Further Analysis on Ramanujan's Continued Fractions

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Abstract

In this paper, we analyze further Ramanujan's continued fractions. We describe the new possible mathematical connections with the MRB Constant and various equations concerning the Dirichlet L-functions and some sectors of String Theory

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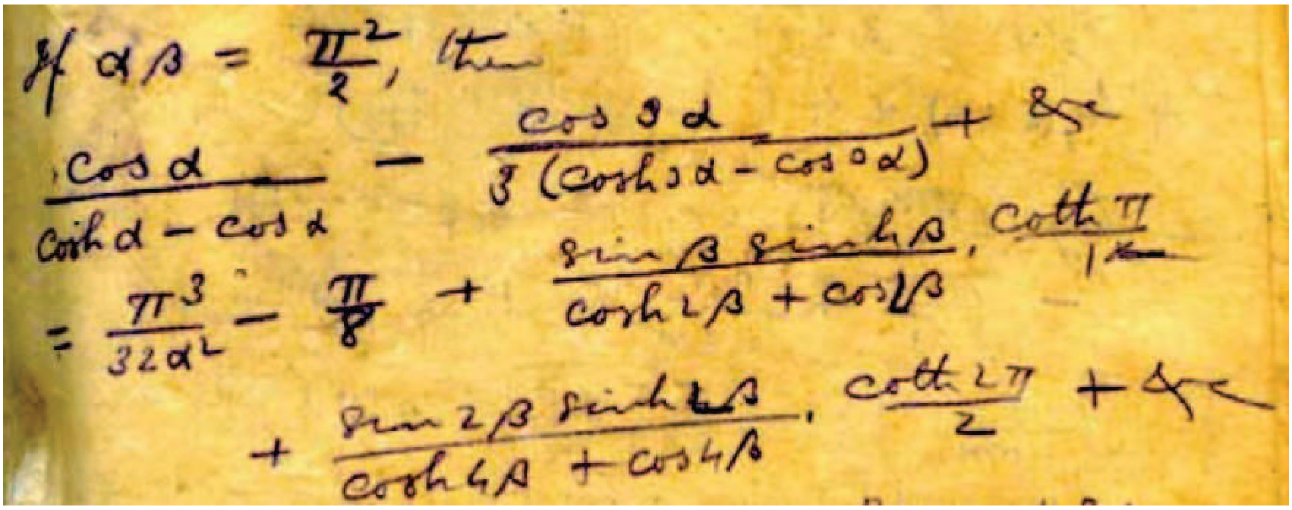
From:

Manuscript Book I - *Srinivasa Ramanujan*

We have:



If $\alpha\beta = \pi^2$, then

$$\frac{\cos\alpha}{\cosh\alpha - \cos\alpha} - \frac{\cos 3\alpha}{3(\cosh 3\alpha - \cos 3\alpha)} + \&c$$

$$= \frac{\pi^3}{32\alpha^2} - \frac{\pi}{8} + \frac{\sin\beta \sinh\beta}{\cosh 2\beta + \cos 2\beta} \cdot \frac{\coth\pi}{1} + \frac{\sin 2\beta \sinh 2\beta}{\cosh 4\beta + \cos 4\beta} \cdot \frac{\coth 2\pi}{2} + \&c$$

From:

(Pi^3)/(32a^2) – Pi/8 + (sin(b) sinh(b))/(cosh(2b)+cos(2b)) * coth(Pi) + (sin(2b) sinh(2b))/(cosh(4b)+cos(4b)) * (coth(2Pi))/2

Input

$$\frac{\pi^3}{32\,a^2} - \frac{\pi}{8} + \frac{\sin(b)\sinh(b)}{\cosh(2\,b) + \cos(2\,b)}\coth(\pi) + \frac{\sin(2\,b)\sinh(2\,b)}{\cosh(4\,b) + \cos(4\,b)}\left(\frac{1}{2}\coth(2\,\pi)\right)$$

$\sinh(x)$ is the hyperbolic sine function

$\cosh(x)$ is the hyperbolic cosine function

$\coth(x)$ is the hyperbolic cotangent function

Alternate forms

$$-\frac{\pi}{8} + \frac{\pi^3}{32\,a^2} + \frac{\cosh(\pi)\sin(b)\sinh(b)}{(\cos(2\,b) + \cosh(2\,b))\sinh(\pi)} + \frac{\cosh(2\,\pi)\sin(2\,b)\sinh(2\,b)}{2\,(\cos(4\,b) + \cosh(4\,b))\sinh(2\,\pi)}$$

$$\frac{\pi^3}{32 a^2} - \frac{2\left(-1 - e^{2\pi}\right)\sin(b)\sinh(b)}{\left(e^{2\pi} - 1\right)(2\cos(2b) + 2\cosh(2b))} + \frac{\coth(2\pi)\sin(2b)\sinh(2b)}{2\cos(4b) + 2\cosh(4b)} - \frac{\pi}{8}$$

$$\left(-4\pi a^2 \cos(2b) - 4\pi a^2 \cosh(2b) + 32 a^2 \coth(\pi)\sin(b)\sinh(b) + \pi^3 \cos(2b) + \pi^3 \cosh(2b)\right) / \left(32 a^2 (\cos(2b) + \cosh(2b))\right) + \frac{\coth(2\pi)\sin(2b)\sinh(2b)}{2(\cos(4b) + \cosh(4b))}$$

Alternate form assuming a and b are real

$$\frac{\pi^3}{32 a^2} - \frac{\sinh(2\pi)\sin(b)\sinh(b)}{(1 - \cosh(2\pi))(\cos(2b) + \cosh(2b))} - \frac{\sinh(4\pi)\sin(2b)\sinh(2b)}{2(1 - \cosh(4\pi))(\cos(4b) + \cosh(4b))} - \frac{\pi}{8}$$

Derivative

$$\frac{\partial}{\partial a}\left(\frac{\pi^3}{32 a^2} - \frac{\pi}{8} + \frac{(\sin(b)\sinh(b))\coth(\pi)}{\cosh(2b) + \cos(2b)} + \frac{(\sin(2b)\sinh(2b))\coth(2\pi)}{(\cosh(4b) + \cos(4b)) \times 2}\right) = -\frac{\pi^3}{16 a^3}$$

For α = β = π/√2 , from

$$\frac{\pi^3}{32 a^2} - \frac{2\left(-1 - e^{2\pi}\right)\sin(b)\sinh(b)}{\left(e^{2\pi} - 1\right)(2\cos(2b) + 2\cosh(2b))} + \frac{\coth(2\pi)\sin(2b)\sinh(2b)}{2\cos(4b) + 2\cosh(4b)} - \frac{\pi}{8}$$

π^3/(32 (π/√2)^2)-(2(-1-e^(2π)) sin(π/√2) sinh(π/√2))/((e^(2π)-1)(2 cos(2(π/√2))+2 cosh(2(π/√2))))+(coth(2π) sin(2(π/√2)) sinh(2(π/√2)))/(2 cos(4(π/√2))+2 cosh(4(π/√2)))-π/8

Input

$$\frac{\pi^3}{32\left(\frac{\pi}{\sqrt{2}}\right)^2} - \frac{(2(-1-e^{2\pi}))\sin\left(\frac{\pi}{\sqrt{2}}\right)\sinh\left(\frac{\pi}{\sqrt{2}}\right)}{(e^{2\pi}-1)\left(2\cos\left(2\times\frac{\pi}{\sqrt{2}}\right)+2\cosh\left(2\times\frac{\pi}{\sqrt{2}}\right)\right)} + \frac{\coth(2\pi)\sin\left(2\times\frac{\pi}{\sqrt{2}}\right)\sinh\left(2\times\frac{\pi}{\sqrt{2}}\right)}{2\cos\left(4\times\frac{\pi}{\sqrt{2}}\right)+2\cosh\left(4\times\frac{\pi}{\sqrt{2}}\right)} - \frac{\pi}{8}$$

$\sinh(x)$ is the hyperbolic sine function
$\cosh(x)$ is the hyperbolic cosine function

Exact result

$$-\frac{\pi}{16} - \frac{2(-1-e^{2\pi})\sin\left(\frac{\pi}{\sqrt{2}}\right)\sinh\left(\frac{\pi}{\sqrt{2}}\right)}{(e^{2\pi}-1)\left(2\cos\left(\sqrt{2}\,\pi\right)+2\cosh\left(\sqrt{2}\,\pi\right)\right)} + \frac{\sin\left(\sqrt{2}\,\pi\right)\sinh\left(\sqrt{2}\,\pi\right)\coth(2\pi)}{2\cos\left(2\sqrt{2}\,\pi\right)+2\cosh\left(2\sqrt{2}\,\pi\right)}$$

Decimal approximation

−0.115892160002214166410929945506057269985170219509225055989071757
…

-0.11589216….

Alternate forms

$$-\frac{\pi}{16} - \frac{2(-1-e^{2\pi})\sin\left(\frac{\pi}{\sqrt{2}}\right)\sinh\left(\frac{\pi}{\sqrt{2}}\right)}{(-1+e^{2\pi})\left(2\cos\left(\sqrt{2}\,\pi\right)+2\cosh\left(\sqrt{2}\,\pi\right)\right)} + \frac{\cosh(2\pi)\sin\left(\sqrt{2}\,\pi\right)\sinh\left(\sqrt{2}\,\pi\right)}{\left(2\cos\left(2\sqrt{2}\,\pi\right)+2\cosh\left(2\sqrt{2}\,\pi\right)\right)\sinh(2\pi)}$$

$$-\frac{\pi}{16}+\frac{2\sin\left(\frac{\pi}{\sqrt{2}}\right)\sinh\left(\frac{\pi}{\sqrt{2}}\right)}{\left(e^{2\pi}-1\right)\left(2\cos\left(\sqrt{2}\,\pi\right)+2\cosh\left(\sqrt{2}\,\pi\right)\right)}+\frac{2\,e^{2\pi}\sin\left(\frac{\pi}{\sqrt{2}}\right)\sinh\left(\frac{\pi}{\sqrt{2}}\right)}{\left(e^{2\pi}-1\right)\left(2\cos\left(\sqrt{2}\,\pi\right)+2\cosh\left(\sqrt{2}\,\pi\right)\right)}-\frac{\sin\left(\sqrt{2}\,\pi\right)\sinh(4\pi)\sinh\left(\sqrt{2}\,\pi\right)}{(1-\cosh(4\pi))\left(2\cos\left(2\sqrt{2}\,\pi\right)+2\cosh\left(2\sqrt{2}\,\pi\right)\right)}$$

$$-\frac{i\left(-1-e^{2\pi}\right)\left(e^{-(i\pi)/\sqrt{2}}-e^{(i\pi)/\sqrt{2}}\right)\left(e^{\pi/\sqrt{2}}-e^{-\pi/\sqrt{2}}\right)}{2\left(e^{2\pi}-1\right)\left(e^{-\sqrt{2}\,\pi}+e^{-i\sqrt{2}\,\pi}+e^{i\sqrt{2}\,\pi}+e^{\sqrt{2}\,\pi}\right)}+\frac{i\left(e^{-2\pi}+e^{2\pi}\right)\left(e^{-i\sqrt{2}\,\pi}-e^{i\sqrt{2}\,\pi}\right)\left(e^{\sqrt{2}\,\pi}-e^{-\sqrt{2}\,\pi}\right)}{4\left(e^{2\pi}-e^{-2\pi}\right)\left(e^{-2\sqrt{2}\,\pi}+e^{-2i\sqrt{2}\,\pi}+e^{2i\sqrt{2}\,\pi}+e^{2\sqrt{2}\,\pi}\right)}-\frac{\pi}{16}$$

Expanded form

$$-\frac{\pi}{16}+\frac{2\sin\left(\frac{\pi}{\sqrt{2}}\right)\sinh\left(\frac{\pi}{\sqrt{2}}\right)}{\left(e^{2\pi}-1\right)\left(2\cos\left(\sqrt{2}\,\pi\right)+2\cosh\left(\sqrt{2}\,\pi\right)\right)}+\frac{2\,e^{2\pi}\sin\left(\frac{\pi}{\sqrt{2}}\right)\sinh\left(\frac{\pi}{\sqrt{2}}\right)}{\left(e^{2\pi}-1\right)\left(2\cos\left(\sqrt{2}\,\pi\right)+2\cosh\left(\sqrt{2}\,\pi\right)\right)}+\frac{\sin\left(\sqrt{2}\,\pi\right)\sinh\left(\sqrt{2}\,\pi\right)\coth(2\pi)}{2\cos\left(2\sqrt{2}\,\pi\right)+2\cosh\left(2\sqrt{2}\,\pi\right)}$$

Alternative representations

$$\frac{\pi^3}{32\left(\frac{\pi}{\sqrt{2}}\right)^2}-\frac{\left(2\left(-1-e^{2\pi}\right)\right)\sin\left(\frac{\pi}{\sqrt{2}}\right)\sinh\left(\frac{\pi}{\sqrt{2}}\right)}{\left(e^{2\pi}-1\right)\left(2\cos\left(\frac{2\pi}{\sqrt{2}}\right)+2\cosh\left(\frac{2\pi}{\sqrt{2}}\right)\right)}+\frac{\coth(2\pi)\left(\sin\left(\frac{2\pi}{\sqrt{2}}\right)\sinh\left(\frac{2\pi}{\sqrt{2}}\right)\right)}{2\cos\left(\frac{4\pi}{\sqrt{2}}\right)+2\cosh\left(\frac{4\pi}{\sqrt{2}}\right)}-\frac{\pi}{8}=-\frac{\pi}{8}-\frac{\left(-1-e^{2\pi}\right)\left(-e^{-\pi/\sqrt{2}}+e^{\pi/\sqrt{2}}\right)\left(-e^{-(i\pi)/\sqrt{2}}+e^{(i\pi)/\sqrt{2}}\right)}{(2i)\left(\left(-1+e^{2\pi}\right)\left(2\cosh\left(-\frac{2i\pi}{\sqrt{2}}\right)+e^{-(2\pi)/\sqrt{2}}+e^{(2\pi)/\sqrt{2}}\right)\right)}+\frac{\left(-e^{-(2\pi)/\sqrt{2}}+e^{(2\pi)/\sqrt{2}}\right)\left(-e^{-(2i\pi)/\sqrt{2}}+e^{(2i\pi)/\sqrt{2}}\right)\left(1+\frac{2}{-1+e^{4\pi}}\right)}{2(2i)\left(2\cosh\left(-\frac{4i\pi}{\sqrt{2}}\right)+e^{-(4\pi)/\sqrt{2}}+e^{(4\pi)/\sqrt{2}}\right)}+\frac{\pi^3}{32\left(\frac{\pi}{\sqrt{2}}\right)^2}$$

$$\frac{\pi^3}{32\left(\frac{\pi}{\sqrt{2}}\right)^2}-\frac{\left(2\left(-1-e^{2\pi}\right)\right)\sin\left(\frac{\pi}{\sqrt{2}}\right)\sinh\left(\frac{\pi}{\sqrt{2}}\right)}{\left(e^{2\pi}-1\right)\left(2\cos\left(\frac{2\pi}{\sqrt{2}}\right)+2\cosh\left(\frac{2\pi}{\sqrt{2}}\right)\right)}+\frac{\coth(2\pi)\left(\sin\left(\frac{2\pi}{\sqrt{2}}\right)\sinh\left(\frac{2\pi}{\sqrt{2}}\right)\right)}{2\cos\left(\frac{4\pi}{\sqrt{2}}\right)+2\cosh\left(\frac{4\pi}{\sqrt{2}}\right)}-\frac{\pi}{8}=-\frac{\pi}{8}-\frac{\cos\left(\frac{\pi}{2}-\frac{\pi}{\sqrt{2}}\right)\left(-1-e^{2\pi}\right)\left(-e^{-\pi/\sqrt{2}}+e^{\pi/\sqrt{2}}\right)}{\left(-1+e^{2\pi}\right)\left(e^{-(2\pi)/\sqrt{2}}+e^{(2\pi)/\sqrt{2}}+e^{-(2i\pi)/\sqrt{2}}+e^{(2i\pi)/\sqrt{2}}\right)}+\frac{\cos\left(\frac{\pi}{2}-\frac{2\pi}{\sqrt{2}}\right)\left(-e^{-(2\pi)/\sqrt{2}}+e^{(2\pi)/\sqrt{2}}\right)\left(1+\frac{2}{-1+e^{4\pi}}\right)}{2\left(e^{-(4\pi)/\sqrt{2}}+e^{(4\pi)/\sqrt{2}}+e^{-(4i\pi)/\sqrt{2}}+e^{(4i\pi)/\sqrt{2}}\right)}+\frac{\pi^3}{32\left(\frac{\pi}{\sqrt{2}}\right)^2}$$

$$\frac{\pi^3}{32\left(\frac{\pi}{\sqrt{2}}\right)^2}-\frac{\left(2\left(-1-e^{2\pi}\right)\right)\sin\left(\frac{\pi}{\sqrt{2}}\right)\sinh\left(\frac{\pi}{\sqrt{2}}\right)}{\left(e^{2\pi}-1\right)\left(2\cos\left(\frac{2\pi}{\sqrt{2}}\right)+2\cosh\left(\frac{2\pi}{\sqrt{2}}\right)\right)}+\frac{\coth(2\pi)\left(\sin\left(\frac{2\pi}{\sqrt{2}}\right)\sinh\left(\frac{2\pi}{\sqrt{2}}\right)\right)}{2\cos\left(\frac{4\pi}{\sqrt{2}}\right)+2\cosh\left(\frac{4\pi}{\sqrt{2}}\right)}-\frac{\pi}{8}=-\frac{\pi}{8}-\frac{\left(-1-e^{2\pi}\right)\left(-e^{-\pi/\sqrt{2}}+e^{\pi/\sqrt{2}}\right)\left(-e^{-(i\pi)/\sqrt{2}}+e^{(i\pi)/\sqrt{2}}\right)}{(2i)\left(\left(-1+e^{2\pi}\right)\left(2\cos\left(-\frac{2i\pi}{\sqrt{2}}\right)+e^{-(2i\pi)/\sqrt{2}}+e^{(2i\pi)/\sqrt{2}}\right)\right)}+\frac{\left(-e^{-(2\pi)/\sqrt{2}}+e^{(2\pi)/\sqrt{2}}\right)\left(-e^{-(2i\pi)/\sqrt{2}}+e^{(2i\pi)/\sqrt{2}}\right)\left(1+\frac{2}{-1+e^{4\pi}}\right)}{2(2i)\left(2\cos\left(-\frac{4i\pi}{\sqrt{2}}\right)+e^{-(4i\pi)/\sqrt{2}}+e^{(4i\pi)/\sqrt{2}}\right)}+\frac{\pi^3}{32\left(\frac{\pi}{\sqrt{2}}\right)^2}$$

i is the imaginary unit

Series representations

$$\frac{\pi^3}{32\left(\frac{\pi}{\sqrt{2}}\right)^2}-\frac{\left(2\left(-1-e^{2\pi}\right)\right)\sin\left(\frac{\pi}{\sqrt{2}}\right)\sinh\left(\frac{\pi}{\sqrt{2}}\right)}{\left(e^{2\pi}-1\right)\left(2\cos\left(\frac{2\pi}{\sqrt{2}}\right)+2\cosh\left(\frac{2\pi}{\sqrt{2}}\right)\right)}+\frac{\coth(2\pi)\left(\sin\left(\frac{2\pi}{\sqrt{2}}\right)\sinh\left(\frac{2\pi}{\sqrt{2}}\right)\right)}{2\cos\left(\frac{4\pi}{\sqrt{2}}\right)+2\cosh\left(\frac{4\pi}{\sqrt{2}}\right)}-$$

$$\frac{\pi}{8}=-\frac{\pi}{16}-\frac{\pi^3\sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\left(\operatorname{Res}_{s=-j_1}\frac{\left(-\frac{1}{2}\right)^{-s}\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)\left(\operatorname{Res}_{s=-j_2}\frac{2^s\,\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)}{2\left(2\sum_{k=0}^{\infty}\frac{8^k\,\pi^{2k}}{(2k)!}+2\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{2^{-s}\,\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}+$$

$$\frac{\pi^3\sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\left(\operatorname{Res}_{s=-j_1}\frac{\left(-\frac{1}{8}\right)^{-s}\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)\left(\operatorname{Res}_{s=-j_2}\frac{8^s\,\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)}{4\left(-1+e^{2\pi}\right)\left(2\sum_{k=0}^{\infty}\frac{2^k\,\pi^{2k}}{(2k)!}+2\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{2^s\,\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}+$$

$$\frac{e^{2\pi}\,\pi^3\sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\left(\operatorname{Res}_{s=-j_1}\frac{\left(-\frac{1}{8}\right)^{-s}\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)\left(\operatorname{Res}_{s=-j_2}\frac{8^s\,\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)}{4\left(-1+e^{2\pi}\right)\left(2\sum_{k=0}^{\infty}\frac{2^k\,\pi^{2k}}{(2k)!}+2\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{2^s\,\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}-$$

$$\frac{\pi^3\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}q^{2k_1}\left(\operatorname{Res}_{s=-k_2}\frac{\left(-\frac{1}{2}\right)^{-s}\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)\left(\operatorname{Res}_{s=-k_3}\frac{2^s\,\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)}{2\sum_{k=0}^{\infty}\frac{8^k\,\pi^{2k}}{(2k)!}+2\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{2^{-s}\,\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}}$$

for $q=e^{2\pi}$

$$\frac{\pi^3}{32\left(\frac{\pi}{\sqrt{2}}\right)^2} - \frac{\left(2\left(-1-e^{2\pi}\right)\right)\sin\left(\frac{\pi}{\sqrt{2}}\right)\sinh\left(\frac{\pi}{\sqrt{2}}\right)}{\left(e^{2\pi}-1\right)\left(2\cos\left(\frac{2\pi}{\sqrt{2}}\right)+2\cosh\left(\frac{2\pi}{\sqrt{2}}\right)\right)} + \frac{\coth(2\pi)\left(\sin\left(\frac{2\pi}{\sqrt{2}}\right)\sinh\left(\frac{2\pi}{\sqrt{2}}\right)\right)}{2\cos\left(\frac{4\pi}{\sqrt{2}}\right)+2\cosh\left(\frac{4\pi}{\sqrt{2}}\right)} - \frac{\pi}{8} =$$

$$\Bigg(\pi\left(\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{(-2)^{k_1}\pi^{2k_1}}{(2k_1)!}+\frac{2^{k_1}\pi^{2k_1}}{(2k_1)!}\right)\left(\frac{2^{3k_2}\pi^{2k_2}}{(2k_2)!}+\frac{(-1)^{k_2}2^{3k_2}\pi^{2k_2}}{(2k_2)!}\right) - e^{2\pi}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left(\frac{(-2)^{k_1}\pi^{2k_1}}{(2k_1)!}+\frac{2^{k_1}\pi^{2k_1}}{(2k_1)!}\right)\left(\frac{2^{3k_2}\pi^{2k_2}}{(2k_2)!}+\frac{(-1)^{k_2}2^{3k_2}\pi^{2k_2}}{(2k_2)!}\right) +$$

$$4\pi^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\left(\frac{(-2)^{k_1}\pi^{2k_1}}{(2k_1)!}+\frac{2^{k_1}\pi^{2k_1}}{(2k_1)!}\right)\left(\operatorname{Res}_{s=-k_2}\frac{\left(-\frac{1}{2}\right)^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)\left(\operatorname{Res}_{s=-k_3}\frac{2^{s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right) -$$

$$4e^{2\pi}\pi^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\left(\frac{(-2)^{k_1}\pi^{2k_1}}{(2k_1)!}+\frac{2^{k_1}\pi^{2k_1}}{(2k_1)!}\right)\left(\operatorname{Res}_{s=-k_2}\frac{\left(-\frac{1}{2}\right)^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)\left(\operatorname{Res}_{s=-k_3}\frac{2^{s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right) +$$

$$2\pi^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\left(\frac{2^{3k_1}\pi^{2k_1}}{(2k_1)!}+\frac{(-1)^{k_1}2^{3k_1}\pi^{2k_1}}{(2k_1)!}\right)\left(\operatorname{Res}_{s=-k_2}\frac{\left(-\frac{1}{8}\right)^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)\left(\operatorname{Res}_{s=-k_3}\frac{8^{s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right) +$$

$$2e^{2\pi}\pi^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\left(\frac{2^{3k_1}\pi^{2k_1}}{(2k_1)!}+\frac{(-1)^{k_1}2^{3k_1}\pi^{2k_1}}{(2k_1)!}\right)\left(\operatorname{Res}_{s=-k_2}\frac{\left(-\frac{1}{8}\right)^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)\left(\operatorname{Res}_{s=-k_3}\frac{8^{s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right) +$$

$$8\pi^2\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\sum_{k_4=0}^{\infty}q^{2k_1}\left(\frac{(-2)^{k_2}\pi^{2k_2}}{(2k_2)!}+\frac{2^{k_2}\pi^{2k_2}}{(2k_2)!}\right)\left(\operatorname{Res}_{s=-k_3}\frac{\left(-\frac{1}{2}\right)^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)\left(\operatorname{Res}_{s=-k_4}\frac{2^{s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right) -$$

$$8e^{2\pi}\pi^2\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\sum_{k_4=0}^{\infty}q^{2k_1}\left(\frac{(-2)^{k_2}\pi^{2k_2}}{(2k_2)!}+\frac{2^{k_2}\pi^{2k_2}}{(2k_2)!}\right)\left(\operatorname{Res}_{s=-k_3}\frac{\left(-\frac{1}{2}\right)^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)\left(\operatorname{Res}_{s=-k_4}\frac{2^{s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)\Bigg)\Bigg) \Big/$$

$$\left(16\left(-1+e^{\pi}\right)\left(1+e^{\pi}\right)\left(\sum_{k=0}^{\infty}\frac{8^k\left(1+(-1)^k\right)\pi^{2k}}{(2k)!}\right)\sum_{k=0}^{\infty}\frac{\left((-2)^k+2^k\right)\pi^{2k}}{(2k)!}\right)$$

for $q = e^{2\pi}$

$$\frac{\pi^3}{32\left(\frac{\pi}{\sqrt{2}}\right)^2} - \frac{\left(2\left(-1-e^{2\pi}\right)\right)\sin\left(\frac{\pi}{\sqrt{2}}\right)\sinh\left(\frac{\pi}{\sqrt{2}}\right)}{\left(e^{2\pi}-1\right)\left(2\cos\left(\frac{2\pi}{\sqrt{2}}\right)+2\cosh\left(\frac{2\pi}{\sqrt{2}}\right)\right)} + \frac{\coth(2\pi)\left(\sin\left(\frac{2\pi}{\sqrt{2}}\right)\sinh\left(\frac{2\pi}{\sqrt{2}}\right)\right)}{2\cos\left(\frac{4\pi}{\sqrt{2}}\right)+2\cosh\left(\frac{4\pi}{\sqrt{2}}\right)} -$$

$$\frac{\pi}{8} = -\frac{\pi}{16} - \frac{\pi^3 \sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\left(\operatorname{Res}_{s=-j_1}\frac{\left(-\frac{1}{2}\right)^{-s}\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)\left(\operatorname{Res}_{s=-j_2}\frac{2^s\,\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)}{2\left(2\sum_{k=0}^{\infty}\frac{(-8)^k\pi^{2k}}{(2k)!}+2\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{(-2)^{-s}\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)} +$$

$$\frac{\pi^3 \sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\left(\operatorname{Res}_{s=-j_1}\frac{\left(-\frac{1}{8}\right)^{-s}\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)\left(\operatorname{Res}_{s=-j_2}\frac{8^s\,\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)}{4\left(-1+e^{2\pi}\right)\left(2\sum_{k=0}^{\infty}\frac{(-2)^k\pi^{2k}}{(2k)!}+2\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{\left(-\frac{1}{2}\right)^{-s}\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)} +$$

$$\frac{e^{2\pi}\pi^3 \sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\left(\operatorname{Res}_{s=-j_1}\frac{\left(-\frac{1}{8}\right)^{-s}\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)\left(\operatorname{Res}_{s=-j_2}\frac{8^s\,\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)}{4\left(-1+e^{2\pi}\right)\left(2\sum_{k=0}^{\infty}\frac{(-2)^k\pi^{2k}}{(2k)!}+2\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{\left(-\frac{1}{2}\right)^{-s}\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)} -$$

$$\frac{\pi^3 \sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty} q^{2k_1}\left(\operatorname{Res}_{s=-k_2}\frac{\left(-\frac{1}{2}\right)^{-s}\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)\left(\operatorname{Res}_{s=-k_3}\frac{2^s\,\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)}{2\sum_{k=0}^{\infty}\frac{(-8)^k\pi^{2k}}{(2k)!}+2\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{(-2)^{-s}\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}}$$

for $q = e^{2\pi}$

$n!$ is the factorial function

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

Integral representations

$$\frac{\pi^3}{32\left(\frac{\pi}{\sqrt{2}}\right)^2} - \frac{\left(2\left(-1-e^{2\pi}\right)\right)\sin\left(\frac{\pi}{\sqrt{2}}\right)\sinh\left(\frac{\pi}{\sqrt{2}}\right)}{\left(e^{2\pi}-1\right)\left(2\cos\left(\frac{2\pi}{\sqrt{2}}\right)+2\cosh\left(\frac{2\pi}{\sqrt{2}}\right)\right)} + \frac{\coth(2\pi)\left(\sin\left(\frac{2\pi}{\sqrt{2}}\right)\sinh\left(\frac{2\pi}{\sqrt{2}}\right)\right)}{2\cos\left(\frac{4\pi}{\sqrt{2}}\right)+2\cosh\left(\frac{4\pi}{\sqrt{2}}\right)} -$$

$$\frac{\pi}{8} = -\left(\left(\left(\pi\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{-\left(2\pi^2\right)/s+s}\left(1+e^{\left(4\pi^2\right)/s}\right)}{\sqrt{s}}\,ds + \right.\right.\right.$$
$$8\pi^2\left(\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{-\pi^2/(2\,s)+s}}{s^{3/2}}\,ds\right)\left(\int_0^1\cosh\left(\sqrt{2}\,\pi\,t\right)dt\right)\int_{\frac{i\pi}{2}}^{2\pi}\operatorname{csch}^2(t)\,dt -$$
$$16\left(\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{-\left(2\pi^2\right)/s+s}\left(1+e^{\left(4\pi^2\right)/s}\right)}{\sqrt{s}}\,ds\right)$$
$$\left.\int_0^1\frac{\left(1+e^{2\pi}\right)\pi^2\cosh\left(\frac{\pi t}{\sqrt{2}}\right)\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{-\pi^2/(8\,s)+s}}{s^{3/2}}\,ds}{4\left(-1+e^{2\pi}\right)\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{-\pi^2/(2\,s)+s}\left(1+e^{\pi^2/s}\right)}{\sqrt{s}}\,ds}\,dt\right)\Bigg/$$
$$\left.\left(16\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{-\left(2\pi^2\right)/s+s}\left(1+e^{\left(4\pi^2\right)/s}\right)}{\sqrt{s}}\,ds\right)\right) \quad \text{for } \gamma > 0$$

$$\frac{\pi^3}{32\left(\frac{\pi}{\sqrt{2}}\right)^2} - \frac{\left(2\left(-1-e^{2\pi}\right)\right)\sin\left(\frac{\pi}{\sqrt{2}}\right)\sinh\left(\frac{\pi}{\sqrt{2}}\right)}{\left(e^{2\pi}-1\right)\left(2\cos\left(\frac{2\pi}{\sqrt{2}}\right)+2\cosh\left(\frac{2\pi}{\sqrt{2}}\right)\right)} + \frac{\coth(2\pi)\left(\sin\left(\frac{2\pi}{\sqrt{2}}\right)\sinh\left(\frac{2\pi}{\sqrt{2}}\right)\right)}{2\cos\left(\frac{4\pi}{\sqrt{2}}\right)+2\cosh\left(\frac{4\pi}{\sqrt{2}}\right)} - \frac{\pi}{8} =$$

$$-\left(\left(8\pi^2\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\pi^2/(2s)+s}}{s^{3/2}}\,ds\right)\left(\int_0^1\cosh\left(\sqrt{2}\,\pi t\right)dt\right)\int_{\frac{i\pi}{2}}^{2\pi}\operatorname{csch}^2(t)\,dt + \pi\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{e^{(2\pi^2)/s+s}}{\sqrt{s}}+\frac{2^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)ds - 16\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{e^{(2\pi^2)/s+s}}{\sqrt{s}}+\frac{2^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)ds\right)\int_0^1\frac{\left(1+e^{2\pi}\right)\pi^2\cosh\left(\frac{\pi t}{\sqrt{2}}\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\pi^2/(8s)+s}}{s^{3/2}}\,ds}{4\left(-1+e^{2\pi}\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{e^{\pi^2/(2s)+s}}{\sqrt{s}}+\frac{2^{s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)ds}\,dt\right)\Bigg/\left(16\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{e^{(2\pi^2)/s+s}}{\sqrt{s}}+\frac{2^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)ds\right)\right) \quad \text{for } 0<\gamma<\frac{1}{2}$$

$$\frac{\pi^3}{32\left(\frac{\pi}{\sqrt{2}}\right)^2} - \frac{\left(2\left(-1-e^{2\pi}\right)\right)\sin\left(\frac{\pi}{\sqrt{2}}\right)\sinh\left(\frac{\pi}{\sqrt{2}}\right)}{\left(e^{2\pi}-1\right)\left(2\cos\left(\frac{2\pi}{\sqrt{2}}\right)+2\cosh\left(\frac{2\pi}{\sqrt{2}}\right)\right)} + \frac{\coth(2\pi)\left(\sin\left(\frac{2\pi}{\sqrt{2}}\right)\sinh\left(\frac{2\pi}{\sqrt{2}}\right)\right)}{2\cos\left(\frac{4\pi}{\sqrt{2}}\right)+2\cosh\left(\frac{4\pi}{\sqrt{2}}\right)} - \frac{\pi}{8} = -\frac{\pi}{16} - \frac{\pi\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\pi^2/(8s)+s}}{s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{\pi^2/(8s)+s}}{s^{3/2}}\,ds}{16\left(-1+e^{2\pi}\right)\left(-\frac{i}{\sqrt{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\pi^2/(2s)+s}}{\sqrt{s}}\,ds + 2\int_{\frac{i\pi}{2}}^{\sqrt{2}\,\pi}\sinh(t)\,dt\right)} - \frac{e^{2\pi}\pi\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\pi^2/(8s)+s}}{s^{3/2}}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{\pi^2/(8s)+s}}{s^{3/2}}\,ds}{16\left(-1+e^{2\pi}\right)\left(-\frac{i}{\sqrt{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\pi^2/(2s)+s}}{\sqrt{s}}\,ds + 2\int_{\frac{i\pi}{2}}^{\sqrt{2}\,\pi}\sinh(t)\,dt\right)} + \frac{\pi\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\pi^2/(2s)+s}}{s^{3/2}}\,ds\right)\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{\pi^2/(2s)+s}}{s^{3/2}}\,ds\right)\int_{\frac{i\pi}{2}}^{2\pi}\operatorname{csch}^2(t)\,dt}{8\left(-\frac{i}{\sqrt{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-(2\pi^2)/s+s}}{\sqrt{s}}\,ds + 2\int_{\frac{i\pi}{2}}^{2\sqrt{2}\,\pi}\sinh(t)\,dt\right)} \quad \text{for } \gamma>0$$

