert space applies quaternions to specify the values of bra-ket combinations. The values of bra-ket combinations will be used in linear combinations of vectors and as eigenvalues of operators.

To make this possible, the bra-ket method distinguishes the vectors from the underlying vector space into two types of vectors with different arithmetic. The two types represent different views of the underlying simple vector space. The ket $\langle \mathbf{f} |$ is a covariant vector, and the bra $| \mathbf{g} \rangle$ is a contravariant vector. The vectors $\mathbf{f}$ and $\mathbf{g}$ reside in the underlying vector space. The arithmetic of the ket vectors differs from the arithmetic of the bra vectors. The bra-ket combination $\langle \mathbf{f} | \mathbf{g} \rangle$ has a quaternionic value. If the underlying vectors $\mathbf{f}$ and $\mathbf{g}$ are equal, then the bra-ket combination can act as a metric. Since the product of quaternions is not commutative, care must be taken with the format of the formulas.

5.2.1 Ket vectors

The addition of ket vectors is commutative and associative.

$$|\mathbf{f}\rangle + |\mathbf{g}\rangle = |\mathbf{g}\rangle + |\mathbf{f}\rangle = |\mathbf{f} + \mathbf{g}\rangle \tag{5.2.1}$$

$$\left(|\mathbf{f} + \mathbf{g}\rangle\right) + |\mathbf{h}\rangle = |\mathbf{f}\rangle + \left(|\mathbf{g} + \mathbf{h}\rangle\right) = |\mathbf{f} + \mathbf{g} + \mathbf{h}\rangle \tag{5.2.2}$$

Together with quaternions, a set of ket vectors forms a ket vector space. Ket vectors are covariant vectors.

A quaternion α can be used to construct a covariant linear combination with the ket vector $|\mathbf{f}\rangle$

$$|\alpha \mathbf{f}\rangle = |\mathbf{f}\rangle \alpha \tag{5.2.3}$$

5.2.2 Bra vectors

For bra vectors hold

$$\langle \mathbf{f}| + \langle \mathbf{g}| = \langle \mathbf{g}| + \langle \mathbf{f}| = \langle \mathbf{f} + \mathbf{g}| \tag{5.2.4}$$

$$\left(\langle \mathbf{f} + \mathbf{g}|\right) + \langle \mathbf{h}| = \langle \mathbf{f}| + \left(\langle \mathbf{g} + \mathbf{h}|\right) = \langle \mathbf{f} + \mathbf{g} + \mathbf{h}| \tag{5.2.5}$$

Bra vectors are contravariant vectors.

$$\langle \alpha \mathbf{f} | = \alpha^* \langle \mathbf{f} | \tag{5.2.6}$$

Quaternions can constitute linear combinations with bra vectors.

A set of bra vectors form the vector space that is adjunct to the vector space of ket vectors that are the origins of these maps. If the map images the adjunct space onto the original vector space, then the bra vectors may be mapped onto the corresponding ket vector.

5.2.3 Bra-ket combination

For the bra-ket combination holds

$$\langle \mathbf{f} | \mathbf{g} \rangle = \langle \mathbf{g} | \mathbf{f} \rangle^* \tag{5.2.7}$$

For quaternionic numbers α and β hold

$$\langle \alpha \mathbf{f} | \mathbf{g} \rangle = \langle \mathbf{g} | \alpha \mathbf{f} \rangle^* = \left(\langle \mathbf{g} | \mathbf{f} \rangle \alpha \right)^* = \alpha^* \langle \mathbf{f} | \mathbf{g} \rangle \tag{5.2.8}$$

$$\langle \mathbf{f} | \beta \mathbf{g} \rangle = \langle \mathbf{f} | \mathbf{g} \rangle \beta \tag{5.2.9}$$

$$\begin{aligned} \langle (\alpha + \beta) \mathbf{f} | \mathbf{g} \rangle &= \alpha^* \langle \mathbf{f} | \mathbf{g} \rangle + \beta^* \langle \mathbf{f} | \mathbf{g} \rangle \\ &= (\alpha + \beta)^* \langle \mathbf{f} | \mathbf{g} \rangle \end{aligned} \tag{5.2.10}$$

This corresponds with (5.2.3) and (5.2.6)

$$\langle \alpha \mathbf{f} | = \alpha^* \langle \mathbf{f} | \tag{5.2.11}$$

$$| \alpha \mathbf{g} \rangle = | \mathbf{g} \rangle \alpha \tag{5.2.12}$$

We made a choice. Another possibility would be $\langle \alpha \mathbf{f} | = \alpha \langle \mathbf{f} |$ and $| \alpha \mathbf{g} \rangle = | \mathbf{g} \rangle \alpha^*$

5.2.4 Operator construction

$| \mathbf{f} \rangle \langle \mathbf{g} |$ is a constructed operator.

$$| \mathbf{g} \rangle \langle \mathbf{f} | = \left(| \mathbf{f} \rangle \langle \mathbf{g} | \right)^\dagger \tag{5.2.13}$$

The superfix † indicates the adjoint version of the operator.

For the orthonormal base $\{|q_i\rangle\}$ consisting of eigenvectors of the reference operator, holds

$$\langle q_n | q_m \rangle = \delta_{nm} \tag{5.2.14}$$

The ***bra-ket method*** enables the definition of new operators that are defined by quaternionic functions.

$$\langle \mathbf{g} | F | \mathbf{h} \rangle = \sum_{i=1}^{N} \{ \langle \mathbf{g} | q_i \rangle F(q_i) \langle q_i | \mathbf{h} \rangle \} \tag{5.2.15}$$

The symbol F is used both for the operator F and the quaternionic function $F(q)$. This enables the shorthand

$$F \equiv |q_i\rangle F(q_i) \langle q_i| \tag{5.2.16}$$

for operator F. It is evident that for the adjoint operator

$$F^\dagger \equiv |q_i\rangle F^*(q_i) \langle q_i| \tag{5.2.17}$$

For ***reference operator*** $\mathfrak{R}$ holds

$$\mathfrak{R} = |q_i\rangle q_i \langle q_i| \tag{5.2.18}$$

If $\{q_i\}$ consists of all rational values of the version of the quaternionic number system that Hilbert space $\mathfrak{H}$ applies then the eigenspace of $\mathfrak{R}$ represents the natural parameter space of the separable Hilbert space $\mathfrak{H}$. It is also the parameter space of the function $F(q)$ that defines the natural operator F in the formula (5.2.16).

5.2.5 Expected value

Any bra vector $\langle \mathbf{g}|$ can be written as a linear combination of the bra base vectors $\{\langle q_i|\}$.

$$\langle \mathbf{g}| = \sum_{i=1}^{N} \{ \langle \mathbf{g} | q_i \rangle \langle q_i | \} \tag{5.2.19}$$

Any ket vector $|\mathbf{g}\rangle$ can be written as a linear combination of the ket base vectors $\{|q_i\rangle\}$.

$$|\mathbf{g}\rangle = \sum_{i=1}^{N} \{ |q_i\rangle \langle q_i | \mathbf{g} \rangle \} \tag{5.2.20}$$

The eigenvalues are archived as a combination of a real value and a spatial value. These parts take independent dimensions. If the real parts are sequenced, then the sequence of eigenvalues represents an ongoing hopping path. If this ongoing hopping path recurrently regenerates the same hop landing location swarm, then the hop landing locations can be summed over the regeneration period in the cells of a dense spatial grid. The total sum results in a spatial center location. The sums in the cells describe a location density distribution. The center location acts as the expected spatial value of the hop landing locations. A hop landing location distribution will describe the hop landing location swarm. If the swarm covers a larger number of locations, then the description by the location density distribution will be more accurate. If the results for the grid cells are sampled over a larger part of the real numbers, then the describing location density distribution approaches a continuous function.

This means that $|\langle \mathbf{g} | \vec{q}_i \rangle|^2 = \langle \mathbf{g} | \vec{q}_i \rangle \langle \vec{q}_i | \mathbf{g} \rangle$ can take the role of a hop landing location distribution. Here, we only used the spatial parts of the eigenvalues.

The expected spatial value for operator $\mathfrak{R}$ and vector $\mathbf{g}$ is

$$\langle \mathfrak{R} \rangle_{\mathbf{g}} = \langle \mathbf{g} | \mathfrak{R} | \mathbf{g} \rangle = \sum_{i=1}^{N} \{ \langle \mathbf{g} | \vec{q}_i \rangle \vec{q}_i \langle \vec{q}_i | \mathbf{g} \rangle \} \tag{5.2.21}$$

The expected value plays its role in a series of subsequent observations or events. After sequencing the timestamps of the samples, the string of samples represents an ongoing hopping path. If the vector $\mathbf{g}$ aims at a special location inside the parameter space of the Hilbert space, then the mechanism that generates the ongoing hopping path recurrently regenerates a hop landing location swarm that is described by a stable location density distribution. For large values of N the location density distribution approaches a continuous function $\langle \mathbf{g} | \vec{q} \rangle \langle \vec{q} | \mathbf{g} \rangle$, and the distribution $\langle \mathbf{g} | \vec{q} \rangle$ can be interpreted as a probability amplitude. The square of the modulus of this probability amplitude is a probability density distribution. What these continuous functions approximately describe are discrete sets. The approach fits better if the number of elements in the set is larger and there exists a requirement that the coherence of the set is large. If at instant zero the vector $\mathbf{g}$ equals the eigenvector that belongs to eigenvalue zero, and the expectation value of $\mathbf{g}$ also equals zero, then the hop landing locations $\{q_i\}$ will tend to stay awhile about the geometrical center of the Hilbert space. If the tendency lasts, then the vector $\mathbf{g}$ will act as a ***unique state vector*** of the Hilbert space.

To give the location density distribution a statistical sense, a stochastic selection process must be or have been active. That selection process is then represented by a footprint vector $|\mathbf{g}\rangle$ that varies over time. How $|\mathbf{g}\rangle$ varies over time is checked by the characteristic function of the selection process. The footprint vector is represented by a vector $\mathbf{g}$ in the underlying vector space. The Hilbert space can archive the life history of the footprint vector in the form of a cord of quaternionic eigenvalues from a dedicated footprint operator.

The state vector of the Hilbert space is a special footprint vector of the Hilbert space. It is the footprint vector that at every instant of time has the expectation value zero. At instant zero the state vector equals the eigenvector that belongs to location zero. This still does not say everything about the essence of the required underlying stochastic selection mechanism. For example, this description does not explain the value and the stability of the recurrence rate of the hop landing location swarm. It is not clear why the characteristic function of the stochastic mechanism is stable.

5.2.6 Operator types

I is used to indicate the identity operator.

For normal operator N holds $NN^\dagger = NN^\dagger$.

The normed eigenvectors of a normal operator form an orthonormal base of the Hilbert space.

For unitary operator U holds $UU^\dagger = U^\dagger U = I$

For Hermitian operator H holds $H = H^\dagger$

A normal operator N has a Hermitian part $\frac{N + N^\dagger}{2}$ and an anti-Hermitian part $\frac{N - N^\dagger}{2}$

For anti-Hermitian operator A holds $A = -A^\dagger$

A Hermitian operator has real eigenvalues. An anti-Hermitian operator has spatial eigenvalues.

The reference operator $\mathfrak{R}$ is a normal operator.

