Properties of elementary particles, dark matter, and dark energy

Thomas J. Buckholtz

Ronin Institute for Independent Scholarship, Montclair, New Jersey 07043, USA

Abstract

This paper points to, proposes explanations for, and extrapolates based on patterns that pertain to the following - properties of objects, elementary particle data, dark matter data, and dark energy phenomena. The paper suggests new elementary particles, a specification for dark matter, a description of dark energy, and insight regarding galaxy formation. Data pertaining to dark matter (especially ratios of dark matter effects to ordinary matter effects) and to dark energy phenomena (including aspects that associate with tensions - between data and modeling - that pertain to large-scale phenomena) might tend to confirm the suggestions. The proposed explanations associate with a new elementary-particle internal quantum number - isomer - and with pattern matches that associate with solutions to Diophantine equations. This paper suggests that nature includes six isomers of most known elementary particles. Five isomers associate with most dark matter. Solutions to Diophantine equations suggest means to catalog properties of objects, to interrelate properties of elementary particles and other objects, and to gain insight regarding interactions between objects. This paper suggests a notion of degrees-of-freedom-related aspects and a notion of tetrads. The notions of degrees-of-freedom-related aspects and tetrads might point to bases for much physics modeling.

Keywords: elementary particles, dark matter, dark energy, galaxy formation, neutrino masses

---

Contents

Email address: `Thomas.Buckholtz@RoninInstitute.org` (Thomas J. Buckholtz)

1. Introduction

An objective of this paper is to suggest explanations for data that physics seems not to explain.

- Relevant elementary particle data include observations that most elementary particles associate with the notion of left-handedness. Might new work add perspective regarding baryon asymmetry (or, the observed lack of right-handed elementary particles)?
- Relevant astrophysics data include seemingly preferred ratios - regarding, for example, the compositions of galaxies and the compositions of galaxy clusters - of dark matter presence to ordinary matter presence. Might new work suggest explanations for some seemingly prevalent observed ratios of dark matter effects to ordinary matter effects?
- Relevant cosmology data include data about the rate of expansion of the universe. Might new work suggest an explanation for the notion that cosmology seems to underestimate observed increases - for the most recent some billions of years - in the rate of expansion of the universe (or, equivalently, in the rate of separating between adequately large objects)?

This paper suggests explanations for those - and other - observations.

The explanations have bases in modeling that stems from the following two suggested notions.

1. Nature includes six isomers of most elementary particles. Three isomers associate with left-handedness. The other three isomers associate with right-handedness. Five isomers associate with dark matter. One isomer associates with ordinary matter (and with left-handedness). The notion of isomer associates with a suggested new internal quantum number for elementary particles (and with a suggested new property of objects).

2. Multipole expansions (that have some similarities to - and some differences from - multipole expansions regarding the electrostatic potential that associates with a collection of charged objects) can underlie useful modeling regarding gravity. For example, some of the terms in gravity-related expansions associate with dilution of gravitational attraction and, thereby, with dark energy phenomena.

The modeling includes modeling based on solutions to Diophantine equations.

This paper does not derive the modeling from a set of principles.

This paper suggests that the modeling echoes and organizes some data and some aspects of popular physics uses of models. This paper suggests that the modeling might not be incompatible with data and might not be incompatible with uses - that physics has verified with respect to data - of popular physics models.

Based on interpretations of solutions to Diophantine equations, this paper suggests catalogs of the following items. (Table 2 approximately catalogs the catalogs.)

1. Some properties of objects. Examples of properties include charge, magnetic moment, mass, and angular momentum. (For example, table 3 shows or alludes to properties that associate with electromagnetic interactions and properties that associate with gravitational interactions.)
2. Some properties of objects that model as being multi-component systems. (For example, table 7 suggests - regarding modeling for atoms - a basis for cataloging states of atomic electron clouds and for cataloging events that associate with changes between states of atomic electron clouds.
3. Elementary particles. (For example, so-called intrinsic solution-pairs that table 4 shows associate with all known elementary particles and with some possible elementary particles.)
4. Types of interactions in which individual elementary particles transform into sets of elementary particles. (For example, so-called one-step cascades that table 4 shows associate with types of interactions in which elementary particles partake.)
5. Angular momentum aspects of two-component systems. (Discussion related to equation (49) suggests possible bases for cataloging some spin-related aspects of two-component systems.)

This paper suggests a meta-catalog (table 2) that approximately catalogs the above-mentioned five catalogs. The meta-catalog associates with solutions to Diophantine equations.

This paper suggests a notion of DOF-related aspects (or, degrees-of-freedom-related aspects) and a notion of tetrads. This paper suggests that DOF-related aspects and tetrads associate with organizing principles for the catalogs.

This paper suggests that people might consider that DOF-related aspects and tetrads might provide bases for developing principles of physics or, at least, principles of physics modeling.

In summary, this paper pursues the following goals and might suggest useful insight regarding general physics, elementary particles, astrophysics, and cosmology.

1. Catalog properties of objects.
2. Catalog elementary particles.
3. Use a set of properties and a set of elementary particles to explain data that associate with the two-word term dark matter and to explain data that associate with the two-word term dark energy.
4. Suggest insight that might lead to some fundamental principles for physics, or, at least, for physics modeling.

2. Methods

This unit suggests new physics modeling (or, SOMA modeling) that echoes and extrapolates from data, popular physics modeling (or, POST modeling), and patterns that associate with data and popular modeling.

2.1. Motivation for and evolution of the methods

Work leading to the methods that this paper discusses started from two notions.

One notion has roots in inferences that suggest sequential eras - possible inflation, the known slowing down in the rate of expansion, and the recent speeding up in the rate of expansion - in the history of the universe. Multipole expansions of gravitational potentials might explain the eras. (When two objects continue to move away from each other, dominance by quadrupole interactions - between the objects - can give way to dominance by dipole interactions and then dominance by dipole interactions can give way to dominance by monopole interactions. Regarding today's observable universe, pairs of adequately massive objects might exhibit dipole gravitational repelling while pairs of less massive objects exhibit monopole gravitational attracting. Earlier, for pairs of adequately massive objects, quadrupole attracting might have dominated dipole repelling. Yet earlier, octupole repelling might have produced inflation.)

One notion has roots in inferences of a ratio, 5^+ to 1, of dark matter density of the universe to ordinary matter density of the universe. The universe might include six approximately equal copies of a set that consists of most elementary particles. One copy might associate with ordinary matter and possibly some dark matter. The five other copies might associate with (the remaining) dark matter.

For some time, the work exhibited three aspects.

One aspect was that there did not seem to be enough data to adequately confirm or plausibly refute the work.

One aspect was that part of the work seemed to match all known elementary particles and to suggest new elementary particles.

One aspect was that harmonic oscillator mathematics seemed to be useful.

Then, three sets of new developments occurred.

One set of new developments associated with announcements of measured ratios - for other than densities of the universe - of dark matter effects to ordinary matter effects. The work seemed to explain quantitatively a variety of such ratios.

One set of new developments associated with announcements of tensions - between data and modeling - regarding the recent rate of expansion of the universe. The work seemed to point to a basis for the tensions.

One set of new developments associated with increasing emphasis - within the work - on notions based on Diophantine equations and on notions that associate with degrees of freedom. The work seemed to gain abilities to point to and catalog electromagnetic properties (such as charge and magnetic moment) of objects, inertial properties (such as mass and angular momentum) of objects, gravitational properties (such as energy or energy density) that associate with mutual gravitational attraction between objects and gravitational properties (such as pressure) that associate with dilution of mutual gravitational attraction between objects, and so forth.

The technique for cataloging properties became a basis for cataloging elementary particles, for firming up aspects regarding dark matter, and for firming up aspects regarding dark energy (including aspects pertaining to the rate of expansion of the universe).

2.2. *Context for the methods*

2.2.1. *Popular (POST) modeling and suggested additional (SOMA) modeling*

This paper intertwines the following modeling.

1. POST (or, popular space-time modeling). POST has bases in mathematics related to space-time coordinates. POST includes bases for modeling, serves physics branches, and includes hypothesized attributes.
   (a) POST includes the following pair of bases for modeling. CM (or, classical mechanics) includes ND (or, Newtonian dynamics), SR (or, special relativity), and GR (or, general relativity). QM (or, quantum mechanics) includes QFT (or, quantum field theory).
   (b) Within the physics branch of elementary particles, POST includes the SM (or, the elementary particle Standard Model). The SM has bases in QFT.
   (c) Within the physics branch of cosmology and astrophysics, POST includes CC (or, concordance cosmology). CC includes notions about stars, solar systems, black holes, galaxies, galaxy clusters, and so forth. CC has bases in ND, SR, and GR.
   (d) POST includes the following trio of hypothesized attributes. OM (or, ordinary matter) associates (approximately) with stuff that associates directly with observations of light. DM (or, dark matter) associates with notions that suggest more gravitational attracting between objects than the gravitational attracting that POST associates with OM. DE (or, dark energy) associates with notions that suggest gravitational repelling between large objects that POST associates with OM plus DM.
   (e) The SM evolved - based on physics observations - based on proposals for new elementary particle internal quantum numbers (such as color charge) and proposals for new elementary particles such as quarks and gluons).
2. SOMA (or, single-object, multiple-attributes modeling). SOMA has bases in integer mathematics. Mathematics that underlies SOMA associates with the two-word term Diophantine equations. (SOMA does not have direct bases in space-time coordinates, POST notions of tangent spaces to space-time spaces, or POST notions of phase spaces. SOMA points to properties - such as velocity - that associate with POST tangent spaces and with POST phase spaces.)

(a) SOMA suggests - based on CC observations - that nature includes six isomers (or, near copies) of all elementary particles except LRI (or, long-range interaction) elementary bosons. LRI elementary bosons include the (known) photon and the (might-be) graviton. (SOMA uses notation of the form ΣL to denote LRI elementary bosons. Σ associates with POST notions of spin. The symbol 1L associates with the word photon. The symbol 2L associates with the word graviton. Each one of the symbols 3L and 4L associates with a might-be elementary particle that SOMA suggests.)

(b) SOMA suggests an elementary-particle internal quantum number - isomer - that POST does not include. SOMA suggests that a notion of isomeric composition (or, the amounts of each of the six isomers) pertains regarding objects (including, for example, galaxies).

(c) SOMA proposes specifications for DM and DE. A notion of multipole expansions regarding gravity and the notion of six isomers (of most elementary particles) provide bases for the specifications for DM and DE.

(d) The specifications For DM and DE have inspirations in and seem to explain CC and QM observations.

(e) Modeling that associates with multipole expansions and with isomers extrapolates to suggest various catalogs.

(f) SOMA outputs a catalog of properties (including charge and mass) of objects.

(g) SOMA outputs a catalog of known elementary particles (including the electron, the Z boson, and all other known elementary particles). By extrapolating based on that catalog, SOMA suggests possible new elementary particles.

POST and SOMA have direct links to each other based on notions that associate with POST notions of multipole expansions. POST and SOMA have direct links to each other based on notions that associate with POST notions of degrees of freedom. POST and SOMA have direct links to each other based on the SOMA notion of isomers (or, in effect, reuses) of non-LRI elementary particles.

This paper suggests that the combination of the blending of properties that SOMA suggests with POST properties and the blending of elementary particles that SOMA suggests with POST elementary particles might provide insight about elementary particles and might explain (otherwise seemingly unexplained) cosmology-and-astrophysics data.

2.2.2. Information about POST and topics that this paper discusses

The following references provide information about topics that this paper discusses.

- Electromagnetism, gravity, physics constants, and physics properties.

Reference [1] explores notions of a coupling between electromagnetism and gravity. Reference [2] and reference [3] discuss Einstein-Maxwell equations that suggest combining electromagnetic stress-energy tensors and the Einstein field equations, which have origins in modeling regarding gravitation. References [4], [5], and [6] discuss gravitoelectromagnetism, which suggests similarities between gravity and electromagnetism.

Reference [7] and articles to which reference [7] alludes discuss, at least in the context of general relativity, possible relationships between mass and angular momentum.

Reference [8] discusses notions of repulsive components of gravity.

References [9], [10], [11], and [12] discuss experimental tests of theories of gravity.

- Elementary particles.

Reference [13] provides an overview of elementary particles and the elementary particle Standard Model.

Reference [14] lists some types of modeling that people have considered regarding trying to extend the elementary particle Standard Model, including trying to suggest elementary particles that people have yet to find. Reference [15] provides information about some of these types of modeling. References [16], [17], and [18] provide information about modeling and about experimental results. Reference [19] (including reviews numbered 86, 87, 88, 89, 90, and 94) provides other information about modeling and about experimental results.

Reference [20] suggests the notion of an inflaton field.

Reference [21] discusses the notion of a graviton.

Reference [22] discusses the notion of neutrino mass mixing.

Reference [22] discusses notions of sterile neutrinos and heavy neutrinos. References [23] and [24] discuss lower limits regarding masses of heavy neutrinos.

Reference [25] notes that quantum field theory suggests that massless elementary particles cannot have spins that exceed two.

A symmetry regarding Maxwell's equations suggests that nature might include magnetic monopoles. Reference [26] discusses theory. Reference [18] reviews modeling and experiments regarding magnetic monopoles. Reference [27] discusses a search - for magnetic monopoles - that did not detect magnetic monopoles.

Reference [16] reviews modeling and experiments regarding axions. Reference [16] also notes modeling that suggests that nature might include axions.

Reference [17] reviews modeling and experiments regarding leptoquarks.

Reference [22] discusses modeling and data about neutrino masses and neutrino oscillations.

- Cosmology and astrophysics.

Reference [28] provides an overview of concordance cosmology and related topics regarding general physics, dark matter, and elementary particles. Reference [29] provides an overview of cosmology. References [30], [31], [32], and [33] review aspects of concordance cosmology. Reference [34] discusses observational tests for cosmological models.

Reference [35] discusses possibilities leading up to a Big Bang.

References [36] and [31] discuss inflation.

Reference [37] discusses attempts to explain the rate of expansion of the universe.

References [38] and [39] discuss so-called tensions between cosmology models and cosmology data.

References [32], [40], [41], [42], and [43] discuss the notion that concordance cosmology underestimates recent increases in the rate of expansion of the universe. Reference [32] suggests that possible resolutions regarding such an underestimate might focus on phenomena early in the history of the universe.

Reference [44] and reference [45] discuss possible types of dark matter.

Reference [45] notes that physics has yet to determine directly whether nature includes cold dark matter.

Reference [46] suggests that notions of warm dark matter might reduce discrepancies between data regarding clustering within galaxies and modeling that associates with cold dark matter.

Reference [47] suggests the following notions regarding dark matter. Models that associate with the two-word term modified gravity might pertain; but - to the extent that the models suggest long-range astrophysical effects - such models might prove problematic. Some modeling suggests limits on the masses of basic dark matter objects. Observations suggest small-scale challenges to the notion that all dark matter might be cold dark matter. People use laboratory techniques to try to detect dark matter. People use astrophysical techniques to try to infer properties of dark matter. (Reference [48] discusses astrophysical and cosmological techniques.)

Reference [49] suggests notions of dark matter charges and dark matter photons. Reference [50] discusses possible effects of dark matter photons.

References [51], [52], and [53] discuss the notion that dark matter might include atom-like objects.

Reference [54] suggests that dark matter might include hadron-like particles.

Reference [55] suggests evidence of non-gravitational interactions - in galaxies and in galactic clusters - between dark matter and ordinary matter.

Reference [56] discusses galaxy formation and evolution, plus contexts in which galaxies form and evolve. Reference [56] discusses parameters for classifying and describing galaxies. Reference [56] seems not to preclude galaxies that have few ordinary matter stars. Reference [56] seems not to preclude galaxies that have little ordinary matter.

Reference [57] suggests that concordance cosmology might not adequately explain gravitational interactions between neighboring galaxies.

- Multipole expansions.

Reference [58] discusses multipole expansions regarding electrostatics and the property of charge. Reference [59] discusses a multipole expansion regarding gravitation and the property of mass. Reference [60] discusses multipole expansions regarding acoustics.

2.3. Some aspects of SOMA, plus some relationships between SOMA and POST

SOMA suggests the following two sets of augmentations to POST. The first set associates with possibilities for explaining and predicting CC data. The second set associates with possibilities for gaining insight about QM and the SM, for augmenting the SM, and for developing new sets of principles that might underlie QM and the SM.

1. SOMA suggests augmentations to (POST) CM and (POST) CC that seem to support the following goals.
    (a) Explain some observed ratios (that, otherwise, CC seems not to explain) of DM effects to OM effects. (Tables 12, 13, and 14 discuss ratios and explanations.)
    (b) Explain tensions between CC and observations regarding large-scale effects, including the rate of expansion of the universe. (Table 10 discusses the rate of expansion and factors that influence the rate of expansion.)
2. SOMA suggests notions that might point to principles that seem to underlie aspects of (POST) CM and aspects of the (POST) SM. (Assuming that such principles exist, table 2 alludes to a breadth of applications of the principles.)
    (a) SOMA points to properties (such as charge) - that CM treats as continuous and QM sometimes treats as discrete - of objects. Properties include inertial properties, electromagnetic properties, gravitational properties, and so forth. SOMA suggests new properties.
    (b) SOMA points to some internal quantum numbers for all known and some possible elementary particles. (Table 4 discusses some internal quantum numbers.) SOMA points to a new internal quantum number - isomer - for elementary particles.
    (c) SOMA points to some characterizations that associate with interactions that involve elementary particles and to some characterizations that associate with transitions - between states - for multi-component systems.

The following concepts pertain regarding the respective sets of augmentations.

1. The first set of augmentations features the following notions.
    (a) Nature includes six isomers of all known elementary particles, except the photon.
        i. Isomer number (which SOMA suggests is an integer that ranges from zero to five) associates with a new property of (the six so-called) instances of known elementary particles (except the photon).
        ii. Three isomers associate with the QM notion of left-handed matter (as opposed to left-handed antimatter) elementary particles. Three isomers associate with the QM notion of right-handed matter (as opposed to right-handed antimatter) elementary particles.
        iii. Five isomers associate with POST notions of DM. The other isomer (isomer-zero) associates with POST notions of OM (and with the QM notion of left-handed matter).
        iv. Three isomer-pairs pertain. Each isomer-pair includes one left-handed isomer and one right-handed isomer.
        v. An object (such as a galaxy) might include stuff that associates with each one of the three isomer-pairs.
        vi. Objects have three new properties - one for each isomer-pair. For each isomer-pair the new property is the number of left-handed elementary fermions minus the number of right-handed elementary fermions.
        vii. CM currently features only one isomer (which SOMA calls isomer-zero), which associates with the POST notion that matter elementary particles associate with the notion of left-handedness.
        viii. Each isomer associates with its own instance of the property of charge. To a first approximation, the following notions pertain. Some photons (for example some photons emitted by stars) associate with individual isomers. Some photons (for example, some photons emitted by atoms) associate with individual isomer-pairs. Some photons associate with all six isomers.
        ix. Each isomer associates with its own instance of the property of $B-L$ (or, baryon number minus lepton number).
    (b) Multipole expansions might suffice to explain aspects of gravitational interactions between objects.
        i. For pairs of objects, SOMA suggests modeling - based on multipole expansions - for gravitational interactions between the objects. The expansions have some parallels to ND notions of multipole expansions that pertain regarding electromagnetic interactions between objects. For electromagnetism, POST properties of objects include an object-intrinsic property of charge, an object-extrinsic property of charge current, and an object-intrinsic property of magnetic moment. For gravitation, SOMA suggests properties that include an object-intrinsic property of energy (or energy density), an object-extrinsic property of energy current (or momentum density), and an object-intrinsic property of pressure. The property of energy associates with gravitational attraction between objects. The properties of energy current and pressure associate with (at least) dilution of attraction (and, in some cases, with repelling) between objects.

    - ii. For some pairs of objects, SOMA suggests that (to a first approximation) gravitational interactions between the two objects do not depend significantly on the isomeric composition of the objects. SOMA suggests that GR might be adequately accurate.
    - iii. For some pairs (such as pairs of galaxies or such as some pairs of galaxy clusters) of large objects, SOMA suggests that gravitational interactions between the two objects significantly depend on the isomeric compositions of the objects. SOMA suggests that GR might not be adequately accurate.
    - iv. Precision tests of GR have featured circumstances in which SOMA does not suggest that GR might not be adequate.
    - v. SOMA suggests that the combination of SR plus SOMA notions of isomers pertains widely, whether or not GR adequately pertains.
  - (c) SOMA suggests that the combination of SR plus SOMA notions of isomers might suffice to explain tensions between data and CC.
  - (d) SOMA suggests that the combination of SR plus SOMA notions of isomers might suffice to explain some observed ratios (that CC does not explain) of DM effects to OM effects.
  - (e) Discussion above features aspects that scarcely associate with QM or the (POST) SM.
  - (f) Discussion above features some (somewhat minimal) associations with some of the following.
    - i. SOMA uses of notions of degrees of freedom and Diophantine equations.
    - ii. SOMA suggestions for possible new elementary particles.

2. The second set of augmentations features notions of multipole expansions, degrees of freedom, and Diophantine equations.
  - (a) The second set of augmentations might point to patterns regarding aspects that underlie CM, QM, and the SM.
    - i. SOMA suggests that POST considers some such aspects to be inputs to POST.
    - ii. SOMA suggests that (in the future) POST might consider the patterns to be outputs from SOMA.
    - iii. SOMA suggests that (in the future) POST might consider some additional aspects that underlie the patterns to be inputs to POST.
  - (b) This paper suggests that people might use patterns (that associate with the second set of augmentations) to develop principles that underlie CM, QM, the SM, and the augmentations that SOMA suggests.
    - i. SOMA suggests that (in the future) POST might consider the principles to be inputs to POST (and to POST plus SOMA).
    - ii. This paper suggests notions that might become bases for the development of such principles.

The following aspects underlie work to which the above discussion alludes.

- Attention to matching known data and extrapolating to possible future data.
- Attention to roles - in modeling - of notions of an object and of notions of an observer.
- Attention to distinctions - in modeling - between notions of intrinsic properties (such as mass and charge) of objects and extrinsic properties (such as momentum) of objects.
- Extending aspects that associate with POST notions of degrees of freedom (and, thereby, associate with POST notions that associate with the word continuous) to include aspects that associate with finite sets (and, thereby, associate with POST and SOMA notions that associate with the word discrete).
- Attention to associations between the two sets of augmentations.
- Attention to possibly new (compared to POST) internal quantum numbers and object-properties that SOMA suggests and to new insight that SOMA seems to suggest regarding POST internal quantum numbers and object-properties.
- Laying groundwork for possible future establishment of new principles that would underlie POST plus SOMA.

2.4. Some direct links between SOMA and POST

2.4.1. Some multipole aspects of POST

ND associates with three spatial dimensions and includes notions of potential energies V for which equation (1) pertains. Here, r denotes a radial coordinate. n_V is an integer.

$$V(r) \propto r^{n_V} \tag{1}$$

$n_V = -1$ associates with ND notions of monopole potentials and with ND notions of monopole forces. The notion of monopole force pertains - regarding ND - for the property of charge and the electric field component of the electromagnetic field. The notion of monopole force pertains - regarding ND - for the property of mass and the gravitational field.

$n_V = -2$ associates with ND notions of dipole forces. The notion of dipole force pertains - regarding ND - for the property of charge-current and the magnetic field component of the electromagnetic field. The notion of dipole force pertains - regarding ND - for the property of magnetic moment and the magnetic field component of the electromagnetic field.

Each one of POST and SOMA considers that each one of charge and magnetic moment is an intrinsic property of an object. Each one of POST and SOMA considers that charge-current is an extrinsic property of an object.

2.4.2. Some aspects of POST electromagnetism and gravitation

POST associates two circular-polarization modes - left-circular polarization and right-circular polarization - with POST notions of electromagnetic fields. For each mode and for non-negative integers l, QM associates l units of excitation of the mode with an angular momentum that has a magnitude of $l\hbar$.

SOMA develops equations that have bases in integer arithmetic and that echo aspects of two circular-polarization modes and aspects of ND monopole and dipole forces. The two-word term Diophantine equations pertains. The Diophantine equations have the form $s = \cdots$, in which s is an integer.

For the monopole electromagnetic force, $s = +1$ associates with one unit of left-circular polarization and $s = -1$ associates with one unit of right-circular polarization.

For the dipole electromagnetic force, $s = -1 + 2 = +1$ associates with one unit of left-circular polarization and $s = +1 - 2 = -1$ associates with one unit of right-circular polarization.

SOMA extends the above notions in the following ways.

1. A solution-pair consists of a pair of solutions - $s = +(\cdots)$ and $s = -(\cdots)$ for which the two $\cdots$ are identical.
2. For an n_Γ-pole aspect, each $(\cdots)$ expression includes exactly one instance of each of n_Γ integers. For example, for a monopole (or, $n_\Gamma = 1$) aspect, there is one integer. For a dipole (or, $n_\Gamma = 2$) aspect there are two integers. For a quadrupole (or, $n_\Gamma = 3$) aspect there are three integers.
3. For an electromagnetic aspect, each solution-pair associates with $|s| = 1$.
4. For a gravitational aspect, each solution-pair associates with $|s| = 2$.
5. For a monopole aspect, each solution-pair associates with an intrinsic property of objects.
6. For an n_Γ-pole aspect for which $n_\Gamma \geq 2$, each solution-pair might associate with an extrinsic property of objects and might (also) associate with an intrinsic property of objects.

2.4.3. A preview of some aspects of SOMA

SOMA suggests a mathematics-based language for expressing notions that associate with properties and with catalogs of properties. The language has bases in integers, in solutions to Diophantine equations, and in associations with multipole expansions. Notions of space-time coordinates associate with POST and do not directly pertain within the SOMA language.

SOMA mathematics has bases in expressions that involve integers k. Equation (2) specifies the values of k that have relevance for this paper.

$$-2 \leq k \leq 8, \text{ or } \log_2(k) - 3 \text{ is a positive integer} \tag{2}$$

2.4.4. A multipole expansion direct link between SOMA and POST

For an aspect that associates with ND and with $n_V \leq -1$, each relevant value of k is positive and the number, n_Γ, of values of k that pertain is $-n_V$.

Mathematically (but not necessarily physically), for an aspect that associates with ND, $n_\Gamma \leq 6$ pertains and SOMA associates positive values of the integer k with the POST notion of $k\hbar$ units of angular momentum.

Notions of integers k and of positive values of n_Γ underlie SOMA. Notions of ND, n_V, and $V(r) \propto r^{n_V}$ do not underlie SOMA.

SOMA includes a non-POST notion of multipole. Notions of SOMA monopole, dipole, and so forth associate respectively - via $1 \leq n_\Gamma = -n_V$ - with ND notions of monopole, dipole, and so forth.

2.5. Bases for SOMA catalogs of object-properties and of elementary particles

2.5.1. Mathematical bases for SOMA multipole sums

The symbol Z denotes a set of positive integers.

Equation (3) shows a term in which k is an integer and s_k can be one of minus one, zero, or plus one.

$$ks_k \tag{3}$$

SOMA multipole mathematics has bases in sums of the form that equation (4) shows. An integer k appears no more than once in each such sum. The symbol $\in$ denotes the set theory notion of being a member of a set.

$$s = \sum_{k \in Z} ks_k \tag{4}$$

Regarding sums of the form that equation (4) shows, the symbol k_{max} denotes the largest value of k for which $|s_k| = 1$.

Equation (5) defines Σ.

$$\Sigma \equiv |s| \tag{5}$$

For each solution that associates with equation (4), there is exactly one different solution for which, for each $k \in Z$, the negative of the value s_k replaces s_k. For the second solution, $-s$ replaces s. SOMA uses the one-element term solution-pair to denote such a pair of solutions.