$\text{csch}(x)$ is the hyperbolic cosecant function

From which, after some calculations, we obtain:

-14(-π/16-(2(-1-e^(2π))sin(π/sqrt(2))sinh(π/sqrt(2)))/((e^(2π)-1)(2cos(sqrt(2)π)+2cosh(sqrt(2)π)))+(sin(sqrt(2)π) sinh(sqrt(2)π) coth(2π))/(2cos(2sqrt(2)π)+2cosh(2sqrt(2)π)))-4(MRB const)^(1-1/(4π)+π)

Input

$$-14\left(-\frac{\pi}{16}-\frac{2\left(-1-e^{2\pi}\right)\sin\left(\frac{\pi}{\sqrt{2}}\right)\sinh\left(\frac{\pi}{\sqrt{2}}\right)}{\left(e^{2\pi}-1\right)\left(2\cos\left(\sqrt{2}\,\pi\right)+2\cosh\left(\sqrt{2}\,\pi\right)\right)}+\frac{\sin\left(\sqrt{2}\,\pi\right)\sinh\left(\sqrt{2}\,\pi\right)\coth(2\pi)}{2\cos\left(2\sqrt{2}\,\pi\right)+2\cosh\left(2\sqrt{2}\,\pi\right)}\right)-4\,C_{\text{MRB}}^{1-1/(4\pi)+\pi}$$

$\sinh(x)$ is the hyperbolic sine function
$\cosh(x)$ is the hyperbolic cosine function
$\coth(x)$ is the hyperbolic cotangent function
C_{MRB} is the MRB constant

Exact result

$$-4\,C_{\text{MRB}}^{1-1/(4\pi)+\pi}-14\left(-\frac{\pi}{16}-\frac{2\left(-1-e^{2\pi}\right)\sin\left(\frac{\pi}{\sqrt{2}}\right)\sinh\left(\frac{\pi}{\sqrt{2}}\right)}{\left(e^{2\pi}-1\right)\left(2\cos\left(\sqrt{2}\,\pi\right)+2\cosh\left(\sqrt{2}\,\pi\right)\right)}+\frac{\sin\left(\sqrt{2}\,\pi\right)\sinh\left(\sqrt{2}\,\pi\right)\coth(2\pi)}{2\cos\left(2\sqrt{2}\,\pi\right)+2\cosh\left(2\sqrt{2}\,\pi\right)}\right)$$

Decimal approximation

1.6179990585048698203064385945984120074285458810256459533426326114
...

1.6179990585…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternate forms

$$-4\, {C_{\text{MRB}}}^{1-1/(4\pi)+\pi} - 14\left(-\frac{\pi}{16} - \frac{2\left(-1-e^{2\pi}\right)\sin\left(\frac{\pi}{\sqrt{2}}\right)\sinh\left(\frac{\pi}{\sqrt{2}}\right)}{\left(-1+e^{2\pi}\right)\left(2\cos\left(\sqrt{2}\,\pi\right)+2\cosh\left(\sqrt{2}\,\pi\right)\right)} + \frac{\cosh(2\pi)\sin\left(\sqrt{2}\,\pi\right)\sinh\left(\sqrt{2}\,\pi\right)}{\left(2\cos\left(2\sqrt{2}\,\pi\right)+2\cosh\left(2\sqrt{2}\,\pi\right)\right)\sinh(2\pi)}\right)$$

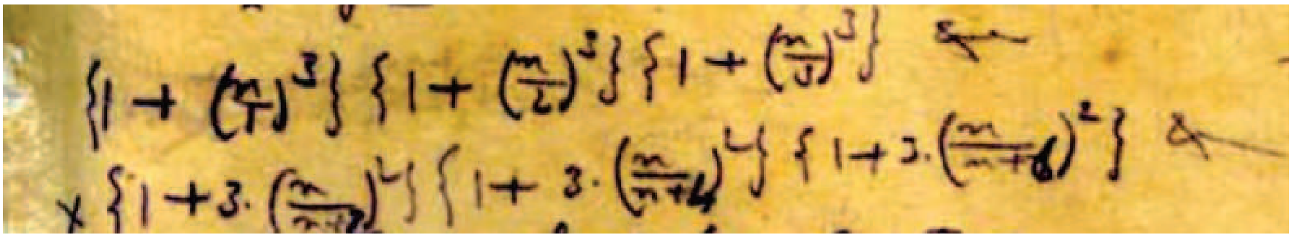
$$-4\, {C_{\text{MRB}}}^{1-1/(4\pi)+\pi} + \frac{7\pi}{8} - \frac{28\sin\left(\frac{\pi}{\sqrt{2}}\right)\sinh\left(\frac{\pi}{\sqrt{2}}\right)}{\left(e^{2\pi}-1\right)\left(2\cos\left(\sqrt{2}\,\pi\right)+2\cosh\left(\sqrt{2}\,\pi\right)\right)} - \frac{28\, e^{2\pi}\sin\left(\frac{\pi}{\sqrt{2}}\right)\sinh\left(\frac{\pi}{\sqrt{2}}\right)}{\left(e^{2\pi}-1\right)\left(2\cos\left(\sqrt{2}\,\pi\right)+2\cosh\left(\sqrt{2}\,\pi\right)\right)} + \frac{14\sin\left(\sqrt{2}\,\pi\right)\sinh(4\pi)\sinh\left(\sqrt{2}\,\pi\right)}{(1-\cosh(4\pi))\left(2\cos\left(2\sqrt{2}\,\pi\right)+2\cosh\left(2\sqrt{2}\,\pi\right)\right)}$$

$$-4\, {C_{\text{MRB}}}^{1-1/(4\pi)+\pi} - 14\left(-\frac{i\left(-1-e^{2\pi}\right)\left(e^{-(i\pi)/\sqrt{2}}-e^{(i\pi)/\sqrt{2}}\right)\left(e^{\pi/\sqrt{2}}-e^{-\pi/\sqrt{2}}\right)}{2\left(e^{2\pi}-1\right)\left(e^{-\sqrt{2}\,\pi}+e^{-i\sqrt{2}\,\pi}+e^{i\sqrt{2}\,\pi}+e^{\sqrt{2}\,\pi}\right)} + \frac{i\left(e^{-2\pi}+e^{2\pi}\right)\left(e^{-i\sqrt{2}\,\pi}-e^{i\sqrt{2}\,\pi}\right)\left(e^{\sqrt{2}\,\pi}-e^{-\sqrt{2}\,\pi}\right)}{4\left(e^{2\pi}-e^{-2\pi}\right)\left(e^{-2\sqrt{2}\,\pi}+e^{-2i\sqrt{2}\,\pi}+e^{2i\sqrt{2}\,\pi}+e^{2\sqrt{2}\,\pi}\right)} - \frac{\pi}{16}\right)$$

Expanded form

$$-4\, {C_{\text{MRB}}}^{1-1/(4\pi)+\pi} + \frac{7\pi}{8} - \frac{28\sin\left(\frac{\pi}{\sqrt{2}}\right)\sinh\left(\frac{\pi}{\sqrt{2}}\right)}{\left(e^{2\pi}-1\right)\left(2\cos\left(\sqrt{2}\,\pi\right)+2\cosh\left(\sqrt{2}\,\pi\right)\right)} - \frac{28\, e^{2\pi}\sin\left(\frac{\pi}{\sqrt{2}}\right)\sinh\left(\frac{\pi}{\sqrt{2}}\right)}{\left(e^{2\pi}-1\right)\left(2\cos\left(\sqrt{2}\,\pi\right)+2\cosh\left(\sqrt{2}\,\pi\right)\right)} - \frac{14\sin\left(\sqrt{2}\,\pi\right)\sinh\left(\sqrt{2}\,\pi\right)\coth(2\pi)}{2\cos\left(2\sqrt{2}\,\pi\right)+2\cosh\left(2\sqrt{2}\,\pi\right)}$$

We have:



From

[1+(n/1)^3] [1+(n/2)^3] [1+(n/3)^3] * [1+3(n/(n+2))^2] [1+3(n/(n+4))^2] [1+3(n/(n+6))^2]

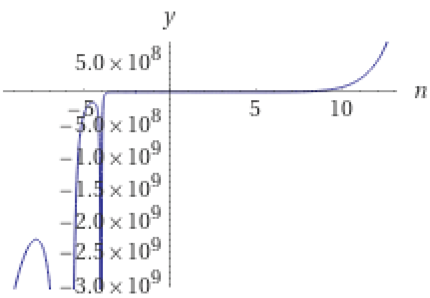
Input

$$\left(1+\left(\frac{n}{1}\right)^3\right)\left(1+\left(\frac{n}{2}\right)^3\right)\left(\left(1+\left(\frac{n}{3}\right)^3\right)\left(1+3\left(\frac{n}{n+2}\right)^2\right)\right)\left(1+3\left(\frac{n}{n+4}\right)^2\right)\left(1+3\left(\frac{n}{n+6}\right)^2\right)$$

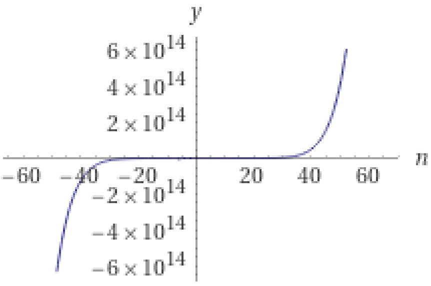
Result

$$\left(\frac{n^3}{27}+1\right)\left(\frac{n^3}{8}+1\right)(n^3+1)\left(\frac{3\,n^2}{(n+2)^2}+1\right)\left(\frac{3\,n^2}{(n+4)^2}+1\right)\left(\frac{3\,n^2}{(n+6)^2}+1\right)$$

Plots **(figures that can be related to the open strings)**



(n from −9.2 to 12.8)



(n from −64.2 to 67.8)

Values

n	1	2	3	4	5
$\left(\frac{n^3}{27}+1\right)\left(\frac{n^3}{8}+1\right)(n^3+1)\left(\frac{3n^2}{(n+2)^2}+1\right)\left(\frac{3n^2}{(n+4)^2}+1\right)\left(\frac{3n^2}{(n+6)^2}+1\right)$	$\frac{832}{225}$	$\frac{4655}{72}$	$\frac{15\,808}{15}$	$\frac{2\,144\,779}{180}$	$\frac{912\,469\,376}{9801}$
approximation	3.69778	64.6528	1053.87	11 915.4	93 099.6

Alternate forms

$$\left(8(n+3)\left(n^2-3n+9\right)\left(n^2+n+1\right)(n+1)\left(n^2-n+1\right)\left(n^2-2n+4\right)\left(n^2+2n+4\right)\left(n^2+3n+9\right)\right)/\left(27(n+4)^2(n+6)^2(n+2)\right)$$

$$\frac{8((n-2)n+4)\left(n^2+n+1\right)\left(n^3+1\right)\left(n^3+27\right)(n(n+2)+4)(n(n+3)+9)}{27(n+2)(n+4)^2(n+6)^2}$$

$$\left(8(n+1)(n+3)\left(n^2-3n+9\right)\left(n^2-2n+4\right)\left(n^2-n+1\right)\left(n^2+n+1\right)\left(n^2+2n+4\right)\left(n^2+3n+9\right)\right)/\left(27(n+2)(n+4)^2(n+6)^2\right)$$

Expanded forms

$$n^3 + \frac{3\,n^2}{n^2+4\,n+4} + \frac{3\,n^5}{n^2+4\,n+4} + \frac{35}{216}\left(n^3 + \frac{3\,n^2}{n^2+4\,n+4} + \frac{3\,n^5}{n^2+4\,n+4} + 1\right)n^3 +$$
$$\frac{1}{216}\left(n^3 + \frac{3\,n^2}{n^2+4\,n+4} + \frac{3\,n^5}{n^2+4\,n+4} + 1\right)n^6 +$$
$$\frac{1}{n^2+8\,n+16}3\left(n^3 + \frac{3\,n^2}{n^2+4\,n+4} + \frac{3\,n^5}{n^2+4\,n+4} + \right.$$
$$\frac{35}{216}\left(n^3 + \frac{3\,n^2}{n^2+4\,n+4} + \frac{3\,n^5}{n^2+4\,n+4} + 1\right)n^3 +$$
$$\left.\frac{1}{216}\left(n^3 + \frac{3\,n^2}{n^2+4\,n+4} + \frac{3\,n^5}{n^2+4\,n+4} + 1\right)n^6 + 1\right)n^2 + \frac{1}{n^2+12\,n+36}$$
$$3\left(n^3 + \frac{3\,n^2}{n^2+4\,n+4} + \frac{3\,n^5}{n^2+4\,n+4} + \frac{35}{216}\left(n^3 + \frac{3\,n^2}{n^2+4\,n+4} + \frac{3\,n^5}{n^2+4\,n+4} + 1\right)\right.$$
$$\left.n^3 + \frac{1}{216}\left(n^3 + \frac{3\,n^2}{n^2+4\,n+4} + \frac{3\,n^5}{n^2+4\,n+4} + 1\right)n^6 + 1\right)n^2 +$$
$$\left(9\left(n^3 + \frac{3\,n^2}{n^2+4\,n+4} + \frac{3\,n^5}{n^2+4\,n+4} + \frac{35}{216}\left(n^3 + \frac{3\,n^2}{n^2+4\,n+4} + \frac{3\,n^5}{n^2+4\,n+4} + 1\right)\right.\right.$$
$$\left.\left.n^3 + \frac{1}{216}\left(n^3 + \frac{3\,n^2}{n^2+4\,n+4} + \frac{3\,n^5}{n^2+4\,n+4} + 1\right)n^6 + 1\right)n^4\right)\Big/$$
$$\left(\left(n^2+8\,n+16\right)\left(n^2+12\,n+36\right)\right) + 1$$

$$\frac{n^{15}}{8\,(n+2)^2\,(n+4)^2\,(n+6)^2} + \frac{n^{13}}{24\,(n+2)^2\,(n+4)^2} + \frac{n^{13}}{24\,(n+2)^2\,(n+6)^2} +$$
$$\frac{n^{13}}{24\,(n+4)^2\,(n+6)^2} + \frac{9\,n^{12}}{2\,(n+2)^2\,(n+4)^2\,(n+6)^2} + \frac{n^{11}}{72\,(n+2)^2} +$$
$$\frac{n^{11}}{72\,(n+4)^2} + \frac{n^{11}}{72\,(n+6)^2} + \frac{3\,n^{10}}{2\,(n+2)^2\,(n+4)^2} + \frac{3\,n^{10}}{2\,(n+2)^2\,(n+6)^2} +$$
$$\frac{3\,n^{10}}{2\,(n+4)^2\,(n+6)^2} + \frac{251\,n^9}{8\,(n+2)^2\,(n+4)^2\,(n+6)^2} + \frac{n^9}{216} + \frac{n^8}{2\,(n+2)^2} +$$
$$\frac{n^8}{2\,(n+4)^2} + \frac{n^8}{2\,(n+6)^2} + \frac{251\,n^7}{24\,(n+2)^2\,(n+4)^2} + \frac{251\,n^7}{24\,(n+2)^2\,(n+6)^2} +$$
$$\frac{251\,n^7}{24\,(n+4)^2\,(n+6)^2} + \frac{27\,n^6}{(n+2)^2\,(n+4)^2\,(n+6)^2} + \frac{n^6}{6} + \frac{251\,n^5}{72\,(n+2)^2} +$$
$$\frac{251\,n^5}{72\,(n+4)^2} + \frac{251\,n^5}{72\,(n+6)^2} + \frac{9\,n^4}{(n+2)^2\,(n+4)^2} + \frac{9\,n^4}{(n+2)^2\,(n+6)^2} +$$
$$\frac{9\,n^4}{(n+4)^2\,(n+6)^2} + \frac{251\,n^3}{216} + \frac{3\,n^2}{(n+2)^2} + \frac{3\,n^2}{(n+4)^2} + \frac{3\,n^2}{(n+6)^2} + 1$$

Alternate form assuming n>0

$$\left(8\left(n^{14} + 4\,n^{13} + 17\,n^{12} + 56\,n^{11} + 189\,n^{10} + 588\,n^{9} + 1055\,n^{8} + 2456\,n^{7} + 3739\,n^{6} + 7588\,n^{5} + 7455\,n^{4} + 7056\,n^{3} + 6588\,n^{2} + 5184\,n + 3888\right)\right) \Big/ \left(27\,(n+2)\,(n+4)^{2}\,(n+6)^{2}\right)$$

Real roots

$$n = -3$$

$$n = -1$$

Complex roots

$$n = -\frac{1}{2} - \frac{i\sqrt{3}}{2}$$

$$n = \frac{1}{2} + \frac{i\sqrt{3}}{2}$$

$$n = \frac{1}{2} - \frac{i\sqrt{3}}{2}$$

$$n = -\frac{1}{2} + \frac{i\sqrt{3}}{2}$$

$$n = -1 - i\sqrt{3}$$

Series expansion at n=0

$$1 + \frac{49\,n^{2}}{48} + \frac{251\,n^{3}}{864} + \frac{1897\,n^{4}}{2304} + O\left(n^{5}\right)$$

(Taylor series)

Series expansion at n=∞

$$\frac{8 n^9}{27} - \frac{16 n^8}{3} + \frac{200 n^7}{3} - \frac{2032 n^6}{3} + \frac{54968 n^5}{9} - 50736 n^4 + \frac{10736248 n^3}{27} - \frac{8954512 n^2}{3} + \frac{65044952 n}{3} - 153517776 + O\left(\left(\frac{1}{n}\right)^1\right)$$

(Taylor series)

Derivative

$$\frac{d}{dn}\left(\left(1+\left(n \times \frac{1}{1}\right)^3\right)\left(1+\left(\frac{n}{2}\right)^3\right)\left(\left(1+\left(\frac{n}{3}\right)^3\right)\left(1+3\left(\frac{n}{n+2}\right)^2\right)\right)\left(1+3\left(\frac{n}{n+4}\right)^2\right)\left(1+3\left(\frac{n}{n+6}\right)^2\right)\right) = \left(8 n \left(3 n^{15} + 54 n^{14} + 381 n^{13} + 1614 n^{12} + 5531 n^{11} + 17054 n^{10} + 48319 n^9 + 117346 n^8 + 224857 n^7 + 377170 n^6 + 577499 n^5 + 773666 n^4 + 874068 n^3 + 599400 n^2 + 339984 n + 127008\right)\right) / \left(9 (n+2)^2 (n+4)^3 (n+6)^3\right)$$

Indefinite integral

$$\int \left(1+\frac{n^3}{27}\right)\left(1+\frac{n^3}{8}\right)\left(1+n^3\right)\left(1+\frac{3 n^2}{(2+n)^2}\right)\left(1+\frac{3 n^2}{(4+n)^2}\right)\left(1+\frac{3 n^2}{(6+n)^2}\right) dn = \frac{4 n^{10}}{135} - \frac{16 n^9}{27} + \frac{25 n^8}{3} - \frac{2032 n^7}{21} + \frac{27484 n^6}{27} - \frac{50736 n^5}{5} + \frac{2684062 n^4}{27} - \frac{8954512 n^3}{9} + \frac{32522476 n^2}{3} - 153517776 n + \frac{4902352}{n+4} + \frac{917050680}{n+6} - \frac{1862}{3} \log(n+2) + \frac{63820120}{3} \log(n+4) + 1043878626 \log(n+6) - \frac{22702282160}{63} + \text{constant}$$

(assuming a complex-valued logarithm)

$\log(x)$ is the natural logarithm

Local minima

$$\min\left\{\left(1+\left(n\,\frac{1}{1}\right)^3\right)\left(1+\left(\frac{n}{2}\right)^3\right)\left(\left(1+\left(\frac{n}{3}\right)^3\right)\left(1+3\left(\frac{n}{n+2}\right)^2\right)\right)\left(1+3\left(\frac{n}{n+4}\right)^2\right)\left(1+3\left(\frac{n}{n+6}\right)^2\right)\right\}=1 \text{ at } n=0$$

$$\min\left\{\left(1+\left(n\,\frac{1}{1}\right)^3\right)\left(1+\left(\frac{n}{2}\right)^3\right)\left(\left(1+\left(\frac{n}{3}\right)^3\right)\left(1+3\left(\frac{n}{n+2}\right)^2\right)\right)\left(1+3\left(\frac{n}{n+4}\right)^2\right)\left(1+3\left(\frac{n}{n+6}\right)^2\right)\right\}\approx 7674.7 \text{ at } n\approx -2.2297$$

From the result

$$\left(\frac{n^3}{27}+1\right)\left(\frac{n^3}{8}+1\right)(n^3+1)\left(\frac{3\,n^2}{(n+2)^2}+1\right)\left(\frac{3\,n^2}{(n+4)^2}+1\right)\left(\frac{3\,n^2}{(n+6)^2}+1\right)$$

for n = 11915.4 :

((11915.4)^3/27+1) ((11915.4)^3/8+1) ((11915.4)^3+1) ((3(11915.4)^2)/((11915.4)+2)^2+1)((3(11915.4)^2)/((11915.4)+4)^2+1)((3 (11915.4)^2)/((11915.4)+6)^2+1)

Input interpretation

$$\left(\frac{11\,915.4^3}{27}+1\right)\left(\frac{11\,915.4^3}{8}+1\right)(11\,915.4^3+1)\left(\left(\frac{3\times 11\,915.4^2}{(11\,915.4+2)^2}+1\right)\left(\frac{3\times 11\,915.4^2}{(11\,915.4+4)^2}+1\right)\left(\frac{3\times 11\,915.4^2}{(11\,915.4+6)^2}+1\right)\right)$$

Result

$1.432346147322289081891519319834100439820998720930729125726 01\ldots \times 10^{36}$

1.43234614732...*10^{36}

From which, after some calculations:

(144+((3 π)/11))*1/(ln(((11915.4)^3/27+1) ((11915.4)^3/8+1) ((11915.4)^3+1) ((3(11915.4)^2)/((11915.4)+2)^2+1)((3(11915.4)^2)/((11915.4)+4)^2+1)((3 (11915.4)^2)/((11915.4)+6)^2+1))+2π)

Input interpretation

$$\left(144+\frac{3\pi}{11}\right)\times 1\Big/\left(\log\left(\left(\frac{11915.4^3}{27}+1\right)\left(\frac{11915.4^3}{8}+1\right)\left(11915.4^3+1\right)\left(\left(\frac{3\times 11915.4^2}{(11915.4+2)^2}+1\right)\left(\frac{3\times 11915.4^2}{(11915.4+4)^2}+1\right)\left(\frac{3\times 11915.4^2}{(11915.4+6)^2}+1\right)\right)\right)+2\pi\right)$$

log(x) is the natural logarithm

Result

1.617868856637355900167565915982289497581010373426263602098829273 7 …

1.6178688566…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternative representations

$$\left(144+\frac{3\pi}{11}\right)\Big/\left(\log\left(\left(\frac{11915.4^3}{27}+1\right)\left(\frac{3\times 11915.4^2}{(11915.4+2)^2}+1\right)\left(\left(\frac{3\times 11915.4^2}{(11915.4+4)^2}+1\right)\left(\frac{3\times 11915.4^2}{(11915.4+6)^2}+1\right)\right)\left(\frac{11915.4^3}{8}+1\right)\left(11915.4^3+1\right)\right)+2\pi\right)=$$

$$\left(144+\frac{3\pi}{11}\right)\Big/\left(2\pi+\log_e\left(\left(1+11915.4^3\right)\left(1+\frac{11915.4^3}{8}\right)\left(1+\frac{11915.4^3}{27}\right)\left(1+\frac{3\times 11915.4^2}{11917.4^2}\right)\left(1+\frac{3\times 11915.4^2}{11919.4^2}\right)\left(1+\frac{3\times 11915.4^2}{11921.4^2}\right)\right)\right)$$

$$\left(144+\frac{3\pi}{11}\right)\Big/\left(\log\left(\left(\frac{11915.4^3}{27}+1\right)\left(\frac{3\times 11915.4^2}{(11915.4+2)^2}+1\right)\left(\left(\frac{3\times 11915.4^2}{(11915.4+4)^2}+1\right)\left(\frac{3\times 11915.4^2}{(11915.4+6)^2}+1\right)\right)\left(\frac{11915.4^3}{8}+1\right)\left(11915.4^3+1\right)\right)+2\pi\right)=$$
$$\left(144+\frac{3\pi}{11}\right)\Big/\left(2\pi+\log(a)\log_a\left(\left(1+11915.4^3\right)\left(1+\frac{11915.4^3}{8}\right)\left(1+\frac{11915.4^3}{27}\right)\left(1+\frac{3\times 11915.4^2}{11917.4^2}\right)\left(1+\frac{3\times 11915.4^2}{11919.4^2}\right)\left(1+\frac{3\times 11915.4^2}{11921.4^2}\right)\right)\right)$$

$$\left(144+\frac{3\pi}{11}\right)\Big/\left(\log\left(\left(\frac{11915.4^3}{27}+1\right)\left(\frac{3\times 11915.4^2}{(11915.4+2)^2}+1\right)\left(\left(\frac{3\times 11915.4^2}{(11915.4+4)^2}+1\right)\left(\frac{3\times 11915.4^2}{(11915.4+6)^2}+1\right)\right)\left(\frac{11915.4^3}{8}+1\right)\left(11915.4^3+1\right)\right)+2\pi\right)=$$
$$\left(144+\frac{3\pi}{11}\right)\Big/\left(2\pi-\text{Li}_1\left(1-\left(1+11915.4^3\right)\left(1+\frac{11915.4^3}{8}\right)\left(1+\frac{11915.4^3}{27}\right)\left(1+\frac{3\times 11915.4^2}{11917.4^2}\right)\left(1+\frac{3\times 11915.4^2}{11919.4^2}\right)\left(1+\frac{3\times 11915.4^2}{11921.4^2}\right)\right)\right)$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations

$$\left(144+\frac{3\pi}{11}\right)\Big/\left(\log\left(\left(\frac{11915.4^3}{27}+1\right)\left(\frac{3\times 11915.4^2}{(11915.4+2)^2}+1\right)\left(\left(\frac{3\times 11915.4^2}{(11915.4+4)^2}+1\right)\left(\frac{3\times 11915.4^2}{(11915.4+6)^2}+1\right)\right)\left(\frac{11915.4^3}{8}+1\right)\left(11915.4^3+1\right)\right)+2\pi\right)=$$
$$\frac{3(528+\pi)}{11\left(2\pi+\log\left(1.43235\times 10^{36}\right)-\sum_{k=1}^{\infty}\frac{(-1)^k e^{-83.2524k}}{k}\right)}$$

$$\left(144+\frac{3\pi}{11}\right)\Big/\left(\log\left(\left(\frac{11915.4^3}{27}+1\right)\left(\frac{3\times 11915.4^2}{(11915.4+2)^2}+1\right)\left(\left(\frac{3\times 11915.4^2}{(11915.4+4)^2}+1\right)\left(\frac{3\times 11915.4^2}{(11915.4+6)^2}+1\right)\right)\left(\frac{11915.4^3}{8}+1\right)\left(11915.4^3+1\right)\right)+2\pi\right)= \frac{3(528+\pi)}{11\left(2\pi+2i\pi\left\lfloor\frac{\arg(1.43235\times 10^{36}-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(1.43235\times 10^{36}-x)^k x^{-k}}{k}\right)} \text{ for } x<0$$

$$\left(144+\frac{3\pi}{11}\right)\Big/\left(\log\left(\left(\frac{11915.4^3}{27}+1\right)\left(\frac{3\times 11915.4^2}{(11915.4+2)^2}+1\right)\left(\left(\frac{3\times 11915.4^2}{(11915.4+4)^2}+1\right)\left(\frac{3\times 11915.4^2}{(11915.4+6)^2}+1\right)\right)\left(\frac{11915.4^3}{8}+1\right)\left(11915.4^3+1\right)\right)+2\pi\right)=(3(528+\pi))\Big/\left(11\left(2\pi+\log(z_0)+\left\lfloor\frac{\arg(1.43235\times 10^{36}-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(1.43235\times 10^{36}-z_0)^k z_0^{-k}}{k}\right)\right)$$