5.3 Non-separable Hilbert space

Every infinite-dimensional separable Hilbert space owns a unique non-separable companion Hilbert space that embeds its separable partner. The non-separable Hilbert space allows operators that maintain eigenspaces that in every dimension and every spatial direction contain closed sets of rational and irrational eigenvalues. These eigenspaces are uncountable and behave as dynamic sticky continuums. These continuums can vibrate, deform, and expand.

Gelfand triple and ***Rigged Hilbert space*** are other names for the general non-separable Hilbert spaces.

In the non-separable Hilbert space, for operators with continuum eigenspaces, the bra-ket method turns from a summation into an integration.

$$\langle \mathbf{g} | F | \mathbf{h} \rangle \equiv \int\iiint \{ \langle \mathbf{g} | q \rangle F(q) \langle q | \mathbf{h} \rangle \} dV d\tau \qquad (5.3.1)$$

Here we omitted the enumerating subscripts that were used in the countable base of the separable Hilbert space. Instead, the integration applies the infinitesimal $dVd\tau$ that is taken from the continuum in the private parameter space.

The shorthand for the operator F is now

$$F \equiv |q\rangle F(q) \langle q| \qquad (5.3.2)$$

For eigenvectors $|q\rangle$, the function $F(q)$ defines as

$$F(q) = \langle q | Fq \rangle = \int\iiint \{ \langle q | q' \rangle F(q') \langle q' | q \rangle \} dV' d\tau' \qquad (5.3.3)$$

The reference operator $\mathcal{R}$ that provides the continuum natural parameter space as its eigenspace follows from

$$\langle \mathbf{g} | \mathcal{R} \mathbf{h} \rangle \equiv \int\iiint \{ \langle \mathbf{g} | q \rangle q \langle q | \mathbf{h} \rangle \} dV d\tau \qquad (5.3.4)$$

The corresponding shorthand is

$$\mathcal{R} \equiv |q\rangle q \langle q| \tag{5.3.5}$$

The reference operator is a special kind of defined operator. Via the quaternionic functions that specify defined operators, the claim becomes clear that every infinite-dimensional separable Hilbert space owns a unique non-separable companion Hilbert space that can be considered to embed its separable companion.

The reverse bracket method combines Hilbert space operator technology with quaternionic function theory and indirectly with quaternionic differential and integral technology.

5.3.1 Expected spatial value

Like the situation in the separable Hilbert space, a grid overlay of the spatial part of the parameter space is applied to be able to integrate over the grid cells. The expected spatial value is averaged over a part of the real part of the parameter space.

In the non-separable Hilbert space, the expected spatial value is defined as an average over the spatial part of the parameter space.

$$\langle \mathfrak{R} \rangle_{\mathbf{g}} = \langle \mathbf{g} | \mathfrak{R} | \mathbf{g} \rangle = \iiint_0 \{ \langle \mathbf{g} | q \rangle \vec{q} \langle q | \mathbf{g} \rangle \} dV \tag{5.3.6}$$

The real part of the parameter space is usually held fixed, and the integration is done over the spatial part of the parameter space.

The location density distribution is a continuous function with values corresponding to locations in the spatial part of the parameter space.

$$|\langle \mathbf{g} | q \rangle|^2 = \langle \mathbf{g} | q \rangle \langle q | \mathbf{g} \rangle \tag{5.3.7}$$

Thus, the variable $\vec{q}$ can be any value in the spatial part of the parameter space.

5.4 Dynamics

It is possible to interpret multidimensional numbers as a combination of a scalar timestamp in the form of a real number and a one-dimensional or three-dimensional spatial location. The progression indicator runs monotonic with the natural numbers on the real number direction line. In this way, the deformation, vibration, and expansion of the corresponding coordinate system become dynamic behaviors where the timestamps play the role of the progression indicator.

With this interpretation, the continuum eigenspace of an operator can be interpreted as the combination of a scalar field and a vector field. These fields can be described by continuous quaternionic functions. The change of these fields can be described by quaternionic differential calculus.

In a non-separable Hilbert space, the location density distribution that describes a hop landing location swarm is a continuous function. It can correspond with the continuous Fourier transform that acts as the characteristic function of the stochastic process that generated the hopping path that recurrently regenerates the hop landing location swarm. The hopping path and the hop landing location swarm are discrete distributions, but the characteristic function and the location density distribution are continuous functions.

Change

In continuums, all convergent series of numbers end in a limit that is a member of that continuum. This fact enables the differentiation of the continuum. Differential calculus shows that a continuum can change. The continuum shows astonishing behavior. It has the habit to remove deformations. Without disturbing actuators, the continuum stays flat.

5.5 Differentiation

Along a direction line, change can be described by a partial differential. If in a region of the space coverage inside this direction line all converging series of coordinate markers result in a limit that is a coordinate marker, then the partial change of the space coverage along the direction of r is defined as the limit

$$\frac{\partial \psi}{\partial r} = \lim_{\delta r \to 0} \frac{\psi(r + \delta r) - \psi(r)}{\delta r} \tag{6.1.1}$$

If the region is covered by all its irrational numbers, then this limit exists. The existence of the limit is not ensured. If the limit does not exist, then the location represents a singular point. It is also possible that the surrounding region is covered by a discrete set of point-like objects.

If the spatial part of the neighborhood is isotropic and the limit also exists in the real number space, then the total differential change df of field f equals

$$df = \frac{\partial f}{\partial \tau} d\tau + \frac{\partial f}{\partial x} \vec{i} dx + \frac{\partial f}{\partial y} \vec{j} dy + \frac{\partial f}{\partial z} \vec{k} dz \tag{6.1.2}$$

In this equation, the partial differentials $\frac{\partial f}{\partial \tau}, \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial y}$ behave like quaternionic differential operators.

The quaternionic nabla ∇ assumes the ***special condition*** that partial differentials direct along the axes of the Cartesian coordinate system in a natural parameter space of a non-separable Hilbert space. Thus,

$$\nabla = \sum_{i=0}^{4} \vec{e}_i \frac{\partial}{\partial x_i} = \frac{\partial}{\partial \tau} + \vec{i}\,\frac{\partial}{\partial x} + \vec{j}\,\frac{\partial}{\partial y} + \vec{k}\,\frac{\partial}{\partial z} \tag{6.1.3}$$

This will be applied in the next section by splitting both the quaternionic nabla and the function in a scalar part and a vector part.

The first-order partial differential equations divide the first-order change of a quaternionic field into five different parts that each represent a new field. We will represent the quaternionic field change operator by a quaternionic nabla operator. This operator behaves like a quaternionic multiplier.

The first order partial differential follows from

$$\nabla = \left\{ \frac{\partial}{\partial \tau}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} = \nabla_r + \vec{\nabla} \tag{6.1.4}$$

The spatial nabla $\vec{\nabla}$ is well-known as the del operator and is treated in detail in Wikipedia. The partial derivatives in the change operator only use parameters that are taken from the natural parameter space.

$$\begin{aligned} \phi = \nabla \psi &= \left(\frac{\partial}{\partial \tau} + \vec{\nabla} \right) \left(\psi_r + \vec{\psi} \right) \\ &= \nabla_r \psi_r - \left\langle \vec{\nabla}, \vec{\psi} \right\rangle + \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} \end{aligned} \tag{6.1.5}$$

In a selected version of the quaternionic number system, only the corresponding version of the quaternionic nabla is active. In a selected Hilbert space, this version is always and everywhere the same.

The differential $\nabla\psi$ describes the change of field ψ. The five separate terms in the first-order partial differential have a separate physical meaning. All basic fields feature this decomposition. The terms may represent new fields.

$$\phi_r = \nabla_r \psi_r - \langle \vec{\nabla}, \vec{\psi} \rangle \tag{6.1.6}$$

ϕ_r is a scalar field.

$$\vec{\phi} = \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} \tag{6.1.7}$$

$\vec{\phi}$ is a vector field.

$\vec{\nabla} f$ is the gradient of f.

$\langle \vec{\nabla}, \vec{f} \rangle$ is the divergence of $\vec{f}$.

$\vec{\nabla} \times \vec{f}$ is the curl of $\vec{f}$.

Important properties of the del operator are

$$\left(\vec{\nabla}, \vec{\nabla} \right) \psi = \Delta \psi = \nabla^2 \psi \tag{6.1.8}$$

$$\left(\vec{\nabla}, \vec{\nabla} \times \vec{\psi} \right) = 0 \tag{6.1.9}$$

$$\vec{\nabla} \times \left(\vec{\nabla} \psi_r \right) = 0 \tag{6.1.10}$$

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{\psi} \right) = \vec{\nabla} \left(\vec{\nabla}, \vec{\psi} \right) - \left(\vec{\nabla}, \vec{\nabla} \right) \vec{\psi} \tag{6.1.11}$$

Sometimes parts of the change get new symbols

$$\vec{E} = -\nabla_r \vec{\psi} - \vec{\nabla} \psi_r \tag{6.1.12}$$

$$\vec{B} = \vec{\nabla} \times \vec{\psi} \tag{6.1.13}$$

The formula (6.1.5) does not leave room for gauges. In Maxwell equations, the equation (6.1.6) is treated as a gauge.

$$\left(\vec{\nabla}, \vec{B}\right) = 0 \tag{6.1.14}$$

$$\vec{\nabla} \times \vec{E} = -\nabla_r \vec{\nabla} \times \vec{\psi} - \vec{\nabla} \times \vec{\nabla} \psi_r = -\nabla_r \vec{B} \tag{6.1.15}$$

$$\left(\vec{\nabla}, \vec{E}\right) = -\nabla_r \left(\vec{\nabla}, \vec{\psi}\right) - \left(\vec{\nabla}, \vec{\nabla}\right) \psi_r \tag{6.1.16}$$

The conjugate of the quaternionic nabla operator defines another type of field change.

$$\nabla^* = \nabla_r - \vec{\nabla} \tag{6.1.17}$$

$$\begin{aligned} \zeta = \nabla^* \phi &= \left(\frac{\partial}{\partial \tau} - \vec{\nabla}\right)\left(\phi_r + \vec{\phi}\right) \\ &= \nabla_r \phi_r + \left\langle \vec{\nabla}, \vec{\phi} \right\rangle + \nabla_r \vec{\phi} - \vec{\nabla} \phi_r \mp \vec{\nabla} \times \vec{\phi} \end{aligned} \tag{6.1.18}$$

All dynamic quaternionic fields obey the same first-order partial differential equations (6.1.5) and (6.1.18).

$$\nabla^\dagger = \nabla^* = \nabla_r - \vec{\nabla} = \nabla_r + \vec{\nabla}^\dagger = \nabla_r + \vec{\nabla}^* \tag{6.1.19}$$

In the Hilbert space, the quaternionic nabla is a normal operator. The operators

$$\nabla^\dagger \nabla = \nabla \nabla^\dagger = \nabla^* \nabla = \nabla \nabla^* = \nabla_r \nabla_r + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \tag{6.1.20}$$

are normal operators who are also Hermitian operators.

The separate operators $\nabla_r \nabla_r$ and $\langle \vec{\nabla}, \vec{\nabla} \rangle$ are also Hermitian operators.

$\langle \vec{\nabla}, \vec{\nabla} \rangle$ is known as the Laplace operator.

