

# On the solution of the strong gravitational field, the solution of the Singularity problem, the origin of Dark energy and Dark matter

Hyoyoung Choi \* <sup>1</sup>

\*Independent Researhcer, Seoul, Republic of Korea.

## Abstract

In order to apply general relativity to a strong gravitational field, the gravitational self-energy of the object itself must be considered. By considering gravitational self-energy, the biggest problem with general relativity, the singularity problem, can be solved. Also, in cosmology, it is calculated that the effect of dark energy is occurring because matter and galaxies entering the particle horizon contribute to the total gravitational potential energy. As galaxies enter the particle horizon, the positive inertial mass increases, while the negative gravitational potential energy produced by these galaxies also increases. While mass energy is proportional to  $M$ , gravitational self-energy increases faster because gravitational self-energy is proportional to  $M^2$ . Accordingly, an effect of increasing dark energy occurs. This model can be verified because it points to the gravitational potential energy and the expansion of the particle horizon as the causes of dark energy. This model predicts an inflection point where dark energy becomes larger and more important than the energy of matter and radiation.

## I. Field equation in strong gravitational field and their solution

### 1. Problem with Einstein's Field Equation

Einstein's field equations are incomplete, despite great achievements. Since the equation is incomplete, it has a singularity problem as a solution. Einstein's field equations do not conform to the equivalence principle of general relativity, which states that all energy is a gravitational source. The gravitational field must also act as a gravitational source, but this is missing from Einstein's field equation.

*In writing the field equation (48) we have assumed that the quantity  $T^{\mu\nu}$  is the energy-momentum tensor of matter. In order to obtain a linear field equation we have left out the effect of the gravitational field upon itself. Because of this omission, our linear field equatuion has several (related) defects: (1) According to (48) matter acts on the gravitational field (changes the fields), but there is no mutual action of gravitational fields on matter; that is, the gravitational field can acquire energy-momentum from matter, but nevertheless the energy-momentum of matter is conserved ( $\partial_\nu T^{\mu\nu} = 0$ ). This is an inconsistency. (2) Gravitational energy does not act as source of gravitation, in contradiction to the principle of equivalence. Thus, although Eq. (48) may be a fair approximation in the equivalence. Thus, although Eq. (48) may be a fair approximation in the case of weak gravitational fields, it cannot be an exact equation. [1]*

*The obvious way to correct for our sin of omission is to include the energy-momentum tensor of the gravitational field in  $T^{\mu\nu}$ . This means that we take for the quantity  $T^{\mu\nu}$  the total energy-momentum tensor of matter plus gravitation:*

$$T^{\mu\nu} = T_{(m)}^{\mu\nu} + t^{\mu\nu} \tag{1}$$

---

<sup>1</sup>E-mail:7icarus7@gmail.com

Here  $T_{(m)}^{\mu\nu}$  and  $t^{\mu\nu}$  are, respectively, the energy-momentum tensor of matter and gravitation. We assume that the interaction energy of matter and gravitation is always included in  $T_{(m)}^{\mu\nu}$ ; this is a reasonable convention since the interaction energy-density will only differ from zero at those places where there is matter.

Our field equation now becomes

$$\partial_\lambda \partial^\lambda \phi^{\mu\nu} = -\kappa(T_{(m)}^{\mu\nu} + t^{\mu\nu}) \quad (2)$$

As such, since the field equation does not include the interaction between the gravitational field and matter, I know that many people are trying to find a solution by making a new field equation that considers the gravitational field as a source of gravity.

However, it does not seem to be able to present an efficient solution comparable to the existing Schwarzschild equation.

## 2. Hint and solution of new field equation taking into account the energy of the gravitational field

New field equation and solution including the gravitational field as a source of gravity are needed. However, I do not know how to deal with general relativity. Nevertheless, I will try with the mind of a student who has to do the homework somehow.

### 2-1. Take a look at the existing solutions!

Let's take a look at the most famous solution, the Schwarzschild solution.

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right)c^2 dt^2 + \frac{1}{\left(1 - \frac{2GM}{c^2 r}\right)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (3)$$

There is a lot, and it seems complicated. Wait a minute, let's look a little more.

Looking at the equation, this is not a big deal. The only important variables are the mass M and the distance r. So, only the mass M is important. Then, to find the solution of the strong gravitational field, we only need to find the equivalent mass of the gravitational field, right?

### 2-2. Equivalent mass of gravitational potential energy

Now, all we need to do is list all the gravitational potential energies and find their equivalent mass.

$$U_T = \sum_i U_i = \sum_i -M_{gp,i} c^2 \quad (4)$$

$-M_{gp,i}$  is the equivalent mass of gravitational potential energy.  $M_{gp,i}$  is a positive value.

Since gravitational potential energy is negative energy,  $-M_{gp}$  is used to clearly indicate it. Most of the situations we need to analyze are two-body problems. The principle of the multi-body problem is similar.

The gravitational problem we face is to establish and solve the equation of motion when mass M and mass m are separated by a distance r.

The solution is well established. In this solution we just put the energy momentum of the gravitational potential energy. It is only necessary to find the equivalent mass of the gravitational potential energy.

$$U_T = \sum_i U_i = U_{gs-M} + U_{gs-m} + U_{gp-Mm} \quad (5)$$

$U_{gp-M}$ : Total gravitational potential energy of large mass M = gravitational self-energy of large mass M

$U_{gs-m}$ : Total gravitational potential energy of small mass m = gravitational self-energy of small mass m

$U_{gp-Mm}$ : The gravitational potential energy between the large mass M and the small mass m

Although the system contains countless particles, it can be summarized in three terms.

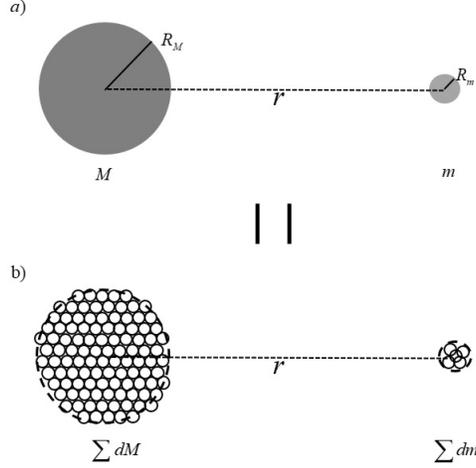


Figure 1: There is a mass  $M$  and mass  $m$ . Mass  $M$  and  $m$  are made up of several particles. If you find the gravitational potential energy of all particles, it looks like there are many terms, but if you organize it, you can organize it into three terms. The gravitational self-energy of the mass  $M$  + the gravitational self-energy of the mass  $m$  + the gravitational potential energy of the mass  $M$  and mass  $m$ .

What is necessary is just to find the equivalent mass corresponding to this gravitational potential energy. This value only differs depending on the specific situation. In individual situations, if the internal structure of the mass  $M$  and the mass  $m$  is presented, it is possible to obtain an accurate value.

