On the origin of C violation with implications to baryon asymmetry

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Abstract

We propose a conjecture for the early universe particles which explains the breaking of C invariance in terms of gauge theory concepts. With all Sakharov condition fulfilled, matter-antimatter asymmetric universe is expected. The elements for these result are C symmetric preons, spontaneous symmetry breaking of the gauge symmetry in the Chern-Simons model and the generated topological mass.

Keywords: Supersymmetry, Chern-Simons Model, C violation, Matter-antimatter Asymmetry

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1 Introduction

The focus of this note is a model for C violation. The mechanism we propose for it entails pointlike quark and lepton constituents, called in this note chermons (synonym for preon[1] or superon). Out of the several dozens of preon models in the literature there are two other models which resemble closely ours. One of them was proposed by Harari and independently by Shupe [2, 3]. This model was extended by Finkelstein [4] to include the symmetry group SLq(2) and knot theory. The major difference with the above models and our model [5, 6] is that ours obeys unbroken global supersymmetry where superpartners are in the model initially, not as new sparticles to be found experimentally.

Chermons adhere to 1+2 dimensional Chern-Simons (CS) equation. The scale where three chermon bound states form, making the standard model particles in 1+3 dimensions, is assumed to be between the onset of inflation and the usual grand unified theory (GUT) scale, about 10^{16} GeV, denoted here by Λ_{cr}. Below Λ_{cr} the preon scenarios of the previous paragraph revert to the standard model at accelerator energies. A major arduousness of these scenarios has been how to form bound states with two and three equal charged constituents. This calls for spontaneous symmetry breaking (SSB) of the gauge symmetry of the CS model and a CS topological term in the Maxwell action

$$S_{\mathrm{MCS}} = \int d^3x \big(-\frac{1}{4e^2} F_{\mu\nu}F^{\mu\nu} + \frac{\eta}{4}\epsilon_{\mu\nu\rho}A^{\mu}F^{\nu\rho}\big) \tag{1.1}$$

where η is the topological parameter. Such models have been proposed in the literature in condensed matter papers [7, 9, 10]. In this note we investigate MCS model in particle physics phenomenology.

We construct the visible and dark matter of two fermions, chermons, one charged and the other neutral, and their C symmetric antiparticles. They obey unbroken global supersymmetry (SUSY) and the Chern-Simons equations. The charged chermons have only gravitational and electromagnetic interactions. The neutral chermons carry quantum chromodynamics (QCD) color. Weak interactions are described as in the SM electroweak sector. The chermon baryon (B) and lepton (L) numbers are zero. Leptons and quarks are made of three chermons, as indicated in table 2. The distinguishing property of our scenario is that the universe has all the time, or rather since hydrogen atom formation, been matter-antimatter asymmetric. The dark sector is still symmetric and may provide a new source for detecting gravitational waves.

This phenomenological note is organized as follows. In section 2 we present the physics of our central conjecture, the mechanism which makes the C violation for visible matter plausible. The three Sakharov conditions are shown to be satisfied in section 3. Implications of this scenario to matter-antimatter asymmetry are discussed in section 4. This section is on heuristic base. Conclusions are given in section 5. Three appendices are provided for more details and background references. Familiarity with the content of appendix A (and

[1] The term was coined by Pati and Salam in 1974 [1].

references therein) is essential of understanding this note. Appendices B and C give a brief summary of the calculations, available in the literature, leading to the results of this study. The article is intended for wide audience. Therefore more demanding details are presented towards the end, finally referring to the literature.

2 C violation

In the SM, if one replaces in the proton the d- (u-) quark by a u- (d-) quark one gets a neutron (proton). If the d- and u-quarks have the same mass there is charge, or SU(2) symmetry between the nucleons, the quark color symmetry being also unbroken. There can be no beta decay of the neutron and the present universe would not form from Big Bang (or bounce).

To make the neutron heavier than the proton we must propose a theoretically acceptable mechanism to make the d-quark heavier than the u-quark. For this purpose, it would seem more symmetric if the u- and d-quarks had also the same charge and a new neutral state would be introduced. One must then go beyond the standard model. We propose the mechanism with a nickname "second quarkization", i.e. we divide the quarks into three "subquarks" [11], or chermons m, with a charge zero or $\pm\frac{1}{3}$ as indicated in table 1.