The two operators can also be combined as $\square = \nabla_r \nabla_r - \langle \vec{\nabla}, \vec{\nabla} \rangle$. This is the d'Alembert operator.

The solutions of $\nabla_r \nabla_r + \langle \vec{\nabla}, \vec{\nabla} \rangle = 0$ and $\nabla_r \nabla_r - \langle \vec{\nabla}, \vec{\nabla} \rangle = 0$ differ. These two equations offer different solutions and for that reason, they deliver different dynamic behavior of the field. The equations control the behavior of the embedding field that physicists call their universe. This dynamic field exists everywhere in the reach of the parameter space of the function. Both equations also control the behavior of the symmetry-related fields. The homogeneous d'Alembert equation is known as the wave equation and offers waves and wave packages as its solutions. Both equations offer shock fronts as solutions but only the operators in (6.1.20) deliver shock fronts that feature a spin or polarization vector. Integration over the time domain turns both equations in the Poisson equation and removes the spin or polarization vector. Shock fronts require a corresponding actuator and occur only in odd numbers of participating dimensions. Spherical shock fronts require an isotropic actuator. Otherwise, the shock front does not appear.

5.6 Continuity equations

Continuity equations are partial quaternionic differential equations.

The dynamic changes of the field are interpreted as field excitations or as field deformations or field expansions.

The field excitations that will be discussed here are solutions of mentioned second-order partial differential equations. Without a corresponding actuator, the field will not react. It appears that spherical pulses are the only actuators that deform the field. The field reacts to these pulses by quickly removing the deformation by sending the deformation away in all directions in the form of shock fronts until these fronts vanish at infinity. This follows from the solutions presented in (6.2.9) and (6.2.11).

One of the second-order partial differential equations results from combining the two first-order partial differential equations $\phi = \nabla \psi$ and $\zeta = \nabla^* \phi$.

$$\begin{aligned}\zeta &= \nabla^* \varphi = \nabla^* \nabla \psi = \nabla \nabla^* \psi = \left(\nabla_r + \vec{\nabla}\right)\left(\nabla_r - \vec{\nabla}\right)\left(\psi_r + \vec{\psi}\right) \\ &= \left(\nabla_r \nabla_r + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle\right)\psi\end{aligned} \tag{6.2.1}$$

All other terms vanish. $\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle$ is known as the Laplace operator.

Integration over the time domain results in the Poisson equation

$$\rho = \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi \tag{6.2.2}$$

Under isotropic conditions, a very special solution of the Poisson equation is the green's function $\frac{1}{4\pi\left|\vec{q} - \overrightarrow{q'}\right|}$ of the affected field. This solution is the spatial Dirac $\delta(\vec{q})$ pulse response of the field under strict isotropic conditions.

$$\nabla \frac{1}{\left|\vec{q} - \overrightarrow{q'}\right|} = -\frac{\left(\vec{q} - \overrightarrow{q'}\right)}{\left|\vec{q} - \overrightarrow{q'}\right|^3} \tag{6.2.3}$$

$$\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \frac{1}{\left| \vec{q} - \overrightarrow{q'} \right|} \equiv \left\langle \vec{\nabla}, \vec{\nabla} \frac{1}{\left| \vec{q} - \overrightarrow{q'} \right|} \right\rangle$$

$$= -\left\langle \vec{\nabla}, \frac{\left(\vec{q} - \overrightarrow{q'} \right)}{\left| \vec{q} - \overrightarrow{q'} \right|^3} \right\rangle = 4\pi\delta\left(\vec{q} - \overrightarrow{q'} \right) \tag{6.2.4}$$

This solution corresponds with an ongoing source or sink that exists in the field. A point-like stationary spatial pulse cannot start a shock front. The stationary spatial point-like object must be a sink or a source. In physics, this means that stationary point-like masses do not exist in physical reality.

Change can take place in one spatial dimension or combined in two or three spatial dimensions.

Under the proper conditions, the dynamic pulse response of the field is a solution of a special form of the equation (6.2.1)

$$\left(\nabla_r \nabla_r + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \right) \psi = 4\pi\delta\left(\vec{q} - \overrightarrow{q'} \right) \theta\left(\tau \pm \tau' \right) \tag{6.2.5}$$

Here $\theta(\tau)$ is a temporal step function and $\delta(\vec{q})$ is a spatial Dirac pulse response. For the spherical pulse response, the pulse must be isotropic.

After the instant τ', the equation turns into a homogeneous equation.

A remarkably simple solution is the shock front in one dimension along the line $\vec{q} - \overrightarrow{q'}$.

$$\psi = f\left(\left| \vec{q} - \overrightarrow{q'} \right| \pm c\left(\tau - \tau' \right) \vec{n} \right) \tag{6.2.6}$$

Here $\vec{n}$ is a normed spatial quaternion. This spatial quaternion has an arbitrary direction that does not vary in time. Here, the normalized vector $\vec{n}$ can be interpreted as the polarization of the solution. We intentionally placed the spatial vector $\vec{n}$ close to speed c. The function f can be a primitive shock front, but it can also be a superposition of primitive shock fronts. The single primitive shock-front solution represents a ***dark energy object***. It represents a quantum of energy.

In isotropic conditions, we better switch to spherical coordinates. Then the equation gets the form

$$\begin{aligned}&\left(\frac{\partial^2}{\partial\tau^2}+\frac{\partial^2}{\partial r^2}+2\frac{\partial}{r\partial r}\right)\psi \\ &=\left(\frac{\partial^2}{\partial\tau^2}+\frac{\partial^2}{\partial r^2}\right)(\psi r)=0\end{aligned} \tag{6.2.7}$$

The second line describes the second-order change of ψr in one dimension along the radius r. That solution is described above. A solution of this equation is

$$\psi r = f\left(r \pm c\tau\vec{n}\right) \tag{6.2.8}$$

The solution of (6.2.7) is described by

$$\psi = \frac{f\left(\left|\vec{q}-\overrightarrow{q'}\right| \pm c\left(\tau-\tau'\right)\vec{n}\right)}{\left|\vec{q}-\overrightarrow{q'}\right|} \tag{6.2.9}$$

The normalized vector $\vec{n}$ can be interpreted as the spin of the solution. It might be related to the direction that is selected when the quaternion-based Hilbert space is temporarily reduced to a subspace that contains a complex-number-based Hilbert space. The spherical pulse response acts either as an expanding or as a contracting spherical shock front. Over time this pulse response integrates into the green's function. This means that the isotropic pulse injects the volume of the green's function into the field. Subsequently, the front spreads this volume over the field. The contracting shock front collects the volume of the green's function and sucks it out of the field. The $\pm$ sign in the equation (6.2.5) selects between injection and subtraction. The shock front moves away from the pulse that caused the front. Finally, it vanishes at infinity The inserted volume expands the field.

Spherical shock fronts are ***dark matter objects***.

Shock fronts only occur in one and three dimensions. A pulse response can also occur in two dimensions, but in that case, the pulse response is a complicated vibration that looks like the result of a throw of a stone in the middle of a pond.

Equations (6.2.1) and (6.2.2) show that the operators $\frac{\partial^2}{\partial \tau^2}$ and $\langle \vec{\nabla}, \vec{\nabla} \rangle$ are valid second-order partial differential operators. These operators combine in the quaternionic equivalent of the wave equation.

$$\varphi = \left(\frac{\partial^2}{\partial \tau^2} - \langle \vec{\nabla}, \vec{\nabla} \rangle \right) \psi = \Box \psi \qquad (6.2.10)$$

This equation also offers one-dimensional and three-dimensional shock fronts as its solutions.

$$\psi = \frac{f\left(\left| \vec{q} - \vec{q'} \right| \pm c(\tau - \tau') \right)}{\left| \vec{q} - \vec{q'} \right|} \qquad (6.2.11)$$

$$\psi = f\left(\left| \vec{q} - \vec{q'} \right| \pm c(\tau - \tau') \right) \qquad (6.2.12)$$

These pulse responses do not contain the normed vector $\vec{n}$. Apart from pulse responses, the wave equation offers waves as its solutions.

If locally the field can be split into a time-dependent part $T(\tau)$and a location-dependent part $A(\vec{q})$, then the homogeneous version of the wave equation can be transformed into the Helmholtz equation.

$$\frac{\partial^2\psi}{\partial\tau^2} = \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi = -\omega^2\psi \qquad (6.2.13)$$

$$\psi(\vec{q},\tau) = A(\vec{q})T(\tau) \qquad (6.2.14)$$

$$\frac{1}{T}\frac{\partial^2 T}{\partial\tau^2} = \frac{1}{A}\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle A = -\omega^2 \qquad (6.2.15)$$

$$\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle A + \omega^2 A = 0 \qquad (6.2.16)$$

$$\frac{\partial^2 T}{\partial\tau^2} + \omega^2 T = 0 \qquad (6.2.17)$$

ω acts as quantum coupling between(6.2.16) and (6.2.17).

The time-dependent part $T(\tau)$ depends on initial conditions, or it indicates the switch of the oscillation mode.

During the switch, the quaternionic Hilbert space temporarily switches to a complex-number-based Hilbert space that is a subspace of the Hilbert space. The switch takes a corresponding interval and during that interval, the subspace emits or absorbs a sequence of equidistant one-dimensional shock fronts. Together, these shock fronts constitute a photon. The one-dimensional shock fronts are discussed above. The switch of the oscillation mode means that temporarily the oscillation is stopped and instead an object is emitted or absorbed that compensates for the difference in potential energy. The location-dependent part of the field $A(\vec{q})$ describes the possible oscillation modes of the field and depends on boundary conditions. The oscillations have a binding effect. They keep moving objects within a bounded region.

For three-dimensional isotropic spherical conditions, the solutions have the form

$$A(r,\theta,\varphi)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\left\{\left(a_{lm}\,j_l\left(kr\right)\right)+b_{lm}Y_l^m\left(\theta,\varphi\right)\right\} \qquad (6.2.18)$$

Here j_l and y_l are the spherical Bessel functions, and Y_l^m are the spherical harmonics. These solutions play a role in the spectra of atomic modules.

Planar and spherical waves are the simpler wave solutions of the equation (6.2.13)

$$\psi\left(\vec{q},\tau\right)=\exp\left\{\vec{n}\left(\left\langle\vec{k},\vec{q}-\vec{q}_0\right\rangle-\omega\tau+\varphi\right)\right\} \qquad (6.2.19)$$

$$\psi\left(\vec{q},\tau\right)=\frac{\exp\left\{\vec{n}\left(\left\langle\vec{k},\vec{q}-\vec{q}_0\right\rangle-\omega\tau+\varphi\right)\right\}}{\left|\vec{q}-\vec{q}_0\right|} \qquad (6.2.20)$$

A more general solution is a superposition of these basic types.

Two quite similar homogeneous second-order partial differential equations exist. They are the homogeneous versions of equations (6.2.5) and (6.2.10). The equation (6.2.5) has spherical shock-front solutions with a spin vector that behaves like the spin of elementary particles. Obviously, the field only reacts dynamically when it gets triggered by corresponding actuators. Pulses may cause shock fronts that after the trigger keep traveling. Oscillations of type (6.2.19) and (6.2.20) must be triggered by periodic actuators.

