Analyzing several equations concerning various aspects of Quantum Mechanics, some Ramanujan parameters and the developments of the MRB Constant. New possible mathematical connections with some parameters of Number Theory

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Abstract

In this paper, we analyze several equations concerning various aspects of Quantum Mechanics, some Ramanujan parameters and the developments of the MRB Constant. We describe new possible mathematical connections with some parameters of Number Theory.

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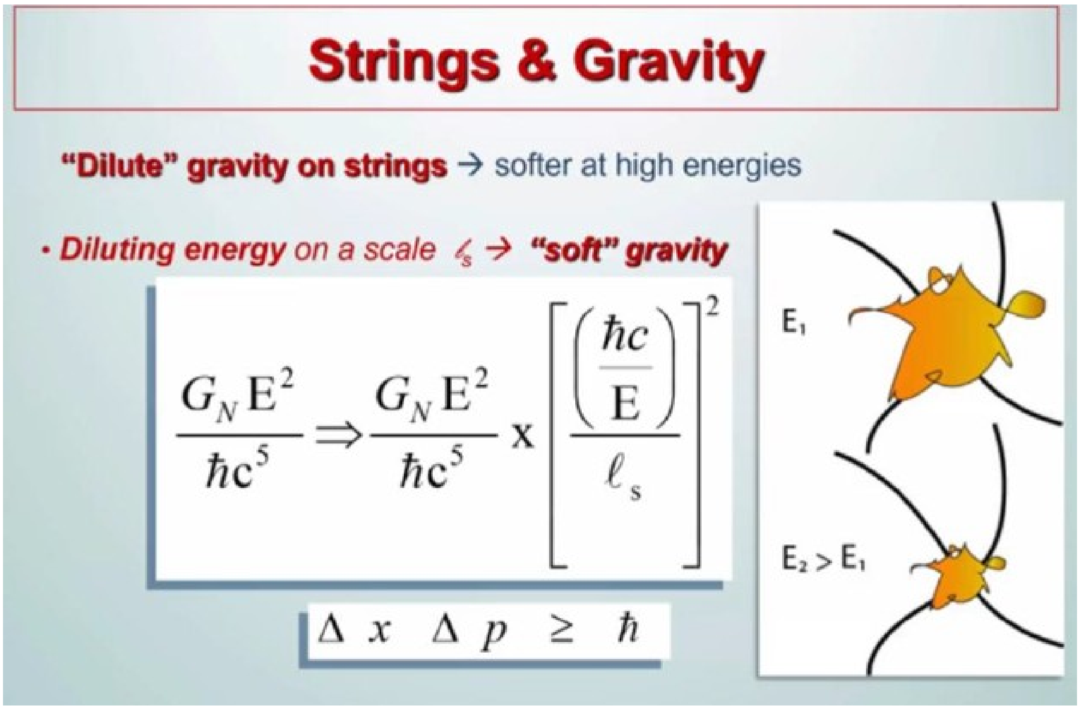
From:

String Theory, Gravity and Particle Physics (*Prof. Augusto Sagnotti* - SNS) - AstronomiAmo 23.04.2020

We have:



(G*E^2)/(h*c^5) [((h*c)/E)*1/l]^2

Input

$$\frac{G e^2}{h c^5} \left(\frac{h c}{e} \times \frac{1}{l}\right)^2$$

Result

$$\frac{G h}{l^2 c^3}$$

Roots

$G = 0, \quad c l \neq 0$

$h = 0, \quad c l \neq 0$

Property as a function
Parity
even

Derivative

$$\frac{\partial}{\partial c}\left(\frac{(G e^2)\left(\frac{h c}{e l}\right)^2}{h c^5}\right) = -\frac{3 G h}{c^4 l^2}$$

Indefinite integral

$$\int \frac{G\,h}{c^3\,l^2}\,dc = -\frac{G\,h}{2\,c^2\,l^2} + \text{constant}$$

Limit

$$\lim_{l\to\pm\infty} \frac{G\,h}{c^3\,l^2} = 0$$

Alternative representation

$$\frac{\left(\frac{h\,c}{e\,l}\right)^2 \left(G\,e^2\right)}{h\,c^5} = \frac{\left(\frac{h\,c}{\exp(z)\,l}\right)^2 \left(G\,\exp^2(z)\right)}{h\,c^5} \quad \text{for } z = 1$$

Series representations

$$\frac{G\,h}{c^3\,l^2} = \sum_{n=0}^{\infty} \frac{(-1+l)^n\,(-1)^n\,(G\,h\,(1+n))}{c^3} \quad \text{for } |-1+l| < 1$$

$$\frac{G\,h}{c^3\,l^2} = \sum_{n=0}^{\infty} \frac{(-1+c)^n\,((-1)^n\,G\,h\,(1+n)\,(2+n))}{2\,l^2} \quad \text{for } |-1+c| < 1$$

$$\frac{G\,h}{c^3\,l^2} = \sum_{n=-\infty}^{\infty} \left(\begin{cases} \frac{G\,h}{l^2} & n = -3 \\ 0 & \text{otherwise} \end{cases}\right) c^n$$

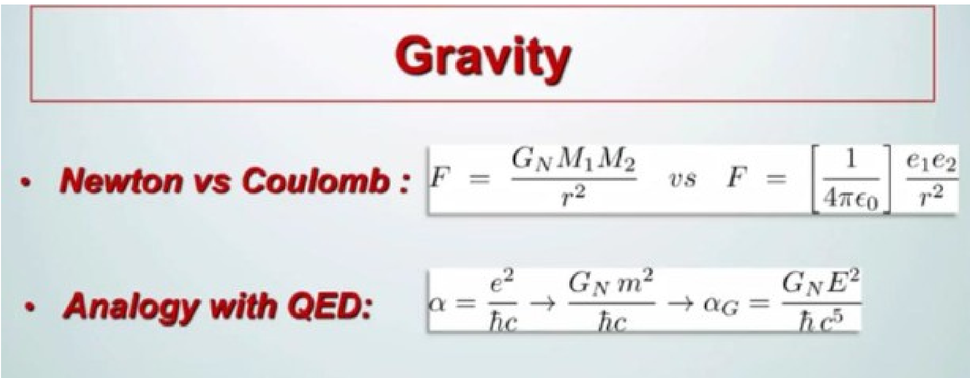
$$\frac{G\,h}{c^3\,l^2} = \sum_{n=-\infty}^{\infty} \left(\begin{cases} \frac{G\,h}{c^3} & n = -2 \\ 0 & \text{otherwise} \end{cases}\right) l^n$$

$|z|$ is the absolute value of z

Definite integral over a hypersphere of radius R

$$\iiiint_{c^2+G^2+h^2+l^2<R^2} \frac{G\,h}{c^3\,l^2}\,dc\,dG\,dh\,dl = 0$$

We have:



For:

α = (G*E^2)/(h*c^5)

we obtain:

(G*E^2)/(h*c^5) [((h*c)/E)*1/l]^2

(G*E^2)/ ((G*E^2)/α) [((h*c)/E)*1/l]^2

Input

$$\frac{G\,e^2}{\frac{G\,e^2}{\alpha}}\left(\frac{h\,c}{e}\times\frac{1}{l}\right)^2$$

Exact result

$$\frac{\alpha\,c^2\,h^2}{e^2\,l^2}$$

Roots

$c = 0\,,\quad l \neq 0$

$h = 0\,,\quad l \neq 0$

$\alpha = 0\,,\quad l \neq 0$

Property as a function

Parity

even

Derivative

$$\frac{\partial}{\partial c}\left(\frac{(G\,e^2)\left(\frac{h\,c}{e\,l}\right)^2}{\frac{G\,e^2}{\alpha}}\right) = \frac{2\,\alpha\,c\,h^2}{e^2\,l^2}$$

Indefinite integral

$$\int \frac{c^2 h^2 \alpha}{e^2 l^2} \, dc = \frac{\alpha c^3 h^2}{3 e^2 l^2} + \text{constant}$$

From:

$$\frac{G h}{l^2 c^3}$$

and:

$$\frac{\alpha c^2 h^2}{e^2 l^2}$$

we obtain:

(G*h)/(l^2*c^3) = (α*c^2*h^2)/(E^2*l^2)

(G*h)/(l^2*c^3) *1/(((α*c^2*h^2)/(E^2*l^2)))

Input

$$\frac{G h}{l^2 c^3} \times \frac{1}{\frac{\alpha c^2 h^2}{e^2 l^2}}$$

Result

$$\frac{G e^2}{\alpha h c^5}$$

Root

$\alpha\, c\, h \neq 0\,, \quad G = 0$

Property as a function

Parity

odd

Derivative

$$\frac{\partial}{\partial c}\left(\frac{G\,h}{\frac{(l^2\,c^3)(\alpha\,c^2\,h^2)}{e^2\,l^2}}\right) = -\frac{5\,e^2\,G}{\alpha\,c^6\,h}$$

Indefinite integral

$$\int \frac{e^2\,G}{c^5\,h\,\alpha}\,dc = -\frac{e^2\,G}{4\,\alpha\,c^4\,h} + \text{constant}$$

Limit

$$\lim_{\alpha\to\pm\infty} \frac{e^2\,G}{c^5\,h\,\alpha} = 0$$

Alternative representation

$$\frac{G\,h}{\frac{(\alpha\,c^2\,h^2)(l^2\,c^3)}{e^2\,l^2}} = \frac{G\,h}{\frac{(\alpha\,c^2\,h^2)(l^2\,c^3)}{\exp^2(z)\,l^2}} \quad \text{for } z = 1$$

Series representations

$$\frac{G\,h}{\frac{(\alpha\,c^2\,h^2)(l^2\,c^3)}{e^2\,l^2}} = \frac{G\sum_{k=0}^{\infty}\frac{2^k}{k!}}{c^5\,h\,\alpha}$$

$$\frac{G\,h}{\frac{(\alpha\,c^2\,h^2)(l^2\,c^3)}{e^2\,l^2}} = \frac{G\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^2}{c^5\,h\,\alpha}$$

$$\frac{G\,h}{\frac{(\alpha\,c^2\,h^2)(l^2\,c^3)}{e^2\,l^2}} = \frac{G}{c^5\,h\,\alpha\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\right)^2}$$

$n!$ is the factorial function

We have

$$\frac{G\,e^2}{\alpha\,h\,c^5}$$

(6.67430*10^-11*E^2)/(1/137*1.054571817×10^−34*299792458^5)

for E = X

(6.67430*10^-11*(X)^2)/(1/137*1.054571817×10^−34*299792458^5)

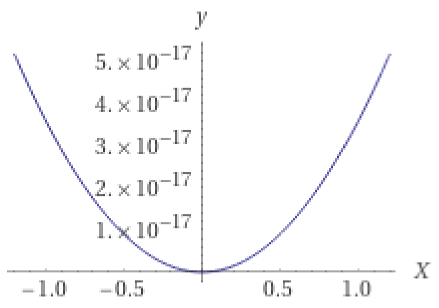
Input interpretation

$$\frac{6.67430\times10^{-11}\,X^2}{\frac{1}{137}\times1.054571817\times10^{-34}\times299\,792\,458^5}$$

Result

$3.58052\times 10^{-17}\ X^2$

Plot **(figure that can be related to an open string)**



(X from −1.2 to 1.2)

Geometric figure

line

Alternate form assuming X is real

$3.58052\times 10^{-17}\ X^2 + 0$

Root

$X = 0$

Polynomial discriminant

$\Delta = 0$

Property as a function

Parity

even

Derivative

$$\frac{d}{dX}\left(3.58052\times 10^{-17}\, X^2\right) = 7.16105\times 10^{-17}\, X$$

Indefinite integral

$$\int 3.58052\times 10^{-17}\, X^2\, dX = 1.19351\times 10^{-17}\, X^3 + \text{constant}$$

Global minimum

$$\min\left\{3.58052\times 10^{-17}\, X^2\right\} = 0 \text{ at } X = 0$$

Definite integral after subtraction of diverging parts

$$\int_0^\infty \left(3.58052\times 10^{-17}\, X^2 - 3.58052\times 10^{-17}\, X^2\right) dX = 0$$

We consider:

E = 0.510998995*299792458^2

from

$$\frac{6.67430\times 10^{-11}\, X^2}{\frac{1}{137}\times 1.054571817\times 10^{-34}\times 299\,792\,458^5}$$

for X = E , we obtain:

(6.67430*10^-11*(0.510998995*299792458^2)^2)/
(1/137*1.054571817×10^−34*299792458^5)

Input interpretation

$$\frac{6.67430\times 10^{-11}\,(0.510998995\times 299\,792\,458^2)^2}{\frac{1}{137}\times 1.054571817\times 10^{-34}\times 299\,792\,458^5}$$

Result

7.552131076414260090719182618970587687631238681741330441873 67… × 10^{16}

7.552131076….*10^{16}

From which, performing the ln and after some calculations, we obtain:

55*1/(ln((6.67430*10^-11*(0.510998995*299792458^2)^2)/
(1/137*1.054571817×10^−34*299792458^5)) -5+(1/π^(1/3))MRB const)

Input interpretation

$$55\times\frac{1}{\log\left(\frac{6.67430\times 10^{-11}\,(0.510998995\times 299\,792\,458^2)^2}{\frac{1}{137}\times 1.054571817\times 10^{-34}\times 299\,792\,458^5}\right)-5+\frac{1}{\sqrt[3]{\pi}}\,C_{\mathrm{MRB}}}$$

log(x) is the natural logarithm
C_{MRB} is the MRB constant

Result

1.618053530854523439970123335266869442291028116025815467329993 5606
…

1.61805353… result that is a very good approximation to the value of the golden ratio 1.618033988749...

