The fine structure constant derived from the broken symmetry of two simple algebraic identities

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The fine structure constant is shown to arise naturally in the course of altering the symmetry of two algebraic identities. Specifically, the symmetry of the identity $M^2 = M^2$ is "broken" by making the substitution $M \to M - y$ on its left side, and the substitution $M^n \to M^n - x^p$ on its right side, where p equals the order of the identity; these substitutions convert the above identity into the equation $(M-y)^2 = M^2 - x^2$. These same substitutions are also applied to the only slightly more complicated identity $(M/N)^3 + M^2 = (M/N)^3 + M^2$ to produce this second equation $(M-y)^3/N^3 + (M-y)^2 = \left(M^3 - x^3\right)/N^3 + M^2 - x^3$. These two equations are then shown to share a mathematical property relating to dy/dx, where, on the second equation's right side this property helps define the special case $\left(M^3 - x^3\right)/N^3 + M^2 - x^3 = \left(10^3 - 0.1^3\right)/3^3 + 10^2 - 0.1^3 = 137.036$, which incorporates a value close to the experimental fine structure constant inverse.

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I. INTRODUCTION

The fine structure constant (FSC) is shown to arise naturally in the course of an investigation of two simple algebraic identities whose symmetry is altered. Specifically, the symmetry of the identities

$$M^2 = M^2 \tag{1.1}$$

and

$$\left(\frac{M}{N}\right)^3 + M^2 = \left(\frac{M}{N}\right)^3 + M^2 \tag{1.2}$$

will be "broken" by making the substitution

$$M \to M - y$$

on their left sides, and the substitution

$$M^n \to M^n - x^p$$

on their right sides, where p equals the order of each identity. The two equations that emerge from applying the above *substitution map* to the above two identities will then be shown to share a property relating to dy/dx, where for this second equation this property gives rise to a value that is close to the experimental FSC.

II. THE SECOND-ORDER IDENTITY

Begin with the symmetric *second-order* identity

$$M^2 = M^2$$

and break its symmetry by making the substitution

$$M \to M - y$$

on its left side, and the substitution

$$M^n \to M^n - x^p$$

on its right side, where $p = 2$, the order of the identity. This produces

$$(M-y)^2 = M^2 - x^2 \tag{2.1}$$

where the constant M is a positive integer and x and y variables.

III. THE FSC CONDITIONS

Now for Eq. (2.1) if

$$x = \frac{1}{M} \quad , \tag{3.1}$$

and if

$$M \gg 1 \tag{3.2}$$

(by at least an order of magnitude) then the value for dy/dx turns out to be simply

$$\frac{dy}{dx} \approx x^p \tag{3.3}$$

with $p = 2$ (see Section V for proof). Because Eqs. (3.1)–(3.3) are all that will be needed to generate the FSC in the next example they will be termed *The FSC Conditions.*

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IV. THE THIRD-ORDER IDENTITY AND THE FINE STRUCTURE CONSTANT

To generate the FSC combine the constant M^2 with the constant $(M/N)^3$ to form the expression

$$\left(\frac{M}{N}\right)^3 + M^2 \quad .$$

Set this expression equal to itself to form this symmetric *third-order* identity

$$\left(\frac{M}{N}\right)^3 + M^2 = \left(\frac{M}{N}\right)^3 + M^2$$

and again apply the substitution

$$M \to M - y$$

on the left, and

$$M^n \to M^n - x^p$$

on the right, with p now equaling 3. This produces *The FSC Equation*

$$\left(\frac{M-y}{N}\right)^3 + (M-y)^2 = \frac{M^3 - x^3}{N^3} + M^2 - x^3 \quad . \tag{4.1}$$

Here the constants M and N are assumed to be positive, and x and y are again variables.

Note that earlier Eq. (2.1) proved consistent with all three FSC Conditions. But it is only for specific values of M and N that Eq. (4.1) will likewise be consistent with all three conditions. To be precise, if Eq. (4.1) fulfills the first two FSC Conditions (Eqs. (3.1) and (3.2)) then it will also fulfill the third FSC Condition (Eq. (3.3)) provided that

$$M = \frac{N^3}{3} + 1 \tag{4.2}$$

(see Section VI for proof). Notably, inspection reveals that *the smallest positive integers* fulfilling Eq. (4.2) are

$$M = 10 \tag{4.3}$$

and

$$N = 3 \quad , \tag{4.4}$$

where substitution into the right side of Eq. (4.1) gives

$$\frac{10^3 - 0.1^3}{3^3} + 10^2 - 0.1^3 = 137.036 \quad . \tag{4.5}$$

The above values, in turn, determine that the left side of Eq. (4.1) gives

$$\left(\frac{10}{3} - \frac{1}{3 \times 29999.932166\ldots}\right)^3 + \left(10 - \frac{1}{29999.932166\ldots}\right)^2 = 137.036 \quad , \tag{4.6}$$

where the 2006 CODATA value for the FSC inverse equals 137.035 999 074, a value differing by just 7 parts per billion from 137.036 [1].

