An alternative cosmological model compatible with the ΛCDM model today.

Stéphane Wojnow*

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Abstract

The theory of quantum mechanics and the theory of general relativity always discuss the initial conditions of the universe and its evolution. We will try to add some simple considerations to this question throught a simple cosmological alternative model based on the the Hubble time, Planck mass flow rate and a variable coefficient α_H*. We find the parameters obtained by the Planck 2018 results with the Planck mass flow rate and Hubble time. In fine, we sketch out a general framework unifying general relativity and quantum field theory.*

Keywords : cosmology, origin of universe, dark energy, Hubble constant, Planck mass flow rate, evolution of universe, quantum mechanics, general relativity, antimatter, Big bang.

**Independent researcher and autodidact, mail : wojnow.stephane@gmail.com*

Introduction

The ΛCDM model based on Einstein's theory of general relativity and on observations is today the most satisfactory theoretical proposal to describe the universe. On the other hand, no quantum description of the universe is now in consensus. We can note that the Planck mass flux is both a relativistic quantity (c^3/G) and a quantum quantity (mPl/tPl). We will use this quantity associated to the Hubble time to propose a quasi complete alternative theoretical framework, relativistic and quantum, of the universe. This alternative theoretical framework, which follows from the ΛCDM model finds values consistent with the results of the Planck 2018 measurements, tries to explain what dark energy is. It proposes an explanation of the disappearance of antimatter in the Big Bang model. Finally it recovers the cosmological diffuse background temperature determined by the WMAP satellite with the Planck 2018 results in a simple and easily affordable cosmological model.

A) A toy cosmological model compatible with the ΛCDM model after the decoupling.

It seems possible to obtain the total mass of the universe from the ΛCDM model otherwise. This could eventually lead to the development of a simple toy cosmological model unknown to the author, built around the Hubble constant , the Hubble time, $t_H = 1 / H$, the Planck mass flow and a variable coefficient α_H.

α_H = **radius of the observable universe** (from calculation of the ΛCDM model for example) **divided by** the **Hubble radius** at time t_H for a flat universe ,:

$$\alpha_H = \frac{c}{H_0}\int_{a=0}^{a=1} \frac{da}{a^2\sqrt{\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda}} / \frac{c}{H_0} \qquad \text{(Equation 1)}$$

where a is the scale factor, c is the speed of light, $H_0 = 67,4 \pm 0,5$ km s^{-1} Mpc^{-1}, is the Hubble parameter measured today[1], the Ω_i are the density parameters of the standard cosmological model, i.e. the ΛCDM model, measured today[1].

$$\alpha_H = \int_{a=0}^{a=1} \frac{da}{a^2\sqrt{\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda}} \qquad \text{(Equation 2)}$$

$\delta = \frac{c^3}{G} = \frac{m_{Pl}}{t_{Pl}}$ is the Planck mass flow rate.

$t_{H_0} = \frac{1}{H_0}$ is the Hubble time ($\approx$ 4,578 10^{17} s = 14,51 billion light years today)

R_{H_0} is the Hubble radius.

$$R_{H_0} = \frac{c}{H_0} = c t_{H_0} \qquad \text{(Equation 3)}$$

The increase of the "total mass of Hubble volume", M_{H_0}, in the sense of the ΛCDM model, i.e. dark energy + matter, is determined for a flat universe by the relation with the critical density $\rho_c = \frac{3}{8\pi G\ t_{H_0}^2}$ and the Hubble volume $V_{H_0} = \frac{4\pi}{3}(c\ t_{H_0})^3$:

$$M_{H_0} = \frac{3}{8\pi G\ t_{H_0}^2}\ \frac{4\pi}{3}(c\ t_{H_0})^3 \qquad \text{(Equation 4)}$$

$$M_{H_0} = \frac{1}{2}\ \frac{c^3}{G}\ t_{H_0} \qquad \text{(Equation 5)}$$

$$M_{H_0} = \frac{1}{2}\ \frac{m_{Pl}}{t_{Pl}}\ t_{H_0} \qquad \text{(Equation 6)}$$

$$M_{H_0} = \frac{1}{2}\ \delta\ t_{H_0} \qquad \text{(Equation 7)}$$

The mass of the observable universe in the sense of the ΛCDM model is :

$$M_{H_0}\alpha_{H_0}^3 = \frac{1}{2}\ \delta\ t_{H_0}\alpha_{H_0}^3 \qquad \text{(Equation 8)}$$

