Viktor Strohm

vfstrohm@yahoo.de

Movement of the ellipsograph ruler with different speeds

Annotation

On the ruler of the ellipsograph, we set point C. We set the movement of the ruler: uniform, uniformly accelerated, elliptical. We calculate the angle of rotation of point C relative to the center of the ellipse, the sectoral velocity and the velocity and acceleration vectors. We compare the sectoral velocity with Kepler's second law.

Introduction

In the scientific literature, the differential equation of an ellipse is derived through dynamic quantities and laws. Then Kepler's laws are derived. However, Kepler's laws are kinematic. This article uses the kinematic equation of an ellipse. The equation is derived through oscillations of a parametric pendulum.

Any point on the ellipsograph ruler moves along an elliptical path.

In order not to refer the reader to the sources, we present the derivation of the formulas necessary for calculating velocities, accelerations, and rotation angles.

Ruler *AB* moves from horizontal to vertical position, figure 1. Point *C* describes ¼ of the ellipse. The direction of the instantaneous rotation of the ruler *AB* around P_{AB} is clockwise in accordance with the direction of the known velocity vector of point *A*.

Speeds of points *B* and *C*:

$$\omega_{AB} = \frac{v_A}{AP_{AB}} \tag{1}$$

$$v_B = \omega_{AB} * BP_{AB} = v_A \frac{BP_{AB}}{AP_{AB}} \tag{2}$$

Vector v_C is directed perpendicular to *CP*.

$$v_C = \omega_{AB} * CP_{AB} = v_A \frac{CP_{AB}}{AP_{AB}} \tag{3}$$

The directions of the velocities of the points $\overrightarrow{v_B}$ and $\overrightarrow{v_C}$ are determined by the instantaneous rotation of the ruler *AB* around the instantaneous center of velocities P_{AB}.