The inhomogeneous pulse activated equations are

$$\left(\nabla_r \nabla_r \pm \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle\right)\psi = 4\pi\delta\left(\vec{q} - \vec{q}'\right)\theta\left(\tau \pm \tau'\right) \qquad (6.2.21)$$

Without the interaction with actuators, all vibrations and deformations of the field keep busy vanishing until the affected field resembles a flat field. Only an ongoing stream of actuators can generate a more persistently deformed field. This is provided by an ongoing embedding of the actuators into the eigenspaces of operators that archive the dynamic fields.

5.7 Isotropic conditions

The two shock-front solutions show an interesting property of the Laplace operator. In isotropic conditions, the Poisson equation can be rewritten as

$$\varphi = \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi = \left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} \right)\psi = \frac{1}{r}\frac{\partial^2}{\partial r^2}\left(r\psi\right) \qquad (6.3.1)$$

The product $\phi = \left(r\psi\right)$ is a solution of a one-dimensional equation in which r plays the variable.

The same thing holds for all differential equations that contain the Laplace operator $\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle$

So, spherical solutions of the second-order differential equations ξ / r can be obtained from the solutions ξ of one-dimensional second-order differential equations by dividing ξ with the distance r to the center.

It looks as if in isotropic conditions the quaternionic differential calculus can be scaled down to complex-number-based differential calculus. This already works at local scales. If on larger scales the isotropic condition is violated, then the coordinates of the complex-number-based abstraction must be adapted to the possibly deformed Cartesian coordinates of the quaternionic platform. This makes sense in the presence of moderate deformations of the quaternionic field. After adaptation, the map of each complex-number-based coordinate line becomes a geodesic.

These tricks are possible because complex-number-based Hilbert spaces can be considered subspaces of quaternionic Hilbert spaces.

If the dimension of the quaternionic Hilbert space is reduced to the dimension of a subspace that contains a complex-number-based Hilbert space, then it might become important whether the selected direction involves a polar angle or an azimuth angle. In mathematics, the range of the polar angle is twice the range of the azimuth angle. In physics, the two ranges are exchanged.

6 Transport of change

The del operator has a direction. This suggests that change moves in that direction.

Enclosure balance equations are quaternionic integral equations that describe the balance between the inside, the border, and the outside of an enclosure.

These integral balance equations base on replacing the del operator $\vec{\nabla}$ with a normed vector $\vec{n}$. The vector $\vec{n}$ is oriented outward and perpendicular to a local part of the closed boundary of the enclosed region.

$$\vec{\nabla}\psi \Leftrightarrow \vec{n}\psi \tag{7.1.1}$$

This approach turns part of the differential continuity equation into a corresponding integral balance equation.

$$\iiint \vec{\nabla}\psi dV = \oiint \vec{n}\psi dS \tag{7.1.2}$$

$\vec{n}\ dS$ plays the role of a differential surface. $\vec{n}$ is perpendicular to that surface.

This result separates into three parts

$$\begin{aligned}\vec{\nabla}\psi &= -\langle\vec{\nabla},\vec{\psi}\rangle + \vec{\nabla}\psi_r \pm \vec{\nabla}\times\vec{\psi} \Leftrightarrow \vec{n}\psi \\ &= -\langle\vec{n},\vec{\psi}\rangle + \vec{n}\psi_r \pm \vec{n}\times\vec{\psi}\end{aligned} \tag{7.1.3}$$

The first part concerns the gradient of the scalar part of the field

$$\vec{\nabla}\psi_r \Leftrightarrow \vec{n}\psi_r \tag{7.1.4}$$

$$\iiint \vec{\nabla}\psi_r dV = \oiint \vec{n}\psi_r dS \tag{7.1.5}$$

The divergence is treated in an integral balance equation that is known as the Gauss theorem. It is also known as the divergence theorem.

$$\left\langle \vec{\nabla}, \vec{\psi} \right\rangle \Leftrightarrow \langle \vec{n}, \vec{\psi} \rangle \tag{7.1.6}$$

$$\iiint \left\langle \vec{\nabla}, \vec{\psi} \right\rangle dV = \oiint \langle \vec{n}, \vec{\psi} \rangle dS \tag{7.1.7}$$

The curl is treated in a corresponding integrated balance equation

$$\vec{\nabla} \times \vec{\psi} \Leftrightarrow \vec{n} \times \vec{\psi} \tag{7.1.8}$$

$$\iiint \vec{\nabla} \times \vec{\psi} dV = \oiint \vec{n} \times \vec{\psi} dS \tag{7.1.9}$$

Equation (7.1.7) and equation (7.1.9) can be combined in the extended theorem

$$\iiint \vec{\nabla} \vec{\psi} dV = \oiint \vec{n} \; \vec{\psi} dS \tag{7.1.10}$$

The method also applies to other partial differential equations. For example

$$\begin{aligned} \vec{\nabla} \times \left(\vec{\nabla} \times \vec{\psi} \right) = \vec{\nabla} \left\langle \vec{\nabla}, \vec{\psi} \right\rangle - \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \vec{\psi} \Leftrightarrow \vec{\nabla} \times \left(\vec{\nabla} \times \vec{\psi} \right) \\ = \vec{n} \langle \vec{n}, \vec{\psi} \rangle - \langle \vec{n}, \vec{n} \rangle \vec{\psi} \end{aligned} \tag{7.1.11}$$

$$\iiint_V \left\{ \vec{\nabla} \times \left(\vec{\nabla} \times \vec{\psi} \right) \right\} dV = \oiint_S \left\{ \vec{\nabla} \left\langle \vec{\nabla}, \vec{\psi} \right\rangle \right\} dS - \oiint_S \left\{ \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \vec{\psi} \right\} dS$$

(7.1.12)

One dimension less, a similar relation exists.

$$\iint_S \left(\left\langle \vec{\nabla} \times \vec{a}, \vec{n} \right\rangle \right) dS = \oint_C \left\langle \vec{a}, d\vec{l} \right\rangle \tag{7.1.13}$$

This is known as the Stokes theorem.

The curl can be presented as a line integral

$$\left\langle \vec{\nabla} \times \vec{\psi}, \vec{n} \right\rangle \equiv \lim_{A \to 0} \left(\frac{1}{A} \oint_C \left\langle \vec{\psi}, d\vec{r} \right\rangle \right) \tag{7.1.14}$$

7 Functions and coordinate systems

This elucidates the sense of introducing the coordinate system. Humans can more easily imagine the dynamic life of coordinate markers than that they can visualize what happens to the target values of a multidimensional function that uses a borderless multidimensional parameter space. If the differential equations that describe the behavior of coordinate markers are everywhere the same, then these equations hold at all scales. Even if spatial expansion plays a role, then its effects can easily be separated from spatial deformation and spatial vibration.

Without triggering by an actuator, the space coverage does not deform or vibrate. This does not exclude the possibility that an encapsulated spatially coherent countable subset of coordinate markers statically deforms the space coverage. This happens for a phenomenon that is called a black hole.

8 Other features of Hilbert spaces

It is already indicated that complex-number-based Hilbert spaces appear as subspaces of quaternionic Hilbert spaces and that real-number-based Hilbert spaces appear as subspaces of complex-number-based Hilbert spaces.

8.1 Position space and change space

This will be used to introduce other orthogonal bases than the natural parameter base. First, we separate the subspace that relates to the real numbers. What is left we call the position space. Next, we introduce the change base, which is an alternative orthogonal base of the position space and that is constituted by the eigenvectors that belong to the change operator. To be able to represent this in a formula we first limit to eigenvectors that belong to a selected direction line. This reduces position space to a single direction. For example, we select the direction $\vec{i}$ along the x coordinate. The change p_x of a field ψ along that direction is $p_x\psi = \frac{\partial\psi}{\partial x}$. The suffix $_x$ indicates the relation with coordinate x.

8.2 Fourier transform

x and p_x are related via a Fourier transform. In this section, we do not indicate in the exponentials the spatial direction number i with a vector cap. Instead, we use the convention that is applied in complex number versions of the exponential function.

The Fourier transform in a separable complex-number-based Hilbert space is given by the relation between $\psi(x)$ and $\tilde{\psi}\left(p_{xn}\right)$ in the sum

$$\psi(x) = \sum_{n=-\infty}^{\infty} \left\{ \tilde{\psi}\left(p_{x,n}\right) e^{2\pi i x p_{x,n}} \left(p_{x,n+1} - p_{x,n}\right) \right\} \qquad (9.2.1)$$

In the limit where $\Delta p_x = \left(p_{x,n+1} - p_{x,n}\right) \to 0$ the sum becomes an integral

$$\psi(x) = \int_{-\infty}^{\infty} \left\{ \tilde{\psi}\left(p_x\right) e^{2\pi i x p_x} \right\} dp_x \tag{9.2.2}$$

The reverse Fourier transform runs as

$$\tilde{\psi}(p_x) = \int_{-\infty}^{\infty} \left\{ \psi\left(x\right) e^{-2\pi i x p_x} \right\} dx \tag{9.2.3}$$

In these formulas, the symbol i represents a normalized spatial number part of a complex number. i corresponds to the spatial direction that was selected for constructing the complex-number-based Hilbert space.

The function $e^{2\pi i x p_x}$ is an eigenfunction of the operator $\vec{p}_x = \vec{i}\,\frac{\partial}{\partial x}$ which is recognizable as part of the change operator (6.1.4).

$$\vec{i}\,\frac{\partial}{\partial x} e^{2\pi i x p_x} = 2\pi\,\vec{p}_x e^{2\pi i x p_x} \tag{9.2.4}$$

The eigenvalue p_x represents the eigenfunction and the eigenvector $\vec{p}_x$ in the change space. In the same sense, the function $e^{-2\pi i x p_x}$ is an eigenfunction of the position operator $-\vec{i}\,\frac{\partial}{\partial p_x}$ and corresponds with the eigenvalue x of that operator.

$$-\vec{i}\,\frac{\partial}{\partial p_x} e^{-2\pi i x p_x} = 2\pi x e^{-2\pi i x p_x} \tag{9.2.5}$$

The eigenvalue x represents the eigenfunction and the eigenvector x in the position space.

The Fourier transform of a Dirac delta function is

$$\tilde{\delta}(p_x) = \int_{-\infty}^{\infty} \left\{ \delta\left(x\right) e^{-2\pi i x p_x} \right\} dx = 1 \tag{9.2.6}$$

The inverse transform tells

$$\delta(x) = \int_{-\infty}^{\infty} \left\{1 \cdot e^{2\pi i x p_x}\right\} dp_x \tag{9.2.7}$$

$$\delta(x-a) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i(x-a)p_x} dp_x \tag{9.2.8}$$

$$e^{2\pi i p_x a} = \int_{-\infty}^{\infty} \delta(x-a) e^{2\pi i x p_x} dx \tag{9.2.9}$$

The operator $\vec{p}_x = \vec{i}\,\frac{\partial}{\partial x}$ is often called the momentum operator for the spatial direction $\vec{i}$ of the coordinate x. $\vec{p}$ differs from the classical momentum that is defined as the product of velocity $\vec{v}$ and mass m. It is important to notice that every orthonormal base vector of the position space is a superposition of ALL orthonormal base vectors of the change space. Further, the norms of the superposition coefficients are all equal. Similarly, every orthonormal base vector of the change space is a superposition of ALL orthonormal base vectors of the position space. Again, the norms of the superposition coefficients are all equal. Thus, jumping between different bases completely randomizes the landing base vector.