$\alpha_{H_0} \approx$ 4.399 10^{26} m / 1.372 10^{26} m $\approx$ 3.175 today if H0 = 67,4km s^{-1} Mpc^{-1}, Ωm=0,315 and ΩΛ=0,685[1].

$$M_{H_0}\alpha_{H_0}^3 \approx 2,959\ 10^{54} kg \qquad \text{(Equation 9)}$$

in other words, the "total mass" of the observable universe ΛCDM mesured today. (e=mc^2)

B) Value of α_H before the decoupling in the cosmological toy model and consequences.
The author hypothesises that, before the decoupling, the radius of the observable universe was equal to the Hubble radius. The ratio α_{H_0} was then equal to 1.

B.1) Thus, the mass of the universe at t_{H0} = Planck time is determined by :

$$M_{H_{t_{Pl}}=\frac{1}{2}\ \frac{m_{Pl}}{t_{Pl}}\ t_{Pl}} \qquad \text{(Equation 10)}$$

$$M_{H_{t_{Pl}}=\frac{1}{2}m_{Pl}} \qquad \text{(Equation 11)}$$

This can be verified with the thermal energy :

$$E_{Th} = \frac{1}{2}\ m_{Pl}\ c^2 = \frac{1}{2}\ k_B\ T_{Pl} \qquad \text{(Equation 12)}$$

where k_B is the Boltzmann constant, with one degree of freedom assumed for the singularity and T_{Pl} the Planck temperature.

B.2) Mass of the universe at Hubble radius in this alternative cosmological model.

Starting from a "Planck time grain mass", the singularity of the Big Bang model, at the beginning of the time of the universe, . Then by making the assumption that for each unit of Planck time that passes, a corresponding mass "Planck time grain mass" is added to the mass of the universe. In our toy cosmological model, the "total mass" (energy) of the universe at the Hubble radius, before and after the decoupling, at time t_{H_0}, grows simply by following the summation :

$$M_{H_0} = \sum_{i=0}^{t_H/t_p} i\frac{m_{Pl}}{2} \qquad \text{(Equation 13)}$$

i.e.

$$M_{H_0} = \frac{1}{2}\ \frac{m_{Pl}}{t_{Pl}}\ t_{H_0} \qquad \text{(Equation 14)}$$

$$M_{H_0} = \frac{1}{2}\ \frac{c^3}{G}\ t_{H_0} \qquad \text{(Equation 15)}$$

t_{H_0} is the Hubble time. H_0 = 67,4 km/s/Mpc[1], t_{H_0} = 4,578 10^{17} seconds today, so $M_{H_0} \approx$ 9,241 10^{52} kg
Note : … and with datas of §2 we have Eq.9 ,$M_{H_0}\alpha^3_{H_0} \approx 2,959\ 10^{54} kg$

with $t_H = 1/H$, so the Hubble radius in this toy universe is the same as the Hubble radius in the ΛCDM model.
This is valid, without recourse to cosmic inflation, from Planck time to the Hubble radius of the universe at the time of decoupling in the standard model (377 700 years) but also beyond. This is made possible by writing the "total mass" and the Hubble radius as sigma summations. This also has the consequence of limiting quantum phenomena in the universe to dimensions of the order of Planck units between t_H and $t_H + t_{Pl}$.