$\arg(z)$ is the complex argument
$\lfloor x\rfloor$ is the floor function
i is the imaginary unit

Integral representations

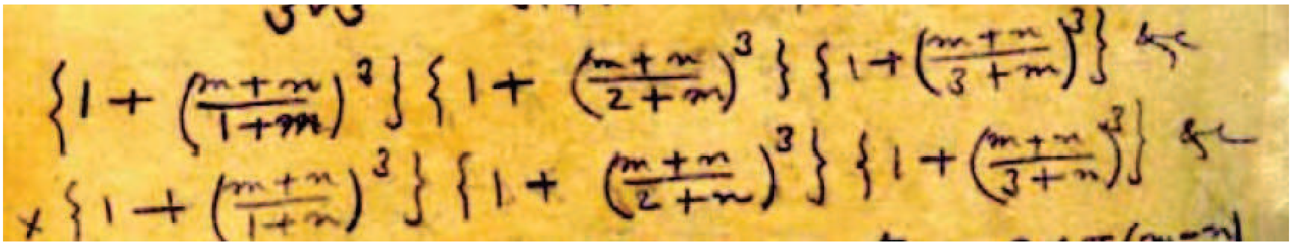
$$\left(144+\frac{3\pi}{11}\right)\Big/\left(\log\left(\left(\frac{11915.4^3}{27}+1\right)\left(\frac{3\times 11915.4^2}{(11915.4+2)^2}+1\right)\left(\left(\frac{3\times 11915.4^2}{(11915.4+4)^2}+1\right)\left(\frac{3\times 11915.4^2}{(11915.4+6)^2}+1\right)\right)\left(\frac{11915.4^3}{8}+1\right)\left(11915.4^3+1\right)\right)+2\pi\right)=\frac{3(528+\pi)}{11\left(2\pi+\int_1^{1.43235\times 10^{36}}\frac{1}{t}\,dt\right)}$$

$$\left(144+\frac{3\pi}{11}\right)\Big/\left(\log\left(\left(\frac{11915.4^3}{27}+1\right)\left(\frac{3\times 11915.4^2}{(11915.4+2)^2}+1\right)\left(\left(\frac{3\times 11915.4^2}{(11915.4+4)^2}+1\right)\left(\frac{3\times 11915.4^2}{(11915.4+6)^2}+1\right)\right)\left(\frac{11915.4^3}{8}+1\right)(11915.4^3+1)\right)+2\pi\right)=$$

$$\frac{6\,i\,\pi\,(528+\pi)}{11\left(4\,i\,\pi^2+\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{-83.2524\,s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)}\quad \text{for } -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

We have:



$$\left\{1+\left(\frac{m+n}{1+m}\right)^3\right\}\left\{1+\left(\frac{m+n}{2+m}\right)^3\right\}\left\{1+\left(\frac{m+n}{3+m}\right)^3\right\} \&c$$
$$\times\left\{1+\left(\frac{m+n}{1+n}\right)^3\right\}\left\{1+\left(\frac{m+n}{2+n}\right)^3\right\}\left\{1+\left(\frac{m+n}{3+n}\right)^3\right\} \&c$$

[1+((m+n)/(1+m))^3] [1+((m+n)/(2+m))^3] [1+((m+n)/(3+m))^3] *
[1+((m+n)/(1+n))^3] [1+((m+n)/(2+n))^3] [1+((m+n)/(3+n))^3]

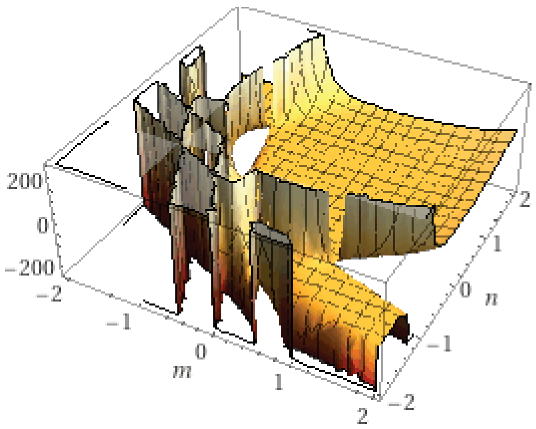
Input

$$\left(1+\left(\frac{m+n}{1+m}\right)^3\right)\left(1+\left(\frac{m+n}{2+m}\right)^3\right)\left(\left(1+\left(\frac{m+n}{3+m}\right)^3\right)\left(1+\left(\frac{m+n}{1+n}\right)^3\right)\right)\left(1+\left(\frac{m+n}{2+n}\right)^3\right)\left(1+\left(\frac{m+n}{3+n}\right)^3\right)$$

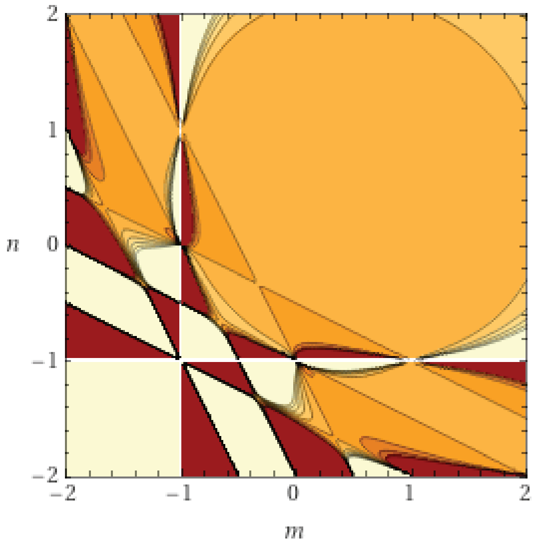
Result

$$\left(\frac{(m+n)^3}{(m+1)^3}+1\right)\left(\frac{(m+n)^3}{(m+2)^3}+1\right)\left(\frac{(m+n)^3}{(m+3)^3}+1\right)\left(\frac{(m+n)^3}{(n+1)^3}+1\right)\left(\frac{(m+n)^3}{(n+2)^3}+1\right)\left(\frac{(m+n)^3}{(n+3)^3}+1\right)$$

3D plot **(figure that can be related to a D-brane/Instanton)**



Contour plot



From the result

$$\left(\frac{(m+n)^3}{(m+1)^3}+1\right)\left(\frac{(m+n)^3}{(m+2)^3}+1\right)\left(\frac{(m+n)^3}{(m+3)^3}+1\right)\left(\frac{(m+n)^3}{(n+1)^3}+1\right)\left(\frac{(m+n)^3}{(n+2)^3}+1\right)\left(\frac{(m+n)^3}{(n+3)^3}+1\right)$$

for m = 4 and n = 8 :

((4 + 8)^3/(4 + 1)^3 + 1) ((4 + 8)^3/(4 + 2)^3 + 1) ((4 + 8)^3/(4 + 3)^3 + 1) ((4 + 8)^3/(8 + 1)^3 + 1) ((4 + 8)^3/(8 + 2)^3 + 1) ((4 + 8)^3/(8 + 3)^3 + 1)

Input

$$\left(\frac{(4+8)^3}{(4+1)^3}+1\right)\left(\frac{(4+8)^3}{(4+2)^3}+1\right)\left(\frac{(4+8)^3}{(4+3)^3}+1\right)\left(\frac{(4+8)^3}{(8+1)^3}+1\right)\left(\frac{(4+8)^3}{(8+2)^3}+1\right)\left(\frac{(4+8)^3}{(8+3)^3}+1\right)$$

Exact result

$$\frac{675837057493}{39703125}$$

Decimal approximation

17022.263549607241243604879968516332152695789059425423061786698150
...

17022.2635496….

From which, after some calculations:

(1/Pi^2)(((4 + 8)^3/(4 + 1)^3 + 1) ((4 + 8)^3/(4 + 2)^3 + 1) ((4 + 8)^3/(4 + 3)^3 + 1) ((4 + 8)^3/(8 + 1)^3 + 1) ((4 + 8)^3/(8 + 2)^3 + 1) ((4 + 8)^3/(8 + 3)^3 + 1))+e+φ

Input

$$\frac{1}{\pi^2}\left(\left(\frac{(4+8)^3}{(4+1)^3}+1\right)\left(\frac{(4+8)^3}{(4+2)^3}+1\right)\left(\frac{(4+8)^3}{(4+3)^3}+1\right)\left(\frac{(4+8)^3}{(8+1)^3}+1\right)\left(\frac{(4+8)^3}{(8+2)^3}+1\right)\left(\frac{(4+8)^3}{(8+3)^3}+1\right)\right)+e+\phi$$

ϕ is the golden ratio

Result

$$\phi + e + \frac{675\,837\,057\,493}{39\,703\,125\,\pi^2}$$

Decimal approximation

1729.0522069352366416370989701793976800365247591439629739416301850

...

1729.05220693….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve (1728 = 8^2 * 3^3). The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1\,351\,674\,114\,986 + 79\,406\,250\,e\,\pi^2 + 39\,703\,125\,\pi^2 + 39\,703\,125\sqrt{5}\,\pi^2}{79\,406\,250\,\pi^2}$$

$$\frac{1}{2}\left(1+\sqrt{5}\right) + e + \frac{675\,837\,057\,493}{39\,703\,125\,\pi^2}$$

$$\frac{39\,703\,125\,\pi^2\,\phi + 675\,837\,057\,493 + 39\,703\,125\,e\,\pi^2}{39\,703\,125\,\pi^2}$$

Alternative representations

$$\frac{\left(\frac{(4+8)^3}{(4+1)^3}+1\right)\left(\frac{(4+8)^3}{(4+2)^3}+1\right)\left(\frac{(4+8)^3}{(4+3)^3}+1\right)\left(\frac{(4+8)^3}{(8+1)^3}+1\right)\left(\frac{(4+8)^3}{(8+2)^3}+1\right)\left(\frac{(4+8)^3}{(8+3)^3}+1\right)}{\pi^2}+e+\phi = e+2\cos\left(\frac{\pi}{5}\right)+\frac{\left(1+\frac{12^3}{5^3}\right)\left(1+\frac{12^3}{6^3}\right)\left(1+\frac{12^3}{7^3}\right)\left(1+\frac{12^3}{9^3}\right)\left(1+\frac{12^3}{10^3}\right)\left(1+\frac{12^3}{11^3}\right)}{(180\,°)^2}$$

$$\frac{\left(\frac{(4+8)^3}{(4+1)^3}+1\right)\left(\frac{(4+8)^3}{(4+2)^3}+1\right)\left(\frac{(4+8)^3}{(4+3)^3}+1\right)\left(\frac{(4+8)^3}{(8+1)^3}+1\right)\left(\frac{(4+8)^3}{(8+2)^3}+1\right)\left(\frac{(4+8)^3}{(8+3)^3}+1\right)}{\pi^2}+e+\phi = e+\frac{\left(1+\frac{12^3}{5^3}\right)\left(1+\frac{12^3}{6^3}\right)\left(1+\frac{12^3}{7^3}\right)\left(1+\frac{12^3}{9^3}\right)\left(1+\frac{12^3}{10^3}\right)\left(1+\frac{12^3}{11^3}\right)}{\pi^2}+ \boxed{\text{root of } -1-x+x^2 \text{ near } x=1.61803}$$

$$\frac{\left(\frac{(4+8)^3}{(4+1)^3}+1\right)\left(\frac{(4+8)^3}{(4+2)^3}+1\right)\left(\frac{(4+8)^3}{(4+3)^3}+1\right)\left(\frac{(4+8)^3}{(8+1)^3}+1\right)\left(\frac{(4+8)^3}{(8+2)^3}+1\right)\left(\frac{(4+8)^3}{(8+3)^3}+1\right)}{\pi^2}+e+\phi = e+\frac{\left(1+\frac{12^3}{5^3}\right)\left(1+\frac{12^3}{6^3}\right)\left(1+\frac{12^3}{7^3}\right)\left(1+\frac{12^3}{9^3}\right)\left(1+\frac{12^3}{10^3}\right)\left(1+\frac{12^3}{11^3}\right)}{(180\,°)^2}+ \boxed{\text{root of } -1-x+x^2 \text{ near } x=1.61803}$$

Series representations

$$\frac{\left(\frac{(4+8)^3}{(4+1)^3}+1\right)\left(\frac{(4+8)^3}{(4+2)^3}+1\right)\left(\frac{(4+8)^3}{(4+3)^3}+1\right)\left(\frac{(4+8)^3}{(8+1)^3}+1\right)\left(\frac{(4+8)^3}{(8+2)^3}+1\right)\left(\frac{(4+8)^3}{(8+3)^3}+1\right)}{\pi^2}+e+\phi = \phi+\frac{675\,837\,057\,493}{39\,703\,125\,\pi^2}+\sum_{k=0}^{\infty}\frac{1}{k!}$$

$$\frac{\left(\frac{(4+8)^3}{(4+1)^3}+1\right)\left(\frac{(4+8)^3}{(4+2)^3}+1\right)\left(\frac{(4+8)^3}{(4+3)^3}+1\right)\left(\frac{(4+8)^3}{(8+1)^3}+1\right)\left(\frac{(4+8)^3}{(8+2)^3}+1\right)\left(\frac{(4+8)^3}{(8+3)^3}+1\right)}{\pi^2}+e+\phi=$$
$$e+\phi+\frac{675\,837\,057\,493}{635\,250\,000\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^2}$$

$$\frac{\left(\frac{(4+8)^3}{(4+1)^3}+1\right)\left(\frac{(4+8)^3}{(4+2)^3}+1\right)\left(\frac{(4+8)^3}{(4+3)^3}+1\right)\left(\frac{(4+8)^3}{(8+1)^3}+1\right)\left(\frac{(4+8)^3}{(8+2)^3}+1\right)\left(\frac{(4+8)^3}{(8+3)^3}+1\right)}{\pi^2}+e+\phi=$$
$$\phi+\frac{675\,837\,057\,493}{39\,703\,125\,\pi^2}+\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}$$

$n!$ is the factorial function

Integral representations

$$\frac{\left(\frac{(4+8)^3}{(4+1)^3}+1\right)\left(\frac{(4+8)^3}{(4+2)^3}+1\right)\left(\frac{(4+8)^3}{(4+3)^3}+1\right)\left(\frac{(4+8)^3}{(8+1)^3}+1\right)\left(\frac{(4+8)^3}{(8+2)^3}+1\right)\left(\frac{(4+8)^3}{(8+3)^3}+1\right)}{\pi^2}+e+\phi=$$
$$e+\phi+\frac{675\,837\,057\,493}{635\,250\,000\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2}$$

$$\frac{\left(\frac{(4+8)^3}{(4+1)^3}+1\right)\left(\frac{(4+8)^3}{(4+2)^3}+1\right)\left(\frac{(4+8)^3}{(4+3)^3}+1\right)\left(\frac{(4+8)^3}{(8+1)^3}+1\right)\left(\frac{(4+8)^3}{(8+2)^3}+1\right)\left(\frac{(4+8)^3}{(8+3)^3}+1\right)}{\pi^2}+e+\phi=$$
$$e+\phi+\frac{675\,837\,057\,493}{158\,812\,500\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^2}$$

$$\frac{\left(\frac{(4+8)^3}{(4+1)^3}+1\right)\left(\frac{(4+8)^3}{(4+2)^3}+1\right)\left(\frac{(4+8)^3}{(4+3)^3}+1\right)\left(\frac{(4+8)^3}{(8+1)^3}+1\right)\left(\frac{(4+8)^3}{(8+2)^3}+1\right)\left(\frac{(4+8)^3}{(8+3)^3}+1\right)}{\pi^2}+e+\phi=$$
$$e+\phi+\frac{675\,837\,057\,493}{158\,812\,500\left(\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt\right)^2}$$

((1/Pi^2)(((4+8)^3/(4+1)^3+1)((4+8)^3/(4+2)^3+1)((4+8)^3/(4+3)^3+1)((4 + 8)^3/(8+1)^3+1)((4+8)^3/(8+2)^3+1)((4+8)^3/(8+3)^3+1))+e+φ)^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\left(\frac{1}{\pi^2}\left(\left(\frac{(4+8)^3}{(4+1)^3}+1\right)\left(\frac{(4+8)^3}{(4+2)^3}+1\right)\left(\frac{(4+8)^3}{(4+3)^3}+1\right)\left(\frac{(4+8)^3}{(8+1)^3}+1\right)\left(\frac{(4+8)^3}{(8+2)^3}+1\right)\left(\frac{(4+8)^3}{(8+3)^3}+1\right)\right)+e+\phi\right)\hat{}\,(1/15)+C_{\text{MRB}}{}^{1-1/(4\pi)+\pi}$$

ϕ is the golden ratio
C_{MRB} is the MRB constant

Exact result

$$C_{\text{MRB}}{}^{1-1/(4\pi)+\pi}+\sqrt[15]{\phi+e+\frac{675837057493}{39703125\,\pi^2}}$$

Decimal approximation

1.644941333069760842085099750433671984360909401172500413854920568 2
...

1.644941333…. ≈ ζ(2) = π²/6 = 1.644934 (trace of the instanton shape)

Alternate forms

$$C_{\text{MRB}}{}^{1-1/(4\pi)+\pi}+\frac{\sqrt[15]{\frac{1}{21}\left(39703125\,\pi^2\,\phi+675837057493+39703125\,e\,\pi^2\right)}}{5^{2/5}\,(11\,\pi)^{2/15}}$$

$$C_{\text{MRB}}{}^{1-1/(4\pi)+\pi}+\frac{\sqrt[15]{\frac{1}{42}\left(1351674114986+39703125\,\pi^2+39703125\sqrt{5}\,\pi^2+79406250\,e\,\pi^2\right)}}{5^{2/5}\,(11\,\pi)^{2/15}}$$

$$\frac{5^{2/5}\times 11^{2/15}\sqrt[15]{21}\ C_{\text{MRB}}{}^{1-1/(4\pi)+\pi}+\frac{\sqrt[15]{39\,703\,125\pi^2\,\phi+675\,837\,057\,493+39\,703\,125\,e\,\pi^2}}{\pi^{2/15}}}{5^{2/5}\times 11^{2/15}\sqrt[15]{21}}$$

Expanded forms

$$C_{\text{MRB}}{}^{1-1/(4\pi)+\pi}+\sqrt[15]{\frac{1}{2}\left(1+\sqrt{5}\right)+e+\frac{675\,837\,057\,493}{39\,703\,125\pi^2}}$$

$$C_{\text{MRB}}{}^{1-1/(4\pi)+\pi}+\sqrt[15]{\frac{1}{2}+\frac{\sqrt{5}}{2}+e+\frac{675\,837\,057\,493}{39\,703\,125\,\pi^2}}$$

(1/27(((1/Pi^2)(((4+8)^3/(4+1)^3+1)((4+8)^3/(4+2)^3+1)((4+8)^3/(4+3)^3+1)((4+8)^3/(8+1)^3+1)((4+8)^3/(8+2)^3+1)((4+8)^3/(8+3)^3+1))+e+φ)-1))^2-3/2MRB const

Input

$$\left(\frac{1}{27}\left(\left(\frac{1}{\pi^2}\left(\left(\frac{(4+8)^3}{(4+1)^3}+1\right)\left(\frac{(4+8)^3}{(4+2)^3}+1\right)\left(\frac{(4+8)^3}{(4+3)^3}+1\right)\left(\frac{(4+8)^3}{(8+1)^3}+1\right)\left(\frac{(4+8)^3}{(8+2)^3}+1\right)\left(\frac{(4+8)^3}{(8+3)^3}+1\right)\right)+e+\phi\right)-1\right)\right)^2-\frac{3}{2}C_{\text{MRB}}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Result

$$\frac{1}{729}\left(\phi-1+e+\frac{675\,837\,057\,493}{39\,703\,125\,\pi^2}\right)^2-\frac{3\,C_{\text{MRB}}}{2}$$

Decimal approximation

4095.965713819903497811682407705656911955973410638090365058162497 3
…

4095.9657138199…. ≈ 4096 = 64^2 , that multiplied by 2 give 8192, indeed:

The total amplitude vanishes for gauge group SO(8192), while the vacuum energy is negative and independent of the gauge group.

The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). (From: "Dilaton Tadpole for the Open Bosonic String " Michael R. Douglas and Benjamin Grinstein - September 2,1986)

Alternate forms

$$\big(-3\,447\,451\,500\,732\,421\,875\,\pi^4\,C_{\text{MRB}} + 913\,511\,456\,561\,593\,174\,890\,098 - 53\,665\,686\,346\,553\,531\,250\,\pi^2 + 53\,665\,686\,346\,553\,531\,250\sqrt{5}\,\pi^2 + 107\,331\,372\,693\,107\,062\,500\,e\,\pi^2 + 4\,729\,014\,404\,296\,875\,\pi^4 - 1\,576\,338\,134\,765\,625\sqrt{5}\,\pi^4 - 3\,152\,676\,269\,531\,250\,e\,\pi^4 + 3\,152\,676\,269\,531\,250\sqrt{5}\,e\,\pi^4 + 3\,152\,676\,269\,531\,250\,e^2\,\pi^4\big) / \left(2\,298\,301\,000\,488\,281\,250\,\pi^4\right)$$

$$\frac{1}{729}\left(\frac{1}{2}\left(\sqrt{5}-1\right)+e+\frac{675\,837\,057\,493}{39\,703\,125\,\pi^2}\right)^2-\frac{3\,C_{\text{MRB}}}{2}$$

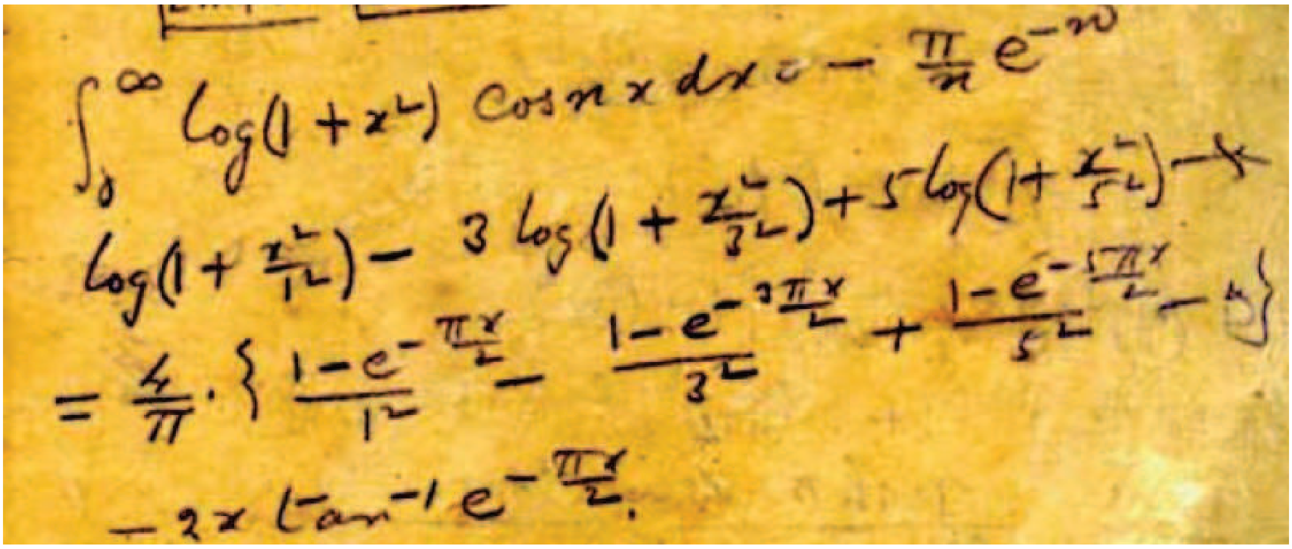
$$\frac{1}{729}\left(-1+\frac{1}{2}\left(1+\sqrt{5}\right)+e+\frac{675\,837\,057\,493}{39\,703\,125\,\pi^2}\right)^2-\frac{3\,C_{\text{MRB}}}{2}$$

Expanded forms

$$-\frac{3\,C_{\text{MRB}}}{2}+\frac{1}{486}-\frac{\sqrt{5}}{1458}-\frac{e}{729}+\frac{\sqrt{5}\,e}{729}+\frac{e^2}{729}+\frac{456\,755\,728\,280\,796\,587\,445\,049}{1\,149\,150\,500\,244\,140\,625\,\pi^4}-\frac{675\,837\,057\,493}{28\,943\,578\,125\,\pi^2}+\frac{675\,837\,057\,493}{5\,788\,715\,625\sqrt{5}\,\pi^2}+\frac{1\,351\,674\,114\,986\,e}{28\,943\,578\,125\,\pi^2}$$

$$-\frac{3\,C_{\text{MRB}}}{2}-\frac{2\,\phi}{729}+\frac{2\,e\,\phi}{729}+\frac{\phi^2}{729}+\frac{1\,351\,674\,114\,986\,\phi}{28\,943\,578\,125\,\pi^2}+\frac{1}{729}-\frac{2\,e}{729}+\frac{e^2}{729}+\frac{456\,755\,728\,280\,796\,587\,445\,049}{1\,149\,150\,500\,244\,140\,625\,\pi^4}-\frac{1\,351\,674\,114\,986}{28\,943\,578\,125\,\pi^2}+\frac{1\,351\,674\,114\,986\,e}{28\,943\,578\,125\,\pi^2}$$

From:



4/Pi [(1-e^((-Pi*x)/2))/(1^2) – (1-e^((-3Pi*x)/2))/(3^2) + (1-e^((-5Pi*x)/2))/(5^2)] – 2x tan^-1(e^((-Pi*x)/2))

Input

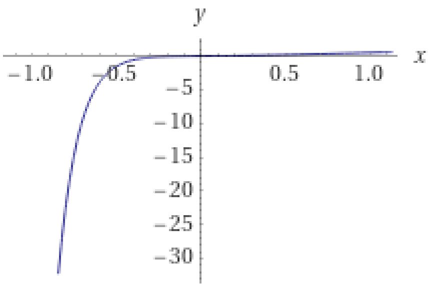
$$\frac{4}{\pi}\left(\frac{1-e^{-(\pi x)/2}}{1^2}-\frac{1-e^{1/2(-3\pi x)}}{3^2}+\frac{1-e^{1/2(-5\pi x)}}{5^2}\right)-(2\,x)\tan^{-1}\left(e^{-(\pi x)/2}\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

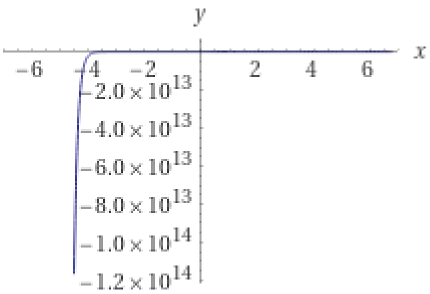
Exact result

$$\frac{4\left(\frac{1}{25}\left(1-e^{-(5\pi x)/2}\right)-e^{-(\pi x)/2}+\frac{1}{9}\left(e^{-(3\pi x)/2}-1\right)+1\right)}{\pi}-2\,x\tan^{-1}\left(e^{-(\pi x)/2}\right)$$

Plots **(figures that can be related to the open strings)**



(x from −1.1 to 1.1)



(x from −6.7 to 6.7)

Alternate forms

$$\frac{4\,e^{-(5\pi x)/2}\left(25\,e^{\pi x}-225\,e^{2\pi x}+209\,e^{(5\pi x)/2}-9\right)}{225\,\pi}-2\,x\tan^{-1}\left(e^{-(\pi x)/2}\right)$$

$$-\frac{2\left(2\left(9\,e^{-(5\pi x)/2}-25\,e^{-(3\pi x)/2}+225\,e^{-(\pi x)/2}-209\right)+225\,\pi\,x\tan^{-1}\left(e^{-(\pi x)/2}\right)\right)}{225\,\pi}$$

$$-\frac{1}{225\,\pi}2\,e^{-(5\pi x)/2}\left(-50\,e^{\pi x}+450\,e^{2\pi x}-418\,e^{(5\pi x)/2}+225\,\pi\,e^{(5\pi x)/2}\,x\tan^{-1}\left(e^{-(\pi x)/2}\right)+18\right)$$

Expanded form

$$-\frac{4\,e^{-(5\pi x)/2}}{25\,\pi}+\frac{4\,e^{-(3\pi x)/2}}{9\,\pi}-\frac{4\,e^{-(\pi x)/2}}{\pi}-2\,x\tan^{-1}\left(e^{-(\pi x)/2}\right)+\frac{836}{225\,\pi}$$

Series expansion at x=0

$$\left(\frac{26}{15}-\frac{\pi}{2}\right)x+\frac{\pi^2 x^3}{4}-\frac{19\pi^3 x^4}{96}+\frac{33\pi^4 x^5}{320}+O\left(x^6\right)$$

(Taylor series)

Series expansion at x = -i

$$\left\lfloor \frac{\arg\left(-\frac{e^{-(\pi x)/2}\left(e^{(\pi x)/2}\pi x-2e^{(\pi x)/2}+i e^{(\pi x)/2}\pi-2i\right)}{x+i}\right)}{2\pi} \right\rfloor \left(\log\left(\frac{2}{\pi}\right)+i\log\left(\frac{2}{\pi}\right)(x+i)+O\left((x+i)^6\right)\right)+$$
$$\left\lfloor \frac{\arg\left(-\frac{e^{-(\pi x)/2}\left(e^{(\pi x)/2}\pi x-2e^{(\pi x)/2}+i e^{(\pi x)/2}\pi-2i\right)}{x+i}\right)}{2\pi} \right\rfloor \left(\log\left(\frac{\pi}{2}\right)+i\log\left(\frac{\pi}{2}\right)(x+i)+O\left((x+i)^6\right)\right)+$$
$$\left\lfloor \frac{-\arg\left(\frac{e^{-(\pi x)/2}\left(i+e^{(\pi x)/2}\right)}{x+i}\right)-\arg(x+i)+\pi}{2\pi} \right\rfloor \left(2i\pi-2\pi(x+i)+O\left((x+i)^6\right)\right)+$$
$$\left(\frac{225\pi\log(x+i)+225\pi\log\left(\frac{\pi}{2}\right)-225\pi\log(2)+(836-1036i)}{225\pi}-\frac{1}{15}i\left(-15\log(x+i)-15\log\left(\frac{\pi}{2}\right)+15\log(2)-46\right)(x+i)+\frac{1}{48}\left(-72i\pi-\pi^2\right)(x+i)^2+\frac{35}{48}i\pi^2(x+i)^3+\frac{7\left(-1200i\pi^3+\pi^4\right)(x+i)^4}{23040}+\frac{3679i\pi^4(x+i)^5}{23040}+O\left((x+i)^6\right)\right)$$

arg(z) is the complex argument
$\lfloor x \rfloor$ is the floor function
log(x) is the natural logarithm