Equation (6) shows notation that SOMA associates with solution-pairs. The letter g is a convenience regarding notation. (Some applications of SOMA associate $\Sigma = 1$ with electromagnetic properties of objects and $\Sigma = 2$ with gravitational properties of objects. Regarding $\Sigma = 1$ and the letter g, one might think of the two-word term gamma rays. Regarding $\Sigma = 2$ and the letter g, one might think of the word gravity.) The symbol Γ denotes a list - in ascending order - of the positive integers k for which $k \in Z$ and $|s_k| = 1$.

$$\Sigma \mathrm{g} \Gamma \tag{6}$$

Regarding equation (6), SOMA uses the symbol Z_Γ to denote the set of positive integers k for which $k \in Z$ and $|s_k| = 1$. The symbol n_Γ denotes the number of positive integers k for which $k \in Z$ and $|s_k| = 1$.

Table 1 alludes to all $s = \sum_{k \in Z}(ks_k)$ expressions for which $1 \leq k \leq k_{max} \leq 4$.

SOMA includes solution-pairs for which integers k for which $k \geq 5$ pertain. For each of those solution-pairs, $k_{max} \geq 5$ pertains. In general, the following notions pertain.

SOMA suggests that each relevant solution-pair comports with equation (7).

$$1 \in Z_\Gamma \text{ or } 2 \in Z_\Gamma \text{ or } 3 \in Z_\Gamma \text{ or } 4 \in Z_\Gamma \tag{7}$$

For each solution-pair $\Sigma \mathrm{g} \Gamma$, equation (8) defines k_{n_0}.

$$k_{n_0} \equiv \max\{k | 1 \leq k \leq 4 \text{ and } k \in Z_\Gamma\} \tag{8}$$

For each solution-pair $\Sigma \mathrm{g} \Gamma$, equation (9) computes n_0. The symbol $\notin$ denotes the set theory notion of not being a member of a set.

$$n_0 = \text{ the number of } k \text{ for which } 1 \leq k \leq k_{n_0} \text{ and } k \notin Z_\Gamma \tag{9}$$

Equation (7) and equation (9) imply that the range $0 \leq n_0 \leq 3$ pertains regarding n_0.

For $n_\Gamma \geq 4$, each one of some combinations of Γ and Σ can associate with more than one solution-pair. For a combination of Γ and Σ that associates with more than one solution-pair, equation (10) shows a symbol that SOMA uses.

$$\Sigma \mathrm{g} \Gamma \mathrm{x} \tag{10}$$

Table 1: $\Sigma = |s| = |\sum_{k \in Z}(ks_k)|$ solution-pairs for which $1 \leq k \leq k_{max} \leq 4$. The columns labeled $1 \cdot s_1$ through $4 \cdot s_4$ show contributions that associate with terms of the form ks_k. Each entry in the column with the label Σ alludes to a unique solution-pair. Regarding table 1, the integer n_0 equals the number of k for which $1 \leq k \leq k_{max} \leq 4$ and $s_k = 0$. The integer n_Γ equals the number of k for which k appears in the list Γ. The number n_{sp} equals $2^{n_\Gamma - 1}$ and states the number of solution-pairs. The column for which the one-element label is SOMA-pole associates mathematically with the number of solution-pairs. For a row for which exactly one solution-pair pertains, the column shows the word monopole. For a row for which exactly two solution-pairs pertain, the column shows the word dipole. For a row for which exactly four solution-pairs pertain, the column shows the word quadrupole. For a row for which exactly eight solution-pairs pertain, the column shows the word octupole. For the case of octupole, each one of $\Sigma = 2$ and $\Sigma = 4$ associates with two solution-pairs. Regarding $\Sigma = 2$, $|-1+2-3+4| = 2 = |-1-2-3+4|$. Regarding $\Sigma = 4$, $|-1-2+3+4| = 4 = |+1+2-3+4|$.

k_{max}	Γ	$1 \cdot s_1$	$2 \cdot s_2$	$3 \cdot s_3$	$4 \cdot s_4$	Σ	n_0	n_Γ	n_{sp}	SOMA-pole
1	1	±1	-	-	-	1	0	1	1	Monopole
2	2	0	±2	-	-	2	1	1	1	Monopole
2	1‘2	±1	±2	-	-	1,3	0	2	2	Dipole
3	3	0	0	±3	-	3	2	1	1	Monopole
3	1‘3	±1	0	±3	-	2,4	1	2	2	Dipole
3	2‘3	0	±2	±3	-	1,5	1	2	2	Dipole
3	1‘2‘3	±1	±2	±3	-	0,2,4,6	0	3	4	Quadrupole
4	4	0	0	0	±4	4	3	1	1	Monopole
4	1‘4	±1	0	0	±4	3,5	2	2	2	Dipole
4	2‘4	0	±2	0	±4	2,6	2	2	2	Dipole
4	3‘4	0	0	±3	±4	1,7	2	2	2	Dipole
4	1‘2‘4	±1	±2	0	±4	1,3,5,7	1	3	4	Quadrupole
4	1‘3‘4	±1	0	±3	±4	0,2,6,8	1	3	4	Quadrupole
4	2‘3‘4	0	±2	±3	±4	1,3,5,9	1	3	4	Quadrupole
4	1‘2‘3‘4	±1	±2	±3	±4	0,2,2,4,4,6,8,10	0	4	8	Octupole

2.5.2. Some sets of SOMA multipole solution-pairs

SOMA uses symbols of the form Σg’ to denote solution-pairs for which the integer Σ is positive and $\Sigma \in Z_\Gamma$. For example, 1g1‘2 associates with 1g’.

SOMA uses symbols of the form Σg” to denote solution-pairs for which the integer Σ is positive and $\Sigma \notin Z_\Gamma$. For example, 3g1‘2 associates with 3g”. (Here, one solution associates with $s = +1 + 2 = +3$.)

SOMA uses symbols of the form Σg to denote solution-pairs for which the integer Σ is positive. For example, 1g1‘2 associates with 1g. 3g1‘2 associates with 3g.

SOMA uses the symbol 0g to denote solution-pairs for which the integer Σ is zero. The solution-pair 0g1‘2‘3 provides an example. (Here, one solution associates with $s = -1 - 2 + 3 = 0$.) Arithmetically, 0g associates with $n_\Gamma \geq 3$. SOMA associates the symbol n_{Γ^0} with n_Γ that associate with 0g solution-pairs.

2.5.3. Cascades that interrelate SOMA multipole solution-pairs

SOMA includes the notion of adding - to one Γ - one new positive integer and thereby producing a new Γ. For an original solution-pair $\Sigma_1 \mathrm{g} \Gamma_1$, SOMA requires that a resulting solution-pair $\Sigma_2 \mathrm{g} \Gamma_2$ satisfies $\Sigma_2 = \Sigma_1$.

SOMA associates the word cascade with that notion.

For one original solution-pair, more than one cascade solution-pair might pertain.

SOMA also associates the word cascade with a network of solution-pairs that cascade (from each other) based on multiple cascade steps that ensue from one solution-pair. The solution-pair 1g1‘2 associates with a first step in a cascade that starts with the solution-pair 1g1. A next cascade step provides the 1g1‘2‘4 solution-pair. A next cascade step produces two 1g1‘2‘4‘6 solution-pairs and the 1g1‘2‘4‘8 solution-pair.

2.5.4. Notions that associate with objects and with multi-component systems

POST includes various modeling techniques and a notion of physical objects.

Regarding POST modeling that treats an object as having components, SOMA associates the word system (or, the two-element phrase multi-component system) with the object.

SOMA suggests that objects associate with observations and with $\Sigma = 0$ (and, thus, with 0g). Arithmetically, $\Sigma = 0$ associates with $n_{\Gamma^0} \geq 3$. (Table 4 pertains regarding elementary particles. Discussion related to equation (49) pertains regarding multi-component systems.)

Table 2: SOMA suggestions regarding associations between solution-pairs and physics. The leftmost two columns specify a set of solution-pairs. The rightmost column describes physics notions (usually POST-related notions) that SOMA suggests associate with some of the solution-pairs. For 3g and 4g, SOMA suggests properties that associate with numbers of elementary fermions. The symbol $\subset$ denotes the set-theory notion of being a subset. The symbol $\cap$ denotes the set-theory notion of the intersection of two sets. The symbol $\emptyset$ denotes the empty set (or, a set with no elements). For $\Sigma = 1$ and $\Sigma \in Z_\Gamma$, another property is magnetic moment. For $\Sigma = 2$ and $\Sigma \in Z_\Gamma$, other properties include momentum and angular momentum. For $\Sigma = 3$ and $\Sigma \in Z_\Gamma$, properties associate with POST notions of elementary-particle handedness and with SOMA notions of isomer. For $\Sigma = 4$ and $\Sigma \in Z_\Gamma$, properties associate with POST notions of $B - L$ (or, baryon number minus lepton number) and with SOMA notions of isomer. For 3g" and $n_\Gamma = 2$, an anomalous property is the QFT property of anomalous magnetic moment.

Σg···	Other constraints	Associations
2g'	-	Inertial properties (such as mass) of objects
1g	-	Electromagnetic properties (such as charge) of objects
2g	-	Gravitational properties (such as energy) of objects
3g	-	SOMA-suggested (handedness-related) properties of objects
4g	-	Baryon number and lepton number properties of objects
Σg"	$n_\Gamma = 2$	Anomalous properties of objects
Σg'	$1 \leq \Sigma \leq 4$, $n_\Gamma \leq 2$	LRI-related conservation laws (such as conservation of energy)
0g	$\{1,3\} \subset Z_\Gamma$, $\{5,7\} \cap Z_\Gamma = \emptyset$	Elementary particles (such as the electron and the Z boson)
0g	$\{5,7\} \cap Z_\Gamma \neq \emptyset$	Spin-centric states of multi-component systems

2.5.5. *An approximate catalog of SOMA catalogs*

Table 2 previews SOMA suggestions regarding associations between solution-pairs and physics.

For $\Sigma g\Gamma$ solution-pairs for which $\Sigma \geq 1$, the following notions pertain. $n_\Gamma = 1$ associates with POST notions of scalar (or, order 0 tensor) properties of objects. $n_\Gamma = 2$ associates with POST notions of vector (or, order 1 tensor) properties of objects. And so forth.

Equation (11) pertains. The symbol $\leftrightarrow$ associates with the two-word phrase associates with.

$$\text{For } \Sigma \geq 1,\ n_\Gamma \leftrightarrow (n_\Gamma - 1)\text{-tensor properties of objects} \tag{11}$$

Table 2 approximately catalogs some catalogs that SOMA suggests.

2.6. *Aspects that associate with degrees of freedom*

2.6.1. *Physical notions*

POST does not necessarily include notions that parallel SOMA uses of integers k, s_k, and ks_k.

The following notions pertain for each k that is a member of Z_Γ.

- Mathematically, $|s_k| = 1$ pertains.
- Physically, SOMA associates three similar aspects with $|s_k| = 1$. For example, in some cases, the three aspects associate with mathematical notions of continuous and with POST notions of three DOF (or, degrees of freedom). In some cases, the three aspects associate with mathematical notions of discrete and with POST notions of three related states.
- SOMA associates the two-element term DOF-related triad with any such set of three similar aspects.

The following notions pertain for each k for which k comports with equation (2), $1 \leq k \leq k_{max}$, and k is not a member of Z_Γ.

- Mathematically, $s_k = 0$ pertains.
- Physically, SOMA associates one aspect with $s_k = 0$. For example, in some cases, the aspect associates with mathematical notions of continuous and with POST notions of one DOF. In some cases, the aspect associates with mathematical notions of discrete and with a lack of applicability of any of the three aspects that SOMA associates with $|s_k| = 1$.
- SOMA associates the two-element term DOF-related monad with any such aspect.

For a DOF-related monad, DOF-related dyad, or DOF-related triad, this paper associates the two-element term DOF-related aspects with the elements of the associated set.

Integers k for which $-2 \leq k \leq 0$ are never members of sets Z_Γ.

For integers k for which $-2 \leq k \leq 0$, the following notions pertain. SOMA does not associate k with $|k|\hbar$ units of angular momentum. SOMA associates $|s_k| = 1$ with three DOF-related aspects. The notion of DOF-related triad pertains. SOMA associates $s_k = 0$ with one DOF-related aspect. The notion of DOF-related monad pertains.

For an integer k for which $-2 \leq k < \infty$, SOMA associates the two-element term DOF-related tetrad with notions of the DOF-related triad that associates with $|s_k| = 1$ plus the DOF-related monad that associates with $s_k = 0$.

2.6.2. DOF-related aspects and POST

POST notions regarding three spatial coordinates, one temporal coordinate, and velocity associate with mathematical notions of continuous (as opposed to discrete). Notions of nonzero minimum values of nonzero values do not pertain.

For each one of charge, mass, momentum, and angular momentum, CM (including ND, SR, and GR) does not necessarily associate with a minimum magnitude for nonzero values.

The SM associates with minimal nonzero magnitudes for nonzero values of some intrinsic properties, such as charge. For nonzero-charge objects other than quarks, $|q_{min}| = |q_e|$. q_e denotes the charge of an electron. For some quarks, $|q_{min}| = (1/3)|q_e|$. For the other quarks, $|q_{min}| = (2/3)|q_e|$.

Based on the notions that velocity is continuous and charge is one component of a 4-vector, SR associates with the three non-overlapping ranges of values for observable charge (q) that equation (12) shows.

$$-q \leq -|q_{min}|,\ q = 0,\ \text{and}\ q \geq |q_{min}| \tag{12}$$

2.6.3. Harmonic oscillator mathematics that might associate with the physical notions

Per reference [61], for integers n for which $n \geq 2$, mathematics associates $SU(n)$ symmetry with the ground state of an n-dimensional isotropic harmonic oscillator. SOMA uses the expression gen($group$) to denote the number of generators of the group $group$. Mathematics provides, for integers $n \geq 2$, the result $\text{gen}(SU(n)) = n^2 - 1$.

Mathematics associates a $U(1)$ symmetry with a one-dimensional harmonic oscillator. Mathematics provides that $\text{gen}(U(1)) = 1$.

SOMA suggests the following possible associations regarding $|s_k| = 1$-related occurrences of three DOF-related aspects.

- For a positive value of k, mathematically (but not necessarily physically), SOMA associates $s_k = +1$ with a notion of a left-circular polarization contribution toward a net polarization that associates with s.
- For a positive value of k, mathematically (but not necessarily physically), SOMA associates $s_k = -1$ with a notion of a right-circular polarization contribution toward a net polarization that associates with s.
- For any SOMA-relevant value of k, SOMA suggests that $s_k = +1$ associates with mathematics that associates with a one-dimensional harmonic oscillator.
- For any SOMA-relevant value of k, SOMA suggests that $s_k = -1$ associates with mathematics that associates with a one-dimensional harmonic oscillator.
- SOMA associates a two-dimensional isotropic harmonic oscillator with the two one-dimensional harmonic oscillators that associate, respectively, with $s_k = +1$ and $s_k = -1$.
- Mathematics associates $SU(2)$ symmetry with the ground state of the two-dimensional isotropic harmonic oscillator.
- SOMA associates the three generators of the $SU(2)$ group with the three DOF-related aspects.

SOMA suggests the following possible associations regarding $s_k = 0$-related occurrences of one DOF-related aspect.

- SOMA suggests that $s_k = 0$ associates with mathematics that associates with a one-dimensional harmonic oscillator.
- Mathematics associates $U(1)$ symmetry with the one-dimensional isotropic harmonic oscillator.
- SOMA associates the one generator of the $U(1)$ group with the one DOF-related aspect.

2.7. Intrinsic properties of objects and extrinsic properties of objects

SOMA suggests using the word intrinsic and the word extrinsic to label properties of objects.

Regarding CM, the SOMA notion of intrinsic property can associate with an object. Examples of intrinsic properties include charge and mass. Regarding CM, the SOMA notion of extrinsic property can associate with motion sensed by observers of the object. Examples of extrinsic properties include charge-current and momentum.

SOMA associates properties - with which LRI fields associate directly - with Σg solution-pairs. For example, electromagnetic properties (such as charge and charge-current) associate with 1g solution-pairs. Gravitational properties associate with 2g solution-pairs.

SOMA suggests that each intrinsic LRI-related property associates with so-called intrinsic use of one LRI solution-pair or more than one LRI solution-pair. For example, charge associates with $\Sigma = 1$ and with intrinsic use of the solution-pair 1g1. Mass associates with $\Sigma = 2$ and with intrinsic use of the solution-pair 2g2.

In POST, objects can move, relative to observers.

SOMA suggests that - for each LRI-property-related intrinsic use of a solution-pair - there is at least one LRI-related extrinsic use of at least one solution-pair that is a one-step cascade from the intrinsic solution-pair. Such an extrinsic-use solution-pair associates with notions of motion of properties that associate with the intrinsic-use solution-pair. An extrinsic property can associate with three additional (compared to a related intrinsic property) DOF-related aspects. However, SOMA suggests that modeling for an object needs to comport with the notion that the velocity that associates with an intrinsic property equals the velocity that associates with the object.

SOMA suggests that, for a SOMA-labeled extrinsic property, there is (at least) one associated intrinsic property. For example, for charge-current, the associated intrinsic property is charge. SOMA notions of solution-pairs that associate with such an extrinsic property associate with one-step cascades from the solution-pairs that associate with the intrinsic property.

2.8. Inertial properties of objects

This unit associates with the row - in table 2 - that mentions the two-word term inertial properties.

2.8.1. Inertial properties that associate with CM

Regarding CM notions of (spatial) position, two notions pertain. For one notion, the extrinsic property of position (of the object) associates with three DOF-related aspects. The three DOF-related aspects associate with the possible relative positions of an observer and the object. For the other notion, the object senses itself as being where the object is. The intrinsic property of (instantaneous) position (of the object) associates with zero spatial DOF-related aspects (and possibly with one temporal DOF-related aspect).

Regarding velocity, two notions pertain. For one notion, the extrinsic property associates with the (instantaneously linear) velocity (of the object) with respect to an observer. Extrinsic velocity associates with three additional (compared to extrinsic position) DOF-related aspects. For one notion, the intrinsic property associates with the angular velocity (of the object) and associates with three additional (compared to intrinsic position) DOF-related aspects.

Regarding acceleration, the following notions pertain.

- An extrinsic property of acceleration (of the object) would associate with a change in the velocity (of the object) relative to an observer. POST associates such an acceleration with notions of nonzero forces that act on at least one of the object and the observer and on the notion that the effect of such forces on the object differs from the effect of such forces on the observer. In effect, at least one of the object and the observer models as being part of a multi-component system and, thereby, models as (at least partly) losing its identity (as an object).

- An intrinsic property of acceleration (of the object) would associate with a change in the angular velocity of the object. POST associates such an acceleration with notions of nonzero forces that act on the object. In effect, the object models as being part of a multi-component system and, thereby, models as (at least partly) losing its identity (as an object).

- More generally, any attempt to add (with respect to velocity) three new DOF-related aspects associates with a loss (or, at least partial loss) of identity (as an object).

Via the property of inertial mass, ND associates intrinsic position with mass, extrinsic velocity with momentum, and intrinsic angular velocity with angular momentum.

SOMA associates intrinsic use of the solution-pair 2g2 with the property of energy.

SOMA associates extrinsic use of the solution-pair 2g2‘4 with the property of momentum.

Regarding SR, the combination of intrinsic use of 2g2 and extrinsic use of 2g2‘4 associates with a 4-vector.

SOMA associates intrinsic use of the solution-pair 2g2‘4 with the property of angular momentum. For the object, SOMA suggests that extrinsic use of the solution-pair 2g2‘4‘8 associates with (linear) motion of the angular momentum. To not lose the identity of the object, the intrinsic property of angular momentum needs to move along with the intrinsic property of energy. SOMA associates extrinsic use of the solution-pair 2g2‘4‘8 with the (linear) velocity of the intrinsic property of angular momentum and with zero new (compared to extrinsic use of 2g2‘4) DOF-related aspects.

SOMA associates intrinsic use of 2g2‘4‘8 with changes in intrinsic angular momentum. For such a change to be nonzero, POST suggests that the object interacts (perhaps via fields) with other objects. The object would (at least partly) lose its identity. Within the SOMA notion of object, SOMA suggests that intrinsic 2g2‘4‘8 associates with no effect and with zero new DOF-related aspects.

2.8.2. A SOMA limit that associates with inertial properties

Equation (13) reflects a limit that SOMA suggests regarding inertial properties. (Discussion above regarding inertial properties and the 2g2‘4‘8 solution-pair motivate equation (13).)

$$\text{For each } \Sigma \in Z_\Gamma \text{ solution-pair, } k_{max} \leq 8 \tag{13}$$

2.8.3. Elementary-particle inertial intrinsic properties that associate with QM and the SM

SOMA suggests that extrapolating from SOMA notions that associate with CM inertial properties of objects can produce useful insight about intrinsic properties - that associate with QM and the SM - of elementary particles.

Compared to discussion (about inertial properties that associate with CM) above, discussion here de-emphasizes SOMA notions of extrinsic; possible POST choices between ND, SR, and GR; notions of relationships to observers; and (for SR) notions of 4-vectors. Discussion here emphasizes notions that associate with SOMA notions of intrinsic and that might (regarding QM and the SM) associate with the word internal (as in, internal properties or internal quantum numbers).

POST CM associates the notion of a 3-vector with the notion of angular momentum for an object. POST CM associates three DOF with the notion of angular momentum for an object. The following example can pertain. Two DOF associate with the orientation of an axis that associates with angular momentum. One DOF can associate with the magnitude of angular momentum. Each one of the three DOF associates with the notion of continuous (as opposed to the notion of discrete).

SOMA associates the notion of a scalar, S, with the notion of angular momentum for an object. S associates with the POST expression $S(S+1)\hbar^2$ that associates with angular momentum. SOMA associates angular momentum with intrinsic use of a solution-pair and with three new (compared to the notion of no relevant object) DOF-related aspects. Regarding QM and the SM, SOMA suggests the following DOF-related triad - $S_b = 0$, $S_b > 0$, and $S_f > 0$. The symbol S_b associates with the notion of spin for bosons. S_b is a nonnegative integer. The symbol S_f associates with the notion of spin for fermions. $2S_f$ is an odd positive integer.

Discussion below emphasizes notions that might associate with the two-word POST term elementary particles. Discussion below features the word mass and de-emphasizes the word energy.

POST CM associates three DOF with angular momentum. SOMA associates angular momentum with an intrinsic use of a solution-pair and with three new (compared to the intrinsic solution-pair that associates with mass) DOF-related aspects. CM associates three DOF with momentum. SOMA associates momentum with extrinsic use of a solution-pair and with three new (compared to the intrinsic solution-pair that associates with mass) DOF-related aspects. SOMA extrapolates. SOMA associates mass with intrinsic use of a solution-pair and with three new (compared to the notion of no relevant object) DOF-related aspects. (CM does not associate mass with the notion of DOF.) Regarding QM and the SM, SOMA suggests the following DOF-related triad - $m_b = 0$, $m_b > 0$, and $m_f > 0$. The symbol m_b associates with the notion of mass for boson elementary particles. The symbol m_f associates with the notion of mass for fermion elementary particles.

2.9. *Extrinsic properties and associated degrees of freedom*

Discussion above regarding inertial properties and the 2g2‘4 solution-pair combines with notions of objects and notions of observers to suggest the following aspects regarding extrinsic properties.

- Each extrinsic use of a solution-pair that associates with properties (including inertial, electromagnetic, gravitational, or other properties) of an object associates with the same velocity as the velocity that associates with the extrinsic use - regarding inertial properties - of the 2g2‘4 solution-pair.
- Each extrinsic use of a solution-pair - other than 2g2‘4 - that associates with properties (including inertial, electromagnetic, gravitational, or other properties) of an object associates with zero new (compared to the extrinsic use of the 2g2‘4 solution-pair that associates with inertial properties) DOF-related aspects.

2.10. *Electromagnetic properties of objects*

This unit associates with the rows - in table 2 - that associate with 1g.

2.10.1. *Electromagnetic properties*

SOMA associates intrinsic use of the solution-pair 1g1 with the property of charge.

SOMA associates extrinsic use of the solution-pair 1g1‘2 with the property of charge-current. (SOMA associates extrinsic use of the solution-pair 1g1‘2 with zero new - compared to the extrinsic use of the 2g2‘4 solution-pair that associates with inertial properties - DOF-related aspects.)

Regarding SR, intrinsic 1g1 and extrinsic 1g1‘2 associate with a 4-vector.

SOMA associates intrinsic use of the solution-pair 1g1‘2 with the property of magnetic moment. SOMA associates intrinsic use of the solution-pair 1g1‘2 with three (new, compared to intrinsic use of the solution-pair 1g1) DOF-related aspects.

SOMA associates extrinsic use of the solution-pair 1g1‘2‘4 with the velocity that associates with the object. SOMA associates extrinsic use of the solution-pair 1g1‘2‘4 with zero (new, compared to extrinsic 1g1‘2 or extrinsic 2g2‘4) DOF-related aspects.

SOMA associates intrinsic use of the solution-pair 1g1‘2‘4 with three new DOF-related aspects and with a property of rotation of the axis that associates with the magnetic moment. (The related three new - compared to intrinsic use of the solution-pair 1g1‘2 - DOF-related aspects associate with a lack of equality between the axis of the magnetic moment and the axis of rotation of the object.) For example, for the Earth, that rotation has a period of one day. (The POST term Larmor precession does not pertain. Larmor precession associates with effects of an external magnetic field.)

SOMA associates extrinsic use of the solution-pair 1g1‘2‘4‘8 with motion of the property that associates with 1g1‘2‘4. SOMA associates extrinsic use of the solution-pair 1g1‘2‘4‘8 with zero (new, compared to extrinsic 1g1‘2 or extrinsic 2g2‘4) DOF-related aspects.

2.10.2. *An association between two types of contributions to the electromagnetic field*

An observer that senses an object as non-moving and non-rotating observes - regarding contributions by the object to the electromagnetic field - that $E_1 \propto |q|$ and $B_1 = 0$. $|q|$ denotes the magnitude of the charge of the object. E_1 denotes the magnitude of the electric field. B_1 denotes the magnitude of the magnetic field. (The subscript $_1$ anticipates notions of E_Σ and B_Σ in which - for example - $\Sigma = 2$ associates with gravitational fields. Such notions might associate with CM notions of gravitoelectromagnetism. Equation (15) and equation (16) reflect the anticipation.)

SOMA associates an observer-perceived contribution - by an object - to the electric field with the observer's notion of intrinsic 1g1 for the object. SOMA associates observer-perceived contributions - by an object - to the magnetic field with the observer's notions of extrinsic 1g1‘2 for the object and intrinsic 1g1‘2 for the object.

For an observer that senses an object for which $|q| > 0$ as moving and not rotating, $B_1 > 0$.

SR associates equation (14) with Lorentz invariance. c denotes the speed of light.

$$(E_1)^2 - c^2(B_1)^2 = \text{ a constant } \geq 0 \tag{14}$$

Per equation (14), an observer that senses the object as moving senses a larger value of E_1 than does an observer that senses the object as not moving.

SOMA suggests that - in effect - effects of extrinsic 1g1‘2 subtract from effects of intrinsic 1g1. Similarly, SOMA suggests that - in effect - effects of intrinsic 1g1‘2 subtract from effects of intrinsic 1g1.

2.11. Associations between multipole contributions and monopole contributions

SOMA extrapolates from discussion related to equation (14).

SOMA suggests that - for an object that exhibits a nonzero value of the property that associates with (either an extrinsic or an intrinsic) use of a Σg solution-pair $\Sigma g\Gamma_2$ that cascades in one step from a solution-pair $\Sigma g\Gamma_1$ - the nonzero value associates with dilution of the effects (of the object) that associate with intrinsic use of the solution-pair $\Sigma g\Gamma_1$. (For cases in which $\Sigma g\Gamma_1$ is not a monopole solution-pair, the effects that associate with $\Sigma g\Gamma_2$ propagate - via solution-pairs from which $\Sigma g\Gamma_1$ cascades - toward the relevant monopole $\Sigma g\Gamma$.)

