DOES THE QUANTUM SUM RULE HOLD AT THE BIG BANG AND WHAT ABOUT QUANTUM MEASURES AT COSMIC SINGULARITIES?

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In Dice 2010 Sumati Surya brought up a weaker Quantum sum rule as a biproduct of a quantum invariant measure space. Our question is, does it make sense to have disjoint sets to give us quantum conditions for a measure at the origin of the big bang? We argue that the answer is no, which has implications as to quantum measures and causal set structure.

A Introduction

In the causal set approach, the probabilities are held to be Markovian [1] , label independent and adhere to a casuality called Bells inequality. The author of [1] refers to a sequential growth called a calssical transition percolation model. Then [1] makes an extension of the above idea to complex models involving quantum measures in the definition of a (quantum) complex percolation model which defiines the amplitude of transition as follows [1]. For a quantum measure space defined as triple as given by $\left(\Omega, A, \mu_V\right)$, with μ_V a yet to be defined vector measure, A an event algebra or set of propositions about the system, and Ω is the sample space of histories or space time configurations.

Let $p \ \varepsilon \ C$, for an amplitude of transition, instead of a probability; and set $\psi\left(C^n\right)$ as the amplitude for a transition from an empty set to n element of a causal set C^n, and with $Cyl\left(C^n\right)$, cylinder set, as a sub set of Ω containing labeled past finite causal sets whose first n elements form the sub causal set C^n. Note that the cylinder sets form an event algebra A with measure given by form the sub-causal set C^n. Here, ψ is a complex measure on A, and so then ψ is a vector measure [1] . This is the primary point of break down which occurs in the case of being at a space time singularity. Away from the singularity we will be working with the physics of

$$D\left(Cyl\left(C^n\right), Cyl\left(C'^n\right)\right) = \psi^*\left(C^n\right)\psi\left(C'^n\right) \tag{1}$$

This is done for a cylinder set [1], where γ is a given path, and γ^t as a truncated path, with $cyl\left(\gamma^t\right)$ a subset of Ω and $\mu\left(cyl\left(\gamma^t\right)\right) = P\left(\gamma^t\right)$, with $P\left(\gamma^t\right)$ the probability of a truncated path, with a given initial $\left(x_i, t_i\right)$ to final $\left(x_f, t_f\right)$ spatial and times. Note that the μ measure would be for $\mu : A \to R^+$ obeying the weaker Quantum sum rule [2]

$$\mu\left(\alpha \cup \beta \cup \gamma\right) = \mu\left(\alpha \cup \beta\right) + \mu\left(\alpha \cup \gamma\right) + \mu\left(\beta \cup \gamma\right) - \mu\left(\alpha\right) - \mu\left(\beta\right) - \mu\left(\gamma\right) \quad (2)$$

This probability would be a quantum probability which would NOT be obeying the classical rule of Kolmogrov [1]

$$P\left(\gamma_1 \cup \gamma_2\right) = P\left(\gamma_1\right) + P\left(\gamma_2\right) \quad (3)$$

The actual probability used would have to take into account quantum interference. And That is due to Eq. (1a) , and Kolmogrov probability no longer applying. Leading to [1]

$$cyl\left(\gamma^t\right) \equiv \left\{\gamma \in \Omega \mid \gamma\left(t'\right) = \gamma^t\left(t'\right) \; for \quad all \quad 0 \le t' \le t\right\} \quad (4)$$

Here, $D : \mathrm{A} \times \mathrm{A} \to C$ is a de coherence functional [1] which is (i) Hermitian, (ii) finitely biadditive, and (iii) strongly additive [2] , i.e. the eignvalues of D constructed as a matrix over the histories $\left\{\alpha_i\right\}$ are non negative.

We have that a quantum mesurement is then defined via

$$\mu\left(\alpha\right) = D(\alpha, \alpha) \ge 0 \quad (5)$$

A quantum vector measurment is defined via

$$\mu_V\left(\alpha\right) := \left[\chi_\alpha\right] \in H \quad (6)$$

where

$$\chi_\alpha\left(\beta\right) = \begin{cases} 1 \\ 0 \end{cases}, \qquad \chi_\alpha\left(\beta\right) = 1 \text{ if } \quad \beta = \alpha\,,\ \chi_\alpha\left(\beta\right) = 0 \text{ if } \quad \beta \ne \alpha \quad (7)$$

Also V is the vector space over A with an inner product given by

$$\langle u,v\rangle_V \equiv \sum_{\alpha\in A}\sum_{\beta\in A} u^*(\alpha)v(\beta)\cdot D(\alpha,\beta) \tag{8}$$

with a histories Hilbert space H constructed via taking a sequence of Cauchy sequences $\{u_i\}$ sharing an equivalence relationship

$$\{u_i\} \sim \{v_i\} \text{ if } \lim_{i\to\infty}\|u_i - v_i\|_V = 0 \tag{9}$$