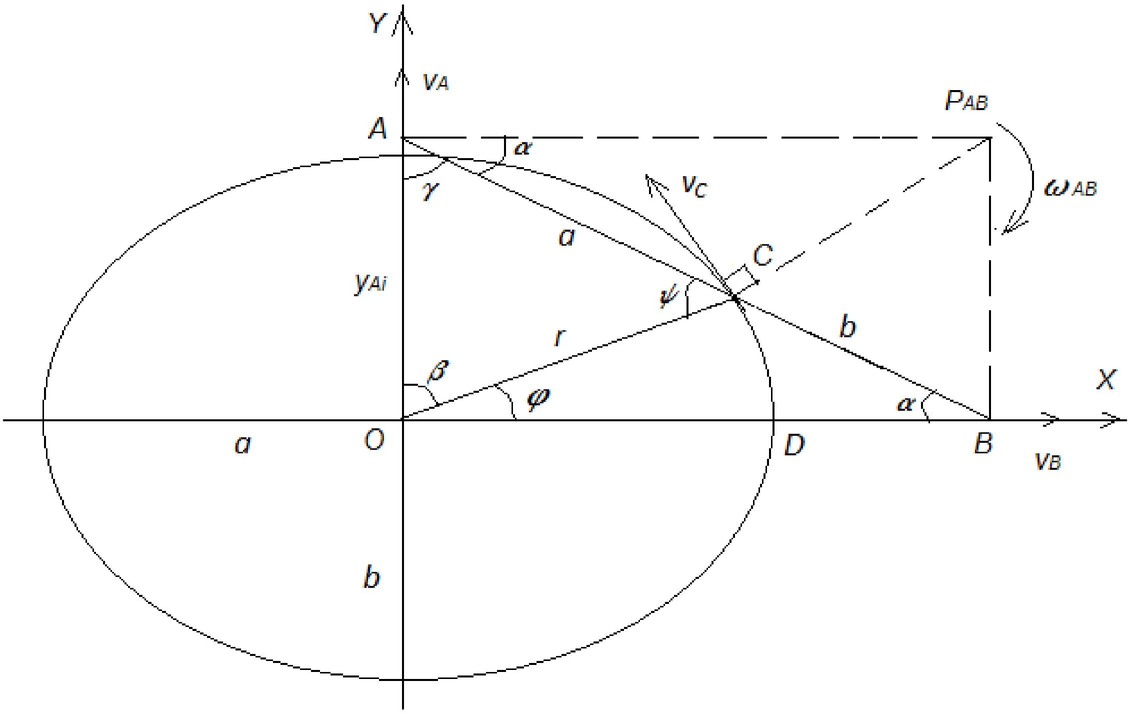

Figure 1

Determination of accelerations of points *B* and *C*

Let's use the theorem - acceleration of points of a flat figure. Point A will be a pole, since the acceleration of point A is known.

The vector equation for the acceleration of point B has the form:

$$\overrightarrow{a_B} = \overrightarrow{a_A} + \overrightarrow{a_{BA}^r} + \overrightarrow{a_{BA}^c} \qquad (4)$$

where $\overrightarrow{a_A}$ is the acceleration of the pole *A* (given);

$\overrightarrow{a_{BA}^r}$ and $\overrightarrow{a_{BA}^c}$ are the rotational and centripetal accelerations of the point *B* in the rotation of the ruler around the pole *A*. In this case:

$$\boldsymbol{a_{BA}^c} = \boldsymbol{\omega_{AB}^2} * BA \qquad (5)$$

The vector $\overrightarrow{a_{BA}^c}$ is located on *BA* and is directed from point *B* to pole *A*.

$$\boldsymbol{a_{BA}^r} = \boldsymbol{\varepsilon_{AB}} * BA \qquad (6)$$

The vector $\overrightarrow{a_{BA}^r}$ is located perpendicular to the ruler *AB*, its direction is unknown, since the direction of the angular acceleration $\boldsymbol{\varepsilon}_{AB}$ is unknown.

In equation (4) there are two unknowns: accelerations $\overrightarrow{a_A}$ and $\overrightarrow{a_{BA}^r}$, which can be determined from the projection equations of vector equality (3) onto the directions of axes *AX* and *AY*:

$$\begin{cases} \boldsymbol{a}_{Bx} = \boldsymbol{a}_{Ax} + \boldsymbol{a}_{BAx}^r + \boldsymbol{a}_{BAx}^c \\ \boldsymbol{a}_{By} = \boldsymbol{a}_{Ay} + \boldsymbol{a}_{BAy}^r + \boldsymbol{a}_{BAy}^c \end{cases} \qquad (7)$$

The direction of the vectors $\overrightarrow{a_B}$ and $\overrightarrow{a_{BA}^r}$ is chosen arbitrarily. System solution (7) allows you to find the numerical value of $\overrightarrow{a_B}$ and $\overrightarrow{a_{BA}^r}$r with a plus or minus sign. A positive value indicates the correctness of the chosen direction of the vectors $\overrightarrow{a_B}$ and $\overrightarrow{a_{BA}^r}$, a negative value indicates the need to change their direction.

System (7) allows us to determine unknown modules:

$$a_A = \sqrt{(a_{Ax})^2 + (a_{Ay})^2}, a_{AB}^r = \sqrt{(a_{ABx}^r)^2 + (a_{ABy}^r)^2} \tag{8}$$

Ruler angular acceleration:

$$\boldsymbol{\varepsilon}_{AB} = \frac{\boldsymbol{a}_{BA}^r}{BA} \tag{9}$$

The acceleration of point *C* is determined by the equation:

$$\overrightarrow{a_C} = \overrightarrow{a_A} + \overrightarrow{a_{CA}^r} + \overrightarrow{a_{CA}^c} \tag{10}$$

