Reinterpretting Solutions of Maxwell's Equations in Vacuum

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The authors show that the solutions of Maxwell's equations in vacuum admit electromagnetic waves having oscillations not only in electromagnetic field but also in spatial displacement. The fact that the existence of such a spatial component also of the electromagnetic wave has remained hidden from observations can be directly related to gauge invariance. It appears that the existence of the spatial oscillations can be attributed to a new basic field which in all probability can be the Higgs field.

PACS numbers: 41.20jb, 42.25Bs, 03.65-w. 03.50 De

KEY WORDS :Spatial amplitude of the electromagnetic wave, Helical structure of the electromagnetic wave, Structure of Photon, Fine structure constant, Compacting, Higgs field, Vector potential

I. INTRODUCTION

We know that electromagnetic field and its interactions have been thoroughly investigated during the last 150 years leaving hardly any scope for further improvement in our understanding. While classical electrodynamics has been able to explain a vast range of phenomena, the accuracy obtained in predicting these phenomena using quantum electrodynamics has been truly astounding. Nevertheless, we should remember that there are certain problems in the theory of electromagnetic fields which have remained intractable to this day. For example, we still do not have a clear picture as to how the electric charge gets compacted to a point without causing the problems of infinity [1]. We still do not know what the fine structure constant actually is except that it in some way or other represents the relative strength of the electromagnetic interactions [2]. We also do not have a clear idea how the amplitudes of the oscillations of electric and magnetic fields of the electromagnetic wave are related to its frequency. Possibly, the mystery of the fine structure constant may get unraveled if we could strike connection between these properties of the electromagnetic wave.

As a first step we propose to re-examine the basic concepts which go to the conventional representation of the electromagnetic wave. It is quite possible that a slight tweaking of the conventional picture while not altering the situation where the current theories reign supreme may provide us with new insights. The one idea we propose to examine in this paper is the implication of introducing spatial amplitude to the electromagnetic wave. Before we proceed further in this direction we shall have a brief review of the solutions of Maxwell's equation in vacuum and see how the conventional picture emerged from it.

We know that the beauty of the Maxwell's equations is that while they are very simple linear equations, classically all aspects of the electromagnetic phenomena can be explained by them. Since we propose to confine ourselves to the study of the transmission of electromagnetic waves in vacuum, we shall confine ourselves to Maxwell's equations in vacuum given by

$$\nabla \cdot \boldsymbol{\xi} = 0, \tag{1a}$$

$$\nabla \times \boldsymbol{\xi} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{1b}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{1c}$$

$$(\nabla \times \mathbf{B}) = \frac{1}{c^2}\frac{\partial \boldsymbol{\xi}}{\partial t}. \tag{1d}$$

We know that for the electric and magnetic fields these equations have solutions in the form (see Appendix)

$$\boldsymbol{\xi} = \boldsymbol{\xi}_0 \sin(\boldsymbol{\omega} t - \mathbf{k} \cdot \mathbf{r}). \tag{2}$$

$$\mathbf{B} = \mathbf{B}_0 \sin(\boldsymbol{\omega} t - \mathbf{k} \cdot \mathbf{r}). \tag{3}$$

Note that the magnetic field will always be perpendicular to the electric field and both will in turn be perpendicular to the direction of propagation. Another important point to be kept in mind is that the solutions represent not a single wave, but a wave front that has the same value for the electric (magnetic) field at any instant in the transverse direction.

II. INTRODUCING SPATIAL AMPLITUDE TO ELECTROMAGNETIC WAVE

The idea of the spatial amplitude of the electromagnetic wave was one of the most actively debated topics towards the end of the 19^{th} century. Going by the analogy with the mechanical waves, it was generally believed by physicists during that period that the electromagnetic wave which is a transverse wave needs a physical medium to propagate in space [5]. They named the medium ether. It was assumed that the oscillations in the electric and magnetic fields were set off by similar spatial oscillations in ether. But the problem with such a medium was that it had to possess all sorts of properties which were quite often incompatible with each other. Besides, in a way ether functioned like an absolute frame of reference against which any motion could be reckoned. Finally when Einstein came up with his theory of relativity in 1905 to explain the constancy of the velocity of light in all inertial frames of references, the concept of ether was given a decent burial. It became generally accepted that the electromagnetic waves were generated by oscillations in electric and magnetic fields only. Such an approach has further justification in that it is based entirely on Maxwell's equations which deal only with variations in the electric and the magnetic fields. These equations do not involve variations in the spatial displacement or any other field.

There is one more important reason for taking the spatial amplitude of the electromagnetic wave as zero. If the electromagnetic wave possessed spatial amplitude, then two identical waves in phase will have twice the spatial amplitude as compared to a single one. This means that the spatial amplitude of the electromagnetic waves in phase will be directly proportional to the intensity of the waves. Therefore, light transmitted through a small aperture should behave differently when the intensity of light is changed. For example, a low intensity coherent light beam may travel through a small aperture without any loss of energy while a high intensity beam should get obstructed substantially by the aperture. But we know light does not behave in that manner. On the other hand, it is observed that the oscillations in the electric (also magnetic) field add up when two such waves occupy the same space. The only way this difference in the behaviour of the spatial and the electric (magnetic) field oscillations could be resolved is by treating the amplitude of the spatial oscillations as zero.

