The Resolution of the Twin Paradox with Three Frames of Reference – A Mathematical and Physical Report

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Abstract

We report how we can use a time dilation-like expression eliminating the twin paradox conundrum that standard formulation is unable to accomplish, since erroneously attribute the meaning of a reciprocal relation to the ageing of the twins. This report result from an approach based on simultaneity and synchronization considering three frames where a preferred frame is Einstein Frame, the frame where the speed of light is isotropic.

Report

In previous works [1-17] particularly in "The physical meaning of synchronization and simultaneity in Special Relativity" [1] it is criticized the approach of Einstein [18] based on the postulates of the isotropy of speed light in every frame and the equivalence of every frame. Several works, some very recent, point out the importance of this discussion about the foundations of Mathematics, Philosophy, Relativity, Quantum Mechanics, Cosmology and Biophysics [19-104]. The consequent Principle of Relativity has been also considered in the articles "On the Consistency between the Assumption of a Special System of Reference and Special Relativity" [10] and "The Principle of Relativity and the Indeterminacy of Special Relativity" [12] and "Special Relativity as a simple geometry problem" introducing "Feynman clock" associated to a preferred frame, Einstein Frame, with time dilation [13]. In a more recent work "Speakable and Unspeakable in Special Relativity: time readings and clock rhythms" [14] it is referred the consequences of these analysis particularly the physical meaning of time dilation and Lorentz-FitzGerald contraction mathematical expressions.

Twin A'' is moving through the x' axis of S' with Einstein speed $|V_E'|$. At $x' = l_1$ the twin return with speed $|V_E'|$ to the origin of S'. The proper times of the twin A'', τ'' for the trip to and τ'' for the trip from, are calculated. The proper times of the twins located at S', τ', is also calculated between the same events. We show how the standard formulation misinterpret the relation of proper times, the ageing of the twins at S'' and S'. For that we calculate through the time dilation – like equation the proper times τ'' with the Lorentzian times. It is easy to show the misinterpretation of the standard formulation through the equality of the two-way trip result, that is consistent with the one-way results, as expected.

Consider S with a rod with length l between O and x of the x axis.

Also consider S' and S'' moving with v_1 and v_2 through the direction of the x axis (Fig. 1).

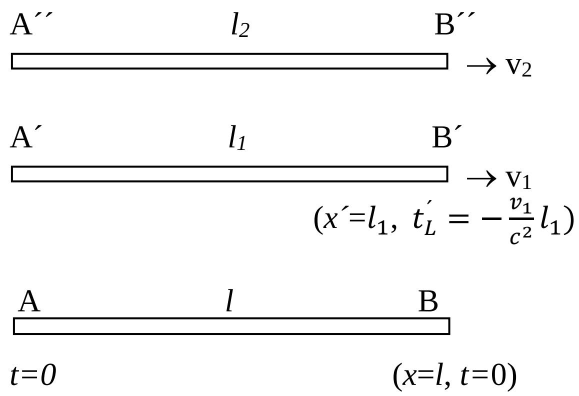

Fig. 1 Frame S' represented by a rod with length l_1 is moving with speed v_1 in relation to frame S, EF, rod with length l. The extremities of the rods coincide simultaneously and therefore, can synchronize clocks at A, A', A'' and B, B', B'' [49, 50]. A twin located at A' we designate by twin A'. The same rule for the other positions.

S is Einstein Frame (EF) as we previously designate it [8], the frame where the one-way speed of light is isotropic (in vacuum) with the value c, the two-way speed of light experimentally measured. It is also assumed that the speed of light in S is independent of the movement of the source. With these assumptions a Lorentz transformation has been obtained by us introducing an intrinsic desynchronization, as we designate it, in the IST transformation [1, 6, 8]

$$t_L' = t' - \frac{v_1}{c^2} x' \qquad (1)$$

where t_L' is the Lorentzian time and t' is the synchronized time. Also by Georgy I. Burde more recently [37]. This has been achieved previously (2002-EPS12 Trends in Physics) [1-3] and the published results about the time dilation meaning and also the meaning of the Lorentz-FitzGerald contraction are also referred by by Zbigniew Oziewicz in relation to the resolution of the twin paradox conundrum in several works, particularly [33]. In relation to the Preferred Frame the time dilation formula has a clear mathematical and physical meaning. Indeed

$$d\tau' = dt \sqrt{1 - \frac{{v_1}^2}{c^2}} \qquad (2)$$

$$d\tau'' = dt \sqrt{1 - \frac{{v_2}^2}{c^2}} \qquad (3)$$

where $d\tau'$ and $d\tau''$ are the diferential of proper times of S' and S'' in relation to the preferred frame with $dt = d\tau$ since for the preferred frame there is no intrinsic desynchronization when $v_1 = 0$

$$t_L^{'} = t' - \frac{v_1}{c^2}x' = t' = t \Rightarrow dt = d\tau \qquad (4)$$

This is eventually a remarkable result that standard formulation is not aware [97, 98] originating the Twin Paradox conundrum.