So then as given in [1] the following happen, namely

$$\left[\{u_i\}\right] + \left[\{v_i\}\right] \equiv \left[\{u_i + v_i\}\right] \tag{10}$$

$$\left[\{\lambda u_i\}\right] \equiv \lambda\left[\{u_i\}\right] \tag{11}$$

$$\left\langle \left[\{u_i\}\right], \left[\{v_i\}\right]\right\rangle \equiv \lim_{i\to\infty}\langle u_i, v_i\rangle_V \tag{12}$$

This for all $\left[\{u_i\}\right], \left[\{v_i\}\right] \in H$ and $\lambda \in C$ so then that the quantum measure is defined for $\mu_V : A \to H$ so that for the inner product on H

$$\left\langle \mu_V(\alpha), \mu_V(\beta)\right\rangle = D(\alpha,\beta) \tag{13}$$

The claim associated with Eq. (1) above is that since ψ is a complex measure on A that Eq. (1) corresponds to what is called an unconditional convergence of the vector measure over all partitions. Secondly, according to the Caratheodary-Hahn theorm there is unconditional convergence for classical stochastic growth, but this is not necessarily always true for a quantum growth process.

Main point of the formalism going to Eq. (13) is of bi-additivity of D leading to the finite addivity of μ_V

$$\mu_V\left(\bigcup_{i=1}^{n}\alpha_i\right) = \sum_{i=1}^{n}\mu_V(\alpha_i) \tag{14}$$

B. Looking at Arguments against Eq. (1) in the vicinity/ orignin of the big bang singularity.

The pre condition for a quantum measure μ_V for a quantum measurement [1] is that for **n** disjoint sets $\alpha_i \ \varepsilon \ \mathrm{A}$, is given by Eq. (14) above.

This Eq. (14) is a pre condition for μ_V being a vector measure over A Eq (14) above right at the point of the big bang, cannot insure the existence of **n** disjoint sets $\alpha_i \ \varepsilon \ \mathrm{A}$. Therefore at the loci of the big bang one would instead get, due to non definable disjoint sets $\alpha_i \ \varepsilon \ \mathrm{A}$ a situation definable as, at best.

$$\mu_V\left(\bigcup_{i=1}^{n}\alpha_i\right) \neq \sum_{i=1}^{n}\mu_V\left(\alpha_i\right) \tag{15}$$

Not being able to have a guarantee of having **n** disjoint sets $\alpha_i \ \varepsilon \ \mathrm{A}$ because of singular conditions at the big bang will bring into question if Eq. (1) can hold and the overall program of analyzing the existence of quantum measures μ_V. I.e. the triple $\left(\Omega, A, \mu_V\right)$ for quantum measures μ_V cannot be guaranteed to exist. More importantly, the statement that there exists $\psi\left(C^n\right)$ from an empty set to a nth element causal set cannot be adhered to, and Eq. (1) cannot exist since there would be no causal set structure at the loci of the big bang.

C. Making sense out of QM and also wave-particle duality.

So what can be inferred ? If discontinuous set structures do not exist at the onset of the big bang in effectively measure zero space, then what is left ? We get into all sorts of difficulties. Our assumption is that a break down of a quantum measure would probably be congruent with the break down of use of QM, in the onset of the big bang. The bottom below is a simple quantum argument. i.e. how QM falls falls apart, i.e. the wave-particle duality structure.I.e. assume that we have ultra light gravitons, with a tiny rest mass, then a simple quantum argument will give us [4]

$$\begin{aligned} & m_{graviton}\Big|_{RELATIVISTIC} < 4.4\times10^{-22}h^{-1}eV/c^2 \\ & \Leftrightarrow \lambda_{graviton} \equiv \frac{\hbar}{m_{graviton}\cdot c} < 2.8\times10^{-8}meters \end{aligned} \tag{16}$$

i.e. the smaller the R.H.S. if Eq. (3) gets, the heavier the rest graviton mass is, which would get us into problems if we look at ultra short wave lengths.

The obvious generalization of Eq. (16) would be for a mass M

$$M \;\Big|_{RELATIVISTIC} < \#\,given \quad h^{-1}eV / c^2$$
$$\Leftrightarrow \lambda \equiv \frac{\hbar}{M \cdot c} < \#\,given \quad 10^{\beta}\,meters \tag{17}$$

One could then get, as in Eq. 16 and Eq, 17 , a sitaution in which

$$m_{graviton}, M \to \infty \Leftrightarrow \lambda \to 0^{+} \tag{18}$$

If we went to a point source, i.e. an infinitely small wave length, the effective mass would go to huge, unphysical values. Since Eq.(16) and Eq. (17) is based upon Quantum structure, the shorter the wave length got, the less physical the problem becomes as in Erq. (18) , until we get to the absurdity of an infinitely massive gravition or an infinitely massive particle for an infinitely short wave length. i.e. not only there would be as we go to a point structure, no disjoint causal structure, our very physics as we understand QM insight would become not tendable. This will lead to a problem with the causal set discretization proceedure brought into analysis, next.