Series expansion at x = i

$$\left\lfloor \frac{\arg\left(-\frac{e^{-(\pi x)/2}\left(e^{(\pi x)/2}\pi x-2e^{(\pi x)/2}-i e^{(\pi x)/2}\pi+2i\right)}{x-i}\right)}{2\pi} \right\rfloor \left(\log\left(\frac{2}{\pi}\right)-i\log\left(\frac{2}{\pi}\right)(x-i)+O\left((x-i)^6\right)\right)+$$
$$\left\lfloor \frac{\arg\left(-\frac{e^{-(\pi x)/2}\left(e^{(\pi x)/2}\pi x-2e^{(\pi x)/2}-i e^{(\pi x)/2}\pi+2i\right)}{x-i}\right)}{2\pi} \right\rfloor \left(\log\left(\frac{\pi}{2}\right)-i\log\left(\frac{\pi}{2}\right)(x-i)+O\left((x-i)^6\right)\right)+$$
$$\left\lfloor \frac{-\arg\left(\frac{e^{-(\pi x)/2}\left(-i+e^{(\pi x)/2}\right)}{x-i}\right)-\arg(x-i)+\pi}{2\pi} \right\rfloor \left(2i\pi+2\pi(x-i)+O\left((x-i)^6\right)\right)+$$
$$\left(\frac{225\pi\log(x-i)+225\pi\log\left(\frac{\pi}{2}\right)-225\pi\log(2)+(836+1036i)}{225\pi}+\right.$$
$$\frac{1}{15}i\left(-15\log(x-i)-15\log\left(\frac{\pi}{2}\right)+15\log(2)-46\right)(x-i)+$$
$$\frac{1}{48}\left(72i\pi-\pi^2\right)(x-i)^2-\frac{35}{48}i\pi^2(x-i)^3+$$
$$\left.\frac{7\left(1200i\pi^3+\pi^4\right)(x-i)^4}{23040}-\frac{3679i\pi^4(x-i)^5}{23040}+O\left((x-i)^6\right)\right)$$

Series expansion at x=∞

$$\frac{1}{225\pi}\left(\log\left(1-i e^{-(\pi x)/2}\right)\left(-225i\pi x+O\left(\left(\frac{1}{x}\right)^{13}\right)\right)+\right.$$
$$\log\left(1+i e^{-(\pi x)/2}\right)\left(225i\pi x+O\left(\left(\frac{1}{x}\right)^{13}\right)\right)-$$
$$\left.36e^{-(5\pi x)/2}+100e^{-(3\pi x)/2}-900e^{-(\pi x)/2}+836\right)$$

Derivative

$$\frac{d}{dx}\left(\frac{4\left(\frac{1-e^{-(\pi x)/2}}{1^2}-\frac{1-e^{-(3\pi x)/2}}{3^2}+\frac{1-e^{-(5\pi x)/2}}{5^2}\right)}{\pi}-(2x)\tan^{-1}\left(e^{-(\pi x)/2}\right)\right)=$$
$$\frac{1}{15\,(e^{\pi x}+1)}e^{-(5\pi x)/2}\left(15\,e^{3\pi x}(\pi x+2)-4\,e^{\pi x}+20\,e^{2\pi x}-30\,e^{(5\pi x)/2}\left(e^{\pi x}+1\right)\tan^{-1}\left(e^{-(\pi x)/2}\right)+6\right)$$

Indefinite integral

$$\int\left(\frac{4\left(1-e^{-(\pi x)/2}+\frac{1}{25}\left(1-e^{-(5\pi x)/2}\right)+\frac{1}{9}\left(-1+e^{-(3\pi x)/2}\right)\right)}{\pi}-2x\tan^{-1}\left(e^{-(\pi x)/2}\right)\right)dx=$$
$$\frac{2\,i\,x\,\mathrm{Li}_2\left(-i\,e^{(\pi x)/2}\right)}{\pi}-\frac{2\,i\,x\,\mathrm{Li}_2\left(i\,e^{(\pi x)/2}\right)}{\pi}-\frac{4\,i\,\mathrm{Li}_3\left(-i\,e^{(\pi x)/2}\right)}{\pi^2}+\frac{4\,i\,\mathrm{Li}_3\left(i\,e^{(\pi x)/2}\right)}{\pi^2}-$$
$$\frac{1}{2}\,i\,x^2\log\left(1-i\,e^{(\pi x)/2}\right)+\frac{1}{2}\,i\,x^2\log\left(1+i\,e^{(\pi x)/2}\right)+x^2\left(-\tan^{-1}\left(e^{-(\pi x)/2}\right)\right)+$$
$$\frac{836\,x}{225\,\pi}+\frac{8\,e^{-(5\pi x)/2}}{125\,\pi^2}-\frac{8\,e^{-(3\pi x)/2}}{27\,\pi^2}+\frac{8\,e^{-(\pi x)/2}}{\pi^2}+\text{constant}$$

(assuming a complex-valued logarithm)

$\mathrm{Li}_n(x)$ is the polylogarithm function

From the exact result

$$\frac{4\left(\frac{1}{25}\left(1-e^{-(5\pi x)/2}\right)-e^{-(\pi x)/2}+\frac{1}{9}\left(e^{-(3\pi x)/2}-1\right)+1\right)}{\pi}-2x\tan^{-1}\left(e^{-(\pi x)/2}\right)$$

for x = 1.1 and 6.7 , we obtain:

(4 (1/25 (1 - e^(-(5 π*1.1)/2)) - e^(-(π*1.1)/2) + 1/9 (e^(-(3 π*1.1)/2) - 1) + 1))/π - 2*1.1 tan^(-1)(e^(-(π*1.1)/2))

Input

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\times1.1)}\right)-e^{-1/2(\pi\times1.1)}+\frac{1}{9}\left(e^{-1/2(3\pi\times1.1)}-1\right)+1\right)}{\pi}-(2\times1.1)\tan^{-1}\left(e^{-1/2(\pi\times1.1)}\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Result

0.570457968576379789561418971562075438624169882618215713033770572 4
…

(result in radians)

0.5704579685….

Alternative representations

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,1.1)}\right)-e^{-1/2(\pi\,1.1)}+\frac{1}{9}\left(e^{-1/2(3\pi\,1.1)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,1.1)}\right)2\times1.1=-2.2\,\mathrm{sc}^{-1}\left(e^{-0.55\pi}\,\middle|\,0\right)+\frac{4\left(1+\frac{1}{25}\left(1-e^{-2.75\pi}\right)+\frac{1}{9}\left(-1+e^{-1.65\pi}\right)-e^{-0.55\pi}\right)}{\pi}$$

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,1.1)}\right)-e^{-1/2(\pi\,1.1)}+\frac{1}{9}\left(e^{-1/2(3\pi\,1.1)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,1.1)}\right)2\times1.1=-2.2\tan^{-1}\left(1,e^{-0.55\pi}\right)+\frac{4\left(1+\frac{1}{25}\left(1-e^{-2.75\pi}\right)+\frac{1}{9}\left(-1+e^{-1.65\pi}\right)-e^{-0.55\pi}\right)}{\pi}$$

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,1.1)}\right)-e^{-1/2(\pi\,1.1)}+\frac{1}{9}\left(e^{-1/2(3\pi\,1.1)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,1.1)}\right)2\times1.1=-2.2\cot^{-1}\left(\frac{1}{e^{-0.55\pi}}\right)+\frac{4\left(1+\frac{1}{25}\left(1-e^{-2.75\pi}\right)+\frac{1}{9}\left(-1+e^{-1.65\pi}\right)-e^{-0.55\pi}\right)}{\pi}$$

$\mathrm{sc}^{-1}(x\,|\,m)$ is the inverse of the Jacobi elliptic function sc

$\tan^{-1}(x, y)$ is the inverse tangent function

$\cot^{-1}(x)$ is the inverse cotangent function

Series representations

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,1.1)}\right)-e^{-1/2(\pi\,1.1)}+\frac{1}{9}\left(e^{-1/2(3\pi\,1.1)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,1.1)}\right)2\times 1.1=\frac{3.71556}{\pi}-\frac{0.16\,e^{-2.75\pi}}{\pi}+\frac{0.444444\,e^{-1.65\pi}}{\pi}-\frac{4\,e^{-0.55\pi}}{\pi}-2.2\sum_{k=0}^{\infty}\frac{e^{i\,k\,\pi}\left(e^{-0.55\pi}\right)^{1+2k}}{1+2k}$$

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,1.1)}\right)-e^{-1/2(\pi\,1.1)}+\frac{1}{9}\left(e^{-1/2(3\pi\,1.1)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,1.1)}\right)2\times 1.1=\frac{836}{225\pi}-\frac{4\,e^{-2.75\pi}}{25\pi}+\frac{4\,e^{-1.65\pi}}{9\pi}-\frac{4\,e^{-0.55\pi}}{\pi}-2.2\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{5}\right)^{k}2^{1+2k}F_{1+2k}\left(\frac{e^{-0.55\pi}}{1+\sqrt{1+\frac{4\,e^{-1.1\pi}}{5}}}\right)^{1+2k}}{1+2k}$$

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,1.1)}\right)-e^{-1/2(\pi\,1.1)}+\frac{1}{9}\left(e^{-1/2(3\pi\,1.1)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,1.1)}\right)2\times 1.1=\frac{836}{225\pi}-\frac{4\,e^{-2.75\pi}}{25\pi}+\frac{4\,e^{-1.65\pi}}{9\pi}-\frac{4\,e^{-0.55\pi}}{\pi}-2.2\tan^{-1}(x)-2.2\pi\left\lfloor\frac{\arg\left(i\left(e^{-0.55\pi}-x\right)\right)}{2\pi}\right\rfloor-1.1\,i\sum_{k=1}^{\infty}\frac{\left(-(-i-x)^{-k}+(i-x)^{-k}\right)\left(e^{-0.55\pi}-x\right)^{k}}{k}\quad\text{for } (i\,x\in\mathbb{R}\text{ and } i\,x<-1)$$

F_n is the n^{th} Fibonacci number
$\arg(z)$ is the complex argument
$\lfloor x\rfloor$ is the floor function
i is the imaginary unit
$\mathbb{R}$ is the set of real numbers

Integral representations

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,1.1)}\right)-e^{-1/2(\pi\,1.1)}+\frac{1}{9}\left(e^{-1/2(3\pi\,1.1)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,1.1)}\right)2\times 1.1=\frac{3.71556}{\pi}-\frac{0.16\,e^{-2.75\pi}}{\pi}+\frac{0.444444\,e^{-1.65\pi}}{\pi}-\frac{4\,e^{-0.55\pi}}{\pi}-2.2\,e^{-0.55\pi}\int_0^1\frac{1}{1+e^{-1.1\pi}\,t^2}\,dt$$

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,1.1)}\right)-e^{-1/2(\pi\,1.1)}+\frac{1}{9}\left(e^{-1/2(3\pi\,1.1)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,1.1)}\right)2\times 1.1=\frac{3.71556}{\pi}-\frac{0.16\,e^{-2.75\pi}}{\pi}+\frac{0.444444\,e^{-1.65\pi}}{\pi}-\frac{4\,e^{-0.55\pi}}{\pi}+\frac{0.55\,e^{-0.55\pi}\,i}{\pi^{3/2}}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\left(1+e^{-1.1\pi}\right)^{-s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds\quad\text{for } 0<\gamma<\frac{1}{2}$$

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,1.1)}\right)-e^{-1/2(\pi\,1.1)}+\frac{1}{9}\left(e^{-1/2(3\pi\,1.1)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,1.1)}\right)2\times 1.1=\frac{3.71556}{\pi}-\frac{0.16\,e^{-2.75\pi}}{\pi}+\frac{0.444444\,e^{-1.65\pi}}{\pi}-\frac{4\,e^{-0.55\pi}}{\pi}-\frac{0.55\,e^{-0.55\pi}}{i\,\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\left(e^{-1.1\pi}\right)^{-s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds\quad\text{for } 0<\gamma<\frac{1}{2}$$

$\Gamma(x)$ is the gamma function

Continued fraction representations

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi 1.1)}\right)-e^{-1/2(\pi 1.1)}+\frac{1}{9}\left(e^{-1/2(3\pi 1.1)}-1\right)+1\right)}{\pi} - \tan^{-1}\left(e^{-1/2(\pi 1.1)}\right) 2\times 1.1 = \frac{\frac{836}{225}-\frac{4e^{-2.75\pi}}{25}+\frac{4e^{-1.65\pi}}{9}-4e^{-0.55\pi}}{\pi} - \frac{2.2\,e^{-0.55\pi}}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{e^{-1.1\pi}k^2}{1+2k}} = 0.957277 - \frac{0.390855}{1+\frac{0.0315636}{3+\frac{0.126254}{5+\frac{0.284072}{7+\frac{0.505017}{9+\dots}}}}}$$

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi 1.1)}\right)-e^{-1/2(\pi 1.1)}+\frac{1}{9}\left(e^{-1/2(3\pi 1.1)}-1\right)+1\right)}{\pi} - \tan^{-1}\left(e^{-1/2(\pi 1.1)}\right) 2\times 1.1 = \frac{\frac{836}{225}-\frac{4e^{-2.75\pi}}{25}+\frac{4e^{-1.65\pi}}{9}-4e^{-0.55\pi}}{\pi} - 2.2\left(e^{-0.55\pi}-\frac{e^{-1.65\pi}}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{e^{-1.1\pi}\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}\right) = 0.957277 - 2.2\left(0.177661-\frac{0.00560763}{3+\frac{0.284072}{5+\frac{0.126254}{7+\frac{0.789089}{9+\frac{0.505017}{11+\dots}}}}}\right)$$

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,1.1)}\right)-e^{-1/2(\pi\,1.1)}+\frac{1}{9}\left(e^{-1/2(3\pi\,1.1)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,1.1)}\right)2\times 1.1=$$

$$\frac{\frac{836}{225}-\frac{4e^{-2.75\pi}}{25}+\frac{4e^{-1.65\pi}}{9}-4e^{-0.55\pi}}{\pi}-\frac{2.2e^{-0.55\pi}}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{e^{-1.1\pi}(1-2k)^2}{1+e^{-1.1\pi}(1-2k)+2k}}=$$

$$0.957277-\cfrac{0.390855}{1+\cfrac{0.0315636}{2.96844+\cfrac{0.284072}{4.90531+\cfrac{0.789089}{6.84218+\cfrac{1.54661}{8.77906+\ldots}}}}}$$

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,1.1)}\right)-e^{-1/2(\pi\,1.1)}+\frac{1}{9}\left(e^{-1/2(3\pi\,1.1)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,1.1)}\right)2\times 1.1=\frac{\frac{836}{225}-\frac{4e^{-2.75\pi}}{25}+\frac{4e^{-1.65\pi}}{9}-4e^{-0.55\pi}}{\pi}-$$

$$\frac{2.2e^{-0.55\pi}}{1+e^{-1.1\pi}+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{2e^{-1.1\pi}\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1+\frac{1}{2}\left(1+(-1)^k\right)e^{-1.1\pi}\right)(1+2k)}}=$$

$$0.957277-\cfrac{0.390855}{1.03156+-\cfrac{0.0631271}{3-\cfrac{0.0631271}{5.15782-\cfrac{0.378763}{7-\cfrac{0.378763}{9.28407+\ldots}}}}}$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}}\ a_k / b_k$ is a continued fraction

And:

(4 (1/25 (1 - e^(-(5 π*6.7)/2)) - e^(-(π*6.7)/2) + 1/9 (e^(-(3 π*6.7)/2) - 1) + 1))/π - 2*6.7 tan^(-1)(e^(-(π*6.7)/2))

Input

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\times 6.7)}\right)-e^{-1/2(\pi\times 6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi\times 6.7)}-1\right)+1\right)}{\pi}-(2\times 6.7)\tan^{-1}\left(e^{-1/2(\pi\times 6.7)}\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Result

1.1823037310922895181896352495964569927736953049433851344648686226
...

(result in radians)

1.182303731….

Alternative representations

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi 6.7)}\right)-e^{-1/2(\pi 6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi 6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi 6.7)}\right)2\times 6.7=-13.4\,\text{sc}^{-1}\left(e^{-3.35\pi}\mid 0\right)+\frac{4\left(1+\frac{1}{25}\left(1-e^{-16.75\pi}\right)+\frac{1}{9}\left(-1+e^{-10.05\pi}\right)-e^{-3.35\pi}\right)}{\pi}$$

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi 6.7)}\right)-e^{-1/2(\pi 6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi 6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi 6.7)}\right)2\times 6.7=-13.4\tan^{-1}\left(1,e^{-3.35\pi}\right)+\frac{4\left(1+\frac{1}{25}\left(1-e^{-16.75\pi}\right)+\frac{1}{9}\left(-1+e^{-10.05\pi}\right)-e^{-3.35\pi}\right)}{\pi}$$

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi 6.7)}\right)-e^{-1/2(\pi 6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi 6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi 6.7)}\right)2\times 6.7=-13.4\cot^{-1}\left(\frac{1}{e^{-3.35\pi}}\right)+\frac{4\left(1+\frac{1}{25}\left(1-e^{-16.75\pi}\right)+\frac{1}{9}\left(-1+e^{-10.05\pi}\right)-e^{-3.35\pi}\right)}{\pi}$$

$\text{sc}^{-1}(x\mid m)$ is the inverse of the Jacobi elliptic function sc

$\tan^{-1}(x, y)$ is the inverse tangent function

$\cot^{-1}(x)$ is the inverse cotangent function

Series representations

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi 6.7)}\right)-e^{-1/2(\pi 6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi 6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi 6.7)}\right)2\times 6.7=\frac{836}{225\pi}-\frac{4e^{-16.75\pi}}{25\pi}+\frac{4e^{-10.05\pi}}{9\pi}-\frac{4e^{-3.35\pi}}{\pi}-13.4\sum_{k=0}^{\infty}\frac{(-1)^k\left(e^{-3.35\pi}\right)^{1+2k}}{1+2k}$$

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi 6.7)}\right)-e^{-1/2(\pi 6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi 6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi 6.7)}\right)2\times 6.7=\frac{836}{225\pi}-\frac{4e^{-16.75\pi}}{25\pi}+\frac{4e^{-10.05\pi}}{9\pi}-\frac{4e^{-3.35\pi}}{\pi}-13.4\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{5}\right)^k 2^{1+2k}F_{1+2k}\left(\frac{e^{-3.35\pi}}{1+\sqrt{1+\frac{4e^{-6.7\pi}}{5}}}\right)^{1+2k}}{1+2k}$$

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi 6.7)}\right)-e^{-1/2(\pi 6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi 6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi 6.7)}\right)2\times 6.7=\frac{836}{225\pi}-\frac{4e^{-16.75\pi}}{25\pi}+\frac{4e^{-10.05\pi}}{9\pi}-\frac{4e^{-3.35\pi}}{\pi}-13.4\tan^{-1}(x)-13.4\pi\left\lfloor\frac{\arg\left(i\left(e^{-3.35\pi}-x\right)\right)}{2\pi}\right\rfloor-6.7i\sum_{k=1}^{\infty}\frac{\left(-(-i-x)^{-k}+(i-x)^{-k}\right)\left(e^{-3.35\pi}-x\right)^k}{k}\quad \text{for } (i x\in\mathbb{R} \text{ and } i x<-1)$$

F_n is the n^{th} Fibonacci number
$\arg(z)$ is the complex argument
$\lfloor x\rfloor$ is the floor function
i is the imaginary unit
$\mathbb{R}$ is the set of real numbers

Integral representations

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,6.7)}\right)-e^{-1/2(\pi\,6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi\,6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,6.7)}\right)2\times6.7=\frac{3.71556}{\pi}-\frac{0.16\,e^{-16.75\pi}}{\pi}+\frac{0.444444\,e^{-10.05\pi}}{\pi}-\frac{4\,e^{-3.35\pi}}{\pi}-13.4\,e^{-3.35\pi}\int_0^1\frac{1}{1+e^{-6.7\pi}t^2}\,dt$$

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,6.7)}\right)-e^{-1/2(\pi\,6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi\,6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,6.7)}\right)2\times6.7=\frac{3.71556}{\pi}-\frac{0.16\,e^{-16.75\pi}}{\pi}+\frac{0.444444\,e^{-10.05\pi}}{\pi}-\frac{4\,e^{-3.35\pi}}{\pi}+\frac{3.35\,e^{-3.35\pi}\,i}{\pi^{3/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(1+e^{-6.7\pi}\right)^{-s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds \quad \text{for } 0<\gamma<\frac{1}{2}$$

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,6.7)}\right)-e^{-1/2(\pi\,6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi\,6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,6.7)}\right)2\times6.7=\frac{3.71556}{\pi}-\frac{0.16\,e^{-16.75\pi}}{\pi}+\frac{0.444444\,e^{-10.05\pi}}{\pi}-\frac{4\,e^{-3.35\pi}}{\pi}-\frac{3.35\,e^{-3.35\pi}}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\left(e^{-6.7\pi}\right)^{-s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds \quad \text{for } 0<\gamma<\frac{1}{2}$$

$\Gamma(x)$ is the gamma function

Continued fraction representations

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi 6.7)}\right)-e^{-1/2(\pi 6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi 6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi 6.7)}\right)2\times 6.7=$$

$$\frac{\frac{836}{225}-\frac{4e^{-16.75\pi}}{25}+\frac{4e^{-10.05\pi}}{9}-4e^{-3.35\pi}}{\pi}-\frac{13.4\,e^{-3.35\pi}}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{e^{-6.7\pi}k^2}{1+2k}}=$$

$$1.18266-\cfrac{0.000360117}{1+\cfrac{7.22235\times10^{-10}}{3+\cfrac{2.88894\times10^{-9}}{5+\cfrac{6.50011\times10^{-9}}{7+\cfrac{1.15558\times10^{-8}}{9+\dots}}}}}$$

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi 6.7)}\right)-e^{-1/2(\pi 6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi 6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi 6.7)}\right)2\times 6.7=\frac{\frac{836}{225}-\frac{4e^{-16.75\pi}}{25}+\frac{4e^{-10.05\pi}}{9}-4e^{-3.35\pi}}{\pi}-$$

$$13.4\left(e^{-3.35\pi}-\frac{e^{-10.05\pi}}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{e^{-6.7\pi}\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}\right)=$$

$$1.18266-13.4\left(0.0000268744-\cfrac{1.94096\times10^{-14}}{3+\cfrac{6.50011\times10^{-9}}{5+\cfrac{2.88894\times10^{-9}}{7+\cfrac{1.80559\times10^{-8}}{9+\cfrac{1.15558\times10^{-8}}{11+\dots}}}}}\right)$$

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,6.7)}\right)-e^{-1/2(\pi\,6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi\,6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,6.7)}\right)2\times 6.7=$$

$$\frac{\frac{836}{225}-\frac{4e^{-16.75\pi}}{25}+\frac{4e^{-10.05\pi}}{9}-4e^{-3.35\pi}}{\pi}-\frac{13.4\,e^{-3.35\pi}}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{e^{-6.7\pi}(1-2k)^2}{1+e^{-6.7\pi}(1-2k)+2k}}=$$

$$1.18266-\cfrac{0.000360117}{1+\cfrac{7.22235\times10^{-10}}{3.+\cfrac{6.50011\times10^{-9}}{5.+\cfrac{1.80559\times10^{-8}}{7.+\cfrac{3.53895\times10^{-8}}{9.+\ldots}}}}}$$

$$\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,6.7)}\right)-e^{-1/2(\pi\,6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi\,6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,6.7)}\right)2\times 6.7=\frac{\frac{836}{225}-\frac{4e^{-16.75\pi}}{25}+\frac{4e^{-10.05\pi}}{9}-4e^{-3.35\pi}}{\pi}-$$

$$\frac{13.4\,e^{-3.35\pi}}{1+e^{-6.7\pi}+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{2e^{-6.7\pi}\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1+\frac{1}{2}\left(1+(-1)^k\right)e^{-6.7\pi}\right)(1+2k)}}=$$

$$1.18266-\cfrac{0.000360117}{1.+-\cfrac{1.44447\times10^{-9}}{3-\cfrac{1.44447\times10^{-9}}{5.-\cfrac{8.66682\times10^{-9}}{7-\cfrac{8.66682\times10^{-9}}{9.+\ldots}}}}}$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}}\,a_k/b_k$ is a continued fraction

From the algebraic sum of the two expressions, after some calculations, we obtain:

Input interpretation

$$1+\left(\left(\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\times6.7)}\right)-e^{-1/2(\pi\times6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi\times6.7)}-1\right)+1\right)}{\pi}-(2\times6.7)\tan^{-1}\left(e^{-1/2(\pi\times6.7)}\right)\right)-0.57045796857\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Result

1.61184576…

(result in radians)

1.61184576…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternative representations

$$1+\left(\left(\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,6.7)}\right)-e^{-1/2(\pi\,6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi\,6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,6.7)}\right)2\times6.7\right)-0.570457968570000\right)=$$

$$0.429542031430000-13.4\,\mathrm{sc}^{-1}\left(e^{-3.35\pi}\,\middle|\,0\right)+\frac{4\left(1+\frac{1}{25}\left(1-e^{-16.75\pi}\right)+\frac{1}{9}\left(-1+e^{-10.05\pi}\right)-e^{-3.35\pi}\right)}{\pi}$$

$$1+\left(\left(\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,6.7)}\right)-e^{-1/2(\pi\,6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi\,6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,6.7)}\right)2\times 6.7\right)-0.570457968570000\right)=$$
$$0.429542031430000-13.4\tan^{-1}\left(1,e^{-3.35\pi}\right)+\frac{4\left(1+\frac{1}{25}\left(1-e^{-16.75\pi}\right)+\frac{1}{9}\left(-1+e^{-10.05\pi}\right)-e^{-3.35\pi}\right)}{\pi}$$

$$1+\left(\left(\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,6.7)}\right)-e^{-1/2(\pi\,6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi\,6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,6.7)}\right)2\times 6.7\right)-0.570457968570000\right)=$$
$$0.429542031430000-13.4\cot^{-1}\left(\frac{1}{e^{-3.35\pi}}\right)+\frac{4\left(1+\frac{1}{25}\left(1-e^{-16.75\pi}\right)+\frac{1}{9}\left(-1+e^{-10.05\pi}\right)-e^{-3.35\pi}\right)}{\pi}$$

$\mathrm{sc}^{-1}(x\,|\,m)$ is the inverse of the Jacobi elliptic function sc

$\tan^{-1}(x, y)$ is the inverse tangent function

$\cot^{-1}(x)$ is the inverse cotangent function

Series representations

$$1+\left(\left(\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,6.7)}\right)-e^{-1/2(\pi\,6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi\,6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,6.7)}\right)2\times 6.7\right)-0.570457968570000\right)=$$
$$0.429542031430000+\frac{836}{225\pi}-\frac{4e^{-16.75\pi}}{25\pi}+\frac{4e^{-10.05\pi}}{9\pi}-\frac{4e^{-3.35\pi}}{\pi}-13.4\sum_{k=0}^{\infty}\frac{(-1)^k\left(e^{-3.35\pi}\right)^{1+2k}}{1+2k}$$

$$1+\left(\left(\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,6.7)}\right)-e^{-1/2(\pi\,6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi\,6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,6.7)}\right)2\times 6.7\right)-0.570457968570000\right)=$$

$$0.429542031430000+\frac{836}{225\pi}-\frac{4e^{-16.75\pi}}{25\pi}+\frac{4e^{-10.05\pi}}{9\pi}-\frac{4e^{-3.35\pi}}{\pi}-13.4\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{5}\right)^k 2^{1+2k}F_{1+2k}\left(\frac{e^{-3.35\pi}}{1+\sqrt{1+\frac{4e^{-6.7\pi}}{5}}}\right)^{1+2k}}{1+2k}$$

$$1+\left(\left(\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,6.7)}\right)-e^{-1/2(\pi\,6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi\,6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,6.7)}\right)2\times 6.7\right)-0.570457968570000\right)=$$

$$0.429542031430000+\frac{836}{225\pi}-\frac{4e^{-16.75\pi}}{25\pi}+\frac{4e^{-10.05\pi}}{9\pi}-\frac{4e^{-3.35\pi}}{\pi}-13.4\tan^{-1}(x)-13.4\pi\left\lfloor\frac{\arg\left(i\left(e^{-3.35\pi}-x\right)\right)}{2\pi}\right\rfloor-6.7\,i\sum_{k=1}^{\infty}\frac{\left(-(-i-x)^{-k}+(i-x)^{-k}\right)\left(e^{-3.35\pi}-x\right)^k}{k}\quad \text{for } (i\,x\in\mathbb{R} \text{ and } i\,x<-1)$$

F_n is the n^{th} Fibonacci number

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

i is the imaginary unit

$\mathbb{R}$ is the set of real numbers

Integral representations

$$1+\left(\left(\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi 6.7)}\right)-e^{-1/2(\pi 6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi 6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi 6.7)}\right)2\times 6.7\right)-0.570457968570000\right)=$$
$$0.429542031430000+\frac{836}{225\pi}-\frac{4e^{-16.75\pi}}{25\pi}+\frac{4e^{-10.05\pi}}{9\pi}-\frac{4e^{-3.35\pi}}{\pi}-13.4\,e^{-3.35\pi}\int_0^1\frac{1}{1+e^{-6.7\pi}t^2}\,dt$$

$$1+\left(\left(\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi 6.7)}\right)-e^{-1/2(\pi 6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi 6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi 6.7)}\right)2\times 6.7\right)-0.570457968570000\right)=$$
$$0.429542031430000+\frac{836}{225\pi}-\frac{4e^{-16.75\pi}}{25\pi}+\frac{4e^{-10.05\pi}}{9\pi}-\frac{4e^{-3.35\pi}}{\pi}+\frac{3.35\,e^{-3.35\pi}\,i}{\pi^{3/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(1+e^{-6.7\pi}\right)^{-s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds\quad\text{for } 0<\gamma<\frac{1}{2}$$

$$1+\left(\left(\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi 6.7)}\right)-e^{-1/2(\pi 6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi 6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi 6.7)}\right)2\times 6.7\right)-0.570457968570000\right)=$$
$$0.42954203143000+\frac{3.7155555555556}{\pi}-\frac{0.160000000000000\,e^{-16.75\pi}}{\pi}+\frac{0.44444444444444\,e^{-10.05\pi}}{\pi}-\frac{4.0000000000000\,e^{-3.35\pi}}{\pi}-\frac{3.35\,e^{-3.35\pi}}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\left(e^{-6.7\pi}\right)^{-s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds\quad\text{for } 0<\gamma<\frac{1}{2}$$