For example, suppose that the mass is uniformly distributed in a spherical shape as shown in the figure above.

$$U_{gs-M} = -\frac{3}{5} \frac{GM^2}{R_M} \quad (6)$$

$$U_{gs-m} = -\frac{3}{5} \frac{Gm^2}{R_m} \quad (7)$$

$$U_{gs-Mm} = -\frac{GMm}{r} \quad (8)$$

Now, let's compare how much each size has compared to the mass energy  $Mc^2$ . Knowing the size of each, we can decide whether we should consider or ignore the corresponding physical quantity.

### 2-2-1. The gravitational self-energy of the gravitational source $M$

The magnitude of gravitational self-energy at the Schwarzschild radius

$$U_{gs-M} = -\frac{3}{5} \frac{GM^2}{R_M} = -\frac{3}{5} \frac{GM^2}{R_S} = -\frac{3}{5} \frac{GM^2}{\frac{2GM}{c^2}} = -0.3Mc^2 \quad (9)$$

$$-M_{gp-M} = -0.3M \quad (10)$$

It can be seen that the gravitational self-energy effect is quite large. At the Schwarzschild radius of an object, it can be seen that the magnitude of negative gravitational self-energy is 30% of the free state.

Since the gravitational self-energy is negative, the mass  $M$  does not fully work in a black hole, but acts as much as the gravitational potential energy is subtracted, and as a result, it becomes the same as the state with a mass of  $0.7M$ .

As experienced in elementary particle physics, you can think of mass defect due to binding energy. The bound state is a state in which the total mass is reduced by the difference in binding energy compared to the free state.

The good thing about gravitational self-energy is that the principle that assumes that all mass is gathered at the center of mass can be used for the equivalent mass formed by gravitational self-energy.

### 2-2-2. The gravitational self-energy of a small mass m or test mass m

1) The magnitude of gravitational self-energy at the Schwarzschild radius

$$U_{gs-m} = -\frac{3}{5} \frac{Gm^2}{R_m} = -\frac{3}{5} \frac{Gm^2}{R_S} = -\frac{3}{5} \frac{Gm^2}{\frac{2Gm}{c^2}} = -0.3mc^2 \quad (11)$$

$$-M_{gp-m} = -0.3m \quad (12)$$

2) Situations in which the gravitational self-energy of a small mass m is negligible

- In the normal case (unless the object has a strong gravitational field), gravitational self-energy can be neglected because it is very small compared to mass energy. In the case of Earth, the gravitational self-energy is  $10^{10}$  times less than the mass energy.

- If the mass m is taken as the test mass, the gravitational self-energy can be neglected.

- If the total mass of mass m is regarded as a mass including gravitational self-energy, the gravitational self-energy of mass m can be neglected.

### 2-2-3. Gravitational potential energy between mass M and mass m

The magnitude of gravitational self-energy at the Schwarzschild radius

$$U_{gs-Mm} = -\frac{GMm}{r} = -\frac{GMm}{R_S} = -\frac{GMm}{\frac{2GM}{c^2}} = -\left(\frac{m}{2M}\right)Mc^2 \quad (13)$$

$$-M_{gp-Mm} = -\frac{m}{2} \quad (14)$$

In the case of  $M \gg m$ , the  $U_{gs-Mm}$  term is negligible.

When considering the motion of particles near a black hole, it can be seen that this value is very small compared to the mass energy  $Mc^2$  of the gravitational source and can be ignored.

However, in cases such as collisions of black holes of similar mass, this term should be taken into account. This term has a maximum value when  $R = R_S$ , and since r is in the denominator, if it is considerably farther than the distance of  $R_S$ , it is also small compared to the gravitational self-energy of the gravitational source M.

In summary, in the situation where  $M \gg m$  or  $r \gg R_S$ , only the first term, the gravitational self-energy of the gravitational source M, needs to be considered, and M and m form a high-density object such as a black hole,  $r \approx R_S$ . In the case of, all three terms should be considered.

### 2-2-4. Equivalent mass of gravitational potential energy in the Earth, the Sun, and a black hole

1) Earth

Earth's mass :  $M_E = 5.972 \times 10^{24} \text{kg}$ , Earth's average radius :  $R_E = 6.371 \times 10^6 \text{m}$

$$U_{gs-Earth} = -\frac{3}{5} \frac{GM_E^2}{R_E} = -2.241 \times 10^{32} [\text{kgm}^2 \text{s}^{-2}] = -M_{gs-Earth} c^2 \quad (15)$$

$$-M_{gs-Earth} = -2.493 \times 10^{15} [\text{kg}] \quad (16)$$

Comparison of Earth's mass energy and gravitational self-energy

$$\frac{|U_{gs-Earth}|}{M_E c^2} = \frac{|-M_{gs-Earth} c^2|}{M_E c^2} = 4.174 \times 10^{-10} \quad (17)$$

It is negligible under normal circumstances.

2) Sun

Sun's mass :  $M_{sun} = 1.988 \times 10^{30} kg$ , Sun's average radius :  $R_{sun} = 6.955 \times 10^8 m$

$$U_{gs-sun} = -\frac{3}{5} \frac{GM_{sun}^2}{R_{sun}} = -2.275 \times 10^{43} [kgm^2s^{-2}] = -M_{gs-sun} c^2 \quad (18)$$

$$-M_{gs-sun} = -2.531 \times 10^{26} [kg] \quad (19)$$

Comparison of Sun's mass energy and gravitational self-energy

$$\frac{|U_{gs-sun}|}{M_{sun} c^2} = \frac{|-M_{gs-sun} c^2|}{M_{\odot} c^2} = 1.273 \times 10^{-4} \quad (20)$$

3) Black hole

$$U_{gs-M} = -\frac{3}{5} \frac{GM^2}{R_M} = -\frac{3}{5} \frac{GM^2}{R_S} = -\frac{3}{5} \frac{GM^2}{\frac{2GM}{c^2}} = -0.3Mc^2 = -M_{gs-M} c^2 \quad (21)$$

Comparison of Black hole's mass energy and gravitational self-energy

$$\frac{|U_{gs-M}|}{Mc^2} = \frac{|-M_{gs-M} c^2|}{Mc^2} = 0.3 \quad (22)$$

These are good results. In the case of the Earth, the magnitude of its gravitational energy was very small ( $10^{-10}$ ), so it was not necessary to consider it, whereas in the case of a neutron star or a black hole, this value appears to be at a meaningful level.

### 2-2-5. When the test particle is near a source of gravity with a strong gravitational field

Since the second and third terms can be ignored, the equivalent mass of the gravitational potential energy is simplified.

$$U_T = \sum_i U_i = U_{gs-M} + U_{gs-m} + U_{gp-Mm} \simeq U_{gs-M} \quad (23)$$

In the case of a black hole, the magnitude of the gravitational potential energy is  $U_T = \sum_i U_i \approx -0.3Mc^2$  and only the gravitational self-energy of the gravitational source M needs to be considered. In particular, the fact that the center of the equivalent mass of the gravitational potential energy coincides with the center of mass of the gravitational source makes the situation convenient.

