A black-hole electron and the sqrt of Planck momentum

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In this article the sqrt of Planck momentum is attributed as a distinct Planck unit and is used to link the mass constants and the charge (electric) constants. From these constants, formulas for a magnetic monopole (AL = ampre-meter) and an electron frequency \( f_p \) are constructed. The electron is seen as oscillating between an electric-state whose duration in units of Planck time \( t_p \) is dictated by \( f_p \), and a mass-state whose duration equals to \( \) (or is defined by) \( 1t_p \). The premise being that after 1 oscillation cycle the magnetic monopoles AL combine and overlap and cancel in this (AL) \(^3\)/T ratio. In mass, time, length terms this ratio is \( MT^{11}/L^{15} \). This overlap permits us to reduce the number of required units to 2 and then define the least precise constants \( G, h, e, m_e, k_B \) in terms of the most precise \( c, \mu_0 \) (exact values), the fine structure constant \( \alpha \) (10-11 digits) and the Rydberg constant \( R \) (12-13 digits). Results are consistent with CODATA 2014.

### Table 1

<table>
<thead>
<tr>
<th>Planck constant</th>
<th>Calculated ((c, \mu_0, \alpha, R))</th>
<th>CODATA 2014</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elementary charge</td>
<td>( c^2 = 1602.176511 \times 10^{-19} )</td>
<td>( e = 1.6021766208(98) \times 10^{-19} )</td>
</tr>
<tr>
<td>Boltzmann’s constant</td>
<td>( k_B = 1.38064852(79) \times 10^{-23} )</td>
<td>( k_B = 6.021766208(98) \times 10^{-23} )</td>
</tr>
<tr>
<td>Mass</td>
<td>( m_e = 9.109382312 \times 10^{-31} )</td>
<td>( m_e = 9.10938356(11) \times 10^{-31} )</td>
</tr>
</tbody>
</table>

#### 1. Sqrt of Planck momentum

In this section I introduce the sqrt of momentum as a distinct Planck unit and suggest how this could be used as a link between the mass and charge domains.

From the formulas for the charge constants I then derive a formula for a magnetic monopole and from this subsequently a formula for an electron \( f_p \).

The electron formula is constructed from dimensioned monopoles (AL = ampre-meter) and from Planck time \( T \) yet it is dimensionless. It is proposed here that the charge and time units are not independent of each other but collapse within the electron according to this ratio; \( f_p = (AL)^3/T \), units = 1. Being unit-less and so independent of any system of units, this electron formula \( f_p \) is a mathematical constant.

1.1 Defining \( Q \) as the sqrt of Planck momentum where Planck momentum \( = m_pc = 2\pi Q^2 = 6.52485... kg.m/s \), and a unit \( q \) whereby \( q^2 = kg.m/s \) giving;

\[
Q = 1.019 113 411..., \; unit = q
\]

Planck momentum: \( 2\pi Q^2 \), units = \( q^2 \)

Planck length: \( l_p \), units = \( m = m/s = q^2/kg \)

1.2. In Planck terms the mass constants are typically defined in terms of Planck mass, here I use Planck momentum;

\[
m_p = \frac{2\pi Q^2}{c}, \; unit = kg
\]

1.3. The charge constants in terms of \( Q^2, c, \alpha, l_p \);

\[
A_0 = \frac{8e^3}{\alpha Q^3}, \; unit \; A = \frac{m^3}{q^3 s^3} = \frac{q^2}{kg^3}
\]

\[
e = A_0 l_p = \frac{8e^3}{\alpha Q^3} \frac{2l_p}{c} = \frac{16e^3}{\alpha Q^3}, \; units = A.s = q^3 kg^{-3}
\]

\[
T_p = A_0 c = \frac{8e^3}{\alpha Q^3} \frac{c}{\pi} = \frac{8e^3}{\pi \alpha Q^3}, \; units = q^2 kg^{-1}
\]

\[
k_B = \frac{E_p}{T_p} = \frac{\pi^2 \alpha Q^3}{4c^3}, \; units = \frac{kg^3}{q}
\]

1.4. As with \( c \), the permeability of vacuum \( \mu_0 \) has been assigned an exact numerical value so it is our next target. The amper is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum,
would produce between these conductors a force equal to exactly 2.10^{-7} newton per meter of length.

\[
\frac{F_{\text{electric}}}{A_Q} = \frac{F_p}{\alpha A_Q} = \frac{2\pi Q^2}{\alpha_p \left( \frac{\alpha Q^2}{5c^3} \right)^2} = \frac{\pi \alpha Q^8}{64\mu_c^6c^5} = \frac{2}{10^3}
\]

(10)

\[
\mu_0 = \frac{\pi^2 \alpha Q^8}{32\mu_c^6c^5} = \frac{4\pi}{10^7}, \text{ units } = \frac{kg \cdot m}{s^2 A^2} = \frac{kg^6}{q^3 s}
\]

(11)

1.5. Rewriting Planck length \( l_p \) in terms of \( Q, c, \alpha, \mu_0; \)

\[
l_p = \frac{\pi^2 \alpha Q^8}{32\mu_0 c^5}, \text{ units } = \frac{q^7 s}{kg} = m
\]