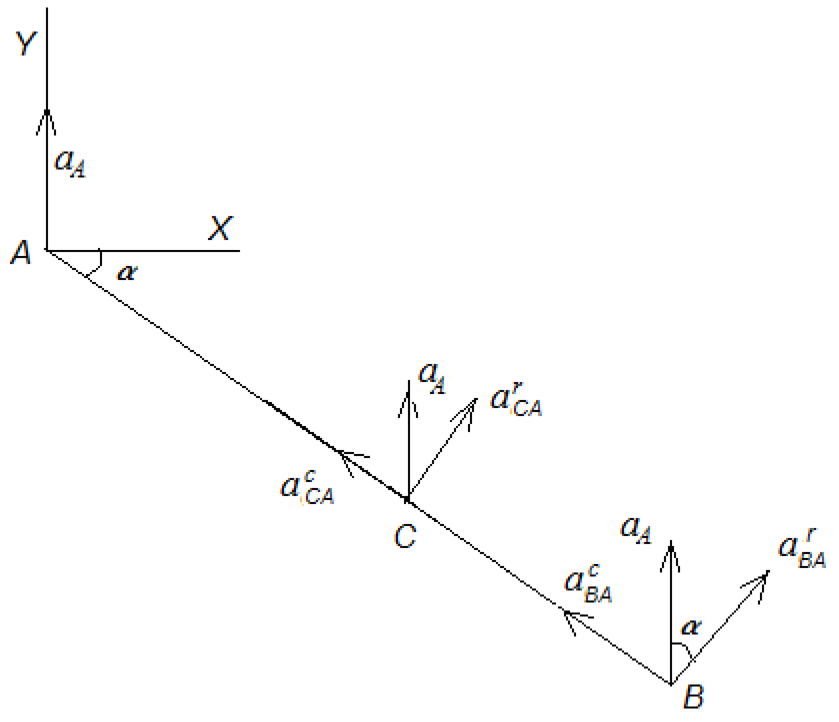

Figure 2

where $\overrightarrow{a_{CA}^r}$ and $\overrightarrow{a_{CA}^c}$ are, respectively, the rotational and centripetal accelerations of the point *C* relative to the pole *A*:

$$\boldsymbol{a}_{CA}^c = \boldsymbol{\omega}_{AB}^2 * AC \tag{11}$$

$$\boldsymbol{a}_{CB}^r = \boldsymbol{\varepsilon}_{AB} * AC \tag{12}$$

Vector $\overrightarrow{a_{CA}^{c}}$ is located on *CA* and directed from point *C* to pole *A*. Vector $\overrightarrow{a_{CA}^{r}}$ is perpendicular to *CA* and directed in the same direction as $\overrightarrow{a_{BA}^{r}}$, Fig. 2.

Equation (8) can be represented in projections on the axes *Ax* and *Ay*:

$$\begin{cases} \boldsymbol{a}_{Cx} = \boldsymbol{a}_{Ax} + \boldsymbol{a}_{CAx}^{c} + \boldsymbol{a}_{CAx}^{r} \\ \boldsymbol{a}_{Cy} = \boldsymbol{a}_{Ay} + \boldsymbol{a}_{CAy}^{c} + \boldsymbol{a}_{CAy}^{r} \end{cases} \tag{13}$$

The acceleration projections of point C are determined from (11). The direction of the vector $\overrightarrow{a_C}$ is determined by the signs of the projections $\boldsymbol{a}_{Cx}$ and $\boldsymbol{a}_{Cy}$.

Vector modulus:

$$\boldsymbol{a}_C = \sqrt{(a_{Cx})^2 + \left(a_{Cy}\right)^2} \tag{14}$$

Let's take a look at the different travel options

T is the period specified by arbitrary units of time. *AB = a+b*, *A(0,* y_A*)* *B(*x_B*,0)*. Initial coordinates of points: *A(0,0)*, *B(a+b,0)*, *C(a,0)*. Initial speed $\boldsymbol{v}_{A0} = 0$.