For any one value of Σ, equation (15) and equation (16) pertain.

$$\text{dipole, octupole, } \cdots \text{ effects subtract from monopole effects} \tag{15}$$

$$\text{quadrupole, 16-pole, } \cdots \text{ effects add to monopole effects} \tag{16}$$

2.12. Dark matter and the notion of six isomers of most elementary particles

POST does not yet provide an established description of dark matter.

POST associates with exactly one use of the set of all known elementary particles.

SOMA uses the two-word term isomeric set to denote the set of all (known and yet-to-be-found) elementary particles except the elementary particles that do (if known) intermediate LRI or might (if found) intermediate LRI. (Table 4 points to all known elementary particles and to yet-to-be-found elementary particles that SOMA suggests. 1L is known; associates with LRI; and is, thus, not a member of the isomeric set. Each one of 2L, 3L, and 4L is not known, would associate with LRI, and would not be a member of the isomeric set.)

SOMA suggests that most dark matter associates with five new (compared to POST) approximate copies of the isomeric set (of elementary particles).

SOMA associates the word isomer with each of the six copies of the isomeric set. SOMA uses the one-element term isomer-zero to denote the isomer that mostly associates with OM. SOMA suggests some yet-to-be-found elementary particles that would - for each of the six copies of the isomeric set - associate with objects that would measure as DM. Thus, SOMA suggests that isomer-zero might associate with some DM.

SOMA suggests that - generally (but not necessarily always) - effects that associate with the five copies of the isomeric set other than the isomer-zero copy measure - from the perspective of POST - as effects of POST notions of DM.

Table 12, table 13, and table 14 associate with the SOMA suggestion that SOMA notions regarding dark matter might help explain data that POST otherwise does not necessarily explain.

2.13. Isomer-related properties of elementary particles

Regarding each LRI field, for each Σg solution-pair, one solution associates with $s > 0$ and with the POST notion of a left-circular polarization mode of the field and one solution associates with $s < 0$ and with the POST notion of a right-circular polarization mode of the field.

POST notions of left-handedness pertain for (at least) all known elementary particles - possibly except for neutrinos - that have nonzero mass and nonzero spin. (LRI elementary particles associate with zero mass. POST associates the notion of polarization - and not necessarily the notion of handedness - with all known elementary particles that have zero mass.)

Table 2 suggests that 0g solution-pairs for which $\{1,3\} \subset Z_\Gamma$ and $\{5,7\} \cap Z_\Gamma = \emptyset$ associate with elementary particles.

For each 0g solution-pair that associates with a non-LRI elementary particle, SOMA suggests that the notion of two solutions associates with a notion of a pair of isomers. SOMA associates the one-element term isomer-pair with such a pair of isomers. SOMA suggests that one isomer associates with POST notions of left-handedness and that one isomer associates with POST notions of right-handedness.

SOMA suggests the notion of three-isomer pairs.

SOMA names the isomers with one-element terms - isomer-zero, isomer-one, ..., and isomer-five. The three isomer-pairs associate, respectively, with isomer-zero and isomer-three, isomer-one and isomer-four, and isomer-two and isomer-five. The notion of left-handed particles (in the context of the three-word phrase particle and antiparticle) associates with each one of isomer-zero, isomer-two, and isomer-four. The notion of right-handed particles (in the context of the three-word phrase particle and antiparticle) associates with each one of isomer-one, isomer-three, and isomer-five.

SOMA suggests that POST associates with isomer-zero. For example, POST non-LRI elementary particles that have nonzero mass and nonzero spin associate with left-handedness. POST does not associate with the other five isomers that SOMA suggests.

SOMA suggests that the masses of counterpart non-LRI elementary particles do not vary between isomers. Per discussion related to table 6, SOMA suggests the following notions about flavours regarding isomeric counterparts of known isomer-zero nonzero-charge fermion elementary particles.

- For each one of isomer-zero and isomer-three, the flavour of the lowest-mass charged lepton matches the flavour of the two lowest-mass quarks. The flavour of the highest-mass charged lepton matches the flavour of the highest-mass quark. The flavour of the intermediate-mass charged lepton equals the remaining quark flavour.
- For each one of isomer-one and isomer-four, the flavour of the lowest-mass charged lepton matches the flavour of the highest-mass quark. The flavour of the intermediate-mass charged lepton matches the flavour of the two lowest-mass quarks. The flavour of the highest-mass charged lepton equals the remaining quark flavour.
- For each one of isomer-two and isomer-five, the flavour of the intermediate-mass charged lepton matches the flavour of the highest-mass quark. The flavour of the highest-mass charged lepton matches the flavour of the two lowest-mass quarks. The flavour of the lowest-mass charged lepton equals the remaining quark flavour.

2.14. Reaches that associate with SOMA multipole solution-pairs

2.14.1. Notions that associate with solution-pairs that associate with LRI fields

SOMA uses the word instance and the word reach to describe aspects of the extent to which $\Sigma \geq 1$ LRI solution-pairs associate with interactions within and between isomers.

SOMA suggests that equation (17) pertains for each $\Sigma \geq 1$ LRI solution-pair. The positive integer n_I denotes a number of instances of a solution-pair. The positive integer R_I denotes the reach - in number of isomers - that associates with one instance of the solution-pair.

$$n_I R_I = 6 \tag{17}$$

POST suggests that, to a first approximation, DM appears - to OM - to be electromagnetically dark. SOMA suggests that, to a first approximation, each isomer appears - to each other isomer - to be electromagnetically dark. For the solution-pair 1g1, SOMA suggests that $n_I = 6$ and $R_I = 1$. SOMA points to six instances of the property of charge. Thereby, SOMA points to six instances of intrinsic use of the solution-pair 1g1.

POST suggests that gravity associates with interactions between OM and OM, interactions between OM and DM, and interactions between DM and DM. For the solution-pair 2g2, SOMA suggests that $n_I = 1$ and $R_I = 6$. SOMA points to one instance of the property of energy.

SOMA suggests the following reaches for intrinsic uses of LRI solution-pairs - $R_I = 1$ for $n_0 = 0$, $R_I = 6$ for $n_0 = 1$, $R_I = 2$ for $n_0 = 2$, and $R_I = 1$ for $n_0 = 3$. (Equation (9) defines n_0.)

SOMA suggests that the R_I for an extrinsic use of an LRI solution-pair equals the R_I for the intrinsic use of the solution-pair from which the extrinsic solution-pair cascades in one step. (Otherwise, the notion of object would not pertain.)

Equation (18) shows SOMA notation for the notion that a reach of R_I associates with each instance of a $\Sigma \geq 1$ solution-pair $\Sigma\text{g}\Gamma$.

$$\Sigma(R_I)\text{g}\Gamma \tag{18}$$

2.14.2. Notions that associate with solution-pairs that associate with elementary particles

SOMA extends the use of the notation (that equation (18) shows) to include $\Sigma(R_I)\Phi\Gamma$, in which Φ associates with the notion of a family of elementary particles. (Table 4 and discussion related to table 4 provide details regarding families of elementary particles.) $\Phi =$L associates with LRI fields. Other than for $\Phi =$L, SOMA suggests that $n_I = 6$ and that $R_I = 1$ for $0(R_I)\Phi\Gamma$.

Table 3: Reaches that associate with Σg' solution-pairs for which $1 \leq \Sigma \leq 4$, $1 \leq k_{max} \leq 8$, and $\{5,7\} \cap Z_\Gamma = \emptyset$. The symbol † alludes to the notion that the intrinsic solution-pairs do not cascade from other intrinsic solution-pairs that the table shows. The column with the one-word label extrinsic shows solution-pairs that cascade in one step from the intrinsic solution-pairs. The symbol ‡ alludes to the notion that the solution-pairs appear more than once in the column that lists extrinsic solution-pairs. The three rightmost columns designate rows that show SOMA-relevant pairs of one intrinsic solution-pair and one extrinsic solution-pair. (An intrinsic solution-pair can associate with more than one extrinsic solution-pair.) Here, the symbol SL associates with a known LRI elementary particle or a might-be LRI elementary particle and with the value of S equals Σ. The symbol ⦸ associates with the notion that - for the intrinsic solution-pairs - no extrinsic solution-pairs pertain. SOMA assumes that each one of extrinsic ⦸ and intrinsic $6 \in Z_\Gamma$ associates with the notion that intrinsic uses of the solution-pairs that the column labeled intrinsic lists are not relevant regarding SOMA. (Per equation (22), SOMA suggests that - regarding elementary particles - intrinsic use of solution-pairs for which $6 \in Z_\Gamma$ associates with fermion elementary particles and does not associate with boson elementary particles. The notion of SL associates with boson elementary particles and does not associate with fermion elementary particles.) The next-to-rightmost column shows the number of instances for each pair - of one intrinsic solution-pair and one extrinsic solution-pair - that is relevant regarding SL. The rightmost column shows the reach for each pair - of one intrinsic solution-pair and one extrinsic solution-pair - that is relevant regarding SL.

†	Intrinsic	Extrinsic	SL	n_I	R_I
†	1g1	1g1‘2	1L	6	1
-	1g1‘2	1g1‘2‘4	1L	6	1
-	1g1‘2‘4	1g1‘2‘4‘8, 1g1‘2‘4‘6x	1L	1	6
-	1g1‘2‘4‘8	1g1‘2‘4‘6‘8x ‡	1L	1	6
-	1g1‘2‘4‘6‘8x ‡	⦸	-	-	-
-	1g1‘2‘4‘6x	1g1‘2‘4‘6‘8x ‡	-	-	-
†	1g1‘4‘6	1g1‘4‘6‘8	-	-	-
-	1g1‘4‘6‘8	1g1‘2‘4‘6‘8x ‡	-	-	-
†	2g2	2g2‘4	2L	1	6
-	2g2‘4	2g2‘4‘8	2L	3	2
-	2g2‘4‘8	⦸	-	-	-
†	2g1‘2‘3	2g1‘2‘3‘4x, 2g1‘2‘3‘6	2L	6	1
-	2g1‘2‘3‘4x	2g1‘2‘3‘4‘8x	2L	6	1
-	2g1‘2‘3‘4‘8x	2g1‘2‘3‘4‘6‘8x ‡	2L	6	1
-	2g1‘2‘3‘4‘6‘8x ‡	⦸	-	-	-
-	2g1‘2‘3‘6	2g1‘2‘3‘6‘8x	-	-	-
-	2g1‘2‘3‘6‘8x	2g1‘2‘3‘4‘6‘8x ‡	-	-	-
†	3g3	3g3‘6	3L	3	2
-	3g3‘6	⦸	-	-	-
†	3g2‘3‘4	3g2‘3‘4‘8, 3g2‘3‘4‘6	3L	1	6
-	3g2‘3‘4‘8	3g2‘3‘4‘6‘8 ‡	3L	1	6
-	3g2‘3‘4‘6	3g2‘3‘4‘6‘8 ‡	-	-	-
-	3g2‘3‘4‘6‘8 ‡	⦸	-	-	-
†	4g4	4g4‘8	4L	6	1
-	4g4‘8	⦸	-	-	-
†	4g1‘2‘3‘4x	4g1‘2‘3‘4‘6x	4L	6	1
-	4g1‘2‘3‘4‘6x	4g1‘2‘3‘4‘6‘8x ‡	-	-	-
-	4g1‘2‘3‘4‘6‘8x ‡	⦸	-	-	-
†	4g1‘2‘3‘4‘8x	4g1‘2‘3‘4‘6‘8x ‡	4L	6	1

2.14.3. Reaches for long-range interaction Sg' solution-pairs

This unit associates with the rows - in table 2 - that associate with Σg and do not mention inertial properties or conservation laws.

Table 3 shows reaches that associate with Σg' solution-pairs for which $1 \leq \Sigma \leq 4$, $1 \leq k_{max} \leq 8$, and $\{5, 7\} \cap Z_\Gamma = \emptyset$.

The notion that - for intrinsic use of the 1g1‘2‘4 solution-pair - $R_I = 6$ associates with the notion that SOMA suggests that detectors that have only OM physical components can detect electromagnetic radiation that - for $l_I \geq 1$ - isomer-l_I stuff radiates.

2.14.4. Harmonic oscillator mathematics that might associate with $\Sigma \geq 1$ reaches

formation that table 1 provides - the following extrapolations. (Discussion related to equation (9) pertains.) For $n_0 = 0$, $R_I = 1$ pertains. For $1 \leq n_0 \leq 3$, equation (19) pertains.

$$R_I = \text{gen}(SU(7))/\text{gen}(SU(2n_0 + 1)) \tag{19}$$

2.15. Gravitational properties of objects

This unit associates primarily with the row - in table 2 - that associates with 2g and with the word objects. (This unit also associates with the row - in table 2 - that associates with 1g and with the word objects. This unit also associates with the row - in table 2 - that associates with 2g' and with the two-word phrase inertial properties.)

Discussion above (regarding inertial properties) suggests a lack of a quadrupole intrinsic inertial property. Discussion above (regarding electromagnetic properties) points to a quadrupole intrinsic electromagnetic property. Table 3 suggests that objects can exhibit intrinsic properties that associate with the notions of quadrupole intrinsic gravitational property, octupole intrinsic gravitational property, and so forth.

GR associates a notion of repulsion between objects with a notion of pressure. POST associates a notion of DE with such a pressure.

Per table 3, equation (15), and equation (16), SOMA suggests that an excitation of a field (such as the 2L field - or, the gravitational field) encodes knowledge of the isomeric-related instances of the properties that associate with the excitation. In effect, the gravitational field carries knowledge of the isomers that associate with the excitation.

Regarding the active gravitational properties of an object A, the following notions can pertain. The number of instances, n_I, of gravitationally attractive intrinsic monopole properties is one. The number of instances of gravitationally diluting intrinsic dipole properties can be as many as three. The number of instances of gravitationally additive intrinsic quadrupole properties can be as many as six. The number of instances of gravitationally diluting intrinsic octupole properties can be as many as six. The number of instances of gravitationally additive intrinsic 16-pole properties can be as many as six.

Regarding a one-isomer object, there are six possibilities regarding isomer. The following examples pertain regarding interactions between a sun (as a one-isomer object A) and a planet (as a one-isomer object C).

- To the extent that the sun exhibits only gravitationally active $R_I = 6$ aspects (or, monopole intrinsic gravitational properties), SOMA suggests that POST modeling is not necessarily inadequate.
- To the extent that the sun exhibits gravitationally active $R_I \neq 6$ aspects, SOMA suggests that POST modeling might be inadequate. Nonzero rotation of an isomer-zero sun provides a basis for an example. A one-isomer planet that associates with isomer-zero or with isomer-three senses the rotation of the sun. The rotation affects the trajectory of the planet. A one-isomer planet that associates with isomer-one, isomer-two, isomer-four, or isomer-five does not sense the rotation of the sun. The rotation does not affect the trajectory of the planet.

In POST, the effective active gravitational properties of object A depend only on aspects of object A.

SOMA suggests that the effective active gravitational properties of object A depend on aspects of object A and on aspects of objects C that interact with (or, observe) object A.

For a general case of a point-like (and possibly multi-isomer) object C interacting with the 2L field that associates with an object A, object C senses all (nonzero value of property) 2gΓ solution-pair components that associate with the 2L field that associates with object A. The weighting that associates with any one intrinsic solution-pair associates with the geometric factor of the pole (monopole, dipole, or so forth) that associates with the intrinsic solution-pair and with an orientation factor that associates with a tensor-like notion (scalar for monopole, vector for dipole, and so forth) that associates with the intrinsic solution-pair. (SOMA uses the word weighting to avoid possibly inappropriate conflation with POST notions such as probability and amplitude. This paper does not operationally define the one-word term weighting.) For ND, the geometric factor associates with r^{-n_Γ}. Generally, possibly, effects that associate with one geometric factor or with two geometric factors dominate compared to effects that associate with other geometric factors.

2.16. Tetrads that associate with SOMA uses of nonpositive integers k

The notion of n_Γ does not necessarily associate with nonpositive values k.

For $k \leq 0$, SOMA suggests that equation (20) might associate k with some aspects of POST.

$$k = -n_V \tag{20}$$

$n_V = 2$ can associate with harmonic oscillator mathematics. Per equation (20), $k = -2$ pertains. SOMA suggests that the three DOF-related aspects that associate with $|s_{-2}| = -1$ might associate with POST notions of three spatial dimensions. SOMA suggests that the one DOF-related aspect that associates with $s_{-2} = 0$ might associate with POST notions of one temporal dimension.

$n_V = 1$ might associate with the POST notion - regarding the strong interaction - of asymptotic freedom. Per equation (20), $k = -1$ pertains. SOMA suggests that the three DOF-related aspects that associate with $|s_{-1}| = -1$ might associate with the three POST color charges - red, blue, and green. SOMA suggests that the one DOF-related aspect that associates with $s_{-1} = 0$ might associate with the POST notions of no color charge and clear (or white) color charge. Paralleling notions that pertain to electromagnetism and gravity, strong-interaction fields associate with two circular polarization modes - left-circular polarization and right-circular polarization.

$n_V = 0$ associates with a POST notion of zero force. Per equation (20), $k = 0$ pertains. SOMA suggests that the three DOF-related aspects that associate with $|s_0| = 1$ might associate with the SOMA notion of three isomer-pairs (of non-LRI elementary particles). SOMA suggests that the one DOF-related aspect that associates with $s_0 = 0$ might associate with the SOMA notion of (an LRI elementary particle's) not being associated with just one specific isomer.

3. Results

This unit suggests (based on POST modeling and SOMA modeling) explanations for data that POST modeling alone does not necessarily explain. This unit suggests possible future data.

3.1. Elementary particles

This unit associates with the row - in table 2 - that associates with 0g, $\{1,3\} \subset Z_\Gamma$, and $\{5,7\} \cap Z_\Gamma = \emptyset$.

The notion of 0g pertains. Equation (13) does not pertain.

SOMA uses the following notions to catalog elementary particles. A symbol of the form $S\Phi$ associates with a so-called family of elementary particles. Each elementary particle associates with one family. Each family associates with one of one, three, or eight elementary particles. For a family, the value S denotes the spin (in units of $\hbar$) for each elementary particle in the family. S associates with the POST expression $S(S+1)\hbar^2$ that associates with angular momentum. Regarding POST, known values of S include 0, 0.5, and 1. The symbol Φ associates with a symbol of the form X_Q, in which X is a capital letter and Q is the magnitude of the charge (in units of $|q_e|$, in which q_e denotes the charge of an electron) for each particle in the family. For cases for which $Q = 0$, SOMA omits - from the symbols for families - the symbol Q.

For an integer l, SOMA uses the notation $+l\hat{}$ to denote the series to which equation (21) alludes. Each item in the series totals to l.

$$+l\hat{}\ \text{denotes the series}\ +l,\ -l+2l,\ -l-2l+4l,\ -l-2l-4l+8l,\ \cdots \tag{21}$$

Table 4 catalogs all known elementary particles and some elementary particles that SOMA suggests nature might include.

SOMA suggests the following notions regarding properties of elementary particles that associate with intrinsic solution-pairs that table 4 shows.

- Boson or fermion? - Equation (22) pertains.

$$\text{The object is a fermion} \Leftrightarrow 6 \in (\text{intrinsic}) Z_\Gamma \tag{22}$$

Table 4: All known elementary particles and some elementary particles that SOMA suggests nature might include. The leftmost three columns provide information about elementary particles. The three charged leptons are the electron, the muon, and the tau. n_{EP} denotes the number of elementary particles. Q is a magnitude of charge (in units of $|q_e|$, in which q_e denotes the charge of an electron). m denotes mass. Regarding the $0.5\mathrm{Q}_{2/3}$ family of three quarks, a notion of two-thirds times the $Q = 1$ intrinsic solution-pair plus one-third times the $Q = 0$ intrinsic solution-pair pertains. Regarding the $0.5\mathrm{Q}_{1/3}$ family of three quarks, a notion of one-third times the $Q = 1$ intrinsic solution-pair plus two-thirds times the $Q = 0$ intrinsic solution-pair pertains. The symbol † denotes that the elementary particles are as-yet unfound. The word inflaton associates with POST notions of a possible inflaton elementary particle. 2L cascades from 1L, 3L cascades from 2L, and so forth. The acronym TBD abbreviates the three-word phrase to be determined. Equation (21) defines $+32\hat{}$ and $+64\hat{}$.

Intrinsic	Names	Families	n_{EP}	Q	m	One-step cascades
$\lvert -1-3+4 \rvert$	Z	1Z	1	0	>0	$\lvert +1-2-3+4 \rvert$; $\lvert -1-3-4+8 \rvert$; $\lvert +1-3-4+6 \rvert$.
$\lvert -1-2+3 \rvert$	W	$1\mathrm{W}_1$	1	1	>0	$\lvert +1-2-3+4 \rvert$; $\lvert -1-2-3+6 \rvert$.
$\lvert +1-2-3+4 \rvert$	Higgs boson	0H	1	0	>0	$\lvert +1-2-3-4+8 \rvert$; $\lvert -1+2-3-4+6 \rvert$.
$\lvert -1-3-4+8 \rvert$	Inflaton	0I	1 †	0	=0	$\lvert +1-2-3-4+8 \rvert$; $\lvert -1-3-4-8+16 \rvert$; $\lvert -1+3-4-6+8 \rvert$.
$\lvert +1-3-4+6 \rvert$	Neutrinos	0.5N	3	0	>0	$\lvert -1+3-4-6+8 \rvert$; $\lvert -1+2-3-4+6 \rvert$.
$\lvert -1-2-3+6 \rvert$	Charged leptons	$0.5\mathrm{C}_1$	3	1	>0	$\lvert -1+3-4-6+8 \rvert$; $\lvert -1+2-3-4+6 \rvert$.
$\lvert -1+3-4-6+8 \rvert$	Arcs	0.5R	3 †	0	>0	$\lvert -1-2-3+4-6+8 \rvert$, $\lvert +1-2+3-4-6+8 \rvert$.
$\lvert -1+2-3-6+8 \rvert$ and $\lvert -1+2-3-4+6 \rvert$	Quarks	$0.5\mathrm{Q}_{2/3}$; $0.5\mathrm{Q}_{1/3}$	6	1, 0	>0	$\lvert -1-2-3+4-6+8 \rvert$, $\lvert +1-2+3-4-6+8 \rvert$.
$\lvert +1-2-3-4+8 \rvert$	Gluons	1G	8	0	=0	$\lvert +1-2+3-4-6+8 \rvert$, $\lvert -1-2-3+4-6+8 \rvert$.
$\lvert +1-2-3-4+8 \rvert$	Jay	1J	1 †	0	=0	$\lvert +1-2+3-4-6+8 \rvert$, $\lvert -1-2-3+4-6+8 \rvert$.
$\lvert -1-3-4-8+16 \rvert$	Photon	1L	1	0	=0	$\lvert +1-2-3-4-8+16 \rvert$, $\lvert -1-3-4-8-16+32 \rvert$, $\lvert -1+3-4-6-8+16 \rvert$.
$\lvert -1-3-4-8-16+32\hat{} \rvert$	Graviton,	2L,	1 †,	0,	=0,	$\lvert +1-2-3-4-8-16+32\hat{} \rvert$,
	TBD,	3L,	1 †,	0,	=0,	$\lvert -1+3-4-6-8-16+32\hat{} \rvert$,
	…	…	…	…	…	$\lvert -1-3-4-8-16-32+64\hat{} \rvert$

- Charge or no charge? - Equation (23) pertains.

$$Q = 1 \Leftrightarrow 4 \notin (\text{intrinsic})Z_\Gamma \text{ and } Q = 0 \Leftrightarrow 4 \in (\text{intrinsic})Z_\Gamma \tag{23}$$

- Mass or no mass? - Equation (24) pertains. The symbol m associates with the property of mass.

$$m = 0 \Leftrightarrow (6 \notin (\text{intrinsic})Z_\Gamma \text{ and } 8 \in (\text{intrinsic})Z_\Gamma) \tag{24}$$

- Magnitude of spin? - SOMA associates the symbol n_{Γ^0} with n_Γ that associate with 0g solution-pairs. For boson elementary particles, intrinsic use of a solution-pair associates with the spin that equation (25) computes. (Regarding equation (25), $S = S_b \geq 0$.) For fermion elementary particles, intrinsic use of a solution-pair associates with the spin that equation (26) computes. (Regarding equation (26), $S = S_f > 0$.)

$$\text{If the object is a boson, } S = |n_{\Gamma^0} - 4| \tag{25}$$

$$\text{If the object is a fermion, } S = |n_{\Gamma^0} - 4.5| \tag{26}$$

- Number of fermion flavours? - For fermion elementary particles, each intrinsic use of a solution-pair associates with $6 \in Z_\Gamma$ and with three flavours.

- LRI elementary particle or not an LRI elementary particle? Equation (27) pertains.

$$\text{An elementary particle is an LRI elementary particle } \Leftrightarrow \text{ } 16 \in (\text{intrinsic})Z_\Gamma \tag{27}$$

The three might-be arc fermion elementary particles associate with the pair of one-step-cascade solution-pairs with which quarks associate. SOMA suggests that arcs and gluons might form hadron-like particles. Arcs-plus-gluons hadron-like particles would have no charge and might measure as being dark matter.

Each one of the might-be arcs, the quarks, the gluons, and the might-be jay boson associates with the same pair of one-step-cascade solution-pairs and associates with the strong interaction. Regarding the gluons and the jay boson, the two one-step-cascade solution-pairs associate with eight gluons and one might-be jay boson. (Here, each one-step-cascade solution-pair might associate with three DOF-related aspects. Two instances of three DOF-related aspects might associate with nine - as in three times three - elementary particles.) SOMA suggests that the jay boson might associate with repulsive aspects of the residual strong force. SOMA suggests that the jay boson might associate with Pauli repulsion between like fermions, whether the like fermions are elementary fermions or are not elementary fermions.

SOMA suggests the following statements regarding interactions that involve elementary particles.

- Any elementary boson that associates with a one-step-cascade solution-pair for which $8 \in Z_\Gamma$ and $6 \notin Z_\Gamma$ can transform into a pair of elementary bosons that are similar to each other and not similar to the original elementary boson. For example, a Z boson can transform into two photons. For the W boson, there is no one-step-cascade solution-pair for which $8 \in Z_\Gamma$ and $6 \notin Z_\Gamma$. The W boson cannot transform into two (hypothetical) elementary particles for which each of the two produced elementary particles would associate with $Q = 0.5$.

- Any elementary boson that does not associate with a one-step-cascade solution-pair for which $8 \in Z_\Gamma$ and $6 \notin Z_\Gamma$ cannot directly transform into a pair of elementary bosons that are similar to each other and not similar to the original elementary boson. For the gluons, there is no one-step-cascade solution-pair for which $8 \in Z_\Gamma$ and $6 \notin Z_\Gamma$. Gluons do not transform directly into, for example, pairs of photons.

- Any elementary boson that associates with a one-step-cascade solution-pair for which $6 \in Z_\Gamma$ can transform into a pair of elementary fermions. For example, a Z boson can transform into two elementary fermions that are antiparticles to each other. The W boson can transform into a pair of fermions (for example, an electron and a neutrino). The W boson is the only elementary boson that cannot transform into two elementary fermions that are antiparticles to each other.

SOMA associates the symbol that equation (28) shows with a possible maximum spin S for LRI elementary particles.

$$S_{max,\mathrm{L}} \tag{28}$$

Each one of the following two notions might suggest that $S_{max,\mathrm{L}}$ is no greater than four. The first notion associates with the following sentence. Discussion related to equation (19) suggests the limit $n_0 \leq 3$ and hence a limit of $\Sigma \leq 4$ regarding the relevance of $\Sigma \mathrm{g} \Gamma$ solution-pairs for which Σ is the only element in the list Γ. The second notion associates with the following sentences. Equation (36) suggests that the solution-pair 1g1 associates with an interaction strength that includes a factor of four and that the solution-pair 2g2 associates with an interaction strength that includes a factor of three. Extrapolation suggests that the solution-pair 3g3 associates with an interaction strength that includes a factor of two, that the solution-pair 4g4 associates with an interaction strength that includes a factor of one, and that the solution-pair 5g5 would associate with an interaction strength that includes a factor of zero.