### 2-3. Under the general theory of relativity, a solution in a strong gravitational field [2] [3]

**In all existing solutions, the mass term  $M$  must be replaced by  $(M - M_{gs})$ .**

$M \rightarrow (M) + (-M_{gs})$ ,  $-M_{gs}$  is the equivalent mass of gravitational self-energy. In all existing solutions (Schwarzschild, Kerr, Reissner-Nordström, ... ), the mass term  $M$  must be replaced by  $(M - M_{gs})$ .

For example, Schwarzschild solution is,

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \frac{1}{\left(1 - \frac{2GM}{c^2 r}\right)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (24)$$

**Schwarzschild-Choi solution** is

$$ds^2 = -\left(1 - \frac{2G(M - M_{gs})}{c^2 r}\right)c^2 dt^2 + \frac{1}{\left(1 - \frac{2G(M - M_{gs})}{c^2 r}\right)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (25)$$

This solution shows a significant difference between the vicinity of a black hole and the interior of a black hole.

For the sphere with uniform density,

$$-M_{gs} = -\frac{3}{5} \frac{GM^2}{Rc^2} \quad (26)$$

1) If  $M \gg |-M_{gs}|$ , in other words if  $r \gg R_S$ , we get the Schwarzschild solution.

2) If  $M = |-M_{gs}|$  (At  $r = R_{gs} = 0.3R_S$ )

Looking for the size in which gravitational self-energy becomes equal to rest mass energy by comparing both,

$$U_{gs} = \left| -\frac{3}{5} \frac{GM^2}{R_{gs}} \right| = Mc^2 \quad (27)$$

$$R_{gs} = \frac{3}{5} \frac{GM}{c^2} \quad (28)$$

Comparing  $R_{gs}$  with  $R_S$ , the radius of Schwarzschild black hole,

$$R_{gs} = \frac{3}{5} \frac{GM}{c^2} < R_S = \frac{2GM}{c^2} \quad (29)$$

$$R_{gs} = 0.3R_S \quad (30)$$

At  $r = R_{gs} = 0.3R_S$ ,  $M = |-M_{gs}|$ . So,

$$ds^2 = -c^2 dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (31)$$

**The singularity disappears, and a flat space-time is obtained.**

#### 2-4. Create a new field equation containing the energy of the gravitational field

It is thought that the method applied above can be used to find new field equation and solution.

1) Establish a universal field equation in which the energy of the gravitational field is entered.

$$T^{\mu\nu} = T_{(m)}^{\mu\nu} + t^{\mu\nu} \quad (32)$$

$T^{\mu\nu}$  is the total energy-momentum tensor of matter and gravity. Here  $T_{(m)}^{\mu\nu}$  and  $t^{\mu\nu}$  are, respectively, the energy-momentum tensor of matter and gravitation.

Our field equation now becomes

$$\partial_\lambda \partial^\lambda \phi^{\mu\nu} = -\kappa (T_{(m)}^{\mu\nu} + t^{\mu\nu}) \quad (33)$$

2) It develops through the existing general relativity principle, and expresses three energy terms of the gravitational field in an appropriate part.

$$U_T = \sum_i U_i = U_{gs-M} + U_{gs-m} + U_{gp-Mm} \quad (34)$$

Find the equivalent mass of the gravitational potential energy and approximate it.

$$\frac{U_T}{c^2} = \frac{\sum_i U_i}{c^2} = \frac{U_{gs-M}}{c^2} + \frac{U_{gs-m}}{c^2} + \frac{U_{gp-Mm}}{c^2} \simeq \frac{U_{gs-M}}{c^2} \quad (35)$$

3) Put it in the Schwarzschild-Choi equation.  
**Schwarzschild-Choi solution** is

$$ds^2 = -\left(1 - \frac{2G(M - M_{gs})}{c^2 r}\right)c^2 dt^2 + \frac{1}{\left(1 - \frac{2G(M - M_{gs})}{c^2 r}\right)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (36)$$

## II. The solution to the black hole singularity problem [2] [3]

### 1. Binding energy in the mass defect problem

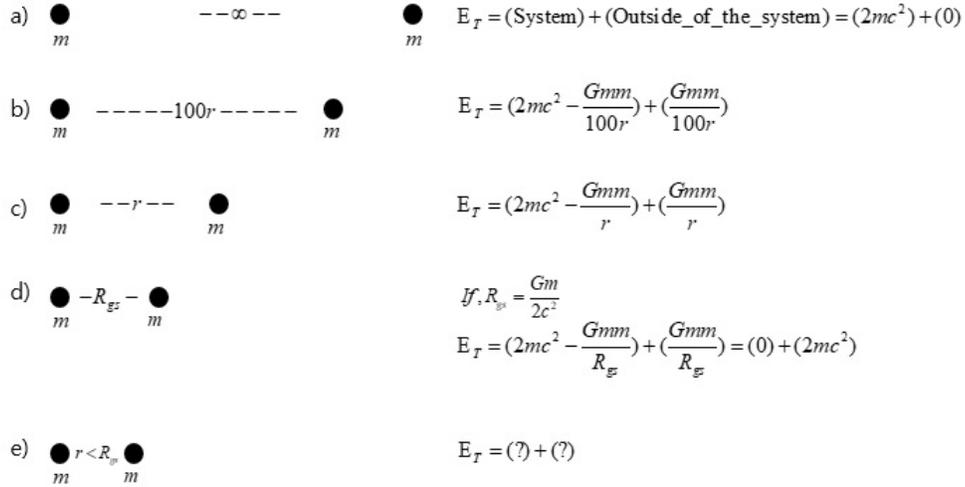


Figure 2: Description from the mass defect. What if we compress more than  $R_{gs}$  ?

When two particles form a binding state, energy corresponding to the binding energy must be released from the system to the outside of the system. In order to keep the two particles close enough so that  $r = R_{gs}$ , the total energy of the system must be zero and the initial (in free state) total mass energy of the system must be released to the outside of the system. Now, in order for these two particles to compress further and achieve a stable state, positive energy must be released from the system to the outside as much as the difference in binding energy. In the case of allowing only positive energy, this compression must be inhibited because there is no more positive energy to withdraw from the system. In other words, gravitational potential energy, a type of binding energy, has the potential to solve the singularity problem.

### 2. Gravitational self-energy

The concept of gravitational self-energy ( $U_{gs}$ ) is the total of gravitational potential energy possessed by a certain object M itself. Since a certain object M itself is a binding state of infinitesimal mass  $dM$ s, it involves the existence of gravitational potential energy among these  $dM$ s and is the value of adding up these.  $M = \sum dM$ .