Uniform movement

Given: point *C* divides *AB* into segments *a* and *b*, *A(0,*y_A*)*, *B(*x_B*,0)*, initial *A(0,0)*, *B(AB,0*). *A* moves uniformly from *O* → *Y*. Accelerations $\boldsymbol{a}_A = 0, \boldsymbol{a}_B = 0$, velocity $\boldsymbol{v}_A = \frac{AB*4}{T}$ (15)

Find: y_{A_i}, x_{C_i}, y_{C_i}, v_{C_i}, $\boldsymbol{a}_{C_i}$, φ_i

Solution

Coordinates *A(0 ,* y_{A_i} *)*:

$$y_{A_i} = \boldsymbol{v}_A * i \tag{16}$$

Further, according to equations (4) - (14)

Coordinates *B(*x_{B_i}*, 0)*:

$$\sin\alpha = \frac{y_{A_i}}{AB}, \alpha = \operatorname{asin}\frac{y_{A_i}}{AB} \tag{17}$$

$$x_{B_i} = \cos\alpha * \mathrm{AB}, y_{B_i} = 0 \tag{18}$$

$$\boldsymbol{\omega}_{AB} = \frac{v_A}{AP_{AB}} = \frac{v_A}{x_{B_i}} \tag{19}$$

$$\boldsymbol{v}_B = \boldsymbol{\omega}_{AB} * BP_{AB} = \boldsymbol{\omega}_{AB} * y_{A_i} \tag{20}$$

From equation (5) $\boldsymbol{a}_{BA}^{c} = \boldsymbol{\omega}_{AB}^{2} * BA$

$$\begin{cases} \boldsymbol{a}_{Bx} = \boldsymbol{a}_{BA}^{c} * \cos\alpha + \boldsymbol{a}_{BA}^{r} * \sin\alpha \\ 0 = \boldsymbol{a}_{Ay} + \boldsymbol{a}_{BA}^{c} * \sin\alpha + \boldsymbol{a}_{BA}^{r} * \cos\alpha \end{cases} \tag{21}$$

Solving the resulting equations, we find $\boldsymbol{a}_B$,

$$\boldsymbol{a}_{BA}^r = \frac{-\boldsymbol{a}_{Ay}-\boldsymbol{a}_{BA}^c*\sin\alpha}{\cos\alpha} = \frac{-\boldsymbol{a}_{BA}^c*\sin\alpha}{\cos\alpha} \quad (22)$$

$$\boldsymbol{\varepsilon}_{AB} = \frac{\boldsymbol{a}_{BA}^r}{AB} \quad (23)$$

Coordinates $P_{AB}\left(x_{B_i}, y_{A_i}\right)$.

Coordinates $C\left(x_{C_i}, y_{C_i}\right)$

$$\frac{a}{AB} = \frac{x_{C_i}}{x_{B_i}}, \frac{b}{AB} = \frac{y_{C_i}}{y_{A_i}} \quad (24)$$

$$x_{C_i} = \frac{a}{AB} * x_{B_i}, y_{C_i} = \frac{b}{AB} * y_{A_i} \quad (25)$$

$$CP_{AB} = \sqrt{{x_{B_i}}^2 + a^2 - 2\left(a * x_{B_i}\right)\cos\alpha} \quad (26)$$

$$\boldsymbol{v}_C = \boldsymbol{\omega}_{AB} * CP_{AB} = \boldsymbol{\omega}_{AB} * \sqrt{{x_{B_i}}^2 + a^2 - 2 * \left(a * x_{B_i}\right) * \cos\alpha} \quad (27)$$

$$\varphi = \operatorname{atan}\frac{y_{C_i}}{x_{C_i}} \quad (28)$$

Point C acceleration is determined by equation (10):): $\overrightarrow{a_C} = \overrightarrow{a_A} + \overrightarrow{a_{CA}^r} + \overrightarrow{a_{CA}^c}$

$$\boldsymbol{a}_{CA}^c = \boldsymbol{\omega}_{AB}^2 * AC = \boldsymbol{\omega}_{AB}^2 * a \quad (29)$$

$$\boldsymbol{a}_{CA}^r = \boldsymbol{\varepsilon}_{AB} * AC = \boldsymbol{\varepsilon}_{AB} * a \quad (30)$$