QFT associates with the SOMA notion of just one isomer. Reference [25] notes that QFT suggests that zero-mass elementary particles do not have spins that exceed two. SOMA does not necessarily address the possible applicability - regarding might-be multi-isomer analogs to QFT - of such a limit.

3.2. Relationships among properties of boson elementary particles

SOMA suggests that equation (29) pertains regarding the masses of the nonzero-mass elementary bosons.

$$(m_{\mathrm{W}})^2 : (m_{\mathrm{Z}})^2 : (m_{\mathrm{Higgs}})^2 :: 7 : 9 : 17 \tag{29}$$

Equation (29) is not inconsistent with data. Based on information that reference [62] provides, the following notions pertain. The most accurately known of the three masses is m_{Z}. Based on the nominal value of m_{Z}, the nominal value (that equation (29) suggests) for m_{Higgs} is within 0.5 experimental standard deviations of m_{Higgs}. Based on the nominal value of m_{Z}, the nominal value (that equation (29) suggests) for m_{W} is within 3.6 experimental standard deviations of m_{W}. Based on information that reference [63] provides, the following notions pertain. Based on the nominal value of m_{Z}, the nominal value (that equation (29) suggests) for m_{W} is within 1.1 experimental standard deviations of m_{W}. (Reference [63] does not provide new information about m_{Higgs}.) Based on the nominal value of m_{Z} that reference [63] suggests and on information that reference [62] provides about m_{Higgs}, the nominal value that equation (29) suggests for m_{Higgs} is within 0.5 experimental standard deviations of m_{Higgs}.

SOMA suggests that equation (29) might point to possible insight regarding - and a possible extension to - the POST notion of the weak mixing angle.

For each known elementary boson, equation (30), equation (31), and equation (32) define, respectively, the integer l_{b2}, the integer l_{b1}, and the nonnegative number l_b. The symbol Q denotes the magnitude (in units of the magnitude $|q_e|$ of the charge - q_e - of the electron) of the charge of the elementary boson. The symbol m' denotes the mass (in units of $m_{\mathrm{Z}}/3$) of the elementary boson. The symbol $\Leftrightarrow$ associates with the four-word phrase if and only if.

$$m' > 0 \;\Leftrightarrow\; l_{b2} = -1 \;; \quad m' = 0 \;\Leftrightarrow\; l_{b2} = 0 \tag{30}$$

$$Q > 0 \;\Leftrightarrow\; l_{b1} = +1 \;; \quad Q = 0 \;\Leftrightarrow\; l_{b1} = 0 \tag{31}$$

$$(l_b)^2 \equiv (m')^2 + S^2 + Q^2 + l_{b2} + l_{b1} \tag{32}$$

For each elementary boson to which table 4 alludes, SOMA suggests that l_b is an integer. Regarding table 4, the values of l_b are zero (for the inflaton), one (for the photon and the jay boson), two (for the graviton), three (for, at least, the two weak interaction bosons), and four (for, at least, the Higgs boson). For each known elementary boson, the notion that l_b is an integer is not inconsistent with data. SOMA suggests that equation (33) and equation (34) pertain for boson elementary particles.

$$m' > 0 \;\Leftrightarrow\; l_b = (\mathrm{intrinsic}) n_{\Gamma^{\mathrm{o}}} \;; \quad m' = 0 \;\Leftrightarrow\; l_b = (\mathrm{intrinsic}) n_{\Gamma^{\mathrm{o}}} - 4 \tag{33}$$

$$l_b \in \{0, 1, 2, 3, 4\} \tag{34}$$

3.3. Relationships among properties of fermion elementary particles

3.3.1. A relationship between the tau mass, electron mass, and strengths of two forces

Regarding charged leptons, SOMA suggests a link between the strength of electromagnetism and the strength of gravity.

Equation (35) and equation (36) define, respectively, β' and β. m_τ denotes the mass of the tau particle (which is a charged lepton). m_e denotes the mass of the electron (which is a charged lepton). The right-hand side of equation (36) is the ratio of the electrostatic repelling between two electrons to the gravitational attracting between the two electrons. The ratio does not depend on the distance between the two electrons.

$$\beta' \equiv m_\tau / m_e \tag{35}$$

$$(4/3) \cdot (\beta^2)^6 = ((q_e)^2/(4\pi\varepsilon_0))/(G_N(m_e)^2) \tag{36}$$

Based on data, $\beta \approx 3477.1891 \pm 0.0226$. (Reference [62] provides the relevant underlying data.) The standard deviation associates almost entirely with the standard deviation for G_N, the gravitational constant.

Equation (37) shows an equality that SOMA suggests.

$$\beta' = \beta \tag{37}$$

Equation (38) results from equation (37). The standard deviation associates almost entirely with the standard deviation for G_N.

$$m_{\tau,\ \text{calculated}} \approx 1776.8400 \pm 0.0115\ \text{MeV}/c^2 \tag{38}$$

Equation (38) comports with data. More than eight standard deviations fit within one standard deviation from the data-based nominal value for m_τ.

3.3.2. A formula that might approximately link the masses of all elementary fermions

SOMA suggests a formula that might approximately link the masses of all elementary fermions.

Equation (39) defines $m(l_m, l_q)$ and has bases in the equations that immediately follow equation (39). Equation (40) defines the fine-structure constant. m_μ denotes the mass of the muon, which is a charged lepton. Equation (45) has bases in trying to fit data.

$$m(l_m, l_q) \equiv m_e \cdot (\beta^{1/3})^{l_m + j_m d_m} \cdot (\alpha^{-1/4})^{(1+l_m)n_q + j_q d_q(l_m)} \tag{39}$$

$$\alpha = ((q_e)^2/(4\pi\varepsilon_0))/(\hbar c) \tag{40}$$

$$j_m = 0, +1, 0, -1 \text{ for, respectively, } l_m \bmod 3 = 0, 1, 3/2, 2\text{; with } 3/2 \bmod 3 \equiv 3/2 \tag{41}$$

$$d_m = (2 - (\log(m_\mu/m_e)/\log(\beta^{1/3}))) \approx 3.840613 \times 10^{-2} \tag{42}$$

$$n_q = 0, 3/2, 3/2, 3/2, 3/2, \text{ for, respectively, } l_q = 3, 2, 3/2, 1, 0 \tag{43}$$

$$j_q = 0, -1, 0, +1, +3 \text{ for, respectively, } l_q = 3, 2, 3/2, 1, 0 \tag{44}$$

$$d_q(0) \sim 0.324,\ d_q(1) \sim -1.062,\ d_q(2) \sim -1.509 \tag{45}$$

$$d_q(l_m) = 0 \text{ for } l_m \leq -1 \text{ and for } l_m \geq 3 \tag{46}$$

SOMA suggests that the number of seemingly independent irrational numbers input into the above equations is seven. For example, the list consisting of m_e, m_μ, β, α, $d_q(0)$, $d_q(1)$, and $d_q(2)$ includes seven irrational numbers.

Table 5: Approximate values of $\log_{10}(m(l_m, l_q)/m_e)$ for all known charged fermion elementary particles. Regarding flavour, this table generalizes, based on POST terminology that associates with charged leptons and with neutrinos. (For example, POST uses the term electron-neutrino.) In table 5, the symbol l_f numbers the three flavours. The "l_f (0.5C$_1$)" terms pertain for fermions in the 0.5C$_1$ family. The symbol 0.5Q$_{>0}$ denotes the pair 0.5Q$_{2/3}$ and 0.5Q$_{1/3}$. The "l_f (0.5Q$_{>0}$)" terms pertain for quarks (or, elementary particles in the two families 0.5Q$_{2/3}$ and 0.5Q$_{1/3}$). l_m is an integer parameter. The domain $-6 \le l_m \le 18$ might have relevance regarding modeling. Q denotes the magnitude of charge, in units of $|q_e|$. The family 0.5C$_1$ associates with $Q = 1$. The family 0.5Q$_{2/3}$ associates with $Q = 2/3$. The family 0.5Q$_{1/3}$ associates with $Q = 1/3$. Regarding table 5, $l_q = 3Q$ pertains. Regarding the rightmost four columns, items show $\log_{10}(m(l_m, l_q)/m_e)$ and - for particles that nature includes - the name of an elementary fermion. For each † case, no particle pertains. Each number in the column with the label $Q = 1/2$ equals the average of the number in the $Q = 2/3$ column and the number in the $Q = 1/3$ column. The notion of geometric mean pertains regarding the mass of the $Q = 2/3$ particle and the mass of the $Q = 1/3$ particle. Regarding each † case, equation (39) provides the number $m(l_m, l_q)$.

l_f (0.5C$_1$)	l_f (0.5Q$_{>0}$)	l_m	$Q = 1$	$Q = 2/3$	$Q = 1/2$	$Q = 1/3$
1 (Electron)	1 (Up, Down)	0	0.00 Electron	0.63 Up	0.80 †	0.97 Down
-	2 (Charm, Strange)	1	1.23 †	3.40 Charm	2.83 †	2.26 Strange
2 (Mu)	3 (Top, Bottom)	2	2.32 Muon	5.53 Top	4.72 †	3.91 Bottom
3 (Tau)	-	3	3.54 Tau	-	-	-

3.3.3. Nominal properties of known charged elementary fermions

Table 5 shows information about properties of all known charged fermion elementary particles. (Reference [62] provides the data that underlie table 5.) Regarding similar tables for each one of isomer-one, isomer-two, isomer-four, and isomer-five, SOMA suggests (per table 6) that the values of l_f that table 5 shows for the charged leptons are not appropriate. For example, for isomer-two, the l_f values in the leftmost column would be 3 (for the row for which - for quarks - $l_f = 1$), blank (for the row for which - for quarks - $l_f = 2$), 1 (for the row for which - for quarks - $l_f = 3$), and 2 (for the remaining row).

For each charged elementary fermion except the top quark, equation (39) suggests a mass that is within one experimental standard deviation of the nominal mass that reference [62] reports. Reference [62] alludes to three estimates for the mass of the top quark. Equation (39) provides a mass (for the top quark) that is within 4.4 standard deviations below the nominal mass that associates with direct measurements, within 4.3 upward standard deviations above the nominal mass that associates with cross-section measurements, and within 1.6 standard deviations below the nominal mass that associates with the four-element phrase pole from cross-section measurements.

3.3.4. Anomalous magnetic moments of charged leptons

This unit associates with the row - in table 2 - that associates with Σg" and with $n_\Gamma = 2$.

QFT associates with two complementary aspects of magnetic moment - nominal magnetic moment and anomalous magnetic moment. QFT calculates anomalous magnetic moments that match data regarding the electron and the muon. The calculations feature notions of virtual photons.

SOMA associates two solution-pairs - 1g1‘2 and 3g1‘2 - with the Γ that equals 1‘2.

SOMA suggests the possibility that intrinsic use of the solution-pair 1g1‘2 associates with the intrinsic property of nominal magnetic moment and that intrinsic use of the solution-pair 3g1‘2 associates with the intrinsic property of anomalous magnetic moment.

Two extrinsic solution-pairs associate with intrinsic use of the 3g1‘2 solution-pair. The 3g1‘2‘6 extrinsic solution-pair associates with $6 \in Z_\Gamma$. SOMA suggests the possibility that the strength of 3g1‘2‘6 can vary based on elementary fermion flavour. The 3g1‘2‘4 extrinsic solution-pair associates with $6 \notin Z_\Gamma$. SOMA suggests the possibility that the strength of 3g1‘2‘4 does not vary based on elementary fermion flavour.

SOMA suggests the possibility that equation (47) approximates a_{cl}, the anomalous magnetic moment for the cl charged lepton. Here, each one of a_4 and a_6 is a constant with respect to a choice between $cl = e$ (for the electron), $cl = \mu$ (for the muon), and $cl = \tau$ (for the tau). a_4 associates with 3g1‘2‘4. a_6 associates with 3g1‘2‘6.

$$a_{cl} \approx a_4 + a_6 t_{cl} \tag{47}$$

Aspects of equation (36) feature squares of properties. (Regarding equation (35), equation (36), and equation (37), $\beta^2 = (G_N(m_\tau)^2)/(G_N(m_e)^2)$.) Squares of properties might associate with notions of self-interactions.

SOMA suggests the possibility that t_{cl} is $(\log(m_{\text{cl}}/m_e))^2$.

Based on data that reference [62] provides regarding the electron and the muon, SOMA calculates a_4 and a_6. Then, SOMA calculates a value, $a_{\tau,\text{SOMA}}$, for a_τ. Reference [64] provides, based on the SM, a first-order result - which SOMA calls $a_{\tau,\text{SM}}$ - for a_τ. Here, SM denotes the two-word term Standard

Model. The value of $a_{\tau,\mathrm{SOMA}}$ results in a value of $(a_{\tau,\mathrm{SOMA}} - a_{\tau,\mathrm{SM}})/a_{\tau,\mathrm{SM}}$ of approximately -0.00228. Each one of $a_{\tau,\mathrm{SOMA}}$ and $a_{\tau,\mathrm{SM}}$ comports with data that reference [62] provides.

3.3.5. Notions that associate with neutrino oscillations

The SM suggests that neutrino oscillations associate with a notion that flavour eigenstates do not necessarily match mass eigenstates. Flavour eigenstates associate with the creation of neutrinos via the weak interaction. Mass eigenstates associate with the motion - after the creation of a neutrino - of a neutrino.

SOMA associates the weak interaction bosons (and the weak interaction) with 0g solution-pairs. Per table 4, neutrino flavour eigenstates associate with intrinsic use of 0g1‘3‘4‘6. SOMA associates mass - and mass eigenstates - with intrinsic use of 2g2.

SOMA suggests the possibility that intrinsic use of the 6g2‘4 solution-pair (and extrinsic use of 6g2‘4‘8) associates with a notion of anomalous angular momentum (including for zero-charge elementary fermions). Paralleling discussion regarding equation (47) and anomalous magnetic moments for charged leptons, there might be differences - among the three neutrino mass eigenstates - regarding anomalous angular momentum. SOMA suggests that such differences might associate with SM notions of differences - regarding masses - between neutrinos.

Reference [22] notes that observations explore the extent to which neutrino oscillations associate with interactions - between neutrinos and the environments through which neutrinos pass. SOMA suggests that such interactions might associate with 2g” (and notions that associate with mass), with 3g” (and notions that associate with flavour), or with 4g” (and notions that associate with flavour). SOMA suggests that properties and events that associate with interactions between neutrinos and their environments might associate (for example) with intrinsic use of 2g1‘4‘5‘6 (for which 2g1‘4‘5‘6‘8 is a one-step cascade), with intrinsic use of solution-pair 3g2‘4‘5‘6 (for which 3g2‘4‘5‘6‘8 is a one-step cascade), or with intrinsic use of solution-pair 4g1‘2‘5‘6 (for which 4g1‘2‘5‘6‘8 is a one-step cascade). (These notions parallel notions that table 7 suggests regarding 1g” - or, electromagnetic - properties and events.)

3.3.6. Neutrino masses

SOMA suggests neutrino masses.

Reference [19] suggests that data point to the notion that the sum of the three neutrino rest energies is at least approximately 0.06 eV and not more than approximately 0.12 eV. Reference [65] discusses data and modeling regarding upper bounds for the sum of the masses of the three neutrinos. Reference [66] discusses a lower bound of 0.06 eV, an upper bound of 0.15 eV, and a possible upper bound of 0.12 eV. Reference [62] suggests that an upper bound might be approximately 0.10 eV.

Neutrinos associate with $Q = 0$. SOMA suggests that some $m(l_m, 0)$ solutions associate with neutrino masses. For $l_m \leq -1$ and for $l_m \geq 3$, no quarks pertain and SOMA suggests that $d_q(l_m) = 0$.

Equation (48) shows a result from equation (39).

$$mc^2 = m(-4,0)c^2 \approx 3.448 \times 10^{-2} \text{ eV} \tag{48}$$

SOMA suggests the following two possibilities.

1. $mc^2 = m(-4,0)c^2 \approx 3.448 \times 10^{-2}$ eV pertains for each of the three neutrinos.
2. $mc^2 = m(-4,0)c^2 \approx 3.448 \times 10^{-2}$ eV pertains for each of two neutrinos. For one neutrino, one of $m(-6,0)c^2 \approx 4.2 \times 10^{-6}$ eV and $m(-5,0)c^2 \approx 4.4 \times 10^{-4}$ eV might pertain.

This paper does not try to explore the extent to which SOMA notions - such as notions regarding anomalous angular momentum and the 6g2‘4 solution-pair or such as notions regarding interactions that associate with Σg” properties for which $n_\Gamma \geq 3$ - might suffice to explain neutrino oscillations, including for the case in which just one mass pertains for all three neutrinos.

3.3.7. Masses of the might-be arc zero-charge elementary fermions

Per equation (39), SOMA suggests that the three flavors of the might-be arcs might associate respectively with the following rest energies - $m(0,0)c^2 \sim 10.7$ MeV, $m(1,0)c^2 \sim 6.8$ MeV, and $m(2,0)c^2 \sim 102$ MeV.

Table 6: Matches between masses and flavours, for isomers of charged elementary fermions. The symbol l_I denotes the isomer number. The symbol f_{l-r} denotes - for fermion elementary particles - whether a matter particle (in the context of matter particle and antimatter particle) elementary fermion is left-handed ($f_{l-r} = +1$), does not associate with handedness ($f_{l-r} = 0$, in which case the elementary fermion is its own antiparticle), or is right-handed ($f_{l-r} = -1$). The symbol $0.5\text{Q}_{>0}$ denotes the pair $0.5\text{Q}_{1/3}$ and $0.5\text{Q}_{2/3}$. As in table 5, here the symbol l_f numbers the three flavours.

l_I	f_{l-r}	l_m ($0.5\text{Q}_{>0}$)	Respective l_f ($0.5\text{Q}_{>0}$)	l_m (0.5C_1)	Respective l_f (0.5C_1)
0	$+1$	$0, 1, 2$	1,2,3	$0, 2, 3$	1,2,3
1	-1	$3, 4, 5$	1,2,3	$3, 5, 6$	3,1,2
2	$+1$	$6, 7, 8$	1,2,3	$6, 8, 9$	2,3,1
3	-1	$9, 10, 11$	1,2,3	$9, 11, 12$	1,2,3
4	$+1$	$12, 13, 14$	1,2,3	$12, 14, 15$	3,1,2
5	-1	$15, 16, 17$	1,2,3	$15, 17, 18$	2,3,1

3.4. Differences - between isomers - regarding properties of fermion elementary particles

If the stuff that associates with each of the five all-DM isomers evolved similarly to the stuff that associates with isomer-zero, SOMA suggestions regarding DM might not adequately comport with observations regarding the Bullet Cluster collision of two galaxy clusters. (Discussion - below - that cites reference [67] provides more information.)

SOMA uses the symbol l_I to number the isomers. The notion of isomer-l_I pertains.

Per discussion (including discussion regarding table 5) above, regarding each l_I that is at least one, SOMA suggests that the instances (of elementary particles) that associate with isomer-l_I match - with respect to mass - the instances (of the counterpart elementary particles) that associate with isomer-zero.

For modeling regarding flavours (and not - for $0 \leq l_I \leq 5$ - for modeling regarding masses), SOMA associates the quarks in isomer-l_I with three values of l_m. The values are $3l_I + 0$, $3l_I + 1$, and $3l_I + 2$. (Table 5 shows the associations for $l_I = 0$.) Across the six isomers, quarks associate with each value of l_m that is in the range $0 \leq l_m \leq 17$. Regarding quarks and flavours, SOMA suggests that - within isomer-l_I - flavour 1 associates with $l_m = 3l_I$, flavour 2 associates with $l_m = 3l_I + 1$, and flavour 3 associates with $l_m = 3l_I + 2$.

Aspects of table 5 point to the notion that means for matching flavours and masses for charged leptons do not match means for matching flavours and masses for quarks. For charged leptons, isomer-zero does not have a charged lepton that associates with $l_m = 1$ and does have a charged lepton that associates with $l_m = 3$. SOMA suggests that - for each l_I - a charged lepton associates with each of $l_m = 3l_I + 0$, $l_m = 3l_I + 2$, and $l_m = 3l_I + 3$.

SOMA suggests that - for each isomer-l_I such that $1 \leq l_I \leq 5$ - the charged-lepton flavour that associates with $l_m = 3(l_I) + 0$ equals the flavour that associates with the isomer-$(l_I - 1)$ charged lepton that associates with the same value of l_m and - thus - with $l_m = 3(l_I - 1) + 3$. SOMA suggests that, across the six isomers, one cyclical order pertains regarding flavours for charged leptons.

Table 6 shows, for isomers of charged elementary fermions, matches between masses and flavours.

3.5. Isomer, as a new internal quantum number for elementary particles

In table 3, each row that associates with $n_I > 1$ associates with the SOMA notion that isomer is a property of objects.

Regarding elementary particles for which $\Phi \neq$L, SOMA suggests that $n_I = 6$.

Table 6 suggests associations between some internal quantum numbers (for some elementary fermions) and l_I (or, isomer number).

SOMA suggests that the SOMA notion of l_I (or, isomer number) associates with a new (compared to QM) internal elementary-particle quantum number.

3.6. Electromagnetic properties and events that associate with atoms or stars

This unit associates primarily with the row - in table 2 - that associates with 1g. This unit associates also with the row - in table 2 - that associates with 0g and $\{5, 7\} \cap Z_\Gamma \neq \emptyset$.

3.6.1. Some spin-related properties of a two-component system and its two components

This unit associates with the row - in table 2 - that associates with 0g and with $\{5, 7\} \cap Z_\Gamma \neq \emptyset$.

The notion of 0g pertains. Equation (13) does not pertain. The notion of an upper limit on k_{max} does not pertain. Regarding spin S, the notion that $2S$ is a nonnegative integer pertains. The range $0 \leq S < \infty$ pertains.

Per equation (25) and equation (49), the solution-pair 0g1‘3‘5‘7 associates with $S = 0$.

$$0 = |+1-3-5+7| \tag{49}$$

The following three sets of solution-pairs associate with $\Sigma = 0$ and with $4 \notin Z_\Gamma$. Equation (21) defines $+8\hat{}$ and $+16\hat{}$. Regarding the notions of 0g and ΣgΓ, for each solution-pair, the integers shown below (or alluded to by the series just above) appear in Γ and no other integers appear in Γ. For each set, for other than the first solution-pair, each solution-pair cascades from the first solution-pair.

1. $|-1-2-5+8|$, $|-1-2-5-8+16\hat{}|$,$|+1+2-5-6+8|$, and $|+1+2-5-6-8+16\hat{}|$
2. $|+2-3-7+8|$, $|+2-3-7-8+16\hat{}|$, $|+2+3-6-7+8|$, and $|+2+3-6-7-8+16\hat{}|$.
3. $|+1-3-5+7|$, $|-1-3+5-7+8\hat{}|$, $|-1-3+5-6+7|$, and $|-1-3-5+6-7+8\hat{}|$.

For each set, SOMA suggests that equation (25) pertains regarding the first two of the four expressions. Thus, each set includes exactly one expression for each nonnegative S for which $2S$ is an even integer. For each set, SOMA suggests that equation (26) pertains regarding the second two of the four expressions. Thus, each set includes exactly one expression for each nonnegative S for which $2S$ is an odd integer.

SOMA suggests, regarding a two-component system, that $5 \in Z_\Gamma$ and $7 \notin Z_\Gamma$ can associate with one component, $5 \notin Z_\Gamma$ and $7 \in Z_\Gamma$ can associate with the other component, and $5 \in Z_\Gamma$ and $7 \in Z_\Gamma$ can associate with the system.

Removal of (just) the criterion that $4 \notin Z_\Gamma$ results in the following notions. Regarding $5 \in Z_\Gamma$ and $7 \notin Z_\Gamma$, $n_{\Gamma^o} = 4$ solution-pairs that might associate with $S = 0$ associate with $|+1-2-4+5|$, $|-1-2-5+8|$, $|+1-4-5+8|$, and $|+2-3-4+5|$. Regarding $5 \notin Z_\Gamma$ and $7 \in Z_\Gamma$, $n_{\Gamma^o} = 4$ solution-pairs that might associate with $S = 0$ associate with $|-1-2-4+7|$, $|+2-3-7+8|$, and $|+3-4-7+8|$. Regarding $5 \in Z_\Gamma$ and $7 \in Z_\Gamma$, $n_{\Gamma^o} = 4$ solution-pairs that might associate with $S = 0$ associate with $|+1-3-5+7|$ and $|+2-4-5+7|$. The numbers of $S = 0$ solution-pairs are four for the case of $5 \in Z_\Gamma$ and $7 \notin Z_\Gamma$, three for the case of $5 \notin Z_\Gamma$ and $7 \in Z_\Gamma$, and two for the case of $5 \in Z_\Gamma$ and $7 \in Z_\Gamma$.

3.6.2. Spins S regarding atoms

SOMA suggests that modeling can treat an atom as a two-component system. One component is the nucleus of the atom. The other component is the electron cloud.

For an atom, each one of the nucleus and the electron cloud has nonzero charge. Based on extrapolating from the notion that - for 0g solution-pairs that associate with elementary particles - $4 \in Z_\Gamma$ associates with zero charge, SOMA suggests that the following notions might pertain. Solution-pairs that associate with (zero-step or more-than-zero-step) cascades from the solution-pair that associates with $|-1-2-5+8|$ associate with spins S of the electron cloud. The solution-pair that associates with $|+1-2-4+5|$ associates with the spin $S = 0$ state of the electron cloud if the atom is an ion that has no electrons. Solution-pairs that associate with (zero-step or more-than-zero-step) cascades from the solution-pair that associates with $|+2-3-7+8|$ associate with spins S of the nucleus. Solution-pairs that associate with (zero-step or more-than-zero-step) cascades from the solution-pair that associates with $|+2-4-5+7|$ associate with spins S of the atom, if the atom (is not an ion and thus) has a charge of zero. Solution-pairs that associate with (zero-step or more-than-zero-step) cascades from the solution-pair that associates with $|+1-3-5+7|$ associate with spins S of the atom, if the atom (is an ion and thus) has a nonzero charge.

3.6.3. Electromagnetic events that associate with two-component systems such as atoms

SOMA suggests that, for $n_\Gamma \geq 4$, 1g” solution-pairs can associate with electromagnetic properties of objects that model as being parts of multi-component systems.

Table 7 shows some aspects of some cascades that associate with some Σg” solution-pairs for which $n_\Gamma \geq 4$, $\Sigma = 1$, and $1 \leq k_{max} \leq 8$. (Discussion related to equation (49) pertains. The notion that $4 \in Z_\Gamma$ associates with zero charge pertains regarding solution-pairs for which $\Sigma = 0$ and does not necessarily pertain regarding table 7.)

3.6.4. Electromagnetic events that associate with stars

SOMA suggests that stars tend to associate with single isomers. For a one-isomer star, SOMA suggests the following contributions to the electromagnetic field.

If the star has a net nonzero charge, some contributions to the electromagnetic field associate with the star’s intrinsic 1g1 and extrinsic 1g1‘2. The notion of 1g’ pertains. The relevant reach, R_I, for the emitted electromagnetic radiation is one isomer.