Possible closed forms

$\Phi + 1 \approx 1.618033988$

$\sqrt{\frac{3\,\mathcal{K}_{-1}}{2}} \approx 1.618057011 8$

$93\,(\overline{V}) \approx 1.618036249$

$\frac{61}{12\,\pi} \approx 1.618075254$

$\frac{\log^3(3)}{e^2 \log^6(2)} \approx 1.618048844$

$\frac{3\,\sqrt{2}}{L} \approx 1.618057803 5$

$\frac{\log(6)}{-1 + \sqrt{2} + \log(2)} \approx 1.618044960$

Φ is the golden ratio conjugate
$\mathcal{K}_{-1}$ is the Khinchin harmonic mean
$\overline{V}$ is the mean tetrahedron-in-tetrahedron volume
$\log(x)$ is the natural logarithm
L is the lemniscate constant

From:

Classical and Quantum Statistical Physics - Fundamentals and Advanced Topics - *CARLO HEISSENBERG, AUGUSTO SAGNOTTI* - Cambridge University Press, First published 2022

We analyze some equations concerning various aspects of Quantum Mechanics

From

$$\Delta \;=\; 2\hbar\,\mathcal{K}\,e^{-\frac{1}{\hbar}\mathcal{S}_0},$$

and

$$\begin{aligned} K_E(\eta,-\eta;T) &= \sqrt{\frac{m\omega}{\pi\hbar}}\,e^{-\frac{\omega T}{2}}\sum_{n\ \mathrm{odd}}\frac{\left(\mathcal{K}\,T\,e^{-\frac{1}{\hbar}\mathcal{S}_0}\right)^n}{n!} \\ &= \frac{1}{2}\sqrt{\frac{m\omega}{\pi\hbar}}\left[e^{-\frac{T}{\hbar}\left(\frac{\hbar\omega}{2}-\frac{\Delta}{2}\right)} \;-\; e^{-\frac{T}{\hbar}\left(\frac{\hbar\omega}{2}+\frac{\Delta}{2}\right)}\right], \end{aligned}$$

(3.349)

For : $m = 9.109*10^{-31}$ $T = 300$ $\omega = 7.81*10^{20}$ ħ = 6.582119569*10^-16

$\Delta = -3.7$ $S = 5.46296...*10^{-30}$, we obtain:

exp(1/2(6.582119569*10^-16*7.81*10^20-x))-exp(1/2(6.582119569*10^-16*7.81*10^20+x))

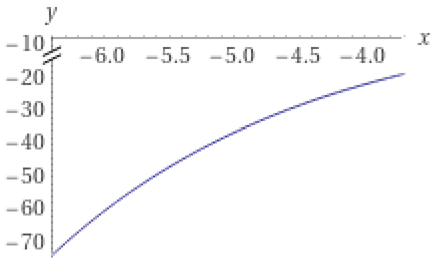
Input interpretation

$$\exp\left(\frac{1}{2}\left(6.582119569\times 10^{-16}\times 7.81\times 10^{20}-x\right)\right)-\exp\left(\frac{1}{2}\left(6.582119569\times 10^{-16}\times 7.81\times 10^{20}+x\right)\right)$$

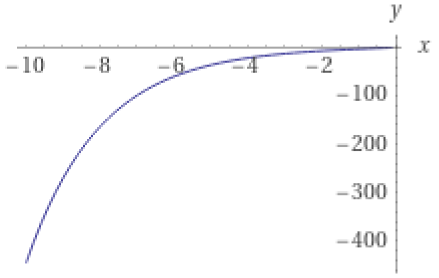
Result

$$e^{1/2\,(514064.-x)}-e^{(x+514064.)/2}$$

Plots **(figures that can be related to the open strings)**



(x from -6.4 to -3.7)



(x from -10 to 0)

Alternate forms

$$-3.0131733835\times 10^{111627}\,e^{-x/2}\left(e^{x}-1\right)$$

$e^{257032. - x/2} - e^{x/2 + 257032.}$

$3.0131733835 \times 10^{111627}\ e^{-x/2} - 3.0131733835 \times 10^{111627}\ e^{x/2}$

Alternate form assuming x is real

$3.0131733835 \times 10^{111627} \sqrt{e^{-x}} - 3.0131733835 \times 10^{111627} \sqrt{e^{x}}$

Real root

$x = 0. \times 10^{-11} \approx 0$

Roots

$x \approx 12.56637061435917\, i\, n\,, \quad n \in \mathbb{Z}$

$x \approx 2\, i\, (6.283185307179586\, n + 3.141592653589793)\,, \quad n \in \mathbb{Z}$

$\mathbb{Z}$ is the set of integers

Integer root

$x = 0$

Properties as a real function

Domain

$\mathbb{R}$ (all real numbers)

Range

$\mathbb{R}$ (all real numbers)

Bijectivity

bijective from its domain to ℝ

Parity

odd

ℝ is the set of real numbers

Series expansion at x=0

$-3.0131733835\times 10^{111627}\,x - 1.2554889098\times 10^{111626}\,x^3 - 1.5693611373\times 10^{111624}\,x^5 + O(x^6)$

(Taylor series)

Indefinite integral

$\int \left(e^{1/2\,(514064.\,-x)} - e^{1/2\,(514064.\,+x)}\right) dx = -6.026346767\times 10^{111627}\,(2.71828^x + 1)\,e^{-0.5\,x}$ + constant

(-300/6.582119569*10^-16) ((e^(1/2 (514064. +3.7)) - e^(1/2 (514064. -3.7))))

Input interpretation

$-\frac{300}{6.582119569\times 10^{-16}}\left(e^{1/2\,(514064.+3.7)} - e^{1/2\,(514064.-3.7)}\right)$

Result

$-1.07300\ldots\times 10^{111646}$

-1.07300...*10^{111646}

1/2*sqrt((9.109*10^-31*7.81*10^20)/(Pi*6.582119569*10^-16)) (-300/6.582119569*10^-16) ((e^(1/2 (514064. +3.7)) - e^(1/2 (514064. -3.7))))

Input interpretation

$$\left(\frac{1}{2}\sqrt{\frac{9.109\times 10^{-31}\times 7.81\times 10^{20}}{\pi\times 6.582119569\times 10^{-16}}}\right)\left(-\frac{300}{6.582119569\times 10^{-16}}\right)\left(e^{1/2(514\,064.+3.7)}-e^{1/2(514\,064.-3.7)}\right)$$

Result

$-3.14683\ldots\times 10^{111648}$

$-3.14683...*10^{111648}$

From:

$$K_E(-\eta,-\eta;T)=\sqrt{\frac{m\omega}{\pi\hbar}}\,e^{-\frac{\omega T}{2}}\sum_{n\text{ even}}\frac{\left(\mathcal{K}\,T\,e^{-\frac{1}{\hbar}\mathcal{S}_0}\right)^n}{n!}$$
$$=\frac{1}{2}\sqrt{\frac{m\omega}{\pi\hbar}}\left[e^{-\frac{T}{\hbar}\left(\frac{\hbar\omega}{2}-\frac{\Delta}{2}\right)}+e^{-\frac{T}{\hbar}\left(\frac{\hbar\omega}{2}+\frac{\Delta}{2}\right)}\right],\qquad(3.350)$$

we obtain:

1/2*sqrt((9.109*10^-31*7.81*10^20)/(Pi*6.582119569*10^-16)) (-300/6.582119569*10^-16) ((e^(1/2 (514064. +3.7)) + e^(1/2 (514064. -3.7))))

Input interpretation

$$\left(\frac{1}{2}\sqrt{\frac{9.109\times 10^{-31}\times 7.81\times 10^{20}}{\pi\times 6.582119569\times 10^{-16}}}\right)\left(-\frac{300}{6.582119569\times 10^{-16}}\right)\left(e^{1/2\,(514\,064.+3.7)}+e^{1/2\,(514\,064.-3.7)}\right)$$

Result

$-3.30638\ldots\times 10^{111648}$

-3.30638...*10^{111648}

Dividing the two above expressions, after some calculations, we obtain:

((-3.30638*10^111648*1/((1/2*sqrt((9.109*10^-31*7.81*10^20)/(Pi*6.582119569*10^-16)) (-300/6.582119569*10^-16) ((e^(1/2 (514064. +3.7)) - e^(1/2 (514064. -3.7))))))))^10+4(MRB const)^(1-1/(4π)+π)

Input interpretation

$$\left(-\left(3.30638\times 10^{111648}\right)\times 1\Big/\left(\left(\frac{1}{2}\sqrt{\frac{9.109\times 10^{-31}\times 7.81\times 10^{20}}{\pi\times 6.582119569\times 10^{-16}}}\right)\left(-\frac{300}{6.582119569\times 10^{-16}}\right)\left(e^{1/2\,(514\,064.+3.7)}-e^{1/2\,(514\,064.-3.7)}\right)\right)\right)^{10}+4\,C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}$$

C_{MRB} is the MRB constant

Result

1.6442927402693669388036515455064835670520692448518418012427260702
...

1.64429274026.... ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape)

Considering:

$m = 9.109*10^{-31}$; T = 300 ; ω = 495.672 ; $ħ = 6.582119569*10^{-16}$

Δ = - 3.7 ; $S = 5.46296...*10^{-30}$; η = 1 and t = 1 , we obtain:

where:

Table 46
Phi^(n/7) scale (octave = 4)

#	Phi^(n/7)	Frequency (Hz)
1	1.0000000	306.342
2	1.0711625	328.142
3	1.1473892	351.494
4	1.2290403	376.508
5	1.3165020	403.300
6	1.4101876	432.000
7	1.5105401	462.742
8	1.6180340	495.672
9	1.7331774	530.945
10	1.8565147	568.729
11	1.9886290	609.201
		612.684

Note. Author's calculation with data From Lange, Nardelli, & Bini (2013, p.3). ©

Table of Frequency System based on Phi

From:

$$\xi_1 = \sqrt{\frac{m}{\mathcal{S}_0}} \frac{\eta \omega}{2} \frac{1}{\cosh^2\left(\frac{\omega t}{2}\right)},$$

(3.355)

we obtain:

sqrt((9.109*10^-31)/(5.46296*10^-30))*1/2*(495.672)*1/(cosh^2(1/2(495.672)))

Input interpretation

$$\sqrt{\frac{9.109\times 10^{-31}}{5.46296\times 10^{-30}}}\times \frac{1}{2}\times 495.672\times \frac{1}{\cosh^2\left(\frac{1}{2}\times 495.672\right)}$$

cosh(x) is the hyperbolic cosine function

Result

$2.18591\ldots \times 10^{-213}$

2.18591...*10^{-213}

We have that

$$\alpha = \sqrt{\frac{m}{\mathcal{S}_0}}\, 2\,\eta\,\omega.$$

sqrt((9.109*10^-31)/(5.46296*10^-30)) * 2 * 495.672

Input interpretation

$$\sqrt{\frac{9.109\times 10^{-31}}{5.46296\times 10^{-30}}}\times 2\times 495.672$$

Result

404.805…

404.805….

From

$$\xi_2 \sim_{t\to\pm\infty} \pm\,\alpha\, e^{\omega|t|},$$

we obtain:

((sqrt((9.109*10^-31)/(5.46296*10^-30)) * 2 * 495.672))*e^(495.672)

Input interpretation

$$\left(\sqrt{\frac{9.109\times 10^{-31}}{5.46296\times 10^{-30}}}\times 2\times 495.672\right) e^{495.672}$$

Result

$7.49653\ldots \times 10^{217}$

$7.49653...*10^{217}$

Multiplying the two above expressions, we obtain:

[((((sqrt((9.109*10^-31)/(5.46296*10^-30)) * 2 * 495.672))*e^(495.672)))] * [(((sqrt((9.109*10^-31)/(5.46296*10^-30))*1/2*(495.672)*1/(cosh^2(1/2(495.672))))))]

Input interpretation

$$\left(\left(\sqrt{\frac{9.109\times10^{-31}}{5.46296\times10^{-30}}}\times2\times495.672\right)e^{495.672}\right)\left(\sqrt{\frac{9.109\times10^{-31}}{5.46296\times10^{-30}}}\times\frac{1}{2}\times495.672\times\frac{1}{\cosh^2\left(\frac{1}{2}\times495.672\right)}\right)$$

cosh(x) is the hyperbolic cosine function

Result

$1.63867\ldots\times10^5$

$1.63867\ldots*10^5$

From which:

((([((((sqrt((9.109*10^-31)/(5.46296*10^-30)) * 2 * 495.672))*e^(495.672)))] * [(((sqrt((9.109*10^-31)/(5.46296*10^-30))*1/2*(495.672)*1/(cosh^2(1/2(495.672))))))]))^1/24-4(MRB const)^(1-1/(4π)+π)

Input interpretation

$$\left(\left(\left(\sqrt{\frac{9.109\times10^{-31}}{5.46296\times10^{-30}}}\times2\times495.672\right)e^{495.672}\right)\left(\sqrt{\frac{9.109\times10^{-31}}{5.46296\times10^{-30}}}\times\frac{1}{2}\times495.672\times\frac{1}{\cosh^2\left(\frac{1}{2}\times495.672\right)}\right)\right)^{\wedge}(1/24)-4\,C_{\mathrm{MRB}}{}^{1-1/(4\pi)+\pi}$$

cosh(x) is the hyperbolic cosine function

C_{MRB} is the MRB constant

Result

1.644697998890043396451927931648632882572023633790990683836916 4055

...