$$\begin{cases}\boldsymbol{a}_{Cx} = \boldsymbol{a}_{Ax} + \boldsymbol{a}_{CAx}^r + \boldsymbol{a}_{CAx}^c \\ \boldsymbol{a}_{Cy} = \boldsymbol{a}_{Ay} + \boldsymbol{a}_{CAy}^r + \boldsymbol{a}_{CAy}^c\end{cases} \quad (31)$$

$$\begin{cases}\boldsymbol{a}_{Cx} = 0 + \boldsymbol{a}_{CA}^r * \sin\alpha + \boldsymbol{a}_{CA}^c * \cos\alpha \\ \boldsymbol{a}_{Cy} = 0 + \boldsymbol{a}_{CA}^r * \cos\alpha + \boldsymbol{a}_{CA}^c * \sin\alpha\end{cases} \quad (32)$$

$$\boldsymbol{a}_C = \sqrt{\boldsymbol{a}_{Cx}^2 + \boldsymbol{a}_{Cy}^2} \quad (33)$$

Uniformly accelerated motion

Given: point *C* divides *AB* into segments *a* and *b*, *A* moves with uniform acceleration from *O* → *Y, A(0,y_A) B(x_B,0)*, initial *A(0,0), B(AB,0)*, 〖$\boldsymbol{a}_{A_i} = const, \boldsymbol{v}_{A_0} = 0$.

Find: y_{A_i}, $\left(x_{C_i}, y_{C_i}\right)$, v_{C_i}, $\boldsymbol{a}_{C_i}$, φ_i

Solution

$$\boldsymbol{v}_{A_i} = \frac{\boldsymbol{a}_A * i^2}{2}; \;\; i = 1 \ldots n = \frac{T}{4} \quad (34)$$

$$AB = \boldsymbol{v}_{An} = \frac{\boldsymbol{a}_A * n^2}{2} \tag{35}$$

$$\boldsymbol{a}_{A_i} = \boldsymbol{a}_A = \frac{2AB}{n^2} \tag{36}$$

Coordinates $A(0, y_{A_i})$:

$$y_{A_i} = \frac{\boldsymbol{a}_A * i^2}{2} \tag{37}$$

Further on the equations (4) – (14)

Coordinates $B(x_{B_i}, 0)$:

$$x_{B_i} = \sqrt{AB^2 - {y_{A_i}}^2} \tag{38}$$

Coordinates $C\left(x_{C_i}, y_{C_i}\right)$:

$$\frac{a}{AB} = \frac{x_{C_i}}{x_{B_i}}, \frac{b}{AB} = \frac{y_{C_i}}{y_{A_i}} \tag{39}$$

$$x_{C_i} = \frac{a}{AB} * x_{B_i}, y_{C_i} = \frac{b}{AB} * y_{A_i} \tag{40}$$

$$\boldsymbol{\omega}_{AB} = \frac{\boldsymbol{v}_{A_i}}{AP_{AB}} = \frac{\boldsymbol{v}_{A_i}}{x_{B_i}} \tag{41}$$

$$\boldsymbol{a}_{BA}^c = \boldsymbol{\omega}_{AB}^2 * AB \tag{42}$$

$$\boldsymbol{a}_{BA}^r = \boldsymbol{\varepsilon}_{AB} * BA \tag{43}$$

The vector $\overrightarrow{a_{BA}^r}$ is located perpendicular to the ruler *AB*, its direction is unknown, since the direction of the angular acceleration $\boldsymbol{\varepsilon}_{AB}$ is unknown.