Table 7: Some aspects of some cascades that associate with some Σg" solution-pairs for which $n_\Gamma \geq 4$, $\Sigma = 1$, and $1 \leq k_{max} \leq 8$. The column with the two-element label $n_\Gamma = 4$ solution-pair associates with states of a two-component system. The column with the two-element label one-step cascade associates with states to which the $n_\Gamma = 4$ solution-pair state can transit via an interaction with the electromagnetic field. (Electromagnetic interactions can also associate with transitions from one-step cascade states to $n_\Gamma = 4$ solution-pair states.) The column with the one-element label R_I shows the reach for each $n_\Gamma = 4$ solution-pair and for each one-step cascade solution-pair. SOMA suggests that notions - in this table - of $n_\Gamma = 4$ solution-pair and of one-step cascade solution-pair can associate with properties of objects that model as being parts of multi-component systems. For example, for an atom, $5 \in Z_\Gamma$ and $7 \notin Z_\Gamma$ can associate with internal energy-related and angular-momentum-related properties of the atom's electron cloud. $5 \notin Z_\Gamma$ and $7 \in Z_\Gamma$ can associate with possible external influences on the atom's electron cloud. $n_\Gamma = 4$ solution-pair use of $6 \in \Gamma$ can associate with aspects that associate with single electrons. $n_\Gamma = 4$ solution-pair use of 1g2'4'5'8 can associate with the atom's fine-structure state. $n_\Gamma = 4$ solution-pair use of 1g2'4'7'8 can associate with the atom's hyperfine state. The column with the two-element label atom-related event suggests phenomena - related to an atom - that can associate with excitations or de-excitations of the 1L (or, electromagnetic) LRI field.

$n_\Gamma = 4$ solution-pair	One-step cascade	SL	R_I	Atom-related event
1g2'4'5'6	1g2'4'5'6'8x	1L	2	An electron transits to a new principal energy.
1g2'4'5'8	1g2'4'5'6'8x	1L	2	An atomic fine-structure change occurs.
1g2'4'6'7	1g2'4'6'7'8x	1L	2	The atom adds or subtracts an electron.
1g2'4'7'8	1g2'4'6'7'8x	1L	2	An atomic hyperfine transition occurs.

SOMA suggests that stellar radiation (such as thermal radiation) might associate - for example - with intrinsic 1g1'2'3'4'5x and with extrinsic 1g1'2'3'4'5'6x or extrinsic 1g1'2'3'4'5'8x. (Another example associates with intrinsic 1g1'2'3'4'7x. Another example associates with 1g", with intrinsic 1g4'5, and with extrinsic 1g2'4'5. For the last example, the extrinsic solution-pair - in effect - inherits a reach, $R_I = 1$, from the intrinsic solution-pair.) The relevant reach, R_I, for the emitted electromagnetic radiation is one isomer.

3.6.5. Implications regarding cosmic background radiation and sensing dark matter

Per table 7, components of cosmic (electromagnetic) background radiation that associate with creation (of electromagnetic radiation) via atomic phenomena can associate with a reach R_I of two isomers. Components of cosmic (electromagnetic) background radiation that associate with creation via other phenomena can associate with a reach R_I of one isomer.

SOMA suggests that electromagnetic radiation that associates with creation via single-isomer atomic phenomena can associate with a reach R_I of two. SOMA suggests that detectors that are made out of OM stuff can detect such radiation that was created by isomer-zero atomic phenomena or by isomer-three atomic phenomena. Discussion related to table 12 suggests that detectors that are made out of OM stuff may have detected electromagnetic radiation that was created by isomer-three atomic phenomena.

3.7. Tetrads, properties of elementary particles, and conservation laws

3.7.1. Tetrads and elementary-particle aspects other than angular momentum states

SOMA suggests that the following notions pertain (compared to the notion of no object or the notion of not an elementary particle) regarding intrinsic monopole solution-pairs to which table 3 alludes.

- The following notions pertain regarding 1g1. For each one of the six isomers, there are three DOF-related aspects - $Q = 1/3$, $Q = 2/3$, and $Q = 1$. For SL elementary particles and other zero-charge elementary particles, there is one DOF-related aspect - $Q = 0$.
- The following notions pertain regarding 2g2. For each one of the six isomers, there are three DOF-related aspects - $m_b = 0$, $m_b > 0$, and $m_f > 0$. For SL elementary particles, there is one DOF-related aspect - $m_b = 0$.
- The following notions pertain regarding 3g3. For each one of the three isomer-pairs, there are three DOF-related aspects - $f_{l-r} = +1$, $f_{l-r} = 0$, and $f_{l-r} = -1$. The symbol f_{l-r} denotes - for fermion elementary particles - whether a matter particle (in the context of matter particle and antimatter particle) elementary fermion is left-handed ($f_{l-r} = +1$), does not associate with handedness ($f_{l-r} = 0$, in which case the elementary fermion is its own antiparticle), or is right-handed ($f_{l-r} = -1$). For example, the isomer-zero electron associates with $f_{l-r} = +1$ and the isomer-zero positron associates with $f_{l-r} = -1$. For non-fermion elementary particles, there is one DOF related aspect - $f_{l-r} = 0$.

Table 8: Some tetrads that SOMA suggests pertain regarding elementary particles. The row for which $k = -2$ associates with notions of space-time dimensions that are relevant for CM modeling and QM modeling. The acronym EP abbreviates the two-word term elementary particle. The acronym EF abbreviates the two-word term elementary fermion. The symbol † associates with the QM notion of Majorana fermion. The symbol ‡ associates with a possibility regarding the might-be 0.5R fermion elementary particles.

Parameter	Triad notion	n_I	R_I	Monad notion
$k = -2$	Three spatial dimensions	1	$6\uplus$	One temporal dimension
$k = -1$	Color charges (red, blue, green)	6	1	The EP is not an EF
$k = 0$	Three isomer-pairs	3	2	The EP associates with one SL
$\Sigma = 1$	Charge ($Q = 1/3$, $Q = 2/3$, $Q = 1$)	6	1	The EP associates with $Q = 0$
$\Sigma = 2$	Mass ($m_b = 0$, $m_b > 0$, $m_f > 0$)	1	6	The EP associates with one SL
$\Sigma = 3$	EF f_{l-r} ($+1$, $0^\dagger$, -1)	3	2	The EP is not an EF
$\Sigma = 4$	EF $\lvert f_{B-L}\rvert$ ($0^\ddagger$, 1/3, 1)	6	1	The EP is not an EF
$k = 6$	Three EF flavours	6	1	The EP is not an EF

- The following notions pertain regarding 4g4. For each one of the six isomers, there are three DOF-related aspects - $|f_{B-L}| = 0$, $|f_{B-L}| = 1/3$, and $|f_{B-L}| = 1$. The symbol f_{B-L} denotes the QM notion of baryon number minus lepton number. For non-fermion elementary particles, there is one DOF related aspect - $f_{B-L} = 0$.

Paralleling discussion related to equation (12), SOMA suggests that - for SR - f_{l-r} associates with notions of being the scalar component of a 4-vector. Paralleling discussion related to equation (12), SOMA suggests that - for SR - f_{B-L} associates with notions of being the scalar component of a 4-vector.

Equation (50) shows notation that SOMA uses to describe a reach that includes all six isomers and all LRI phenomena.

$$R_I = 6\uplus \tag{50}$$

Table 8 lists some tetrads that SOMA suggests pertain regarding elementary particles.

SOMA suggests relevance - regarding the following - for the notion of the existence of two solutions per one solution-pair.

- Two isomers (in the context of isomer-pair).
- Two handednesses (in the context of isomers).
- Two handednesses (in the context of nonzero spin, nonzero mass elementary particles).
- Two circular polarizations (in the context of zero mass elementary particles).

3.7.2. Tetrads and conservation laws

Table 9 lists some conserved quantities that POST includes or that SOMA suggests.

For example, for an object, absent interactions - that exchange charge - with other objects, each of the six instances of charge is conserved.

Each of the instances - in table 9 - for which the parameter is a value of Σ associates with a quantity that adds across components (including SL aspects) of a multi-component system.

3.8. Possibilities for conversions between isomers

Equation (51) symbolizes an interaction in which an isomer-l_I object transits from a state X_{l_I} to a state Y_{l_I} and the interaction produces a pair of isomer-l'_I elementary fermions. The symbol FLH denotes a left-handed fermion elementary particle and the symbol FRH denotes a right-handed fermion elementary particle. The left-handed fermion elementary particle and the right-handed fermion elementary particle are antiparticles for each other.

$$X_{l_I} \rightarrow Y_{l_I} + FLH_{l'_I} + FRH_{l'_I} \tag{51}$$

For each one of 1L, 2L, and 3L, extending table 7 would result in at least one intrinsic Σg” solution-pair for which $R_I = 6$, there is a one-step-cascade solution-pair for which $6 \in Z_\Gamma$, and there is a one-step-cascade solution-pair for which $8 \in Z_\Gamma$. Examples of such intrinsic solution-pairs include 1g2‘3‘4‘5‘7x, 2g1‘3‘4‘5‘7x, and 3g1‘2‘4‘5‘7x.

Table 9: Some conserved quantities that POST includes or that SOMA suggests. The acronym EF abbreviates the one-element term elementary-fermion. Conserved notions that SOMA suggests and POST does not necessarily include are numbers of dimensions and EF f_{l-r}. Regarding each row for which $1 \leq \Sigma \leq 4$ pertains, equation (11) pertains regarding notions of scalar properties and vector properties.

(a) SOMA interpretation.

Parameter	Conserved triad notion	n_I	R_I	Conserved monad notion
$k = -2$	Three spatial dimensions	1	$6\uplus$	One temporal dimension
$k = -1$	Color charge	6	1	-
$\Sigma = 1$ - Intrinsic use of 1g1	Charge	6	1	-
$\Sigma = 2$ - Intrinsic use of 2g2	Energy	1	$6\uplus$	-
$\Sigma = 2$ - Extrinsic use of 2g2‘4	Momentum	1	$6\uplus$	-
$\Sigma = 2$ - Intrinsic use of 2g2‘4	Angular momentum	1	$6\uplus$	-
$\Sigma = 3$ - Intrinsic use of 3g3	EF f_{l-r}	3	2	-
$\Sigma = 4$ - Intrinsic use of 4g4	EF f_{B-L}	6	1	-

(b) POST-like interpretation. POST does not necessarily include notions that associate with (at least) $n_I \geq 3$ and f_{l-r}.

Parameter	Conserved vector notion	n_I	R_I	Conserved scalar notion
$k = -2$	Three spatial dimensions	1	$6\uplus$	One temporal dimension
$k = -1$	-	6	1	Color charge
$\Sigma = 1$ - Intrinsic use of 1g1	-	6	1	Charge
$\Sigma = 2$ - Intrinsic use of 2g2	-	1	$6\uplus$	Energy
$\Sigma = 2$ - Extrinsic use of 2g2‘4	Momentum	1	$6\uplus$	-
$\Sigma = 2$ - Intrinsic use of 2g2‘4	Angular momentum	1	$6\uplus$	-
$\Sigma = 3$ - Intrinsic use of 3g3	-	3	2	EF f_{l-r}
$\Sigma = 4$ - Intrinsic use of 4g4	-	6	1	EF f_{B-L}

SOMA suggests that interactions that associate with equation (51) can result in any isomer-l_1 creating stuff that associates with any isomer-l'_1. The notion that 16-pole pertains regarding the relevant solution-pairs might suggest that such interactions occur mainly in situations in which stuff is dense (or, mainly, in the early history of the universe).

Equation (52) symbolizes an interaction in which an isomer-l_I matter-and-antimatter pair of similar elementary fermions produce an isomer-l_I left-handed elementary particle and an isomer-l'_I right-handed similar elementary particle. (Similar interactions could produce $FLH_{l'_I} + FRH_{l_I}$.) Here, based on the notion - in table 9 - of conservation (for each one of the three isomer-pairs) of elementary fermion f_{l-r}, SOMA suggests that $|l_I - l'_I| = 3$.

$$FLH_{l_I} + FRH_{l_I} \rightarrow FLH_{l_I} + FRH_{l'_I} \qquad (52)$$

For each one of 1L and 3L, table 3 lists at least one intrinsic Σg' solution-pair for which both $R_I \geq 2$ and there is a one-step-cascade solution-pair for which $6 \in Z_\Gamma$. Examples of such intrinsic solution-pairs include 1g1‘2‘4 and 3g3. For 1g1‘2‘4, there is also a one-step-cascade solution-pair for which $8 \in Z_\Gamma$.

For l_I=0 and $l'_I = 3$, equation (52) associates with a decrease in the number of isomer-zero right-handed elementary fermions and an increase in the number of isomer-three right-handed elementary fermions.

3.9. Eras in the evolution of the universe

Reference [68] discusses CC notions regarding cyclic cosmology. SOMA includes the possibility that the present universe arose from an implosion of energy. SOMA does not yet consider either aspects that may have created the energy that would have imploded or whether the present universe might eventually implode.

Reference [35] discusses possibilities that might lead to a Big Bang.

CC suggests three eras in the rate of expansion of the universe. The eras feature, respectively, rapid expansion; continued expansion, with the rate of expansion decreasing; and continued expansion, with the rate of expansion increasing.

SOMA suggests using the notion of eras regarding the separating from each other of clumps - that, today, POST would consider to be large - of stuff. Examples of such clumps might include galaxy

Table 10: Six possible eras regarding the rate of separating of large clumps. The rightmost three columns suggest eras. The leftmost four columns describe phenomena that SOMA suggests as noteworthy causes for the eras. Generally, a noteworthy cause associates with notions of acceleration. Generally, an era associates with a range of velocities. The symbol → associates with the notion that a noteworthy cause may gain prominence before an era starts. The two-element term intrinsic s-p abbreviates the two-element phrase intrinsic solution-pairs. Subsequent rows associate with later eras. CC suggests notions of a Big Bang (or - at least - of a time that CC associates with the two-word term Big Bang). The symbol † denotes a possible association between the relevant era and some CC notions of a Big Bang. SOMA points to the possibility for the first two eras that the table discusses. Intrinsic use of the solution-pair 0g1‘2‘3‘4‘8 associates with the Pauli exclusion principle (and with the might-be jay boson). The other intrinsic solution-pairs to which the table alludes associate with gravitation. CC uses the word inflation (or, the two-word term inflationary epoch) to name the era that associates with the third row in the table. CC suggests that the inflationary epoch started about 10^{-36} seconds after the Big Bang. CC suggests that the inflationary epoch ended between 10^{-33} seconds after the Big Bang and 10^{-32} seconds after the Big Bang. Possibly, no direct evidence exists for the inflationary epoch. TBD denotes to be determined. The following notions pertain regarding the column with the one-word label notes. The symbol 1 denotes the notion that CC interpretations of data support the notions of each one of the two billions-of-years eras. The symbol 2 denotes the notion that CC suggests the era. The symbol 3 denotes the notion that SOMA suggestions regarding resolving CC tensions (between data and modeling) that associate with the fifth row do not necessarily depend on the existence of the era.

Force	Intrinsic s-p	SOMA-pole	R_I	→	Rate of separating	Duration	Notes
Attractive	2g1‘2‘3‘4‘8x	16-pole	6	→	Is negative	TBD	3
Repulsive	0g1‘2‘3‘4‘8	16-pole	1	→	Turns positive †	TBD	3
Repulsive	2g1‘2‘3‘4x	Octupole	1	→	Increases rapidly	Less than a second	2, 3
Attractive	2g1‘2‘3	Quadrupole	1	→	Decreases	Billions of years	1
Repulsive	2g2‘4	Dipole	2	→	Increases	Billions of years	1
Attractive	2g2	Monopole	6	→	Would decrease	-	3

clusters and even larger clumps. SOMA suggests (based on equation (15) and equation (16)) that, for a pair of similar objects that always move away from each other, the dominating gravitational effects transit (over time) all or a portion of the following sequence: 16-pole attracting, octupole repelling, quadrupole attracting, dipole repelling, and monopole attracting.

Table 10 discusses six possible eras regarding the rate of separating of large clumps. (References [69], [20], [36], and [70] discuss the possible inflationary epoch. References [71], [72], [73], and [74] provide data and discussion about the two multi-billion-years eras. Reference [37] discusses attempts to explain the rate of expansion of the universe.)

SOMA suggests that some SOMA notions regarding eras that follow the inflationary epoch might not necessarily depend significantly on SOMA notions regarding the inflationary epoch or regarding eras that might precede the inflationary epoch.

This paper does not try to explore the possibility that (or to estimate a time at which) a transition - for the largest observable objects - from repelling based on 2g2‘4 to attracting based on 2g2 might occur.

3.10. Baryon asymmetry

The two-word term baryon asymmetry associates with the POST notion that - regarding known stuff - there are many more left-handed (or matter) fermion elementary particles than right-handed (or antimatter) fermion elementary particles. CC suggests that baryon asymmetry arose early in the history of the universe. From the perspective of SOMA, such known stuff associates with isomer-zero.

Discussion related to equation (52) points to a means that may have produced baryon asymmetry. Possibly, POST notions of lasing pertained regarding relevant excitations of LRI fields. SOMA suggests that processes leading to baryon asymmetry led to isomer-three stuff having fewer left-handed (or antimatter, from the perspective of isomer-three) fermion elementary particles than right-handed (or matter, from the perspective of isomer-three) fermion elementary particles.

This paper does not address the topic of the extent to which steps leading to a predominance in isomer-zero stuff of left-handed elementary particles (and not to a predominance of right-handed elementary particles) have a basis other than random chance.

3.11. Evolution of stuff that associates with dark matter isomers

SOMA uses the two-element term isomer-l_I stuff to denote objects (including hadron-like particles, atom-like objects, and stars) that associate with the isomer-l_I.

3.11.1. Evolution of isomer-1, isomer-2, isomer-4, and isomer-5 stuff

Here, SOMA uses the one-element term alt-isomer to designate an isomer other than isomer-zero and isomer-three.

For each one of the six isomers, a charged baryon that includes exactly three flavour 3 quarks is more massive than the counterpart zero-charge baryon that includes exactly three flavour 3 quarks. (For example, the hadron that includes just two tops and one bottom has a larger total mass than does the hadron that includes just one top and two bottoms.)

Per table 5 and table 6, alt-isomer flavour 3 charged leptons are less massive than isomer-zero flavour 3 charged leptons. When flavour 3 quark states are much populated (and based on interactions mediated by W bosons), the stuff that associates with an alt-isomer converts more charged baryons to zero-charge baryons than does the stuff that associates with isomer-zero. Eventually, regarding the stuff that associates with the alt-isomer, interactions that entangle multiple W bosons result in the stuff that associates with the alt-isomer having more neutrons and fewer protons than does the stuff that associates with isomer-zero. The sum of the mass of a proton and the mass of an alt-isomer flavour 1 charged lepton exceeds the mass of a neutron. Compared to isomer-zero neutrons, alt-isomer neutrons scarcely decay. The IGM (or, intergalactic medium) that associates with the alt-isomer scarcely interacts with itself via electromagnetism.

3.11.2. Evolution of isomer-3 stuff

The following two possibilities pertain. In one possibility, the evolution of isomer-three stuff parallels the evolution of isomer-zero stuff. In the second possibility, the evolution of isomer-three stuff does not parallel the evolution of isomer-zero stuff. The second possibility might associate with - for example - a difference in handedness - with respect to charged leptons or with respect to W bosons - between isomer-three and isomer-zero.

This paper nominally assumes that the evolution of isomer-three stuff parallels the evolution of isomer-zero stuff.

3.12. Tensions - among data and models - regarding large-scale phenomena

SOMA suggests means to resolve tensions - between data and CC - regarding the rate of expansion of the universe, regarding large-scale clumping of matter, and regarding gravitational interactions between neighboring galaxies.

3.12.1. The rate of expansion of the universe

Table 10 lists two known eras in the history of the universe.

CC underestimates - for the second multi-billion-years era - increases in the rate of expansion of the universe. (References [40], [41], [42], [43], [75], [76], [77], and [78] provide further information. Reference [79] suggests that the notion that DM is similar to OM might help resolve the relevant tension. Reference [80] discusses various possible resolutions.)

SOMA suggests the following explanation for such underestimates.

When using modeling based on GR, CC might try to extend the use of an equation of state (or the use of a cosmological constant) that works well regarding early in the first multi-billion-years era. Regarding the first multi-billion-years era, SOMA suggests dominance by attractive effects that associate with intrinsic use of the 2g1‘2‘3 component of gravity. The notion of a reach of one pertains. The symbol 2(1)g1‘2‘3 pertains. SOMA suggests that - later in the first multi-billion-years era - repulsive effects that associate with intrinsic use of 2(2)g2‘4 become significant. Dominance by 2(2)g2‘4 pertains by the time the second multi-billion-years era starts. However, use of an equation of state that has roots in the time period in which 2(1)g1‘2‘3 dominates might - at best - extrapolate based on a notion of 2(1)g2‘4 (and not based on a notion of 2(2)g2‘4) and would underestimate the strength of the key gravitational driver - of expansion - by a factor of two.

SOMA points - conceptually - to the following possible remedy.

CC might change (regarding the stress-energy tensor or the cosmological constant) the aspects that would associate with repelling and the 2g2‘4 component of gravity. The contribution - to the pressure - that associates with intrinsic use of 2g2‘4 might need to double (compared to the contribution that would associate with intrinsic use of 2(1)g2‘4).

3.12.2. Large-scale clumping of matter

CC overestimates large-scale clumping of matter - OM and DM. (References [81], [82], [83], and [43] provide data and discussion.)

SOMA suggests that CC modeling associates with a repulsive component - 2(1)g2‘4 - of gravity. SOMA suggests that 2(2)g2‘4 pertains. (That is, for each instance of 2g2‘4, a reach of two isomers pertains.) The additional (compared to CC modeling) repelling might explain the overestimating that CC suggests.

Table 11: Stages and other information regarding the evolution of a galaxy for which a notion of a one-isomer original clump pertains. The table suggests stages, with subsequent rows associating with later stages. The next-to-rightmost column describes aspects of the stage. The leftmost three columns in the table describe a component of 2L that is a noteworthy cause for the stage. The two-element term intrinsic s-p abbreviates the two-element phrase intrinsic solution-pairs. The symbol → associates with the notion that a noteworthy cause may gain prominence before a stage-starts. Table 11 associates with a scenario in which a galaxy forms based on one original one-isomer clump and initially does not significantly collide with other galaxies. The galaxy might retain some stuff that associates with the repelled isomer. The rightmost column in table 11 suggests terminology regarding the evolution of galaxies. (A galaxy can include stuff from more than one earlier galaxy.)

Force	Intrinsic s-p	R_I	→	Stage	Aspects of the stage	Era
Attractive	2g1‘2‘3	1	→	1	A one-isomer original clump forms.	First
Repulsive	2g2‘4	2	→	2	The original clump repels (some) stuff that associates with the isomer that associates with the original clump and (most) stuff that associates with one other isomer.	First
Attractive	2g2	6	→	3	The original clump attracts stuff that associates with the four not-repelled isomers and stuff that associates with the isomer that associates with the original clump.	Second
Attractive	2g2	6	→	4	Another galaxy subsumes the original clump and might subsequently merge with yet other galaxies.	Third

3.12.3. Effects - within galaxies - of the gravity associated with nearby galaxies

CC might not account for some observations about effects - within individual galaxies - of the gravity associated with nearby galaxies. (Reference [57] provides further information.)

SOMA suggests that CC modeling associates with a repulsive component - 2(1)g2‘4 - of gravity. SOMA suggests that 2(2)g2‘4 pertains. The additional (compared to CC modeling) repelling might explain at least some aspects of the data that reference [57] discusses.

3.13. Formation and evolution of galaxies

3.13.1. Mechanisms regarding the formation and evolution of galaxies

Reference [45] suggests that galaxies form around early clumps of stuff. Reference [45] associates the word halo with such clumps.

SOMA suggests that each one of many early halos associates with one isomer. SOMA associates with such early halos the three-element term one-isomer original clump. Clumping occurs based on gravitational effects. Differences - between the evolution of stuff associating with any one of isomer-zero and isomer-three and the evolution of stuff associating with any one of isomer-one, isomer-two, isomer-four, and isomer-five are not necessarily significant regarding this gravitationally based clumping. The six isomers might form such clumps approximately equally.

Table 11 discusses SOMA suggestions regarding the formation and evolution of a galaxy for which a notion of a one-isomer original clump pertains.

Presumably, some galaxies form based on two or more clumps, for which all the clumps associate with just one isomer. Possibly, some galaxies form based on two or more clumps, for which some clumps associate with isomers that are not the same as the isomers that associate with some other clumps.

3.13.2. Aspects regarding the evolution of galaxies

Table 11 suggests three eras regarding the evolution of galaxies. The first era associates with the first two rows in table 11. The second era associates with the 2g2 attractive force that associates with the third row in table 11. The third era associates with collisions between and mergers of galaxies.

SOMA suggests the possibility that some galaxies do not exit the first era and do not significantly collide with other galaxies.

SOMA suggests that some galaxies result from aspects associating with the 2g2 attractive force that associates with the third row in table 11. Here, this paper discusses three cases. (Mixed cases and other cases might pertain.)

- Each one of some era-one galaxies does not collide with other galaxies. Such a galaxy accumulates (via 2g2 attracting) stuff associating with various isomers that have representation in nearby IGM.

Table 12: Ratios - that pertain to light that dates to about 400,000 years after the Big Bang - of observed effects to effects that POST estimates. The three-word phrase cosmic optical background associates with radiation that - recently - measures as optical radiation or measures as close (with respect to wavelengths) to optical radiation. The acronym CMB associates with radiation that - recently - measures as cosmic microwave background radiation. DM:OM denotes a ratio of DM effects to OM effects that this paper suggests.

Aspect	Observed : POST-CC-expected	SOMA-suggested DM:OM
Amount of cosmic optical background	2 : 1	1 : 1
Some absorption of CMB	2 : 1	1 : 1

The galaxy becomes an era-two galaxy. The galaxy might include stuff that significantly associates with as many as five isomers.

- Each one of some era-two galaxies merges (via 2g2 attracting) mainly just with galaxies that feature the same five isomers. The galaxy that merged, in effect, loses it status of being a galaxy. The resulting larger object is an era-two galaxy. The galaxy might include stuff that significantly associates with as many as five isomers.
- Each one of some era-one or era-two galaxies merges (via 2g2 attracting) with other galaxies. The galaxy that merged, in effect, loses its status of being a galaxy. The resulting larger object is an era-three galaxy. The galaxy might include stuff that significantly associates with as many as six isomers.

3.14. Ratios of dark matter effects to ordinary matter effects

3.14.1. Ratios that might pertain regarding the cosmic electromagnetic background

Table 12 lists ratios - that pertain to light that dates to about 400,000 years after the Big Bang - of observed effects to effects that POST estimates. The acronym CMB abbreviates the three-word term cosmic microwave background (or, the four-word term cosmic microwave background radiation). (References [84], [85] and [86] provide data and discussion regarding the amount of cosmic optical background. References [87], [88], and [89] provide data and discussion regarding absorption of CMB.)

The following two paragraphs provide SOMA-suggested explanations for the observations to which table 12 alludes.

The three-word phrase cosmic optical background associates with now nearly-optical light remaining from early in the universe. CC suggests that atomic transitions produced radiation that today measures as cosmic (optical and microwave) background radiation. SOMA associates POST atomic transitions with isomer-zero. Observations found twice as much light as CC expected. SOMA suggests that isomer-one, isomer-two, isomer-four, and isomer-five stuff did not result in much stuff that is similar to isomer-zero atoms. SOMA suggests that isomer-three stuff evolved similarly to isomer-zero stuff. For four types of changes in atomic energy levels, table 7 alludes to 1L-producing events that associate with $R_I = 2$. SOMA suggests that such events explain the two-to-one observed-to-expected ratios regarding the cosmic optical background. Isomer-zero (or, OM) stuff produced half of the observed light. Isomer-three (or, DM) stuff produced half of the observed light.

The four-element phrase some absorption of CMB associates with the notion that measurements of some specific depletion of CMB indicate twice as much depletion as CC expected based solely on hyperfine interactions with (isomer-zero) hydrogen atoms. SOMA suggests (per table 7) that isomer-three (or, DM) hydrogen-like atoms account for the half of the absorption for which isomer-zero (or, OM) hydrogen atoms do not account.