1.64469799889…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape)

Now, we have

$$\mathcal{K} \;=\; \sqrt{\frac{\mathcal{S}_0}{2\pi\hbar}}\,\sqrt{2\,\omega}\;\alpha \;=\; 2\sqrt{\frac{\eta^2\,\omega^3\,m}{\pi\,\hbar}}. \qquad (3.372)$$

$$\Delta \;=\; \frac{\hbar\,\omega}{\pi}\;e^{-\frac{1}{\hbar}\mathcal{S}_0}\,\sqrt{\frac{2\,\pi\,\omega^3\,m^2}{\hbar\,\lambda}}, \qquad (3.374)$$

From (3.372), we obtain:

For : m = $9.109*10^{-31}$; T = 300 ; ω = 495.672 ; ħ = 6.582119569*10^-16

Δ = - 3.7 ; S = $5.46296...*10^{-30}$; η = 1

$$\mathcal{K} \;=\; \sqrt{\frac{\mathcal{S}_0}{2\pi\hbar}}\,\sqrt{2\,\omega}\;\alpha \;=\; 2\sqrt{\frac{\eta^2\,\omega^3\,m}{\pi\,\hbar}}. \qquad (3.372)$$

2*sqrt(((((495.672)^3*(9.109*10^-31))/(Pi*6.582119569*10^-16))))

Input interpretation

$$2\sqrt{\frac{495.672^3 \times 9.109 \times 10^{-31}}{\pi \times 6.582119569 \times 10^{-16}}}$$

Result

0.000463233…

0.000463233...

We have:

$$\lambda \simeq 2 \mathrm{w} e^{-\omega T} = 4 \omega \alpha^2 e^{-\omega T},$$

4*495.672*((((sqrt((9.109*10^-31)/(5.46296*10^-30)) * 2 * 495.672))))^2*exp(-495.672*300)

Input interpretation

$$4 \times 495.672 \left(\sqrt{\frac{9.109 \times 10^{-31}}{5.46296 \times 10^{-30}}} \times 2 \times 495.672\right)^2 \exp(-495.672 \times 300)$$

Result

$1.68817\ldots \times 10^{-64572}$

$1.68817\ldots.*10^{-64572}$

From:

$$\Delta = \frac{\hbar\,\omega}{\pi}\, e^{-\frac{1}{\hbar}S_0} \sqrt{\frac{2\,\pi\,\omega^3\, m^2}{\hbar\,\lambda}}, \qquad (3.374)$$

1/Pi*(6.582119569*10^-16*495.672)*e^(-5.46296*10^-30/6.582119569*10^-16)* sqrt((((2*Pi*495.672^3*(9.109*10^-31)^2)/((6.582119569*10^-16 *(1.68817*10^-64572))))))

Input interpretation

$$\frac{1}{\pi}\left(6.582119569\times 10^{-16}\times 495.672\right) \exp\left(-\frac{5.46296\times 10^{-30}}{6.582119569\times 10^{-16}}\right)\sqrt{\frac{2\,\pi\times 495.672^3\,(9.109\times 10^{-31})^2}{6.582119569\times 10^{-16}\times \frac{1.68817}{10^{64\,572}}}}$$

Result

$\tilde{\infty}$

$\tilde{\infty}$ is complex infinity

Decimal approximation

$7.85003\ldots \times 10^{32254}$

$7.85003...*10^{32254}$

From the two previous expressions,

$$4\times 495.672\left(\sqrt{\frac{9.109\times 10^{-31}}{5.46296\times 10^{-30}}}\times 2\times 495.672\right)^2 \exp(-495.672\times 300)$$

$1.68817\ldots \times 10^{-64572}$

and

$$\frac{1}{\pi}\left(6.582119569\times 10^{-16}\times 495.672\right)$$
$$\exp\left(-\frac{5.46296\times 10^{-30}}{6.582119569\times 10^{-16}}\right)\sqrt{\frac{2\pi\times 495.672^3\left(9.109\times 10^{-31}\right)^2}{6.582119569\times 10^{-16}\times \frac{1.68817}{10^{64\,572}}}}$$

$7.85003\ldots \times 10^{32254}$

after some calculations, we obtain:

(7-9/e)*11161√(1/((1.68817e-64572))*1/((((((1/Pi*(6.582119569e-16*495.672)*e^(-5.46296e-30/6.582119569e-16)*
√((((2*Pi*495.672^3*(9.109e-31)^2)/((6.582119569e-16 *(1.68817e-64572))))))))))^2)

Input interpretation

$$\left(7-\frac{9}{e}\right)\times 11\,161\sqrt{\left(\frac{1}{\frac{1.68817}{10^{64\,572}}}\times 1\Bigg/\left(\frac{1}{\pi}\left(6.582119569\times 10^{-16}\times 495.672\right)\exp\left(-\frac{5.46296\times 10^{-30}}{6.582119569\times 10^{-16}}\right)\sqrt{\frac{2\pi\times 495.672^3\left(9.109\times 10^{-31}\right)^2}{6.582119569\times 10^{-16}\times \frac{1.68817}{10^{64\,572}}}}\right)^2\right)}$$

Result

4.03685×10^{35}

$4.03685*10^{35}$ ~ $4.036978*10^{35}$ (Planck mass flow)

We observe that 11161 is given by the following Ramanujan taxicab number:

$$11161^3 + 11468^3 = 14258^3 + 1$$

11161 = (14258^3-11468^3+1)^(1/3)

$$\sqrt[3]{14258^3 - 11468^3 + 1}$$

11161

Furthermore, from

$$\frac{1}{\pi}\left(6.582119569\times 10^{-16}\times 495.672\right)$$
$$\exp\left(-\frac{5.46296\times 10^{-30}}{6.582119569\times 10^{-16}}\right)\sqrt{\frac{2\,\pi\times 495.672^3\left(9.109\times 10^{-31}\right)^2}{6.582119569\times 10^{-16}\times \frac{1.68817}{10^{64\,572}}}}$$

$7.85003\ldots \times 10^{32254}$

we obtain:

1/43ln(7.85003 × 10^32254)+2-MRB const

Input interpretation

$$\frac{1}{43}\log\left(7.85003\times 10^{32254}\right) + 2 - C_{\text{MRB}}$$

$\log(x)$ is the natural logarithm
C_{MRB} is the MRB constant

Result
1729.0130731…

1729.0130731….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

and again:

(1/43ln(7.85003 × 10^32254)+2-MRB const)^1/15+(MRB const)^(1-1/(4π)+π)

Input interpretation

$$\sqrt[15]{\frac{1}{43}\log(7.85003\times10^{32\,254})+2-C_{\mathrm{MRB}}}+C_{\mathrm{MRB}}^{1-1/(4\pi)+\pi}$$

$\log(x)$ is the natural logarithm
C_{MRB} is the MRB constant

Result

1.64493885273…

1.64493885273…. ≈ $\zeta(2) = \pi^2/6 = 1.644934$ (trace of the instanton shape)

(1/27((1/43ln(7.85003 × 10^32254)+2-MRB const)-1))^2

Input interpretation

$$\left(\frac{1}{27}\left(\left(\frac{1}{43}\log(7.85003\times10^{32\,254})+2-C_{\mathrm{MRB}}\right)-1\right)\right)^2$$

$\log(x)$ is the natural logarithm
C_{MRB} is the MRB constant

Result

4096.0619763…

4096.0619763…. ≈ $4096 = 64^2$

Now, we have:

$$E(\theta) = \frac{\hbar\,\omega}{2} - \frac{\hbar\,\omega}{\pi}\,e^{-\frac{1}{\hbar}S_0}\sqrt{\frac{2\,\pi\,\omega^3\,m^2}{\hbar\,\lambda}}\,\cos\theta. \qquad (3.379)$$

For : $m = 9.109*10^{-31}$; T = 300 ; ω = 495.672 ; ħ = 6.582119569*10^-16

Δ = - 3.7 ; $S = 5.46296...*10^{-30}$; η = 1 ; $\lambda = 1.68817\ldots \times 10^{-64572}$

(1/2*6.582119569e-16*495.672)-(1/Pi* 6.582119569e-16*495.672)*exp(-5.46296e-30 / 6.582119569e-16) * sqrt(((1/(6.582119569e-16*1.68817e-64572)*(2Pi*(495.672)^3*(9.109e-31)^2)))) * cos(Pi/6)

Input interpretation

$$\frac{1}{2}\times 6.582119569\times 10^{-16}\times 495.672 - \left(\frac{1}{\pi}\times 6.582119569\times 10^{-16}\times 495.672\right)\exp\left(-\frac{5.46296\times 10^{-30}}{6.582119569\times 10^{-16}}\right)\sqrt{\frac{1}{6.582119569\times 10^{-16}\times \frac{1.68817}{10^{64572}}}\left(2\,\pi\times 495.672^3\,(9.109\times 10^{-31})^2\right)}\,\cos\left(\frac{\pi}{6}\right)$$

Result

$\tilde{\infty}$

$\tilde{\infty}$ is complex infinity

Decimal approximation

$-6.79833\ldots \times 10^{32254}$

$-6.79833...*10^{32254}$

Dividing the previous expression

$$\frac{1}{\pi}\left(6.582119569\times 10^{-16}\times 495.672\right)\exp\left(-\frac{5.46296\times 10^{-30}}{6.582119569\times 10^{-16}}\right)\sqrt{\frac{2\,\pi\times 495.672^3\left(9.109\times 10^{-31}\right)^2}{6.582119569\times 10^{-16}\times \frac{1.68817}{10^{64\,572}}}}$$

by

$$\frac{1}{2}\times 6.582119569\times 10^{-16}\times 495.672-\left(\frac{1}{\pi}\times 6.582119569\times 10^{-16}\times 495.672\right)\exp\left(-\frac{5.46296\times 10^{-30}}{6.582119569\times 10^{-16}}\right)\sqrt{\frac{1}{6.582119569\times 10^{-16}\times \frac{1.68817}{10^{64\,572}}}\left(2\,\pi\times 495.672^3\left(9.109\times 10^{-31}\right)^2\right)}\cos\left(\frac{\pi}{6}\right)$$

that is equal to

$$-6.79833\ldots\times 10^{32254}$$

we obtain:

-1/(-6.79833*10^32254)*(1/Pi*(6.582119569*10^-16*495.672)*e^(-5.46296*10^-30/6.582119569*10^-16)*
sqrt((((2*Pi*495.672^3*(9.109*10^-31)^2)/((6.582119569*10^-16 *(1.68817*10^-64572)))))))

Input interpretation

$$\left(-\left(\frac{1}{\pi}\left(6.582119569\times 10^{-16}\times 495.672\right)\exp\left(-\frac{5.46296\times 10^{-30}}{6.582119569\times 10^{-16}}\right)\sqrt{\frac{2\,\pi\times 495.672^3\left(9.109\times 10^{-31}\right)^2}{6.582119569\times 10^{-16}\times \frac{1.68817}{10^{64\,572}}}}\right)\right)\Big/\left(-\left(6.79833\times 10^{32\,254}\right)\right)$$

Result

$\tilde{\infty}$

$\tilde{\infty}$ is complex infinity

Decimal approximation

1.154700311467067121648881238309917005599559160069666097566150198 2

...

1.154700311467….

From which, after some calculations, we obtain:

(-1/(-6.79833e+32254)*(1/Pi*(6.582119569e-16*495.672)*e^(-5.46296e-30/6.582119569e-16)* sqrt((((2*Pi*495.672^3*(9.109e-31)^2)/((6.582119569e-16 (1.68817e-64572)))))))^(7/2)-9(MRB const)^(1-1/(4π)+π)

Input interpretation

$$\left(\left(-\left(\frac{1}{\pi}\left(6.582119569\times 10^{-16}\times 495.672\right)\exp\left(-\frac{5.46296\times 10^{-30}}{6.582119569\times 10^{-16}}\right)\sqrt{\frac{2\pi\times 495.672^3\left(9.109\times 10^{-31}\right)^2}{6.582119569\times 10^{-16}\times\frac{1.68817}{10^{64572}}}}\right)\right)\Big/\left(-\left(6.79833\times 10^{32254}\right)\right)\right)^{7/2}-9\,C_{MRB}^{\,1-1/(4\pi)+\pi}$$

C_{MRB} is the MRB constant

Result

$\tilde{\infty}$

$\tilde{\infty}$ is complex infinity

Decimal approximation

1.644302342082840517706878984035801037863145326441697699249251880 3

...