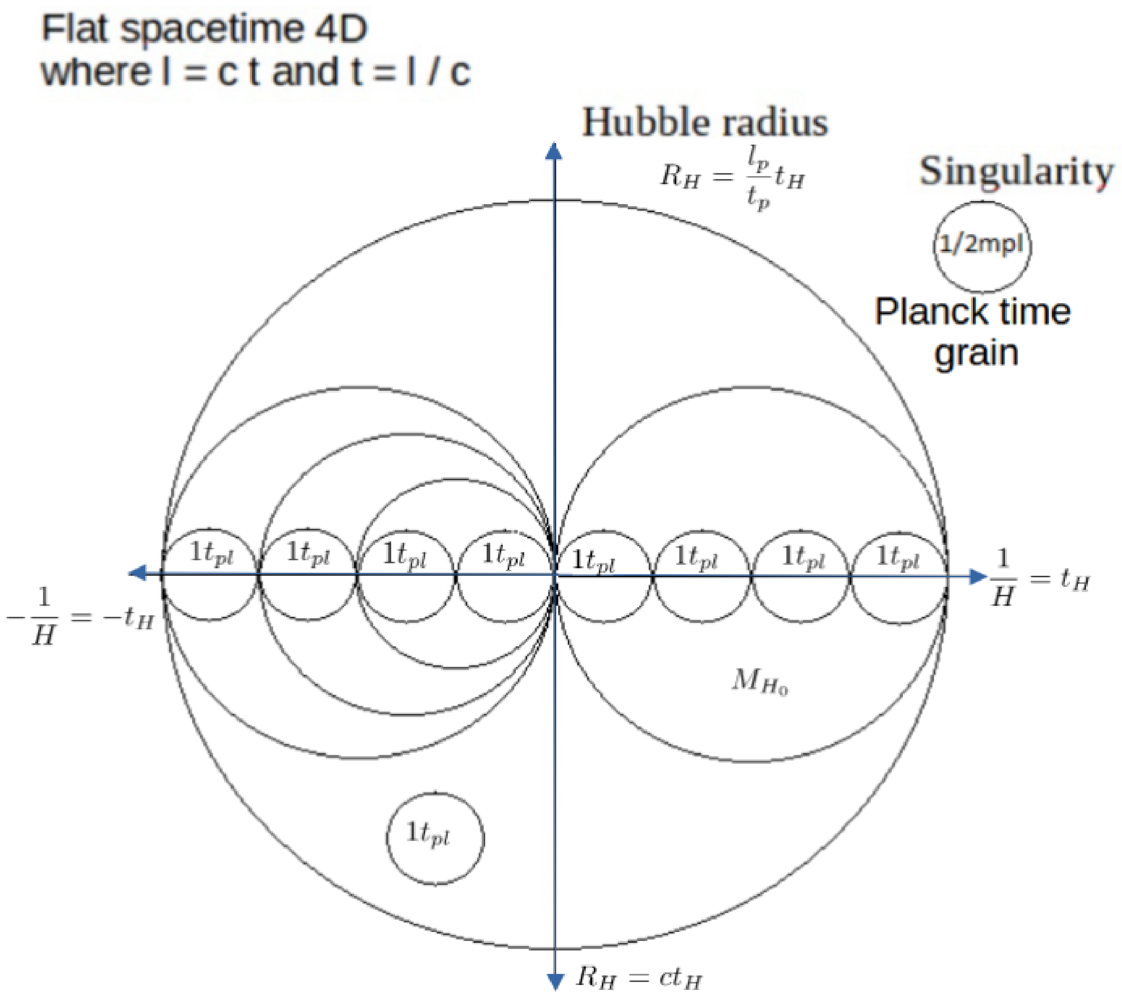

Figure 1: Hubble sphère

C) Proposal of determination of the cosmological constant in this toy cosmological model.

C.1) The Hubble sphere seen as a black hole.

Figure 1 shows that the observation of the Hubble sphere is always done in a given direction whether along the axes t_H or along the axes R_H. When we look in the opposite direction, we observe a universe with the same characteristics, namely a Hubble universe whose mass increases as a function of t_H. This toy model is therefore by construction isotropic, i.e. identical whatever the direction of observation. It is also homogeneous on a large scale by construction, i.e. for any considered time interval t_{Pl}, it contains a Planck half-mass. For a large number of Planck half-masses, the latter agglomerate to form stars first under the effect of dark matter and then under the gravitational effect of the Planck half-masses agglomerated under the effect of dark matter.