(12)

1.6. A magnetic monopole in terms of \( Q, c, \alpha, l_p; \)

The ampere-meter is the SI unit for pole strength (the product of charge and velocity) in a magnet \((Am = ec)\). A magnetic monopole \( \sigma_e \), is a hypothetical particle that is a magnet with only 1 pole [4]. I propose a magnetic monopole \( \sigma_e \) from \( \alpha, e, c \) (\( \sigma_e = 0.13708563 \times 10^{-6} \));

\[
\sigma_e = \frac{3\alpha^2 ec}{2\pi^2}, \text{ units } = \frac{q^5 s}{kg^3}
\]

(13)

I then use this monopole to construct an electron frequency function \( f_e \) (\( f_e = 0.2389545 \times 10^{12} \));

\[
f_e = \frac{\sigma_e^2}{l_p} = \frac{2 \pi^3 \alpha^3 \mu_c^{10}}{\pi^6 Q^9} = \frac{3 \pi^3 Q^7}{4 \pi^2 \mu_0^8}, \text{ units } = \frac{q^{15} s^2}{kg^{12}}
\]

(14)

1.7. The most precisely measured of the natural constants is the Rydberg constant \( R_\infty \) (see table) and so it is important to this model. The unit for \( R_\infty \) is 1/m. For \( m_e \) see eq(22);

\[
R_\infty = \frac{m_e e^4 \mu_c^6 c^3}{8\hbar^3} = \frac{2 \pi c^5 \mu_0^8}{3^{3} \pi^2 Q^{15}}, \text{ units } = \frac{1}{m} = \frac{kg^{10}}{q^{17} s^3}
\]

(15)

This however now gives us 2 solutions for length \( m; \) see eq(1) and eq(15), if they are both valid then there must be a ratio whereby the units \( q, s, kg \) overlap and cancel;

\[
m = \frac{q^2 s}{kg} \cdot \frac{q^{15} s^2}{kg^{12}} = \frac{q^{17} s^3}{kg^{13}}; \text{ thus } \frac{q^{15} s^2}{kg^{12}} = 1
\]

(16)

and so we can further reduce the number of units required, for example we can define \( s \) in terms of \( kg, q; \)

\[
s = \frac{kg^6}{q^{15/2}}
\]

(17)

\[
\mu_0 = \frac{kg^6}{q^7}, \text{ units } = \frac{q^{15} s^2}{kg^{12}} = 1
\]

(18)

1.8. We find that this ratio is embedded in that electron function \( f_e \) (eq 14), and so \( f_e \) is a dimensionless mathematical constant whose function appears to be dictating the frequency of the Planck units;

\[
f_e = \frac{\sigma_e^2}{l_p}; \text{ units } = \frac{q^{15} s^2}{kg^{12}} = 1
\]

(19)

Replacing \( q \) with the more familiar \( m \) gives this ratio;

\[
q^2 = \frac{k g \cdot m}{s}; q^{15} = \left( \frac{k g \cdot m}{s} \right)^{15} = \frac{k g^{24}}{s^5}
\]

(20)

units = \( \frac{k g^9 s^{11}}{m^{15}} = 1 \)

(21)

Electron mass as frequency of Planck mass:

\[
m_e = \frac{m_p}{f_e}, \text{ unit } = kg
\]

(22)

Electron wavelength via Planck length:

\[
\lambda_e = 2\pi l_p f_e, \text{ units } = m = \frac{q^7 s}{kg}
\]

(23)

Gravitation coupling constant:

\[
\alpha_G = \left( \frac{m_e}{m_p} \right)^2 = \frac{1}{f_e}, \text{ units } = 1
\]

(24)

1.9. The Rydberg constant \( R_\infty = 10973731.568508(65) \) [5] has been measured to a 12 digit precision. The known precision of Planck momentum and so \( Q \) is low, however with the solution for the Rydberg eq(15) we may re-write \( Q \) as \( Q^{15} \) in terms of \( c, \mu_0, R \) and \( \alpha \);

\[
Q^{15} = \frac{2 \pi c^5 q_0}{3^{5} \pi^3 \alpha R}, \text{ units } = \frac{kg^{10}}{s^5} = q^{15}
\]

(25)

Using the formulas for \( Q^{15} \) eq(25) and \( l_p \) eq(12) we can rewrite the least accurate dimensioned constants in terms of the most accurate constants; \( R, c, \mu_0, \alpha \). 1 first convert the constants until they include a \( Q^{15} \) term which can then be replaced by eq(25). Setting unit \( x \) as;

\[
unit x = \frac{kg^{12}}{q^{15} s^2} = 1
\]

(26)

Elementary charge \( e = 1.602 176 51130 e-19 \) (table p1)

\[
e = \frac{16 \pi^3}{\alpha Q^3} = \frac{\pi^2 Q^5}{2 \mu_0 c^3}, \text{ units } = \frac{q^3}{kg^3}
\]