Gravitational self-energy or Gravitational binding energy ( $-U_{gs}$ ) in case of uniform density is given by

$$U_{gs} = -\frac{3}{5} \frac{GM^2}{R} \quad (37)$$

**In the generality of cases, the value of gravitational self-energy is small enough to be negligible, compared to mass energy  $Mc^2$ . So generally, there was no need to consider gravitational self-energy. However the smaller R becomes, the higher the absolute value of  $U_{gs}$ . For this reason, we can see that  $U_{gs}$  is likely to offset the mass energy in a certain radius.**

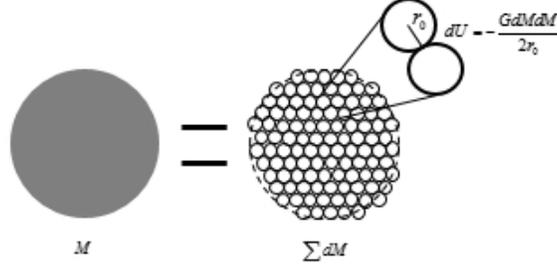


Figure 3: Since all mass  $M$  is a set of infinitesimal mass  $dM$ s and each  $dM$  is gravitational source, too. There exists gravitational potential energy among each of  $dM$ s. Generally, mass of an object measured from its outside corresponds to the value of dividing the total of all energy into  $c^2$ .

Thus, looking for the size in which gravitational self-energy becomes equal to rest mass energy by comparing both,

$$U_{gs} = \left| -\frac{3}{5} \frac{GM^2}{R_{gs}} \right| = Mc^2 \quad (38)$$

$$R_{gs} = \frac{3}{5} \frac{GM}{c^2} \quad (39)$$

This equation means that if mass  $M$  is uniformly distributed within the radius  $R_{gs}$ , gravitational self-energy for such an object equals mass energy in size. So, in case of such an object, mass energy and gravitational self-energy can be completely offset while total energy is zero. Since total energy of such an object is 0, gravity exercised on another object outside is also 0.

Comparing  $R_{gs}$  with  $R_S$ , the radius of Schwarzschild black hole,

$$R_{gs} = \frac{3}{5} \frac{GM}{c^2} < R_S = \frac{2GM}{c^2} \quad (40)$$

$$R_{gs} = 0.3R_S \quad (41)$$

This means that there exists the point where gravitational self-energy becomes equal to mass energy within the radius of black hole, and that, supposing a uniform distribution, the value exists at the point  $0.3R_S$ , a 30% level of the black hole radius.

Even with kinetic energy and virial theorem applied only the radius diminishes as negative energy counterbalances positive energy, but no effects at all on this point: “There is a zone which cannot be compressed anymore due to the negative gravitational potential energy.” Although potential energy changes to kinetic energy, in order to achieve a stable bonded state, a part of the kinetic energy must be released to the outside of the system.

Considering the virial theorem ( $K=-U/2$ ),

$$R_{gs-vir} = \frac{1}{2}R_{gs} = 0.15R_S \quad (42)$$

Since this value is on a level not negligible against the size of black hole, we should never fail to consider “gravitational self-energy” for case of black hole. In case of the smallest black hole with three times the solar mass,  $R_S = 9km$ .  $R_{gs}$  of this black hole is as far as 3km. In other words, even in a black hole with smallest size that is made by the contraction of a star, the mass distribution can’t be reduced to at least radius 3km ( $R_{gs-vir} = 1.5km$ ).

### 3. Black hole does not have a singularity, but it has a Zero Energy Zone

From the equation above, even if some particle comes into the radius of black hole, it is not a fact that it contracts itself infinitely to the point  $R = 0$ . From the point  $R_{gs}$  (or  $R_{gs-vir}$ ), gravity is 0, and when it enters into the area of  $R_{gs}$  (or  $R_{gs-vir}$ ), total energy within  $R_{gs}$  (or  $R_{gs-vir}$ ) region corresponds to negative

values enabling anti-gravity to exist. This  $R_{gs}$  (or  $R_{gs-vir}$ ) region comes to exert repulsive gravity effects on the particles outside of it, therefore it interrupting the formation of singularity at the near the area  $R=0$ .

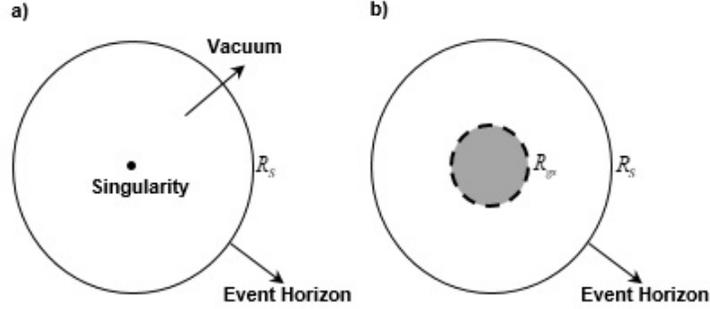


Figure 4: a) Existing Model. b) New Model. The area of within  $R_{gs}$ (or  $R_{gs-vir}$ ) has gravitational self-energy(potential energy) of negative value, which is larger than mass energy of positive value. If  $r$  is less than  $R_{gs}$ , this area becomes negative energy (mass) state. There is a repulsive gravitational effect between the negative masses, which causes it to expand again. This area (within  $R_{gs}$ ) exercises anti-gravity on all particles entering this area, and accordingly prevents all masses from gathering to  $r=0$ . Therefore the distribution of mass (energy) can't be reduced to at least radius  $R_{gs}$  (or  $R_{gs-vir}$ ).

#### 4. In a strong gravitational field, the solution of the general theory of relativity In all existing solutions, the mass term $M$ must be replaced by $(M - M_{gs})$ .

We can solve the problem of singularity by separating the term  $(-M_{gs})$  of gravitational self-energy from mass and including it in the solutions of field equation.

$M \rightarrow (M) + (-M_{gs})$ ,  $-M_{gs}$  is the equivalent mass of gravitational self-energy. In all existing solutions (Schwarzschild, Kerr, Reissner-Nordström, ...), the mass term  $M$  must be replaced by  $(M - M_{gs})$ .

For example, Schwarzschild solution is,

$$ds^2 = -(1 - \frac{2GM}{c^2r})c^2dt^2 + \frac{1}{(1 - \frac{2GM}{c^2r})}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \quad (43)$$

**Schwarzschild-Choi solution is**

$$ds^2 = -(1 - \frac{2G(M - M_{gs})}{c^2r})c^2dt^2 + \frac{1}{(1 - \frac{2G(M - M_{gs})}{c^2r})}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \quad (44)$$

For the sphere with uniform density,

$$-M_{gs} = -\frac{3}{5} \frac{GM^2}{Rc^2} \quad (45)$$

- 1) If  $M \gg | -M_{gs} |$ , in other words if  $r \gg R_S$ , we get the Schwarzschild solution.
- 2) If  $M = | -M_{gs} |$

$$ds^2 = -c^2dt^2 + dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \quad (46)$$

At  $r = R_{gs}$ , a flat space-time is obtained.