In other words, there is always an observational bias that makes a part of the Hubble Universe not available to our observation, but it is there. We have therefore a Hubble universe of mass $\rho_c V_{H0}$ composed of two mini Hubble spheres that could have a radius of $R_{H0} = c\ t_{H0}$ and a mass $M_{H0} = \frac{1}{2}\frac{c^3}{G} t_{H0}$ because $R_{H0} = c\ t_{H0}$ is proportional to M_{H0}. These two mini Hubble spheres "touch" each other at the observer's original location and time. The observer sees only one of these mini-spheres at any given time

The invariant gravitational force that attracts these masses of the two mini Hubble spheres is therefore $FM_{H_0}^{\pm}$:

$$FM_{H_0}^{\pm} = \frac{GM_{H_0}^{+} M_{H_0}^{-}}{R_{H_0}^2} \quad \text{(Equation 16)}$$

$$FM_{H_0}^{\pm} = \frac{c^4}{4G} \quad \text{(Equation 17)}$$

$$FM_{H_0}^{\pm} = \frac{F_{Pl}}{4} \quad \text{(Equation 18)}$$

where F_{Pl} is Planck's force and where N is the Newton.

$$FM_{H_0}^{\pm} = 3,02564\ 10^{43}\ N \quad \text{(Equation 19)}$$

The Planck force characterizes a property of space-time according to Barrow and Gibbons[2]. In general relativity, the limiting value it represents does not correspond to the Planck unit, but to the reduced Planck unit, where G is replaced by $4G$. The resulting reduced Planck force is four times weaker and is equal to Eq.16 to Eq.19 . This is a maximum limit in general relativity, attainable only at the horizon of a black hole. As the radius of a Schwarzschild black hole R_s is also its horizon R_h where $R_s = R_h = R_{H_0}$ it is permissible to assimilate the Hubble universe to a Schwarzschild black hole. Its barycenter is the center of the Hubble sphere.

$$R_h = \frac{2GM_{H_0}}{c^2} \quad \text{(Equation 20)}$$

Considering the Hubble sphere as a Schwarzschild black hole will be essential in a following paragraph to theorize the temperature of the cosmic microwave background, i.e. the CMB.

The two mini Hubble spheres can also be two complete Hubble spheres.We will simply note that the possibility of a double universe with two opposite time arrows proposed by the Soviet physicist Andreï Sakharov in 1967 is taken up here. The ideas that follow from Andreï Sakharov's hypothesis should be re-examined according to the author, especially to account for dark matter and dark energy. The hypothesis of Andreï Sakharov has given rise to few scientific works. Among the scientists who have worked on his hypothesis are Nathan Rosen, Jean Pierre Pettitt, Gabriel Chardin, Michael Boris Green, John Henry Schwarz, Abdus Salam (Nobel Prize in Physics in 1979), or Sabine Hossenfelder. Finally we note by examining two mini Hubble spheres or two complete Hubble spheres, that $M_{H_0}^{+}$ and $M_{H_0}^{-}$ can be seen as a mass of matter and a mass of antimatter.

C.2) Proposal of determination of the cosmological constant.

Here, we try to deal with dark energy. In classical mechanics, the gravitational interaction between two masses is instantaneous, but in general relativity this interaction cannot be faster than the speed of light. We will use this property of general relativity theory to propose a value for the cosmological constant. The value, which is questionable from a dimensional point of view, is nevertheless consistent with the results of Planck 2018, as we'll point out below. We'll finish by showing the dimensional consistency of this proposition in §F).