1.644302342…. ≈ $\zeta(2) = \pi^2/6 = 1.644934$ (trace of the instanton shape)

From:

Schulman, Lawrence S - **Techniques and applications of path integration** -

We have the following equation:

(from: **CRITICAL DROPLETS, ALIAS INSTANTONS, AND METASTABILITY**)

$$\frac{1}{\sqrt{\pi}}\int da_1 = \frac{1}{\sqrt{\pi}}\int dz \left\| \frac{d\varphi_z}{dz} \right\| = \frac{1}{\sqrt{\pi}}\left[\int \left(\frac{d\varphi_z}{dz}\right)^2 dx\right]^{1/2}\int dz$$
$$= \frac{(2\varepsilon)^{3/4}}{(-3\alpha)^{1/2}}\frac{L}{\sqrt{\pi}} \qquad (29.15)$$

(2ε)^(3/4) / (-3α)^(1/2) * L/(sqrtPi)

Input

$$\frac{(2\,\varepsilon)^{3/4}}{\sqrt{-3\,\alpha}} \times \frac{L}{\sqrt{\pi}}$$

Exact result

$$\frac{2^{3/4}\,\varepsilon^{3/4}\,L}{\sqrt{3\,\pi}\,\sqrt{-\alpha}}$$

Alternate form assuming L, α, and ε are positive

$$-\frac{i\, 2^{3/4}\, \varepsilon^{3/4}\, L}{\sqrt{3\,\pi}\, \sqrt{\alpha}}$$

Real roots

$L < 0\,, \quad \alpha < 0\,, \quad \varepsilon = 0$

$L = 0\,, \quad \alpha < 0\,, \quad \varepsilon \geq 0$

$L > 0\,, \quad \alpha < 0\,, \quad \varepsilon = 0$

Root for the variable ε

$\varepsilon = 0$

Derivative

$$\frac{\partial}{\partial L}\left(\frac{(2\,\varepsilon)^{3/4}\, L}{\sqrt{-3\,\alpha}\, \sqrt{\pi}}\right) = \frac{2^{3/4}\, \varepsilon^{3/4}}{\sqrt{3\,\pi}\, \sqrt{-\alpha}}$$

Indefinite integral

$$\int \frac{2^{3/4}\, L\, \varepsilon^{3/4}}{\sqrt{3\,\pi}\, \sqrt{-\alpha}}\, dL = \frac{\varepsilon^{3/4}\, L^2}{\sqrt[4]{2}\, \sqrt{3\,\pi}\, \sqrt{-\alpha}} + \text{constant}$$

Series representations

$$\frac{L(2\varepsilon)^{3/4}}{\sqrt{\pi}\ \sqrt{-3\alpha}} = -\frac{2^{3/4} L \sqrt{-\alpha}\ \varepsilon^{3/4}}{\sqrt{3}\ \alpha \sqrt{-1+\pi}\ \sum_{k=0}^{\infty} (-1+\pi)^{-k} \binom{\frac{1}{2}}{k}}$$

$$\frac{L(2\varepsilon)^{3/4}}{\sqrt{\pi}\ \sqrt{-3\alpha}} = -\frac{2^{3/4} L \sqrt{-\alpha}\ \varepsilon^{3/4}}{\sqrt{3}\ \alpha \sqrt{-1+\pi}\ \sum_{k=0}^{\infty} \frac{(-1)^k (-1+\pi)^{-k} \left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{L(2\varepsilon)^{3/4}}{\sqrt{\pi}\ \sqrt{-3\alpha}} = -\frac{2\times 2^{3/4} L \sqrt{-\alpha}\ \varepsilon^{3/4} \sqrt{\pi}}{\sqrt{3}\ \alpha \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} (-1+\pi)^{-s}\, \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

From the alternate form assuming L, α, and ε are positive

$$-\frac{i\, 2^{3/4} \varepsilon^{3/4} L}{\sqrt{3\pi}\ \sqrt{\alpha}}$$

for L = 4, α = 8 and ε = 16 , we obtain:

-(i 2^(3/4) 4 16^(3/4))/(sqrt(3 π) sqrt(8))

Input

$$-\frac{i \times 2^{3/4} \times 4 \times 16^{3/4}}{\sqrt{3\pi}\ \sqrt{8}}$$

i is the imaginary unit

Exact result

$$-\frac{16\, i \sqrt[4]{2}}{\sqrt{3\pi}}$$

Decimal approximation

$-6.197862224673643520629297058255197404422150677697486283971455 50\ldots\, i$

-6.19786222467…. *i*

Property

$-\frac{16\, i \sqrt[4]{2}}{\sqrt{3\pi}}$ is a transcendental number

Alternate complex forms

$$\frac{16 \sqrt[4]{2} \left(\cos\left(-\frac{\pi}{2}\right) + i \sin\left(-\frac{\pi}{2}\right)\right)}{\sqrt{3\pi}}$$

$$\frac{16 \sqrt[4]{2}\, e^{-(i\pi)/2}}{\sqrt{3\pi}}$$

Polar coordinates

$r = \frac{16 \sqrt[4]{2}}{\sqrt{3\pi}}$ (radius), $\theta = -\frac{\pi}{2}$ (angle)

Series representations

$$-\frac{i\,2^{3/4}\times 4\times 16^{3/4}}{\sqrt{3\pi}\,\sqrt{8}} = -\frac{32\times 2^{3/4}\,i}{\sqrt{7}\,\sqrt{-1+3\pi}\left(\sum_{k=0}^{\infty} 7^{-k}\binom{\frac{1}{2}}{k}\right)\sum_{k=0}^{\infty}(-1+3\pi)^{-k}\binom{\frac{1}{2}}{k}}$$

$$-\frac{i\,2^{3/4}\times 4\times 16^{3/4}}{\sqrt{3\pi}\,\sqrt{8}} = -\frac{32\times 2^{3/4}\,i}{\sqrt{7}\,\sqrt{-1+3\pi}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{7}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k(-1+3\pi)^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

$$-\frac{i\,2^{3/4}\times 4\times 16^{3/4}}{\sqrt{3\pi}\,\sqrt{8}} = -\frac{32\times 2^{3/4}\,i}{\sqrt{z_0}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(8-z_0)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3\pi-z_0)^k z_0^{-k}}{k!}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

From the exact result

$$-\frac{16\,i\,\sqrt[4]{2}}{\sqrt{3\pi}}$$

after some calculations, we obtain:

(-(16 i 2^(1/4))/sqrt(3 π))^4+233+21-3MRB const

Input

$$\left(-\frac{16\, i \sqrt[4]{2}}{\sqrt{3\,\pi}}\right)^4 + 233 + 21 - 3\, C_{\text{MRB}}$$

i is the imaginary unit
C_{MRB} is the MRB constant

Exact result

$$-3\, C_{\text{MRB}} + 254 + \frac{131\,072}{9\,\pi^2}$$

Decimal approximation

1729.0331080024467296050510019562173010289559922531551003405581002

…

1729.033108….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$-\frac{27\,\pi^2\, C_{\text{MRB}} - 131\,072 - 2286\,\pi^2}{9\,\pi^2}$$

$$-\frac{27\,\pi^2\, C_{\text{MRB}} - 2\left(65\,536 + 1143\,\pi^2\right)}{9\,\pi^2}$$

((-(16 i 2^(1/4))/sqrt(3 π))^4+233+21-3MRB const)^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\sqrt[15]{\left(-\frac{16\, i \sqrt[4]{2}}{\sqrt{3\,\pi}}\right)^4 + 233 + 21 - 3\, C_{\text{MRB}}} + C_{\text{MRB}}^{1-1/(4\,\pi)+\pi}$$

i is the imaginary unit
C_{MRB} is the MRB constant

Exact result

$$C_{\text{MRB}}^{1-1/(4\,\pi)+\pi} + \sqrt[15]{-3\, C_{\text{MRB}} + 254 + \frac{131072}{9\,\pi^2}}$$

Decimal approximation

1.6449401225668244275775435722193236292830603854580912317706104089
…

1.644940122566…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape)

Alternate forms

$$C_{\text{MRB}}^{1-1/(4\,\pi)+\pi} + \frac{\sqrt[15]{-27\,\pi^2\, C_{\text{MRB}} + 131072 + 2286\,\pi^2}}{(3\,\pi)^{2/15}}$$

$$C_{\text{MRB}}^{1-1/(4\,\pi)+\pi} + \frac{\sqrt[15]{2\left(65536 + 1143\,\pi^2\right) - 27\,\pi^2\, C_{\text{MRB}}}}{(3\,\pi)^{2/15}}$$

$$\frac{1}{3\,\pi^{2/15}} C_{\text{MRB}}^{-1/(4\,\pi)} \left(3\,\pi^{2/15}\, C_{\text{MRB}}^{1+\pi} + 3^{13/15} \sqrt[4\,\pi]{C_{\text{MRB}}} \sqrt[15]{-27\,\pi^2\, C_{\text{MRB}} + 131072 + 2286\,\pi^2}\right)$$

(1/27(((-(16 i 2^(1/4))/sqrt(3 π))^4+233+21-3MRB const)-1))^2-MRB const

Input

$$\left(\frac{1}{27}\left(\left(\left(-\frac{16\,i\,\sqrt[4]{2}}{\sqrt{3\,\pi}}\right)^4 + 233 + 21 - 3\,C_{\text{MRB}}\right) - 1\right)\right)^2 - C_{\text{MRB}}$$

i is the imaginary unit
C_{MRB} is the MRB constant

Exact result

$$\frac{1}{729}\left(-3\,C_{\text{MRB}} + 253 + \frac{131\,072}{9\,\pi^2}\right)^2 - C_{\text{MRB}}$$

Decimal approximation

4095.969098317202848555622232825332226458413541001901077761921160 7
…

4095.9690983172…. ≈ 4096 = 64^2

Alternate forms

$$\frac{1}{729}\left(-2247\,C_{\text{MRB}} + 9\,C_{\text{MRB}}^2 + 64\,009\right) - \frac{262\,144\,(3\,C_{\text{MRB}} - 253)}{6561\,\pi^2} + \frac{17\,179\,869\,184}{59\,049\,\pi^4}$$

$$\frac{1}{59\,049\,\pi^4}\left(-7\,077\,888\,\pi^2\,C_{\text{MRB}} - 182\,007\,\pi^4\,C_{\text{MRB}} + 729\,\pi^4\,C_{\text{MRB}}^2 + 17\,179\,869\,184 + 596\,901\,888\,\pi^2 + 5\,184\,729\,\pi^4\right)$$

$$\frac{729\,\pi^4\,C_{\text{MRB}}^2 - 27\,\pi^2\left(262\,144 + 6741\,\pi^2\right)C_{\text{MRB}} + \left(131\,072 + 2277\,\pi^2\right)^2}{59\,049\,\pi^4}$$

Expanded form

$$-\frac{749\,C_{\text{MRB}}}{243}+\frac{C_{\text{MRB}}{}^2}{81}-\frac{262144\,C_{\text{MRB}}}{2187\,\pi^2}+\frac{64009}{729}+\frac{17179869184}{59049\,\pi^4}+\frac{66322432}{6561\,\pi^2}$$

We have:

Our main interest throughout has been the imaginary part of ψ which, because Z_0 is real, has the form

$$\operatorname{Im}\psi(\alpha)=-\frac{1}{L}\lim\ \operatorname{Im}\left(\frac{Z_1}{Z_0}\right)$$
$$=\pm\frac{1}{2}\sqrt{\frac{1}{3\varepsilon\pi}}\left[\frac{2(2\varepsilon)^{3/4}}{(-3\alpha)^{1/2}}\right]\exp\left[\frac{-(2\varepsilon)^{3/2}}{(-3\alpha)}\right]\frac{\Pi'\lambda_j^{-1/2}}{\Pi\lambda_j^{(0)-1/2}} \tag{29.24}$$

Bringing our expression for the ratio of the product of continuum states from the appendix we have

$$\operatorname{Im}\psi(\alpha)=\pm\frac{2^{7/4}\varepsilon^{5/4}}{(-\pi\alpha)^{1/2}}\exp\left[-\frac{(2\varepsilon)^{3/2}}{(-3\alpha)}\right] \tag{29.25}$$

From the right-hand side:

$$\mathrm{Im}\,\psi(\alpha) = \pm\frac{2^{7/4}\varepsilon^{5/4}}{(-\pi\alpha)^{1/2}}\exp\left[-\frac{(2\varepsilon)^{3/2}}{(-3\alpha)}\right]$$

we obtain:

(1/(-Pi*α)^(1/2))*(2)^(7/4)*ε^(5/4)*exp((-(2ε)^(3/2)/(-3α))

Input

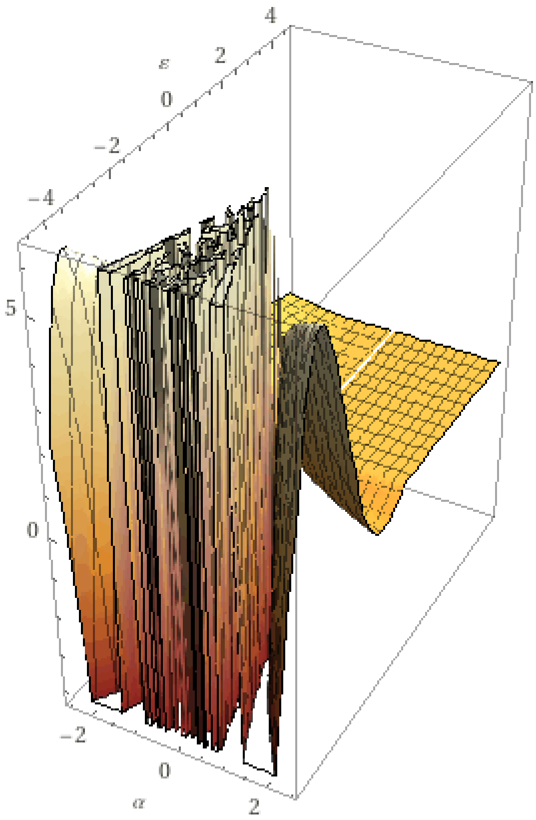
$$\frac{1}{\sqrt{-\pi\,\alpha}}\times 2^{7/4}\,\varepsilon^{5/4}\exp\left(\frac{-(2\,\varepsilon)^{3/2}}{-3\,\alpha}\right)$$

Exact result

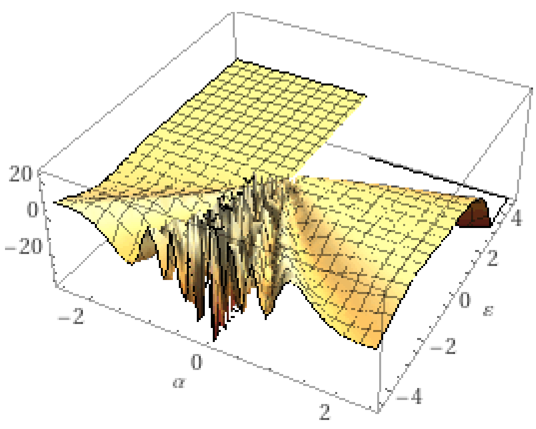
$$\frac{2\times 2^{3/4}\,\varepsilon^{5/4}\,e^{\left(2\sqrt{2}\,\varepsilon^{3/2}\right)/(3\,\alpha)}}{\sqrt{\pi}\,\sqrt{-\alpha}}$$

3D plots

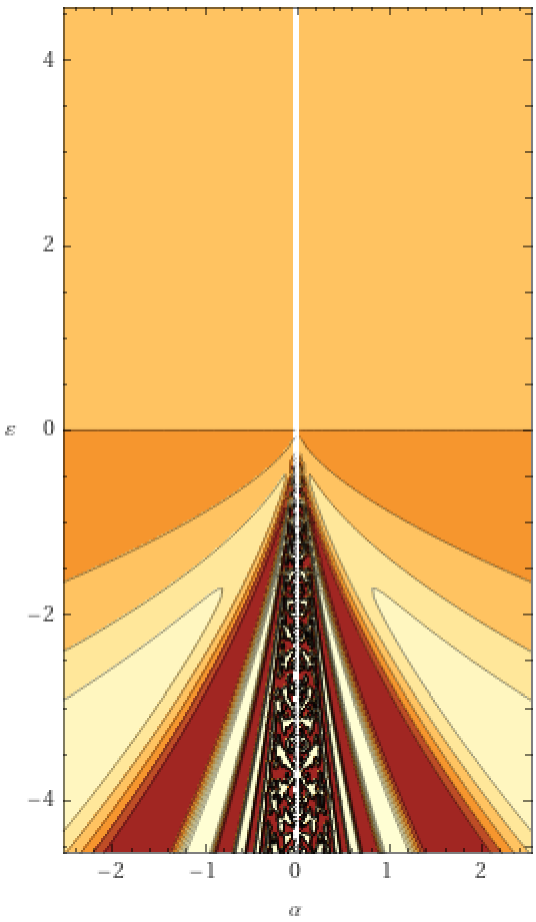
Real part **(figures that can be related to the D-branes/Instantons)**



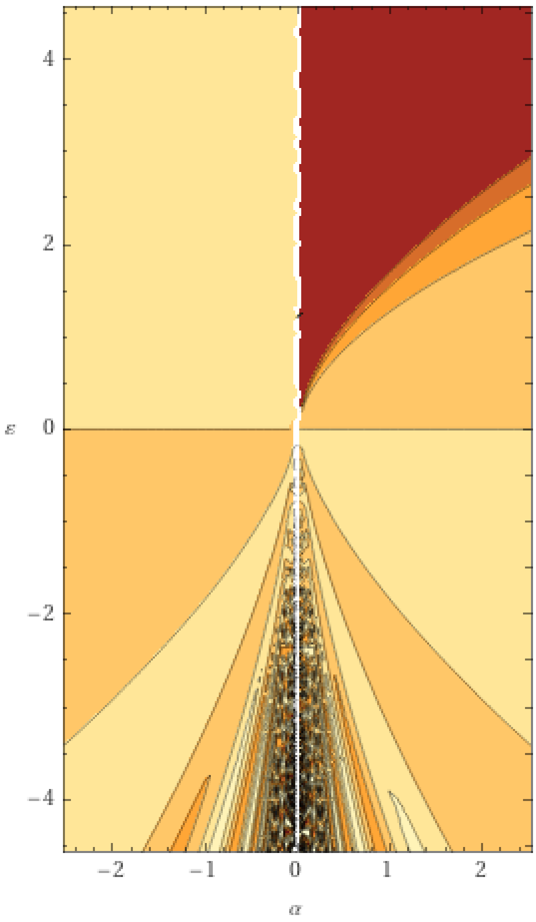
Imaginary part



Contour plots
Real part



Imaginary part



Alternate form assuming α and ε are positive

$$-\frac{2\,i\,2^{3/4}\,\varepsilon^{5/4}\,e^{\left(2\sqrt{2}\ \varepsilon^{3/2}\right)/(3\alpha)}}{\sqrt{\pi}\ \sqrt{\alpha}}$$

Roots

$\alpha = 0$

$\varepsilon = 0$

Series expansion at α=0

$$\begin{cases} e^{(2\sqrt{2}\,\varepsilon^{3/2})/(3\alpha)}\left(\frac{2\times 2^{3/4}\,\varepsilon^{5/4}}{\sqrt{\pi}\,\sqrt{-\alpha}} + O\left(\alpha^{79}\right)\right) & \text{Im}(\alpha) \le 0 \\ e^{(2\sqrt{2}\,\varepsilon^{3/2})/(3\alpha)}\left(\frac{2\times 2^{3/4}\,\varepsilon^{5/4}\left(\frac{1}{\sqrt{-\alpha}}\right)^*}{\sqrt{\pi}} + O\left(\alpha^{79}\right)\right) & \text{(otherwise)} \end{cases}$$

Im(z) is the imaginary part of z

z^* is the complex conjugate of z

Series expansion at α=∞

$$\frac{2\times 2^{3/4}\sqrt{\alpha}\,\sqrt{\frac{1}{\alpha}}\,\varepsilon^{5/4}}{\sqrt{\pi}\,\sqrt{-\alpha}} + \frac{8\sqrt[4]{2}\,\sqrt{\alpha}\,\left(\frac{1}{\alpha}\right)^{3/2}\varepsilon^{11/4}}{3\sqrt{\pi}\,\sqrt{-\alpha}} + \frac{8\times 2^{3/4}\sqrt{\alpha}\,\left(\frac{1}{\alpha}\right)^{5/2}\varepsilon^{17/4}}{9\sqrt{\pi}\,\sqrt{-\alpha}} + \frac{32\sqrt[4]{2}\,\sqrt{\alpha}\,\left(\frac{1}{\alpha}\right)^{7/2}\varepsilon^{23/4}}{81\sqrt{\pi}\,\sqrt{-\alpha}} + \frac{16\times 2^{3/4}\sqrt{\alpha}\,\left(\frac{1}{\alpha}\right)^{9/2}\varepsilon^{29/4}}{243\sqrt{\pi}\,\sqrt{-\alpha}} + O\left(\left(\frac{1}{\alpha}\right)^5\right)$$

(generalized Puiseux series)

Derivative

$$\frac{\partial}{\partial\alpha}\left(\frac{2^{7/4}\,\varepsilon^{5/4}\exp\left(\frac{-(2\varepsilon)^{3/2}}{-3\alpha}\right)}{\sqrt{-\pi\,\alpha}}\right) = -\frac{\sqrt[4]{2}\,\varepsilon^{5/4}\,e^{(2\sqrt{2}\,\varepsilon^{3/2})/(3\alpha)}\left(3\sqrt{2}\,\alpha + 8\,\varepsilon^{3/2}\right)}{3\sqrt{\pi}\,(-\alpha)^{5/2}}$$

Indefinite integral

$$\int \frac{2\times 2^{3/4}\,e^{(2\sqrt{2}\,\varepsilon^{3/2})/(3\alpha)}\,\varepsilon^{5/4}}{\sqrt{\pi}\,\sqrt{-\alpha}}\,d\alpha = -4\sqrt{\frac{2}{3\pi}}\,\sqrt{-\alpha}\,\varepsilon^{5/4}\sqrt{-\frac{\varepsilon^{3/2}}{\alpha}}\;\Gamma\left(-\frac{1}{2}, -\frac{2\sqrt{2}\,\varepsilon^{3/2}}{3\alpha}\right) + \text{constant}$$

$\Gamma(a, x)$ is the incomplete gamma function

Limit

$$\lim_{\alpha\to\pm\infty} \frac{2\times 2^{3/4}\, e^{\left(2\sqrt{2}\ \varepsilon^{3/2}\right)/(3\,\alpha)}\ \varepsilon^{5/4}}{\sqrt{\pi}\ \sqrt{-\alpha}} = 0$$

From

$$\frac{2\times 2^{3/4}\, \varepsilon^{5/4}\, e^{\left(2\sqrt{2}\ \varepsilon^{3/2}\right)/(3\,\alpha)}}{\sqrt{\pi}\ \sqrt{-\alpha}}$$

for ε = -4 and α = -2 , we obtain:

(2 2^(3/4) e^((2 sqrt(2) (-4)^(3/2))/(3 -2)) (-4)^(5/4))/(sqrt(π) sqrt(2))

Input

$$\frac{2\times 2^{3/4}\, e^{\left(2\sqrt{2}\ (-4)^{3/2}\right)/(3-2)}\ (-4)^{5/4}}{\sqrt{\pi}\ \sqrt{2}}$$

Exact result

$$-\frac{8\sqrt[4]{-1}\ 2^{3/4}\, e^{-16\,i\sqrt{2}}}{\sqrt{\pi}}$$

Decimal approximation

7.506547812785244722804783274829031749770751411880234423982135857…
+
1.127822755766103078679078537022844541792814424120995524201372348…
i

Alternate complex forms

$$\frac{-8\sqrt[4]{2}\sin(16\sqrt{2})-8\sqrt[4]{2}\cos(16\sqrt{2})}{\sqrt{\pi}}+\frac{i\left(8\sqrt[4]{2}\sin(16\sqrt{2})-8\sqrt[4]{2}\cos(16\sqrt{2})\right)}{\sqrt{\pi}}$$

$$\frac{8\times 2^{3/4}\left(\cos\left(\tan^{-1}\left(\frac{\frac{\sin(16\sqrt{2})}{\sqrt{2}}-\frac{\cos(16\sqrt{2})}{\sqrt{2}}}{-\frac{\sin(16\sqrt{2})}{\sqrt{2}}-\frac{\cos(16\sqrt{2})}{\sqrt{2}}}\right)\right)+i\sin\left(\tan^{-1}\left(\frac{\frac{\sin(16\sqrt{2})}{\sqrt{2}}-\frac{\cos(16\sqrt{2})}{\sqrt{2}}}{-\frac{\sin(16\sqrt{2})}{\sqrt{2}}-\frac{\cos(16\sqrt{2})}{\sqrt{2}}}\right)\right)\right)}{\sqrt{\pi}}$$

$$\frac{8\times 2^{3/4}\exp\left(i\tan^{-1}\left(\frac{\frac{\sin(16\sqrt{2})}{\sqrt{2}}-\frac{\cos(16\sqrt{2})}{\sqrt{2}}}{-\frac{\sin(16\sqrt{2})}{\sqrt{2}}-\frac{\cos(16\sqrt{2})}{\sqrt{2}}}\right)\right)}{\sqrt{\pi}}$$

