The New Prime theorem（7）

$$P,\ jP+15-j(j=1,2,4,7,8,11,13,14)$$

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Abstrat

Using Jiang function we prove that there exist infinitely many primes P such that each $jP+15-j$ is a prime.

Theorem.

$$P,\ jP+15-j(j=1,2,4,7,8,11,13,14). \qquad (1)$$

There exist infinitely many primes P such that each of $jP+15-j$ is a prime.

Proof. We have Jiang function[1]

$$J_2(\omega)=\prod_P[P-1-\chi(P)], \qquad (2)$$

where $\omega=\prod_P P$,

$\chi(P)$ is the number of solutions of congruence

$$\prod(jq+15-j)(j=1,2,4,7,8,11,13,14)\equiv 0 \pmod P \qquad (3)$$

$q=1,\cdots,P-1$.

From (3) we have $\chi(2)=0$, $\chi(3)=1$, $\chi(5)=1$, $\chi(7)=3$, $\chi(11)=5$, $\chi(13)=5$, $\chi(P)=8$ otherwise.

From (3) and (2) we have

$$J_2(\omega)=315\prod_{17\le P}(P-9)\ne 0. \qquad (4)$$

We prove that there exist infinitely many primes P such that $jP+15-j$ is a prime.

We have the best asymptotic formula [1]

$$\pi_9(N,2)=\left|\{P\le N: jP+15-j=prime\}\right|\sim\frac{J_2(\omega)\omega^8}{\phi^9(\omega)}\frac{N}{\log^9 N}, \qquad (5)$$

where $\phi(\omega)=\prod_P(P-1)$.

Reference

[1] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. http://www. wbabin.net/math /xuan2. pdf.