8.3 Uncertainty principle

The uncertainty principle states

$$\left(\int_{-\infty}^{\infty} (x-x_0)^2 \left|\psi(x)\right|^2 dx\right)\left(\int_{-\infty}^{\infty} (p_x - p_{x,0})^2 \left|\tilde{\psi}(p_x)\right|^2 dp_x\right) \geq \frac{1}{16\pi^2}$$

(9.3.1)

For a Gaussian distribution, the equality sign holds. The Fourier transform of a Gaussian distribution is again a Gaussian distribution that has a different standard deviation.

If $\psi(x)$ spreads, then $\tilde{\psi}(p_x)$ shrinks and vice versa.

8.4 Stochastic processes

In this way, the characteristic function of a stochastic process that resides in the change space can control the spread of the location density distribution of the produced location swarm that resides in position space.

The stochastic process consists of a Poisson process that regulates the distribution in the real-number-based Hilbert space that is a subspace of the quaternionic Hilbert space and a binomial process that regulates the distribution in position space. This distribution is described by a location density distribution.

The production of the stochastic process is archived in the eigenspace of a dedicated footprint operator that stores its eigenvalues in quaternionic storage bins that consists of a real number valued timestamp and a three-dimensional spatial number value that represents a hop landing location. After sequencing the timestamps, the hop landing locations represent a hopping path of a point-like object. The hopping path regularly regenerates a coherent hop landing location swarm. The location density distribution describes this swarm.

If this location density distribution is a Gaussian distribution, then its Fourier transform determines exactly the location density distribution of the swarm. The Fourier transform is again a Gaussian distribution but it has different characteristics.

The described stochastic process can deliver the actuators that generate the pulse responses that may deform the dynamic universe field. In some way, an ongoing embedding process must map the eigenspace of the footprint operator onto the embedding field. As previously argued, the footprint operator's eigenspace corresponds to a dynamic footprint vector that defines a location density function and a probability amplitude. The footprint vector resides in the underlying vector space and has a representation in Hilbert space via the footprint operator.

The stochastic processes that own a characteristic function which are described here, are in common use in the qualification of imaging quality via the Optical Transfer Function of an imaging process or imaging equipment. The Optical Transfer Function is the Fourier transform of the point spread function. For point-like objects, the PSF acts as a location density distribution.

A system of Hilbert spaces that share the same underlying vector space can perform the job of the imaging platform. In this system, the imaging process will be called the embedding process. This explanation still says nothing about the essence of the necessary underlying stochastic selection process. That remains a mystery

8.4.1 Deformation and the universe

If the hop landings of a particle cause a spherical shock front, then the deformation that is caused by the footprint is roughly defined by the convolution of the location density distribution that describes the footprint and the green's function of the embedding field. This formulation is not exact because each spherical shock front quickly fades away. The pulses occur in a sequence and not in a single instant. This effect weakens the deformation. Still, due to the huge number of hops that constitute the swarm, the spherical pulse response will blur the hop landing location swarm such that its image becomes a smooth function. This smooth function describes the local deformation potential of the considered particle. Far from the geometric center at distance r from the particle, the particle looks point-like, and the deformation potential $V(r)$ can be described by

$$V(r) = MG / r \qquad (9.4.1)$$

Here M is the mass of the particle. G is a constant.

The embedding field is a superposition of deformation potentials. A formula like (9.4.1) does not directly show that deformation leads to the attraction between massive objects. The deformation potential does not own a point of engagement. Or that point must be given by the geometrical center of local deformation.

9 Hilbert repository

A system of Hilbert spaces that all share the same underlying vector space can act as a modeling platform that not only supports dynamic fields that obey quaternionic differential equations.

The Hilbert repository applies the structured storage capacity of the Hilbert spaces that are members of the system. The requirement that all member Hilbert spaces must share the same underlying vector space restricts the types of Hilbert spaces that can be a member of the Hilbert repository. It appears that the coordinate systems that determine the symmetry type of the Hilbert spaces must have the Cartesian coordinate axes in parallel. This restriction enables the determination of differences in symmetry. Only the sequence along the axis can be freely selected up or down. This means that only a small set of symmetry types will be tolerated. One of the Hilbert spaces will act as the background platform and its symmetry will act as background symmetry. Its natural parameter space will act as background parameter space. All other members of the system will float with the geometric center of their parameter space over the background parameter space. This already generates a dynamic system. The symmetry differences generate symmetry-related sources or sinks that will be located at the geometric center of the natural parameter space of the corresponding floating Hilbert space. The sources and sinks correspond to symmetry-related charges that generate symmetry-related fields.

Not the symmetries of the floating Hilbert spaces are important. Instead, the differences between the symmetry of the floating member and the background symmetry are important for establishing the type of the member Hilbert space. The counts of the differences in symmetry restrict to the shortlist -3, -2, -1, 0, +1, +2, +3.

All floating Hilbert spaces are separable. The background Hilbert space is an infinite-dimensional separable Hilbert space. It owns a non-separable companion Hilbert space that embeds its separable partner.

The Hilbert repository supports the containers of footprints that can map into the quaternionic fields. The vectors that represent the footprint vectors originate in the underlying vector field. They act as state vectors for the Hilbert spaces that act as containers for the footprints. The state vector represents the vector from the underlying vector space that aims at the geometric center of the floating Hilbert space. This enables the maps of these state vectors and the corresponding footprint into the dynamic universe field. The state vector represents a vector from the underlying vector space that tries to locate the position of the geometric center of the floating platform in the parameter space of the background platform. State vectors are special footprint vectors. Together this entwined locator installs an ongoing embedding process that acts as an imaging process of the geometric center of the floating platform onto the background parameter space. The eigenspace of a dedicated operator maps this image into the dynamic field that represents the universe.

In this way, a huge amount of ongoing hopping paths are mapped onto the embedding field. Physicists call this dynamic field the universe. On the floating platforms, the hopping paths are closed. The movement of the floating platforms breaks the closure of the images of the hopping paths.

9.1 Standard Model

The structure and behavior of the purely mathematical Hilbert repository show a striking resemblance with the structure of the Standard Model of the elementary fermions. The Standard Model of the elementary fermions is part of the Standard Model of particle physics that experimental particle physicists treat as their workbook. This does not include the physical theories that are often considered as part of the Standard Model of particle physics. These theories are Quantum Field Theory, Quantum Electro Dynamics, and Quantum Chromo Dynamics. QFT, QED, and QCD seek their foundation in the Lagrangian that is derived from the least action principle. The author considers this principle a high-level concept that follows from the behavior of the coverage of space by an uncountable set of point-like objects.

The first-order change equations (6.1.5) and (6.1.18) already reflect this typical behavior.

The least action principle does not imply the ongoing recurrent regeneration of the elementary fermions.

The shortlist of counts of the differences in symmetry corresponds to a shortlist of electric charges -1, -2/3, -1/3, 0, +1/3, +2/3, +1 in the Standard Model. These values are due to the choice of physicists to attribute electric charge -1 to the electrons.

The mathematical model allows but does not predict that each elementary fermion owns a private footprint vector that also acts as a state vector. The state vectors synchronize the beginnings of the ongoing hopping paths. Each fermion type has a fixed mass. This means that the private stochastic mechanisms very regularly regenerate the same deformation. The mathematical model does not yet predict this regularity.

9.2 Conglomerates

Elementary fermions appear to behave as elementary modules. The conglomerates of these elementary modules populate the dynamic field that we call our universe. All massive objects, except black holes, are conglomerates of elementary fermions. All elementary fermions own mass. This means that the universe is covered by massive modular systems.

A private stochastic process determines the complete local life story of each elementary fermion. That stochastic process is controlled in the change space of its private Hilbert space. The private stochastic process produces an ongoing hopping path and corresponds to a footprint vector that consists of a dynamically changing superposition of the reference operator's eigenvectors. This is explained in the formula (5.2.21). Each floating platform of the Hilbert repository owns a single private footprint vector. The footprint vector acts as the state vector of the elementary fermion and the probability amplitude corresponds to what physicists call the wavefunction of the particle.

This invites the idea that conglomerates of elementary fermions are defined by stochastic processes whose characteristic functions are defined in the change space of the background platform. In this change space, the characteristic function of a stochastic process that defines a conglomerate is a superposition of the characteristic functions of the components of the conglomerate. The dynamic superposition coefficients act as displacement generators. This means that these displacement generators define the internal oscillations of the components within the conglomerates. It might not hold for higher order conglomerates, but it holds for the lower order conglomerates.

Since in change space, the position is not defined, the fact that a component belongs to a conglomerate does not restrict the distance between the components. This way of defining the membership of a conglomerate introduces entanglement. Independent of their mutual distance, components of a conglomerate must still obey the Pauli exclusion principle.

9.3 Interaction with black holes

Field excitations cannot enter or leave black holes, but the Hilbert spaces that represent elementary fermions may hover over the enclosed region of the black hole. So, part of the footprint of the elementary particle may be mapped into the region of the black hole. The mass of the black hole attracts nearby elementary fermions. Together with the effect of hovering this may enable the growth of black holes and the merge of approaching black holes. It may also explain the merge of a black hole and a dense star.

9.4 Hadrons

Hadrons can be mesons or baryons. They are conglomerates of quarks. Quarks can only bind via oscillations and via the attraction that is induced by their electric charges. Since the symmetry of quarks does not differ from the background symmetry in an isotropic way, the footprint of quarks does not deform the embedding field. So, mass does not help to bind the quarks until they reach an isotropic symmetry difference. This phenomenon is called color confinement. Hadrons feature mass. Thus, these conglomerates are sufficiently isotropic to deform the embedding field. Once configured, the mutual binding of baryons is very strong. The nuclei of atoms are constituted by baryons.

9.5 Atoms

Compound modules are composite modules for which the images of the geometric centers of the platforms of the components coincide in the background platform. The charges of the platforms of the elementary modules establish the primary binding of the corresponding platforms. Physicists and chemists call these compound modules atoms or atomic ions.

In free compound modules, the geometric symmetry-related charges do not take part in the internal oscillations. The targets of the private stochastic processes of the elementary modules oscillate. This means that the hopping path of the elementary module folds around the oscillation path and the hop landing location swarm gets smeared along the oscillation path. The oscillation path is a solution to the Helmholtz equation. Each fermion must use a different oscillation mode. A change of the oscillation mode goes together with the emission or absorption of a photon. As suggested earlier the emission or absorption of a photon involves a switch from the quaternionic Hilbert space to a subspace that is represented by a complex-number-based Hilbert space. The duration of the switch lasts a full particle regeneration cycle. During that cycle, the stochastic mechanism does not produce a swarm of hop landing locations that produce pulses that generate spherical shock fronts, but instead, it produces a one-dimensional string of equidistant pulse responses that cause one-dimensional shock fronts. The center of emission coincides with the geometrical center of the compound module. This ensures that the emitted photon does not lose its integrity. All photons will share the same emission duration, and that duration will coincide with the regeneration cycle of the hop landing location swarm. This is the reason that photons obey the Planck-Einstein relation $E = h\nu$. Absorption cannot be interpreted so easily. It can only be comprehended as a time-reversed emission act. Otherwise, the absorption would require an incredible aiming precision for the photon. The number of one-dimensional pulses in the string corresponds to the step in the energy of the Helmholtz oscillation.