We project the vector equation (4) on the coordinate axis:

$$\begin{cases} \boldsymbol{a}_{Bx} = \boldsymbol{a}_{BA}^c * \cos\alpha + \boldsymbol{a}_{BA}^r * \sin\alpha \\ 0 = \boldsymbol{a}_{Ay} + \boldsymbol{a}_{BA}^c * \sin\alpha + \boldsymbol{a}_{BA}^r * \cos\alpha \end{cases} \tag{44}$$

Solving the resulting equations, we find $\boldsymbol{a}_B$:

$$\boldsymbol{a}_{BA}^r = \frac{-\boldsymbol{a}_{Ay} - \boldsymbol{a}_{BA}^c * \sin\alpha}{\cos\alpha} \tag{45}$$

$$\boldsymbol{\varepsilon}_{AB} = \frac{\boldsymbol{a}_{BA}^r}{AB} \tag{46}$$

The acceleration of point C is determined by equation (10): $\overrightarrow{a_C} = \overrightarrow{a_A} + \overrightarrow{a_{CA}^r} + \overrightarrow{a_{CA}^c}$

$$\boldsymbol{a}_{CA}^c = \boldsymbol{\omega}_{AB}^2 * AC = \boldsymbol{\omega}_{AB}^2 * a \tag{47}$$

$$\boldsymbol{a}_{CA}^r = \boldsymbol{\varepsilon}_{AB} * AC = \boldsymbol{\varepsilon}_{AB} * a \tag{48}$$

Equation (10) can be represented in projections on the axes *Ax* and *Ay*:

$$\begin{cases} \boldsymbol{a}_{Cx} = \boldsymbol{a}_{Ax} + \boldsymbol{a}_{CAx}^{r} + \boldsymbol{a}_{CAx}^{c} \\ \boldsymbol{a}_{Cy} = \boldsymbol{a}_{Ay} + \boldsymbol{a}_{CAy}^{r} + \boldsymbol{a}_{CAy}^{c} \end{cases} \tag{49}$$

$$\begin{cases} \boldsymbol{a}_{Cx} = 0 + \boldsymbol{a}_{CA}^{r} * \sin\alpha + \boldsymbol{a}_{CA}^{c} * \cos\alpha \\ \boldsymbol{a}_{Cy} = \boldsymbol{a}_{A} + \boldsymbol{a}_{CA}^{r} * \cos\alpha + \boldsymbol{a}_{CA}^{c} * \sin\alpha \end{cases} \tag{50}$$

$$a_C = \sqrt{a_{Cx}^2 + a_{Cy}^2} \tag{51}$$

Elliptical movement

There is a system of equations for a parametric pendulum (52):

$$\begin{cases} x = r(\varphi(t))\cdot cos(\varphi(t)) \\ \boldsymbol{y} = r(\varphi(t))\cdot sin(\varphi(t)) \end{cases} \tag{52}$$

Substitute the radius of the ellipse about the center $r(\varphi(t)) = \frac{b}{\sqrt{1-e^2 cos^2\varphi(t)}}$ (53)

We solve the system with respect to the rotation angle φ. We obtain the kinematic equation of curves of the second order with respect to the center:

$$\ddot{\varphi} = \frac{2*e^2*cos(\varphi)*sin(\varphi)*\dot{\varphi}^2}{1-e^2*cos(\varphi)^2} \tag{54}$$

Given: point *C* divides *AB* into segments *a* and *b*, *A* moves elliptically according to formula (54), *from O → Y*, A *A(0,y_A)*, *B(x_B,0)*, initial *A(0,0,) B(AB,0)*, $\boldsymbol{v}_{A_0} = 0$ is calculated by formula (54). Here, the calculations are carried out by the program [1] from the application.

Find: y_{A_i}, x_{C_i}, y_{C_i}, v_{C_i}, $\boldsymbol{a}_{C_i}$.