3.14.2. Ratios that pertain for some galaxies

Table 13 suggests explanations for some ratios - that pertain to some galaxies - of DM effects to OM effects. (References [90] and [91] provide data and discussion. Reference [90] influenced the choice - that this paper reflects - of a time range to associate with the word early. Regarding the combination of 0^+:1 and later, references [92], [93], [94], [95], [96], [97], and [98] provide data and discussion. Reference [99] discusses a galaxy that might have started as containing mostly OM. Regarding observed DM galaxies, references [45], [100], [101], and [102] provide data and discussion. Current techniques might not be capable of observing early DM galaxies. References [103] and [104] suggest, regarding galaxy clusters, the existence of clumps of DM that might be individual galaxies. Extrapolating from results that references [45] and [105] discuss regarding ultrafaint dwarf galaxies that orbit the Milky Way galaxy might suggest that the universe contains many DM:OM $1 : 0^+$ later galaxies. Reference [106] discusses a trail of

Table 13: Suggested explanations for some ratios - that pertain to some galaxies - of DM effects to OM effects. DM:OM denotes a ratio of DM effects to OM effects. Inferences of DM:OM ratios come from interpreting data. Regarding galaxies, the notion of early associates with observations that pertain to galaxies that associate with (or, would, if people could detect the galaxies, associate with) high redshifts. High might associate with $z > 7$ and possibly with smaller values of z. Here, z denotes redshift. The word later associates with the notion that observations pertain to objects later in the history of the universe. The two-element term DM galaxy denotes a galaxy that contains much less OM than DM. Possibly, people have yet to directly detect early DM galaxies. Table 11 provides information about the explanations.

Objects	DM:OM	Examples	Explanation
Some early galaxies	$0^+ : 1$	Reported	OM original clump. Stage-1 or stage-2.
Some later galaxies	$0^+ : 1$	Reported	OM original clump. Stage-1 or stage-2.
Some early galaxies	$1 : 0^+$	No known reports	DM-isomer(s) original clump. Stage-1 or stage-2.
Some later galaxies	$1 : 0^+$	Reported	DM-isomer(s) original clump. Stage-1 or stage-2.
Some later galaxies	$\sim 4 : 1$	Reported	Non-isomer-three original clump. Stage-3.
Many later galaxies	$5^+ : 1$	Reported	Any-isomer(s) original clump(s). Stage-4.

galaxies for which at least two galaxies have little DM. Reference [106] suggests that the little-dark-matter galaxies result from a collision that would have some similarities to the Bullet Cluster collision. Regarding galaxies for which DM:OM ratios of ~4:1 pertain, references [107] and [108] provide data and discussion. Regarding later galaxies for which DM:OM ratios of 5^+:1 pertain, reference [45] provides data and discussion. References [109] and [110] provide data about collisions of galaxies.)

Table 13 does not rule out the notion that galaxies somewhat fully populate DM:OM ranges within the interval of $0 : 1$ to, say, $6 : 1$. For DM:OM ratios of less than (say) ten, table 11 suggests that each range of DM:OM ratios to which table 13 alludes might stand out statistically (in terms of numbers of galaxies) from ranges (of positive-number ratios) near to the range to which table 13 alludes.

Table 13 does not rule out the notion that galaxies somewhat fully populate a DM:OM range of $10^{p_1} : 1$ to $10^{p_2} : 1$ for which SOMA does not suggest a value of p_1; p_1 exceeds, say, three; p_2 exceeds p_1; and SOMA does not suggest a value of p_2.

3.14.3. Ratios that pertain regarding phenomena that are bigger than galaxies

SOMA suggests two possible contributions toward the notion that measurements of large-scale presences of DM might exceed five times measurements of large-scale presences of OM.

- Asymmetric evolution. Here, the term alt-isomer refers to isomer-one, isomer-two, isomer-four, or isomer-five. The evolution of alt-isomer stuff might deviate - compared to the evolution of isomer-zero stuff - early enough that (nominally) isomer-zero high-energy excitations of the electromagnetic field produce alt-isomer stuff significantly more copiously than (nominally) alt-isomer excitations of the electromagnetic field produce isomer-zero stuff. (Discussion related to equation (51) pertains.)
- Asymmetric measurement. Isomer-zero stuff might include hadron-like particles that include just zero-charge elementary fermions and gluons. (Table 4 points to - for example - might-be arc fermion elementary particles.) These isomer-zero hadron-like particles might measure as DM. The stuff associated with isomers other than isomer-zero might also include hadron-like particles that include just zero-charge elementary fermions and gluons. Those non-isomer-zero hadron-like particles would measure as DM.

Table 14 suggests explanations for observed ratios - that pertain to larger-than-galaxies-scale phenomena - of DM effects to OM effects. (Reference [62] provides data and discussion regarding densities of the universe. References [111], [112], [113], and [114] provide data and discussion regarding galaxy clusters.)

3.14.4. Aspects related to collisions of pairs of galaxy clusters

Reference [67] discusses the Bullet Cluster collision of two galaxy clusters.

CC suggests two general types of trajectories for stuff. Most DM - from either one of the clusters - exits the collision with trajectories that are consistent with having interacted just gravitationally with the other cluster. Also, OM stars - from either cluster - exit the collision with trajectories that are consistent with having interacted just gravitationally with the other cluster. However, OM IGM - from either cluster - lags the cluster's OM stars and DM. CC suggests that the OM IGM interacted electromagnetically with the other cluster's OM IGM, as well as gravitationally with the other cluster.

SOMA suggests that SOMA might comport (regarding each cluster) with the interpretations of data, with one possible exception. The possible exception associates with the notion that SOMA suggests that

Table 14: Suggested explanations for observed ratios - that pertain to larger-than-galaxies-scale phenomena - of DM effects to OM effects. DM:OM denotes a ratio of DM effects to OM effects. Inferences of DM:OM ratios come from interpreting data. To the extent that only asymmetric evolution pertains, SOMA suggests that nature might not include significant amounts of isomer-zero stuff (such as the possible hadron-like particles that include just zero-charge elementary fermions and gluons) that would measure as DM. To the extent that only asymmetric measurement pertains, counterpart (to the isomer-zero stuff that measures as DM) stuff across all six isomers associates with the plus in DM:OM $5^+ : 1$.

Aspect	DM:OM	Comment
Densities of the universe	$5^+ : 1$	Each one of asymmetric evolution and asymmetric measurement might pertain.
Some galaxy clusters	$5^+ : 1$	SOMA suggests that galaxy clusters (that have not collided with other galaxy clusters) associate with DM:OM ratios that are similar to DM:OM ratios for densities of the universe. (Similarity to DM:OM ratios for many stage-4 galaxies also pertains.)

isomer-three IGM interacts electromagnetically and follows trajectories that are consistent with OM IGM trajectories.

Regarding the possible exception, at least three possibilities arise.

For one possibility, per table 7, the light that CC associates with OM IGM might include light that SOMA associates with OM IGM and light that SOMA associates with isomer-three IGM.

For one possibility, isomer-three IGM measures as DM and CC does not adequately report (or otherwise account for) lagging isomer-three IGM.

For one possibility, isomer-three IGM follows trajectories that are consistent with other DM trajectories.

SOMA suggests that interpretations of data may not be sufficient to rule out each one of the first two possibilities or to rule out a combination of the first two possibilities.

SOMA notions of DM are not necessarily incompatible with constraints - that have bases in observations of collisions of galaxy clusters - regarding DM.

4. Discussion

This unit suggests additional (compared to previous units) associations between POST modeling and SOMA modeling. This unit suggests additional (compared to previous units) associations between possible future data and SOMA modeling. This unit suggests (based on POST modeling and SOMA modeling) bases that might point to principles that – in the future – might underlie physics or physics modeling.

4.1. Possible elementary particles that POST has yet to include

SOMA might provide insight regarding the existence and properties of some as-yet-unfound elementary particles that POST hypothesizes or that SOMA might suggest.

4.1.1. Elementary particles that SOMA suggests

Table 4 catalogs as-yet-unfound elementary particles that SOMA suggests.

4.1.2. Right-handed W boson

Reference [115] discusses a fraction of decays - of OM top quarks for which the decay products include W bosons - that might produce right-handed W bosons. The fraction, f_+, is 3.6×10^{-4}. Reference [19] provides a confidence level of 90 percent that the rest energy of a might-be W_R (or, right-handed W boson) exceeds 715 GeV. Reference [116] provides other information.

SOMA suggests that W_R bosons associate only with isomers one, three, and five. SOMA suggests possibilities for inter-isomer interactions and conversions.

Aspects of SOMA might approximately reproduce the above result that SM modeling suggests.

Aspects related to equation (39) suggest values of calculated masses that do not associate with masses of known or suggested elementary particles. For example, SOMA does not suggest that $m(5,3)$ associates with the inertial mass of an isomer-one charged lepton. However, perhaps such mass-like quantities associate with some measurable aspects of nature. For charged leptons and $0 \leq l_I \leq 4$ and $0 \leq l'_f \leq 3$, $m(3(l_I+1)+l'_f,3) = \beta m(3(l_I+0)+l'_f,3)$. One might conjecture that isomer-zero observations of some aspects of isomer-one phenomena associate with notions of non-inertial mass-like quantities that are β times the inertial masses for isomer-zero elementary particles (and that are β times inertial masses for the counterpart isomer-one elementary particles).

Based on notions of scaling that might calculate non-inertial mass-like quantities, SOMA might suggest that $f_+ \sim e^{(\beta^{-1})} - 1 \approx \beta^{-1} \approx 2.9 \times 10^{-4}$. This estimate might not be incompatible with results that reference [115] discusses. A notion of $m_{\text{non-inertial,W}_R\text{isomer one}}c^2 = \beta m_{\text{W}}c^2 \approx 2.8 \times 10^5$ GeV might pertain. Here, the notion of a non-inertial mass-like quantity might associate with data that associate with interactions that associate with 1L or 1W_1. The interactions do not necessarily associate directly with 2L.

4.1.3. Magnetic monopole

Table 1 seems not to suggest a 1L interaction with a monopole other than an electric monopole. SOMA does not suggest a property that would associate with a magnetic monopole.

4.1.4. Some possible elementary particles that SOMA does not necessarily suggest

The notion of 0g and results that equation (26) suggest do not necessarily rule out possibilities for 1.5R and $1.5\text{Q}_{>0}$ elementary particles.

- 1.5R would associate with intrinsic $|-2-4+6|$ and with extrinsic $|-2+4-6+8|$.
- 1.5 $\text{Q}_{>0}$ would associate with intrinsic $|-2-6+8|$ and $|-2-4+6|$ and with extrinsic $|-2+4-6+8|$.

The relevant one-step cascade shares - with the one-step cascades that associate with the elementary particles that associate with the strong interaction - the notion that each one of two, four, six, and eight is a member of Z_Γ. Each other elementary particle that does not associate with the strong interaction does not associate with the notion that each one of two, four, six, and eight is a member of each one-step-cascade Z_Γ. SOMA does not necessarily rule out the notion that some dark matter consists of hadron-like particles that include gluons and 1.5R elementary particles.

The following notions might suggest that the might-be 1.5R and $1.5\text{Q}_{>0}$ elementary particles do not exist. For all elementary particles to which table 4 alludes, $1 \in Z_\Gamma$ and $3 \in Z_\Gamma$. For the might-be 1.5R and $1.5\text{Q}_{>0}$ elementary particles, neither $1 \in Z_\Gamma$ nor $3 \in Z_\Gamma$. Notions that associate with equation (32) and notions that associate with equation (39) might extrapolate to suggest that the might-be 1.5R and $1.5\text{Q}_{>0}$ elementary particles would associate with negative masses. Regarding the existence of might-be $1.5\text{Q}_{>0}$ elementary particles and not necessarily regarding the existence of the might-be 1.5R elementary particle, people have not found evidence of the might-be $1.5\text{Q}_{>0}$ elementary particles, which would have charge and might measure as OM.

4.2. Phenomena that might involve the SOMA-suggested jay boson elementary particle

SOMA points to interactions that might involve the might-be jay boson.

4.2.1. Pauli repulsion

POST includes the notion that two identical fermions cannot occupy the same state. Regarding QM, one notion is that repelling between identical fermions associates with overlaps of wave functions. Another QM notion features wave functions that are antisymmetric with respect to the exchange of two identical fermions.

SOMA might be compatible with such aspects of POST and, yet, not necessitate - regarding POST dynamics modeling - the use of wave functions. QM based on jay bosons might suffice. CM based on potentials that would associate with effects of jay bosons might suffice.

SOMA suggests that QM or CM based on jay bosons might suggest that the prevention of two identical fermions from occupying the same state might associate with, in effect, interactions - mediated by jay bosons - that try to change aspects related to the fermions. Notions of changing a spin orientation might pertain. For elementary fermions, notions of changing a flavour might pertain.

4.2.2. Energy levels in positronium

Reference [117] discusses the transition - between two states of positronium - characterized by the expression that equation (53) shows.

$$2^3S_1 \to 2^3P_0 \tag{53}$$

Four standard deviations below the nominal observed value of the energy that associates with the transition approximately equals four standard deviations above the nominal value of the energy that POST suggests.

SOMA notions regarding jay bosons might explain the might-be discrepancy regarding positronium. Compared to QFT, a new notion of virtual charge exchange or a new notion of virtual flavour change might pertain.

To the extent that QFT does not suffice to explain positronium energy levels, SOMA notions related to the jay boson might help to close the gap between observations and modeling.

4.2.3. Pauli crystals

Reference [118] reports detection of Pauli crystals. SOMA suggests that modeling based on the notion of jay bosons might help explain relevant phenomena.

4.3. Some phenomena that associate with galaxies

4.3.1. Some quenching of star formation

Some galaxies seem to stop forming stars. (Reference [119] and reference [120] discuss examples.) Such quenching might take place within three billion years after the Big Bang, might associate with a lack of hydrogen atoms, and might (per reference [120]) pertain to half of the galaxies that associate with the notion of a certain type of galaxy.

SOMA suggests that some such quenching might associate with repelling that associates with 2(2)g2‘4. Some quenching might associate with galaxies for which original clumps featured isomer-zero stuff or isomer-three stuff.

4.3.2. Some stopping of the accrual of matter

Reference [121] discusses a galaxy that seems to have stopped accruing both OM and DM about four billion years after the Big Bang.

The galaxy that reference [121] discusses might (or might not) associate with the notion of significant presence early on of one of isomer-zero and isomer-three, one of isomer-one and isomer-four, and one of isomer-two and isomer-five. Such early presences might associate with a later lack of nearby stuff for the galaxy to accrue.

4.3.3. Aspects regarding stellar stream GD-1 in the Milky Way galaxy

Data regarding stellar stream GD-1 suggest the possibility of effects from a yet-to-be-detected non-OM clump - in the Milky Way galaxy - with a mass of 10^6 to 10^8 solar masses. (References [122] and [123] provide data and discussion regarding the undetected object. Reference [123] cites reference [124] and reference [125].) SOMA suggests that the undetected object might be a clump of DM.

4.4. Some possibilities for detecting non-isomer-zero dark matter

Table 7 points to electromagnetic phenomena that associate with reaches of two and, thereby, suggests that OM equipment might be able to catalyze or detect transitions within isomer-three atoms. Discussion related to table 12 suggests that data point to detection, by OM equipment, of light emitted by transition events that associate with isomer-three atoms. Presumably, some isomer-three atoms pass (essentially unimpeded by isomer-zero stuff) through isomer-zero stuff that is near to and includes the Earth. SOMA suggests that experiments - based on light produced by OM stuff and light detected by OM stuff - might be able to detect (via transition events that associate with isomer-three atoms) isomer-three atomic stuff. This paper does not discuss notions regarding whether techniques are now or when techniques might become sufficiently sensitive that such experiments would be feasible.

4.5. Some information that gravitational waves might convey

Reference [126] discusses opportunities for research regarding gravitational waves.

Extending notions that associate with table 7 suggests intrinsic uses of 2g” solution-pairs that might associate with the producing of gravitational waves.

SOMA suggests that intrinsic use of the 2g1‘3‘4 solution-pair might be relevant. Intrinsic use of the 2g1‘3‘4 solution-pair associates with a reach of six. SOMA suggests that intrinsic use of 2g1‘3‘4 associates with significant aspects of the production of gravitational waves.

Intrinsic use of 2g4‘5‘7 associates with a reach of one. Intrinsic use of any one of 2g1‘4‘5, 2g3‘4‘5, and 2g1‘4‘7 associates with a reach of two. To the extent that intrinsic use of at least one such 2g” solution-pair is relevant regarding the producing of gravitational waves, SOMA suggests that adequately detailed analyses of the gravitational signatures that associate with collisions of objects - such as black holes - might enable the development of data that associate with the extents to which the colliding objects include stuff that associates with more than one isomer or more than one isomer-pair.

4.6. Modeling regarding gravity

Present GR is not necessarily adequately compatible with data.

Present GR is not necessarily adequately compatible with SOMA notions of DM. SOMA suggests that POST modeling based on GR can be less than adequately accurate.

Tests of GR have featured phenomena that associate with the isomer-pair that includes isomer-zero and isomer-three. Each one of the Sun, the planet Mercury, and the Earth associates with isomer-zero. Relevant radiation from distant stars and galaxies associates essentially just with isomer-zero stuff and isomer-three stuff.

For cases in which POST suggests that uses of general relativity adequately (or nearly adequately) comport with data, SOMA suggests that the following notions - about uses of SOMA solution-pairs and about the GR stress-energy tensor - might help bridge from SOMA to GR or from GR to SOMA. (The GR stress-energy tensor is symmetric.) Intrinsic 2g2 associates with the one stress-energy-tensor component (T^{00}) that associates with energy density. Extrinsic 2g2‘4 associates with the three components ($T^{01}, T^{02},$ and T^{03}) that associate with momentum density and also associates with the three components ($T^{10}, T^{20},$ and T^{30}) that associate with energy flux. Intrinsic 2g2‘4 associates with the three components ($T^{11}, T^{22},$ and T^{33}) that associate with pressure. Extrinsic 2g2‘4‘8 associates with the three components ($T^{21}, T^{31},$ and T^{32}) that associate with momentum flux and also associates with the three components ($T^{12}, T^{13},$ and T^{23}) that associate with shear stress. (This paper does not further explore the usefulness of such notions.)

Intrinsic 2g1‘2‘3 associates with $R_I = 1$. SOMA suggests that intrinsic 2g1‘2‘3 might associate with (at least some aspects of) the ND notion of a unique major axis of (rotational) inertia and a unique minor axis of (rotational) inertia. SOMA does not necessarily yet suggest associations between 2L solution-pairs that associate with $R_I = 1$ and the GR stress-energy tensor.

4.7. Patterns regarding elementary particles

Table 4 and discussion related to table 4 underlie this unit.

For this discussion, Z_Γ associates only with 0g intrinsic solution-pairs that associate with known elementary particles or with SOMA-suggested elementary particles.

For SOMA modeling regarding the W boson, the following aspects pertain for the one relevant intrinsic solution-pair.

- The notion of $4 \notin Z_\Gamma$ pertains. The notion of $Q = 1$ pertains.
- The notion of $6 \notin Z_\Gamma$ pertains. The notion of $m_b > 0$ pertains. The notion ofm_f does not pertain.
- The notions of $6 \notin Z_\Gamma$ and $n_{\Gamma^0} \neq 4$ pertain. The notion of $S_b > 0$ pertains. The notion of S_f does not pertain.
- The notion of $8 \notin Z_\Gamma$ pertains. The notion of $m_b = 0$ does not pertain.
- The notion of $16 \notin Z_\Gamma$ pertains. The notion of $n_I = 6$ pertains. The notion of $R_I = 1$ pertains.
- The notions of $6 \notin Z_\Gamma$ and $8 \notin Z_\Gamma$ pertain. POST models can treat the elementary particle as modeling as an object that is not part of a multi-component system.

Compared to aspects that pertain regarding the W boson, the following aspects pertain for intrinsic solution-pairs that associate with known elementary particles or with SOMA-suggested elementary particles.

- A change to $4 \in Z_\Gamma$ removes the notion of $Q = 1$ and installs the notion of $Q = 0$.
- A change to $6 \in Z_\Gamma$ removes the notion of m_b and installs the notion of $m_f > 0$.
- A change to $6 \notin Z_\Gamma$ and $n_{\Gamma^0} = 4$ removes the notion of $S_b > 0$ and installs the notion of $S_b = 0$.
- A change to $6 \in Z_\Gamma$ removes the notion of S_b and installs the notion of $S_f > 0$.
- A change to $8 \in Z_\Gamma$ and $6 \notin Z_\Gamma$ removes the notion of $m_b > 0$ and installs the notion of $m_b = 0$.

- A change to $16 \in Z_\Gamma$ and $128 \notin Z_\Gamma$ removes the notion of $n_I = 6$ and installs the notion of $R_I > 1$.
- A change to $6 \in Z_\Gamma$ and $8 \in Z_\Gamma$ removes the notion that POST models treat the elementary particle as modeling as an object that is not part of a multi-component system.

4.8. Tetrads, angular momentum states, and minimally comprehensive models

4.8.1. Tetrads and angular momentum states

Table 8 does not discuss spin.

Per discussion related to equation (49), the following four items might associate with a set that has a basis for which zero spin associates with $n_\Gamma = 3$ - $|-1-2+3|$, $|+1-2-3+4^\wedge|$, $|-1-2-3-6|$, and $|-1+2-3-4^\wedge+6|$.

The following notions suggest caution regarding the extent of the possible relevance for that set.

- $4 \in Z_\Gamma$ pertains for exactly two (and not zero or all) of the items that would be in the set.
- For elementary particles, per equation (25), zero spin associates with $n_\Gamma = 4$.

SOMA suggests that the set that associates with $|-1-2+3|$ might associate with spin states for a notion of no object (and no multi-component system).

4.8.2. Tetrads and minimally comprehensive models

SOMA suggests that the notion that $n_\Gamma \geq 4$ pertains for zero spin states (for objects) associates with a notion that some physics models need to associate with at least four tetrads. SOMA suggests that - for such models - the relevant tetrads need to include tetrads that associate with space-time dimensions or space-time coordinates (s_{-2}), isomer-pairs and objects (s_0), (electromagnetic) charge (s_1), and mass (s_2).

SOMA suggests the possibility that models that do not associate with - at least - the tetrads that associate with space-time dimensions (s_{-2}), isomer-pair and objects (s_0), (electromagnetic) charge (s_1), and mass (s_2) might not suffice for so-called minimally comprehensive modeling regarding DM phenomena and DE phenomena.

4.9. Tetrads and interrelations regarding properties of elementary particles

Regarding elementary bosons and equation (32), SOMA suggests the following notions. Each one of m', S, and Q associates with a tetrad. l_{b2} associates with a choice - that associates with gravity - between notions that might associate with the two-word term longitudinal aspects and the three-word term no longitudinal aspects. l_{b1} associates with a choice - that associates with electromagnetism - between notions that might associate with the two-word term longitudinal aspects and the three-word term no longitudinal aspects. SOMA suggests that equation (32) associates with three tetrads and two so-called (off-or-on) switches that associate with space-time coordinates. (Equation (30) and equation (31) associate with the respective switches.)

Regarding elementary fermions and equation (39), SOMA suggests the following notions. The set that consists of the four elements l_m, $j_m d_m$, $(1+l_m)n_q$, and $j_q d_q(l_m)$ associates with a notion of four tetrads. $S = 0.5$ associates with an implicit tetrad. (Each elementary fermion associates with $S = 0.5$. S does not appear in equation (39).) SOMA suggests that equation (39) associates with five tetrads.

SOMA suggests a notion that - for elementary bosons - three tetrads (instead of four tetrads for minimally comprehensive modeling regarding DM phenomena and DE phenomena) associates with a notion of $F_T = +1$ free tetrad. SOMA notes that $\int_0^n D^{F_T} dD = \int_0^n D^1 dD = n^2$ might associate with the notion that adding squares of integers interrelates properties of elementary bosons. Equation (32) tends to comport with that notion.

SOMA suggests a notion that - for elementary fermions - five tetrads (instead of four tetrads for minimally comprehensive modeling regarding DM phenomena and DE phenomena) associates with a notion of $F_T = -1$ free tetrads. SOMA notes that $\int_{n_{ref}}^n D^{F_T} dD = \int_{n_{ref}}^n D^{-1} dD = \log(n/n_{ref})$ might associate with the notion that adding logarithms of integer-related numbers interrelates (at least approximately) properties of elementary fermions. The five integer-related numbers feature (at least as factors) respectively l_m, j_m, $(1+l_m)$ and $2n_q$, j_q, and S. Equation (39) tends to comport with that notion.

This paper does not try to explore thoroughly the notion that there might be - for elementary fermions - an analog to the notions that equation (34) might pertain regarding all boson elementary particles and that the analogous set (for elementary fermions) would be a set of integers. (Discussion related to equation (67) might have relevance.)

4.10. Harmonic oscillator mathematics and relationships among elementary-particle properties

4.10.1. Some harmonic oscillator mathematics

Modeling for a j-dimensional isotropic harmonic oscillator can feature j linear coordinates $x_{k'}$ - each with a domain $-\infty < x_{k'} < \infty$ - and an operator that is the sum - over k' - of j operators of the form that equation (54) shows. The number C is positive and is common to all j uses of equation (54). The word isotropic associates with the commonality - across all j uses of equation (54) - of the number C.

$$-\frac{\partial^2}{\partial(x_{k'})^2} + C \cdot (x_{k'})^2 \tag{54}$$

For $j \geq 2$, one can split the overall operator into pieces. Equation (55) associates with a split into two pieces. Here, each of j_1 and j_2 is a positive integer.

$$j = j_1 + j_2 \tag{55}$$

In discussion below, the symbol D might be any one of j, j_1, and j_2.

For $D \geq 2$, mathematics related to isotropic harmonic oscillators can feature partial differential equations, a radial coordinate, and $D-1$ angular coordinates. Equation (56) defines a radial coordinate.

$$x = (\sum_{k'} (x_{k'})^2)^{1/2} \tag{56}$$

SOMA suggests replacing x via the expression that equation (57) shows. Here, r_{HO} denotes the radial coordinate and has dimensions of length. The parameter η has dimensions of length. The parameter η is a nonzero real number. The magnitude $|\eta|$ associates with a scale length. (Here, r_{HO} associates with mathematics for HO - or, harmonic oscillators - and does not necessarily associate with uses of r elsewhere - for example, in equation (1) - in this paper.)

$$x = r_{HO}/\eta \tag{57}$$

Applications of equations (58) and (59) can associate with POST. Each of ξ and ξ' is an as-yet unspecified constant. The symbol $\phi_R(r_{HO})$ denotes a function of r_{HO}. The symbol ${\nabla_{r_{HO}}}^2$ denotes a Laplacian operator. Ω associates with aspects that associate with angular coordinates. (For $D = 3$, reference [127] shows a representation for Ω in terms of an operator that is a function of spherical coordinates.)

$$\xi\phi_R(r_{HO}) = (\xi'/2)(-\eta^2{\nabla_{r_{HO}}}^2 + (\eta)^{-2}r_{HO}^2)\phi_R(r_{HO}) \tag{58}$$

$${\nabla_{r_{HO}}}^2 = r_{HO}^{-(D-1)}(\partial/\partial r_{HO})(r_{HO}^{D-1})(\partial/\partial r_{HO}) - \Omega r_{HO}^{-2} \tag{59}$$

SOMA applications assume that the symbol Ω is a constant. SOMA applications do not necessarily require that D is a positive integer for which $D \geq 2$. SOMA applications include solutions that pertain for the domain that equation (60) shows. With respect to the domain $0 \leq r_{HO} < \infty$, ϕ_R associates with the mathematics notion of having a definition almost everywhere. (Some aspects of POST applications associate with the following notions. D is a nonnegative integer. ϕ_R associates with a radial factor that is part of a representation of a wave function. For $D = 1$, equation (59) might not be appropriate. For $D > 1$, a representation of a wave function may need to include a factor for which angular coordinates play roles. The domain for a representation of such a wave function needs to include $r_{HO} = 0$. For SOMA applications, ϕ_R does not necessarily associate with the notion of a factor in a representation for a wave function and does not necessarily need to have a definition that associates with $r_{HO} = 0$.)

$$0 < r_{HO} < \infty \tag{60}$$

In discussion below, the symbol D might be any real number.