- 3) If  $M \ll | -M_{gs} |$ , in other words if  $0 \leq r \ll R_{gs}$ ,

$$ds^2 \simeq -(1 + \frac{2GM_{gs}}{c^2r})c^2dt^2 + \frac{1}{(1 + \frac{2GM_{gs}}{c^2r})}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \quad (47)$$

In the domain of  $0 \leq r \ll R_{gs}$ ,

The area of within  $R_{gs}$  has gravitational self-energy of negative value, which is larger than mass energy of positive value. Negative mass has gravitational effect which is repulsive to each other. So, we can assume that  $-M_{gs}$  is almost evenly distributed. Therefore  $\rho_{gs}$  is constant. And we must consider the Shell Theorem.

$$-M_{gs} = -\frac{4\pi r^3}{3}\rho_{gs} \quad (48)$$

$$\left(1 + \frac{2GM_{gs}}{c^2 r}\right) = 1 + \frac{2G\left(\frac{4\pi}{3}r^3\rho_{gs}\right)}{c^2 r} = 1 + \frac{8\pi G\rho_{gs}r^2}{3c^2} \quad (49)$$

$$ds^2 \simeq -\left(1 + \frac{8\pi G\rho_{gs}r^2}{3c^2}\right)c^2 dt^2 + \frac{1}{\left(1 + \frac{8\pi G\rho_{gs}r^2}{3c^2}\right)} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \quad (50)$$

If  $r \rightarrow 0$ ,

$$ds^2 \simeq -c^2 dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \quad (51)$$

There is no singularity.

In practice, mass contraction must be stopped at the point where  $M_{shell} = M_{shell-gs}$ .

### 5. Internal structure of black hole considering gravitational self-energy

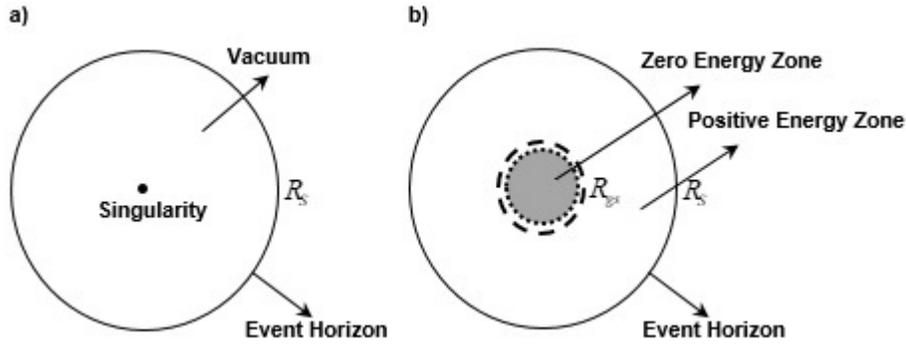


Figure 5: Internal structure of the black hole. a) Existing model b) New model. If, over time, the black hole stabilizes, **the black hole does not have a singularity in the center, but it has a Zero (total) Energy Zone.**

When the mass distribution inside the black hole is reduced from  $R_S$  to  $R_{gs}$  (or  $R_{gs-vir}$ ), the energy must be released from the inside of the system to the outside of the system in order to reach this binding state. Here, the system refers to the mass distribution within the radius  $0 \leq r \leq R_{gs}$  (or  $R_{gs-vir}$ ). Although potential energy changes to kinetic energy, in order to achieve a stable bonded state, a part of the kinetic energy must be released to the outside of the system. We need to consider the virial theorem.

At this time, the emitted energy does not go out of the black hole. This energy is distributed in the  $R_{gs}$  (or  $R_{gs-vir}$ )  $< r \leq R_S$  region.

**If you have only the concept of positive energy, please refer to the following explanation.**

From the point of view of mass defect,  $r = R_{gs}$  (or  $R_{gs-vir}$ ) is the point where the total energy of the system is zero. For the system to compress more than this point, there must be an positive energy release from the system. However, since the total energy of the system is zero, there is no positive energy that the system can release. Therefore, the system cannot be more compressed than  $r = R_{gs}$  (or  $R_{gs-vir}$ ). So black hole doesn't have singularity.

By locking horns between gravitational self-energy and mass energy, particles inside black hole or distribution of energy can be stabilized. As a final state, the black hole does not have a singularity in the center, but it has a Zero (total) Energy Zone (ZEZ).  $R_{gs}$  is the maximum of ZEZ.

**6. Inside the huge black hole, there is enough space for intelligent life to exist [4]**

A black hole has no singularity, has a Zero Energy Zone with a total energy of zero, and this region is very large, reaching 15% ~ 30% of the radius of the black hole. It suggests an internal structure of a black hole that is completely different from the existing model. Inside the huge black hole, there is an area where intelligent life can live.

Therefore, by considering gravitational self-energy, it is possible to solve the problem of singularity of black hole, which is the most important problem in general relativity. And, this discovery provides a logic for why we are surviving in universe black hole (formed when only mass energy is considered) without collapsing into singularity.

Size of mass distribution	The size of the universe black hole created by the mass distribution	Size of Zero Energy Zone (15% ~ 30%)
14.3Gly	14.3Gly	2.2Gly ~ 4.3Gly
20.0Gly	39.1Gly	5.9Gly ~ 11.7Gly
46.5Gly	491.6Gly	73.7Gly ~ 147.5Gly
100.0Gly	4883.9Gly	732.6Gly ~ 1465.2gly

Figure 6: Under the observed average density, the size of Universe Black Hole and Zero Energy Zone. If the size of the mass distribution increases  $R$  times, the Universe Black Hole and ZEZ created by the new mass distribution become  $R^3$  times larger.

For example, if the masses are distributed approximately 46.5Gly with the average density of the current universe, the size of the black hole created by this mass distribution will be 491.6Gly, and the size of the Zero Energy Zone will be approximately 73.7Gly ~ 147.5Gly. In other words, there is no strong tidal force and a region with almost flat space-time that can form a stable galaxy structure is much larger than the observable range of 46.5 Gly. The entire universe is estimated to be much larger than the observable universe, so it may not be at all unusual for us to observe only the Zero Energy Zone (nearly flat space-time).

Even if humans live inside a black hole called the universe, a sufficiently stable survival area for intelligent life is guaranteed.

### III. The sources of dark energy are gravitational potential energy and the expansion of the particle horizon

#### 1. Expansion of the universe by gravitational potential energy or gravitational self-energy

Within  $R_{gs}$ , negative gravitational self-energy is larger than positive mass energy, and the region within  $R_{gs}$  corresponds to a negative mass state, and repulsive force or antigravity exists.

**Now, we have repulsion or anti-gravity on a cosmic scale. Therefore, this force will be applicable to various phenomena that require repulsive force on a cosmic scale.** For example, Inflation, Dark energy, and the Force that displace the expansion of space and move galaxies.

#### 1-1. Expansion within $R_{gs}$ from the initial mass-energy distribution at the birth of the universe [4]

Consider the initial state of the universe. The entire universe is larger than the present observable universe, 46.5Gly. Since we do not know the size of the entire universe, after thinking about the state in which all the mass-energy in the present observable universe is concentrated in a very small area, let's apply this logic to the entire universe.