$\Gamma(x)$ is the gamma function

Continued fraction representations

$$1+\left(\left(\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,6.7)}\right)-e^{-1/2(\pi\,6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi\,6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,6.7)}\right)2\times 6.7\right)-0.570457968570000\right)=$$

$$0.429542031430000+\frac{\frac{836}{225}-\frac{4e^{-16.75\pi}}{25}+\frac{4e^{-10.05\pi}}{9}-4e^{-3.35\pi}}{\pi}-\frac{13.4\,e^{-3.35\pi}}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{e^{-6.7\pi}k^2}{1+2k}}=1.61221-\cfrac{0.000360117}{1+\cfrac{7.22235\times 10^{-10}}{3+\cfrac{2.88894\times 10^{-9}}{5+\cfrac{6.50011\times 10^{-9}}{7+\cfrac{1.15558\times 10^{-8}}{9+\dots}}}}}$$

$$1+\left(\left(\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,6.7)}\right)-e^{-1/2(\pi\,6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi\,6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,6.7)}\right)2\times 6.7\right)-0.570457968570000\right)=$$

$$0.42954203143000+e^{-3.35\pi}\left(-13.4-\frac{4}{\pi}\right)+\frac{3.71555555555556}{\pi}-\frac{4e^{-16.75\pi}}{25\pi}+e^{-10.05\pi}\left(\frac{4}{9\pi}+\frac{13.4}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{e^{-6.7\pi}\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}\right)=$$

$$1.61185+1.94096\times 10^{-14}\left(\frac{4}{9\pi}+\cfrac{13.4}{3+\cfrac{6.50011\times 10^{-9}}{5+\cfrac{2.88894\times 10^{-9}}{7+\cfrac{1.80559\times 10^{-8}}{9+\cfrac{1.15558\times 10^{-8}}{11+\dots}}}}}\right)$$

$$1+\left(\left(\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,6.7)}\right)-e^{-1/2(\pi\,6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi\,6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,6.7)}\right)2\times6.7\right)-0.570457968570000\right)=$$

$$0.429542031430000+\frac{\frac{836}{225}-\frac{4e^{-16.75\pi}}{25}+\frac{4e^{-10.05\pi}}{9}-4e^{-3.35\pi}}{\pi}-\frac{13.4\,e^{-3.35\pi}}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{e^{-6.7\pi}(1-2k)^2}{1+e^{-6.7\pi}(1-2k)+2k}}=$$

$$1.61221-\cfrac{0.000360117}{1+\cfrac{7.22235\times10^{-10}}{3.+\cfrac{6.50011\times10^{-9}}{5.+\cfrac{1.80559\times10^{-8}}{7.+\cfrac{3.53895\times10^{-8}}{9.+\ldots}}}}}$$

$$1+\left(\left(\frac{4\left(\frac{1}{25}\left(1-e^{-1/2(5\pi\,6.7)}\right)-e^{-1/2(\pi\,6.7)}+\frac{1}{9}\left(e^{-1/2(3\pi\,6.7)}-1\right)+1\right)}{\pi}-\tan^{-1}\left(e^{-1/2(\pi\,6.7)}\right)2\times6.7\right)-0.570457968570000\right)=$$

$$0.429542031430000+\frac{\frac{836}{225}-\frac{4e^{-16.75\pi}}{25}+\frac{4e^{-10.05\pi}}{9}-4e^{-3.35\pi}}{\pi}-\frac{13.4\,e^{-3.35\pi}}{1+e^{-6.7\pi}+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{2e^{-6.7\pi}\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1+\frac{1}{2}\left(1+(-1)^k\right)e^{-6.7\pi}\right)(1+2k)}}=$$

$$1.61221-\cfrac{0.000360117}{1.+-\cfrac{1.44447\times10^{-9}}{3-\cfrac{1.44447\times10^{-9}}{5.-\cfrac{8.66682\times10^{-9}}{7-\cfrac{8.66682\times10^{-9}}{9.+\ldots}}}}}$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}}a_k/b_k$ is a continued fraction

From:

Moments of large families of Dirichlet L-functions – *Vorrapan (Fai) Chandee (Kansas state University)*, joint work with *Xiannan Li, Kaisa Matomaki, and Maksym Radziwill* – 24.06.2022 - 50 Years of Number Theory and Random Matrix Theory Conference, June 2022

We have:

Theorem (C., Li, Matomäki, and Radziwiłł (2022+))

We have $\mathcal{M}_8(Q)$ is

$$\sim 24024\ a_4 \sum_{q\sim Q} \prod_{p|q} \frac{\left(1-\frac{1}{p}\right)^7}{\left(1+\frac{9}{p}+\frac{9}{p^2}+\frac{1}{p^3}\right)} \phi^*(q) \frac{(\log q)^{16}}{16!} \times \int_{-\infty}^{\infty} \left|\Gamma\left(\frac{1/2+it}{2}\right)\right|^8 dt$$

$$\sim 24024\ \tilde{a}_4 Q^2 \frac{(\log Q)^{16}}{16!} \int_{-\infty}^{\infty} \left|\Gamma\left(\frac{1/2+it}{2}\right)\right|^8 dt.$$

From:

$$24024\ \tilde{a}_4 Q^2 \frac{(\log Q)^{16}}{16!} \int_{-\infty}^{\infty} \left|\Gamma\left(\frac{1/2+it}{2}\right)\right|^8 dt$$

for Q = 8:

24024*2*64* ((ln(8))^16) (1/16!) integrate((((gamma((1/2+i*t)/2))^8)) dt)

Input

$$\frac{\left(24024\times 2\times 64\log^{16}(8)\right)\int \Gamma\left(\frac{1}{2}\left(\frac{1}{2}+i\,t\right)\right)^8\,dt}{16!}$$

$\log(x)$ is the natural logarithm
$n!$ is the factorial function
$\Gamma(x)$ is the gamma function
i is the imaginary unit

Exact result

$$\frac{\log^{16}(8)\left(\int \Gamma\left(\frac{1}{2}\left(i\,t+\frac{1}{2}\right)\right)^8\,dt\right)}{6804000}$$

Alternate forms

$$\frac{\log^{16}(8)\left(\int \Gamma\left(\frac{i\,t}{2}+\frac{1}{4}\right)^8\,dt\right)}{6804000}$$

$$\frac{\log^{16}(8)\left(\int \Gamma\left(\frac{1}{4}\left(2\,i\,t+1\right)\right)^8\,dt\right)}{6804000}$$

$$\frac{362797056}{875}\log^{16}(2)\left(\int \frac{\left(\left(\frac{1}{4}\left(1+2\,i\,t\right)\right)!\right)^8}{(2\,t-i)^8}\,dt\right)$$

Derivative

$$\frac{d}{dt}\left(\frac{24024\times 2\times 64\log^{16}(8)}{16!}\int \Gamma\left(\frac{1}{2}\left(\frac{1}{2}+i\,t\right)\right)^8\,dt\right)=\frac{\log^{16}(8)\,\Gamma\left(\frac{i\,t}{2}+\frac{1}{4}\right)^8}{6804000}$$

(log^16(8) (integral Γ(1/2 (i t + 1/2))^8 dt))/6804000

Input

$$\frac{\log^{16}(8) \int \Gamma\left(\frac{1}{2}\left(i\,t+\frac{1}{2}\right)\right)^8 dt}{6\,804\,000}$$

log(x) is the natural logarithm
Γ(x) is the gamma function
i is the imaginary unit

Exact result

$$\frac{\log^{16}(8)\left(\int \Gamma\left(\frac{1}{2}\left(i\,t+\frac{1}{2}\right)\right)^8 dt\right)}{6\,804\,000}$$

Alternate forms

$$\frac{\log^{16}(8)\left(\int \Gamma\left(\frac{i\,t}{2}+\frac{1}{4}\right)^8 dt\right)}{6\,804\,000}$$

$$\frac{\log^{16}(8)\left(\int \Gamma\left(\frac{1}{4}\left(2\,i\,t+1\right)\right)^8 dt\right)}{6\,804\,000}$$

$$\frac{362\,797\,056}{875}\log^{16}(2)\left(\int \frac{\left(\left(\frac{1}{4}\left(1+2\,i\,t\right)\right)!\right)^8}{(2\,t-i)^8}\,dt\right)$$

n! is the factorial function

Derivative

$$\frac{d}{dt}\left(\frac{\log^{16}(8) \int \Gamma\left(\frac{1}{2}\left(i\,t+\frac{1}{2}\right)\right)^8 dt}{6\,804\,000}\right)=\frac{\log^{16}(8)\,\Gamma\left(\frac{i\,t}{2}+\frac{1}{4}\right)^8}{6\,804\,000}$$

for t = 4:

(log^16(8) (integral Γ(1/2 (i 4 + 1/2))^8 dt))/6804000

Input

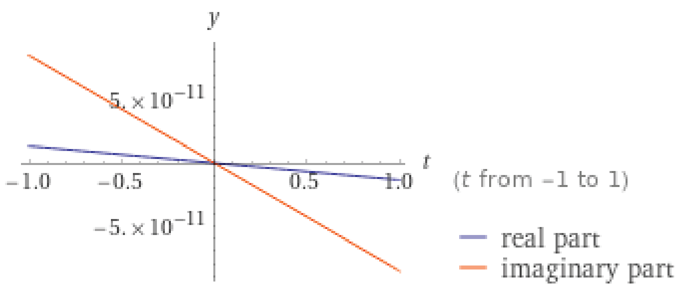
$$\frac{\log^{16}(8) \int \Gamma\left(\frac{1}{2}\left(i \times 4 + \frac{1}{2}\right)\right)^8 \, dt}{6\,804\,000}$$

log(x) is the natural logarithm
Γ(x) is the gamma function
i is the imaginary unit

Exact result

$$\frac{t \log^{16}(8)\, \Gamma\left(\frac{1}{4} + 2\,i\right)^8}{6\,804\,000}$$

Plot



Alternate form assuming t>0

$$\frac{177\,147\, t \log^{16}(2)\, \Gamma\left(\frac{1}{4} + 2\,i\right)^8}{28\,000}$$

Alternate form

$$\left(\frac{3\,527\,153\,974\,499\,328}{278\,814\,211\,279\,296\,875} + \frac{775\,909\,710\,102\,528\, i}{39\,830\,601\,611\,328\,125}\right)\left(\left(\frac{1}{4} + 2\,i\right)!\right)^8 t \log^{16}(2)$$

n! is the factorial function

Indefinite integral

$$\frac{\log^{16}(8) \int \Gamma\left(\frac{1}{2}\left(i\,4+\frac{1}{2}\right)\right)^8 dt}{6804000} = \frac{t \log^{16}(8)\, \Gamma\left(\frac{1}{4}+2 i\right)^8}{6804000} + \text{constant}$$

for t = 2:

(log^16(8) Γ(1/4 + 2 i)^8)/6804000

Input

$$\frac{\log^{16}(8)\, \Gamma\left(\frac{1}{4}+2 i\right)^8}{6804000}$$

log(x) is the natural logarithm
Γ(x) is the gamma function
i is the imaginary unit

Decimal approximation

$$-1.339338138911257904429817657837795920765681519592850 5563\ldots \times 10^{-11} - 8.548017109385472970808745781132926493837033716180794 9073\ldots \times 10^{-11}\, i$$

Result

$$-1.339338138911257904429817657837795920765681519592850 5563\ldots \times 10^{-11} - 8.548017109385472970808745781132926493837033716180794 9072\ldots \times 10^{-11}\, i$$

Alternate complex forms

$$8.652307388939036587662623362498163170462884806226154 2108 \times 10^{-11} (\cos(-1.726216768961865664859541018688798139814977883367231 68150) + i \sin(-1.726216768961865664859541018688798139814977883367231 68150))$$

$8.6523073889390365876626233624981631704628848062261542108 \times 10^{-11}$
$e^{-1.7262167689618656648595410186887981398149778833672316815 0 i}$

Polar coordinates

$r = 8.6523073889390365876626233624981631704628848062261542108 \times 10^{-11}$ (radius), $\theta = -1.7262167689618656648595410186887981398149778833672316815 0$ (angle)

8.652307388...*10^-11

Alternate complex forms

$$i \operatorname{Im}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2\,i\right)^8}{6\,804\,000}\right) + \operatorname{Re}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2\,i\right)^8}{6\,804\,000}\right)$$

$$\frac{\log^{16}(8)\left(\operatorname{Im}\left(\Gamma\left(\frac{1}{4}+2\,i\right)\right)^2+\operatorname{Re}\left(\Gamma\left(\frac{1}{4}+2\,i\right)\right)^2\right)^4(\cos(-1.72622)+i\sin(-1.72622))}{6\,804\,000}$$

$$\frac{e^{-1.72622\,i}\log^{16}(8)\left(\operatorname{Im}\left(\Gamma\left(\frac{1}{4}+2\,i\right)\right)^2+\operatorname{Re}\left(\Gamma\left(\frac{1}{4}+2\,i\right)\right)^2\right)^4}{6\,804\,000}$$

Im(z) is the imaginary part of z
Re(z) is the real part of z

Alternate forms

$$\left(\frac{3\,527\,153\,974\,499\,328}{278\,814\,211\,279\,296\,875}+\frac{775\,909\,710\,102\,528\,i}{39\,830\,601\,611\,328\,125}\right)\left(\left(\frac{1}{4}+2\,i\right)!\right)^8\log^{16}(2)$$

$$\frac{177\,147\log^{16}(2)\,\Gamma\left(\frac{1}{4}+2\,i\right)^8}{28\,000}$$

Alternative representations

$$\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000} = \frac{\log^{16}(8)\left(\exp\left(-\log G\left(2i+\frac{1}{4}\right)+\log G\left(1+2i+\frac{1}{4}\right)\right)\right)^8}{6804000}$$

$$\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000} = \frac{\log^{16}(8)\left(\frac{G\left(1+2i+\frac{1}{4}\right)}{G\left(2i+\frac{1}{4}\right)}\right)^8}{6804000}$$

$$\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000} = \frac{\log_e^{16}(8)\left(\exp\left(-\log G\left(2i+\frac{1}{4}\right)+\log G\left(1+2i+\frac{1}{4}\right)\right)\right)^8}{6804000}$$

$G(z)$ is the Barnes G-function
$\log_b(x)$ is the base- b logarithm

Series representations

$$\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000} = \frac{\Gamma\left(\frac{1}{4}+2i\right)^8\left(\log(7)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{7}\right)^k}{k}\right)^{16}}{6804000}$$

$$\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000} = \frac{\log^{16}(8)}{6804000\left(\sum_{k=1}^{\infty}\left(\frac{1}{4}+2i\right)^k c_k\right)^8}$$

$$\text{for}\left(c_1=1\text{ and }c_2=1\text{ and }c_k=\frac{\gamma\,c_{-1+k}+\sum_{j=1}^{-2+k}(-1)^{1+j+k}\,c_j\,\zeta(-j+k)}{-1+k}\right)$$

$$\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000} = \frac{\Gamma\left(\frac{1}{4}+2i\right)^8\left(\log(z_0)+\left\lfloor\frac{\arg(8-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(8-z_0)^k z_0^{-k}}{k}\right)^{16}}{6804000}$$

$\zeta(s)$ is the Riemann zeta function
γ is the Euler-Mascheroni constant

arg(z) is the complex argument
⌊x⌋ is the floor function

Integral representations

$$\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000} = \frac{\left(\int_0^1 \log^{-3/4+2i}\left(\frac{1}{t}\right)dt\right)^8 \log^{16}(8)}{6804000}$$

$$\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000} = \frac{\exp\left(8\int_0^1 \frac{(3-8i)-4x^{1/4+2i}+(1+8i)x}{4\log(x)-4x\log(x)}\,dx\right)\log^{16}(8)}{6804000}$$

$$\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000} = \frac{\left(\int_1^8 \frac{1}{t}\,dt\right)^{16}\left(\int_0^1 \log^{-3/4+2i}\left(\frac{1}{t}\right)dt\right)^8}{6804000}$$

From

$$i\,\mathrm{Im}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)+\mathrm{Re}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)$$

we obtain, after some calculations:

((1/((i Im((log^16(8) Γ(1/4 + 2 i)^8)/6804000) + Re((log^16(8) Γ(1/4 + 2 i)^8)/6804000)))))^(1/3)

Input

$$\sqrt[3]{\frac{1}{i\,\mathrm{Im}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)+\mathrm{Re}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)}}$$

log(x) is the natural logarithm
Γ(x) is the gamma function
Im(z) is the imaginary part of z
Re(z) is the real part of z

i is the imaginary unit

Exact result

$$\frac{1}{\sqrt[3]{i\,\mathrm{Im}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2\,i\right)^8}{6\,804\,000}\right)+\mathrm{Re}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2\,i\right)^8}{6\,804\,000}\right)}}$$

Result

1896.865964052976487143419058372739529158712673949395862859710111…
+
1230.348176516396610356413464895496953821163975068171026748624506…
i

Alternate complex forms

$$2260.9416447665893082378633982950847684973465603321934927223360\,16\,(\cos(0.575405589653955221619847006229599379938325961122410560502140\,7084) + i\,\sin(0.5754055896539552216198470062295993799383259611224105605021407084))$$

$$2260.9416447665893082378633982950847684973465603321934927223360\,16\,e^{0.5754055896539552216198470062295993799383259611224105605021407084\,i}$$

Polar coordinates

$r = 2260.941644766589308237863398295084768497346560332193492722336016$ (radius), $\theta = 0.5754055896539552216198470062295993799383259611224105605021407084$ (angle)

2260.941644766….

Possible closed forms

$$799\,C + 610 + 131\,\pi - 276\,\pi^2 + 770\,\pi\log(2) + 345\,\pi\log(3) - \frac{2}{9}\,i\left(97\,e^{\pi} - 342\,\pi - 1849\log(\pi) - 2130\log(2\,\pi) - 535\tan^{-1}(\pi)\right) \approx 1896.8659640529764871491135287349743560200085149589511647499362 05 + 1230.3481765163966103592481830798591573182906621129953969155565 54\,i$$

$$\pi\ \boxed{\text{root of } x^5 - 606\,x^4 + 1334\,x^3 - 199\,x^2 - 3599\,x + 400 \text{ near } x = 603.791} - \frac{2}{9}\,i\left(97\,e^{\pi} - 342\,\pi - 1849\log(\pi) - 2130\log(2\,\pi) - 535\tan^{-1}(\pi)\right) \approx 1896.8659640529764871540422319728771802749055603725567166146570 41 + 1230.3481765163966103592481830798591573182906621129953969155565 54\,i$$

$$\frac{318\,453}{200} - \frac{66\,853}{50\,\pi} + \frac{23\,243\,\pi}{100} - \frac{2}{9}\,i\left(97\,e^{\pi} - 342\,\pi - 1849\log(\pi) - 2130\log(2\,\pi) - 535\tan^{-1}(\pi)\right) \approx 1896.8659640529764871295847522659467495703588556292864472440941 73 + 1230.3481765163966103592481830798591573182906621129953969155565 54\,i$$

log(x) is the natural logarithm
$\tan^{-1}(x)$ is the inverse tangent function
C is Catalan's constant

Alternate complex forms

$$i\,\mathrm{Im}\left(\frac{1}{\sqrt[3]{i\,\mathrm{Im}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2\,i\right)^8}{6\,804\,000}\right) + \mathrm{Re}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2\,i\right)^8}{6\,804\,000}\right)}}\right) + \mathrm{Re}\left(\frac{1}{\sqrt[3]{i\,\mathrm{Im}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2\,i\right)^8}{6\,804\,000}\right) + \mathrm{Re}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2\,i\right)^8}{6\,804\,000}\right)}}\right)$$

$$\left(30\times 6^{2/3}\sqrt[3]{7}\left(\cos\left(-\frac{1}{3}\arg\left(i\,\mathrm{Im}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)+\mathrm{Re}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)\right)\right)+ i\sin\left(-\frac{1}{3}\arg\left(i\,\mathrm{Im}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)+\mathrm{Re}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)\right)\right)\right)\right)\Big/ \left(\log^{16/3}(8)\left(\mathrm{Im}\left(\Gamma\left(\frac{1}{4}+2i\right)\right)^2+\mathrm{Re}\left(\Gamma\left(\frac{1}{4}+2i\right)\right)^2\right)^{4/3}\right)$$

$$\frac{30\times 6^{2/3}\sqrt[3]{7}\,\exp\left(-\frac{1}{3}\,i\arg\left(i\,\mathrm{Im}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)+\mathrm{Re}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)\right)\right)}{\log^{16/3}(8)\left(\mathrm{Im}\left(\Gamma\left(\frac{1}{4}+2i\right)\right)^2+\mathrm{Re}\left(\Gamma\left(\frac{1}{4}+2i\right)\right)^2\right)^{4/3}}$$

$\arg(z)$ is the complex argument

Alternate forms

$$-\frac{30\sqrt[3]{-7}\,6^{2/3}}{\log^{16/3}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^{8/3}}$$

$$\frac{21125\times 65^{2/3}}{216\times 6^{2/3}\sqrt[3]{\left(\frac{9722113}{7}+2138688\,i\right)\left(\left(\frac{1}{4}+2i\right)!\right)^8}\,\log^{16/3}(2)}$$

$$\frac{30\times 6^{2/3}}{\log^{16/3}(8)\sqrt[3]{\frac{1}{7}\left(i\,\mathrm{Im}\left(\Gamma\left(\frac{1}{4}+2i\right)^8\right)+\mathrm{Re}\left(\Gamma\left(\frac{1}{4}+2i\right)^8\right)\right)}}$$

$$\frac{1}{\sqrt[3]{i\,\mathrm{Im}\left(\frac{177147\log^{16}(2)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{28000}\right)+\mathrm{Re}\left(\frac{177147\log^{16}(2)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{28000}\right)}}$$

$$\frac{10\left(\frac{2}{3}\right)^{2/3}}{27\log^{16/3}(2)\sqrt[3]{\frac{1}{7}\left(i\,\mathrm{Im}\left(\Gamma\left(\frac{1}{4}+2i\right)^8\right)+\mathrm{Re}\left(\Gamma\left(\frac{1}{4}+2i\right)^8\right)\right)}}$$

$n!$ is the factorial function

All 3rd roots of 1/(i Im((log16(8) Γ(1/4+2 i)8)/6804000)+Re((log16(8) Γ(1/4+2 i)8)/6804000))

$$\frac{\exp\left(\frac{1}{3} i \left(\pi - \tan^{-1}\left(\frac{\operatorname{Im}\left(\frac{\Gamma\left(\frac{1}{4}+2 i\right)^8 \log^{16}(8)}{6\,804\,000}\right)}{\operatorname{Re}\left(\frac{\Gamma\left(\frac{1}{4}+2 i\right)^8 \log^{16}(8)}{6\,804\,000}\right)}\right)\right)\right)}{\sqrt[3]{\left|i \operatorname{Im}\left(\frac{\Gamma\left(\frac{1}{4}+2 i\right)^8 \log^{16}(8)}{6\,804\,000}\right) + \operatorname{Re}\left(\frac{\Gamma\left(\frac{1}{4}+2 i\right)^8 \log^{16}(8)}{6\,804\,000}\right)\right|}} \approx 1.90\times 10^3 + 1.23\times 10^3\, i$$

(principal root)

$$\frac{\exp\left(\frac{1}{3} i \left(3\pi - \tan^{-1}\left(\frac{\operatorname{Im}\left(\frac{\Gamma\left(\frac{1}{4}+2 i\right)^8 \log^{16}(8)}{6\,804\,000}\right)}{\operatorname{Re}\left(\frac{\Gamma\left(\frac{1}{4}+2 i\right)^8 \log^{16}(8)}{6\,804\,000}\right)}\right)\right)\right)}{\sqrt[3]{\left|i \operatorname{Im}\left(\frac{\Gamma\left(\frac{1}{4}+2 i\right)^8 \log^{16}(8)}{6\,804\,000}\right) + \operatorname{Re}\left(\frac{\Gamma\left(\frac{1}{4}+2 i\right)^8 \log^{16}(8)}{6\,804\,000}\right)\right|}} \approx -2.01\times 10^3 + 1.03\times 10^3\, i$$

$$\frac{\exp\left(i \left(-2\pi + \frac{1}{3}\left(5\pi - \tan^{-1}\left(\frac{\operatorname{Im}\left(\frac{\Gamma\left(\frac{1}{4}+2 i\right)^8 \log^{16}(8)}{6\,804\,000}\right)}{\operatorname{Re}\left(\frac{\Gamma\left(\frac{1}{4}+2 i\right)^8 \log^{16}(8)}{6\,804\,000}\right)}\right)\right)\right)\right)}{\sqrt[3]{\left|i \operatorname{Im}\left(\frac{\Gamma\left(\frac{1}{4}+2 i\right)^8 \log^{16}(8)}{6\,804\,000}\right) + \operatorname{Re}\left(\frac{\Gamma\left(\frac{1}{4}+2 i\right)^8 \log^{16}(8)}{6\,804\,000}\right)\right|}} \approx 1.2\times 10^2 - 2.26\times 10^3\, i$$

$|z|$ is the absolute value of z

$\tan^{-1}(x)$ is the inverse tangent function

Alternative representations

$$\sqrt[3]{\frac{1}{i\,\mathrm{Im}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)+\mathrm{Re}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)}} = \left(1\Big/\left(\frac{i^2\left(-1+e^{-2i\arg\left(\left(\Gamma\left(2i+\frac{1}{4}\right)^8\log^{16}(8)\right)/6804000\right)}\right)\Gamma\left(2i+\frac{1}{4}\right)^8\log^{16}(8)}{2\times 6804000} + \frac{1}{2}\left(\left(\frac{\Gamma\left(2i+\frac{1}{4}\right)^8\log^{16}(8)}{6804000}\right)^* + \frac{\Gamma\left(2i+\frac{1}{4}\right)^8\log^{16}(8)}{6804000}\right)\right)\right)\wedge(1/3)$$

$$\sqrt[3]{\frac{1}{i\,\mathrm{Im}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)+\mathrm{Re}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)}} = \left(1\Big/\left(\frac{i^2\left(-1+e^{-2i\arg\left(\left(\Gamma\left(2i+\frac{1}{4}\right)^8\log^{16}(8)\right)/6804000\right)}\right)\Gamma\left(2i+\frac{1}{4}\right)^8\log^{16}(8)}{2\times 6804000} + \frac{\left(1+e^{-2i\arg\left(\left(\Gamma\left(2i+\frac{1}{4}\right)^8\log^{16}(8)\right)/6804000\right)}\right)\Gamma\left(2i+\frac{1}{4}\right)^8\log^{16}(8)}{2\times 6804000}\right)\right)\wedge(1/3)$$

$$\sqrt[3]{\frac{1}{i\,\text{Im}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)+\text{Re}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)}} = \left(1\Big/\left(\frac{\left(1+e^{-2i\arg\left(\left(\Gamma\left(2i+\frac{1}{4}\right)^8\log^{16}(8)\right)/6804000\right)}\right)\Gamma\left(2i+\frac{1}{4}\right)^8\log^{16}(8)}{2\times 6804000} + \frac{i\left(-\left(\frac{\Gamma\left(2i+\frac{1}{4}\right)^8\log^{16}(8)}{6804000}\right)^* + \frac{\Gamma\left(2i+\frac{1}{4}\right)^8\log^{16}(8)}{6804000}\right)}{2i}\right)\right)\wedge(1/3)$$

z^* is the complex conjugate of z

Series representations

$$\sqrt[3]{\frac{1}{i\,\text{Im}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)+\text{Re}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)}} = \left(30\times 6^{2/3}\sqrt[3]{7}\right)\Big/\left(\left(i\,\text{Im}\left(\Gamma\left(\frac{1}{4}+2i\right)^8\left(\log(7)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{7}\right)^k}{k}\right)^{16}\right)+\text{Re}\left(\Gamma\left(\frac{1}{4}+2i\right)^8\left(\log(7)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{7}\right)^k}{k}\right)^{16}\right)\right)\wedge(1/3)\right)$$

$$\sqrt[3]{\frac{1}{i\,\text{Im}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)+\text{Re}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)}} = \frac{30\times 6^{2/3}\sqrt[3]{7}}{\log^{16/3}(8)\sqrt[3]{i\,\text{Im}\left(\frac{1}{\left(\sum_{k=1}^{\infty}\left(\frac{1}{4}+2i\right)^k c_k\right)^8}\right)+\text{Re}\left(\frac{1}{\left(\sum_{k=1}^{\infty}\left(\frac{1}{4}+2i\right)^k c_k\right)^8}\right)}}$$

$$\text{for}\left(c_1=1\text{ and }c_2=1\text{ and }c_k=\frac{\gamma\,c_{-1+k}+\sum_{j=1}^{-2+k}(-1)^{1+j+k}\,c_j\,\zeta(-j+k)}{-1+k}\right)$$

$$\sqrt[3]{\frac{1}{i\,\mathrm{Im}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)+\mathrm{Re}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)}} = \frac{30\times 6^{2/3}\sqrt[3]{7}}{\sqrt[3]{i\,\mathrm{Im}\left(\frac{\left(\log(7)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{7}\right)^k}{k}\right)^{16}}{\left(\sum_{k=1}^{\infty}\left(\frac{1}{4}+2i\right)^k c_k\right)^8}\right)+\mathrm{Re}\left(\frac{\left(\log(7)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{7}\right)^k}{k}\right)^{16}}{\left(\sum_{k=1}^{\infty}\left(\frac{1}{4}+2i\right)^k c_k\right)^8}\right)}}$$

$$\text{for }\left(c_1=1\text{ and }c_2=1\text{ and }c_k=\frac{\gamma\,c_{-1+k}+\sum_{j=1}^{-2+k}(-1)^{1+j+k}\,c_j\,\zeta(-j+k)}{-1+k}\right)$$

$\zeta(s)$ is the Riemann zeta function

γ is the Euler-Mascheroni constant

Integral representations

$$\sqrt[3]{\frac{1}{i\,\mathrm{Im}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)+\mathrm{Re}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)}} = \frac{30\times 6^{2/3}\sqrt[3]{7}}{\log^{16/3}(8)\sqrt[3]{i\,\mathrm{Im}\left(\left(\int_0^1\log^{-3/4+2i}\left(\frac{1}{t}\right)dt\right)^8\right)+\mathrm{Re}\left(\left(\int_0^1\log^{-3/4+2i}\left(\frac{1}{t}\right)dt\right)^8\right)}}$$

$$\sqrt[3]{\frac{1}{i\,\mathrm{Im}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)+\mathrm{Re}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)}} = \left(30\times 6^{2/3}\sqrt[3]{7}\right) \Big/ \left(\log^{16/3}(8)\left(i\,\mathrm{Im}\left(\exp\left(8\int_0^1\frac{(3-8i)-4x^{1/4+2i}+(1+8i)x}{4\log(x)-4x\log(x)}\,dx\right)\right)+\mathrm{Re}\left(\exp\left(8\int_0^1\frac{(3-8i)-4x^{1/4+2i}+(1+8i)x}{4\log(x)-4x\log(x)}\,dx\right)\right)\right)^{\wedge}(1/3)\right)$$

$$\frac{\sqrt[3]{\frac{1}{i\,\mathrm{Im}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)+\mathrm{Re}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)}} = \frac{30\times 6^{2/3}\sqrt[3]{7}}{\sqrt[3]{i\,\mathrm{Im}\left(\left(\int_1^8 \frac{1}{t}\,dt\right)^{16}\left(\int_0^1 \log^{-3/4+2i}\left(\frac{1}{t}\right)dt\right)^8\right)+\mathrm{Re}\left(\left(\int_1^8 \frac{1}{t}\,dt\right)^{16}\left(\int_0^1 \log^{-3/4+2i}\left(\frac{1}{t}\right)dt\right)^8\right)}}$$

2(((((1/((i Im((log^16(8) Γ(1/4 + 2 i)^8)/6804000) + Re((log^16(8) Γ(1/4 + 2 i)^8)/6804000)))))^(1/3))-(199+11+47+(π+1/8MRB const)))+1/2)

Input

$$2\left(\left(\sqrt[3]{\frac{1}{i\,\mathrm{Im}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)+\mathrm{Re}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)}} - \left(199+11+47+\left(\pi+\frac{1}{8}C_{\mathrm{MRB}}\right)\right)\right)+\frac{1}{2}\right)$$

$\log(x)$ is the natural logarithm
$\Gamma(x)$ is the gamma function
$\mathrm{Im}(z)$ is the imaginary part of z
$\mathrm{Re}(z)$ is the real part of z
i is the imaginary unit
C_{MRB} is the MRB constant

Exact result

$$2\left(\frac{1}{\sqrt[3]{i\,\mathrm{Im}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)+\mathrm{Re}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)}} - \frac{C_{\mathrm{MRB}}}{8} - \frac{513}{2} - \pi\right)$$

Decimal approximation

3274.401777888157871029850700495406484241517033326316479380927331…
+
2460.696353032793220712826929790993907642327950136342053497249014…
i

Polar coordinates

$r = 4095.941106127628158615096714812602565753495835328858093239659014$ (radius), $\theta = 0.6444571812045965954248613255514808800251452408034398759263546595$ (angle)

4095.941106127…. ≈ 4096 = 64^2 , that multiplied by 2 give 8192, indeed:

The total amplitude vanishes for gauge group SO(8192), while the vacuum energy is negative and independent of the gauge group.