(27)

\[
e^3 = \frac{\pi^6 Q^{15}}{8 \mu_0 c^3} = \frac{4 \pi^3}{3 \pi c^5 \alpha R}, \text{ units } = \frac{kg^3}{q^6} = \left( \frac{q^3}{kg^3} \right)^{3} x
\]

(28)

Planck constant \( h = 6.626 069 134 e-34 \)

\[
h = 2\pi Q^2 2\pi l_p = \frac{4 \pi^4 \alpha Q^{10}}{8 \mu_0 c^3}, \text{ units } = \frac{q^4 s}{kg}
\]

(29)

\[
h^3 = \left( \frac{4 \pi^4 \alpha Q^{10}}{8 \mu_0 c^3} \right)^3 = \frac{2 \pi^{10} \mu_0^3}{36 \pi^5 \alpha^2 R^2}, \text{ units } = \frac{kg^{21}}{q^{18}} = \left( \frac{q^4}{kg^3} \right)^{3} x^2
\]

(30)

Boltzmann constant \( k_B = 1.379 510 14752 e-23 \)

\[
k_B = \frac{\pi^2 Q^5}{4 c^3}, \text{ units } = \frac{kg^3}{q}
\]

(31)
\[ k_B^3 = \frac{\pi^4 \mu_0^3}{32 c^3 \alpha^3 R}, \quad \text{units} = kg^2 s^3 q^{18} x^2 = (kg^3 q)^3 x \]

Gravitation constant \( G = 6.672 \ 497 \ 19229 \ e^{-11} \)

\[ G = c^2 \rho_{\mu} = \frac{\pi \alpha Q^6}{64 \mu_0 c^2}, \quad \text{units} = \frac{q^6 s}{kg^6} \]

\[ G^2 = \frac{\pi^3 \mu_0}{2 \times 10^{-3} \alpha^2 R}, \quad \text{units} = kg^4 s = \left(\frac{q^6 s}{kg^6}\right)^2 x^2 \]

Planck length

\[ l_p^{15} = \frac{\pi^2 \mu_0^2}{2 \times 10^{-3} c^5 \alpha^4 R^8}, \quad \text{units} = kg^9 s^5 = \left(\frac{q^7 s}{kg^3}\right)^{15} x^8 \]

Planck mass

\[ m_p^{15} = \frac{2^{10} \pi^3 c^{15} \alpha}{\mu_0}, \quad \text{units} = kg^{39} s^{30} = \frac{1}{q^3 s^4} \]

Electron mass \( m_e = 9.109 \ 382 \ 31256 \ e^{-31} \)

\[ m_e^3 = \frac{16 \pi^4 e^{10} R \mu_0^3}{3 e^3 c^3}, \quad \text{units} = kg^7 s = \frac{1}{q^2 s^3} \frac{1}{x^2} \]

Ampere

\[ A_q = \frac{2^{10} \pi^3 c^{15} \alpha}{\mu_0}, \quad \text{units} = q^{30} s^{27} = \left(\frac{q^7 s}{kg^3}\right)^{27} \]

1.10. Alpha the fine structure constant. Note; for convenience I use the commonly recognized value for alpha as \( \alpha \approx 137.036 \).

\[ \alpha = \frac{2h}{\mu_0 e^2 c} = 2.2 \pi Q^2 2\pi l_p \frac{32 l_p e^5}{\pi^2 \alpha Q^6} \frac{a^2 Q^6}{256 l_p^3 c^4} = \alpha \]

1.11. \(( r = \sqrt{q} \))

There is a solution for an \( r^2 = q \), it is the radiation density constant from the Stefan Boltzmann constant \( \sigma \);

\[ \sigma = \frac{2 \pi^3 k_B^3}{15 h^3 c^2}, \quad \text{units} = \frac{4 \sigma}{c}, \quad \text{units} = \frac{4 \sigma}{c} \]

\[ r_d^3 = \frac{3 \pi^3 \mu_0^3}{5 e^{10}}, \quad \text{units} = \frac{kg^{30} s^{20}}{q^2 s^3} \frac{1}{x^2} \frac{kg^6 s}{q^2 s^3} = r^3 \]

2 Notes

A charged rotating black hole is a black hole that possesses angular momentum and charge. In particular, it rotates about one of its axes of symmetry. In physics, there is a speculative notion that if there were a black hole with the same mass and charge as an electron, it would share many of the properties of the electron including the magnetic moment and Compton wavelength. This idea is substantiated within a series of papers published by Albert Einstein between 1927 and 1949. In them, he showed that if elementary particles were treated as singularities in spacetime, it was unnecessary to postulate geodesic motion as part of general relativity [2].

The Dirac Kerr–Newman black-hole electron was introduced by Burinskii using geometrical arguments. The Dirac wave function plays the role of an order parameter that signals a broken symmetry and the electron acquires an extended space-time structure. Although speculative, this idea was corroborated by a detailed analysis and calculation [1].

References
3. Online physical constants calculator http://planckmomentum.com/sqrtQ/
5. Rydberg constant http://physics.nist.gov/cgi-bin/cuu/Value?ryd