Since the velocity of the gravitational interaction $FM_{H_0}^{\pm}$ between $M_{H_0}^{+}$ and $M_{H_0}^{-}$ is limited to c we assume that the power of $FM_{H_0}^{\pm}$ is $PM_{H_0}^{\pm}$ Watts such that:

$$FM_{H_0}^{\pm}c \quad \text{(Equation 21)}$$

$$PM_{H_0}^{\pm} = 9,0706\ 10^{51} kg.m^2.s^{-3} \quad \text{(Equation 22)}$$

We will look for what could balance this power. As we have already used the opposite with $M_{H_0}^{+}$ and $M_{H_0}^{-}$ to find $FM_{H_0}^{\pm}$, this time we will use the inverse of $PM_{H_0}^{\pm}$ to get the neutrality egal to 1 of the mathematical operation:

$$\frac{1}{PM_{H_0}^{\pm}} = 1,1025\ 10^{-52} kg^{-1}.m^{-2}.s^{3} \quad \text{(Equation 23)}$$

The Watt is also the measure of energy flow. The latter is by definition the measure of the total power of electromagnetic radiation emitted or received by a real or virtual surface. We assume that c is an electromagnetic radiation. We will also assume to obtain the dimension [L^{-2}] of the cosmological constant Λ, that the "1" in the numerator of Eq.23 is of dimension [$M\ T^{-3}$], i.e. kgs^{-3}. This is the dimension of a power surface density in electromagnetism. The author admitted in a previous version of this article that this approach to obtaining the dimension of Λ was "most certainly wrong", but the value of retaining the numerical value of Λ from $PM_{H_0}^{\pm}$ seemed more important to him. In a future paragraph, the author will show that this "anomalous" dimension of the cosmological constant applies within the framework of this alternative model, which is intended as a general framework for developing a theory unifying general relativity and quantum field theory.

Note that :

$$\frac{1}{PM_{H_0}^{\pm}} = \frac{4}{P_{Pl}} \quad \text{(Equation 24)}$$

where P_{Pl} is the Planck power.

C.3) Validation of the value of the proposed cosmological constant.

The density parameter of the cosmological constant Ω_Λ in the ΛCDM model is defined by Friedmann equation for a flat universe as follows:

$$\Omega_\Lambda = \frac{c^2\Lambda}{3H_0^2} \quad \text{(Equation 25)}$$

i.e . with Planck 2018 results (H_0 = 67,4 km/s/Mpc[1], t_{H_0} = 4,578 10^{17} seconds today) and the proposed value of Λ:

$$\Omega_\Lambda = \frac{299792458^2\ .\ 1,1025\ 10^{-52}.\ (4,578\ 10^{17})^2}{3} \quad \text{(Equation 26)}$$

$$\Omega_\Lambda = 0,6923 \quad \text{(Equation 27)}$$

By simplifying, today, the matter density parameter $\Omega_m = 1 - \Omega_\Lambda$, i.e. $\Omega_m = 0,3077$.

Planck 2018 results[1] gives Ωm = 0.315 ± 0.007. If Ωm = 0.315 - 0.007, then Ωm = 0.3080. The theoretical value of Λ gives a result extremely close to the lower bound of Ωm with the Planck 2018 results[1]. This is the main reason why the author thinks that the important open question about the determination of the dimension of Λ seems acceptable to him. This alternative cosmological model would give the origin of dark energy where the ΛCDM model fails.

C.4) Proposed solution of the vacuum catastrophe in this alternative cosmological model.

Let's consider the force that attracts our two mini Hubble spheres in contact and expanding, $M_{H_0}^+$ and $M_{H_0}^-$ at a distance R_{H_0}. At the point of origin of figure.1, this force crosses at speed c a quantum surface of Planck scale l_{pl}^2, where l_{pl} is the Planck length. The power $PM_{H_0}^\pm$ or energy flux of the cosmological constant thus crosses orthogonally the virtual surface l_{pl}^2. Mathematically, this gives us :

$$\varphi = \frac{PM_H^\pm}{l_{pl}^2} \approx 3,5\ 10^{121} kg.s^{-3} \quad \text{(Equation 28)}$$