The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). (From: “Dilaton Tadpole for the Open Bosonic String “ Michael R. Douglas and Benjamin Grinstein - September 2,1986)

Possible closed forms

$$\frac{1}{20}\left(2319 e^{\pi} + 2761 \pi - 430 \log(\pi) - 919 \log(2 \pi) + 4223 \tan^{-1}(\pi)\right) + \frac{i\left(343\,625 + 22\,349 e + 25\,840 e^2\right)}{89 e} \approx$$

3274.401777888157871029628100548945623733509288718345041301125081 +
2460.696353032793220673839143612188646270945110134810983706762522
i

$$\frac{1}{20}\left(2319\,e^{\pi} + 2761\,\pi - 430\log(\pi) - 919\log(2\,\pi) + 4223\tan^{-1}(\pi)\right) + i\left(\frac{17856\,e!}{29} + 543 - \frac{136}{e} - \frac{6996\,e}{29}\right) \approx 3274.40177788815787102962810054894562373350928871834504130112508 1 + 2460.6963530327932207132430237301253351150084776932221951122001 07\,i$$

$$\frac{1}{20}\left(2319\,e^{\pi} + 2761\,\pi - 430\log(\pi) - 919\log(2\,\pi) + 4223\tan^{-1}(\pi)\right) + \frac{i\left(-1257 + 169\sqrt{\pi} + 4257\,\pi + 70\,\pi^{3/2} + 269\,\pi^2\right)}{2\,\pi} \approx 3274.40177788815787102962810054894562373350928871834504130112508 1 + 2460.6963530327932207142872200656679497489400186627819297361207 12\,i$$

$\tan^{-1}(x)$ is the inverse tangent function
$\log(x)$ is the natural logarithm
$n!$ is the factorial function

27sqrt(2(((((1/((i Im((log^16(8) Γ(1/4 + 2 i)^8)/6804000) + Re((log^16(8) Γ(1/4 + 2 i)^8)/6804000)))))^(1/3))-(199+11+47+(π+1/8MRB const)))+1/2))+1

Input

$$27\sqrt{\left(2\left(\left(\sqrt[3]{\frac{1}{i\,\mathrm{Im}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2\,i\right)^8}{6804000}\right) + \mathrm{Re}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2\,i\right)^8}{6804000}\right)}} - \left(199 + 11 + 47 + \left(\pi + \frac{1}{8}\,C_{\mathrm{MRB}}\right)\right)\right) + \frac{1}{2}\right)\right)} + 1$$

$\log(x)$ is the natural logarithm
$\Gamma(x)$ is the gamma function
$\mathrm{Im}(z)$ is the imaginary part of z
$\mathrm{Re}(z)$ is the real part of z
i is the imaginary unit
C_{MRB} is the MRB constant

Exact result

$$1+27\sqrt{2\left(\frac{1}{\sqrt[3]{i\,\mathrm{Im}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)+\mathrm{Re}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)}}-\frac{C_{\mathrm{MRB}}}{8}-\frac{513}{2}-\pi\right)}$$

Decimal approximation

1640.051549288110098580729843773684579210148176767548976607298684… + 547.2212396675470050415963188506769111110809709619956746158418146… i

Polar coordinates

$r = 1728.936138052998517353206634733519722056121821478764570992885221$ (radius), $\theta = 0.3220454252269946465892898729637538280144283389689239852184714556$ (angle)

1728.936138052….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve (1728 = 8^2 * 3^3). The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Possible closed forms

$$\frac{-32859-9992\pi+206975\pi^2}{384\pi}+\frac{i\left(-2514\,e\,e!+17186+17770\,e-91\,e^2\right)}{24\,e}\approx$$
$$1640.0515492881100985800679976379172151367930141585708604101300090+547.2212396675470050386991914726032576577154890104804265667464 38\,i$$

$$\frac{5334\, e\, e! - 27\,364 + 13\,929\, e - 3145\, e^2}{11\, e} + \frac{i\left(-2514\, e\, e! + 17\,186 + 17\,770\, e - 91\, e^2\right)}{24\, e} \approx$$
$$1640.051549288110098603675460741621655974651610132668684312267370 + 547.221239667547005038699191472603257657715489010480426566746438\, i$$

$$\frac{-32\,859 - 9992\,\pi + 206\,975\,\pi^2}{384\,\pi} + \frac{1}{300}\, i\left(538\,830\,\zeta(3) + 35\,940\,\zeta(5) - 67\,800\,\pi^2 + 1523\,\pi^4\right) \approx$$
$$1640.051549288110098580067997637917215136793014158570860410130090 + 547.221239667547005028034420145917479943117817724346839012542097\, i$$

$n!$ is the factorial function
$\zeta(s)$ is the Riemann zeta function

(27sqrt(2(((((1/((i Im((log^16(8) Γ(1/4 + 2 i)^8)/6804000) + Re((log^16(8) Γ(1/4 + 2 i)^8)/6804000)))))^(1/3))-(199+11+47+(π+1/8MRB const)))+1/2))+1)^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\left(27\sqrt{\left(2\left(\left(\left(\sqrt[3]{\frac{1}{i\,\mathrm{Im}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6\,804\,000}\right)+\mathrm{Re}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6\,804\,000}\right)}} - \left(199+11+47+\left(\pi+\frac{1}{8}C_{\mathrm{MRB}}\right)\right)\right)+\frac{1}{2}\right)\right)\right)}+1\right) \mathbin{\hat{}} (1/15) + C_{\mathrm{MRB}}^{1-1/(4\pi)+\pi}$$

$\log(x)$ is the natural logarithm
$\Gamma(x)$ is the gamma function
$\mathrm{Im}(z)$ is the imaginary part of z
$\mathrm{Re}(z)$ is the real part of z
i is the imaginary unit
C_{MRB} is the MRB constant

Exact result

$$C_{\mathrm{MRB}}^{1-1/(4\pi)+\pi} + \left(1 + 27 \sqrt{2\left(\frac{1}{\sqrt[3]{i\,\mathrm{Im}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right) + \mathrm{Re}\left(\frac{\log^{16}(8)\,\Gamma\left(\frac{1}{4}+2i\right)^8}{6804000}\right)}} - \frac{C_{\mathrm{MRB}}}{8} - \frac{513}{2} - \pi\right)}\right)\hat{}\,(1/15)$$

Decimal approximation

1.6445551353304644130036036285412035333334484959318542147960838… +
0.0352894134758896364764234075663178852928823613765284684372344OO…
i

Polar coordinates

$r = 1.6449337177665470970693921902176725979965226998190770509267302$ (radius), $\theta = 0.021455041401890843021714290787630884479294782893142391423056293$ (angle)

1.644933717766…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape)

Possible closed forms

$$\frac{187 + 97\pi + 212\pi^2}{-37 + 3\pi + 162\pi^2} + \frac{159473503\, i}{4519018235} \approx$$
$$1.6445551353304644133757183309653130064303122130844089943108816 +$$
$$0.0352894134758896364577294076796306620789725580737781643406004\, i$$

$$\frac{187 + 97\pi + 212\pi^2}{-37 + 3\pi + 162\pi^2} + \frac{i\pi^4}{2^{3/4}\sqrt[4]{7256550384047}} \approx$$
$$1.6445551353304644133757183309653130064303122130844089943108816 +$$
$$0.0352894134758896924917828954288371161341062122623466470990053\, i$$

$$\frac{187 + 97\pi + 212\pi^2}{-37 + 3\pi + 162\pi^2} +$$
$$i \left(\text{root of } 8741 x^3 + 9173 x^2 - 50888 x + 1784 \text{ near } x = 0.0352894\right) \approx$$
$$1.6445551353304644133757183309653130064303122130844089943108816 +$$
$$0.0352894134758896364926182820183163010817797031603288277895555\, i$$

We have:

By the Large Sieve inequality,

$$\sum_{q \asymp Q} \sum^{*}_{\chi \pmod q} \left| \sum_{Q^{2-\epsilon} \leq n \leq Q^2} \frac{d_4(n)}{\sqrt{n}} \chi(n) \right|^2$$

$$\ll (Q^2 + Q^2) \left(\sum_{Q^{2-\epsilon} \leq n \leq Q^2} \frac{d_4(n)^2}{n} \right)$$

$$\ll Q^2 \left((\log Q^2)^{16} - (\log Q^{2-\epsilon})^{16} \right)$$

$$\ll \epsilon Q^2 \log^{16} Q.$$

From:

$$\epsilon Q^2 \log^{16} Q.$$

for ε = 1/24 :

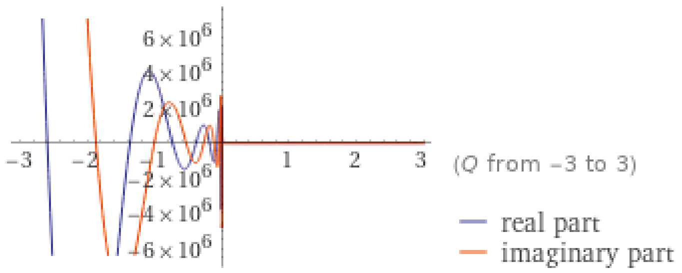
1/24*Q^2*ln^16(Q)

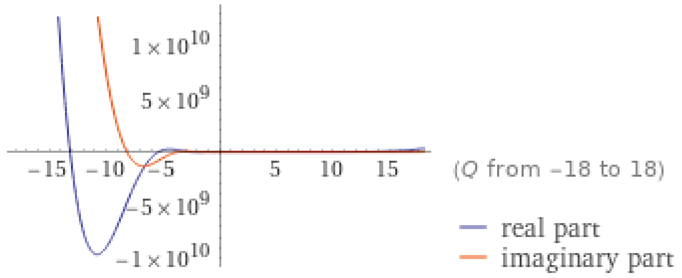
Input

$$\frac{1}{24} Q^2 \log^{16}(Q)$$

$\log(x)$ is the natural logarithm

Plots **(figures that can be related to the open strings)**





Root

$Q = 1$

Derivative

$$\frac{d}{dQ}\left(\frac{1}{24} Q^2 \log^{16}(Q)\right) = \frac{1}{12} Q \log^{15}(Q) (\log(Q) + 8)$$

Indefinite integral

$$\int \frac{1}{24} Q^2 \log^{16}(Q)\, dQ = \frac{1}{4\,251\,528} Q^3 \left(59\,049 \log^{16}(Q) - 314\,928 \log^{15}(Q) + 1\,574\,640 \log^{14}(Q) - 7\,348\,320 \log^{13}(Q) + 31\,842\,720 \log^{12}(Q) - 127\,370\,880 \log^{11}(Q) + 467\,026\,560 \log^{10}(Q) - 1\,556\,755\,200 \log^{9}(Q) + 4\,670\,265\,600 \log^{8}(Q) - 12\,454\,041\,600 \log^{7}(Q) + 29\,059\,430\,400 \log^{6}(Q) - 58\,118\,860\,800 \log^{5}(Q) + 96\,864\,768\,000 \log^{4}(Q) - 129\,153\,024\,000 \log^{3}(Q) + 129\,153\,024\,000 \log^{2}(Q) - 86\,102\,016\,000 \log(Q) + 28\,700\,672\,000\right) + \text{constant}$$

(assuming a complex-valued logarithm)

Local maximum

$$\max\left\{\frac{1}{24} Q^2 \log^{16}(Q)\right\} = \frac{35\,184\,372\,088\,832}{3\,e^{16}} \text{ at } Q = \frac{1}{e^8}$$

Global minimum

$$\min\left\{\frac{1}{24} Q^2 \log^{16}(Q)\right\} = 0 \text{ at } Q = 1$$

Alternative representations

$$\frac{1}{24} Q^2 \log^{16}(Q) = \frac{1}{24} Q^2 \log_e^{16}(Q)$$

$$\frac{1}{24} Q^2 \log^{16}(Q) = \frac{1}{24} Q^2 \left(\log(a) \log_a(Q)\right)^{16}$$

$$\frac{1}{24} Q^2 \log^{16}(Q) = \frac{1}{24} Q^2 (-\text{Li}_1(1-Q))^{16}$$

$\log_b(x)$ is the base- b logarithm
$\text{Li}_n(x)$ is the polylogarithm function

Series representations

$$\frac{1}{24} Q^2 \log^{16}(Q) = \frac{1}{24} Q^2 \left(\sum_{k=1}^{\infty} \frac{(-1)^k (-1+Q)^k}{k}\right)^{16} \quad \text{for } |-1+Q| < 1$$

$$\frac{1}{24} Q^2 \log^{16}(Q) = \frac{1}{24} Q^2 \left(\log(-1+Q) - \sum_{k=1}^{\infty} \frac{(-1)^k (-1+Q)^{-k}}{k}\right)^{16} \quad \text{for } |-1+Q| > 1$$

$$\frac{1}{24} Q^2 \log^{16}(Q) = \frac{1}{24} Q^2 \left(2 i \pi \left\lfloor \frac{\arg(Q-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (Q-x)^k x^{-k}}{k}\right)^{16} \quad \text{for } x < 0$$

$|z|$ is the absolute value of z
$\arg(z)$ is the complex argument
$\lfloor x \rfloor$ is the floor function

Integral representations

$$\frac{1}{24} Q^2 \log^{16}(Q) = \frac{1}{24} Q^2 \left(\int_1^Q \frac{1}{t}\, dt\right)^{16}$$

$$\frac{1}{24} Q^2 \log^{16}(Q) = \frac{Q^2 \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(-1+Q)^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\, ds\right)^{16}}{1572864\,\pi^{16}} \quad \text{for } (-1 < \gamma < 0 \text{ and } |\arg(-1+Q)| < \pi)$$

$\Gamma(x)$ is the gamma function

Definite integral

$$\int_0^1 \frac{1}{24} Q^2 \log^{16}(Q)\, dQ = \frac{3587584000}{531441} \approx 6750.67$$

From:

$$\frac{1}{24} Q^2 \log^{16}(Q)$$

for ε = 1/24 and Q = 8 , we obtain:

1/24*8^2*ln^16(8)

Input

$$\frac{1}{24} \times 8^2 \log^{16}(8)$$

$\log(x)$ is the natural logarithm

Exact result

$$\frac{8 \log^{16}(8)}{3}$$

Decimal approximation

325923.88203480004762538450628659972264384513318993046992517533367...

325923.8820348….

Property

$\frac{8 \log^{16}(8)}{3}$ is a transcendental number

Alternate form

$114\,791\,256 \log^{16}(2)$

Alternative representations

$$\frac{1}{24} \times 8^2 \log^{16}(8) = \frac{1}{24} \times 8^2 \log_e^{16}(8)$$

$$\frac{1}{24} \times 8^2 \log^{16}(8) = \frac{1}{24} \times 8^2 (\log(a) \log_a(8))^{16}$$

$$\frac{1}{24} \times 8^2 \log^{16}(8) = \frac{1}{24} \times 8^2 (-\text{Li}_1(-7))^{16}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations

$$\frac{1}{24} \times 8^2 \log^{16}(8) = \frac{8}{3} \left(\log(7) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{7}\right)^k}{k}\right)^{16}$$

$$\frac{1}{24} \times 8^2 \log^{16}(8) = \frac{8}{3} \left(2 i \pi \left\lfloor \frac{\arg(8-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (8-x)^k x^{-k}}{k}\right)^{16}$$

for $x < 0$

$$\frac{1}{24} \times 8^2 \log^{16}(8) = \frac{8}{3} \left(\log(z_0) + \left\lfloor \frac{\arg(8-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (8-z_0)^k z_0^{-k}}{k}\right)^{16}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representations

$$\frac{1}{24} \times 8^2 \log^{16}(8) = \frac{8}{3} \left(\int_1^8 \frac{1}{t} \, dt\right)^{16}$$

$$\frac{1}{24}\times 8^2 \log^{16}(8) = \frac{\left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{7^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^{16}}{24576\,\pi^{16}} \quad \text{for } -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

From:

$$\frac{8\log^{16}(8)}{3}$$

we obtain:

(1/27((3sqrt((8 log^16(8))/3)+16+2MRB const)-1))^2-1/Pi

Input

$$\left(\frac{1}{27}\left(\left(3\sqrt{\frac{1}{3}\left(8\log^{16}(8)\right)}+16+2\,C_{\text{MRB}}\right)-1\right)\right)^2-\frac{1}{\pi}$$

$\log(x)$ is the natural logarithm

C_{MRB} is the MRB constant

Exact result

$$\frac{1}{729}\left(2\,C_{\text{MRB}}+15+2\sqrt{6}\,\log^{8}(8)\right)^2-\frac{1}{\pi}$$

Decimal approximation

4096.004191243035173426672418035042932382194552921537310071051649 0…

4096.004191243…. ≈ 4096 = 64^2 , that multiplied by 2 give 8192, indeed:

The total amplitude vanishes for gauge group SO(8192), while the vacuum energy is negative and independent of the gauge group.

The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). (From: "Dilaton Tadpole for the Open Bosonic String " Michael R. Douglas and Benjamin Grinstein - September 2,1986)

Alternate forms

$$\frac{1}{729\pi}\left(60\pi C_{\text{MRB}} + 4\pi C_{\text{MRB}}^2 + 8\sqrt{6}\,\pi \log^8(8)\, C_{\text{MRB}} - 729 + 225\pi + 24\pi\log^{16}(8) + 60\sqrt{6}\,\pi\log^8(8)\right)$$

$$\frac{1}{729}\left(60 C_{\text{MRB}} + 4 C_{\text{MRB}}^2 + 8\sqrt{6}\,\log^8(8)\, C_{\text{MRB}} + 225 + 24\log^{16}(8) + 60\sqrt{6}\,\log^8(8)\right) - \frac{1}{\pi}$$

$$\frac{4\pi C_{\text{MRB}}^2 + 12\pi\left(5 + 4374\sqrt{6}\,\log^8(2)\right)C_{\text{MRB}} + 9\left(\pi\left(5 + 4374\sqrt{6}\,\log^8(2)\right)^2 - 81\right)}{729\pi}$$

$$\frac{1}{729}\left(2 C_{\text{MRB}} + 15 + 13\,122\sqrt{6}\,\log^8(2)\right)^2 - \frac{1}{\pi}$$

Expanded form

$$\frac{20 C_{\text{MRB}}}{243} + \frac{4 C_{\text{MRB}}^2}{729} + \frac{8}{243}\sqrt{\frac{2}{3}}\log^8(8)\, C_{\text{MRB}} + \frac{25}{81} - \frac{1}{\pi} + \frac{8\log^{16}(8)}{243} + \frac{20}{81}\sqrt{\frac{2}{3}}\log^8(8)$$

27sqrt((1/27((3sqrt((8 log^16(8))/3)+16+2MRB const)-1))^2-1/Pi)+1

Input

$$27\sqrt{\left(\frac{1}{27}\left(\left(3\sqrt{\frac{1}{3}\left(8\log^{16}(8)\right)}+16+2\,C_{\mathrm{MRB}}\right)-1\right)\right)^2-\frac{1}{\pi}}+1$$

$\log(x)$ is the natural logarithm
C_{MRB} is the MRB constant

Exact result

$$27\sqrt{\frac{1}{729}\left(2\,C_{\mathrm{MRB}}+15+2\sqrt{6}\,\log^8(8)\right)^2-\frac{1}{\pi}}+1$$

Decimal approximation

1729.000884090101570057971654319082722056051288438189762651056673 2

…

1729.00088409….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve ($1728 = 8^2 * 3^3$). The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1}{\sqrt{\frac{\pi}{60\pi C_{\mathrm{MRB}}+4\pi C_{\mathrm{MRB}}^2+8\sqrt{6}\,\pi\log^8(8)\,C_{\mathrm{MRB}}-729+225\pi+24\pi\log^{16}(8)+60\sqrt{6}\,\pi\log^8(8)}}}+1$$

$$\frac{1}{\sqrt{\frac{\pi}{4\pi C_{\mathrm{MRB}}^2+12\pi\left(5+4374\sqrt{6}\,\log^8(2)\right)C_{\mathrm{MRB}}+9\left(\pi\left(5+4374\sqrt{6}\,\log^8(2)\right)^2-81\right)}}}+1$$

$$\frac{\sqrt{\frac{\pi}{60\pi C_{MRB}+4\pi C_{MRB}^2+8\sqrt{6}\,\pi\log^8(8)\,C_{MRB}-729+225\pi+24\pi\log^{16}(8)+60\sqrt{6}\,\pi\log^8(8)}}+1}{\sqrt{\frac{\pi}{60\pi C_{MRB}+4\pi C_{MRB}^2+8\sqrt{6}\,\pi\log^8(8)\,C_{MRB}-729+225\pi+24\pi\log^{16}(8)+60\sqrt{6}\,\pi\log^8(8)}}}$$

$$27\sqrt{\frac{1}{729}\left(2C_{MRB}+15+13122\sqrt{6}\log^8(2)\right)^2-\frac{1}{\pi}}+1$$

(27sqrt((1/27((3sqrt((8 log^16(8))/3)+16+2MRB const)-1))^2-1/Pi)+1)^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\sqrt[15]{27\sqrt{\left(\frac{1}{27}\left(\left(3\sqrt{\frac{1}{3}\left(8\log^{16}(8)\right)}+16+2C_{MRB}\right)-1\right)\right)^2-\frac{1}{\pi}}+1}+C_{MRB}^{1-1/(4\pi)+\pi}$$

log(x) is the natural logarithm

C_{MRB} is the MRB constant

Exact result

$$C_{MRB}^{1-1/(4\pi)+\pi}+\sqrt[15]{27\sqrt{\frac{1}{729}\left(2C_{MRB}+15+2\sqrt{6}\log^8(8)\right)^2-\frac{1}{\pi}}+1}$$

Decimal approximation

1.6449380801657499731509137738114450052977659459359145313129229900

...

1.6449380801…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape)

Alternate forms

$$C_{\text{MRB}}^{-1/(4\pi)} \left(C_{\text{MRB}}^{1+\pi} + \sqrt[4\pi]{C_{\text{MRB}}} \sqrt[15]{27 \sqrt{\frac{1}{729} \left(2 C_{\text{MRB}} + 15 + 2 \sqrt{6} \log^8(8)\right)^2 - \frac{1}{\pi}} + 1} \right)$$

$$C_{\text{MRB}}^{1-1/(4\pi)+\pi} + \sqrt[15]{\frac{1}{\sqrt{\frac{\pi}{4 \pi C_{\text{MRB}}^2 + 12 \pi \left(5 + 4374 \sqrt{6} \log^8(2)\right) C_{\text{MRB}} + 9 \left(\pi \left(5 + 4374 \sqrt{6} \log^8(2)\right)^2 - 81\right)}}} + 1}$$

Now, we have:

Roughly we want to study

$$\frac{Q}{H} \frac{HQ}{N} \sum_{h \asymp H} \sum_{\chi \bmod h}^{*} \int \left| \sum_{m \asymp N} \frac{d_4(m)\chi(m)}{m^{1/2+it}} \right|^2 g\left(\frac{HQt}{N}\right) dt$$

$$= \frac{Q^2}{N} \sum_{h \asymp H} \sum_{\chi \bmod h}^{*} \int \left| \sum_{m} \frac{d_4(m)\chi(m)}{m^{1/2+it}} w\left(\frac{m}{N}\right) \right|^2 g\left(\frac{t}{T}\right) dt,$$

writing $T = \frac{N}{HQ}$.

The hybrid large sieve gives

$$\ll \frac{Q^2}{N}(H^2T+N)\sum_{m\sim N}\frac{d_4(n)^2}{n} \asymp Q^2(\log Q)^{16},$$

which is too big.

From

$$Q^2(\log Q)^{16}$$

we obtain, for Q = 8:

8^2(ln(8))^16

Input

$8^2 \log^{16}(8)$

$\log(x)$ is the natural logarithm

Exact result

$64 \log^{16}(8)$

Decimal approximation

$7.822173168835201143009228150878393343452283196558331278204 20\ldots \times 10^6$

7.8221731688...*10^6

Property

$64 \log^{16}(8)$ is a transcendental number

Alternate form

$2\,754\,990\,144 \log^{16}(2)$

Alternative representations

$$8^2 \log^{16}(8) = 8^2 \log_e^{16}(8)$$

$$8^2 \log^{16}(8) = 8^2 (\log(a) \log_a(8))^{16}$$

$$8^2 \log^{16}(8) = 8^2 (-\text{Li}_1(-7))^{16}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations

$$8^2 \log^{16}(8) = 64 \left(\log(7) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{7}\right)^k}{k}\right)^{16}$$

$$8^2 \log^{16}(8) = 64 \left(2 i \pi \left\lfloor \frac{\arg(8-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (8-x)^k x^{-k}}{k}\right)^{16} \quad \text{for } x < 0$$

$$8^2 \log^{16}(8) = 64 \left(\log(z_0) + \left\lfloor \frac{\arg(8-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (8-z_0)^k z_0^{-k}}{k}\right)^{16}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representations

$$8^2 \log^{16}(8) = 64 \left(\int_1^8 \frac{1}{t} \, dt\right)^{16}$$

$$8^2 \log^{16}(8) = \frac{\left(\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{7^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^{16}}{1024\,\pi^{16}} \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

From which:

1/2sqrt((8^2(ln(8))^16))+322+7+φ

Input

$$\frac{1}{2}\sqrt{8^2 \log^{16}(8)} + 322 + 7 + \phi$$

$\log(x)$ is the natural logarithm
ϕ is the golden ratio

Exact result

$\phi + 329 + 4\log^8(8)$

Decimal approximation

1729.0254468011964399657148939973469884827154522558548361858848636
…

1729.0254468….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve (1728 = 8^2 * 3^3). The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Property

$329 + \phi + 4\log^8(8)$ is a transcendental number

Alternate forms

$$\frac{1}{2}\left(659+\sqrt{5}+8\log^8(8)\right)$$

$$\frac{659}{2}+\frac{\sqrt{5}}{2}+4\log^8(8)$$

$$\frac{1}{2}\left(659+\sqrt{5}\right)+4\log^8(8)$$

$$\phi+329+26244\log^8(2)$$

Alternative representations

$$\frac{1}{2}\sqrt{8^2\log^{16}(8)}+322+7+\phi=329+\phi+\frac{1}{2}\sqrt{8^2\log_e^{16}(8)}$$

$$\frac{1}{2}\sqrt{8^2\log^{16}(8)}+322+7+\phi=329+\phi+\frac{1}{2}\sqrt{8^2\left(\log(a)\log_a(8)\right)^{16}}$$

$$\frac{1}{2}\sqrt{8^2\log^{16}(8)}+322+7+\phi=329+\phi+\frac{1}{2}\sqrt{8^2\left(-\text{Li}_1(-7)\right)^{16}}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations

$$\frac{1}{2}\sqrt{8^2\log^{16}(8)}+322+7+\phi=329+\phi+4\left(\log(7)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{7}\right)^k}{k}\right)^8$$

$$\frac{1}{2}\sqrt{8^2\log^{16}(8)}+322+7+\phi=$$
$$329+\phi+4\left(2i\pi\left\lfloor\frac{\arg(8-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(8-x)^k x^{-k}}{k}\right)^8 \quad \text{for } x<0$$

$$\frac{1}{2}\sqrt{8^2\log^{16}(8)}+322+7+\phi =$$
$$329+\phi+4\left(\log(z_0)+\left\lfloor\frac{\arg(8-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(8-z_0)^k z_0^{-k}}{k}\right)^8$$

arg(z) is the complex argument
⌊x⌋ is the floor function

Integral representations

$$\frac{1}{2}\sqrt{8^2\log^{16}(8)}+322+7+\phi = 329+\phi+4\left(\int_1^8\frac{1}{t}\,dt\right)^8$$

$$\frac{1}{2}\sqrt{8^2\log^{16}(8)}+322+7+\phi = 329+\phi+\frac{\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{7^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^8}{64\pi^8}$$
for $-1<\gamma<0$

Γ(x) is the gamma function

(1/2sqrt((8^2(ln(8))^16))+322+7+φ)^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\sqrt[15]{\frac{1}{2}\sqrt{8^2\log^{16}(8)}+322+7+\phi}+C_{MRB}^{1-1/(4\pi)+\pi}$$

log(x) is the natural logarithm
ϕ is the golden ratio
C_{MRB} is the MRB constant

Exact result

$$C_{MRB}^{1-1/(4\pi)+\pi}+\sqrt[15]{\phi+329+4\log^8(8)}$$

Decimal approximation

1.644939636991337385944592359571203659197711336841326275275693950 5…

1.644939636…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape)

Alternate forms

$$C_{\mathrm{MRB}}^{1-1/(4\pi)+\pi} + \sqrt[15]{\frac{1}{2}\left(659+\sqrt{5}+8\log^8(8)\right)}$$

$$C_{\mathrm{MRB}}^{1-1/(4\pi)+\pi} + \sqrt[15]{\frac{1}{2}\left(659+\sqrt{5}\right)+4\log^8(8)}$$

$$\frac{1}{2} C_{\mathrm{MRB}}^{-1/(4\pi)} \left(2 C_{\mathrm{MRB}}^{1+\pi} + 2^{14/15} \sqrt[15]{659+\sqrt{5}+8\log^8(8)} \sqrt[4\pi]{C_{\mathrm{MRB}}}\right)$$

$$C_{\mathrm{MRB}}^{1-1/(4\pi)+\pi} + \sqrt[15]{\phi+329+26244\log^8(2)}$$

Expanded form

$$C_{\mathrm{MRB}}^{1-1/(4\pi)+\pi} + \sqrt[15]{329+\frac{1}{2}\left(1+\sqrt{5}\right)+4\log^8(8)}$$

(1/27((1/2sqrt((8^2(ln(8))^16))+322+7+φ)-1))^2-MRB const

Input

$$\left(\frac{1}{27}\left(\left(\frac{1}{2}\sqrt{8^2\log^{16}(8)}+322+7+\phi\right)-1\right)\right)^2 - C_{\mathrm{MRB}}$$

$\log(x)$ is the natural logarithm
ϕ is the golden ratio
C_{MRB} is the MRB constant

Exact result

$$\frac{1}{729}\left(\phi+328+4\log^8(8)\right)^2 - C_{\mathrm{MRB}}$$

Decimal approximation

4095.932777932948889879270198432134883740700594955564766763469518 0
…

4095.9327779…. ≈ 4096 = 64^2 , that multiplied by 2 give 8192, indeed:

The total amplitude vanishes for gauge group SO(8192), while the vacuum energy is negative and independent of the gauge group.