The type of stochastic process that controls the binding of components appears to be responsible for the absorption and emission of photons and the change of oscillation modes. If photons arrive with too low energy, then the energy is spent on the kinetic energy of the common platform. If photons arrive with too high energy, then the energy is distributed over the available oscillation modes, and the rest is spent on the kinetic energy of the common platform, or it escapes into free space. The process must somehow archive the modes of the components. It can apply the private platform of the components for that purpose. Most probably, the current value of the dynamic superposition coefficient is stored in the eigenspace of a special superposition operator.

9.6 Molecules

Molecules are conglomerates of compound modules that each keep their private geometrical center. However, electron oscillations are shared among the compound modules. Together with the geometric symmetry-related charges, this binds the compound modules into the molecule.

10 Dynamics in the Hilbert repository

10.1 Embedding in the background platform

The differences in the symmetry between the platforms only become apparent when a floating platform is embedded into the background platform or more specific when eigenvalues of a dedicated footprint operator are mapped to corresponding eigenvectors in the background platform. A special operator in the non-separable Hilbert space of the background platform manages in its eigenspace the dynamic field that embeds discrete eigenvalues that originate from the eigenspace of the footprint operator that resides in the considered floating platform.

The entwined locator concerns a specific background vector and representations of that vector that perform different tasks. The main task of the locator is to locate the position of the geometrical center of a selected floating platform and to embed that position into the target embedding field. This action is blurred by a stochastic detection mechanism and affected by deformations of the embedding field. The second part of the task is to locate the geometrical center in the realm of the floating platform. The result is a hop landing location swarm that will be (or is already) archived as a cord of quaternions in the eigenspace of a footprint operator that resides on the floating platform. Each quaternion in this cord archives a combination of a timestamp and a three-dimensional location. The archival decouples the generation of the landing location swarm from the retrieval of that swarm. If before the retrieval, the timestamps are sequenced, then the retrieved swarm represents an ongoing hopping path. The swarm recurrently regenerates and can then be described by a stable location density distribution. This location density distribution has a Fourier transform that characterizes the detection process. An extra task is that the expected value of the archived data must result in position zero of the parameter space of the floating platform. This makes the locator the state vector of the floating platform. The image of the locator onto the background parameter space is blurred by the stochastic mechanism and by possible new deformation of the target field by the impinging hop landings. The imaging process is also affected by existing deformations of the embedding field.

The coverage of the embedding field lets the field act as a sticky medium. The sticky medium resists the embedding of objects that break the symmetry of the embedding field. It appears that only isotropic symmetry breaks can deform the embedding field. The sticky medium reacts to new deformations by moving the deformation in all directions away from the embedding location until it vanishes at infinity. Differential calculus shows that the sticky medium reacts with a spherical pulse response that behaves as a spherical shock front that diminishes its amplitude with increasing distance from the location of the pulse. The pulse responses can superpose and join into a more persistent and more smoothed local deformation. This occurs when large amounts of nearby point-like actuators cooperate during a long enough time interval.

Aside from this footprint streaming mechanism, the symmetry-related charges represent sources or sinks that generate streams that embed symmetry-related fields into the embedding field. The charges are not spread over the root geometry of the floating platform. Instead, they locate in the geometric center of the floating platform. Thus, for isolated fermions, the map of the footprint spreads around the image of the symmetry-related charge.

Without these streaming processes, not many dynamics would occur in the embedding field.

10.2 Hilbert Book Model

The real part of the parameter space of a Hilbert space is independent of the spatial part of the parameter space. This means that for every value of the real part of the parameter space the Hilbert space offers an archive of the spatial part of the parameter space. In a non-separable Hilbert space, the archived spatial continuums can change. The values of the real part of the parameter space can be interpreted as instants of time. This means that like a book the Hilbert space describes the history of the spatial continuum in a sequence of one page per instant of time.

10.3 Footprint

An ongoing embedding of a stream of symmetry-disturbing eigenvalues may cause a quasi-persistent deformation of the embedding field. The eigenspace of the footprint operator can archive a cord of quaternionic storage bins that contain the timestamps and the landing locations that will be embedded. After sequencing the timestamps, the archive shows an ongoing hopping path that is used in an ongoing embedding process. This embedding process runs during the running episode of the Hilbert repository and acts as an ***imaging process*** in which the image quality is characterized by an Optical Transfer Function. This function is the Fourier transfer of the Point Spread Function. The Point Spread Function can be interpreted as a hop landing location density distribution. Its Fourier transform is the Optical Transfer Function of the imaging process that embeds the footprint of the considered object.

10.3.1 Footprint mechanism

The mechanism that generates the content of the eigenspace of the footprint operator did its work in the creation episode of the Hilbert repository. The private natural parameter space of the Hilbert space already exists in this creation episode. The timestamps and the hopping locations of the hopping path were taken from this private parameter space. The footprint mechanism owns a characteristic function that ensures that the hopping path recurrently regenerates a hop landing location swarm that features a stable location density distribution which is the Fourier transform of the characteristic function of the footprint mechanism. The location density distribution equals the mentioned Point Spread Function, and the characteristic function equals the corresponding Optical Transfer Function.

The footprint generation mechanism replicates the attempt of the state vector to locate the geometrical center of the Hilbert space. For that reason, the state vector represents a vector from the underlying vector space. The mechanism archives its results in the eigenspace of the footprint operator. Once archived, these data can be retrieved as an ongoing sequence. The embedding process reads the data in the order of the archived timestamps.

The hopping path, the hop landing location swarm, the location density distribution, and the Point Spread Function reside in the position space of the Hilbert space. The continuous location density distribution equals the Point Spread Function and describes the discrete hop landing location swarm.

The Optical Transfer Function equals the characteristic function of the footprint mechanism, and both reside in the change space.

The position space and the change space concern the same subspace of the Hilbert space. They distinguish in the orthonormal base with the help of which superpositions are defined. The subspace of the Hilbert space that archives timestamps is complementary to the part that archives the spatial eigenvalues in the private parameter space of the Hilbert space. Only the archival in quaternionic eigenvalues connects timestamps to spatial locations. Retrieval of the eigenvalues retrieves the connection between timestamps and spatial locations. Retrieval can be done by another operator than the operator that archived the quaternionic eigenvalues.

Nothing is said yet about the original distribution of the timestamps. In imaging processes, the distribution of discrete objects in the imaging beam can often be characterized as the result of a combination of a Poisson process and a binomial process, where the binomial process is implemented by a spatial point spread function. In that case, the Poisson process handles the distribution of the timestamps.

10.3.2 Footprint characteristics

After sequencing the timestamps, the footprint generating mechanism recurrently produces a constant stream of potential point-like actuators in the form of a swarm that features a constant location density distribution. The stream takes the form of an ongoing hopping path. The actuators that originate from the same floating separable Hilbert space have a constant symmetry. Some of these actuator symmetries can disturb the symmetry of the embedding field and therefore they can generate pulse responses that at least temporarily deform this field. A symmetry disturbance that generates a spherical pulse response must represent an isotropic difference between the two symmetries. A sufficiently constant and sufficiently dense and coherent stream of such actuators can generate a quasi-persistent deformation.

10.4 Resisting change

The Green’s function, the shock fronts, and the oscillations also demonstrate the stickiness of dynamic quaternionic fields. Discrete sets of quaternions do not show this stickiness.

The stickiness of the field tends to flatten the field and it resists permanent deformations of the field.

11 Deformation potentials

In physics, potential energy is the energy held by an object because of its position relative to other objects.

The deformation potential at a location is equal to the work (energy transferred) per unit mass that would be needed to move an object to that location from a reference location where the value of the potential equals zero.

The spherical shock fronts integrate over time into the green's function of the field. Thus, the shock front injects the content of the green's function into the affected field. All spherical shock fronts spread the contents of the front over the full field.

We consider the deformation potential to be zero at infinity. Thus, if infinity is selected as the reference location, then the deformation potential at a considered location is equal to the work (energy transferred) per unit mass that would be needed to move an object from infinity to that location. The potential at a location represents the reverse action of the combined spherical shock fronts that act at that location.

11.1 Center of deformation

The deformation potential $V(r)$ describes the effect of a local response to an isotropic point-like actuator and reflects the work that must be done by an agent to bring a unit amount of the injected stuff from infinity back to the considered location.

$$V(r) = m_p G / r \tag{12.1.1}$$

Here m_p represents the mass that corresponds to the full pulse response. G takes care for adaptation to physical units. r is the distance to the location of the pulse.

A stream of footprint actuators recurrently regenerates a coherent swarm of embedding locations in the dynamic universe field. Viewed from sufficient distance r that swarm generates a potential

$$V(r) = MG / r \quad (12.1.2)$$

Here M represents the mass that corresponds to the considered swarm of pulse responses. r is the distance to the center of the deformation. This formula is valid at sufficiently large values of r such that the whole swarm can be considered as a point-like object.

In a coherent swarm of massive objects $p_i, i = 1, 2, 3, \ldots n$, each with static mass m_i at locations r_i, the center of mass $\vec{R}$ follows from

$$\sum_{i=1}^{n} m_i \left(\vec{r}_i - \vec{R} \right) = \vec{0} \quad (12.1.3)$$

Thus

$$\vec{R} = \frac{1}{M} \sum_{i=1}^{n} m_i \vec{r}_i \quad (12.1.4)$$

Where

$$M = \sum_{i=1}^{n} m_i \quad (12.1.5)$$

In the following, we will consider an ensemble of massive objects that own a center of mass $\vec{R}$ and a fixed combined mass M as a single massive object that is located at $\vec{R}$. The separate masses m_i may differ because, at the instant of summation, the corresponding deformation might have partly faded away.

$\vec{R}$ can be a dynamic location. In that case, the ensemble must move as one unit. The problem with the treatise in this paragraph is that in physical reality, point-like objects that possess a static mass do not exist. Only pulse responses that temporarily deform the field exist. Except for black holes, these pulse responses constitute all massive objects that exist in the universe.

11.2 Pulse location density distribution

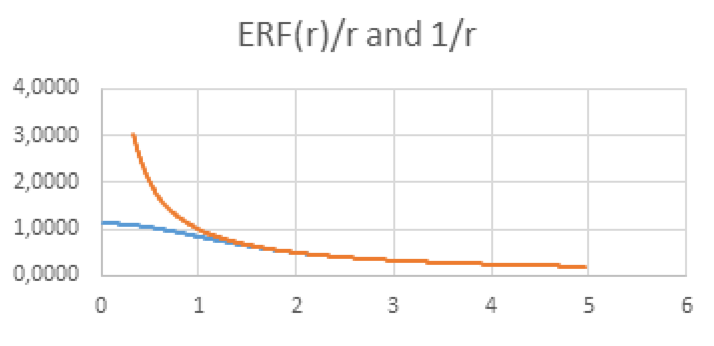
It is false to treat a pulse location density distribution as a set of point-like masses as is done in formulas (12.1.3) and (12.1.4). Instead, the deformation potential follows from the convolution of the location density distribution and the green's function. This calculation is still not correct, because the exact result depends on the fact that the deformation that is due to a pulse response quickly fades away and the result also depends on the density of the distribution. If these effects can be ignored, then the resulting deformation potential of a Gaussian density distribution would be given by

$$g(r) \approx GM \frac{ERF(r)}{r} \qquad (12.2.1)$$

Where $ERF(r)$ is the well-known error function. Here the deformation potential is a perfectly smooth function that at some distance from the center equals the approximated deformation potential that was described above in the equation (12.1.2). As indicated above, the convolution only offers an approximation because this computation does not account for the influence of the density of the swarm, and it does not compensate for the fact that the deformation by the individual pulse responses quickly fades away. Thus, the exact result depends on the duration of the recurrence cycle of the swarm.