The time dilation means that a clock moving in relation to the Preferred Frame has time dilated (see Mohazabbi and Luo article [97, 98, 33]).

But from (2) and (3)

$$d\tau'' = d\tau' \frac{\sqrt{1 - \frac{{v_2}^2}{c^2}}}{\sqrt{1 - \frac{{v_1}^2}{c^2}}} \qquad (5)$$

The Lorentz Transformation between S, S' and S'' is

$$x' = \frac{x - v_1 t}{\sqrt{\left(1 - \frac{{v_1}^2}{c^2}\right)}} \qquad (6)$$

$$t_L^{'} = \frac{t - \frac{v_1}{c^2}x}{\sqrt{\left(1 - \frac{{v_1}^2}{c^2}\right)}} \qquad (7)$$

$$x'' = \frac{x - v_2 t}{\sqrt{\left(1 - \frac{{v_2}^2}{c^2}\right)}} \qquad (8)$$

$$t_L^{''} = \frac{t - \frac{v_2}{c^2}x}{\sqrt{\left(1 - \frac{{v_2}^2}{c^2}\right)}} \qquad (9)$$

From (6), (7), (8) and (9) [6, 8, 99]

$$x'' = \frac{x' - V_E' t_L'}{\sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)}} \qquad (10)$$

$$t_L'' = \frac{t_L' - \frac{V_E'}{c^2} x'}{\sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)}} \qquad (11)$$

with Einstein velocity [6, 8, 33, 99]

$$V_E' = \frac{v_2 - v_1}{1 - \frac{v_1 v_2}{c^2}} \qquad (12)$$

but with a meaning that standard approach cannot accomplished, naturally.

Indeed, from (6) and (7) for $x=l$ and $t=0$

$$x' = l_1 = \frac{l - v_1 \times 0}{\sqrt{\left(1 - \frac{v_1^2}{c^2}\right)}} \qquad (13)$$

$$x'' = l_2 = \frac{l - v_2 \times 0}{\sqrt{\left(1 - \frac{v_2^2}{c^2}\right)}} \qquad (14)$$

Therefore,

$$\frac{l_2}{l_1} = \frac{\sqrt{\left(1 - \frac{v_1^2}{c^2}\right)}}{\sqrt{\left(1 - \frac{v_2^2}{c^2}\right)}} \qquad (15)$$

From (10) for ($x' = l_1, t_L' = -\frac{v_1}{c^2} l_1$) (see Fig. 1)

$$l_2 = \frac{l_1 + V_E' \frac{v_1 l_1}{c^2}}{\sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)}} = \frac{l_1}{\sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)}} \left(1 + V_E' \frac{v_1}{c^2}\right) \qquad (16)$$

From (11)

$$dt_L^{''} = d\tau'' = \frac{dt_L^{'} - \frac{V_E^{'}}{c^2} V_E^{'} dt_L^{'}}{\sqrt{\left(1 - \frac{V_E^{'2}}{c^2}\right)}} = \frac{dt_L^{'}(1 - \frac{V_E^{'2}}{c^2})}{\sqrt{\left(1 - \frac{V_E^{'2}}{c^2}\right)}} = dt_L^{'} \sqrt{\left(1 - \frac{V_E^{'2}}{c^2}\right)} \qquad (17)$$

This seems time dilation (eq. (2)) but it is not (eq. (2), (3) and (5) and the following (eq. (19)).

Notice

$$t_L^{'} = t' - \frac{v_1}{c^2} x'$$

$$dt_L^{'} = dt' - \frac{v_1}{c^2} V_E^{'} dt_L^{'}$$

$$dt_L^{'} \left(1 + \frac{v_1}{c^2} V_E^{'}\right) = dt' = d\,\tau' \quad (18)$$

We assume as obvious $dt' = d\,\tau'$. The assumption of standard relativity is similar but with Lorentzian clocks (eq. (17), with $dt_L^{'} = dt' = d\,\tau'$ *and of course this standard assumption it is not correct).* This standard assumption originates the conundrum.

