Eliminate the Irrelevant to the Subject and Prove Equations and Inequalities related to Beal's Conjecture

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Abstract

The subject of this article is exactly to analyze Beal's conjecture and prove it.

First, we classify mathematical expressions which consist of A^X, B^Y and C^Z, according to the parity of A, B and C, and from this get rid of two kinds of $A^X+B^Y\neq C^Z$, for they have nothing to do with the conjecture.

Next, we exemplify $A^X+B^Y=C^Z$ under main premises plus the prerequisite.

After that, $A^X+B^Y\neq C^Z$ under main premises plus the constraint is proved by us too, according to the fundamental theorem of arithmetic.

Finally, after comparing $A^X+B^Y=C^Z$ and $A^X+B^Y\neq C^Z$ under main premises, we came to the conclusion that Beal's conjecture is true.

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1. Introduction

Beal's conjecture states that if $A^X+B^Y=C^Z$, where A, B, C, X, Y and Z are positive integers, and X, Y and Z are all greater than 2, then A, B and C must have a common prime factor.

The conjecture was discovered by Andrew Beal in 1993. Later, it was announced in December 1997 issue of the Notices of the American Mathematical Society, [1]. However, it remains a conjecture that has neither been proved nor disproved.

The conjecture shows that whoever wants to prove it, must both exemplify $A^X+B^Y=C^Z$ if A, B and C have at least one common prime factor, and prove $A^X+B^Y\neq C^Z$ if A, B and C have no any common prime factor.

First of all, we consider the scope of values of A, B, C, X, Y and Z in $A^X+B^Y=C^Z$ as main premises, then further regard that A, B and C have at least one common prime factor as a prerequisite, and regard that A, B and C have no any common prime factor as a constraint.

2. On Mathematical Expressions which Consist of A^X, B^Y and C^Z

First, we classify mathematical expressions which consist of A^X, B^Y and C^Z, according to the parity of A, B and C, and from this get rid of following two kinds of $A^X+B^Y\neq C^Z$:

1) A, B and C are all odd numbers.

2) A, B and C are two even numbers and an odd number.

After that, we continue to have following two kinds whose each contains the indefinite equation $A^X+B^Y=C^Z$ under main premises:

1) A, B and C are all positive even numbers.

2) A, B and C are two positive odd numbers and one positive even number.

3. Exemplify $A^X+B^Y=C^Z$ Under main Premises plus the Prerequisite

For two retained indefinite equations that differ from each other, in fact, each of them has many sets of solution as positive integers, as shown in the following examples.

If A, B and C are all positive even numbers, let A=B=C=2, X=Y ≥ 3 and Z=X+1, then $A^X+B^Y=C^Z$ is changed to $2^X+2^X=2^{X+1}$, so $A^X+B^Y=C^Z$ after the assignment of values has one set of solution with A, B and C as 2, 2 and 2, also A, B and C have one common prime factor 2.

In addition to this, let A=B=162, C=54, X=Y=3 and Z=4, then $A^X+B^Y=C^Z$ is changed to $162^3+162^3=54^4$, so $A^X+B^Y=C^Z$ after the assignment of values has a set of solution with A, B and C as 162, 162 and 54, also A, B and C have two common prime factors 2 and 3.

If A, B and C are two positive odd numbers and one positive even number, let A=C=3, B=6, X=Y=3 and Z=5, then $A^X+B^Y=C^Z$ is changed to $3^3+6^3= 3^5$, so $A^X+B^Y=C^Z$ after the assignment of values has one set of solution with A, B and C as 3, 6 and 3, also A, B and C have one common prime factor 3.

In addition to this, let A=B=7, C=98, X=6, Y=7 and Z=3, then $A^X+B^Y=C^Z$ is changed to $7^6+7^7=98^3$, so $A^X+B^Y=C^Z$ after the assignment of values has one set of solution with A, B and C as 7, 7 and 98, also A, B and C have one common prime factor 7.

Thus there is surely $A^X+B^Y=C^Z$ under main premises plus the prerequisite.

4. Prove $A^X+B^Y\neq C^Z$ Under main Premises plus the Constraint

Pursuant to the requirement of the conjecture, if we can further prove $A^X+B^Y\neq C^Z$ under main premises plus the constraint, undoubtedly the conjecture is tenable.

When A, B and C are all even numbers, they have at least one common prime factor 2, so A, B and C without common prime factor can only be two odd numbers and one even number.

***Proof*·** If A, B, and C have not a common prime factor, evidently A^X, B^Y, and C^Z have not a common prime factor, then any two of A^X, B^Y, and C^Z have not a common prime factor either.

This is because if two of A^X, B^Y and C^Z have a common prime factor, then we can extract this common prime factor from these two terms to become a prime factor of their sum or difference. While another term has not this common prime factor. Accordingly, this case can only lead up to $A^X+B^Y\neq C^Z$ or $C^Z - A^X \neq B^Y$ or $C^Z - B^Y \neq A^X$, according to the fundamental theorem of arithmetic [or as the unique factorization theorem of natural number], [2].

For $C^Z - A^X \neq B^Y$ and $C^Z - B^Y \neq A^X$, after you transpose a term of each of them, you get $A^X+B^Y\neq C^Z$ too.

Therefore, there is only $A^X+B^Y\neq C^Z$ under main premises plus the constraint.

5. Make a Summary and Reach the Conclusion

To sum up, we have exemplified $A^X+B^Y=C^Z$ under main premises plus the prerequisite, in the section 3, and have proved $A^X+B^Y\neq C^Z$ under main premises plus the constraint, in the section 4.

Now that we have already proved all equations and all inequalities relating to the conjecture, so let us continue to make a comparison between $A^X+B^Y=C^Z$

and $A^X+B^Y \neq C^Z$ under main premises, then, it is able to reach the conclusion that an indispensable prerequisite of $A^X+B^Y=C^Z$ under main premises is exactly that A, B and C must have a common prime factor.

The proof was thus brought to a close. As a consequence, Beal's conjecture is tenable.

P.S. Prove Fermat's Last Theorem from Proven Beal's Conjecture

Since Fermat's last theorem is a special case of Beal's conjecture [3], so we let X=Y=Z, then $A^X+B^Y=C^Z$ is changed to $A^X+B^X=C^X$.

If Beal's conjecture is proved to be true, then we divide each term of $A^X+B^X=C^X$ by the greatest common divisor of three terms of the equation itself, and get a set of solution with A, B and C as positive integers without common prime factor.

It is obvious that such a conclusion is in contradiction with proven Beal's conjecture. As thus, we have proved Fermat's last theorem by reduction to absurdity, as easy as pie.

References

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