SOMA considers solutions of the form that equation (61) shows. (In POST, solutions that associate with equation (54) and with $D = 1$ have the form $H(x)\exp(-x^2)$, in which $H(x)$ is a Hermite polynomial. Mathematics that SOMA suggests can allow for a SOMA-adequately-useful set of solutions for which each solution associates with - in effect - a one-term polynomial.)

$$\phi_R(r_{HO}) \propto (r_{HO}/\eta)^\nu \exp(-r_{HO}^2/(2\eta^2)), \text{ with } \eta^2 > 0 \tag{61}$$

Equations (62) and (63) characterize solutions. The parameter η does not appear in these equations.

$$\xi = (D + 2\nu)(\xi'/2) \tag{62}$$

$$\Omega = \nu(\nu + D - 2) \tag{63}$$

$\phi_R(r_{HO})$ normalizes if and only if equation (64) pertains. The symbol $(\phi_R(r_{HO}))^*$ denotes the complex conjugate of $\phi_R(r_{HO})$.

$$\int_0^\infty (\phi_R(r_{HO}))^* \phi_R(r_{HO}) r_{HO}^{D-1} dr_{HO} < \infty \tag{64}$$

Equation (65) associates with the domains of D and ν for which normalization pertains for $\phi_R(r_{HO})$. For $D + 2\nu = 0$, normalization pertains in the limit $\eta^2 \to 0^+$. Regarding mathematics relevant to normalization for $D + 2\nu = 0$, the delta function that equation (66) shows pertains. Here, $(x')^2$ associates with r_{HO}^2 and 4ϵ associates with η^2. (Reference [128] provides equation (66).) The difference in domains, between $-\infty < x' < \infty$ and equation (60), is not material here.

$$D + 2\nu \geq 0 \tag{65}$$

$$\delta(x') = \lim{}_{\epsilon \to 0^+} (1/(2\sqrt{\pi\epsilon}))e^{-(x')^2/(4\epsilon)} \tag{66}$$

4.10.2. Possible associations with DOF-related aspects

In equation (59), Ω associates with the radial aspects of the Laplacian operator that associates with D dimensions. For $2P$ being a nonnegative integer, the notions of equation (63) and $P = \nu$ combine to produce $\Omega = P(P + D - 2)$. (For $D = 3$, $P = S$, and $2S$ being a nonnegative integer, Ω can associate with the POST notion $S(S+1)\hbar^2$ that POST associates with angular momentum.) SOMA suggests that D might associate with a number of DOF-related aspects.

4.10.3. Possible associations with relationships among properties of elementary particles

For boson elementary particles, equation (32) suggests links between mass, spin, and charge. The following notions might provide useful insight.

Each one of m', S, and Q is always non-negative. Perhaps, regarding each one of m', S, and Q, some modeling associates with two DOF-related aspects - positive quantity and zero quantity. Regarding mathematics associated with Laplacian operators, for $D = 2$ and for each of $P = m'$, $P = S$, and $P = Q$, the factor $P^2 = P(P + D - 2)$ pertains regarding aspects of the mathematics. Equation (32) includes a term $(m')^2$. Equation (32) includes a term S^2. Equation (32) includes a term Q^2.

Whether or not l_{b2} is nonzero associates with the POST notion of whether longitudinal aspects can pertain. Regarding mathematics associated with Laplacian operators, for $D = 2$ and for $P = (l_{b2})^{1/2}$, the factor $P^2 = P(P + D - 2)$ pertains regarding aspects of the mathematics. Equation (32) includes a term $l_{b2} = ((l_{b2})^{1/2})^2$.

Whether or not l_{b1} is nonzero associates with the POST notion of whether longitudinal aspects can pertain. Regarding mathematics associated with Laplacian operators, for $D = 2$ and for $P = (l_{b1})^{1/2}$, the factor $P^2 = P(P + D - 2)$ pertains regarding aspects of the mathematics. Equation (32) includes a term $l_{b1} = ((l_{b1})^{1/2})^2$.

Regarding the masses of boson elementary particles, possibly, $D = 2$ associates with two DOF-related aspects of which one aspect is $m_b > 0$ and the other aspect is $m_b = 0$. Regarding the masses of fermion elementary particles, possibly, $D = 1$ associates with one DOF-related aspect of $m_f > 0$. Here, the factor $P(P + D - 2) = P(P - 1)$ might pertain. SOMA suggests that modeling can consider that a total of three (as in two plus one) DOF-related aspects associate with the notion of elementary particle mass.

Regarding the spins of boson elementary particles, possibly, $D = 2$ associates with two DOF-related aspects of which one aspect is $S_b > 0$ and the other aspect is $S_b = 0$. Regarding the spins of fermion elementary particles, possibly, $D = 1$ associates with one DOF-related aspect of $S_f > 0$. Here, the factor $P(P + D - 2) = P(P - 1)$ might pertain. SOMA suggests that modeling can consider that a total of three (as in two plus one) DOF-related aspects associate with the notion of elementary particle spin.

This insight might provide a basis for modeling that would underlie SOMA.

4.10.4. Possible associations among flavours of charged elementary fermions

Per the numbering that SOMA uses regarding flavours of charged elementary fermions and per relationships that table 5 shows, equation (67) calculates quark flavours $l_f(0.5\mathrm{Q}_{>0})$.

$$l_f(0.5\mathrm{Q}_{>0}) = 1 + l_m \tag{67}$$

For quarks, $|B| = 1/3$. For leptons, $|L| = 1$.

Possibly, modeling regarding the lack - that table 5 shows - of complete alignment between $l_f(0.5\mathrm{C}_1)$ and $l_f(0.5\mathrm{Q}_{>0})$ associates with the notions - for charged leptons - of three values (zero, one, and two) for x, $y = (1+x)/3$ (or, the values 1/3, 2/3, and 1), three DOF-related aspects, $D = 3$, $P(P+D-2)$, and $P = y$. Here, $P(P+D-2) = y(y+1)$. For $x = 0$, $y(y+1) = 4/9 = (2/9)\cdot(1+2) = (2/9)\cdot(1+z)$ pertains, with $z = 2^{l_m}$ and $l_m = 0$. For $x = 1$, $y(y+1) = 10/9 = (2/9)\cdot(1+5) = (2/9)\cdot(1+z)$ pertains, with $z = 2^{l_m}$ and $l_m = 2$. For $x = 2$, $y(y+1) = 18/9 = (2/9)\cdot(1+8) = (2/9)\cdot(1+z)$ pertains, with $z = 2^{l_m}$ and $l_m = 3$.

Equation (68) pertains.

$$l_f(0.5\mathrm{C}_1) = 1 \leftrightarrow l_m = 0,\;\; l_f(0.5\mathrm{C}_1) = 2 \leftrightarrow l_m = 2,\;\; l_f(0.5\mathrm{C}_1) = 3 \leftrightarrow l_m = 3 \tag{68}$$

Each one of equation (67) and equation (68) comports with table 5.

4.11. Harmonic oscillator mathematics, gauge symmetries, and the Higgs mechanism

4.11.1. Possible associations with gauge symmetries

POST associates gauge symmetries with interactions that some elementary bosons intermediate. The strong interaction associates with $SU(3)$ symmetry. Electromagnetism associates with $U(1)$ symmetry. The weak interaction associates with $SU(2) \times U(1)$ symmetry, in which the $SU(2)$ aspect is a broken symmetry.

Table 4 lists one-step cascades that associate with elementary particles. SOMA suggests that such one-step cascades associate with interactions. The following rule might associate four DOF-related aspects with the strong interaction, four DOF-related aspects with the weak interaction, and two DOF-related aspects with electromagnetism.

- For each relevant boson, associate two DOF-related aspects with each one-step cascade for which $6 \in Z_\Gamma$.

The following process might associate aspects of SOMA with POST gauge symmetries.

- Associate a one-dimensional harmonic oscillator with each DOF-related aspect.
- Associate one harmonic oscillator (that associates with the rule above) with the QM notion of excitation state for the boson field.
- For the strong interaction, associate the remaining three harmonic oscillators with one three-dimensional isotropic harmonic oscillator. Associate the ground state of the three-dimensional isotropic harmonic oscillator with $SU(3)$ symmetry.
- For the electromagnetic interaction, associate the remaining one-dimensional harmonic oscillator with $U(1)$ symmetry.
- For each weak interaction boson (that is, the Z boson or the W boson), associate the remaining one-dimensional harmonic oscillator with $U(1)$ symmetry. Associate the two one-dimensional harmonic oscillators that associate with the other weak interaction boson (that is, respectively, the W boson or the Z boson) with one two-dimensional isotropic harmonic oscillator. Associate the ground state of the two-dimensional isotropic harmonic oscillator with $SU(2)$ symmetry. Associate the POST notion of broken symmetry with the difference between 0g1‘3‘4‘6 (which is the one-step cascade relevant to the Z boson) and 0g1‘2‘3‘6 (which is the one-step cascade relevant to the W boson).

4.11.2. A second harmonic oscillator state that associates with zero spin

Regarding positive integers D and negative integers 2ν, equation (65) points to solutions that POST might consider to associate with harmonic-oscillator-state energies that are less than ground-state energies. For example, $D = 3$, $\nu = -1$, $\phi_R(r_{HO}) \propto (r_{HO}/\eta)^{-1} \exp(-r_{HO}^2/(2\eta^2))$, and a lack of angular dependence might associate with $\Omega = 0$ and with a wave function that associates with an energy for which $(D+2\nu)/2 = 0.5$ pertains, whereas a POST ground state might associate with $D = 3$, $\nu = 0$, $\Omega = 0$, a lack of angular dependence, and a wave function that associates with an energy for which $(D+2\nu)/2 = 1.5$ pertains.

4.11.3. Possible associations with the Higgs mechanism

The SM includes the notion of a Higgs mechanism. The SM does not associate a gauge symmetry with interactions that the Higgs boson intermediates.

Regarding the Higgs boson, the above-mentioned rule (regarding DOF-related aspects for three other interactions that elementary bosons intermediate) points to two DOF-related aspects. Each of the other interactions associates with elementary bosons for which $S = 1$. For the Higgs boson, $S = 0$. SOMA suggests that excitations associate with one two-dimensional isotropic harmonic oscillator. Here, $D = 2$. The SM ground state for the Higgs field might associate with $\nu = 0$ and with an energy that associates with $(D + 2\nu)/2 = 1$. The presence of a Higgs boson might associate with $\nu = -1$ and with an energy that associates with $(D + 2\nu)/2 = 0$.

Because zero (the energy factor that associates with $\nu = -1$) is less than one (the energy factor that associates with $\nu = 0$), these SOMA notions might associate with the SM notion of the Higgs mechanism.

4.12. Some other possible applications of harmonic oscillator mathematics

POST includes notions in which non-integer numbers of dimensions D associate with the expression $\lim_{D\to j}(\cdots)$, in which j is a nonnegative integer such as a number of spatial dimensions. (This paper does not further discuss the notion that aspects such as equation (63) might prove useful regarding such POST notions.)

POST includes notions that associate with the two-element phrase point-like particle. Discussion related to equation (65) and equation (66) points to the possibility that notions that associate with $\lim_{(D+2\nu)\to 0^+}$ might associate with notions of point-like. (This paper does not further discuss the notion that aspects that associate with $\lim_{(D+2\nu)\to 0^+}$ might prove useful regarding POST notions of point-like.)

4.13. SOMA notions regarding some challenges to which POST points

4.13.1. Quantum gravity

POST points to difficulties regarding harmonizing GR and possible notions of quantum gravity.

SOMA suggests - based on notions regarding isomers - that applications of GR are not necessarily adequately accurate. SOMA suggests that SR might suffice.

SOMA suggests that - regarding gravity and SR - quantizing gravity might be similar to quantizing electromagnetism.

4.13.2. Unified theories

POST points to difficulties regarding developing a so-called Grand Unified Theory, which would unite - at high energies - into one force the electromagnetic interaction, the weak interaction, and the strong interaction. (Reference [129] provides an overview of grand unification.)

SOMA suggests the notion that modeling based on a combining of aspects of POST with SOMA notions of s_k and $|s|$ might associate with some notions of unifying the four POST forces (including gravitation) with each other.

SOMA suggests the notion that modeling based on a combining of aspects of POST with SOMA notions of s_k and $|s|$ might associate with some notions of unifying forces with other aspects of physics.

4.13.3. Many-body problem

SOMA associates internal states of elementary particles with 0g solution-pairs for which $\{1,3\} \subset Z_\Gamma$ and - for each positive odd number k for which $k \geq 5$ - $k \notin Z_\Gamma$. SOMA associates internal states of some two-component systems with 0g solution-pairs for which $\{5,7\} \cap Z_\Gamma \neq \emptyset$ and - for each positive odd number k for which $k \geq 9$ - $k \notin Z_\Gamma$.

SOMA suggests the notion that internal states of some three-component systems might associate with 0g solution-pairs for which $\{9,11\} \cap Z_\Gamma \neq \emptyset$.

4.13.4. Connections between classical modeling and quantum modeling

POST explores connections between continuous CM modeling and discrete QM modeling. (Reference [130] discusses aspects of this exploration and provides further references. Reference [131] discusses aspects regarding electromagnetism.) Such explorations tend to explore notions of developing discrete modeling from continuous bases.

SOMA features discrete notions. SOMA suggests possible relationships between continuous and discrete.

POST notions of DOF associate with continuous. SOMA notions of DOF-related aspects associate with continuous and with discrete.

Equation (12) includes aspects that associate with continuous and aspects that associate with discrete.

Steps from notions that associate with table 8 to notions that associate with table 9a to notions that associate with table 9b might provide insight regarding transitions - regarding modeling - from discrete (including QM) to continuous (including CM).

This paper does not address the extent to which people might consider that SOMA might provide bases for insight about connections between continuous CM modeling and discrete QM modeling.

4.13.5. Underlying principles or fundamental concepts

POST seeks to catalog underlying principles or fundamental concepts that might underlie physics or physics modeling.

This paper does not address the extent to which people might consider SOMA notions of DOF-related aspects and tetrads as bases for steps that might lead to principles that might underlie physics or physics modeling.

5. Concluding remarks

Each of the following sentences describes a physics challenge that has persisted for the most recent eighty or more years. Interrelate physics properties, properties of objects, and physics constants. Provide, for elementary particles, an analog to the periodic table for chemical elements. Describe bases for phenomena that POST (or, modeling that has bases in popular modeling notions of space-time coordinates) associates with the two-word term dark matter. Describe bases for phenomena that POST associates with the two-word term dark energy. Explain the overall evolution of the universe. Interrelate physics models. Develop a list of principles that underlie physics or physics modeling.

Physics amasses data that people can use as bases for developing and evaluating modeling aimed at addressing the challenges.

SOMA (or, modeling that - for a single object - points to multiple attributes) addresses those physics challenges and has bases in notions of degrees of freedom and in the following mathematics - integer arithmetic, multipole expansions, Diophantine equations, and multidimensional harmonic oscillators.

SOMA unites and decomposes aspects of electromagnetism and gravity. For each of those two long-range interactions, the decomposition associates with properties - of objects - that people can measure and that POST features. For electromagnetism, the properties include charge and magnetic moment. For gravity, the properties include energy.

SOMA points to all known elementary particles and to some might-be elementary particles. SOMA includes a notion of isomers of elementary particles that do not mediate long-range interactions. SOMA suggests that isomer is a new elementary-particle internal quantum number. SOMA features a notion of instances of components of long-range interactions. SOMA features a candidate description of dark matter.

SOMA suggests explanations for data regarding dark matter. SOMA points to possible resolutions for tensions - between data and POST - regarding effects of dark energy. SOMA suggests insight regarding galaxy formation and evolution.

SOMA matches data that POST matches, suggests explanations for data that POST seems not to explain, suggests results regarding data that people have yet to gather, and points to possible opportunities to develop models that unite aspects of physics and physics modeling.

In summary, SOMA suggests augmentations - to POST - that might achieve the following results. Extend the list of elementary particles. Predict masses for at least two neutrinos. Predict masses - that would be more accurate than known masses - for some other elementary particles. Describe dark matter. Explain ratios of dark matter effects to ordinary matter effects. Provide insight regarding galaxy formation. Describe bases for phenomena that associate with the two-word term dark energy. Explain eras in the history of the universe. Link properties of objects. Interrelate physics models. Point to steps toward improving notions regarding principles of physics.

Acknowledgments

The following people provided comments regarding the effectiveness of drafts that led to parts of this paper: Charles K. Chui, Mohamed Nassef Hussein Comsan, Immanuel Freedman, Ervin Goldfain, Vesselin Gueorguiev, Ali Khaledi-Nasab, Tom Lawrence, and Mak Tafazoli.

References

[1] Maria Becker, Adam Caprez, and Herman Batelaan. On the Classical Coupling between Gravity and Electromagnetism. *Atoms*, 3(3):320–338, June 2015. DOI 10.3390/atoms3030320. 2.2.2

[2] Maximo Banados, Glenn Barnich, Geoffrey Compere, and Andrés Gomberoff. Three-dimensional origin of Gödel spacetimes and black holes. *Phys. Rev. D*, 73:044006, February 2006. DOI 10.1103/PhysRevD.73.044006. 2.2.2

[3] Glenn Barnich and Andrés Gomberoff. Dyons with potentials: Duality and black hole thermodynamics. *Phys. Rev. D*, 78:025025, July 2008. DOI 10.1103/PhysRevD.78.025025. 2.2.2

[4] Jairzinho Ramos Medina. *Gravitoelectromagnetism (GEM): A Group Theoretical Approach*. PhD thesis, Drexel University, August 2006. Link: https://core.ac.uk/download/pdf/190333514.pdf. 2.2.2

[5] David Delphenich. Pre-Metric Electromagnetism as a Path to Unification. In *Unified Field Mechanics*. World Scientific, September 2015. Link: https://arxiv.org/ftp/arxiv/papers/1512/1512.05183.pdf. 2.2.2

[6] Giorgio Papini. Some Classical and Quantum Aspects of Gravitoelectromagnetism. *Entropy*, 22(10):1089, September 2020. DOI: 10.3390/e22101089. 2.2.2

[7] Steve Nadis. Mass and Angular Momentum, Left Ambiguous by Einstein, Get Defined. *Quanta Magazine*, July 2022. Link: https://www.quantamagazine.org/mass-and-angular-momentum-left-ambiguous-by-einstein-get-defined-20220713. 2.2.2

[8] Nick Gorkavyi and Alexander Vasilkov. A repulsive force in the Einstein theory. *Monthly Notices of the Royal Astronomical Society*, 461(3):2929–2933, July 2016. DOI 10.1093/mnras/stw1517. 2.2.2

[9] T. Damour. 21: Experimental Tests of Gravitational Theory. In P. A. Zla and others (Particle Data Group), Prog. Theor. Exp. Phys, 083C01 (2020) and 2021 update, 2019. DOI 10.1093/ptep/ptaa104. 2.2.2

[10] M. Kramer, I. H. Stairs, R. N. Manchester, N. Wex, A. T. Deller, W. A. Coles, M. Ali, M. Burgay, F. Camilo, I. Cognard, et al. Strong-Field Gravity Tests with the Double Pulsar. *Phys. Rev. X*, 11(4):041050, December 2021. DOI 10.1103/physrevx.11.041050. 2.2.2

[11] C. W. F. Everitt, D. B. DeBra, B. W. Parkinson, J. P. Turneaure, J. W. Conklin, M. I. Heifetz, G. M. Keiser, A. S. Silbergleit, T. Holmes, J. Kolodziejczak, et al. Gravity Probe B: Final Results of a Space Experiment to Test General Relativity. *Phys. Rev. Lett.*, 106:221101, May 2011. DOI 10.1103/PhysRevLett.106.221101. 2.2.2

[12] Pierre Touboul, Gilles Metris, Manuel Rodrigues, Joel Berge, Alain Robert, Quentin Baghi, Yves Andre, Judicael Bedouet, Damien Boulanger, et al. *MICROSCOPE* Mission: Final Results of the Test of the Equivalence Principle. *Phys. Rev. Lett.*, 129:121102, September 2022. DOI 10.1103/PhysRevLett.129.121102. 2.2.2

[13] Robert Mann. *An Introduction to Particle Physics and the Standard Model*. CRC Press, November 2009. DOI 10.1201/9781420083002. 2.2.2

[14] S. Gasiorowicz and P. Langacker. Elementary Particles in Physics. University of Pennsylvania. Link: https://www.physics.upenn.edu/ pgl/e27/E27.pdf. 2.2.2

[15] A. Hebecker and J. Hisano. 94: Grand Unified Theories. In P. A. Zyla and others (Particle Data Group), Prog. Theor. Exp. Phys, 083C01 (2020) and 2021 update, 2019. DOI 10.1093/ptep/ptaa104. 2.2.2

[16] A. Ringwald, L. J. Rosenberg, and G. Rybka. 91: Axions and Other Similar Particles. In P. A. Zyla and others (Particle Data Group), Prog. Theor. Exp. Phys, 083C01 (2020) and 2021 update, 2019. DOI 10.1093/ptep/ptaa104. 2.2.2

[17] S. Rolli and M. Tanabashi. 95: Leptoquarks. In P. A. Zyla and others (Particle data Group), Prog. Theor. Exp. Phys, 083C01 (2020) and 2021 update, 2019. DOI 10.1093/ptep/ptaa104. 2.2.2

[18] D. Milstead and E. J. Weinberg. 96: Magnetic Monopoles. In P. A. Zyla and others (Particle Data Group), Prog. Theor. Exp. Phys, 083C01 (2020) and 2021 update, 2019. DOI 10.1093/ptep/ptaa104. 2.2.2

[19] P. A. Zyla et al. Review of Particle Physics. *PTEP*, 2020(8):083C01, 2020. DOI 10.1093/ptep/ptaa104. 2.2.2, 3.3.6, 4.1.2

[20] Brian Green. *Until the End of Time: Mind, Matter, and Our Search for Meaning in an Evolving Universe.* Alfred A. Knopf, February 2020. Link: https://www.penguinrandomhouse.com/books/549600/until-the-end-of-time-by-brian-greene/. 2.2.2, 3.9

[21] Charles W. Misner, Kip S. Thorne, and John Archibald Wheeler. *Gravitation.* University of Princeton Press, October 2017. Link: https://press.princeton.edu/books/hardcover/9780691177793/gravitation. 2.2.2

[22] M. C. Gonzalez-Garcia and M. Yokoyama. 14: Neutrino Masses, Mixing, and Oscillations. In P. A. Zyla and others (Particle Data Group), Prog. Theor. Exp. Phys, 083C01 (2020) and 2021 update, 2019. DOI 10.1093/ptep/ptaa104. 2.2.2, 3.3.5

[23] P. Vogel and A. Piepke. Neutrino Properties. In P. A. Zyla and others (Particle Data Group), Prog. Theor. Exp. Phys, 083C01 (2020) and 2021 update, August 2019. DOI 10.1093/ptep/ptaa104. 2.2.2

[24] E. Elfgren and S. Fredriksson. Mass limits for heavy neutrinos. *Astronomy and Astrophysics*, 479(2):347–353, December 2007. DOI 10.1051/0004-6361:20078898. 2.2.2

[25] Matthew D. Schwartz. *Quantum Field Theory and the Standard Model.* Cambridge University Press, December 2013. DOI 10.1017/9781139540940. 2.2.2, 3.1

[26] P. A. M. Dirac. The Theory of Magnetic Poles. *Phys. Rev.*, 74:817–830, October 1948. DOI 10.1103/PhysRev.74.817. 2.2.2

[27] R. Abbasi, M. Ackermann, J. Adams, J. A. Aguilar, M. Ahlers, M. Ahrens, C. Alispach, A. A. Alves, N. M. Amin, R. An, et al. Search for Relativistic Magnetic Monopoles with Eight Years of IceCube Data. *Phys. Rev. Lett.*, 128:051101, February 2022. DOI 10.1103/PhysRevLett.128.051101. 2.2.2

[28] Silvio Bonometto, Vittorio Gorini, and Ugo Moschella, editors. *Modern Cosmology.* Institute of Physics Publishing, 2002. ISBN: 10.3847/1538-4357/ab2873 Link: https://www.routledge.com/Modern-Cosmology/Bonometto-Gorini-Moschella/p/book/9780750308106. 2.2.2

[29] Kip S. Thorne and Roger D. Blandford. *Relativity and Cosmology.* Princeton University Press, 2021. ISBN 9780691207391. 2.2.2

[30] K. A. Olive and J. A. Peacock. 22: Big-Bang Cosmology. In P. A. Zyla and others (Particle Data Group), Prog. Theor. Exp. Phys, 083C01 (2020) and 2021 update, 2019. DOI 10.1093/ptep/ptaa104. 2.2.2

[31] J. Ellis and D. Wands. 23: Inflation. In P. A. Zyla and others (Particle Data Group), Prog. Theor. Exp. Phys, 083C01 (2020) and 2021 update, 2019. DOI 10.1093/ptep/ptaa104. 2.2.2

[32] D. H. Weinberg and M. White. 28: Dark Energy. In P. A. Zyla and others (Particle Data Group), Prog. Theor. Exp. Phys, 083C01 (2020) and 2021 update, 2019. DOI 10.1093/ptep/ptaa104. 2.2.2

[33] Wendy L. Freedman and Barry F. Madore. The Hubble Constant. *Annu Rev Astron Astrophys*, 48(1):673–710, 2010. DOI 10.1146/annurev-astro-082708-101829. 2.2.2

[34] I. Olivares-Salaverri and Marcelo B. Ribeiro. Testing cosmological models with the brightness profile of distant galaxies. *Astrophysics and Space Science*, 366(11), November 2021. DOI 10.1007/s10509-021-04016-3. 2.2.2

[35] Justin Khoury, Burt A. Ovrut, Nathan Seiberg, Paul J. Steinhardt, and Neil Turok. From big crunch to big bang. *Phys. Rev. D*, 65:086007, April 2002. DOI 10.1103/PhysRevD.65.086007. 2.2.2, 3.9

[36] Tao Zhu, Anzhong Wang, Gerald Cleaver, Klaus Kirsten, and Qin Sheng. Pre-inflationary universe in loop quantum cosmology. *Phys. Rev. D*, 96:083520, October 2017. DOI 10.1103/PhysRevD.96.083520. 2.2.2, 3.9

[37] Alessandra Silvestri and Mark Trodden. Approaches to understanding cosmic acceleration. *Rep. Prog. Phys.*, 72(9):096901, August 2009. DOI 10.1088/0034-4885/72/9/096901. 2.2.2, 3.9

[38] Eleonora Di Valentino. Challenges of the Standard Cosmological Model. *Universe*, 8(8), August 2022. DOI 10.3390/universe8080399. 2.2.2

[39] Marc Kamionkowski and Adam G. Riess. The Hubble Tension and Early Dark Energy, 2022. DOI: 10.48550/ARXIV.2211.04492. 2.2.2

[40] L. Verde, T. Treu, and A. G. Riess. Tensions between the early and late Universe. *Nature Astronomy*, 3(10):891–895, September 2019. DOI 10.1038/s41550-019-0902-0. 2.2.2, 3.12.1

[41] Johanna L. Miller. Gravitational-lensing measurements push Hubble-constant discrepancy past 5σ. *Physics Today*, 2020(1):0210a, February 2020. DOI 10.1063/pt.6.1.20200210a. 2.2.2, 3.12.1

[42] Thomas Lewton. What Might Be Speeding Up the Universe's Expansion? *Quanta Magazizne*, May 2020. Link: https://www.quantamagazine.org/why-is-the-universe-expanding-so-fast-20200427/. 2.2.2, 3.12.1