Therefore from (17) and (18)

$$d\tau'' = \frac{d\tau'}{1 + \frac{v_1}{c^2} V_E^{'}} \sqrt{\left(1 - \frac{V_E^{'2}}{c^2}\right)} \quad (19)$$

And from (5)

$$d\tau'' = d\tau' \frac{\sqrt{1 - \frac{v_2^2}{c^2}}}{\sqrt{1 - \frac{v_1^2}{c^2}}} = \frac{d\tau'}{1 + \frac{v_1}{c^2} V_E^{'}} \sqrt{\left(1 - \frac{V_E^{'2}}{c^2}\right)} \qquad (20)$$

$$\frac{\sqrt{1 - \frac{v_2^2}{c^2}}}{\sqrt{1 - \frac{v_1^2}{c^2}}} = \frac{1}{1 + \frac{v_1}{c^2} V_E^{'}} \sqrt{\left(1 - \frac{V_E^{'2}}{c^2}\right)} = \frac{d\tau''}{d\tau'} \qquad (21)$$

Therefore if

$$\lceil v_2 \rceil > \lceil v_1 \rceil \qquad d\tau'' < d\tau' \qquad (23)$$

$$\lceil v_2 \rceil < \lceil v_1 \rceil \qquad d\tau'' > d\tau' \qquad (24)$$

$$v_2 = -v_1 \qquad d\tau'' = d\tau' \qquad (25\ )$$

Pirooz Mohazzabi and Qinghua Luo are partially right. Standard relativity seems … does not resolve the twin paradox. The ageing of the twins is given by (20) and the statement of reciprocity based on the time dilation-like expression given by (17) is inaccurate based on the error of the synchronization of Lorentzian clocks.

Indeed, if $v_2 > v_1$ and twin A´´ is moving from A´ to B´ with

$$V_E^{'} = \frac{v_2 - v_1}{1 - \frac{v_1 v_2}{c^2}} > 0 \qquad (26)$$

When A´´ pass by A´, B´´ pass by B´ and all twins has proper times zero. When twin A´´ arrive at twin B´ the ageing of twin A´´ is τ''

$$\tau'' = \frac{l_1}{V_E^{'}}\sqrt{\left(1 - \frac{V_E^{'2}}{c^2}\right)} = \Delta t_L^{'}\sqrt{\left(1 - \frac{V_E^{'2}}{c^2}\right)} \qquad (27)$$

and B´ simultaneously moves through S´´ from B´´ to A´´ ageing, using the time dilation-like expression (17),

$$\tau' = \frac{l_2}{V_E^{'}}\sqrt{\left(1 - \frac{V_E^{'2}}{c^2}\right)} = \Delta t_L^{''}\sqrt{\left(1 - \frac{V_E^{'2}}{c^2}\right)} \qquad (28)$$

From (16)

$$\tau' = \frac{l_2}{V_E^{'}}\sqrt{\left(1 - \frac{V_E^{'2}}{c^2}\right)} = \frac{l_1\left(1 + V_E^{'}\frac{v_1}{c^2}\right)}{V_E^{'}\sqrt{\left(1 - \frac{V_E^{'2}}{c^2}\right)}}\sqrt{\left(1 - \frac{V_E^{'2}}{c^2}\right)} \qquad (29)$$

and from (27)

$$\tau' = \tau''\frac{\left(1 + V_E^{'}\frac{v_1}{c^2}\right)}{\sqrt{\left(1 - \frac{V_E^{'2}}{c^2}\right)}} \qquad (30)$$

Consistently with

$$\tau'' = \tau' \frac{\sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)}}{\left(1 + V_E' \frac{v_1}{c^2}\right)} = \Delta t_L' \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} \qquad (31)$$

The Lorentzian times $\Delta t_L''$ and $\Delta t_L'$, $\frac{l_2}{V_E'}$ and $\frac{l_1}{V_E'}$ are not the proper times τ' and τ''. This is what standard interpretation assume and Pirooz Mohazzabi and Qinghua Luo also affirm, questioning however the complete accuracy of the standard interpretation.