The dimension of is that of a surface power density, i.e., that of the energy flux $PM_{H_0}^\pm$ that starts from the origin of the Hubble sphere to interact with its surface. This $kg.s^{-3}$ dimension is also the one that was missing from the numerator when we determined the value of the cosmological constant Λ in Eq.23. And l_{pl}^{-2} is the assumed value of quantum energy suggested by quantum field theory with a cutoff at l_{pl}. One writing of the vacuum catastrophe is divide the energy of the cosmological constant Λ by the vacuum energy suggested by quantum field theory is :

$$\frac{l_{pl}^{-2}}{\Lambda} \quad \text{(Equation 29)}$$

Considering the zero point energy suggested by the quantum field theory l_{pl}^{-2} as the inverse of a surface and not as an energy should fit easily into the mirror theories that follow from Andreï Sakharov's hypothesis.

D) Proposal of explanation of the disappearance of antimatter in the Big Bang model.

To make this proposal we must refer to figure 1. To facilitate the understanding of what follows without having to navigate in this file, I make a copy below

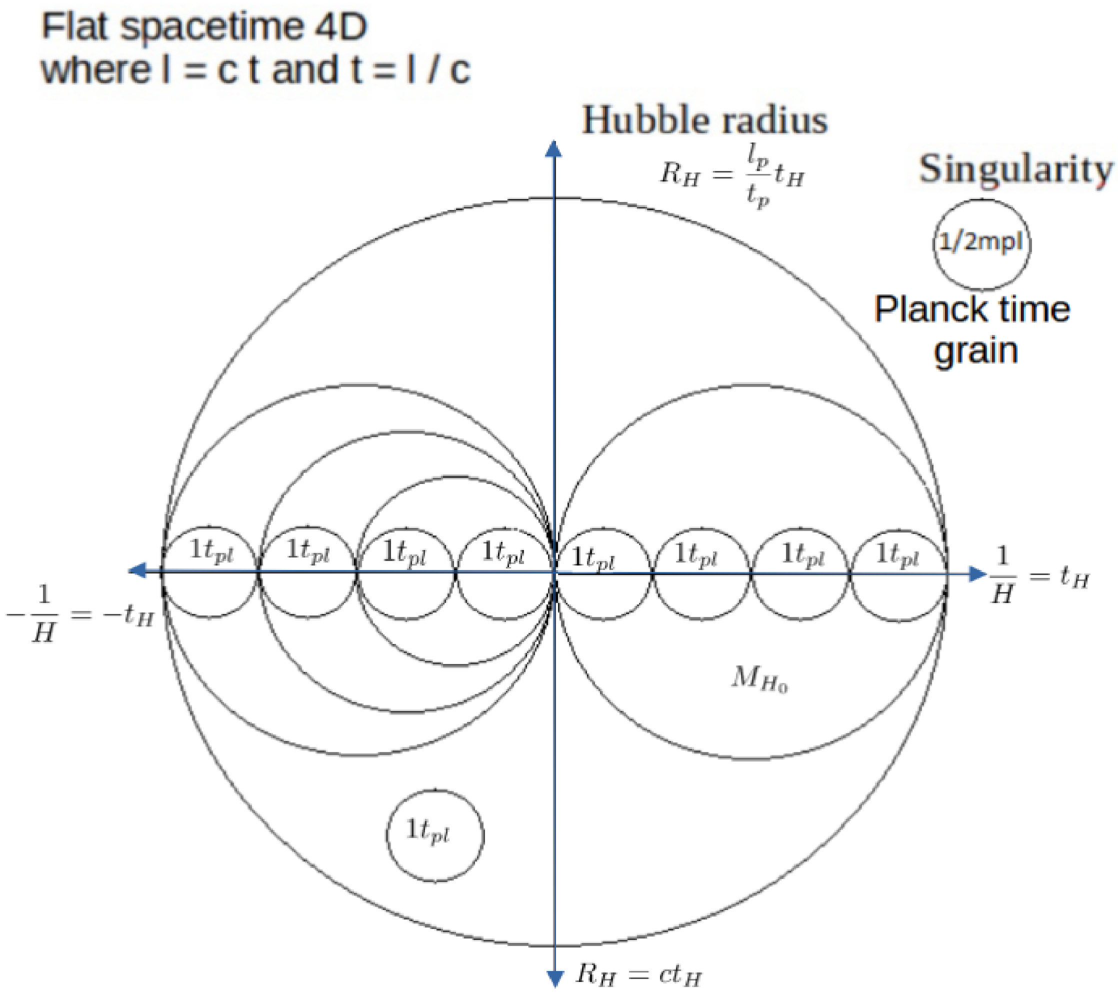

Figure: Hubble sphère

The resolution of this problem comes naturally when the human or instrumental observational bias is identified:

When we observe the Hubble sphere in the "up" or "down" direction, the two masses M_{H0} of the two Hubble mini-spheres are not included in the field of view. They are there but the observer placed at the origin of the 4 directions of the figure does not see them. It is Schrödinger's cat that is both dead and alive as long as we look in these directions. The time can pass as much as one wants, as long as the observer does not change direction of observation, the observer does not know if he will see the matter ***M+*** or the antimatter ***M-*** .The cat of the matter is thus at the same time dead ***M-*** and alive ***M+***. When the observer chooses to make the observation by turning 90°, he will see a dead cat or a living cat. This model of universe starts at $t_{H0} = 0$, the time being signed + or -, the observer will see , either matter $M - /t^{-}$, or