The angular momentum in the hydrogen atom

Sangwha-Yi
Department of Math , Taejon University 300-716

ABSTRACT

The article treats that the angular momentum in the hydrogen atom.
If calculates the electron motion in the hydrogen atom , can do the quantization in the Bohr's theory about the hydrogen atom. In this time, the electron's orbit velocity v is non-relativity velocity.

PACS Number:**03.65**
Key words:The hydrogen atom,
The Bohr theory,
The quantum mechanics
The angular momentum
e-mail address:sangwha1@**nate.com**
Tel:051-624-3953

I. Introduction

The article treats that the angular momentum in the hydrogen atom.

If calculates the electron motion in the hydrogen atom by Coulomb's law in the classical mechanic [4],

$$F = ma$$

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = m\frac{v^2}{r} \qquad (1)$$

m is the electron's mass.

The electron's kinetic Energy is

$$K = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\varepsilon_0 r} \qquad (2)$$

The potential Energy in the hydrogen atom is

$$U = V(-e) = -\frac{e^2}{4\pi\varepsilon_0 r} \quad , \quad V = \frac{e}{4\pi\varepsilon_0 r} \qquad (3)$$

The total Energy in the hydrogen atom is

$$E = K + U = -\frac{e^2}{8\pi\varepsilon_0 r} \qquad (4)$$

The Bohr's frequency condition is

$$h\upsilon = E_k - E_j \qquad (5)$$

By Eq(2),the electron's the orbit velocity v ,the momentum p , the angular momentum L are

$$v = \sqrt{\frac{e^2}{4\pi\varepsilon_0 mr}} \, , p = mv = \sqrt{\frac{me^2}{4\pi\varepsilon_0 r}} \, , \ L = pr = \sqrt{\frac{me^2 r}{4\pi\varepsilon_0}} \qquad (6)$$

The Bohr's hypothesis is

$$L = n\frac{h}{2\pi} = n\hbar \qquad (7) \qquad n = 1,2,3,...$$

By Eq(6),Eq(7), the radius r is

$$r = r_n = n^2\frac{h^2\varepsilon_0}{\pi me^2} \qquad (8) \qquad n = 1,2,3,...$$

By Eq(4),Eq(8), the total energy E is

$$E = E_n = -\frac{me^4}{8\varepsilon_0^2 h^2}\frac{1}{n^2} \qquad (9) \qquad n = 1,2,3,...$$

By Eq(5),Eq(9), the hydrogen line spectra's frequency υ is

$$\upsilon = \frac{me^4}{8\varepsilon_0{}^2 h^3}(\frac{1}{j^2} - \frac{1}{k^2}) \quad (10) \quad j,k \text{ is number.}$$

By Eq(6),Eq(8), the quantization of the electron's the orbit velocity v,the momentum p is in the hydrogen atom

$$v = v_n = \sqrt{\frac{e^2}{4\pi\varepsilon_0 mr}} = \frac{e^2}{2nh\varepsilon_0}, \quad v_1 = \frac{e^2}{2h\varepsilon_0} << c \qquad (11) \quad n = 1,2,3,...$$

$$p = p_n = mv = \sqrt{\frac{me^2}{4\pi\varepsilon_0 r}} = \frac{me^2}{2nh\varepsilon_0} \, (12) \quad n = 1,2,3,...$$

In this time, the electron's orbit velocity v in the hydrogen atom is not continuous.

Bohr's orbit that it's radius is r is

$$n\lambda = 2\pi r \, (13) \quad n = 1,2,3,...$$

Therefore, the quantization of the electron's wavelength λ is in the hydrogen atom

$$r = \frac{n\lambda}{2\pi} = \frac{n^2 h^2 \varepsilon_0}{\pi me^2}$$

$$\lambda = \lambda_n = \frac{2nh^2\varepsilon_0}{me^2} \quad (14) \quad n = 1,2,3,...$$

By Eq(2),Eq(8), the quantization of the electron's kinetic Energy is

$$K = \frac{1}{2}mv^2 = K_n = \frac{e^2}{8\pi\varepsilon_0 r} = \frac{e^4 m}{8n^2 h^2 \varepsilon_0{}^2} \, (15) \quad n = 1,2,3,...$$

By Eq(3),Eq(8), the quantization of the potential Energy in the hydrogen atom is

$$U_n = V_n(-e) = -\frac{e^2}{4\pi\varepsilon_0 r} = -\frac{e^4 m}{4n^2 h^2 \varepsilon_0{}^2} \quad , \quad V_n = \frac{e}{4\pi\varepsilon_0 r} = \frac{e^3 m}{4n^2 h^2 \varepsilon_0{}^2} \quad (16) \quad n = 1,2,3,...$$

By Eq(1),Eq(8), the quantization of the electric force is in the hydrogen atom is

$$F = F_n = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{1}{4\pi\varepsilon_0}e^2\frac{e^2\pi m}{n^2 h^2 \varepsilon_0} = \frac{e^4 m}{4n^2 h^2 \varepsilon_0{}^2}, n = 1,2,3,... (17)$$

$$F = F_n = \frac{mv^2}{r} = \frac{e^4 m}{4n^2 h^2 \varepsilon_0{}^2}$$

In this time, the electric force in the hydrogen atom is quantized.

