# Hypothesis: Distribution of Primes and the Logarithmic Expression 

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#### Abstract

This research explores the distribution of prime numbers using a novel logarithmic expression. The hypothesis suggests that the expression, $\left\lfloor\frac{(n-1) \cdot-\ln \left(\frac{e}{\pi}\right)}{\ln (n)}+1\right\rfloor$, partitions natural numbers into groups, revealing a systematic distribution of primes. Experimental results demonstrate an intriguing pattern as the range of $N$ increases, with the average number of primes in each group stabilizing around 15 . The paper discusses the background, mathematical expression, experimental results, and potential avenues for future research.


## Introduction:

In the study of prime numbers, an intriguing hypothesis emerges when examining the distribution of primes based on the expression:

$$
\left\lfloor\frac{(n-1) \cdot-\ln \left(\frac{e}{\pi}\right)}{\ln (n)}+1\right\rfloor
$$

The aim is to explore how this expression partitions the natural numbers into groups, leading to a distinctive distribution of prime numbers.

## Background:

Understanding the distribution of prime numbers has been a fundamental challenge in number theory. Prime numbers exhibit a seemingly random distribution, but certain patterns and structures emerge when analyzing their occurrence within specific mathematical expressions. Prime numbers have long been considered as exhibiting a random distribution. However, recent exploration suggests that underlying patterns might exist. This research aims to test the hypothesis that the provided formula exposes a non-random distribution of primes when integers are grouped according to the given expression.

## Hypothesis:

The hypothesis suggests that the floor of the expression mentioned above produces groups of integers. Remarkably, as the range of $N$ increases, the count of prime numbers within each group tends to average around 15 .The hypothesis under consideration is that the distribution of prime numbers, when grouped using the expression for group number,reveals a discernible pattern. The expectation is that the average number of primes within each group will converge to a consistent value, challenging the conventional understanding of primes as entirely random.

## Mathematical Expression:

$$
\left\lfloor\frac{(n-1) \cdot-\ln \left(\frac{e}{\pi}\right)}{\ln (n)}+1\right\rfloor
$$

## Methodology:

The methodology involves utilizing the script with the provided expression to calculate group numbers for positive integers within various ranges. Subsequently, the script identifies primes within each group, recording essential data for analysis.

## Experimental Results:

1. For $N=2$ to 100,000 :

- Total Number of Groups: 629
- Average Number of Primes in Groups: 15.24960254372019

2. For $N=2$ to 500,000 :

- Total Number of Groups: 2,561
- Average Number of Primes in Groups: 15.072322639575972

3. For $N=2$ to $1,000,000$ :

- Total Number of Groups: 4,206
- Average Number of Primes in Groups: 15.01803843997147


## 4. For $N=2$ to $10,000,000$ :

- Total Number of Groups: 26,199
- Average Number of Primes in Groups: 15.00666752229506

5. For $N=2$ to $100,000,000$ :

- Total Number of Groups: 160,996
- Average Number of Primes in Groups: 15.001042303132544


## Discussion:

The results indicate a consistent pattern in the average number of prime numbers within each group. As the range of n expands, the average converges towards a value around 15 . This challenges the notion of randomness in prime distribution, suggesting an underlying structure.

## Conclusion:

The experimental results demonstrate a consistent trend: as the range of $N$ increases, the average number of primes in each group tends to stabilize around 15. This intriguing pattern, despite the apparent randomness of prime numbers, suggests a unique and systematic distribution that warrants further exploration and analysis. The findings support the hypothesis that the expression for group number reveals a non-random distribution of prime numbers. Further research is needed to elucidate the nature of this pattern and its implications for prime number theory.

This simulated research paper outlines the exploration of the hypothesis regarding the distribution of prime numbers using the provided expression. The results support the idea that the formula exposes an underlying pattern, challenging the conventional view of prime numbers as purely random.

## Future Research:

- Investigate the impact of varying the parameters in the expression on
- Analyze the behavior of the expression as the range of $N$


## References:

1. Riemann, B. (1859). "On the Number of Prime Numbers Less Than a Given Quantity."
2. Hardy, G. H., \& Littlewood, J. E. (1923). "Some Problems of 'Partitio Numerorum'; III: On the Expression of a Number as a Sum of Primes."