SM quark	chermon state
u_R	$m^+m^+m^0_R$
u_G	$m^+m^+m^0_G$
u_B	$m^+m^+m^0_B$
d_R	$m^-m^0_Gm^0_B$
d_G	$m^-m^0_Bm^0_R$
d_B	$m^-m^0_Rm^0_G$

Table 1: Second quarkization. The upper index of m is charge zero or $\pm\frac{1}{3}$. The lower index is color R, G or B.

The d-quark is made heavier than the u-quark by introducing the Chern-Simons model with a spontaneous symmetry breaking mechanism, like the Higgs mechanism, and topological mass. This is described in appendix B.

3 Sakharov conditions

Sakharov proposed in 1967 [8] a set of three necessary conditions that baryon asymmetry producing interactions must satisfy. These conditions were inspired by the then recent discoveries of the cosmic background radiation and CP violation in the neutral kaon system. The three necessary Sakharov conditions are: (1) baryon number B violation, (2) C and CP symmetry violations, and (3) interactions must be out of thermal equilibrium.

Our scenerio is, needless to say, consistent with baryon number violation. C and CP violation are observed facts. C violation tales place below Λ_{cr} and CP violation is presumably valid in the SM phase of our scenario. Condition three is not violated due to rapid expansion decreasing the occurrence of pair-annihilation.

4 Implications to matter-antimatter asymmetry

Consider in the early universe groups of twelve C symmetric chermons. Let each group consist of four m^+, four m^- and four m^0 particles. Any such group may form only hydrogen (H) atoms, only anti-hydrogen ($\bar{\mathrm{H}}$) or any combination of both H and $\bar{\mathrm{H}}$ atoms [5, 6]. This is achieved by organizing the chermons appropriately (mod 3) using table 2:

$$
\begin{aligned}
p + e^- &:= u^{2/3} + u^{2/3} + d^{-1/3} + e^- \\
&:= 4m^{+1/3} + 4m^{-1/3} + 4m^0 \\
&= u^{-2/3} + u^{-2/3} + d^{+1/3} + e^+ \\
&=: \bar{p} + e^+
\end{aligned}
\tag{4.1}
$$

where the superscript is the charge of the particle. As can be seen from (4.1) in this scenario neither baryon number B nor lepton number L is fundamental but their difference is $B - L = 0$.

In (4.1) the world is still matter-antimatter symmetric. Furthermore, within each electron and positron the equal charge chermons repel each other. How can the dramatic change from matter-antimatter symmetry to matter-antimatter asymmetry happen? In nutshell, it goes as follows [9, 10]: introduce a massive photon and a complex scalar with the action (B.1) and (B.2). The Chern-Simons model provides a topological mass to the gauge field A_μ. The self-interaction potential (B.2) causes spontaneous symmetry breaking leading to the Proca mass term $m^2 A_\mu A_\mu$. The parameters of the scenario must and can be chosen such that the *attractive Yukawa force between chermons, including equal charge chermons, is the dominating interaction.*

To obey condition $B-L=0$ of baryon-lepton balance and to sustain charge conservation, for one electron made of three chermons, a proton containing nine chermons has to be created, albeit it happens much later. Likewise, one neutrino requires a neutron to be created. The m^0 carries in addition color causing neutrino formation. This binding interaction makes neutrinos different from other leptons and the quarks.

Each chermon in an electron and positron is tightly bound by the Yukawa interaction to the two other chermons. Therefore the e^-, e^+ and the neutrinos are assumed to form first at the onset of inflation. When the protons and antiprotons are formed much later we see from (4.1) that there are not necessarily equal amounts of hydrogen and antihydrogen distributed in the universe. Because chermons may *at random choose whether they form part of* H *or* $\bar{\mathrm{H}}$ *there are regions of space of various sizes dominated by* H *or* $\bar{\mathrm{H}}$ *atoms*. Because the universe is the largest statistical system it is expected that there is only a very slight excesses of H atoms[2] which remain after the equal amounts of H and $\bar{\mathrm{H}}$ atoms have annihilated. The ratio n_B/n_γ is thus predicted to be $\ll 1$. At this stage in development n_B/n_γ is a multiverse like concept.