The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). (From: "Dilaton Tadpole for the Open Bosonic String " Michael R. Douglas and Benjamin Grinstein - September 2,1986)

Alternate forms

$$\frac{1}{729}\left(\frac{1}{2}\left(657+\sqrt{5}\right)+4\log^8(8)\right)^2-C_{\text{MRB}}$$

$$\frac{1}{729}\left(328+\frac{1}{2}\left(1+\sqrt{5}\right)+4\log^8(8)\right)^2-C_{\text{MRB}}$$

$$\frac{1}{729}\left(-729\,C_{\text{MRB}}+\phi^2+8\,\phi\left(82+6561\log^8(2)\right)+16\left(82+6561\log^8(2)\right)^2\right)$$

$$\frac{1}{729}\left(\phi+328+26244\log^8(2)\right)^2-C_{\text{MRB}}$$

Expanded forms

$$-C_{\text{MRB}}+\frac{656\,\phi}{729}+\frac{\phi^2}{729}+\frac{8}{729}\,\phi\log^8(8)+\frac{107584}{729}+\frac{16\log^{16}(8)}{729}+\frac{2624\log^8(8)}{729}$$

$$-C_{\mathrm{MRB}}+\frac{215827}{1458}+\frac{73\sqrt{5}}{162}+\frac{16\log^{16}(8)}{729}+\frac{292\log^{8}(8)}{81}+\frac{4}{729}\sqrt{5}\log^{8}(8)$$

We have:

$$\sum_m \frac{d_4(m)\chi(m)}{m^{1/2+it}} w\left(\frac{m}{N}\right) \sim L^4(1/2+it,\chi).$$

and we apply the functional equation for $L^4(1/2+it)$ (Voronoi summation).

$$\frac{Q^2}{N}\sum_{h\asymp H}\ \sideset{}{^*}\sum_{\chi \bmod h}\int\left|\sum_{m\ll\frac{(TH)^4}{N}}\frac{d_4(m)\bar{\chi}(m)}{m^{1/2-it}}\right|^2 g\left(\frac{t}{T}\right)dt.$$

$$\frac{(TH)^4}{N}=\frac{1}{N}\left(\frac{N}{Q}\right)^4 \asymp Q^{2-3\epsilon},$$

and this is shorter than the original sum!

From

$$Q^{2-3\epsilon}$$

we obtain:

8^(2-3*1/24)

Input

$8^{2-3\times 1/24}$

Result

$8^{15/8}$

Decimal approximation

49.350746413054106355593339586903514921818024374146317145211514839...

49.350746413….

From which:

76*1/((8^(2-3*1/24))-2-2MRB const)

Input

$$76\times\frac{1}{8^{2-3\times 1/24}-2-2\,C_{\text{MRB}}}$$

C_{MRB} is the MRB constant

Result

$$\frac{76}{-2\,C_{\text{MRB}}-2+32\times 2^{5/8}}$$

Decimal approximation

1.6178809177204084018288622624594185244628022903784172826524761267...

1.61788091772…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternate forms

$$\frac{38}{-C_{\text{MRB}} + 16 \times 2^{5/8} - 1}$$

$$-\frac{38}{C_{\text{MRB}} + 1 - 16 \times 2^{5/8}}$$

$$\frac{1}{-\frac{C_{\text{MRB}}}{38} - \frac{1}{38} + \frac{8 \times 2^{5/8}}{19}}$$

And, we have:

By the hybrid large sieve, we have the above is bounded by

$$\ll \frac{Q^2}{Q^{2-\epsilon}} \left(H^2 T + Q^{2-3\epsilon}\right) \sum_{m \ll \frac{(TH)^4}{N}} \frac{d_4^2(n)}{n}$$

$$= Q^{\epsilon}(Q^{2-2\epsilon} + Q^{2-3\epsilon})(\log Q)^{16}$$

$$\ll Q^{2-\epsilon_0}.$$

From

$$Q^{2-\epsilon_0}$$

we obtain:

8^(2-1/24)

Input

$8^{2-1/24}$

Result

$8^{47/24}$

Decimal approximation

58.688258765098958831586662066825243394473930651957743986666372438
…

58.688258765….

From which:

89*1/((8^(2-1/24))-1/2-Pi)+(MRB const)^(1-1/(4π)+π)

Input

$$89\times\frac{1}{8^{2-1/24}-\frac{1}{2}-\pi}+C_{\text{MRB}}{}^{1-1/(4\pi)+\pi}$$

C_{MRB} is the MRB constant

Exact result

$$C_{\text{MRB}}{}^{1-1/(4\pi)+\pi}+\frac{89}{-\frac{1}{2}+32\times 2^{7/8}-\pi}$$

Decimal approximation

1.6179327910988180108508111158907655472000104331397632177579661925
…

1.617932791…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternate forms

$$C_{\mathrm{MRB}}^{1-1/(4\pi)+\pi} + \frac{178}{-1 + 64 \times 2^{7/8} - 2\pi}$$

$$C_{\mathrm{MRB}}^{1-1/(4\pi)+\pi} - \frac{178}{1 - 64 \times 2^{7/8} + 2\pi}$$

$$C_{\mathrm{MRB}}^{1-1/(4\pi)+\pi} + \frac{89}{\frac{1}{2}\left(64 \times 2^{7/8} - 1\right) - \pi}$$

We have:

Conrey, Iwaniec and Soundararajan's work

$$\sum_{q\sim Q} \sum_{\chi \ (\mathrm{mod}\ q)}^{*} \int_{-\infty}^{\infty} \left|L(\tfrac{1}{2}+it,\chi)\right|^6 \left|\Gamma\left(\frac{1/2+it}{2}\right)\right|^6 dt$$

$$\sim 42 a_3 \sum_{q\sim Q} \prod_{p|q} \frac{\left(1-\frac{1}{p}\right)^5}{\left(1+\frac{4}{p}+\frac{1}{p^2}\right)} \phi^*(q) \frac{(\log q)^9}{9!} \int_{-\infty}^{\infty} \left|\Gamma\left(\frac{1/2+it}{2}\right)\right|^6 dt$$

$$\sim 42\, \tilde{a}_3 Q^2 \frac{(\log Q)^9}{9!} \int_{-\infty}^{\infty} \left|\Gamma\left(\frac{1/2+it}{2}\right)\right|^6 dt.$$

From

$$42\ \tilde{a}_3 Q^2 \frac{(\log Q)^9}{9!} \int_{-\infty}^{\infty} \left| \Gamma\left(\frac{1/2+it}{2}\right)\right|^6 dt$$

for Q = 8 and t = 4:

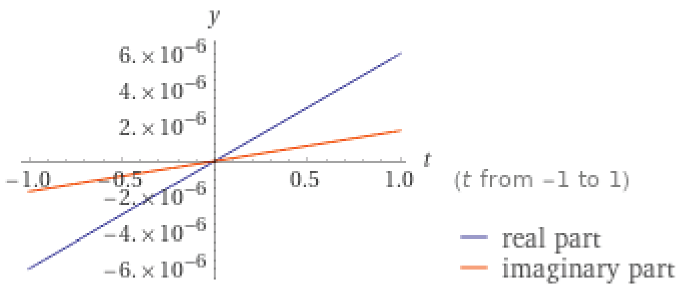
42*2*64* ((ln(8))^9) (1/9!) integrate((((gamma((1/2+i*4)/2))^6)) dt)

Indefinite integral

$$\frac{(42 \times 2 \times 64 \log^9(8)) \int \Gamma\left(\frac{1}{2}\left(\frac{1}{2}+i\,4\right)\right)^6 dt}{9!} = \frac{2}{135}\, t \log^9(8)\, \Gamma\left(\frac{1}{4}+2\,i\right)^6 + \text{constant}$$

log(x) is the natural logarithm
n! is the factorial function
Γ(x) is the gamma function
i is the imaginary unit

Plot



Alternate form assuming t>0

$$\frac{1458}{5}\, t \log^9(2)\, \Gamma\left(\frac{1}{4}+2\,i\right)^6$$

Alternate form

$$\left(-\frac{1204324982784}{377094453125} - \frac{1113270386688\,i}{377094453125}\right)\left(\left(\frac{1}{4} + 2\,i\right)!\right)^6 t \log^9(2)$$

From the right-hand side, i.e. the solution, we obtain, for t = 4 :

2/135 * 4 * Γ(1/4 + 2 i)^6 log^9(8)

Input

$$\frac{2}{135} \times 4\,\Gamma\left(\frac{1}{4} + 2\,i\right)^6 \log^9(8)$$

$\Gamma(x)$ is the gamma function
$\log(x)$ is the natural logarithm
i is the imaginary unit

Exact result

$$\frac{8}{135} \log^9(8)\,\Gamma\left(\frac{1}{4} + 2\,i\right)^6$$

Decimal approximation

$$0.000023964369861822107222992458581910735025788170052622226896 18\ldots + 6.7908556943457271332657367129983096991944770351751243374 23\ldots \times 10^{-6}\,i$$

Polar coordinates

$r = 0.0000249079654716244439837934740112415406161285546594743698 4260$ (radius), $\theta = 0.27613375007349737058666592762315283723735128716212914935 77$ (angle)

0.00002490796547….

Alternate complex forms

$$i\,\mathrm{Im}\left(\frac{8}{135}\log^9(8)\,\Gamma\left(\frac{1}{4}+2\,i\right)^6\right)+\mathrm{Re}\left(\frac{8}{135}\log^9(8)\,\Gamma\left(\frac{1}{4}+2\,i\right)^6\right)$$

$$\frac{8}{135}\log^9(8)\left(\mathrm{Im}\left(\Gamma\left(\frac{1}{4}+2\,i\right)\right)^2+\mathrm{Re}\left(\Gamma\left(\frac{1}{4}+2\,i\right)\right)^2\right)^3(\cos(0.276134)+i\sin(0.276134))$$

$$\frac{8}{135}\,e^{0.276134\,i}\log^9(8)\left(\mathrm{Im}\left(\Gamma\left(\frac{1}{4}+2\,i\right)\right)^2+\mathrm{Re}\left(\Gamma\left(\frac{1}{4}+2\,i\right)\right)^2\right)^3$$

$\mathrm{Im}(z)$ is the imaginary part of z
$\mathrm{Re}(z)$ is the real part of z

Alternate forms

$$\left(-\frac{4\,817\,299\,931\,136}{377\,094\,453\,125}-\frac{4\,453\,081\,546\,752\,i}{377\,094\,453\,125}\right)\left(\left(\frac{1}{4}+2\,i\right)!\right)^6\log^9(2)$$

$$\frac{5832}{5}\log^9(2)\,\Gamma\left(\frac{1}{4}+2\,i\right)^6$$

$n!$ is the factorial function

Alternative representations

$$\frac{4}{135}\times 2\,\Gamma\left(\frac{1}{4}+2\,i\right)^6\log^9(8)=\frac{8}{135}\log^9(8)\left(\exp\left(-\log G\left(2\,i+\frac{1}{4}\right)+\log G\left(1+2\,i+\frac{1}{4}\right)\right)\right)^6$$

$$\frac{4}{135}\times 2\,\Gamma\left(\frac{1}{4}+2\,i\right)^6\log^9(8)=\frac{8}{135}\log^9(8)\left(\frac{G\left(1+2\,i+\frac{1}{4}\right)}{G\left(2\,i+\frac{1}{4}\right)}\right)^6$$

$$\frac{4}{135}\times 2\,\Gamma\left(\frac{1}{4}+2\,i\right)^6\log^9(8)=\frac{8}{135}\log_e^9(8)\left(\exp\left(-\log G\left(2\,i+\frac{1}{4}\right)+\log G\left(1+2\,i+\frac{1}{4}\right)\right)\right)^6$$

$\log G(z)$ gives the logarithm of the Barnes G-function
$G(z)$ is the Barnes G-function
$\log_b(x)$ is the base- b logarithm

Series representations

$$\frac{4}{135}\times 2\,\Gamma\left(\frac{1}{4}+2\,i\right)^6 \log^9(8) = \frac{8}{135}\,\Gamma\left(\frac{1}{4}+2\,i\right)^6 \left(\log(7)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{7}\right)^k}{k}\right)^9$$

$$\frac{4}{135}\times 2\,\Gamma\left(\frac{1}{4}+2\,i\right)^6 \log^9(8) = \frac{8\log^9(8)}{135\left(\sum_{k=1}^{\infty}\left(\frac{1}{4}+2\,i\right)^k c_k\right)^6}$$

$$\text{for }\left(c_1 = 1 \text{ and } c_2 = 1 \text{ and } c_k = \frac{\gamma\, c_{-1+k} + \sum_{j=1}^{-2+k}(-1)^{1+j+k}\, c_j\, \zeta(-j+k)}{-1+k}\right)$$

$$\frac{4}{135}\times 2\,\Gamma\left(\frac{1}{4}+2\,i\right)^6 \log^9(8) = \frac{8}{135}\,\Gamma\left(\frac{1}{4}+2\,i\right)^6 \left(\log(z_0)+\left\lfloor\frac{\arg(8-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(8-z_0)^k\, z_0^{-k}}{k}\right)^9$$

$\zeta(s)$ is the Riemann zeta function
γ is the Euler-Mascheroni constant
$\arg(z)$ is the complex argument
$\lfloor x\rfloor$ is the floor function

Integral representations

$$\frac{4}{135}\times 2\,\Gamma\left(\frac{1}{4}+2\,i\right)^6 \log^9(8) = \frac{8}{135}\left(\int_0^1 \log^{-3/4+2\,i}\left(\frac{1}{t}\right)dt\right)^6 \log^9(8)$$

$$\frac{4}{135}\times 2\,\Gamma\left(\frac{1}{4}+2\,i\right)^6 \log^9(8) = \frac{8}{135}\exp\left(6\int_0^1 \frac{(3-8\,i)-4\,x^{1/4+2\,i}+(1+8\,i)\,x}{4\log(x)-4\,x\log(x)}\,dx\right)\log^9(8)$$

$$\frac{4}{135}\times 2\,\Gamma\left(\frac{1}{4}+2\,i\right)^6 \log^9(8) = \frac{8}{135}\left(\int_1^8 \frac{1}{t}\,dt\right)^9\left(\int_0^1 \log^{-3/4+2\,i}\left(\frac{1}{t}\right)dt\right)^6$$

From which:

1/23((1/((2/135 * 4 * Γ(1/4 + 2 i)^6 log^9(8)))))-17-MRb const

Input

$$\frac{1}{23} \times \frac{1}{\frac{2}{135} \times 4\,\Gamma\left(\frac{1}{4} + 2\,i\right)^6 \log^9(8)} - 17 - C_{\text{MRB}}$$

$\Gamma(x)$ is the gamma function
$\log(x)$ is the natural logarithm
i is the imaginary unit
C_{MRB} is the MRB constant

Exact result

$$-C_{\text{MRB}} - 17 + \frac{135}{184 \log^9(8)\,\Gamma\left(\frac{1}{4} + 2\,i\right)^6}$$

Decimal approximation

$1662.241215898250842517392327012154871276141750873435970954188100\ldots - 475.904877392769503703828753496015846167087780709274866347563449 0\ldots\, i$

Polar coordinates

$r = 1729.026116678728003343028119967632766995980596844090054640137598$ (radius), $\theta = -0.278843985536988712755186537075777621832251136045783627394768146 6$ (angle)

1729.026116678….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve ($1728 = 8^2 * 3^3$). The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate complex forms

$$i \operatorname{Im}\left(\frac{135}{184 \log^9(8)\, \Gamma\left(\frac{1}{4} + 2 i\right)^6}\right) - C_{\mathrm{MRB}} + \operatorname{Re}\left(\frac{135}{184 \log^9(8)\, \Gamma\left(\frac{1}{4} + 2 i\right)^6}\right) - 17$$

$$\sqrt{\left(\left(\left(135\left(-15\,\mathrm{Im}\left(\Gamma\left(\frac{1}{4}+2\,i\right)\right)^4\mathrm{Re}\left(\Gamma\left(\frac{1}{4}+2\,i\right)\right)^2+15\,\mathrm{Im}\left(\Gamma\left(\frac{1}{4}+2\,i\right)\right)^2\mathrm{Re}\left(\Gamma\left(\frac{1}{4}+2\,i\right)\right)^4+\mathrm{Im}\left(\Gamma\left(\frac{1}{4}+2\,i\right)\right)^6-\mathrm{Re}\left(\Gamma\left(\frac{1}{4}+2\,i\right)\right)^6\right)\right)\Big/\left(184\log^9(8)\left(\mathrm{Im}\left(\Gamma\left(\frac{1}{4}+2\,i\right)\right)^2+\mathrm{Re}\left(\Gamma\left(\frac{1}{4}+2\,i\right)\right)^2\right)^6\right)+C_{\mathrm{MRB}}+17\right)^2+\left(18\,225\left(3\,\mathrm{Im}\left(\Gamma\left(\frac{1}{4}+2\,i\right)\right)^5\mathrm{Re}\left(\Gamma\left(\frac{1}{4}+2\,i\right)\right)-10\,\mathrm{Im}\left(\Gamma\left(\frac{1}{4}+2\,i\right)\right)^3\mathrm{Re}\left(\Gamma\left(\frac{1}{4}+2\,i\right)\right)^3+3\,\mathrm{Im}\left(\Gamma\left(\frac{1}{4}+2\,i\right)\right)\mathrm{Re}\left(\Gamma\left(\frac{1}{4}+2\,i\right)\right)^5\right)^2\right)\Big/\left(8464\log^{18}(8)\left(\mathrm{Im}\left(\Gamma\left(\frac{1}{4}+2\,i\right)\right)^2+\mathrm{Re}\left(\Gamma\left(\frac{1}{4}+2\,i\right)\right)^2\right)^{12}\right)\right)}$$

$$\left(\cos\left(\tan^{-1}\left(\frac{\mathrm{Im}\left(\frac{135}{184\log^9(8)\,\Gamma\left(\frac{1}{4}+2\,i\right)^6}\right)}{-C_{\mathrm{MRB}}+\mathrm{Re}\left(\frac{135}{184\log^9(8)\,\Gamma\left(\frac{1}{4}+2\,i\right)^6}\right)-17}\right)\right)+i\sin\left(\tan^{-1}\left(\frac{\mathrm{Im}\left(\frac{135}{184\log^9(8)\,\Gamma\left(\frac{1}{4}+2\,i\right)^6}\right)}{-C_{\mathrm{MRB}}+\mathrm{Re}\left(\frac{135}{184\log^9(8)\,\Gamma\left(\frac{1}{4}+2\,i\right)^6}\right)-17}\right)\right)\right)$$

$$\sqrt{\left(\left(\left(135\left(-15\operatorname{Im}\left(\Gamma\left(\frac{1}{4}+2i\right)\right)^4\operatorname{Re}\left(\Gamma\left(\frac{1}{4}+2i\right)\right)^2+15\operatorname{Im}\left(\Gamma\left(\frac{1}{4}+2i\right)\right)^2\operatorname{Re}\left(\Gamma\left(\frac{1}{4}+2i\right)\right)^4+\operatorname{Im}\left(\Gamma\left(\frac{1}{4}+2i\right)\right)^6-\operatorname{Re}\left(\Gamma\left(\frac{1}{4}+2i\right)\right)^6\right)\right)\Big/\left(184\log^9(8)\left(\operatorname{Im}\left(\Gamma\left(\frac{1}{4}+2i\right)\right)^2+\operatorname{Re}\left(\Gamma\left(\frac{1}{4}+2i\right)\right)^2\right)^6\right)+C_{\mathrm{MRB}}+17\right)^2+\left(18225\left(3\operatorname{Im}\left(\Gamma\left(\frac{1}{4}+2i\right)\right)^5\operatorname{Re}\left(\Gamma\left(\frac{1}{4}+2i\right)\right)-10\operatorname{Im}\left(\Gamma\left(\frac{1}{4}+2i\right)\right)^3\operatorname{Re}\left(\Gamma\left(\frac{1}{4}+2i\right)\right)^3+3\operatorname{Im}\left(\Gamma\left(\frac{1}{4}+2i\right)\right)\operatorname{Re}\left(\Gamma\left(\frac{1}{4}+2i\right)\right)^5\right)^2\right)\Big/\left(8464\log^{18}(8)\left(\operatorname{Im}\left(\Gamma\left(\frac{1}{4}+2i\right)\right)^2+\operatorname{Re}\left(\Gamma\left(\frac{1}{4}+2i\right)\right)^2\right)^{12}\right)\right)}$$

$$\exp\left(i\tan^{-1}\left(\frac{\operatorname{Im}\left(\frac{135}{184\log^9(8)\,\Gamma\left(\frac{1}{4}+2i\right)^6}\right)}{-C_{\mathrm{MRB}}+\operatorname{Re}\left(\frac{135}{184\log^9(8)\,\Gamma\left(\frac{1}{4}+2i\right)^6}\right)-17}\right)\right)$$

Im(z) is the imaginary part of z

Re(z) is the real part of z

$\tan^{-1}(x)$ is the inverse tangent function

((1/23((1/((2/135 * 4 * Γ(1/4 + 2 i)^6 log^9(8)))))-17-MRb const))^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\sqrt[15]{\frac{1}{23}\times\frac{1}{\frac{2}{135}\times 4\,\Gamma\left(\frac{1}{4}+2i\right)^6\log^9(8)}-17-C_{\mathrm{MRB}}}+C_{\mathrm{MRB}}^{1-1/(4\pi)+\pi}$$

$\Gamma(x)$ is the gamma function

$\log(x)$ is the natural logarithm

i is the imaginary unit

C_{MRB} is the MRB constant

Exact result

$$C_{\mathrm{MRB}}^{1-1/(4\pi)+\pi} + \sqrt[15]{-C_{\mathrm{MRB}} - 17 + \frac{135}{184 \log^9(8)\, \Gamma\left(\frac{1}{4} + 2 i\right)^6}}$$

Decimal approximation

1.6446556581042231559201366106826400355264320657352001962408198… −
0.030556136795178886199446752210193307458517896118184870182908372…
i

Polar coordinates

$r = 1.6449394855830049820473160142271844169691720848159600758952563$ (radius), $\theta = -0.018576910961864587334390749033633432198510527416483133073726364$ (angle)

1.644939485583…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape)

(1/27((1/23((1/((2/135 * 4 * Γ(1/4 + 2 i)^6 log^9(8)))))-17-MRb const)-1))^2-2MRB const

Input

$$\left(\frac{1}{27}\left(\left(\frac{1}{23} \times \frac{1}{\frac{2}{135} \times 4\, \Gamma\left(\frac{1}{4} + 2 i\right)^6 \log^9(8)} - 17 - C_{\mathrm{MRB}}\right) - 1\right)\right)^2 - 2\, C_{\mathrm{MRB}}$$

$\Gamma(x)$ is the gamma function
$\log(x)$ is the natural logarithm
i is the imaginary unit
C_{MRB} is the MRB constant

Exact result

$$-2\,C_{\text{MRB}} + \frac{1}{729}\left(-C_{\text{MRB}} - 18 + \frac{135}{184\log^9(8)\,\Gamma\left(\frac{1}{4}+2\,i\right)^6}\right)^2$$

Decimal approximation

3474.572051734104526870993134550249033589751691443222103130426839… − 2168.978867412544306238921761246152064246444138890460202466664193… i

Polar coordinates

$r = 4095.988314189134524188145419914369913883842158111194459420508970$ (radius), $\theta = -0.558055099068006594286894827932655340921819381633152237013940012 7$ (angle)

4095.988314189…. ≈ 4096 = 64^2 , that multiplied by 2 give 8192, indeed:

The total amplitude vanishes for gauge group SO(8192), while the vacuum energy is negative and independent of the gauge group.

The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). (From: “Dilaton Tadpole for the Open Bosonic String “ Michael R. Douglas and Benjamin Grinstein - September 2,1986)

Possible closed forms

$$1606\,e! + i\left(18\,C + 847 - 56\,\pi + 66\,\pi^2 + 20\,\pi\log(2) - 1029\,\pi\log(3)\right) + \frac{18\,756}{5} - \frac{8258}{5\,e} - \frac{11\,978\,e}{5} \approx 3474.5720517341045268341229616098911103823167428989801784440 26548 - 2168.9788674125443062452363629871011225052345046680958627767023 77\,i$$

$$1606\,e! + \frac{18\,756}{5} - \frac{8258}{5\,e} - \frac{11\,978\,e}{5} + i\left(-176\,e^{\pi} - 2594\,\pi + 604\log(\pi) + 1702\log(2\,\pi) + 4937\tan^{-1}(\pi)\right) \approx 3474.572051734104526834122961609891110382316742898980178444026548 - 2168.978867412544306212925467282672190707454404527617065771822527\,i$$

$$\frac{11\,239\,\pi\,\pi! + 25\,008 - 8578\,\pi + 17\,615\,\pi^2}{39\,\pi} + i\left(18\,C + 847 - 56\,\pi + 66\,\pi^2 + 20\,\pi\log(2) - 1029\,\pi\log(3)\right) \approx 3474.572051734104526872762455132443201911196541963805744752906926 - 2168.978867412544306245236362987101122505234504668095862776702377\,i$$

$n!$ is the factorial function
$\log(x)$ is the natural logarithm
C is Catalan's constant
$\tan^{-1}(x)$ is the inverse tangent function

Alternate complex forms

$$\frac{2}{729}\,i\,\mathrm{Im}\left(\frac{135}{184\log^9(8)\,\Gamma\left(\frac{1}{4}+2\,i\right)^6}\right)\left(-C_{\mathrm{MRB}} + \mathrm{Re}\left(\frac{135}{184\log^9(8)\,\Gamma\left(\frac{1}{4}+2\,i\right)^6}\right) - 18\right) + \frac{1}{729}\left(\left(-C_{\mathrm{MRB}} + \mathrm{Re}\left(\frac{135}{184\log^9(8)\,\Gamma\left(\frac{1}{4}+2\,i\right)^6}\right) - 18\right)^2 - \mathrm{Im}\left(\frac{135}{184\log^9(8)\,\Gamma\left(\frac{1}{4}+2\,i\right)^6}\right)^2\right) - 2\,C_{\mathrm{MRB}}$$

We have:

The sixth moment without the t-average

- Deriving an analogous result without the average over t is challenging due to certain "unbalanced" sums.

Theorem (C., X. Li, K. Matomäki, and M. Radziwiłł (2022+))

$$\sum_{q\sim Q}\ \sideset{}{^*}\sum_{\chi \pmod q} \left|L(\tfrac{1}{2},\chi)\right|^6 \sim 42a_3 \sum_{q\sim Q}\prod_{p|q}\frac{\left(1-\frac{1}{p}\right)^5}{\left(1+\frac{4}{p}+\frac{1}{p^2}\right)}\phi^*(q)\frac{(\log q)^9}{9!}$$
$$\sim 42\ \tilde{c}_3 Q^2\frac{(\log Q)^9}{9!}.$$

From:

$$42\ \tilde{c}_3 Q^2\frac{(\log Q)^9}{9!}.$$

we obtain:

42*2*64*((ln(8))^9)/9!

Input

$$42\times 2\times 64\times\frac{\log^9(8)}{9!}$$

$\log(x)$ is the natural logarithm
$n!$ is the factorial function

Exact result

$$\frac{2\log^9(8)}{135}$$

Decimal approximation

10.770023949611942923417349658814869222726620141203458761301988673 …

10.770023949….

Property

$\frac{2\log^9(8)}{135}$ is a transcendental number

Alternate form

$$\frac{1458\log^9(2)}{5}$$

Alternative representations

$$\frac{42\log^9(8)\,2\times 64}{9!} = \frac{5376\,(\log(a)\log_a(8))^9}{\Gamma(10)}$$

$$\frac{42\log^9(8)\,2\times 64}{9!} = \frac{5376\log^9(8)}{8!!\times 9!!}$$

$$\frac{42\log^9(8)\,2\times 64}{9!} = \frac{5376\log_e^9(8)}{(1)_9}$$

$\Gamma(x)$ is the gamma function
$\log_b(x)$ is the base- b logarithm
$n!!$ is the double factorial function
$(a)_n$ is the Pochhammer symbol (rising factorial)

Series representations

$$\frac{42 \log^9(8)\, 2 \times 64}{9!} = \frac{2}{135} \left(\log(7) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{7}\right)^k}{k} \right)^9$$

$$\frac{42 \log^9(8)\, 2 \times 64}{9!} = \frac{2}{135} \left(2 i \pi \left\lfloor \frac{\arg(8-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (8-x)^k x^{-k}}{k} \right)^9$$

for $x < 0$

$$\frac{42 \log^9(8)\, 2 \times 64}{9!} = \frac{2}{135} \left(\log(z_0) + \left\lfloor \frac{\arg(8-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (8-z_0)^k z_0^{-k}}{k} \right)^9$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representations

$$\frac{42 \log^9(8)\, 2 \times 64}{9!} = \frac{2}{135} \left(\int_1^8 \frac{1}{t}\, dt \right)^9$$

$$\frac{42 \log^9(8)\, 2 \times 64}{9!} = -\frac{i \left(\int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{7^{-s}\, \Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds \right)^9}{34\,560\, \pi^9} \quad \text{for } -1 < \gamma < 0$$

From which:

1+1/(1/7*((42*2*64*((ln(8))^9)/9!)+3/2MRB const))+Pi^2(MRB const)^(1-1/(4π)+π)

Input

$$1 + \frac{1}{\frac{1}{7}\left(42\times 2\times 64\times \frac{\log^9(8)}{9!} + \frac{3}{2} C_{\text{MRB}}\right)} + \pi^2 C_{\text{MRB}}^{1-1/(4\pi)+\pi}$$

$\log(x)$ is the natural logarithm
$n!$ is the factorial function
C_{MRB} is the MRB constant

Exact result

$$\pi^2 C_{\text{MRB}}^{1-1/(4\pi)+\pi} + \frac{7}{\frac{3 C_{\text{MRB}}}{2} + \frac{2\log^9(8)}{135}} + 1$$

Decimal approximation

1.6444617652330945381767646948176433161229835107473505239984989462

...