D. QM, wave lengths, and problems with Quantum measure Eq.(14), and justification of Eq. (19)

As stated by [1], one can think of Causal sets as part of a partial ordering of space time, and to replace the space time continuum with locally finite partially ordered sets [5] ,[6]. We assert that in place of Eq. (1) which will involve the notion of partially ordered sets that instead one has in the immediate neighborhood of a singularity, where we are using the ideas of the beginning of this manuscript. So at the singularity.

$$D\Big(Cyl\big(C^{n}\big), Cyl\big(C'^{n}\big)\Big) \neq \psi^{*}\big(C^{n}\big)\psi\big(C'^{n}\big) \tag{19}$$

That Eq. (19) may happen is due to what may happen in the finite dimensional H and what happens with total variation [1] as given by looking at finite partitions [8]

$$\pi(\alpha) = \{\alpha_{\rho}\}, \alpha \in A \tag{20}$$

Here the supremum is over all finite partitions as given in Eq. (20) above. And then we look at if there is a sufficiently convergent behavior for μ_V , so that uniqueness would be guaranteed by the Caratheodary-Hahn –Huvanek theorem. We will be looking at then having the following supremum expression for all FINITE partitions as of Eq. (20) and

$$\left|\mu_V\left(\alpha\right)\right| = \sup_{\pi(\alpha)} \sum_{\rho} \left\|\mu_V\left(\alpha_\rho\right)\right\| \tag{21}$$

Having a singularity removes applications of Eq. (20) , and of having uniqueness itself by[7] challenged ? What happens if we have instead of Eq. (21) a sitaution for which we no longer have finite partitions, ordered sets, but instead

$$\left|\mu_V\left(\alpha\right)\right| \neq \sup_{\pi(\alpha)} \sum_{\rho} \left\|\mu_V\left(\alpha_\rho\right)\right\| \tag{22}$$

Or worse, a situation where there is no finite partially ordered set, i.e. no CAUSAL set ? Such a situation may

What could go wrong? Suppose that Eq. (20) no longer holds Suppose we cannot even write partitions or ordered causal sets at a singularity so Eq(21) no longer holds and we cannot even write Eq.(22)?

Eq. (1) as given in the beginning depends upon having [1] an "*unconditional convergence of the vector measure over all partitions* ". Replace partitions with causal set structure, and one still has the same requirement of an *unconditional convergence of the vector set over all 'causal set structure'* within a finite geometric regime of space time.

Our entire supposition as to Eq. (1), Eq. (21) and even Eq. (22) becomes untendable at the singularity. So then, we cannot force QM , with an infinitely small 'wavelength', i.e. infintely small measure back upon a cosmic singularity, i.e. the big bang itself.

E. Conclusion? Back to a deterministic treatment of QM, as suggested by t'Hooft QM

[1] Suggests a way out of the impasse. If we look at **unconditional convergence over all partitions, if we cannot do this for a point, in which we tried to have Quantum measures constructed, then we have to look at how the singular point , for the big bang, is embedded via higher dimensional analogs to a non singular structure.**

Secondly, is to not insist upon forcing the situation given in Eq. (16) and Eq. (17) to its extremes. I.e. looking at what was said "as to " ***real" complex percolation models in which one accepts that a quantum measure is not additive, as in Eq.(15), but that " the observables of the theory are identical to those of the classical transitive percolations. In particular, the observables can be characterized by "stem sets" " .***

If we can put the surrounding the big bang singular point classical ***transitive percolations and relate that to observables identical to classical transitive percolations,***

we are on the way to fixing the problem of the Quantum measure [8]. I.e. this may be a way to be in fedelity with working with t'Hooft's embedding of Quantum mechanics within a higher dimensional theory, as would show up in fixing the problems with the Quantum measure[8] and QM as given in the limits as to Eq. (16) and Eq. (17) above.

We can assert though our arguments in 4 space cosmology would contravene [9] 's structure at the extreme limits of singular big bang physics, as well as lead to the untendability of the quantum sum rule (due to vanishing of disjoint set structure). **That is if we stick to 4 dimensional space and no higher dimensions.**

The only way about the above stated problems for 4 dimensions and a tradtional big bang singularity would be using **singular point classical *transitive percolations and relate that to observables identical to classical transitive percolations and giving up t he additivity of quantum measure.***

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