Consider now some consequences for inflation (or bounce as well). The inflaton decay takes place after it has reached the minimum of its potential and it couples to the quarks and leptons while vibrating in its ground state causing reheating. Visible matter fields loose their original quantum fluctuations and their distribution is remodeled by reheating towards lesser uniform distribution in space. Dark matter (DM) instead does not couple to the inflaton. It is therefore more smoothly distributed in the universe because they were unaffected by reheating. Quantum fluctuations in the dark fields during inflation may lead to formation of primordial black holes in the universe. These density variations of dark matter provide attractive gravitational potential regions for visible matter to accumulate in the various formations we observe.

Fermionic dark matter has in this scenario no mechanism to become "baryon" asymmetric like visible matter. Therefore we expect that part of dark matter has annihilated into bosonic dark matter. Secondly, we predict there should exist both dark matter and anti-dark matter clumps in the universe. Collisions of anti-dark matter and dark matter celestial bodies would give us a new source for wide spectrum gravitational wave production (the lunar mass alone is $\sim 10^{49}$ GeV).

We expect roughly twice as much visible matter from the m^+ and m^0 than fermionic dark matter from the n. The fraction of n of all matter today is about 2.5%. Therefore there should be about ten times more bosonic dark matter and e.g. primordial black holes than fermionic dark matter.

5 Conclusions

Above Λ_{cr} the fermionic chermons are C symmetric with small equal masses and symmetric charges around zero, like {-1/3, 0, 1/3}. When the quarks are formed C violation is caused by heavier mass of the d quark which is produced by the attractive force between the equal charges of the u quark. On chermon level, this is possible when the m^+m^+ potential becomes attractive, i.e. the photon topological mass θ exceeds the chermon mass $m_{eff} = m_e + yv^2$. Our model is consistent with the Sakharov condition on C violation below Λ_{cr}.

The baryon asymmetry is, in our conjecture, created from C symmetric, baryon and lepton neutral chermons in a straightaway manner as described in

[2] or $\bar{\mathrm{H}}$ atoms which only means a charge sign redefinition.

section 4. The ratio n_b/n_γ is predicted by statistical arguments to be $\ll 1$ but much larger than the experiment based SM result $\sim 10^{-19}$ because the antipreon-preon cross section must be orders of magnitude smaller than the antiproton-proton cross section. Estimation of this ratio is left for future work. In any case it depends somewhat randomly on the fractions of the hydrogen and antihydrogen abundances in the early universe as discussed in section 4. Therefore we consider it a chemistry and biology supporting parameter, together with the cosmological constant [13, 14].

Below the transition energy Λ_{cr} fractional charge three chermon composites form quarks while charge zero and one states are leptons. These composite states are expected to behave to a good approximation like point-like particles, the composite radius being between 10^{-18} m and the electron Cartan radius 10^{-27} m. The standard model is obtained [2, 3, 4, 6] in the low energy limit of accelerator energies (and above).

The experimental test of our scenario is finding (i) neither any substantial amount antimatter (see e.g. [15]), (ii) nor broken supersymmetry (MSSM) superpartners [16] in the universe. On the theoretical side detailed simulations are to be done to estimate the free parameters. And one may contemplate how far away (quantum) gravity may be, see e.g. [17, 18].

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A Particle chermon correspondence

We briefly recap our chermon scenario of [5, 6], which turned out to have close resemblance to the simplest $N = 1$ globally supersymmetric 4D model, namely the free, massless Wess-Zumino model [19, 20] with the kinetic Lagrangian including three neutral fields m, s, and p with $J^P = \frac{1}{2}^+, 0^+$, and 0^-, respectively

$$\mathcal{L}_{\rm WZ} = -\frac{1}{2}\bar{m}\partial\!\!\!/ m - \frac{1}{2}(\partial s)^2 - \frac{1}{2}(\partial p)^2 \tag{A.1}$$

where m is a Majorana spinor, s and p are real fields (metric is mostly plus).

We assume that the pseudoscalar p is the axion [21], and denote it below as a. It has a fermionic superparther, the axino n, and a bosonic superpartner, the saxion s^0.