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$r \approx 7.5908$ (radius), $\theta \approx 0.14913$ (angle)

7.5908

Alternate forms

$$-\frac{(8+8i)\sqrt[4]{2}\,e^{-16i\sqrt{2}}}{\sqrt{\pi}}$$

$$-\frac{8\sqrt[4]{2}\left(\sin(16\sqrt{2})+\cos(16\sqrt{2})\right)}{\sqrt{\pi}}+\frac{8i\sqrt[4]{2}\left(\sin(16\sqrt{2})-\cos(16\sqrt{2})\right)}{\sqrt{\pi}}$$

$$-\frac{8\times 2^{3/4}\,e^{(i\pi)/4-16i\sqrt{2}}}{\sqrt{\pi}}$$

Series representations

$$\frac{2\left(2^{3/4}\, e^{\left(2\sqrt{2}\,(-4)^{3/2}\right)/(3-2)}\,(-4)^{5/4}\right)}{\sqrt{\pi}\,\sqrt{2}} = -\frac{16\sqrt[4]{-2}\,\exp\left(-16\, i\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}\right)}{\sqrt{z_0}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (\pi-z_0)^k z_0^{-k}}{k!}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{2\left(2^{3/4}\, e^{\left(2\sqrt{2}\,(-4)^{3/2}\right)/(3-2)}\,(-4)^{5/4}\right)}{\sqrt{\pi}\,\sqrt{2}} = -\left(\left(16\sqrt[4]{-2}\,\exp\left(-16\, i\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\Big/\left(\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(i\pi\left\lfloor\frac{\arg(\pi-x)}{2\pi}\right\rfloor\right)\sqrt{x}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k (2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k (\pi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{2\left(2^{3/4}\, e^{\left(2\sqrt{2}\,(-4)^{3/2}\right)/(3-2)}\,(-4)^{5/4}\right)}{\sqrt{\pi}\,\sqrt{2}} =$$

$$-\left(\left(16\sqrt[4]{-2}\,\exp\left(-16\,i\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}\right) \left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor-1/2\lfloor\arg(\pi-z_0)/(2\pi)\rfloor} z_0^{-1-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor-1/2\lfloor\arg(\pi-z_0)/(2\pi)\rfloor}\right) \Big/ \left(\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (\pi-z_0)^k z_0^{-k}}{k!}\right)\right)$$

$n!$ is the factorial function
$(a)_n$ is the Pochhammer symbol (rising factorial)
$\mathbb{R}$ is the set of real numbers
$\arg(z)$ is the complex argument
$\lfloor x\rfloor$ is the floor function

From which:

4(((2 2^(3/4) e^((2 sqrt(2) (-4)^(3/2))/(3 -2)) (-4)^(5/4))/(sqrt(π) sqrt(2))))^3-21-φ

Input

$$4\left(\frac{2\times 2^{3/4}\, e^{\left(2\sqrt{2}\,(-4)^{3/2}\right)/(3-2)}\,(-4)^{5/4}}{\sqrt{\pi}\,\sqrt{2}}\right)^3 - 21 - \phi$$

ϕ is the golden ratio

Exact result

$$-\phi - 21 - \frac{8192\,(-1)^{3/4}\sqrt[4]{2}\, e^{-48\,i\sqrt{2}}}{\pi^{3/2}}$$

Decimal approximation

1554.727079682360636453971104808202854670254148244754266873044136…
+
756.8719086376487841989095898825767787404492654199866677743388393…
i

Alternate complex forms

$$-\phi - 21 + \frac{i\left(-4096\times 2^{3/4}\sin\left(48\sqrt{2}\right) - 4096\times 2^{3/4}\cos\left(48\sqrt{2}\right)\right)}{\pi^{3/2}} + \frac{4096\times 2^{3/4}\cos\left(48\sqrt{2}\right) - 4096\times 2^{3/4}\sin\left(48\sqrt{2}\right)}{\pi^{3/2}}$$

$$\left(\cos\left(\tan^{-1}\left(\frac{-4096\times 2^{3/4}\sin\left(48\sqrt{2}\right) - 4096\times 2^{3/4}\cos\left(48\sqrt{2}\right)}{\pi^{3/2}\left(-\phi - 21 + \frac{4096\times 2^{3/4}\cos\left(48\sqrt{2}\right) - 4096\times 2^{3/4}\sin\left(48\sqrt{2}\right)}{\pi^{3/2}}\right)}\right)\right) + i\sin\left(\tan^{-1}\left(\frac{-4096\times 2^{3/4}\sin\left(48\sqrt{2}\right) - 4096\times 2^{3/4}\cos\left(48\sqrt{2}\right)}{\pi^{3/2}\left(-\phi - 21 + \frac{4096\times 2^{3/4}\cos\left(48\sqrt{2}\right) - 4096\times 2^{3/4}\sin\left(48\sqrt{2}\right)}{\pi^{3/2}}\right)}\right)\right)\right) / \left(\pi^{3/2}\sqrt{\left(2/\left(134\,217\,728\sqrt{2} + \left(927 + 43\sqrt{5}\right)\pi^3 - 8192\times 2^{3/4}\left(43 + \sqrt{5}\right)\pi^{3/2}\left(\cos\left(48\sqrt{2}\right) - \sin\left(48\sqrt{2}\right)\right)\right)\right)}\right)$$

$$\frac{\exp\left(i\tan^{-1}\left(\frac{-4096\times 2^{3/4}\sin\left(48\sqrt{2}\right) - 4096\times 2^{3/4}\cos\left(48\sqrt{2}\right)}{\pi^{3/2}\left(-\phi - 21 + \frac{4096\times 2^{3/4}\cos\left(48\sqrt{2}\right) - 4096\times 2^{3/4}\sin\left(48\sqrt{2}\right)}{\pi^{3/2}}\right)}\right)\right)}{\pi^{3/2}\sqrt{\frac{2}{134\,217\,728\sqrt{2} + \left(927 + 43\sqrt{5}\right)\pi^3 - 8192\times 2^{3/4}\left(43 + \sqrt{5}\right)\pi^{3/2}\left(\cos\left(48\sqrt{2}\right) - \sin\left(48\sqrt{2}\right)\right)}}}$$

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$r \approx 1729.2$ (radius), $\theta \approx 0.45305$ (angle)

1729.2

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$-\phi - 21 - \frac{8192\, i \sqrt[4]{-2}\, e^{-48 i \sqrt{2}}}{\pi^{3/2}}$$

$$\frac{1}{2}\left(-43 - \sqrt{5}\right) - \frac{8192\, i \sqrt[4]{-2}\, e^{-48 i \sqrt{2}}}{\pi^{3/2}}$$

$$-\frac{\pi^{3/2}(\phi + 21) + 8192\,(-1)^{3/4} \sqrt[4]{2}\, e^{-48 i \sqrt{2}}}{\pi^{3/2}}$$

$$-\phi - 21 - \frac{8192 \sqrt[4]{2}\, e^{(3 i \pi)/4 - 48 i \sqrt{2}}}{\pi^{3/2}}$$

Expanded form

$$-\frac{43}{2} - \frac{\sqrt{5}}{2} - \frac{8192\,(-1)^{3/4} \sqrt[4]{2}\, e^{-48 i \sqrt{2}}}{\pi^{3/2}}$$

Series representations

$$4\left(\frac{2\left(2^{3/4}\,e^{\left(2\sqrt{2}\,(-4)^{3/2}\right)/(3-2)}\,(-4)^{5/4}\right)}{\sqrt{\pi}\,\sqrt{2}}\right)^3-21-\phi=$$

$$-\left(\left(\exp\left(-48\,i\,\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)\right.\right.$$
$$\left(16384\,(-2)^{3/4}+21\exp\left(48\,i\,\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)\right.$$
$$\sqrt{z_0}^{\,6}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^3$$
$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\pi-z_0)^k\,z_0^{-k}}{k!}\right)^3+$$
$$\exp\left(48\,i\,\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)\phi$$
$$\sqrt{z_0}^{\,6}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^3$$
$$\left.\left.\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\pi-z_0)^k\,z_0^{-k}}{k!}\right)^3\right)\right)\Bigg/$$
$$\left.\left(\sqrt{z_0}^{\,6}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\pi-z_0)^k\,z_0^{-k}}{k!}\right)^3\right)\right)$$

for (not ($z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0$))

$$4\left(\frac{2\left(2^{3/4}\, e^{\left(2\sqrt{2}\,(-4)^{3/2}\right)/(3-2)}\,(-4)^{5/4}\right)}{\sqrt{\pi}\,\sqrt{2}}\right)^3 - 21 - \phi =$$

$$-\Bigg(\Bigg(\exp\left(-48\, i \exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\Bigg(16384\,(-2)^{3/4} + 21\exp\left(48\, i \exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\right.$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\exp^3\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)$$

$$\exp^3\left(i\,\pi\left\lfloor\frac{\arg(\pi-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,6}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3$$

$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(\pi-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3 +$$

$$\exp\left(48\, i \exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\phi\exp^3\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp^3\left(i\,\pi\left\lfloor\frac{\arg(\pi-x)}{2\pi}\right\rfloor\right)$$

$$\sqrt{x}^{\,6}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3$$

$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(\pi-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3\Bigg)\Bigg)\Bigg/$$

$$\Bigg(\exp^3\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp^3\left(i\,\pi\left\lfloor\frac{\arg(\pi-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,6}$$

$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3$$

$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(\pi-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3\Bigg)\Bigg) \quad \text{for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$4\left(\frac{2\left(2^{3/4}\, e^{\left(2\sqrt{2}\;(-4)^{3/2}\right)/(3-2)}\;(-4)^{5/4}\right)}{\sqrt{\pi}\;\sqrt{2}}\right)^3-21-\phi=$$

$$-\left(\left(e^{-48\,i\left(\frac{1}{z_0}\right)^{1/2\left\lfloor \arg(2-z_0)/(2\pi)\right\rfloor} z_0^{1/2+1/2\left\lfloor \arg(2-z_0)/(2\pi)\right\rfloor} \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}\right.\right.$$

$$\left(\frac{1}{z_0}\right)^{-3/2\left\lfloor \arg(2-z_0)/(2\pi)\right\rfloor-3/2\left\lfloor \arg(\pi-z_0)/(2\pi)\right\rfloor}$$

$$z_0^{-3-3/2\left\lfloor \arg(2-z_0)/(2\pi)\right\rfloor-3/2\left\lfloor \arg(\pi-z_0)/(2\pi)\right\rfloor}\left(16384\,(-2)^{3/4}+21\right.$$

$$e^{48\,i\left(\frac{1}{z_0}\right)^{1/2\left\lfloor \arg(2-z_0)/(2\pi)\right\rfloor} z_0^{1/2+1/2\left\lfloor \arg(2-z_0)/(2\pi)\right\rfloor} \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}$$

$$\left(\frac{1}{z_0}\right)^{3/2\left\lfloor \arg(2-z_0)/(2\pi)\right\rfloor+3/2\left\lfloor \arg(\pi-z_0)/(2\pi)\right\rfloor}$$

$$z_0^{3+3/2\left\lfloor \arg(2-z_0)/(2\pi)\right\rfloor+3/2\left\lfloor \arg(\pi-z_0)/(2\pi)\right\rfloor}$$

$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^3$$

$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\pi-z_0)^k z_0^{-k}}{k!}\right)^3+$$

$$e^{48\,i\left(\frac{1}{z_0}\right)^{1/2\left\lfloor \arg(2-z_0)/(2\pi)\right\rfloor} z_0^{1/2+1/2\left\lfloor \arg(2-z_0)/(2\pi)\right\rfloor} \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}$$

$$\phi\left(\frac{1}{z_0}\right)^{3/2\left\lfloor \arg(2-z_0)/(2\pi)\right\rfloor+3/2\left\lfloor \arg(\pi-z_0)/(2\pi)\right\rfloor}$$

$$z_0^{3+3/2\left\lfloor \arg(2-z_0)/(2\pi)\right\rfloor+3/2\left\lfloor \arg(\pi-z_0)/(2\pi)\right\rfloor}$$

$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^3$$

$$\left.\left.\left.\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\pi-z_0)^k z_0^{-k}}{k!}\right)^3\right)\right)\right/$$

$$\left(\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\pi-z_0)^k z_0^{-k}}{k!}\right)^3\right)\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

((4(((2 2^(3/4) e^((2 sqrt(2) (-4)^(3/2))/(3 -2)) (-4)^(5/4))/(sqrt(π) sqrt(2))))^3-21-φ))^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\sqrt[15]{4\left(\frac{2\times 2^{3/4}\, e^{\left(2\sqrt{2}\,(-4)^{3/2}\right)/(3-2)}\,(-4)^{5/4}}{\sqrt{\pi}\,\sqrt{2}}\right)^3 - 21 - \phi} + C_{\mathrm{MRB}}{}^{1-1/(4\pi)+\pi}$$