1.644461765233…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape)

Alternate forms

$$\pi^2 C_{\text{MRB}}^{1-1/(4\pi)+\pi} + \frac{1890}{405 C_{\text{MRB}} + 4\log^9(8)} + 1$$

$$\pi^2 C_{\text{MRB}}^{1-1/(4\pi)+\pi} + \frac{70}{3\left(5 C_{\text{MRB}} + 972\log^9(2)\right)} + 1$$

$$\frac{15 C_{\text{MRB}} + 15\pi^2 C_{\text{MRB}}^{2-1/(4\pi)+\pi} + 2916\pi^2\log^9(2) C_{\text{MRB}}^{1-1/(4\pi)+\pi} + 70 + 2916\log^9(2)}{3\left(5 C_{\text{MRB}} + 972\log^9(2)\right)}$$

$$\pi^2 C_{\text{MRB}}^{1-1/(4\pi)+\pi} + \frac{70}{15 C_{\text{MRB}} + 2916\log^9(2)} + 1$$

$$\pi^2 C_{\text{MRB}}^{1-1/(4\pi)+\pi} + \frac{7}{\frac{3 C_{\text{MRB}}}{2} + \frac{1458\log^9(2)}{5}} + 1$$

Observations

We note that, from the number 8, we obtain as follows:

8^2

64

$8^2 \times 2 \times 8$

1024

$8^4 = 8^2 \times 2^6$

True

$8^4 = 4096$

$8^2 \times 2^6 = 4096$

$2^{13} = 2 \times 8^4$

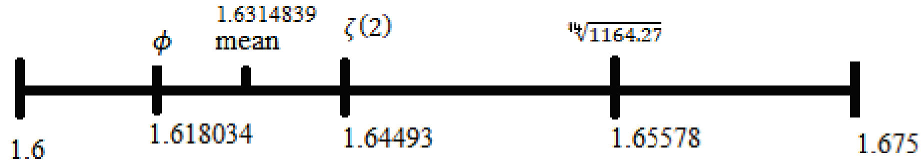
True

$2^{13} = 8192$

$2 \times 8^4 = 8192$

We notice how from the numbers 8 and 2 we get 64, 1024, 4096 and 8192, and that 8 is the fundamental number. In fact $8^2 = 64$, $8^3 = 512$, $8^4 = 4096$. We define it "fundamental number", since 8 is a Fibonacci number, which by rule, divided by the previous one, which is 5, gives 1.6 , a value that tends to the golden ratio, as for all numbers in the Fibonacci sequence

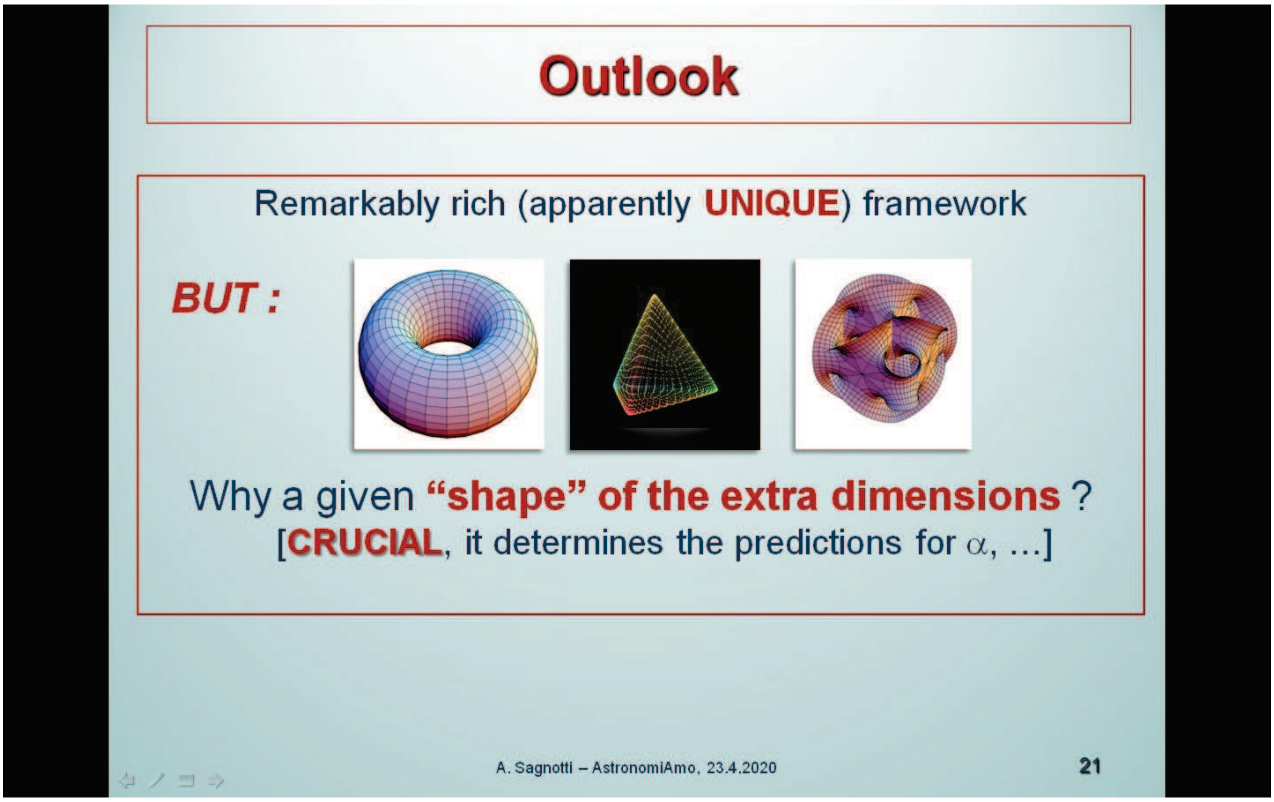
“Golden” Range



Finally we note how $8^2 = 64$, multiplied by 27, to which we add 1, is equal to 1729, the so-called "Hardy-Ramanujan number". Then taking the 15th root of 1729, we obtain a value close to ζ(2) that 1.6438 ..., which, in turn, is included in the range of what we call "golden numbers"

Furthermore for all the results very near to 1728 or 1729, adding $64 = 8^2$, one obtain values about equal to 1792 or 1793. These are values almost equal to the Planck multipole spectrum frequency 1792.35 and to the hypothetical Gluino mass

Appendix



From: *A. Sagnotti – AstronomiAmo, 23.04.2020*

In the above figure, it is said that: "why a given shape of the extra dimensions? Crucial, it determines the predictions for α".
We propose that whatever shape the compactified dimensions are, their geometry must be based on the values of the golden ratio and ζ(2), (the latter connected to 1728 or 1729, whose fifteenth root provides an excellent approximation to the above mentioned value) which are recurrent as solutions of the equations that we are going to develop. It is important to specify that the initial conditions are **always** values belonging to a fundamental chapter of the work of S. Ramanujan "Modular equations and Appoximations to Pi" (see references). These values are some multiples of 8 (64 and 4096), 276, which added to 4096, is equal to 4372, and finally $e^{\pi\sqrt{22}}$

We have, in certain cases, the following connections:

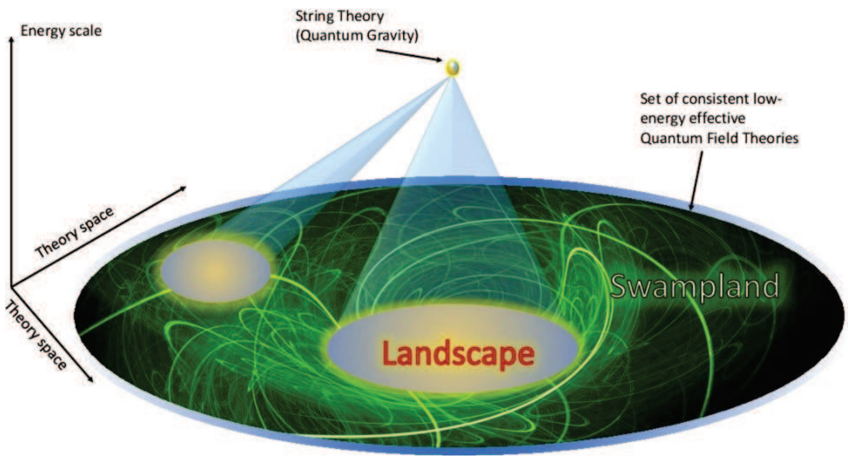


Fig. 1

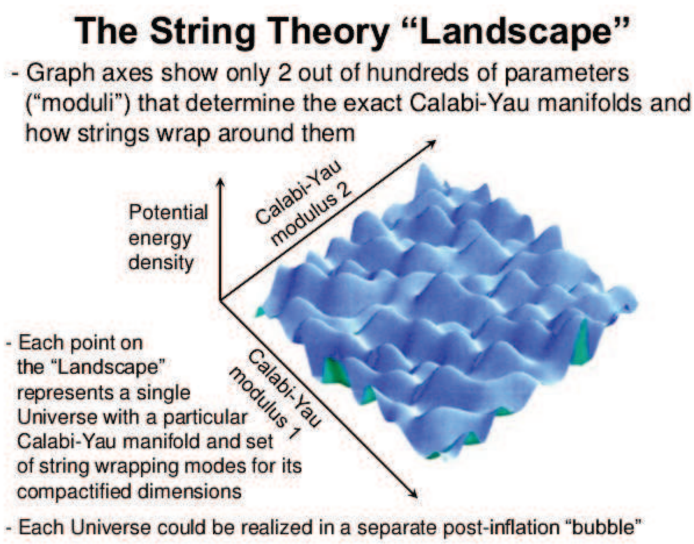


Fig. 2

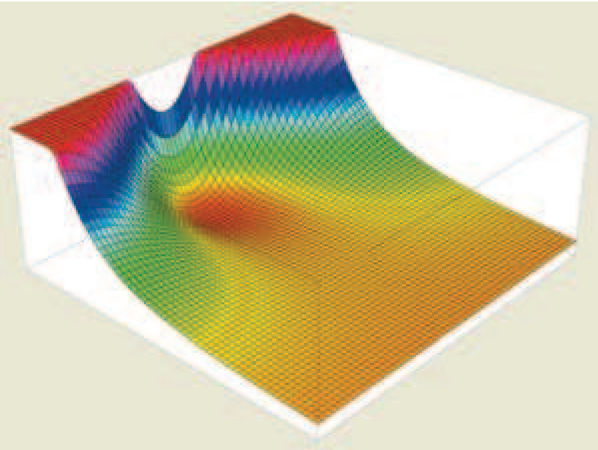


Fig. 3
Stringscape - a small part of the string-theory landscape showing the new de Sitter solution as a local minimum of the energy (vertical axis). The global minimum occurs at the infinite size of the extra dimensions on the extreme right of the figure.

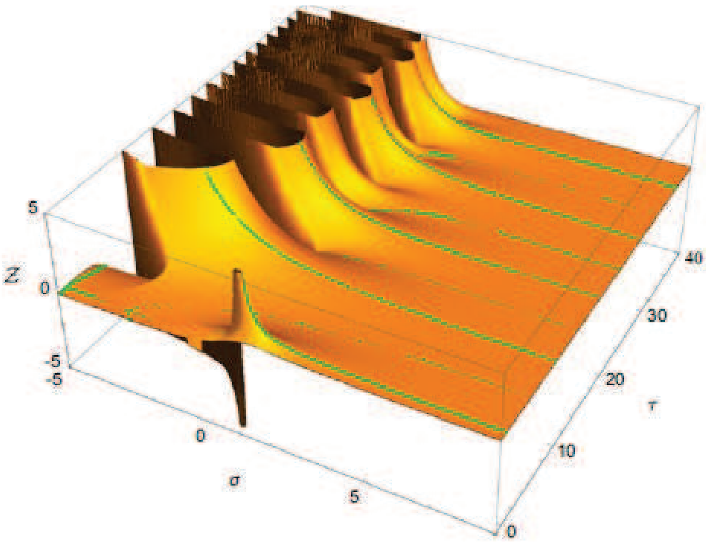


Figure 2. Lines in the complex plane where the Riemann zeta function ζ is real (green) depicted on a relief representing the positive absolute value of ζ for arguments $s \equiv \sigma + i\tau$ where the real part of ζ is positive, and the negative absolute value of ζ where the real part of ζ is negative. This representation brings out most clearly that the lines of constant phase corresponding to phases of integer multiples of 2π run down the hills on the left-hand side, turn around on the right and terminate in the non-trivial zeros. This pattern repeats itself infinitely many times. The points of arrival and departure on the right-hand side of the picture are equally spaced and given by equation (11).

Fig. 4

From: https://www.mdpi.com/2227-7390/6/12/285/htm

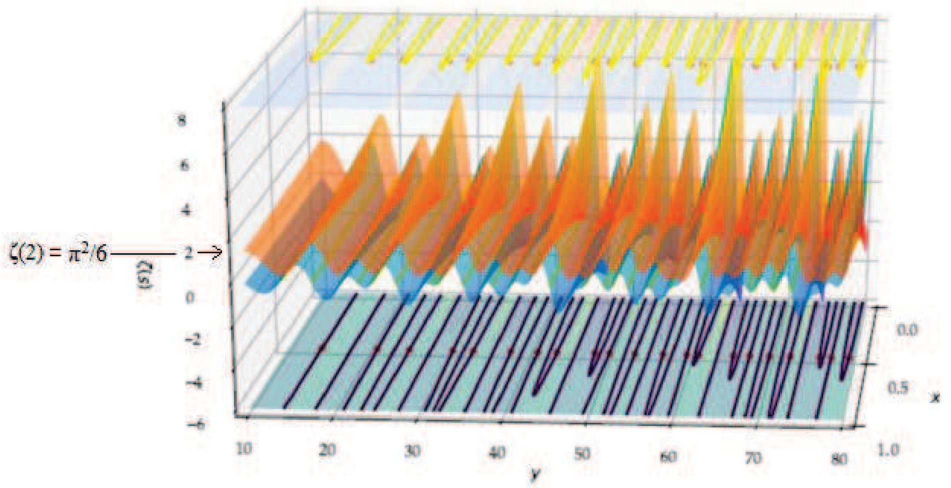


Figure 1. $C\,(x, y)$ and $S\,(x, y)$ surfaces of the Riemann $\zeta\,(x, y) = C - i\,S$ function, in the critical strip $\boldsymbol{s}$: $0 \leqslant x \leqslant 1; 10 \leqslant y \leqslant 80$. On the top and bottom planes, the C and S common zeros are the red points.

Fig. 5

3D plot	$\zeta(2 + i\,t)$

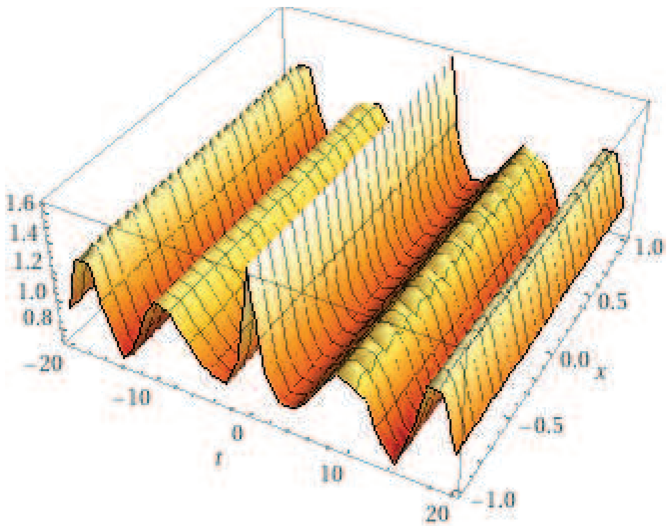


Fig. 6

Where ζ(2+it) :

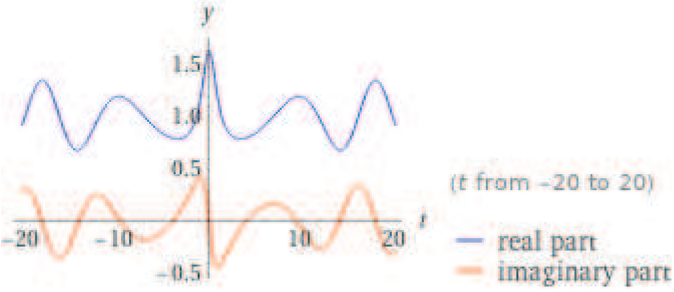
Input

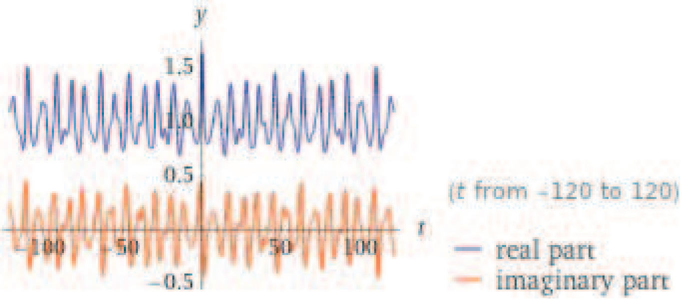
$\zeta(2 + i\, t)$

$\zeta(s)$ is the Riemann zeta function

i is the imaginary unit

Plots





Roots

$t = 2\, i\,(n + 1)\,, \quad n \in \mathbb{Z}\,, \quad n \geq 1$

$t = -i\,(\rho_n - 2)\,, \quad n \neq 0\,, \quad n \in \mathbb{Z}$

$\mathbb{Z}$ is the set of integers

ρ_n is the nontrivial n^{th} zero of the Riemann zeta function

Series expansion at t=0

$$\frac{\pi^2}{6} + i\, t\, \zeta'(2) - \frac{1}{2}\, t^2\, \zeta''(2) - \frac{1}{6}\, i\, \zeta^{(3)}(2)\, t^3 + \frac{1}{24}\, \zeta^{(4)}(2)\, t^4 + O\left(t^5\right)$$

(Taylor series)

Alternative representations

$$\zeta(2+i\,t) = \zeta(2+i\,t,\,1)$$

$$\zeta(2+i\,t) = S_{1+i\,t,1}(1)$$

$$\zeta(2+i\,t) = \frac{\zeta\left(2+i\,t,\,\frac{1}{2}\right)}{-1+2^{2+i\,t}}$$

$\zeta(s,\,a)$ is the generalized Riemann zeta function

$S_{n,p}(x)$ is the Nielsen generalized polylogarithm function

Series representations

$$\zeta(2+i\,t) = \sum_{k=1}^{\infty} k^{-2-i\,t} \quad \text{for } \operatorname{Im}(t) < 1$$

$$\zeta(2+i\,t) = \frac{\sum_{k=0}^{\infty} (1+2\,k)^{-2-i\,t}}{1-2^{-2-i\,t}} \quad \text{for } \operatorname{Im}(t) < 1$$

$$\zeta(2+i\,t) = e^{\sum_{k=1}^{\infty} P(k\,(2+i\,t))/k} \quad \text{for } \operatorname{Im}(t) < 1$$

$\operatorname{Im}(z)$ is the imaginary part of z

$P(z)$ gives the prime zeta function

Integral representations

$$\zeta(2+i\,t) = \frac{1}{\Gamma(2+i\,t)} \int_0^{\infty} \frac{\tau^{1+i\,t}}{-1+e^{\tau}}\,d\tau \quad \text{for } \operatorname{Im}(t) < 1$$

$$\zeta(2+i\,t) = \frac{2^{1+i\,t}}{\Gamma(3+i\,t)} \int_0^{\infty} \tau^{2+i\,t} \operatorname{csch}^2(\tau)\,d\tau \quad \text{for } \operatorname{Im}(t) < 1$$

$$\zeta(2+i\,t) = \frac{2^{1+i\,t}}{\Gamma(2+i\,t)} \int_0^{\infty} e^{-\tau}\,\tau^{1+i\,t} \operatorname{csch}(\tau)\,d\tau \quad \text{for } \operatorname{Im}(t) < 1$$

$\Gamma(x)$ is the gamma function

$\operatorname{csch}(x)$ is the hyperbolic cosecant function

Functional equations

$$\zeta(2+it) = -i\,2^{2+it}\,\pi^{1+it}\,\Gamma(-1-it)\sinh\left(\frac{\pi t}{2}\right)\zeta(-1-it)$$

$$\zeta(2+it) = \frac{\pi^{3/2+it}\,\Gamma\left(-\frac{1}{2}-\frac{it}{2}\right)\zeta(-1-it)}{\Gamma\left(1+\frac{it}{2}\right)}$$

$$\zeta(2+it) = -\frac{i\sum_{k=0}^{\infty}\frac{\Gamma\left(k-\frac{it}{2}\right)\sum_{j=0}^{k}(-1)^j\,(1+2j)\binom{k}{j}\zeta(2+2j)}{k!}}{(-i+t)\,\Gamma\left(-\frac{it}{2}\right)}$$

With regard the Fig. 4 the points of arrival and departure on the right-hand side of the picture are equally spaced and given by the following equation:

$$\tau'_k \equiv k\,\frac{\pi}{\ln 2},$$

with $k = \ldots, -2, -1, 0, 1, 2, \ldots$

we obtain:

2Pi/(ln(2))

Input:

$$2\times\frac{\pi}{\log(2)}$$

Exact result:

$$\frac{2\pi}{\log(2)}$$

Decimal approximation:

9.0647202836543876192553658914333336203437229354475911683720330958
…

9.06472028365….

Alternative representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log_e(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(a)\log_a(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2\coth^{-1}(3)}$$

Series representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2 i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

Integral representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\int_1^2 \frac{1}{t}\, dt}$$

$$\frac{2\pi}{\log(2)} = \frac{4 i \pi^2}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds} \quad \text{for } -1 < \gamma < 0$$

From which:

(2Pi/(ln(2)))*(1/12 π log(2))

Input:

$$\left(2 \times \frac{\pi}{\log(2)}\right)\left(\frac{1}{12} \pi \log(2)\right)$$

log(x) is the natural logarithm

Exact result:

$$\frac{\pi^2}{6}$$

Decimal approximation:

1.6449340668482264364724151666460251892189499012067984377355582293
…

1.6449340668…. = ζ(2) = $\frac{\pi^2}{6}$ = 1.644934…

From:

Modular equations and approximations to π **-** *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

We note that, with regard 4372, we can to obtain the following results:

27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.0526944170905625170637208637148763684189306538457854815447023...

1729.0526944….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = 8^2 * 3^3) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10 - 2\sqrt{5}} - 2\right)}{2\left(\sqrt{5} - 1\right)}$$

Minimal polynomial

$$256 x^8 + 95\,744 x^7 - 3\,248\,750\,080 x^6 - 914\,210\,725\,504 x^5 + 15\,498\,355\,554\,921\,184 x^4 + 2\,911\,478\,392\,539\,914\,656 x^3 - 32\,941\,144\,911\,224\,677\,091\,680 x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456 x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Bigg/$$
$$\left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\Bigg/$$
$$\left(2\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Or:

27((4096+276)^1/2-2-1/2(((√(10-2√5) -2))⁄((√5-1))))+φ

Input

$$27\left(\sqrt{4096+276}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3...

1729.0526944…. as above

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256 x^8 + 95744 x^7 - 3248750080 x^6 - 914210725504 x^5 + 15498355554921184 x^4 + 2911478392539914656 x^3 - 32941144911224677091680 x^2 - 3092528914069760354714456 x + 26320050609744039027169013041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4096+276}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=$$

$$\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Bigg/\left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4096+276}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=$$

$$\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\Bigg/\left(2\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)$$

for (not ($z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0$))

From which:

(27((4372)^1/2-2-1/2(((√(10-2√5) -2))⁄((√5-1))))+φ)^1/15

Input

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi}$$

ϕ is the golden ratio

Exact result

$$\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation

1.6438185685849862799902301317036810054185756873505184804834183124
…

1.64381856858…. ≈ ζ(2) = $\frac{\pi^2}{6}$=1.6449 34…

Alternate forms

$$\sqrt[15]{\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}}$$

$$\frac{1}{\sqrt[15]{\frac{2\left(\sqrt{5}-1\right)}{166-108\sqrt{5}-108\sqrt{1093}+108\sqrt{5465}-27\sqrt{2\left(5-\sqrt{5}\right)}}}}$$

$$\sqrt[15]{\text{root of } 256x^8 + 95744x^7 - 3248750080x^6 - 914210725504x^5 + 15498355554921184x^4 + 2911478392539914656x^3 - 32941144911224677091680x^2 - 3092528914069760354714456x + 26320050609744039027169013041 \text{ near } x = 1729.05}$$

Minimal polynomial

$$256x^{120} + 95744x^{105} - 3248750080x^{90} - 914210725504x^{75} + 15498355554921184x^{60} + 2911478392539914656x^{45} - 32941144911224677091680x^{30} - 3092528914069760354714456x^{15} + 26320050609744039027169013041$$

Expanded forms

$$\sqrt[15]{\frac{1}{2}(1+\sqrt{5}) + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}} - 2}{2(\sqrt{5}-1)}\right)}$$

$$\sqrt[15]{-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5(10-2\sqrt{5})}}$$

All 15th roots of ϕ + 27 (-2 + 2 sqrt(1093) - (sqrt(10 - 2 sqrt(5)) - 2)/(2 (sqrt(5) - 1)))

$$e^0 \sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}} - 2}{2(\sqrt{5}-1)}\right)} \approx 1.64382 \text{ (real, principal root)}$$

$$e^{(2i\pi)/15} \sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}} - 2}{2(\sqrt{5}-1)}\right)} \approx 1.50170 + 0.6686i$$

$$e^{(4i\pi)/15} \sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}} - 2}{2(\sqrt{5}-1)}\right)} \approx 1.0999 + 1.2216i$$

$$e^{(2 i \pi)/5} \sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2\left(\sqrt{5} - 1\right)}\right)} \approx 0.5080 + 1.5634\, i$$

$$e^{(8 i \pi)/15} \sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2\left(\sqrt{5} - 1\right)}\right)} \approx -0.17183 + 1.63481\, i$$

Series representations

$$\sqrt[15]{27\left(\sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right) 2}\right) + \phi} = \frac{1}{\sqrt[15]{2}} \left(\left(\left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 2\phi\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(9 - 2\sqrt{5}\right)^{-k}\right) \Big/ \left(-1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)\right) \hat{}\ (1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right) 2}\right) + \phi} = \frac{1}{\sqrt[15]{2}} \left(\left(\left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + 108\sqrt{1093}\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + 2\phi\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - 27\sqrt{9 - 2\sqrt{5}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(9 - 2\sqrt{5}\right)^{-k}}{k!}\right) \Big/ \left(-1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)\right) \hat{}\ (1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi} =$$

$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.$$
$$108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/$$
$$\left.\left.\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\hat{}\,(1/15)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Integral representation

$$(1+z)^a = \frac{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)}$$ for ($0 < \gamma < -\text{Re}(a)$ and $|\arg(z)| < \pi$)

From:

An Update on Brane Supersymmetry Breaking
J. Mourad and A. Sagnotti - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E\, \phi} \;=\; -\,\frac{\beta_E^{(p)}\, h^2}{\gamma_E}\; e^{-2\,(8-p)\,C+2\,\beta_E^{(p)}\,\phi}$$

$$16\, k'\, e^{-2C} \;=\; \frac{h^2\left(p\,+\,1\,-\,\frac{2\,\beta_E^{(p)}}{\gamma_E}\right)e^{-2\,(8-p)\,C+2\,\beta_E^{(p)}\,\phi}}{(7\,-\,p)}$$

$$(A')^2 \;=\; k\, e^{-2A} \;+\; \frac{h^2}{16(p\,+\,1)}\left(7\,-\,p\,+\,\frac{2\,\beta_E^{(p)}}{\gamma_E}\right)e^{-2\,(8-p)\,C+2\,\beta_E^{(p)}\,\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-2\,(8-p)\,C+2\,\beta_E^{(p)}\,\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning p, C, β_E and ϕ correspond to the exponents of e (i.e. of exp). Thence we obtain for p = 5 and $\beta_E = 1/2$:

$$e^{-6C+\phi} = 4096\, e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while $-6C+\phi$ is equal to $-\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18)) we obtain:

Input:

$\exp\left(-\pi \sqrt{18}\right)$

Exact result:

$e^{-3 \sqrt{2} \pi}$

Decimal approximation:

$1.6272016226072509292942156739117979541838581136954016... \times 10^{-6}$

$1.6272016... * 10^{-6}$

Property:

$e^{-3 \sqrt{2} \pi}$ is a transcendental number

Series representations:

$$e^{-\pi \sqrt{18}} = e^{-\pi \sqrt{17} \sum_{k=0}^{\infty} 17^{-k} \binom{1/2}{k}}$$

$$e^{-\pi \sqrt{18}} = \exp\left(-\pi \sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi \sqrt{18}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}{2 \sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi}=4096\,e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^}{-6}$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016\ldots * 10\text{^}{-6}$$

$$0.000244140625\ e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^}{-6}$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right)=-13.328648814475=-\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6}\times\frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\ e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785…

0.00666501785…

Series representations:

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096 \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} 17^{-k} \binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096 \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096 \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\, \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}} \times \frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:

ln(0.00666501784619)

Input interpretation:

log(0.00666501784619)

Result:

-5.010882647757...

-5.010882647757…

Alternative representations:

$$\log(0.006665017846190000) = \log_e(0.006665017846190000)$$

$$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$$

$$\log(0.006665017846190000) = -\text{Li}_1(0.993334982153810000)$$

Series representations:

$$\log(0.00666501784619000) = -\sum_{k=1}^{\infty} \frac{(-1)^k (-0.993334982153810000)^k}{k}$$

$$\log(0.00666501784619000) = 2 i \pi \left\lfloor \frac{\arg(0.00666501784619000 - x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.00666501784619000 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.00666501784619000) = \left\lfloor \frac{\arg(0.00666501784619000 - z_0)}{2 \pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.00666501784619000 - z_0)}{2 \pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.00666501784619000 - z_0)^k z_0^{-k}}{k}$$

Integral representation:

$$\log(0.00666501784619000) = \int_1^{0.00666501784619000} \frac{1}{t} \, dt$$

In conclusion:

$$-6C+\phi=-5.010882647757\ldots$$

and for C = 1, we obtain:

$\phi=-5.010882647757+6=0.989117352243 = \phi$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1} = 1 - \frac{e^{-\pi}}{1+\frac{e^{-2\pi}}{1+\frac{e^{-3\pi}}{1+\frac{e^{-4\pi}}{1+\ldots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value $0.989117352243 = \phi$ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684$$

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References

Manuscript Book I - *Srinivasa Ramanujan*

Moments of large families of Dirichlet L-functions – *Vorrapan (Fai) Chandee (Kansas state University)*, joint work with *Xiannan Li, Kaisa Matomaki, and Maksym Radziwill* – 24.06.2022 - 50 Years of Number Theory and Random Matrix Theory Conference, June 2022

***The Geometry of the MRB constant* by Marvin Ray Burns**

https://www.academia.edu/22271085/The_Geometry_of_the_MRB_constant

Modular equations and approximations to π - *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

An Update on Brane Supersymmetry Breaking
Jihad Mourad and Augusto Sagnotti - arXiv:1711.11494v1 [hep-th] 30 Nov 2017