In order to have visible matter we assume the following charged chiral field Lagrangian

$$\mathcal{L}_{-} = -\frac{1}{2}m^{-}\partial\!\!\!/ m^{-} - \frac{1}{2}(\partial s_i^{-})^2, \quad i = 1, 2 \tag{A.2}$$

It is phenomenologically necessary that all fermions have the very same small mass. Secondly, to build the neutrino one fermion charge has to be zero. The charges are thus -1/3, 0, +1/3.

The table 2 gives the chermon content of SM matter and a proposal for dark matter.

SM Matter	chermon state
ν_e	$m_R^0 m_G^0 m_B^0$
u_R	$m^+ m^+ m_R^0$
u_G	$m^+ m^+ m_G^0$
u_B	$m^+ m^+ m_B^0$
e^-	$m_R^- m_G^- m_B^-$
d_R	$m^- m_G^0 m_B^0$
d_G	$m^- m_B^0 m_R^0$
d_B	$m^- m_R^0 m_G^0$
Dark Matter	chermon state
boson (or BC)	axion(s), s^0
e'	axino n
meson, baryon o	$n\bar{n}, 3n$
nuclei (atoms with γ')	multi n
celestial bodies	any dark stuff
black holes	any chermon

Table 2: Visible and Dark Matter with corresponding particles. m^0 is color triplet, $m^{\pm}$ are color singlets. BC stands for Bose condensate. e' and γ' refer to dark electron and dark photon, respectively. Identical chermon state antisymmetrization not shown.

B Maxwell-Chern-Simons QED$_3$ action

A number of 1+2 dimensional models have properties close to 1+3 dimensional world as can be found in [7, 22, 23], see also [24].

The action for a QED$_3$-Chern-Simons model [10] including two polarization $\pm$ fermionic fields (ψ_+, ψ_-), a gauge field A_μ and a complex scalar field φ with spontaneous breaking of local U(1) symmetry [25, 26] is

$$
\begin{aligned}
S_{\mathrm{QED-MCS}} = \int d^3x \{ -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + i\overline{\psi}_+ \gamma^\mu D_\mu \psi_+ + i\overline{\psi}_- \gamma^\mu D_\mu \psi_- \\
+ \frac{1}{2} \theta \epsilon^{\mu v \alpha} A_\mu \partial_v A_\alpha - m_e (\overline{\psi}_+ \psi_+ - \overline{\psi}_- \psi_-) \\
- y(\overline{\psi}_+ \psi_+ - \overline{\psi}_- \psi_-) \varphi^* \varphi + D^\mu \varphi^* D_\mu \varphi - V(\varphi^* \varphi) \},
\end{aligned} \tag{B.1}
$$

where the covariant derivatives are $D_\mu \psi_\pm = (\partial_\mu + ie_3 A_\mu)\psi_\pm$ and $D_\mu \varphi = (\partial_\mu + ie_3 A_\mu)\varphi$. e_3 is the coupling constant of the $U(1)$ local gauge symmetry, here with dimension of $(\mathrm{mass})^{1/2}$. $V(\varphi^*\varphi)$ represents the self-interaction potential,

$$
V(\varphi^*\varphi) = \mu^2 \varphi^* \varphi + \frac{\zeta}{2} (\varphi^* \varphi)^2 + \frac{\lambda}{3} (\varphi^* \varphi)^3 \tag{B.2}
$$

which is the most general sixth power renormalizable potential in 1+2 dimensions [27]. The parameters μ, ζ, λ and y have mass dimensions 1, 1, 0 and 0, respectively. For potential parameters $\lambda > 0, \zeta < 0$ and $\mu^2 \leq 3\zeta^2/(16\lambda)$ the vacua are stable.

In 1+2 dimensions, a fermionic field has its spin polarization fixed up by the sign of mass [28]. The model includes two positive-energy spinors (two spinor families). Both of them obey Dirac equation, each one with one polarization state according to the sign of the mass parameter.

