A simple mathematical solution of the cosmological constant problem.

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Abstract / Introduction

Assuming a constant cosmological vacuum density in quantum mechanics, we provide a simple mathematical solution to the cosmological constant problem, i.e. the disagreement of the order of a factor 10^{122} between the theoretical and the measured value of the vacuum energy . We give a non-exclusive route for our solution to make physical sense.

Keywords : Cosmological constant problem, cosmological constant, zero point energy.

With :

m_p Planck mass,

l_p Planck length,

$\hbar$ reduced Planck constant,

c speed of light in vacuum,

Λ cosmological constant,

A energy density of the zero point energy in Quantum fied theory.

B energy density of the vacuum assumed for the cosmological constant in quantum mechanics.

C cosmological constant's energy density of the ΛCDM model,

Finally ***A/C*** which is the usual value of the vacuum catastrophe (="cosmological constant problem"), egal about 10^{122}.We will show that ***A/C=C/B*** so ***C²=A*B***.

– let's consider ***A***, the energy density of the zero point exprimed in J/m³ :

$$A = m_p c^2 / l_p^3 = \hbar(l_p^{-4}).c\ ^*$$

$$= \hbar(l_p^{-2})^2.c$$

$$B = \frac{1}{(8\pi)^2}.\hbar(\Lambda_{m^{-2}})^2.c$$

-* $m_p.l_p = \frac{\hbar}{c}$

– The energy density of the cosmological constant ***C***, with J/m³, is the geometric mean of A and B :

$$A/C = C/B$$

$$C = \sqrt{A.B} = \sqrt{\hbar(l_p^{-2})^2.c.\hbar(\Lambda_{m^{-2}})^2.c/(8\pi)^2}$$

$$= \sqrt{\hbar^2(l_p^{-2})^2.c^2(\Lambda_{m^{-2}})^2/(8\pi)^2}$$

$$= \frac{\hbar c.\Lambda_{m^{-2}}}{l_p^2 8\pi} \;{}^{**}$$

$$= \frac{F_p.\Lambda_{m^{-2}}}{8\pi}$$

where

$F_p = \frac{c^4}{G}$ is the Planck force,

$$\ldots = \frac{c^4.\Lambda_{m^{-2}}}{8\pi G} = \rho_\Lambda c^2$$

in other words, the classical formula of the energy density of the cosmological constant in the ΛCDM model.

with this addition to simplify the verification :

$$-^{**}\; l_p = \sqrt{\frac{\hbar G}{c^3}}$$

$$l_p^2 = \frac{\hbar G}{c^3}$$

so

$$\frac{\hbar.c}{l_p^2} = \frac{\hbar.c.c^3}{\hbar G} = \frac{c^4}{G} = F_p$$

Method used:

- We write density energy exprimed in J/m^3 of the zero point energy in the quantum field theory ***A***, with the reduced Planck constant to make appear a unit of dimension $[L^{-2}]$
- We assume a vacuum volume density of the cosmological constant in quantum mechanics, ***B***, always with the reduced Planck constant, on the same dimensional model as ***A*** . *the cosmological constant of dimension* $[L^{-2}]$ *is of the same dimension as* lp^{-2}
- It is shown that the vacuum volume density of the cosmological constant of general relativity, ***C***, is the geometric mean of ***A*** and ***B***.

One can criticize this solution because it is only mathematical and does not make physical sense unlike other solutions like the one solution proposed by Unruh and its doctoral students. Nevertheless it is much more affordable from a mathematical point of view.

However, a question arises: what is the physical meaning of the square root of an energy density (for ***A*** or ***B***)?

There is no reference on this subject for cosmology. But there is one for the "cohesive energy density" in relation with an ideal gas. cf. the English Wikipedia : Hildebrand solubility parameter. This could possibly be a new approach to the question of the cosmological constant problem.

References :

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