Hence, as was the case earlier for Eq. (2.1), the first two FSC Conditions imply the third FSC Condition, *but for Eq. (4.1) this is true provided that M and N fulfill Eq. (4.2), where, surprisingly, the smallest positive integers that fulfill Eq. (4.2)—10 and 3, respectively—generate the FSC inverse as a by-product.* Accordingly, the FSC inverse arises naturally from the analysis of the "broken symmetry" of two simple mathematical identities, making 137.036 of interest to pure mathematicians independent of its important role as a fundamental constant of physics.

V. PROOF FOR THE SECOND-ORDER IDENTITY AND EQUATION

Proof that for Eq. (2.1), if Eqs. (3.1) and (3.2) are true so is Eq. (3.3).

Equation (2.1)

$$(M-y)^2 = M^2 - x^2$$

simplifies to

$$2My - y^2 = x^2 \tag{5.1}$$

so that

$$(2M - 2y)\, dy = 2x dx \tag{5.2}$$

$$\frac{dy}{dx} = \frac{x}{M-y} \quad . \tag{5.3}$$

Equation (3.1) implies that $M = \frac{1}{x}$ so that by substitution

$$\frac{dy}{dx} \approx \frac{x}{\frac{1}{x} - y} \approx \frac{x^2}{1 - xy} \quad , \tag{5.4}$$

Given Eqs. (3.1) and (3.2), the cross-terms on the left side of Eq. (2.1) guarantee that $y < x^2$, while Eqs. (3.1) and (3.2) also determine that $x \ll 1$. Accordingly,

$$\frac{dy}{dx} \approx x^2 \quad . \tag{5.5}$$

In this way Eq. (3.3) is recovered for $p = 2$. This confirms that, for Eq. (2.1), if Eqs. (3.1) and (3.2) (the first two FSC Conditions) are true then so is Eq. (3.3) (the third FSC Condition).

VI. PROOF FOR THE THIRD-ORDER IDENTITY AND THE FSC EQUATION

Proof that for Eq. (4.1), if its values for M and N fulfill Eq. (4.2), then if Eqs. (3.1) and (3.2) are true so is Eq. (3.3).

If the higher-order powers of y in the FSC Equation (Eq. (4.1))

$$\left(\frac{M-y}{N}\right)^3 + (M-y)^2 = \frac{M^3-x^3}{N^3} + M^2 - x^3$$

are ignored, then it can be simplified to

$$\frac{3M^2y}{N^3} + 2My \approx \frac{x^3}{N^3} + x^3 \tag{6.1}$$

$$3M^2y + 2N^3My \approx x^3 + N^3x^3 \tag{6.2}$$

$$\left(3M + 2N^3\right)My \approx \left(N^3+1\right)x^3 \tag{6.3}$$

and solved for y

$$\begin{aligned} y &\approx \frac{N^3+1}{3M+2N^3}\frac{x^3}{M} \\ &\approx \frac{N^3+1}{M+\frac{2}{3}N^3}\frac{x^3}{3M} \quad , \end{aligned} \tag{6.4}$$

so that

$$dy \approx \frac{N^3+1}{M+\frac{2}{3}N^3}\frac{x^2}{M}dx \tag{6.5}$$

and

$$\frac{dy}{dx} \approx \frac{N^3+1}{M+\frac{2}{3}N^3}\frac{x^2}{M} \quad . \tag{6.6}$$

Although Eq. (6.6) is approximate, the terms it ignores are small as y is small (which y will be, given the first two FSC Conditions).

Now in order to assure that for Eq. (4.1), if the first two FSC Conditions are met then the third FSC Condition will also hold, combine Eqs. (6.6) and Eq. (3.3) to give

$$x^p \approx \frac{dy}{dx} \approx \frac{N^3+1}{M+\frac{2}{3}N^3}\frac{x^2}{M} \quad . \tag{6.7}$$

For Eq. (4.1) $p = 3$, so

$$x^3 \approx \frac{N^3+1}{M+\frac{2}{3}N^3}\frac{x^2}{M} \tag{6.8}$$

and

$$x \approx \frac{N^3+1}{M+\frac{2}{3}N^3}\frac{1}{M} \quad . \tag{6.9}$$

Equation (3.1) provides that $x = \dfrac{1}{M}$ so that by substitution

$$\frac{1}{M} \approx \frac{N^3+1}{M+\frac{2}{3}N^3}\frac{1}{M} \quad . \tag{6.10}$$

This gives

$$1 \approx \frac{N^3+1}{M+\frac{2}{3}N^3} \tag{6.11}$$

or

$$M + \frac{2}{3}N^3 \approx N^3 + 1 \quad , \tag{6.12}$$

which gives

$$M \approx \frac{N^3}{3} + 1 \quad . \tag{6.13}$$

Hence, Eq. (4.2) does constrain Eq. (4.1)'s values for M and N in such a way that if Eqs. (3.1) and (3.2) (the first two FSC Conditions) are true then so is Eq. (3.3) (the third FSC Condition).

[1] P.J. Mohr, B.N. Taylor, and D.B. Newell (2011), "The 2010 CODATA Recommended Values of the Fundamental Physical Constants" (Web Version 6.0). This database was developed by J. Baker, M. Douma, and S. Kotochigova. Available: `http://physics.nist.gov/constants` [Friday, 22-Jul-2011 10:04:27 EDT]. National Institute of Standards and Technology, Gaithersburg, MD 20899.