In the example, we apply a normalized location density distribution, but the actual location density distribution might have a higher amplitude.

This might explain why some elementary module types exist in multiple generations. These generations appear to have their own mass. For example, elementary fermions exist in three generations. The two more massive generations usually get the name muon or tau generation.



This might also explain why different first-generation elementary particle types show different masses. Due to the convolution, and the coherence of the location density distribution, the blue curve does not show any sign of the singularity that is contained in the red curve, which shows the green's function.

In physical reality, no point-like static mass object exists. The most important lesson of this investigation is that far from the deformation center of the distribution the deformation of the field is characterized by the here shown simplified form of the deformation potential

$$\phi(r) \approx \frac{GM}{r} \tag{12.2.2}$$

Warning: This simplified form shares its shape with the green's function of the deformed field. This does not mean that the green's function owns a mass that equals $M_G = \frac{1}{G}$. The functions only share the form of their tail.

11.3 Rest mass

The weakness in the definition of the deformation potential is the definition of the unit of mass and the fact that shock fronts move with a fixed finite speed. Thus, the definition of the deformation potential only works properly if the geometric center location of the swarm of injected spherical pulses is at rest in the affected embedding field. The consequence is that the mass that follows from the definition of the deformation potential is the ***rest mass*** of the considered swarm. We will call the mass that is corrected for the motion of the observer relative to the observed scene the ***inertial mass***.

11.4 Observer

The inspected location is the location of a hypothetical test object that owns an amount of mass. It can represent an elementary particle or a conglomerate of such particles. This location is the target location in the embedding field. The embedding field is supposed to be deformed by the embedded objects.

Observers can access information that is retrieved from storage locations that for them have a historic timestamp. That information is transferred to them via the dynamic universe field. This dynamic field embeds both the observer and the observed event. The dynamic geometric data of point-like objects are archived in Euclidean format as a combination of a timestamp and a three-dimensional spatial location. The embedding field affects the format of the transferred information. The observers perceive in spacetime format. A hyperbolic Lorentz transform converts the Euclidean coordinates of the background parameter space into the spacetime coordinates that are perceived by the observer.

11.4.1 Lorentz transform

In dynamic fields, shock fronts move with speed c. In the quaternionic setting, this speed is unity.

$$x^2 + y^2 + z^2 = c^2\tau^2 \qquad (12.4.1)$$

In flat dynamic fields, swarms of triggers of spherical pulse responses move with lower speed v.

For the geometric centers of these swarms still holds:

$$x^2 + y^2 + z^2 - c^2\tau^2 = x'^2 + y'^2 + z'^2 - c^2\tau'^2 \qquad (12.4.2)$$

If the locations $\{x, y, z\}$ and $\{x', y', z'\}$ move with uniform relative speed v, then

$$ct' = ct\cosh(\omega) - x\sinh(\omega) \qquad (12.4.3)$$

$$x' = x\cosh(\omega) - ct\sinh(\omega) \qquad (12.4.4)$$

$$\cosh(\omega) = \frac{\exp(\omega) + \exp(-\omega)}{2} = \frac{c}{\sqrt{c^2 - v^2}} \qquad (12.4.5)$$

$$\sinh(\omega) = \frac{\exp(\omega) - \exp(-\omega)}{2} = \frac{v}{\sqrt{c^2 - v^2}} \qquad (12.4.6)$$

$$\cosh(\omega)^2 - \sinh(\omega)^2 = 1 \qquad (12.4.7)$$

This is a hyperbolic transformation that relates two coordinate systems, which is known as a Lorentz boost.

This transformation can concern two platforms P and P' on which swarms reside and that move with uniform relative speed.

However, it can also concern the storage location P that contains a timestamp τ and spatial location $\{x, y, z\}$ and platform P' that has coordinate time t' and location $\{x', y', z'\}$.

In this way, the hyperbolic transform relates two platforms that move with uniform relative speed. One of them may be a floating Hilbert space on which the observer resides. Or it may be a cluster of such platforms that cling together and move as one unit. The other may be the background platform on which the embedding process produces the image of the footprint.

The Lorentz transform converts a Euclidean coordinate system consisting of a location $\{x, y, z\}$ and proper timestamps τ into the perceived coordinate system that consists of the spacetime coordinates $\{x', y', z', ct'\}$ in which t' plays the role of coordinate time. The uniform velocity v causes time dilation $\Delta t' = \frac{\Delta\tau}{\sqrt{1-\frac{v^2}{c^2}}}$ and length contraction

$\Delta L' = \Delta L\sqrt{1-\frac{v^2}{c^2}}$

11.4.2 Minkowski metric

Spacetime is ruled by the Minkowski metric.

In flat field conditions, proper time τ is defined by

$$\tau = \pm\frac{\sqrt{c^2t^2 - x^2 - y^2 - z^2}}{c} \quad (12.4.8)$$

And in deformed fields, still

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 \quad (12.4.9)$$

Here ds is the spacetime interval and $d\tau$ is the proper time interval. dt is the coordinate time interval

11.4.3 Schwarzschild metric

Polar coordinates convert the Minkowski metric to the Schwarzschild metric. The proper time interval $d\tau$ obeys

$$c^2 d\tau^2 = \left(1-\frac{r_s}{r}\right)c^2 dt^2 - \left(1-\frac{r_s}{r}\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2 d\varphi^2\right) \quad (12.4.10)$$

Under pure isotropic conditions, the last term on the right side vanishes.

According to mainstream physics, in the environment of a black hole, the symbol r_s stands for the Schwarzschild radius.

$$r_s = \frac{2GM}{c^2} \tag{12.4.11}$$

The variable r equals the distance to the center of mass of the massive object with mass M .

The Hilbert Book model finds a different value for the boundary of a spherical black hole. That radius is a factor of two smaller.

11.4.4 Event horizon

The deformation potential energy $U(r)$

$$U(r) = \frac{mMG}{r} \tag{12.4.12}$$

at the event horizon $r = r_{eh}$ of a black hole is supposed to be equal to the mass-energy equivalent of an object that has unit mass $m = 1$ and is brought by an agent from infinity to that event horizon. Dark energy objects are energy packages in the form of one-dimensional shock fronts that are a candidate for this role. Photons are strings of equidistant samples of these energy packages. The energy equivalent of the unit mass objects is

$$E = mc^2 = \frac{mMG}{r_{eh}} \tag{12.4.13}$$

Or

$$r_{eh} = \frac{MG}{c^2} \tag{12.4.14}$$

At the event horizon, all energy of the dark energy object is consumed to compensate for the deformation potential energy at that location. No field excitation and in particular no shock front can pass the event horizon.

11.5 Inertial mass

The Lorentz transform also gives the transform of the rest mass to the mass that is relevant when the embedding field moves relative to the floating platform of the observed object with uniform speed $\vec{v}$.

In that case, the inertial mass M relates to the test mass M_0 as

$$M = \gamma M_0 = \frac{M_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{12.5.1}$$

This indicates that the formula (12.1.2) for the deformation potential at distance r must be changed to

$$V(r) = \frac{M_0 G}{r\sqrt{1 - \frac{v^2}{c^2}}} \tag{12.5.2}$$

11.6 Inertia

The relation between inertia and mass is complicated. We apply an artificial field that resists its changing. The condition that for each type of massive object, the deformation potential is a static function, and the condition that in free space, the massive object moves uniformly, establish that inertia rules the dynamics of the situation. These conditions define an artificial quaternionic field that resists change. The scalar part of the artificial field is represented by the deformation potential, and the uniform speed of the massive object represents the vector part of the field.

The first-order change of the quaternionic field can be divided into five separate partial changes. Some of these parts can compensate for each other.

Mathematically, the statement that in the first approximation nothing in the field ξ changes indicates that locally, the first-order partial differential $\nabla\xi$ will be equal to zero.

$$\zeta = \nabla\xi = \nabla_r\xi_r - \left\langle \vec{\nabla}, \vec{\xi} \right\rangle + \vec{\nabla}\xi_r + \nabla_r\vec{\xi} \pm \vec{\nabla}\times\vec{\xi} = 0 \qquad (12.6.1)$$

Thus

$$\zeta_r = \nabla_r\xi_r - \left\langle \vec{\nabla}, \vec{\xi} \right\rangle = 0 \qquad (12.6.2)$$

$$\vec{\zeta} = \vec{\nabla}\xi_r + \nabla_r\vec{\xi} \pm \vec{\nabla}\times\vec{\xi} = 0 \qquad (12.6.3)$$

These formulas can be interpreted independently. For example, according to the equation (12.6.2), the variation in time of ξ_r can compensate the divergence of $\vec{\xi}$. The terms that are still eligible for change must together be equal to zero. For our purpose, the curl $\vec{\nabla}\times\vec{\xi}$ of the vector field $\vec{\xi}$ is expected to be zero. The resulting terms of the equation (12.6.3) are

$$\nabla_r\vec{\xi} + \vec{\nabla}\xi_r = 0 \qquad (12.6.4)$$

In the following text plays $\vec{\xi}$ the role of the vector field and ξ_r plays the role of the scalar deformation potential of the considered object. For elementary modules, this special field concerns the effect of the hop landing location swarm that resides on the floating platform on its image in the embedding field. It reflects the activity of the stochastic process and the uniform movement of the geometric center of the floating platform over the embedding field in the background platform. It is characterized by a mass value and by the uniform velocity of the floating platform with respect to the background platform. The real (scalar) part conforms to the deformation that the stochastic process causes. The vector part conforms to the speed of movement of the floating platform. The main characteristic of this field is that it tries to keep its overall change zero. The author calls ξ the ***conservation field***.

At a large distance r, we approximate this potential by using the formula

$$\zeta_r(r) \approx \frac{GM}{r} \tag{12.6.5}$$

Here M is the inertial mass of the object that causes the deformation. The new artificial field $\xi = \left\{\frac{GM}{r}, \vec{v}\right\}$ considers a uniformly moving mass as a normal situation. It is a combination of scalar potential $\frac{GM}{r}$ and speed $\vec{v}$. This speed of movement is the relative speed between the floating platform and the background platform. At rest this speed is uniform.

If this object accelerates, then the new field $\left\{\frac{GM}{r}, \vec{v}\right\}$ tries to counteract the change of the vector field $\vec{v}$ by compensating this with an equivalent change of the scalar part $\frac{GM}{r}$ of the new field ξ. According to the equation (12.6.4), this equivalent change is the gradient of the real part of the field.

$$\vec{a} = \dot{\vec{v}} = -\vec{\nabla}\left(\frac{GM}{r}\right) = \frac{GM\,\vec{r}}{|\vec{r}|^3} \tag{12.6.6}$$

This generated vector field acts on masses that appear in its realm.