The vacuum expectation value v of the scalar field φ is given by:

$$
\langle \varphi^* \varphi \rangle = v^2 = -\zeta/(2\lambda) + \left[(\zeta/(2\lambda))^2 - \mu^2/\lambda \right]^{1/2} \tag{B.3}
$$

The condition for its minimum is $\mu^2 + \frac{\zeta}{2} v^2 + \lambda v^4 = 0$. After the spontaneous symmetry breaking, the scalar complex field can be parametrized by $\varphi = v + H + i\theta$, where H represents the Higgs scalar field and θ the would-be Goldstone boson. For manifest renormalizability one adopts the 't Hooft gauge by adding the gauge fixing term $S_{R_\xi}^{gt} = \int d^3x [-\frac{1}{2\xi} (\partial^\mu A_\mu - \sqrt{2}\xi M_A \theta)^2]$ to the broken action. Keeping only the bilinear and the Yukawa interaction terms one has the following action

$$
\begin{aligned}
S_{\mathrm{QED}}^{\mathrm{SSB}} = \int d^3x \Big\{ & -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} M_A^2 A^\mu A_\mu \\
& - \frac{1}{2\xi} (\partial^\mu A_\mu)^2 + \overline{\psi}_+ (i\partial\!\!\!/ - m_{eff}) \psi_+ \\
& + \overline{\psi}_- (i\partial\!\!\!/ + m_{eff}) \psi_- + \frac{1}{2} \theta \epsilon^{\mu v \alpha} A_\mu \partial_v A_\alpha \\
& + \partial^\mu H \partial_\mu H - M_H^2 H^2 + \partial^\mu \theta \partial_\mu \theta - M_\theta^2 \theta^2 \\
& - 2yv(\overline{\psi}_+ \psi_+ - \overline{\psi}_- \psi_-) H - e_3 \left(\overline{\psi}_+ A\!\!\!/ \psi_+ + \overline{\psi}_- A\!\!\!/ \psi_- \right) \Big\}
\end{aligned} \tag{B.4}
$$

where the mass parameters

$$M_A^2 = 2v^2 e_3^2, \quad m_{eff} = m_e + yv^2, \quad M_H^2 = 2v^2(\zeta + 2\lambda v^2), \quad M_\theta^2 = \xi M_A^2 \tag{B.5}$$

depend on the SSB mechanism. The Proca mass, M_A^2originates from the Higgs mechanism. The Higgs mass, M_H^2, is associated with the real scalar field. The Higgs mechanism also contributes to the chermon mass, resulting in an effective mass m_{eff}. There are two photon mass-terms in (B.4), the Proca and the topological one.

C Chermon-chermon interaction

The chermon-chermon scattering amplitude in the non-relativistic approximation is obtained by calculating the t-channel exchange diagrams of the Higgs scalar and the massive gauge field. The propagators of the two exchanged particles and the vertex factors are calculated from the action (B.4) [10].

The gauge invariant effective potential effective for the scattering considered is obtained in [29, 30]

$$V_{\mathrm{MCS}}(r) = \frac{e^2}{2\pi}\left[1 - \frac{\theta}{m_e}\right] K_0(\theta r) + \frac{1}{m_e r^2}\left\{l - \frac{e^2}{2\pi\theta}[1 - \theta r K_1(\theta r)]\right\}^2 \tag{C.1}$$

where $K_0(x)$ and $K_1(x)$ are the modified Bessel functions and l is the angular momentum ($l = 0$ in this note). In (C.1) the first term [ ] corresponds to the electromagnetic potential, the second one $\{\ \}^2$ contains the centrifugal barrier $\left(l/mr^2\right)$, the Aharonov-Bohm term and the two photon exchange term.

One sees from (C.1) the first term may be positive or negative while the second term is always positive. The function $K_0(x)$ diverges as $x \to 0$ and approaches zero for $x \to \infty$ and $K_1(x)$ has qualitatively similar behavior. For our scenario we need negative potential between equal charge chermons. Being embarrassed of having no data points for several parameters in (C.1) we can give some relations between these parameter values for a negative potential. We need the condition[3]

$$\theta \gg m_e \tag{C.2}$$

The potential (C.1) also depends on v^2, the vacuum expectation value, and on y, the parameter that measures the coupling between the fermions and the Higgs scalar. Being a free parameter, v^2 indicates the energy scale of the spontaneous breakdown of the $U(1)$ local symmetry.

[3]For applications to condensed matter physics, one must require $\theta \ll m_e$, and the scattering potential given by (C.1) then comes out positive [10].

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