If we consider the "returning twin" with equal $\lceil V_E' \rceil$ but with, from (24)

$$\lceil v_2 \rceil < \lceil v_1 \rceil \quad d\tau'' > d\tau' \qquad 32)$$

This can be described with B´´ moving with $V_E' < 0$ from B´ to A´ (Fig.1) with

$$\tau'' = \frac{l_1}{\lceil V_E' \rceil} \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} = \Delta t_L' \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} \qquad (33)$$

and simultaneously A´ moving from A´´ to B´´ (Fig. 1) with

$$\tau' = \frac{l_2}{\lceil V_E' \rceil} \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} = \frac{l_1 \left(1 - \lceil V_E' \rceil \frac{v_1}{c^2}\right)}{V_E' \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)}} \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} \qquad (34)$$

$$\tau' = \tau'' \frac{\left(1 - \lceil V_E' \rceil \frac{v_1}{c^2}\right)}{\sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)}} \qquad (35)$$

Therefore, we obtain as expected for the two-way proper times τ'' *and* τ' (the addition of proper times for the trip to and fro)

$$\tau'' = \tau' \sqrt{1 - \frac{V_E'^2}{c^2}} = \frac{2 l_1}{|V_E'|} \sqrt{1 - \frac{V_E'^2}{c^2}} \qquad (36)$$

The standard formulation consider (17)

$$d\tau'' = dt_L' \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)}$$

the relation of the ageing of the twins [97] – this is the origin of the conundrum:

"The twin paradox is the consequence of the following thought experiment. System *O* is at rest and system *O´* is moving. Therefore, the clock in *O´* ticks slower than that in *O*. Thus, for example, if the two clocks are initially synchronized to read *t=t´=0*, after a while they may show *t=10* (some arbitrary unit of time) but *t´=6*. Therefore, an observer moving with system *O´* will be younger than that in system *O*. However, as seen by the observer in *O´*, she is at rest and system *O* is moving away from her. Therefore, according to the observer in *O´*, the observer in *O* should be younger. This is the foundation of the twin paradox, wich is stated as follows: Twin *A* is on Earth and twin *B* travels to a distant star with a speed close to the speed of light. Afterward, she returns to Earth with the same speed. When they reunite, according to twin *A*, twin *B* must be younger, but according to twin *B*, twin *A* must be younger "[97]. This is not so. The time dilation-like (time dilation is valid in relation to a preferred frame) and considered by the standard formulation the time dilation expression, valid reciprocally. This conundrum has been eliminated. We have shown how we can use this time dilation-like expression eliminating the twin paradox conundrum that standard formulation is unable to accomplish, since erroneously attribute the meaning of a reciprocal relation to the ageing of the twins. Therefore, we calculate the classic example of the twins whatever the frames considered. The twin that returns is the younger because the cumulative effect of the ageing is not reciprocal. Since the time dilation-like exist and can be used, originating the idea of "seeing the other twin ageing slower"- the origin of the conundrum. This cannot subsist because the relation between ageing is a relation between proper times. The time dilation-like expression is a relation between proper times only for the preferred frame.

Another example of the conundrum is " **Twins Approaching Each Other" [97]:**

"Consider twins, *A* and *B*, both initially at rest with respect to an inertial frame and separated by distance *d*. They synchronize their clocks according to the following method. When the clock of twin *A* reads $t_A=0$, she sends a light signal towards twin *B*. This light signal takes a time d/c to reach twin *B*. So, when twin *B* receives the light signal, she sets her clock to $t_B=d/c$ [2]. Then at a time that the two twins had previously agreed upon, they start moving towards each other with equal accelerations relative to an inertial frame *O* at their midpoint. The accelerations are very large but take place in a very short time (essentially a Dirac δ function) resulting in relativistic speeds. The two twins then start moving towards each other, each with a constant speed *v* relative to the other, as shown in Figure 2.

According to twin *A*, twin *B* is moving with speed *v*. Therefore, when they reach each other at the midpoint *O*, the clock of *B* should show a shorter time than the clock of *A*, *i.e.*, $t_B<t_A$. On the other hand, according to twin *B*, twin *A* is moving with speed *v*. Therefore, when they reach each other, the clock of *A* should show a shorter time than the clock of *B*, *i.e.*, $t_A<t_B$. In this situation, the system is completely symmetric;

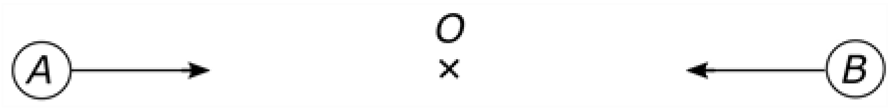

Figure 2. Twins *A* and *B* approaching each other with relative speed *v*.
Neither twin leaves her reference frame, and both have the same initial acceleration. Therefore, none of the suggested explanations can resolve the paradox in this case"

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