matter $M + /t^{+}$, i.e. $0,5m_{pl}^{+}$. The mass $0,5m_{pl}^{-}$ is on the time line $-t_H$ from the origin. It is located on the other side of the observer's time origin. He does not see it. This explains the infinitesimal amount of antimatter in the observed universe in the Bing bang model wich begin at $t_{H0} = t_{pl}$.

E) Proposal of determination of the CMB temperature in this alternative cosmological model.

I had stressed the importance of considering the Hubble sphere as a black hole at the end of paragraph C.1). Here, I will partially repeat the work of the article "A Rotating Model of a Light Speed Expanding Hubble-Hawking Universe" by U. V. Satya Seshavatharam and S. Lakshminarayana[3] because they give, with an approximation that I would not use, the CMB temperature from the Hawking temperature of black holes.
"(3) Following Hawking's formula for the temperature of black holes [26], the current cosmic temperature can be expressed as follows: "

$$T_{H_0} = \frac{\hbar c^3}{8\pi k_B G\sqrt{m_{Pl} M_{H_0}}} \qquad \text{(Equation 30)}$$

where $\hbar$ is the reduced Planck constant, or Dirac constant and k_B the Boltzmann constant. The Planck 2018 results give a value of H0 = (67.40 ± 0.5) km s^{-1} Mpc^{-1}. Taking the lower bound of the confidence index, we obtain H0 = 66.90 km s^{-1} Mpc^{-1} i.e. $t_{H0} = 4,6124\ 10^{17}s$. We obtain with Eq,14, $M_{H0} = 9,310\ 10^{52}kg$.

$$T_{H_0} = \frac{1,0545718\ 10^{34} * 299792458^3}{8\pi * 1.380649\ 10^{23} * 6.6743\ 10^{11}\sqrt{2,176434\ 10^{-8} * 9,310}\ 10^{52}} \qquad \text{(Equation 31)}$$

note: an error in my latex editor makes it impossible to set the Boltzmann constant to the correct power of 10. Please rexify as follows for your calculations:
Boltzmann constant = 1.380649 10^{-23} m^2 kg s^{-2} K^{-1}

$$T_{H0} = 2,7256\ K \qquad \text{(Equation 32)}$$

The CMB temperature measured today, i.e. for z=0, is :
TCMB(z = 0) = 2.72548 ±0.0005 7 K[3]. The upper bound of the uncertainty error is 2.72605 K. This measurement is therefore in perfect agreement with the calculation made by assimilating the Hubble sphere to a black hole and calculating its Hawking temperature.