We know that at any point on the path of the electromagnetic wave the divergence of the electric field (also magnetic field) in the transverse direction at any instant is zero. In fact the electromagnetic wave has to be seen as progressing as a wave front, rather than as a single wave progressing along a straight line. The progression of the wave may be represented by successive wave fronts in the direction of motion. Note that the wave front is assumed to extend to infinity in the transverse plane. In other words, the introduction of the spatial amplitude does not appear to be incompatible with Maxwell's equations. In the next section we shall go a step further and show that the electromagnetic wave could be attributed a helical structure which will provide us with a simple picture of photon which accounts for its spin angular momentum.

It is reasonable to assume that the spatial oscillations propagating at luminal velocities constitute the basic wave that transports the electromagnetic oscillations and these two oscillations could be having a phase difference of zero or π. If we denote the spatial amplitude by $\boldsymbol{\eta}_0$, then we may represent the plane polarized spatial and electromagnetic waves by

$$\boldsymbol{\eta} = \pm\boldsymbol{\eta}_0 \sin(\boldsymbol{\omega} t - \mathbf{k}\cdot\mathbf{r}) \tag{4a}$$

$$\boldsymbol{\xi} = \boldsymbol{\xi}_0 \sin(\boldsymbol{\omega} t - \mathbf{k}\cdot\mathbf{r}) \tag{4b}$$

The reason why the oscillations in the electric and the spatial displacement could combine only in two ways may be due to the fact that they represent the coupling between two different fields. If the spatial oscillations are allowed to couple with the oscillations of the electric (and magnetic) field in all possible phases, then in the case where the wave is trapped between two reflecting mirrors we may have a situation with the standing wave formed is entirely by the oscillations in the electromagnetic field while the spatial oscillations get destroyed completely. It could be the other way round also. Such coupling between the oscillations may be disallowed in order to avoid such situations.

Note that the introduction of the spatial oscillations above does not alter the situation represented by the solutions given in equations 2 and 3. The electromagnetic wave represented by equations 2 and 3 continues to represent the electric and magnetic fields in the transverse planes and their variation with time. In other words, the spatial wave defined by equations 4a and 4b is consistent with Maxwell's equations.

III. PHOTON AS A HELICAL CIRCULARLY POLARIZED WAVE

We saw that the solution of Maxwell's equations in vacuum is expressed as a sine function in equation 2. We could have taken the solution as a cosine function also. In fact, we could have taken the linear combination of the sine and cosine waves which would represent a circularly polarized wave. In that case treating the wave as progressing along the $Z-direction$, we may represent the circularly polarized (left handed when viewed head on) spatial and electromagnetic field oscillations by

$$\boldsymbol{\eta}_x = \boldsymbol{\eta}_0 \sin[\boldsymbol{\omega} t - \mathbf{k} z] \tag{5a}$$

$$\boldsymbol{\eta}_y = \boldsymbol{\eta}_0 \cos[\boldsymbol{\omega} t - \mathbf{k} z] \tag{5b}$$

$$\boldsymbol{\xi}_x = \boldsymbol{\xi}_0 \sin[\boldsymbol{\omega} t - \mathbf{k} z] \tag{5c}$$

$$\boldsymbol{\xi}_y = \boldsymbol{\xi}_0 \cos[\boldsymbol{\omega} t - \mathbf{k} z] \tag{5d}$$

Initially when the concept of photon was introduced by Einstein, it was treated as the particle aspect of the electromagnetic waves. The wave nature becomes relevant when we take a large group of the photons which can be studied classically. But the structure of photon itself has remained an enigma. The classical picture of the wave and the quantum picture of the particle have remained irreconcilable. There is one more reason for this incompatibility. The classical picture deals with only the propagation of the wave front. The idea of a wave train is not properly defined in the classical approach. With the introduction of the spatial amplitude as explained in the previous section, photon could be attributed the internal structure of a helical wave. To be exact, photon has to be treated as a wave train formed by helical waves with the electric (and magnetic) field riding on it making it circularly polarized.

We shall now show that the helical structure of the electromagnetic wave will force spatial amplitude $\boldsymbol{\eta}_0$ to possess only a certain value. We know that the projection of the helical wave given in equations 5a and 5b on to the transverse $X-Yplane$ will be a circle $(figure1)$. Let us now estimate the velocity of the circular motion. The simplest case that comes to our mind is one where the circular motion occurs at the velocity of light. Any other velocity will involve introduction of a new attribute to the electromagnetic wave. Since it is obvious that by the time the wave travels one wavelength along the $Z-axis$,it would have executed one full circle in the