ϕ is the golden ratio
C_{MRB} is the MRB constant

Exact result

$$C_{\mathrm{MRB}}{}^{1-1/(4\pi)+\pi} + \sqrt[15]{-\phi - 21 - \frac{8192\,(-1)^{3/4}\,\sqrt[4]{2}\,e^{-48\,i\,\sqrt{2}}}{\pi^{3/2}}}$$

Decimal approximation

1.6441991249471788605975599334062970061045560543879695550911969... + 0.0496412535303249660276658753457205590877871190946559903618244442... i

Alternate complex forms

1.6449483324801209948584330505812372965477409736855405361375516 (cos(0.030182584128644325496256574568702212537463614853715470269394229) + i sin(0.030182584128644325496256574568702212537463614853715470269394229))

$$1.6449483324801209948584330505812372965477409736855405361375516\, e^{0.030182584128644325496256574568702212537463614853715470269394229\, i}$$

Polar coordinates

$r = 1.6449483324801209948584330505812372965477409736855405361375516$ (radius), $\theta = 0.030182584128644325496256574568702212537463614853715470269394229$ (angle)

1.64494833248….≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape)

Alternate complex forms

$$i \operatorname{Im}\left(\sqrt[15]{-\phi - 21 - \frac{8192\,(-1)^{3/4}\,\sqrt[4]{2}\,e^{-48\,i\,\sqrt{2}}}{\pi^{3/2}}}\right) + C_{\mathrm{MRB}}{}^{1-1/(4\pi)+\pi} + \operatorname{Re}\left(\sqrt[15]{-\phi - 21 - \frac{8192\,(-1)^{3/4}\,\sqrt[4]{2}\,e^{-48\,i\,\sqrt{2}}}{\pi^{3/2}}}\right)$$

$$
\begin{aligned}
&\sqrt{\Bigg(\frac{1}{4}\, C_{\text{MRB}}{}^{-1/(2\pi)}} \\
&\quad \Bigg(2\, C_{\text{MRB}}{}^{1+\pi} + \frac{1}{\sqrt[10]{\pi}} 2^{29/30} \big(134\,217\,728\sqrt{2} + \big(927 + 43\sqrt{5}\big)\pi^3 - 8192 \times \\
&\qquad\qquad 2^{3/4}\big(43 + \sqrt{5}\big)\pi^{3/2}\big(\cos\big(48\sqrt{2}\big) - \sin\big(48\sqrt{2}\big)\big)\big)\,\hat{}\, \\
&\qquad\qquad (1/30)\cos\Bigg(\frac{1}{15}\tan^{-1}\Bigg(\\
&\qquad\qquad\qquad \frac{8192 \times 2^{3/4}\big(\sin\big(48\sqrt{2}\big) + \cos\big(48\sqrt{2}\big)\big)}{8192 \times 2^{3/4}\big(\cos\big(48\sqrt{2}\big) - \sin\big(48\sqrt{2}\big)\big) - \big(43 + \sqrt{5}\big)\pi^{3/2}} \\
&\qquad\qquad\qquad \Bigg)\Bigg) \sqrt[4\pi]{C_{\text{MRB}}}\Bigg)^2 + \\
&\quad \sin^2\Bigg(\frac{1}{15}\tan^{-1}\Bigg(\frac{8192 \times 2^{3/4}\big(\sin\big(48\sqrt{2}\big) + \cos\big(48\sqrt{2}\big)\big)}{8192 \times 2^{3/4}\big(\cos\big(48\sqrt{2}\big) - \sin\big(48\sqrt{2}\big)\big) - \big(43 + \sqrt{5}\big)\pi^{3/2}}\Bigg)\Bigg) \Big/ \\
&\quad \Big(\sqrt[5]{\pi}\,\big(-\big(2\big/\big(-134\,217\,728\sqrt{2} - \big(927 + 43\sqrt{5}\big)\pi^3 + \\
&\qquad\qquad 8192 \times 2^{3/4}\big(43 + \sqrt{5}\big)\pi^{3/2} \\
&\qquad\qquad\quad \big(\cos\big(48\sqrt{2}\big) - \sin\big(48\sqrt{2}\big)\big)\big)\big)\big)\,\hat{}\,(1/15)\Big)\Bigg) \\
&\Bigg(\cos\Bigg(\tan^{-1}\Bigg(\frac{\operatorname{Im}\Bigg(\sqrt[15]{-\phi - 21 - \frac{8192\,(-1)^{3/4}\sqrt[4]{2}\,e^{-48\,i\sqrt{2}}}{\pi^{3/2}}}\Bigg)}{C_{\text{MRB}}{}^{1-1/(4\pi)+\pi} + \operatorname{Re}\Bigg(\sqrt[15]{-\phi - 21 - \frac{8192\,(-1)^{3/4}\sqrt[4]{2}\,e^{-48\,i\sqrt{2}}}{\pi^{3/2}}}\Bigg)}\Bigg)\Bigg) + \\
&\quad i \\
&\quad \sin\Bigg(\tan^{-1}\Bigg(\frac{\operatorname{Im}\Bigg(\sqrt[15]{-\phi - 21 - \frac{8192\,(-1)^{3/4}\sqrt[4]{2}\,e^{-48\,i\sqrt{2}}}{\pi^{3/2}}}\Bigg)}{C_{\text{MRB}}{}^{1-1/(4\pi)+\pi} + \operatorname{Re}\Bigg(\sqrt[15]{-\phi - 21 - \frac{8192\,(-1)^{3/4}\sqrt[4]{2}\,e^{-48\,i\sqrt{2}}}{\pi^{3/2}}}\Bigg)}\Bigg)\Bigg)\Bigg)
\end{aligned}
$$

$$\sqrt{\left(\frac{1}{4} C_{\text{MRB}}{}^{-1/(2\pi)}\right.}$$

$$\left(2 C_{\text{MRB}}{}^{1+\pi} + \frac{1}{\sqrt[10]{\pi}} 2^{29/30} \left(134\,217\,728 \sqrt{2} + \left(927 + 43 \sqrt{5}\right) \pi^3 - 8192 \times\right.\right.$$

$$\left. 2^{3/4} \left(43 + \sqrt{5}\right) \pi^{3/2} \left(\cos\left(48 \sqrt{2}\right) - \sin\left(48 \sqrt{2}\right)\right)\right) \text{^}$$

$$(1/30) \cos\left(\frac{1}{15} \tan^{-1}\left(\right.\right.$$

$$\frac{8192 \times 2^{3/4} \left(\sin\left(48 \sqrt{2}\right) + \cos\left(48 \sqrt{2}\right)\right)}{8192 \times 2^{3/4} \left(\cos\left(48 \sqrt{2}\right) - \sin\left(48 \sqrt{2}\right)\right) - \left(43 + \sqrt{5}\right) \pi^{3/2}}$$

$$\left.\left.\left.\right)\right) \sqrt[4\pi]{C_{\text{MRB}}}\right)^2 +$$

$$\sin^2\left(\frac{1}{15} \tan^{-1}\left(\frac{8192 \times 2^{3/4} \left(\sin\left(48 \sqrt{2}\right) + \cos\left(48 \sqrt{2}\right)\right)}{8192 \times 2^{3/4} \left(\cos\left(48 \sqrt{2}\right) - \sin\left(48 \sqrt{2}\right)\right) - \left(43 + \sqrt{5}\right) \pi^{3/2}}\right)\right) \Big/$$

$$\left(\sqrt[5]{\pi} \left(-\left(2 \Big/ \left(-134\,217\,728 \sqrt{2} - \left(927 + 43 \sqrt{5}\right) \pi^3 + \right.\right.\right.\right.$$

$$8192 \times 2^{3/4} \left(43 + \sqrt{5}\right) \pi^{3/2}$$

$$\left.\left.\left.\left.\left(\cos\left(48 \sqrt{2}\right) - \sin\left(48 \sqrt{2}\right)\right)\right)\right)\right) \text{^} (1/15)\right)$$

$$\exp\left(i \tan^{-1}\left(\frac{\operatorname{Im}\left(\sqrt[15]{-\phi - 21 - \frac{8192 (-1)^{3/4} \sqrt[4]{2}\, e^{-48 i \sqrt{2}}}{\pi^{3/2}}}\right)}{C_{\text{MRB}}{}^{1-1/(4\pi)+\pi} + \operatorname{Re}\left(\sqrt[15]{-\phi - 21 - \frac{8192 (-1)^{3/4} \sqrt[4]{2}\, e^{-48 i \sqrt{2}}}{\pi^{3/2}}}\right)}\right)\right)$$

$\operatorname{Im}(z)$ is the imaginary part of z

$\operatorname{Re}(z)$ is the real part of z

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Alternate forms

$$C_{\text{MRB}}^{1-1/(4\pi)+\pi} + \sqrt[15]{\frac{1}{2}\left(-43-\sqrt{5}\right) - \frac{8192\, i\, \sqrt[4]{-2}\; e^{-48 i \sqrt{2}}}{\pi^{3/2}}}$$

$$C_{\text{MRB}}^{1-1/(4\pi)+\pi} + \frac{\sqrt[15]{-\pi^{3/2}(\phi+21) - 8192\,(-1)^{3/4}\sqrt[4]{2}\; e^{-48 i \sqrt{2}}}}{\sqrt[10]{\pi}}$$

$$\frac{1}{2\sqrt[10]{\pi}} C_{\text{MRB}}^{-1/(4\pi)} \left(2\sqrt[10]{\pi}\; C_{\text{MRB}}^{1+\pi} + 2^{14/15}\sqrt[15]{e^{-48 i \sqrt{2}}\left(-16384\,(-1)^{3/4}\sqrt[4]{2} - 43\, e^{48 i \sqrt{2}}\, \pi^{3/2} - \sqrt{5}\; e^{48 i \sqrt{2}}\, \pi^{3/2}\right)} \sqrt[4\pi]{C_{\text{MRB}}}\right)$$

$$C_{\text{MRB}}^{1-1/(4\pi)+\pi} + \sqrt[15]{-\phi - 21 - \frac{8192\sqrt[4]{2}\; e^{(3 i \pi)/4 - 48 i \sqrt{2}}}{\pi^{3/2}}}$$

Expanded form

$$C_{\text{MRB}}^{1-1/(4\pi)+\pi} + \sqrt[15]{-\frac{43}{2} - \frac{\sqrt{5}}{2} - \frac{8192\,(-1)^{3/4}\sqrt[4]{2}\; e^{-48 i \sqrt{2}}}{\pi^{3/2}}}$$

(1/27(((4(((2 2^(3/4) e^((2 sqrt(2) (-4)^(3/2))/(3 -2)) (-4)^(5/4))/(sqrt(π) sqrt(2))))^3-21-φ))-1))^2-2Φ-4MRB const

Input

$$\left(\frac{1}{27}\left(\left(4\left(\frac{2\times 2^{3/4}\,e^{(2\sqrt{2}\,(-4)^{3/2})/(3-2)}\,(-4)^{5/4}}{\sqrt{\pi}\,\sqrt{2}}\right)^3-21-\phi\right)-1\right)\right)^2-2\,\Phi-4\,C_{\mathrm{MRB}}$$

ϕ is the golden ratio
Φ is the golden ratio conjugate
C_{MRB} is the MRB constant

Exact result

$$-4\,C_{\mathrm{MRB}}-2\,\Phi+\frac{1}{729}\left(-\phi-22-\frac{8192\,(-1)^{3/4}\,\sqrt[4]{2}\,e^{-48\,i\,\sqrt{2}}}{\pi^{3/2}}\right)^2$$

Decimal approximation

2523.681563484859445529621508795021364495211915626822662739330821…
+
3226.261674351683161024392445998532953064773855082397897805763417…
i