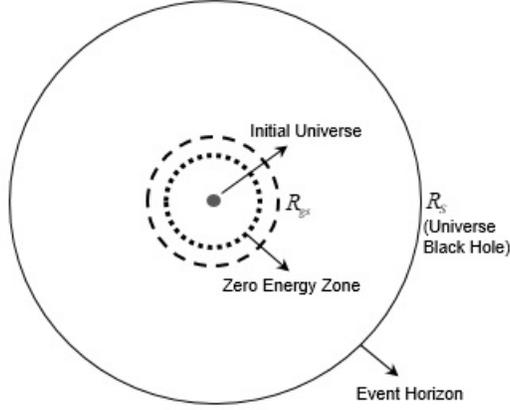


Figure 7: Comparison of the size of the initial universe, the size of the ZEZ, and the size of the Universe Black Hole

As calculated above, the size of the ZEZ produced by all mass-energy in the observable universe is approximately  $73.7 \text{ Gly} \sim 147.5 \text{ Gly}$ , and the size of the universe black hole is  $491.6 \text{ Gly}$ . Since these materials are concentrated in a very small area, the negative gravitational potential energy of this area exceeds the positive mass energy and corresponds to a negative mass state as a whole. Because there is a repulsive gravitational effect between negative masses, it expands. [3]

This expansion is accelerated up to at least ZEZ ( $73.7 \text{ Gly} \sim 147.5 \text{ Gly}$ ), and since it is in an accelerated state, expansion continues beyond ZEZ. As time passes, when the distribution of mass is outside the ZEZ, the mass state within the ZEZ is a state in which the positive mass energy is greater than the negative gravitational potential energy, so the total mass (within the ZEZ) is a positive mass, and the attraction is applied to the masses outside the ZEZ. This will have the effect of slowing the expansion.

**The universe expansion at the time of the big bang is because all matter started in a region smaller than the ZEZ**, and there is a possibility that it corresponds to the accelerated expansion process up to the ZEZ. The size of the ZEZ created by the mass distribution of the observable universe is  $73.7 \text{ Gly} \sim 147.5 \text{ Gly}$ , but the present observable universe is passing  $46.5 \text{ Gly}$ .

**1) In the early days of the universe, why didn't the universe become a singularity or black hole?**

In mainstream models, this is explained by the expansion of space. In this model, the explanation is different. If the total mass of the universe is collected in a very small area, the negative gravitational potential energy is greater than the positive mass energy, and the whole is placed in a negative mass state. There is a repulsive gravitational effect between the negative masses, so expansion occurs away from each other. At least up to the  $R_{gs}$  (Maximum of ZEZ) region, there is an expansion. The singularity problem can be explained as "The singularity itself cannot be formed because of negative gravitational potential energy."

**2) At the beginning of the universe, the problem of escape from a black hole created by the total mass of the universe?**

At the beginning of the universe, when considering the expansion in a high-density state, there is a problem that people mistakenly think that this event is the escape of matter from the inside of the black hole created by the total mass of the universe to the outside to form galaxies or stars.

The black hole event horizon created by the total mass of the universe is very large compared to the area where the total mass of the universe is gathered. In other words, in the Black Hole Cosmology model, **matter does not escape the universe black hole, but has not yet reached the event horizon of the universe black hole (formed when only mass energy is considered without considering gravitational potential energy).**

### 3) About the inflation mechanism

In the above analysis, we hypothesized that all matter in the present observable universe was gathered in a very small area.

By relaxing the conditions, one can assume the sequential birth of mass, or take advantage of the fact that the propagation speed of the field is limited to the speed of light. Also, because of the finite time after birth, the range of interaction is limited. In other words, not everything interacts at the same moment, but it has certain characteristics sequentially according to the birth of the field, the propagation speed of the field, and time (age of the universe). Such circumstances makes it possible to adjust the size of  $R_{gs}$  or ZEZ in the early universe to be smaller than the current  $73.7 \sim 147.5\text{Gly}$ .

In the standard cosmology, we postulate a rapid accelerated expansion process called inflation before the big bang model. **The accelerated expansion caused by negative gravitational potential energy may be used to explain the inflation mechanism.**

At the birth of the universe, with all matter gathered in a very small area, rapid accelerated expansion occurred due to negative gravitational potential energy, and this accelerated expansion may be terminated due to several factors (end of inflation mechanism). For example, inflation may dissipate after expansion to  $R_{gs}$  made by initial materials, or it may end up with problems such as particle horizons.

Since the force from the mass distribution within  $R_{gs}$  is anti-gravity, it will ensure the expansion and uniform density of the universe. Anyway, repulsion is now possible on a cosmic scale, so please try using it for various purposes.

#### 1-2. Decelerating expansion after $R_{gs}$ [4]

If we do not assume the birth or influx of new mass-energy, then slowed expansion occurs after  $R_{gs}$ . However, if we assume the birth or influx of new mass-energy, such as vacuum energy or a cosmological constant, the situation becomes more complex and dependent on assumptions.

Also, when the particle horizon becomes large, the  $R_{gs}$  also becomes large. Accordingly, continuous accelerated expansion may occur.

## 2. The sources of dark energy are gravitational potential energy and the expansion of the particle horizon

### 2-1. Comparison of magnitudes of mass energy and total gravitational potential energy in the observable universe

#### 1) Total mass energy of the observable universe (particle horizon)

For density, the critical density value was used.  $\rho_c = 8.50 \times 10^{-27}[\text{kgm}^{-3}]$  [5], observable universe radius :  $46.5\text{Gly}$

$$Mc^2 = \frac{4\pi r^3 \rho c^2}{3} = 2.75 \times 10^{71}[\text{kgm}^2 \text{s}^{-2}] \quad (52)$$

#### 2) Total gravitational potential energy of the observable universe (particle horizon)

$$U = -\frac{3}{5} \frac{GM^2}{R} = -\frac{16\pi^2 GR^5 \rho^2}{15} = -8.35 \times 10^{71}[\text{kgm}^2 \text{s}^{-2}] \quad (53)$$

### 3) In the observable universe, the ratio of total mass energy to total gravitational potential energy

$$\frac{U}{Mc^2} = \frac{-8.35 \times 10^{71}}{2.75 \times 10^{71}} = -3.03 \quad (54)$$

The repulsive force component is approximately 3.03 times the attractive force component. The universe is accelerating expansion.

### 4) In the observable universe, the ratio of total mass energy to total gravitational potential energy

The following equation is an approximate form of the energy density of the standard cosmology model. [5]

$$\frac{M_{repulsive}}{M_{attractive}} = \frac{-3\rho_{\Lambda}}{\rho_m + \rho_{\Lambda}} = \frac{-3(0.683\rho)}{0.317\rho + 0.683\rho} = -2.049 \quad (55)$$

Although there is a slight difference between the two values, it is not an astronomical error, so it is likely to be resolved later. Kinetic energy, general relativity, or cosmological effects are likely to explain the difference between the two values.

## 2-2. Comparison of magnitudes of mass energy and total gravitational potential energy in the cosmic event horizon

### 1) The magnitude of the total mass energy in the cosmic event horizon

Cosmic event horizon : 16.7Gly

$$Mc^2 = \frac{4\pi r^3 \rho c^2}{3} = 1.275 \times 10^{70} [kgm^2s^{-2}] \quad (56)$$

### 2) The magnitude of the total gravitational potential energy in the cosmic event horizon

$$U = -\frac{3}{5} \frac{GM^2}{R} = -\frac{16\pi^2 GR^5 \rho^2}{15} = -4.987 \times 10^{69} [kgm^2s^{-2}] \quad (57)$$

### 3) Comparison of magnitudes of total mass energy and total gravitational potential energy in the cosmic event horizon

$$\frac{U}{Mc^2} = \frac{-4.987 \times 10^{69}}{1.275 \times 10^{70}} = -0.39 \quad (58)$$

In the calculation, the current critical density value was used, but when the particle horizon is 16.7 Gly, the density is different from now. So, just look at the logic.