F) Numerical and dimensional consistency of zero-point energy in this alternative model.

The dimension and value corresponding to vacuum energy in quantum field theory is l_{Pl}^{-2}[4]. The dimension of the cosmological constant in this alternative model (Eq.23 and Eq.24) is

$$kg^{-1}.m^{-2}.s^3 \qquad \text{(Equation 33)}$$

We have the value of l_{Pl}^{-2}, which is totally independent of Λ, so much so that it is referred to as the "worst prediction of theoretical physics" with regard to the gap between its cosmological measurement and the prediction in quantum theory.

It is very easy to reconcile these two values by calculating :

$$\frac{4.t_{Pl}^3}{m_{Pl}.l_{Pl}^2} = 1,1025\ kg^{-1}.m^{-2}.s^3 \qquad \text{(Equation 34)}$$

which is the value and dimension of the cosmological constant in this alternative model, which unifies general relativity and quantum field theory.

Conclusion.

In this alternative model, the mass of the Hubble sphere is egal to $M_{H0} = \sum_{i=0}^{t_H/t_p} \frac{i\ m_{Pl}}{2}$ and appears as a "stacking" of Planck half masses on a Hubble timeline t_H instead of a density multiplied by a spherical volume in the ΛCDM model. This stacking of masses is compatible with the apparent isotropy and homogeneity of the universe. This model starts at $t_H = 0$, contrary to the Big bang model, and goes until today. It gives results consistent with the observations made with the Planck satellite. He takes up the idea of double universes and sketches out lines of thought on the relationship between the infinitely small of quantum mechanics and the infinitely large treated with general relativity. If we hesitated for a while about the value proposed here of the cosmological constant because of its special dimension compared to the usual conventions, we can be much more sure of its foundation ($M_{H_0}^+$ and $M_{H_0}^-$ with $FM_{H_0}^\pm$). It allows us to theorize about the CMB's temperature measurement and the interest in revisiting and renewing Andrei Sakharov's hypothesis.

Furthermore, we note that the idea of Bruno Valeixo Bento and Stav Zalel in their article "If time had no beginning"[5] seems correct. By linking it to quantum space, we can assume that multi-universes could exist everywhere in a flat, infinite 4D spacetime, as proposed in Figure 1 with singularities inside and outside the Hubble sphere. This is true for every unit of Planck time that elapses, but also before the Planck time of the Big Bang.

Finally, we give a solution to the "worst prediction of theoretical physics", also known as the vacuum catastrophe or the cosmological constant problem.

In conclusion, this model is a candidate as a general framework for a theory of everything. Among other things, it validates the zero-energy universe hypothesis.

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Author's final note:

So far, attempts to construct an alternative model to the ΛCDM model have not been able to account for the CMB temperature. To be sure, this alternative model is still incomplete. In particular, it lacks explanations of the power spectrum of the CMB polarization and the power spectrum of galaxies. Since this model does not take dark matter into account, the explanation of the power spectrum of galaxies is currently inaccessible. But according to Chat GPT, the power spectrum of the CMB polarization on the other hand could be accessible with this formula :

$$C_l = \frac{2k_B^4 T_{CMB}^4}{c^2\hbar^3}\frac{\Omega_b^2}{\Omega_m^2}\int_0^{\infty}\frac{\Delta_{\mathcal{R}}^2(k)}{k^2}j_l^2(kr)dk$$

with :

C_l is the angular power spectrum of the CMB

$\Delta_{\mathcal{R}}(k)$ is the amplitude of the primordial power spectrum of density perturbations

k represents the wave number. It is a measure of the wavelength of the fluctuation in the early universe, where the initial density fluctuations were generated. In the case of the CMB, the wavenumber is often expressed in terms of angular scales, measured in degrees on the celestial sphere.

j_l is the spherical Bessel function

r is the comoving distance at redshift z corresponding to the angular scale

The verifications that I asked the GPT Cat to perform on the numerical application of his formula are obviously still open to question and are eventually to be explored by the scientific community.