Alternate complex forms

$$-4\,C_{\mathrm{MRB}}-2\,\Phi+\frac{1}{729\,\pi^{3/2}}2\,i\left(-4096\times 2^{3/4}\sin\left(48\sqrt{2}\right)-4096\times 2^{3/4}\cos\left(48\sqrt{2}\right)\right)\left(-\phi-22+\frac{4096\times 2^{3/4}\cos\left(48\sqrt{2}\right)-4096\times 2^{3/4}\sin\left(48\sqrt{2}\right)}{\pi^{3/2}}\right)+\frac{1}{729}\left(\left(-\phi-22+\frac{4096\times 2^{3/4}\cos\left(48\sqrt{2}\right)-4096\times 2^{3/4}\sin\left(48\sqrt{2}\right)}{\pi^{3/2}}\right)^2-\frac{\left(-4096\times 2^{3/4}\sin\left(48\sqrt{2}\right)-4096\times 2^{3/4}\cos\left(48\sqrt{2}\right)\right)^2}{\pi^3}\right)$$

$$\frac{1}{1458\,\pi^{3}}\sqrt{\left(\left(\pi^{3}\left(5832\,C_{\mathrm{MRB}}+2916\,\Phi-1015-45\sqrt{5}\right)+134\,217\,728\sqrt{2}\,\sin\left(96\sqrt{2}\right)+8192\times 2^{3/4}\left(45+\sqrt{5}\right)\pi^{3/2}\left(\cos\left(48\sqrt{2}\right)-\sin\left(48\sqrt{2}\right)\right)\right)^{2}+134\,217\,728\sqrt{2}\left(\left(45+\sqrt{5}\right)\pi^{3/2}-8192\times 2^{3/4}\left(\cos\left(48\sqrt{2}\right)-\sin\left(48\sqrt{2}\right)\right)\right)^{2}\left(\sin\left(48\sqrt{2}\right)+\cos\left(48\sqrt{2}\right)\right)^{2}\right)}$$

$$\left(\cos\left(\tan^{-1}\left(\left(2\left(-4096\times 2^{3/4}\sin\left(48\sqrt{2}\right)-4096\times 2^{3/4}\cos\left(48\sqrt{2}\right)\right)\left(-\phi-22+\frac{4096\times 2^{3/4}\cos\left(48\sqrt{2}\right)-4096\times 2^{3/4}\sin\left(48\sqrt{2}\right)}{\pi^{3/2}}\right)\right)\Big/\left(729\,\pi^{3/2}\left(-4\,C_{\mathrm{MRB}}-2\,\Phi+\frac{1}{729}\left(\left(-\phi-22+\frac{1}{\pi^{3/2}}\left(4096\times 2^{3/4}\cos\left(48\sqrt{2}\right)-4096\times 2^{3/4}\sin\left(48\sqrt{2}\right)\right)\right)^{2}-\frac{\left(-4096\times 2^{3/4}\sin\left(48\sqrt{2}\right)-4096\times 2^{3/4}\cos\left(48\sqrt{2}\right)\right)^{2}}{\pi^{3}}\right)\right)\right)\right)\right)+i\sin\left(\tan^{-1}\left(\left(2\left(-4096\times 2^{3/4}\sin\left(48\sqrt{2}\right)-4096\times 2^{3/4}\cos\left(48\sqrt{2}\right)\right)\left(-\phi-22+\frac{4096\times 2^{3/4}\cos\left(48\sqrt{2}\right)-4096\times 2^{3/4}\sin\left(48\sqrt{2}\right)}{\pi^{3/2}}\right)\right)\Big/\left(729\,\pi^{3/2}\left(-4\,C_{\mathrm{MRB}}-2\,\Phi+\frac{1}{729}\left(\left(-\phi-22+\frac{1}{\pi^{3/2}}\left(4096\times 2^{3/4}\cos\left(48\sqrt{2}\right)-4096\times 2^{3/4}\sin\left(48\sqrt{2}\right)\right)\right)^{2}-\frac{\left(-4096\times 2^{3/4}\sin\left(48\sqrt{2}\right)-4096\times 2^{3/4}\cos\left(48\sqrt{2}\right)\right)^{2}}{\pi^{3}}\right)\right)\right)\right)\right)\right)$$

$$\frac{1}{1458\,\pi^3}\sqrt{\left(\left(\pi^3\left(5832\,C_{\text{MRB}}+2916\,\Phi-1015-45\sqrt{5}\right)+134\,217\,728\sqrt{2}\,\sin\left(96\sqrt{2}\right)+8192\times 2^{3/4}\left(45+\sqrt{5}\right)\pi^{3/2}\left(\cos\left(48\sqrt{2}\right)-\sin\left(48\sqrt{2}\right)\right)\right)^2+134\,217\,728\sqrt{2}\left(\left(45+\sqrt{5}\right)\pi^{3/2}-8192\times 2^{3/4}\left(\cos\left(48\sqrt{2}\right)-\sin\left(48\sqrt{2}\right)\right)\right)^2\left(\sin\left(48\sqrt{2}\right)+\cos\left(48\sqrt{2}\right)\right)^2\right)}$$

$$\exp\left(i\tan^{-1}\left(\left(2\left(-4096\times 2^{3/4}\sin\left(48\sqrt{2}\right)-4096\times 2^{3/4}\cos\left(48\sqrt{2}\right)\right)\left(-\phi-22+\frac{4096\times 2^{3/4}\cos\left(48\sqrt{2}\right)-4096\times 2^{3/4}\sin\left(48\sqrt{2}\right)}{\pi^{3/2}}\right)\right)\Big/\left(729\,\pi^{3/2}\left(-4\,C_{\text{MRB}}-2\,\Phi+\frac{1}{729}\left(\left(-\phi-22+\frac{4096\times 2^{3/4}\cos\left(48\sqrt{2}\right)-4096\times 2^{3/4}\sin\left(48\sqrt{2}\right)}{\pi^{3/2}}\right)^2-\frac{\left(-4096\times 2^{3/4}\sin\left(48\sqrt{2}\right)-4096\times 2^{3/4}\cos\left(48\sqrt{2}\right)\right)^2}{\pi^3}\right)\right)\right)\right)\right)$$

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$r \approx 4096.1$ (radius), $\theta \approx 0.90698$ (angle)

$4096.1 \approx 4096 = 64^2$

Alternate forms

$$-4\,C_{\text{MRB}}-2\,\Phi+\frac{1}{729}\left(\phi+22-\frac{(4096-4096\,i)\,2^{3/4}\,e^{-48\,i\sqrt{2}}}{\pi^{3/2}}\right)^2$$

$$-4\,C_{\text{MRB}}-2\,\Phi+\frac{1}{729}\left(\frac{1}{2}\left(-45-\sqrt{5}\right)-\frac{8192\,i\sqrt[4]{-2}\,e^{-48\,i\sqrt{2}}}{\pi^{3/2}}\right)^2$$

$$-4\,C_{\text{MRB}} - 2\,\Phi + \frac{1}{729}\left(-22 + \frac{1}{2}\left(-1 - \sqrt{5}\right) - \frac{8192\,(-1)^{3/4}\,\sqrt[4]{2}\;e^{-48\,i\,\sqrt{2}}}{\pi^{3/2}}\right)^2$$

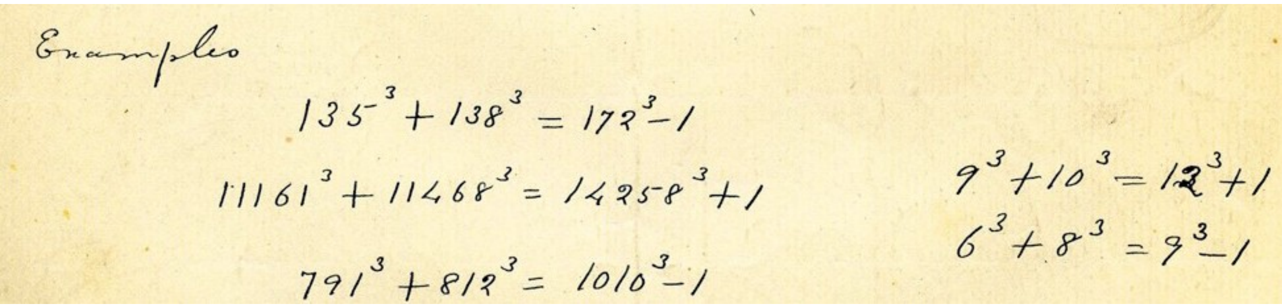
$$-4\,C_{\text{MRB}} - 2\,\Phi + \frac{1}{729}\left(-\phi - 22 - \frac{8192\,\sqrt[4]{2}\;e^{(3\,i\,\pi)/4-48\,i\,\sqrt{2}}}{\pi^{3/2}}\right)^2$$

Expanded form

$$-4\,C_{\text{MRB}} - 2\,\Phi + \frac{1015}{1458} + \frac{5\sqrt{5}}{162} - \frac{67\,108\,864\,i\,\sqrt{2}\;e^{-96\,i\,\sqrt{2}}}{729\,\pi^3} + \frac{40\,960\,(-1)^{3/4}\,\sqrt[4]{2}\;e^{-48\,i\,\sqrt{2}}}{81\,\pi^{3/2}} + \frac{8192\,(-1)^{3/4}\,\sqrt[4]{2}\,\sqrt{5}\;e^{-48\,i\,\sqrt{2}}}{729\,\pi^{3/2}}$$

On the Ramanujan taxicab numbers

We have:


Examples

$135^3 + 138^3 = 172^3 - 1$

$11161^3 + 11468^3 = 14258^3 + 1$

$791^3 + 812^3 = 1010^3 - 1$

$9^3 + 10^3 = 12^3 + 1$

$6^3 + 8^3 = 9^3 - 1$

We observe that:

(172^3-1)^1/31

Input

$$\sqrt[31]{172^3 - 1}$$

Result

$3^{3/31}\sqrt[31]{188461}$

Decimal approximation

1.6456651030212824832898820470765489933345525452175234517615331489

...

1.64566510302...≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape)

Alternate form

root of $x^{31} - 5088447$ near $x = 1.64567$

(14258^3+1)^1/58

Input

$\sqrt[58]{14258^3 + 1}$

Result

$\sqrt[29]{21}\ \sqrt[58]{6572600593}$

Decimal approximation

1.6400802564534048306447584605455239585017857206311322342166961909

...

1.6400802564534…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape)

((1010)^3-1)^1/42

Input

$\sqrt[42]{1010^3 - 1}$

Result

$\sqrt[14]{7}\ \sqrt[42]{3003793}$

Decimal approximation

1.6390582338672179513479881763631782296015896767808489130107044405...

1.639058233867…. result very near to the mean between ζ(2) = $\frac{\pi^2}{6} = 1.644934\ldots$, the value of golden ratio 1.61803398… and the 14th root of the Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3$ = 1164.2696 i.e. 1.65578..., i.e. 1.63958266

((12^3)+1)^1/15

Input

$\sqrt[15]{12^3 + 1}$

Result

$\sqrt[15]{1729}$

Decimal approximation

1.6438152287487281305800880313247695143292831436999401726452126788...

1.6438152287…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape)

(9^3-1)^1/13

Input

$\sqrt[13]{9^3 - 1}$

Result

$2^{3/13}\ \sqrt[13]{91}$

Decimal approximation

1.6602135430335898944465409919904892710280291280277020057159063333
...

1.660213543…. result very near to the 14th root of the following Ramanujan's class invariant $Q=\left(G_{505}/G_{101/5}\right)^3$ = 1164.2696 i.e. 1.65578...

In conclusion, we obtain from the mean of the all previous expressions:

1/5(((9^3-1)^1/13)+(((12^3)+1)^1/15)+((14258^3+1)^1/58)+(((1010)^3-1)^1/42)+((172^3-1)^1/31))

Input

$$\frac{1}{5}\left(\sqrt[13]{9^3-1}+\sqrt[15]{12^3+1}+\sqrt[58]{14258^3+1}+\sqrt[42]{1010^3-1}+\sqrt[31]{172^3-1}\right)$$

Result

$$\frac{1}{5}\left(2^{3/13}\sqrt[13]{91}+\sqrt[15]{1729}+\sqrt[29]{21}\sqrt[58]{6572600593}+\sqrt[14]{7}\sqrt[42]{3003793}+3^{3/31}\sqrt[31]{188461}\right)$$

Decimal approximation

1.6457664730248446580618515414601019933590480428714293554700105585
...

1.6457664730248….≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape)

Alternate forms

$$\frac{1}{5}\sqrt[29]{21}\sqrt[58]{6572600593}+\frac{1}{5}\sqrt[31]{7}\sqrt[42]{13}\left(2^{3/13}\times 7^{18/403}\times 13^{29/546}+7^{16/465}\times 13^{3/70}\sqrt[15]{19}+3^{3/31}\times 13^{11/1302}\sqrt[31]{2071}+7^{17/434}\sqrt[42]{231061}\right)$$

$$\frac{1}{5}\sqrt[31]{7}\sqrt[58]{13}\left(2^{3/13}\times 7^{18/403}\times 13^{45/754}+7^{16/465}\times 13^{43/870}\sqrt[15]{19}+3^{3/31}\times 13^{27/1798}\sqrt[31]{2071}+7^{17/434}\times 13^{4/609}\sqrt[42]{231061}+\sqrt[29]{3}\,7^{2/899}\sqrt[58]{505584661}\right)$$

Expanded form

$$\frac{1}{5}\times 2^{3/13}\sqrt[13]{91}+\frac{\sqrt[15]{1729}}{5}+\frac{1}{5}\times 3^{3/31}\sqrt[31]{188461}+\frac{1}{5}\sqrt[14]{7}\sqrt[42]{3003793}+\frac{1}{5}\sqrt[29]{21}\sqrt[58]{6572600593}$$

It’s interesting to observe that also with regard these Ramanujan’s taxicab numbers, the mean of the various nth roots that we have calculated, is always a result very near to $\zeta(2) = 1.64493\ldots$.

Acknowledgments

We would like to thank Professor **Augusto Sagnotti** theoretical physicist at Scuola Normale Superiore (Pisa – Italy) for his very useful explanations and his availability

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