In the cosmic event horizon(16.7Gly), the repulsive component is smaller than the attractive component. In this period, the universe is decelerating expansion. That is, when the particle horizon is 16.7 Gly, the dark energy component is smaller than that of matter. This period is a period of decelerated expansion.

**At 16.7 Gly, the attraction component is larger than the repulsive component, whereas at 46.5 Gly, the repulsive component is 3.03 times larger than the attraction component. This means that there is an inflection point that changes from decelerated expansion to accelerated expansion.** Let's find the inflection point!

## 2-3. Increase in dark energy

### 2-3-1. Creating a Comparison Expression for Mass Energy and Gravitational Potential Energy

$$\frac{U}{Mc^2} = \frac{-\frac{3}{5} \frac{GM^2}{R}}{Mc^2} = -\left(\frac{4\pi G\rho}{5c^2}\right)R^2 = -\left(\frac{4\pi G\rho}{5c^2}\right)(k \times Gly)^2 = -1.43 \times 10^{-3}k^2 \quad (59)$$

$$R = k \times Gly \quad (60)$$

Expression in the form of expressing even the density, which is a variable

$$\frac{U}{Mc^2} = -\left(\frac{4\pi G\rho}{5c^2}\right)R^2 = (-1.68 \times 10^{23}m^3kg^{-1})\rho k \quad (61)$$

The ratio depends on the average density ( $\rho$ ) of the universe and the size (The range of interaction, the particle horizon R) of the universe.

### 2-3-2. Find the inflection point where attractive and repulsive forces are balanced

$$\frac{U}{Mc^2} = -\left(\frac{4\pi G\rho}{5c^2}\right)R^2 = -1.43 \times 10^{-3}k^2 = -1.00 \quad (62)$$

$$k = \sqrt{\frac{1}{0.00143}} = 26.44 \quad (63)$$

$$R = 26.4Gly \quad (64)$$

The particle horizon at the transition from material dominance to dark energy dominance. The particle horizon at the transition from a period of predominant attraction to a period of predominant repulsion.

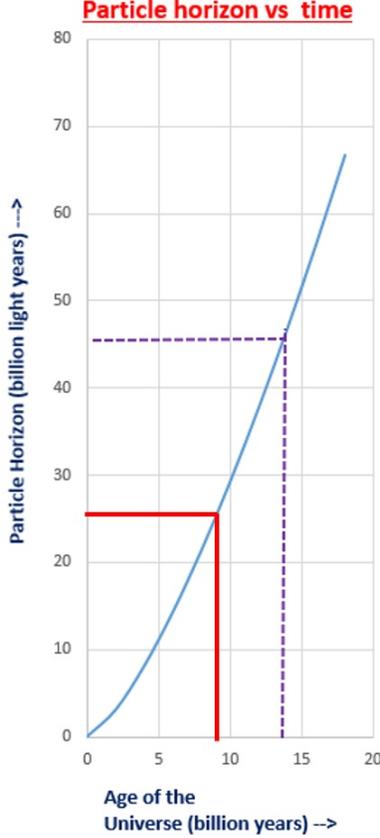


Figure 8: About 5 billion years ago, it passed the inflection point 26.4Gly. [6] The inflection point is affected by the Hubble constant and density. Experts need to calculate with accurate data.

### 2-3-3. The ratio of increase in gravitational self-energy to mass increase

$$\frac{d(Mc^2)}{dR} = 4\pi R^2 \rho c^2 \quad (65)$$

$$\frac{d(U_{gs})}{dR} = -\frac{16\pi^2 G}{3} R^4 \rho^2 = \left(-\frac{4\pi R^3 \rho G}{3Rc^2}\right)(4\pi R^2 \rho c^2) = -\frac{GM}{Rc^2} \left(\frac{d(Mc^2)}{dR}\right) \quad (66)$$

$$\frac{d(U_{gs})}{dR} = -\frac{R_S}{2R} \frac{d(Mc^2)}{dR} \quad (67)$$

$R_S$  is the Schwarzschild radius of the black hole formed by Particle Horizon.

In Figure 6, the size of the event horizon formed by the mass distribution of 46.5 Gly is 491.6 Gly.

$$\frac{d(U_{gs})}{dR} = -\frac{R_S}{2R} \left(\frac{d(Mc^2)}{dR}\right) = -\frac{491.6Gly}{2(46.5Gly)} \left(\frac{d(Mc^2)}{dR}\right) = -(5.29) \left(\frac{d(Mc^2)}{dR}\right) \quad (68)$$

If the particle horizon increases and a positive mass is produced by  $M$ , the equivalent mass of negative gravitational potential energy is produced by  $- 5.29 M$ . This value is not a fixed value, it depends on the density and the size of the particle horizon.

However, the negative mass ratio of 5.29 produced is very similar to the ratio of matter : dark matter.

In my 2009 paper, assuming that the negative mass is 5.06 times greater than the positive mass, it was argued that both dark matter and dark energy could be explained. [7] I also argued that the essence of dark matter is negative mass. [2] [7]

**2-3-4. Increase in dark energy (gravitational potential energy) due to increase in particle horizon**

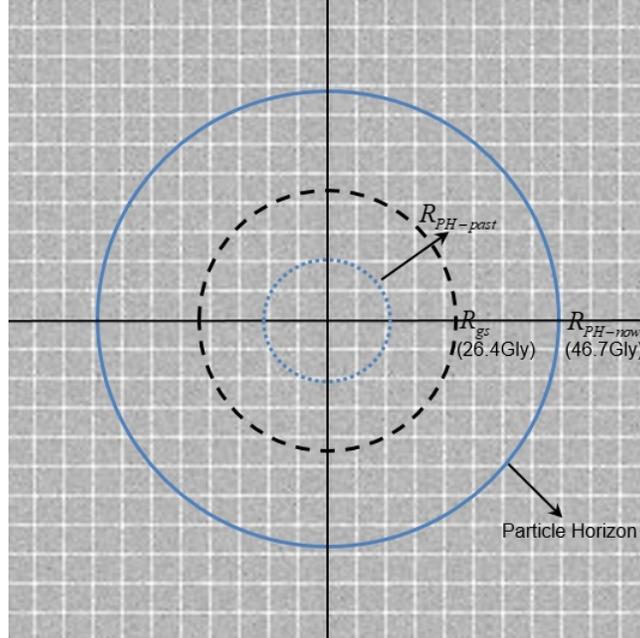


Figure 9: The past Particle Horizon, the present particle horizon, and  $R_{gs}$ , the inflection point at which the magnitudes of repulsive and attractive forces are equal.