Solution

Formula (54) calculates φ_i, x_{C_i}, y_{C_i}:

$$\alpha = \arcsin\frac{y_C}{b} \tag{56}$$

$$\beta = \frac{\pi}{2} - \varphi_i \tag{57}$$

$$\gamma = \arcsin\left(\frac{r_i*\sin\beta}{a}\right) \tag{58}$$

$$\psi = \pi - \gamma - \beta \tag{59}$$

$$y_{A_i} = \frac{y_{C_i} + a*\sin\alpha}{b} \tag{59}$$

$$v_{A_i} = y_{A_i} - y_{A_{i-1}} \tag{60}$$

$$a_{A_i} = v_{A_i} - v_{A_{i-1}} \tag{61}$$

Further, according to equations (4) - (14)

Coordinates *B(x_{B_i}, 0)*:

$$x_{B_i} = \sqrt{AB^2 - y_{A_i}{}^2} \tag{62}$$

Re-find the coordinates $C\left(x_{C_i}, y_{C_i}\right)$:

$$\frac{a}{AB} = \frac{x_{C_i}}{x_{B_i}}, \frac{b}{AB} = \frac{y_{C_i}}{y_{A_i}} \tag{63}$$

$$x_{C_i} = \frac{a}{AB} * x_{B_i}, y_{C_i} = \frac{b}{AB} * y_{A_i} \tag{64}$$

$$\boldsymbol{\omega}_{AB} = \frac{\boldsymbol{v}_{A_i}}{AP_{AB}} = \frac{\boldsymbol{v}_{A_i}}{x_{B_i}} \tag{65}$$

$$\boldsymbol{a}_{BA}^{c} = \boldsymbol{\omega}_{AB}^{2} * AB \tag{66}$$

$$\boldsymbol{a}_{BA}^{r} = \boldsymbol{\varepsilon}_{AB} * BA \tag{67}$$

The vector $\overrightarrow{a_{BA}^{r}}$ is located perpendicular to the ruler *AB*, its direction is unknown, since the direction of the angular acceleration $\boldsymbol{\varepsilon}_{AB}$ is unknown.

We project the vector equation (4) on the coordinate axis:

$$\begin{cases} \boldsymbol{a}_{Bx} = \boldsymbol{a}_{BA}^{c} * \cos\alpha + \boldsymbol{a}_{BA}^{r} * \sin\alpha \\ 0 = \boldsymbol{a}_{Ay} + \boldsymbol{a}_{BA}^{c} * \sin\alpha + \boldsymbol{a}_{BA}^{r} * \cos\alpha \end{cases} \tag{68}$$

Solving the resulting equations, we find $\boldsymbol{a}_{B}$,

$$\boldsymbol{a}_{BA}^{r} = \frac{-\boldsymbol{a}_{Ay} - \boldsymbol{a}_{BA}^{c} * \sin\alpha}{\cos\alpha} \tag{69}$$

$$\boldsymbol{\varepsilon}_{AB} = \frac{\boldsymbol{a}_{BA}^{r}}{AB} \tag{70}$$

Point *C* acceleration is determined by equation (10): $\overrightarrow{a_C} = \overrightarrow{a_A} + \overrightarrow{a_{CA}^{r}} + \overrightarrow{a_{CA}^{c}}$

$$\boldsymbol{a}_{CA}^{c} = \boldsymbol{\omega}_{AB}^{2} * AC = \boldsymbol{\omega}_{AB}^{2} * a \tag{71}$$

$$\boldsymbol{a}_{CA}^{r} = \boldsymbol{\varepsilon}_{AB} * AC = \boldsymbol{\varepsilon}_{AB} * a \tag{72}$$

Equation (10) can be represented in projections on the axes *Ax* and *Ay*:

$$\begin{cases} \boldsymbol{a}_{Cx} = \boldsymbol{a}_{Ax} + \boldsymbol{a}_{CAx}^{r} + \boldsymbol{a}_{CAx}^{c} \\ \boldsymbol{a}_{Cy} = \boldsymbol{a}_{Ay} + \boldsymbol{a}_{CAy}^{r} + \boldsymbol{a}_{CAy}^{c} \end{cases} \tag{73}$$

$$\begin{cases} \boldsymbol{a}_{Cx} = 0 + \boldsymbol{a}_{CA}^{r} * \sin\alpha + \boldsymbol{a}_{CA}^{c} * \cos\alpha \\ \boldsymbol{a}_{Cy} = \boldsymbol{a}_{A} + \boldsymbol{a}_{CA}^{r} * \cos\alpha + \boldsymbol{a}_{CA}^{c} * \sin\alpha \end{cases} \tag{74}$$

$$a_C = \sqrt{a_{Cx}^2 + a_{Cy}^2} \tag{75}$$

The obtained motion parameters allow checking the fulfillment of Kepler's laws. The check is carried out by the program [1] from the application.