[43] Christopher Wanjek. Dark Matter Appears to be a Smooth Operator. *Mercury*, 49(3):10–11, October 2020. Link: https://astrosociety.org/news-publications/mercury-online/mercury-online.html/article/2020/12/10/dark-matter-appears-to-be-a-smooth-operator. 2.2.2, 3.12.1, 3.12.2

[44] A. Del Popolo. Dark matter, density perturbations, and structure formation. *Astronomy Reports*, 51(3):169–196, March 2007. DOI 10.1134/s1063772907030018. 2.2.2

[45] Joshua D. Simon and Marla Geha. Illuminating the darkest galaxies. *Physics Today*, 74(11):30–36, November 2021. DOI 10.1063/pt.3.4879. 2.2.2, 3.13.1, 3.14.2

[46] Paul Bode, Jeremiah P. Ostriker, and Neil Turok. Halo Formation in Warm Dark Matter Models. *The Astrophysical Journal*, 556(1):93–107, July 2001. DOI 10.1086/321541. 2.2.2

[47] L. Baudis and S. Profumo. 27: Dark Matter. In P. A. Zyla and others (Particle Data Group), Prog. Theor. Exp. Phys, 083C01 (2020) and 2021 update, 2019. DOI 10.1093/ptep/ptaa104. 2.2.2

[48] Kimberly K. Boddy, Mariangela Lisanti, Samuel D. McDermott, Nicholas L. Rodd, Christoph Weniger, Yacine Ali-Haimoud, Malte Buschmann, Ilias Cholis, Djuna Croon, et al. Snowmass2021 theory frontier white paper: Astrophysical and cosmological probes of dark matter. *Journal of High Energy Astrophysics*, 35:112–138, August 2022. DOI 10.1016/j.jheap.2022.06.005. 2.2.2

[49] Lotty Ackerman, Matthew R. Buckley, Sean M. Carroll, and Marc Kamionkowski. Dark matter and dark radiation. *Physical Review D*, 79:023519, January 2009. DOI 10.1103/PhysRevD.79.023519. 2.2.2

[50] James S. Bolton, Andrea Caputo, Hongwan Liu, and Matteo Viel. Comparison of Low-Redshift Lyman-α Forest Observations to Hydrodynamical Simulations with Dark Photon Dark Matter. *Phys. Rev. Lett.*, 129:211102, November 2022. DOI: 10.1103/PhysRevLett.129.211102. 2.2.2

[51] David E. Kaplan, Gordan Z. Krnjaic, Keith R. Rehermann, and Christopher M. Wells. Atomic dark matter. *Journal of Cosmology and Astroparticle Physics*, 2010(05):021–021, May 2010. DOI: 10.1088/1475-7516/2010/05/021. 2.2.2

[52] Francis-Yan Cyr-Racine and Kris Sigurdson. Cosmology of atomic dark matter. *Physical Review D*, 87(10):103515, May 2013. DOI: https://doi.org/10.1103/PhysRevD.87.103515. 2.2.2

[53] James Gurian, Michael Ryan, Sarah Schon, Donghui Jeong, and Sarah Shandera. A Lower Bound on the Mass of Compact Objects from Dissipative Dark Matter. *The Astrophysical Journal Letters*, 939(1):L12, October 2022. DOI: 10.3847/2041-8213/ac997c. 2.2.2

[54] Yu-Dai Tsai, Robert McGehee, and Hitoshi Murayama. Resonant Self-Interacting Dark Matter from Dark QCD. *Physical Review Letters*, 128(17):172001, April 2022. DOI 10.1103/physrevlett.128.172001. 2.2.2

[55] Man Ho Chan. Two mysterious universal dark matter–baryon relations in galaxies and galaxy clusters. *Physics of the Dark Universe*, 38:101142, December 2022. DOI: 10.1016/j.dark.2022.101142. 2.2.2

[56] Houjun Mo, Frank van den Bosch, and Simon White. *Galaxy Formation and Evolution*. Cambridge University Press, Cambridge, UK, 2010. Link: https://www.cambridge.org/us/academic/subjects/physics/astrophysics/galaxy-formation-and-evolution-1. 2.2.2

[57] Kyu-Hyun Chae, Federico Lelli, Harry Desmond, Stacy S. McGaugh, Pengfei Li, and James M. Schombert. Testing the Strong Equivalence Principle: Detection of the External Field Effect in Rotationally Supported Galaxies. *The Astrophysical Journal*, 904(1):51, November 2020. DOI 10.3847/1538-4357/abbb96. 2.2.2, 3.12.3

[58] John David Jackson. *Classical Electrodynamics*. WILEY, third edition, August 1998. Link: https://www.wiley.com/en-us/Classical Electrodynamics, 3rd Edition-p-9780471309321. 2.2.2

[59] Ioannis Haranas and Michael Harney. Detection of the Relativistic Corrections to the GravitationalPotential Using a Sagnac Interferometer. *Progress in Physics*, 3:3, July 2008. Link: http://www.ptep-online.com/complete/PiP-2008-03.pdf. 2.2.2

[60] Daniel A. Russell, Joseph P. Titlow, and Ya-Juan Bemmen. Acoustic monopoles, dipoles, and quadrupoles: An experiment revisited. *Am. J. Phys.*, 67(8):660–664, August 1999. DOI 10.1119/1.19349. 2.2.2

[61] Jean-Pierre Amiet and Stefan Weigert. Commensurate harmonic oscillators: Classical symmetries. *Journal of Mathematical Physics*, 43(8):4110–4126, August 2002. DOI 10.1063/1.1488672. 2.6.3

[62] R. L. Workman and Others. Review of Particle Physics. *PTEP*, 2022:083C01, 2022. DOI: 10.1093/ptep/ptac097. 3.2, 3.3.1, 3.3.3, 3.3.3, 3.3.4, 3.3.6, 3.14.3

[63] T. Aaltonen, S. Amerio, D. Amidei, A. Anastassov, A. Annovi, J. Antos, G. Apollinari, J. A. Appel, T. Arisawa, et al. High-precision measurement of the W boson mass with the CDF II detector. *Science*, 376(6589):170–176, April 2022. DOI 0.1126/science.abk1781. 3.2

[64] G. A. Gonzalez-Sprinberg and J. Vidal. Tau magnetic moment. *Proceedings of The International Conference On Nanoscience and Technology*, 912(1):012001, October 2017. DOI 10.1088/1742-6596/912/1/012001. 3.3.4

[65] Isabelle Tanseri, Steffen Hagstotz, Sunny Vagnozzi, Elena Giusarma, and Katherine Freese. Updated neutrino mass constraints from galaxy clustering and CMB lensing-galaxy cross-correlation measurements. *Journal of High Energy Astrophysics*, July 2022. DOI 10.1016/j.jheap.2022.07.002. 3.3.6

[66] Sunny Vagnozzi, Elena Giusarma, Olga Mena, Katherine Freese, Martina Gerbino, Shirley Ho, and Massimiliano Lattanzi. Unveiling ν secrets with cosmological data: Neutrino masses and mass hierarchy. *Phys. Rev. D*, 96:123503, December 2017. DOI 10.1103/PhysRevD.96.123503. 3.3.6

[67] M. Markevitch, A. H. Gonzalez, D. Clowe, A. Vikhlinin, W. Forman, C. Jones, S. Murray, and W. Tucker. Direct Constraints on the Dark Matter Self-Interaction Cross Section from the Merging Galaxy Cluster 1E 0657-56. *Astrophysical Journal*, 606(2):819–824, May 2004. DOI 10.1086/383178. 3.4, 3.14.4

[68] Leonardo Banchi and Francesco Caravelli. Geometric phases and cyclic isotropic cosmologies. *Classical and Quantum Gravity*, 33(10):105003, April 2016. DOI 10.1088/0264-9381/33/10/105003. 3.9

[69] Mark P. Hertzberg. Structure Formation in the Very Early Universe. *Physics Magazine*, 13(26), February 2020. DOI 10.1103/physics.13.16. 3.9

[70] Martin Bucher, Alfred S. Goldhaber, and Neil Turok. Open universe from inflation. *Phys. Rev. D*, 52:3314–3337, September 1995. DOI 10.1103/PhysRevD.52.3314. 3.9

[71] N. G. Busca, T. Delubac, J. Rich, S. Bailey, A. Font-Ribera, D. Kirkby, J.-M. Le Goff, M. M. Pieri, A. Slosar, E. Aubourg, et al. Baryon acoustic oscillations in the Lya forest of BOSS quasars. *Astronomy and Astrophysics*, 552(A96), April 2013. DOI 10.1051/0004-6361/201220724. 3.9

[72] S. Perlmutter, G. Aldering, G. Goldhaber, R. A. Knop, P. Nugent, P. G. Castro, S. Deustua, S. Fabbro, A. Goobar, Groom, et al. Measurements of Ω and Λ from 42 high-redshift supernovae Ω. *Astrophysical Journal*, 517(2):565–586, June 1999. DOI 10.1086/307221. 3.9

[73] Adam G. Riess, Alexei V. Filippenko, Peter Challis, Alejandro Clocchiatti, Alan Diercks, Peter M. Garnavich, Ron L. Gilliland, Craig J. Hogan, Saurabh Jha, Robert P. Kirshner, et al. Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. *Astronomical Journal*, 116(3):1009–1038, September 1998. DOI 10.1086/300499. 3.9

[74] Adam G. Riess, Louis-Gregory Strolger, John Tonry, Stefano Casertano, Henry C. Ferguson, Bahram Mobasher, Peter Challis, Alexei V. Filippenko, Saurabh Jha, Weidong Li, et al. Type Ia Supernova Discoveries at z $>$ 1 from the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution. *Astrophysical Journal*, 607(2):665–687, June 2004. DOI 10.1086/383612. 3.9

[75] Natalie Wolchover. New Wrinkle Added to Cosmology's Hubble Crisis. *Quanta Magazine*, February 2020. Link: https://www.quantamagazine.org/new-wrinkle-added-to-cosmologys-hubble-crisis-20200226/. 3.12.1

[76] Wendy L. Freedman, Barry F. Madore, Taylor Hoyt, In Sung Jang, Rachael Beaton, Myung Gyoon Lee, Andrew Monson, Jill Neeley, and Jeffrey Rich. Calibration of the Tip of the Red Giant Branch (TRGB). *Astrophysical Journal*, 891(1):57, March 2020. DOI 10.3847/1538-4357/ab7339. 3.12.1

[77] Vivian Poulin, Tristan L. Smith, Tanvi Karwal, and Marc Kamionkowski. Early Dark Energy can Resolve the Hubble Tension. *Physical Review Letters*, 122(22):221301, June 2019. DOI 10.1103/physrevlett.122.221301. 3.12.1

[78] Eleonora Di Valentino, Luis A. Anchordoqui, Ozgur Akarsu, Yacine Ali-Haimoud, Luca Amendola, Nikki Arendse, Marika Asgari, Mario Ballardini, Spyros Basilakos, Elia Battistelli, et al. Snowmass2021 - Letter of interest cosmology intertwined II: The hubble constant tension. *Astroparticle Physics*, 131:102605, 2021. DOI 10.1016/j.astropartphys.2021.102605. 3.12.1

[79] Francis-Yan Cyr-Racine, Fei Ge, and Lloyd Knox. Symmetry of Cosmological Observables, a Mirror World Dark Sector, and the Hubble Constant. *Phys. Rev. Lett.*, 128:201301, May 2022. DOI 10.1103/PhysRevLett.128.201301. 3.12.1

[80] Helena Garcia Escudero, Jui-Lin Kuo, Ryan E. Keeley, and Kevork N. Abazajian. Early or phantom dark energy, self-interacting, extra, or massive neutrinos, primordial magnetic fields, or a curved universe: An exploration of possible solutions to the H_0 and σ_8 problems. *Phys. Rev. D*, 106:103517, November 2022. DOI: 10.1103/PhysRevD.106.103517. 3.12.1

[81] Charlie Wood. A New Cosmic Tension: The Universe Might Be Too Thin. *Quanta Magazine*, September 2020. Link: https://www.quantamagazine.org/a-new-cosmic-tension-the-universe-might-be-too-thin-20200908/. 3.12.2

[82] Khaled Said, Matthew Colless, Christina Magoulas, John R. Lucey, and Michael J. Hudson. Joint analysis of 6dFGS and SDSS peculiar velocities for the growth rate of cosmic structure and tests of gravity. *Monthly Notices of The Royal Astronomical Society*, 497(1):1275–1293, July 2020. DOI 10.1093/mnras/staa2032. 3.12.2

[83] Supranta S. Boruah, Michael J. Hudson, and Guilhem Lavaux. Cosmic flows in the nearby Universe: new peculiar velocities from SNe and cosmological constraints. *Monthly Notices of The Royal Astronomical Society*, August 2020. DOI 10.1093/mnras/staa2485. 3.12.2

[84] Tod R. Lauer, Marc Postman, Harold A. Weaver, John R. Spencer, S. Alan Stern, Marc W. Buie, Daniel D. Durda, Carey M. Lisse, A. R. Poppe, et al. New Horizons Observations of the Cosmic Optical Background. *The Astrophysical Journal*, 906(2):77, January 2021. DOI 10.3847/1538-4357/abc881. 3.14.1

[85] Jose Luis Bernal, Gabriela Sato-Polito, and Marc Kamionkowski. Cosmic Optical Background Excess, Dark Matter, and Line-Intensity Mapping. *Physical Review Letters*, 129(23):231301, November 2022. DOI: 10.1103/physrevlett.129.231301. 3.14.1

[86] Tod R. Lauer, Marc Postman, John R. Spencer, Harold A. Weaver, S. Alan Stern, G. Randall Gladstone, Richard P. Binzel, Daniel T. Britt, Marc W. Buie, et al. Anomalous Flux in the Cosmic Optical Background Detected with New Horizons Observations. *The Astrophysical Journal Letters*, 927(1):L8, March 2022. DOI: 10.3847/2041-8213/ac573d. 3.14.1

[87] Judd D. Bowman, Alan E. E. Rogers, Raul A. Monsalve, Thomas J. Mozdzen, and Nivedita Mahesh. An absorption profile centred at 78 megahertz in the sky-averaged spectrum. *Nature*, 555(7694):67–70, March 2018. DOI 10.1038/nature25792. 3.14.1

[88] Rennan Barkana. Possible interaction between baryons and dark-matter particles revealed by the first stars. *Nature*, 555(7694):71–74, March 2018. DOI 10.1038/nature25791. 3.14.1

[89] Paolo Panci. 21-cm line Anomaly: A brief Status. In *33rd Rencontres de Physique de La Vallee d'Aoste*, July 2019. Link: https://cds.cern.ch/record/2688533. 3.14.1

[90] Peter Behroozi, Risa Wechsler, Andrew Hearin, and Charlie Conroy. UniverseMachine: The correlation between galaxy growth and dark matter halo assembly from z = 0-10. *Monthly Notices of The Royal Astronomical Society*, 488(3):3143–3194, May 2019. DOI 10.1093/mnras/stz1182. 3.14.2

[91] R. Genzel, N. M. Forster Schreiber, H. Ubler, P. Lang, T. Naab, R. Bender, L. J. Tacconi, E. Wisnioski, S. Wuyts, T. Alexander, et al. Strongly baryon-dominated disk galaxies at the peak of galaxy formation ten billion years ago. *Nature*, 543(7645):397–401, March 2017. DOI 10.1038/nature21685. 3.14.2

[92] Pieter van Dokkum, Roberto Abraham, Jean Brodie, Charlie Conroy, Shany Danieli, Allison Merritt, Lamiya Mowla, Aaron Romanowsky, and Jielai Zhang. A High Stellar Velocity Dispersion and ˜100 Globular Clusters for the Ultra-diffuse Galaxy Dragonfly 44. *Astrophysical Journal*, 828(1):L6, August 2016. DOI 10.3847/2041-8205/828/1/l6. 3.14.2

[93] Shannon Hall. Ghost galaxy is 99.99 per cent dark matter with almost no stars. *New Scientist*, August 2016. Link: https://www.newscientist.com/article/2102584-ghost-galaxy-is-99-99-per-cent-dark-matter-with-almost-no-stars/. 3.14.2

[94] Pavel E. Mancera Pina, Filippo Fraternali, Elizabeth A. K. Adams, Antonino Marasco, Tom Oosterloo, Kyle A. Oman, Lukas Leisman, Enrico M. di Teodoro, Lorenzo Posti, Michael Battipaglia, et al. Off the Baryonic Tully-Fisher Relation: A Population of Baryon-dominated Ultra-diffuse Galaxies. *Astrophysical Journal*, 883(2):L33, September 2019. DOI 10.3847/2041-8213/ab40c7. 3.14.2

[95] Pavel E. Mancera Pina, Filippo Fraternali, Tom Oosterloo, Elizabeth A. K. Adams, Kyle A. Oman, and Lukas Leisman. No need for dark matter: resolved kinematics of the ultra-diffuse galaxy AGC 114905. *Mon. Not. R. Astron Soc.*, December 2021. DOI 10.1093/mnras/stab3491. 3.14.2

[96] Qi Guo, Huijie Hu, Zheng Zheng, Shihong Liao, Wei Du, Shude Mao, Linhua Jiang, Jing Wang, Yingjie Peng, Liang Gao, et al. Further evidence for a population of dark-matter-deficient dwarf galaxies. *Nature Astronomy*, 4(3):246–251, November 2019. DOI 10.1038/s41550-019-0930-9. 3.14.2

[97] Pieter van Dokkum, Shany Danieli, Roberto Abraham, Charlie Conroy, and Aaron J. Romanowsky. A Second Galaxy Missing Dark Matter in the NGC 1052 Group. *Astrophysical Journal*, 874(1):L5, March 2019. DOI 10.3847/2041-8213/ab0d92. 3.14.2

[98] Kristi A Webb, Alexa Villaume, Seppo Laine, Aaron J Romanowsky, Michael Balogh, Pieter van Dokkum, Duncan A Forbes, Jean Brodie, Christopher Martin, and Matt Matuszewski. Still at odds with conventional galaxy evolution: the star formation history of ultradiffuse galaxy Dragonfly 44. *Monthly Notices of the Royal Astronomical Society*, 516(3):3318–3341, August 2022. DOI 10.1093/mnras/stac2417. 3.14.2

[99] R. Herrera-Camus, N. M. Forster Schreiber, S. H. Price, H. Ubler, A. D. Bolatto, R. L. Davies, D. Fisher, R. Genzel, D. Lutz, T. Naab, et al. Kiloparsec view of a typical star-forming galaxy when the Universe was ∼1 Gyr old. *Astronomy and Astrophysics*, 665:L8, September 2022. DOI: 10.1051/0004-6361/202142562. 3.14.2

[100] Charles Day. A primordial merger of galactic building blocks. *Physics Today*, 2021(1):0614a, June 2021. DOI 10.1063/PT.6.1.20210614a. 3.14.2

[101] Yuta Tarumi, Naoki Yoshida, and Anna Frebel. Formation of an Extended Stellar Halo around an Ultra-faint Dwarf Galaxy Following One of the Earliest Mergers from Galactic Building Blocks. *The Astrophysical Journal Letters*, 914(1):L10, June 2021. DOI 10.3847/2041-8213/ac024e. 3.14.2

[102] Elena Asencio, Indranil Banik, Steffen Mieske, Aku Venhola, Pavel Kroupa, and Hongsheng Zhao. The distribution and morphologies of Fornax Cluster dwarf galaxies suggest they lack dark matter. *Mon Not R Astron Soc*, June 2022. DOI 10.1093/mnras/stac1765. 3.14.2

[103] Massimo Meneghetti, Guido Davoli, Pietro Bergamini, Piero Rosati, Priyamvada Natarajan, Carlo Giocoli, Gabriel B. Caminha, R. Benton Metcalf, Elena Rasia, Stefano Borgani, et al. An excess of small-scale gravitational lenses observed in galaxy clusters. *Science*, 369(6509):1347–1351, September 2020. DOI 10.1126/science.aax5164. 3.14.2

[104] Maria Temming. Dark matter clumps in galaxy clusters bend light surprisingly well. *Science News*, September 2020. Link: https://www.sciencenews.org/article/dark-matter-clumps-galaxy-clusters-bend-light-surprisingly-well. 3.14.2

[105] Joshua D. Simon and Marla Geha. The Kinematics of the Ultra-faint Milky Way Satellites: Solving the Missing Satellite Problem. *Astrophys. J.*, 670(1):313–331, November 2007. DOI 10.1086/521816. 3.14.2

[106] Pieter van Dokkum, Zili Shen, Michael A. Keim, Sebastian Trujillo-Gomez, Shany Danieli, Dhruba Dutta Chowdhury, Roberto Abraham, Charlie Conroy, J. M. Diederik Kruijssen, et al. A trail of dark-matter-free galaxies from a bullet-dwarf collision. *Nature*, 605(7910):435–439, May 2022. DOI 10.1038/s41586-022-04665-6. 3.14.2

[107] J. Jimenez-Vicente, E. Mediavilla, C. S. Kochanek, and J. A. Munoz. Dark Matter Mass Fraction in Lens Galaxies: New Estimates from Microlensing. *Astrophysical Journal*, 799(2):149, January 2015. DOI 10.1088/0004-637x/799/2/149. 3.14.2

[108] J. Jimenez-Vicente, E. Mediavilla, J. A. Munoz, and C. S. Kochanek. A Robust Determination of the Size of Quasar Accretion Disks Using Gravitational Microlensing. *Astrophysical Journal*, 751(2):106, May 2012. DOI 10.1088/0004-637x/751/2/106. 3.14.2

[109] Whitney Clavin. Rotating Galaxies Galore. April 2020. Link: https://www.caltech.edu/about/news/rotating-galaxies-galore. 3.14.2

[110] O. LeFevre, M. Bethermin, A. Faisst, P. Capak, P. Cassata, J. D. Silverman, D. Schaerer, and L. Yan. The ALPINE-ALMA [CII] survey: Survey strategy, observations and sample properties of 118 star-forming galaxies at 4<z<6. October 2019. DOI 10.1051/0004-6361/201936965. 3.14.2

[111] Ewa L. Lokas and Gary A. Mamon. Dark matter distribution in the Coma cluster from galaxy kinematics: breaking the mass-anisotropy degeneracy. *Monthly Notices of The Royal Astronomical Society*, 343(2):401–412, August 2003. DOI 10.1046/j.1365-8711.2003.06684.x. 3.14.3

[112] Elena Rasia, Giuseppe Tormen, and Lauro Moscardini. A dynamical model for the distribution of dark matter and gas in galaxy clusters. *Monthly Notices of The Royal Astronomical Society*, 351(1):237–252, June 2004. DOI 10.1111/j.1365-2966.2004.07775.x. 3.14.3

[113] Lawrence Rudnick. The Stormy Life of Galaxy Clusters: astro version. January 2019. DOI 10.48550/arXiv.1901.09448. 3.14.3

[114] Lawrence Rudnick. The stormy life of galaxy clusters. *Physics Today*, 72(1):46–52, January 2019. DOI 10.1063/pt.3.4112. 3.14.3

[115] V. M. Abazov, B. Abbott, M. Abolins, B. S. Acharya, M. Adams, T. Adams, M. Agelou, J.-L. Agram, S. H. Ahn, M. Ahsan, et al. Search for right-handed W bosons in top quark decay. *Physical Review D*, 72:011104, July 2005. DOI 10.1103/PhysRevD.72.011104. 4.1.2

[116] Paul Langacker and S. Uma Sankar. Bounds on the mass of W sub R and the W sub L - W sub R mixing angle. zeta. in general SU(2) sub L times SU(2) sub R times U(1) models. *Physical Review D*, 40(5):1569–1585, September 1989. DOI 10.1103/PhysRevD.40.1569. 4.1.2

[117] L. Gurung, T. J. Babij, S. D. Hogan, and D. B. Cassidy. Precision Microwave Spectroscopy of the Positronium $n = 2$ Fine Structure. *Physical Review Letters*, 125:073002, August 2020. DOI 10.1103/PhysRevLett.125.073002. 4.2.2

[118] Marvin Holten, Luca Bayha, Keerthan Subramanian, Carl Heintze, Philipp M. Preiss, and Selim Jochim. Observation of Pauli Crystals. *Phys. Rev. Lett.*, 126:020401, Jan 2021. DOI 10.1103/PhysRevLett.126.020401. 4.2.3

[119] Ben Forrest, Marianna Annunziatella, Gillian Wilson, Danilo Marchesini, Adam Muzzin, M. C. Cooper, Z. Cemile Marsan, Ian McConachie, Jeffrey C. C. Chan, Percy Gomez, et al. An Extremely Massive Quiescent Galaxy at z = 3.493: Evidence of Insufficiently Rapid Quenching Mechanisms in Theoretical Models. *Astrophysical Journal*, 890(1):L1, February 2020. DOI 10.3847/2041-8213/ab5b9f. 4.3.1

[120] Katherine E. Whitaker, Christina C. Williams, Lamiya Mowla, Justin S. Spilker, Sune Toft, Desika Narayanan, Alexandra Pope, Georgios E. Magdis, Pieter G. van Dokkum, Mohammad Akhshik, et al. Quenching of star formation from a lack of inflowing gas to galaxies. *Nature*, 597(7877):485–488, September 2021. DOI 10.1038/s41586-021-03806-7. 4.3.1

[121] David A. Buote and Aaron J. Barth. The Extremely High Dark Matter Halo Concentration of the Relic Compact Elliptical Galaxy Mrk 1216. *Astrophysical Journal*, 877(2):91, May 2019. DOI 10.3847/1538-4357/ab1008. 4.3.2

[122] Ana Bonaca, David W. Hogg, Adrian M. Price-Whelan, and Charlie Conroy. The Spur and the Gap in GD-1: Dynamical Evidence for a Dark Substructure in the Milky Way Halo. *Astrophysical Journal*, 880(1):38, July 2019. DOI 10.3847/1538-4357/ab2873. 4.3.3

[123] David Ehrenstein. Mapping Dark Matter in the Milky Way. *Physics Magazine*, 12(51), May 2019. Link: https://physics.aps.org/articles/v12/51. 4.3.3

[124] Lina Necib, Mariangela Lisanti, and Vasily Belokurov. Inferred Evidence for Dark Matter Kinematic Substructure with SDSS–Gaia. *Astrophysical Journal*, 874(1):3, March 2019. DOI 10.3847/1538-4357/ab095b. 4.3.3

[125] V Belokurov, A J Deason, S E Koposov, M Catelan, D Erkal, A J Drake, and N W Evans. Unmixing the Galactic halo with RR Lyrae tagging. *Monthly Notices of the Royal Astronomical Society*, 477(2):1472–1483, March 2018. DOI 10.1093/mnras/sty615. 4.3.3

[126] Gianluca Calcagni. Next Step in Gravity and Cosmology: Fundamental Theory or Data-Driven Models? *Frontiers in Astronomy and Space Sciences*, 7, September 2020. DOI: 10.3389/fspas.2020.00052. 4.5

[127] Anonymous. Digital Library of Mathematical Functions. National Institute of Standards and Technology, 2022. Link: https://dlmf.nist.gov/. 4.10.1

[128] Eric Weisstein. Delta Function. Wolfram MathWorld web page. Link(2020): http://mathworld.wolfram.com/DeltaFunction.html. 4.10.1

[129] Paul Langacker. Grand unification. *Scholarpedia*, 7(10):11419, 2012. DOI 10.4249/scholarpedia.11419. 4.13.2

[130] Richard L. Liboff. The correspondence principle revisited. *Physics Today*, 37(2):50–55, February 1984. DOI: 10.1063/1.2916084. 4.13.4

[131] Janos Polonyi. Quantum-classical crossover in electrodynamics. *Phys. Rev. D*, 74:065014, September 2006. DOI: 10.1103/PhysRevD.74.065014. 4.13.4

Manuscript date: January 29, 2023