- 1) Particles and galaxies spread almost uniformly throughout the universe through the inflation process.
- 2) Galaxies move according to the Hubble-Lemaitre law.
- 3) On the other hand, the propagation speed of the field, the range of interaction (particle horizon), has the fastest speed, the speed of light in expanding space.
- 4) Thus, over time, many new substances (matters and galaxies) enter the particle horizon. In other words, **the newly entering materials undergo gravitational interaction, resulting in an increase in mass and an increase in gravitational potential energy in the region within the particle horizon.**
- 5) By the way, **while mass energy is proportional to  $M$ , total gravitational potential energy (gravitational self-energy) is proportional to  $M^2$ . That is, the gravitational potential energy increases faster. Accordingly, the repulsive force component increases faster than the attractive force component.**
- 6) The increase in gravitational potential energy due to the newly incorporated matter into the particle horizon is causing the dark energy. The same principle is applicable to the birth of energy within a particle horizon. That is, when the mass energy increases by  $M$ , the gravitational self-energy increases by  $M^2$ .
- 7) In the present universe, it is predicted that the dark energy effect (repulsive effect) surpassed the attractive effect of matter and dark matter about 5 billion years ago. Estimating the inflection point with this model, the particle horizon is approximately 26.4 Gly.

8) This model includes the point where the dark energy is smaller than the attractive force term, and becomes larger than the attractive force term over time, and presents a result very similar to the point of inflection 5 billion years ago.

9) Gravitational potential energy is a concept that already exists and is negative energy that can create repulsive force. **This model produces similar results to the phenomenon of applying negative pressure while having positive inertial mass. As the particle horizon expands, the positive mass (new influx of matter) increases, but the negative gravitational potential created by these positive masses is greater. While having a positive inertial mass, it is creating a negative gravitational mass that is larger than the positive inertial mass.**

10) Precise calculations should be performed by experts in this field.

#### IV. It is possible that the source of dark matter is also gravitational potential energy.

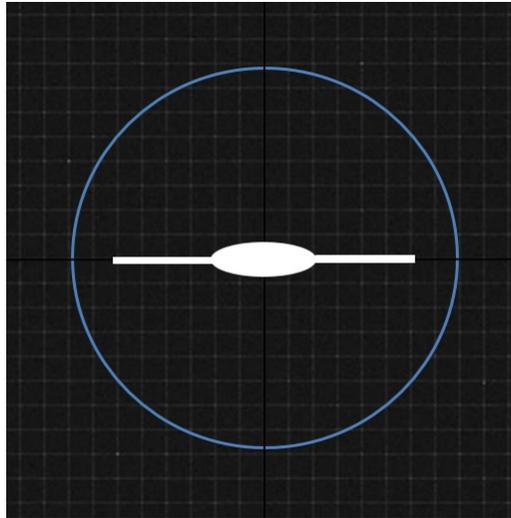


Figure 10: Galaxies are trapped in an ocean full of negative energy (mass). It is possible that negative energies outside the galactic structure are preventing the galaxies from changing their orbits.

The present universe is a universe in which the negative gravitational potential energy is greater than the positive mass energy. And, the magnitude of negative energy (mass) is about 3.3 times the amount of the sum of matter + dark matter, and about 20 times when only matter is taken into account.

Thus, galaxies can be viewed as structures trapped in an ocean full of negative mass.

Because the average density of the galaxy itself is higher than the average density of the universe, the galaxy itself is greater than the mass density of negative energy, and thus can retain its structure.

However, the current structure of the galaxy is not a structure without anything, but a structure with internal and external pressures.

Thus, even if the stars in the spiral arms of galaxies have higher velocities than their current orbits, it is likely that they will not be able to transition to orbits with a greater radius than their current orbits.

In other words, the dark matter effect may be a phenomenon that occurs because the structure of the galaxy is already in a structure that is under pressure from all sides.

Deep-sea fish living in high water pressure maintain their shape and live in the deep sea, but when they come out of the water with low air pressure, they expand and lose their shape or explode.

If the effect of the gravitational potential energy of the entire universe was the essence of dark matter, then there would be no particle-like substance, and therefore no electromagnetic interaction.

## V. Conclusion <sup>2</sup>

In order to apply the general theory of relativity to the strong gravitational field, it is sufficient to consider the gravitational self-energy, which is the binding energy of the object itself.

By considering the gravitational self-energy, it is possible to solve the singularity problem, which is the biggest problem with general relativity, and to rescue general relativity that collapses into a singularity created by oneself.

Also, in the field of cosmology, the effect of dark energy occurs because matter and galaxies entering the particle horizon contribute to the total gravitational potential energy. Since the total mass in the particle horizon is proportional to  $M$ , whereas the gravitational self-energy is proportional to  $M^2$ , the repulsive force component increases faster and accelerated expansion occurs.

This model can be verified because it points to the gravitational potential energy and the particle horizon as the causes.

This model predicts an inflection point where dark energy becomes larger and more important than the energy of matter and radiation. Through this model, the past, present and future of the universe can be predicted. Therefore, I think experts need to review this model.

## References

- [1] Hans C. Ohanian, *Gravitation and Spacetime*, (W. W. Norton & Company, New York).(1976).
- [2] Hyoyoung Choi. On Problems and Solutions of General Relativity (Commemoration of the 100th Anniversary of General Relativity), (2015).  
[<https://www.researchgate.net/publication/286935998>][<http://vixra.org/abs/1511.0240>].
- [3] Hyoyoung Choi. Solution of the Singularity Problem of Black Hole, (2016).  
[<https://www.researchgate.net/publication/313314666>][<http://vixra.org/abs/1701.0667>].
- [4] Hyoyoung Choi. Problems and Solutions of Black Hole Cosmology, (2022).  
[<https://www.researchgate.net/publication/359192496>][<https://vixra.org/abs/2203.0073>].
- [5] Planck Collaboration et al., Planck 2015 results. XIII. Cosmological parameters, (2015).  
[<https://arxiv.org/abs/1502.01589>]
- [6] Steve Hurley's website. Cosmic Horizons, (2021).  
[<https://explainingscience.org/2021/04/30/cosmic-horizons/>]
- [7] Hyoyoung Choi. Hypothesis of dark matter and dark energy with negative mass, (2009).  
[<https://www.researchgate.net/publication/228610043>]

---

<sup>2</sup>Journals have very high barriers to entry, and I understand that. In order to be listed on arXiv, endorsement by other scholars is required. Although endorsed by other scholars, arXiv's administrators are arbitrarily deleting papers. I've experienced it a few times, and it's painful.

Some scientific communities prevent even discussion of content not published in journals. A scientific community that does not allow hypotheses, it was something I could never have imagined when I was a physics student. Who made the rule that only professional soccer players can play soccer? Is the Internet Science Community a Pro League?

No matter how ugly a paper is, it is very careful to delete it or format it.

What we have to keep in mind is that neither Newton's nor Einstein's theory. What we must protect are freedom of thought, freedom of research, and freedom of speech and publication. We have the discriminators of the truth of the grand universe and time.