Kepler's second law

Uniform movement

```
 Enter char =
 if char = "y" then the source data is specified:
y
 a = 0.500; b = 0.450; T = 360

 Second law of Kepler
Point bypasses 1/4 ellipse counterclockwise in  89 time units
  Input 0 - uniform motion OR
  Input 1-uniformly accelerated motion OR,
  Input 2- elliptical motion):
0
 UNIFORM MOTION
 Set the start of the first sector (1,...,  89): 3
 Set the end of the first sector (   3< end <  89):17
 Set the start of the second sector (1,...,  89): 55
  first sector: angle(start)=  0.03; angle(end)=  0.17
  second secto: angle(start)=  0.61; angle(end) =  0.82
  intervals of time t1=  14; t2=  14
Area of the first sector:    0.1767757E-01
IERR:     0
Area of the second sector:    0.2445188E-01
IERR:     0
```

Figure 3

Uniformly accelerated motion

```
 Enter char =
 if char = "y" then the source data is specified:
y
 a = 0.500; b = 0.450; T = 360

 Second law of Kepler
Point bypasses 1/4 ellipse counterclockwise in  89 time units
  Input 0 - uniform motion OR
  Input 1-uniformly accelerated motion OR,
  Input 2- elliptical motion):
1
 UNIFORMLY ACCELERATEM MOTION
 Set the start of the first sector (1,...,  89): 3
 Set the end of the first sector (   3< end <  89):17
 Set the start of the second sector (1,...,  89): 55
  first sector: angle(start)=  0.00; angle(end)=  0.03
  second secto: angle(start)=  0.35; angle(end) =  0.59
  intervals of time t1=  14; t2=  14
Area of the first sector:    0.3933465E-02
IERR:     0
Area of the second sector:    0.2803914E-01
IERR:     0
```

Figure 4

Elliptical movement

```
 Enter char =
 if char = "y" then the source data is specified:
y
 a = 0.500; b = 0.450; T = 360

 Second law of Kepler
Point bypasses 1/4 ellipse counterclockwise in  89 time units
  Input 0 - uniform motion OR
  Input 1-uniformly accelerated motion OR,
  Input 2- elliptical motion):
2
 ELLEPTICAL MOTION
 Set the start of the first sector (1,...,  89): 3
 Set the end of the first sector (   3< end <  89):17
 Set the start of the second sector (1,....,  89): 55
  first sector: angle(start)=  0.03; angle(end)=  0.25
  second secto: angle(start)=  0.89; angle(end) =  1.15
  intervals of time t1=  14; t2=  14
Area of the first sector:    0.2748870E-01
IERR:       0
Area of the second sector:    0.2748918E-01
IERR:       0
```

Figure 5

Equality of the areas of sectors is carried out only with elliptical motion.

Conclusion

We see that the ellipsograph is a good tool for studying the laws of motion along an ellipse.

The article used materials from textbooks on mechanics. Perhaps the derivation of formulas (54), is rarely given, so the article [1] is proposed

Literature

Viktor Strohm, Kepler's laws as properties of the kinematic equations of motion of a point along curves of the second order, https://www.academia.edu/60717349/Keplers_laws_as_properties_of_the_kinematic_equations_of_motion_of_a_point_along_curves_of_the_second_order

Applications

1. Viktor Strohm, программа вычисления секторной скорости, Kepler2_centerEllipsograph, https://drive.google.com/file/d/113FjF69mgsyfh8hb-s0OMddukcRlSaiT/view?usp=sharing