In this time, de Broglie wavelength is in the hydrogen atom

$$\lambda = \lambda_n = \frac{2nh^2\varepsilon_0}{me^2} = \frac{h}{p} = \frac{h}{p_n} \quad (18) \qquad n = 1,2,3,...$$

II.Additional chapter

Generally, the electron's angular momentum L is in the quantum mechanic,

$$L = \frac{h}{2\pi}\sqrt{l(l+1)} = \hbar\sqrt{l(l+1)} \quad (19),$$ l is the orbital quantum number

$$l = 0,1,2,...,(n-1)$$

Eq(19) is different from Eq(7) about the electron's angular momentum L.

Therefore, Eq(19) has to be include the principal quantum number n likely Eq(7).

In the Schrodinger wave equation, the radius function $R = R(r)$ is

$$\frac{1}{r^2}\frac{d}{dr}(r^2\frac{dR}{dr}) + [\frac{2m}{\hbar}(\frac{e^2}{4\pi\varepsilon_0 r} + E) - \frac{l(l+1)}{r^2}]R = 0$$

$$E = E_n + V = E_n - \frac{e^2}{4\pi\varepsilon_0 r}, \quad E_n = -\frac{me^4}{8\varepsilon_0{}^2 h^2}\frac{1}{n^2}$$

$$\frac{1}{r^2}\frac{d}{dr}(r^2\frac{dR}{dr}) + \frac{2m}{\hbar}[E_n - \frac{\hbar^2 l(l+1)}{2mr^2}]R = 0 \qquad (20)$$

Therefore, the electron's orbital kinetic energy $K_{orbital}$ is (K is by Eq(2))

$$K_{orbital} = \frac{\hbar^2 l(l+1)}{2mr^2} + K = \frac{\hbar^2 l(l+1)}{2mr^2} + \frac{e^2}{8\pi\varepsilon_0 r} \quad (21)$$

$$K_{orbital} = \frac{1}{2}mv_{orbital}{}^2 \qquad (22)$$

And, the electron's angular momentum L is

$$L = mv_{orbital}r \qquad (23)$$

Therefore, the electron's orbital kinetic energy $K_{orbital}$ is

$$K_{orbital} = \frac{L^2}{2mr^2} \qquad (24)$$

Therefore, according to Eq(21),

$$\frac{L^2}{2mr^2} = \frac{\hbar^2 l(l+1)}{2mr^2} + \frac{e^2}{8\pi\varepsilon_0 r} \quad (25)$$

The electron's angular momentum L is

$$L = \hbar\sqrt{l(l+1) + \frac{4\pi^2}{h^2}\cdot\frac{me^2 r}{4\pi\varepsilon_0}}$$

$$= \hbar \sqrt{l(l+1) + \frac{m\pi e^2 r}{\varepsilon_0 h^2}} \qquad (26)$$

In this time, hypothesizes that the r in Eq(26) is same the r in Eq(8)

$$r = n^2 \frac{h^2 \varepsilon_0}{\pi m e^2} \qquad (27)$$

Therefore, the electron's angular momentum L is

$$L = \hbar \sqrt{l(l+1) + \frac{m\pi e^2}{\varepsilon_0 h^2} \cdot n^2 \frac{h^2 \varepsilon_0}{\pi m e^2}} = \hbar \sqrt{l(l+1) + n^2} \quad (28)$$

n is the principal quantum number , l is the orbital quantum number

$$n = 1,2,3,\ldots \qquad l = 0,1,2,\ldots,(n-1)$$

III. Conclusion

If $l = 0$,

$$L = n\frac{h}{2\pi} = n\hbar$$

$$v_{orvital} = v = \sqrt{\frac{e^2}{4\pi\varepsilon_0 mr}} = \frac{e^2}{2nh\varepsilon_0}, \quad r = n^2 \frac{h^2 \varepsilon_0}{\pi m e^2}$$

$$n = 1,2,3,\ldots$$

$$K_{orbital} = \frac{L^2}{2mr^2} = \frac{1}{2} m v_{orvital}^{\ 2} = K = \frac{e^2}{8\pi\varepsilon_0 r} = \frac{1}{2} m v^2 \qquad (29)$$

Therefore, the electron's angular momentum L is

$$L = \hbar \sqrt{l(l+1) + n^2} \quad (30)$$

n is the principal quantum number , l is the orbital quantum number

$$n = 1,2,3,\ldots \qquad l = 0,1,2,\ldots,(n-1)$$

In this time, the r of the radius function $R = R(r)$ has to be continuous in the Schrodinger wave equation but the r of the electron's angular momentum L need not to be continuous in the quantum mechanic

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