Thus, if two uniformly moving masses m and M exist in each other's neighborhood, then any disturbance of the situation will cause the deformation force

$$\vec{F}\left(\vec{r}_1-\vec{r}_2\right)=m_0\vec{a}=\frac{Gm_0M\left(\vec{r}_1-\vec{r}_2\right)}{\left|\vec{r}_1-\vec{r}_2\right|^3}=\gamma\frac{Gm_0M_0\left(\vec{r}_1-\vec{r}_2\right)}{\left|\vec{r}_1-\vec{r}_2\right|^3} \qquad (12.6.7)$$

Here $M=\gamma M_0$ is the inertial mass of the object that causes the deformation. m_0 is the rest mass of the observer.

The inertial mass M relates to its rest mass M_0 as

$$M=\gamma M_0=\frac{M_0}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (12.6.8)$$

This formula holds for all elementary particles except for quarks.

The problem with quarks is that these particles do not provide an isotropic symmetry difference. They must first combine into hadrons to be able to generate an isotropic symmetry difference. This phenomenon is known as ***color confinement***.

11.7 Momentum

In the formula (12.6.7) that relates mass to force the factor γ that corrects for the relative speed can be attached to m_0 or to M_0

$$\vec{F}\left(\vec{r}_1-\vec{r}_2\right)=\gamma\frac{Gm_0M_0\left(\vec{r}_1-\vec{r}_2\right)}{\left|\vec{r}_1-\vec{r}_2\right|^3} \qquad (12.7.1)$$

The force relates to the temporal change of the momentum vector $\vec{P}$ of the observer

$$\vec{F}=\dot{\vec{P}}=\frac{d\vec{P}}{dt} \qquad (12.7.2)$$

The momentum vector $\vec{P}$ is part of a quaternionic momentum P. The momentum depends on the relative speed of the moving object that causes the deformation which defines the mass. The speed is determined relative to the field that embeds the object and that gets deformed by the investigated object. For free elementary particles, the speed equals the floating speed of the platform on which the particle resides.

$$P = P_r + \vec{P} \tag{12.7.3}$$

$$\|P\|^2 = P_r^2 + \|\vec{P}\|^2 \tag{12.7.4}$$

$$\vec{P} = \gamma m_0 \vec{v} \tag{12.7.5}$$

$$\|\vec{P}\|^2 = \gamma^2 m_0^2 \|\vec{v}\|^2 \tag{12.7.6}$$

$$\|P\|^2 = \gamma^2 m_0^2 c^2 = P_r^2 + \gamma^2 m_0^2 \|\vec{v}\|^2 \tag{12.7.7}$$

$$\|P\| = \gamma m_0 c = E / c \tag{12.7.8}$$

$$E = \gamma m_0 c^2 \tag{12.7.9}$$

$$\begin{aligned} P_r^2 &= \gamma^2 m_0^2 c^2 - \gamma^2 m_0^2 \|\vec{v}\|^2 \\ &= \gamma^2 m_0^2 \left(c^2 - \|\vec{v}\|^2\right) = \gamma^2 m_0^2 c^2 \left(1 - \left\|\frac{\vec{v}}{c}\right\|^2\right) = m_0^2 c^2 \end{aligned} \tag{12.7.10}$$

$$P_r = m_0 c = \frac{E}{\gamma c} \tag{12.7.11}$$

$$\|\vec{P}\| = \gamma m_0 \|\vec{v}\| \tag{12.7.12}$$

$$P = P_r + \vec{P} = m_0 c + \gamma m_0 \vec{v} = \frac{E}{\gamma c} + \gamma m_0 \vec{v} \tag{12.7.13}$$

If $\vec{v}=\vec{0}$ then $\vec{P}=\vec{0}$ and $\|P\|=P=P_r=m_0c$

Here Einstein's famous mass-energy equivalence is involved.

$$E=\gamma m_0c^2=mc^2 \tag{12.7.14}$$

The disturbance by the ongoing expansion of the embedding field suffices to put the deformation force into action. The description also holds when the field ξ describes a conglomerate of platforms and M represents the mass of the conglomerate.

The artificial field ξ represents the habits of the underlying model that ensures the constancy of the deformation potential and the uniform floating of the considered massive objects in free space.

Inertia ensures that the third-order differential (the third-order change) of the deformed field is minimized. It does that by varying the speed of the platforms on which the massive objects reside.

Inertia bases mainly on the definition of mass that applies to the region outside the sphere where the deformation potential behaves like the green's function of the field. There, the formula $\xi_r=\frac{GM}{r}$ applies.

Further, it bases on the intention of modules to keep the deformation potential inside the mentioned sphere constant. At least that holds when this potential is averaged over the regeneration period. In that case, the overall change $\nabla\zeta$ in the conservation field ζ equals zero. Next, the definition of the conservation field supposes that the swarm which causes the deformation moves as one unit. Further, the fact is used that the solutions of the homogeneous second-order partial differential equation can superpose in new solutions of that same equation.

The popular sketch in which the deformation of our living space is presented by smooth dips is obviously false. The story that is represented in this paper shows the deformations as local extensions of the field, which represents the universe. In both sketches, the deformations elongate the information path, but none of the sketches explain why two masses attract each other. The above explanation founds on the habit of the stochastic process to recurrently regenerate the same time average of the deformation potential, even when that averaged potential moves uniformly. Without the described habit of the stochastic processes, inertia would not exist.

The applied artificial field also explains the deformation attraction by black holes.

The artificial field that implements mass inertia also plays a role in other fields. Similar tricks can be used to explain the electrical force from the fact that the electrical field is produced by sources and sinks that can be described with the green's function.

11.7.1 Forces

In the Hilbert repository, all symmetry-related charges are located at the geometric center of an elementary particle and all these particles own a footprint that for isotropic symmetry differences can deform the embedding field. In that case, the particle features mass and forces might be coupled to acceleration via

$$F = m\vec{a} \qquad (12.7.15)$$

Or to momentum via

$$F = \dot{\vec{P}} \qquad (12.7.16)$$

12 Conclusions

The Hilbert Book Model applies the system of Hilbert spaces that all share the same underlying vector space. The author calls this system the Hilbert repository. This approach differs on several essential points from the approach that mainstream physics follows. Still, an astonishing agreement exists between the Standard Model of the elementary fermions that is contained in the Stand Model of the experimental particle physicists and the Hilbert repository.

In the Hilbert Book Model (HBM), the footprints of all massive objects are recurrently regenerated with a high repetition rate that corresponds with the duration of the emission of photons.

Mainstream physics still has not found a suitable explanation for dark matter objects and dark energy objects. The HBM explains these objects as field excitations that behave as shock fronts and are described in detail by solutions of second-order quaternionic partial differential equations. The spherical shock fronts are the only field excitations that deform the field that embeds them. Photons are strings of equidistant one-dimensional shock-fronts.

Elementary fermions are complicated objects that are represented by a private quaternionic separable Hilbert space that manages the properties of the fermion. These Hilbert spaces own a private parameter space and a private symmetry. The separable Hilbert spaces float with the geometric center of their parameter space over a background parameter space that is managed by a background separable Hilbert space. This background Hilbert space owns a non-separable Hilbert space The non-separable Hilbert space embeds its separable companion. The non-separable Hilbert space manages several continuums in the eigenspace of a corresponding dedicated operator. One of the continuums is a dynamic field that physicists call our universe. The universe field embeds the images of the geometric centers of the floating separable Hilbert spaces. This map is blurred by stochastic disturbances of the locator vector that resides in the underlying vector space and points to the geometric center of the floating Hilbert space. Depending on the difference in symmetry, the embedding of the image may cause a spherical shock response that will temporarily deform the universe field. The corresponding shock front moves away in all directions until it vanishes at infinity. The content of the shock front expands the covered volume of the field. An isotropic symmetry difference with the background platform is required for the generation of the spherical shock front. Only few fermions feature an isotropic symmetry difference. Isolated quarks do not possess the required isotropic symmetry difference and will not produce a deformation of the universe. However, combined in a hadron such that the combination features an isotropic symmetry difference, the hadron can cause deformation. This phenomenon is known as color confinement.

The non-separable Hilbert space embeds its separable partner. Consequently, the parameter space of the non-separable Hilbert space is the parameter space of the separable companion Hilbert space where the irrational numbers are added to the rational numbers. The result is a continuum. The parameter spaces are not affected by deforming actuators. However, the continuum eigenspaces of other operators than the reference operator of the non-separable can be vibrated, deformed, and expanded.

Symmetry-related charges are located at the geometric centers of the floating Hilbert spaces. The charges depend on the difference in the symmetry between the floating platform and the background platform. The charges act as sources or sinks of corresponding symmetry-related fields. These fields differ fundamentally from the universe field. However, both types of fields obey the same quaternionic field equations. They differ in their start and boundary conditions.

The archival of the footprint in the floating separable Hilbert space enables the independent retrieval of that footprint at a later instance. Thus, the footprint can have been generated in an episode before the beginning of the flow of time. The retrieval can occur as a function of the flow of time and uses the archived timestamps for synchronizing the retrieval. This means that at the instant of time zero, none of the archived footprint data was retrieved. Without deforming actuators, the embedding field stays flat. Thus, at the beginning of the flow of time, the embedding field was in its maiden state. The function that described the universe field was equal to its parameter space. Immediately after that instant the locator landings started, distributed randomly over that parameter space, to mark the locations of the geometrical centers of the floating Hilbert spaces. Depending on the symmetry of the floating Hilbert space this resulted in a corresponding spherical shock front. This certainly does not look like the Big Bang that mainstream physics promotes. Instead, already at its start the ongoing embedding was a quiet imaging process.

The background non-separable Hilbert space defines in change space the conglomerates of elementary fermions as superpositions. For that reason, it applies the characteristic functions of the stochastic mechanisms that generate the footprints of the elementary fermions. In change-space, position is not defined. This is the reason for the existence of entanglement. The Pauli exclusion principle works independently of the distance between the elements of the conglomerate.

Elementary fermions act like elementary modules. Together they constitute all massive objects that occur in the universe. The notorious exception is formed by the black holes. For the rest, the contents of the universe is one large modular system that produces a huge number and enormous diversity of modular subsystems. Atoms, molecules, rocks, planets, stars, galaxies, living species are all examples of modular systems. Every human is a modular system. On planet earth, before the arrival of humans, the modularization happened in a stochastic way. Since the arrival of the humans, the modularization can happen in an intelligent way. Computers are excellent examples of this development.

Once the elementary fermions were formed, the rest of the content of the universe followed automatically. Modular systems that care for their own community and that take care for the modular systems on which they depend have the greatest chance to survive. See “A law of nature” in https://vixra.org/author/j_a_j_van_leunen .

Mainstream physics usually bases on the steady action principle. The steady action principle does not request a recurrent regeneration of the objects that occur in the universe. It does not request that conglomerates be generated in a modular way. It also does not oppress the strange reaction of continuums on disruptions by actuators.

Forces require a point of engagement. Fields do not own a point of engagement. For quaternionic functions, the first-order change already connects the gradient of a scalar field to the time variation of the corresponding vector part of the field. It suffices that the universe field shows a gradient in its scalar part and that the spatial part of the field moves uniformly. Thus, a gravitational potential raises an acceleration of the moving spatial field. Intuition cannot tell you this. But mathematics does.