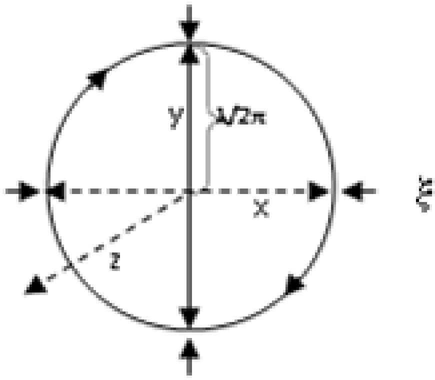

FIG. 1: This is the projection of a circularly polarized wave on a transverse plane. The vertical and the horizontal lines stand for two spatial waves having a phase difference of $\frac{\pi}{2}$. Here we have also shown arrows pointing inward representing the electric field at these points having a phase difference of π with spatial oscillations.

transverse directions. There fore, we may conclude that the radius , R of the circle will be given by

$$R = \frac{\lambda}{2\pi} = \frac{1}{\mathbf{k}} \tag{6}$$

Here we have a clearer idea of the term helicity used in the case of the electromagnetic wave because now we have the case of the spatial amplitude of the wave spinning around an axis. This allows us to understand the concept of spin of the electromagnetic wave classically on the basis of the helical structure of the wave. Since a point on the electromagnetic wave executes rotational motion in the transverse direction, we may take the magnitude of its angular momentum for this motion to be

$$S = \mid \mathbf{r} \times \mathbf{p} \mid = \left(\frac{\lambda}{2\pi}\right)\left(\frac{h}{\lambda}\right) = \hbar \tag{7}$$

Note that the momentum p used in the above expression represents the momentum of the circular motion in the transverse direction. Of course, the magnitude of the momentum in the direction of progression also has the same value.

IV. COMPACTING OF THE SPATIAL AMPLITUDE INTO THE INTERNAL SPACE

We shall now examine how the superposition of two waves affects the spatial amplitude. Let us consider two identical waves both in phase represented by equations 5a , 5b , 5c and 5d. If we go by the classical theory of the mechanical waves, resultant wave will be given by

$$\boldsymbol{\eta}_x^{'} = 2\boldsymbol{\eta}_0 \sin[\boldsymbol{\omega} t - \mathbf{k}z] \tag{8a}$$

$$\boldsymbol{\eta}_y^{'} = 2\boldsymbol{\eta}_0 \cos[\boldsymbol{\omega} t - \mathbf{k}z] \tag{8b}$$

$$\boldsymbol{\xi}_x^{'} = 2\boldsymbol{\xi}_0 \sin[\boldsymbol{\omega} t - \mathbf{k}z] \tag{8c}$$

$$\boldsymbol{\xi}_y^{'} = 2\boldsymbol{\xi}_0 \cos[\boldsymbol{\omega} t - \mathbf{k}z] \tag{8d}$$

We observe that the resultant wave has amplitude which is twice that of the individual component. Although prima-facie 8a and 8b appear to be in order, on detailed scrutiny we encounter a serious problem. We observe that while the spatial amplitude has doubled, the wave length of the wave has remained unchanged. This means that when we project the helical wave on to the transverse plane, the circle so obtained will have a radius of $\frac{\lambda}{\pi}$. Therefore, if the integrity of the wave is to be retained in propagation, the velocity of the circular motion will have to be twice that of light which is not acceptable. Note that this problem arises only in the case of the spatial oscillations. It does not arise in the case of the oscillations in the electric and magnetic fields.

The solution to this problem can be found if we treat electromagnetic waves as constituted by a large number of photons. Note that we represent a photon as a wave train of circularly polarized helical waves. Here the spatial oscillations of the photons do not interfere with each other by superposition. Only the electric and magnetic fields need to be affected by superposition. The spatial amplitude remains unaffected by superposition for reason that the velocity of circulation in the transverse plane can only be c. In other words two photons travelling in the same direction cannot be represented by waves in the classical sense by adding up their spatial amplitudes. In fact, it will be impossible to resolve the helical wave into two plane polarized spatial waves orthogonal to each other. Two helical waves of opposite helicity will travel along the same path without affecting the spatial amplitude of each other by the process of superposition. Note that this property is peculiar to only the spatial oscillations. As regards the oscillations in the electric and magnetic fields, they will behave in the classical manner interfering with each other constructively or destructively according to the phase difference between the waves.

The picture described above will be identical to the conventional picture provided we treat the diameter of the helical wave as negligibly small and equate it to zero for all practical purposes. But once the radius is treated as zero, it becomes impossible to account for the spin angular momentum of the wave. Therefore, spin will have to be accounted for by introducing an internal space to the wave and defining it there. Note that this procedure is nothing but treating the radius of the helical wave as zero by compacting it into the internal coordinates. In such a mathematical construct the helical path of the electromagnetic wave becomes a straight line and the internal space will have the form of a cylindrical tube.

We should understand that compacting the spatial amplitude into the internal space is a mathematical program which accounts for the fact that the spatial oscillations of two waves occupying the same region in space-time do not interfere with each other. If the mechanical properties of the wave propagation are to hold good such a situation would arise only when the spatial amplitude is zero. In the process of compacting the amplitude into the internal space it becomes possible to treat its value in the external coordinates as zero and this way the compatibility to the mechanics of the wave propagation is taken care of. We should keep in mind that the so called internal space is actually an innate part of the external space which is spanned by the laboratory coordinate system. The process of compacting of the amplitude into the internal coordinates should be understood as just a mathematical procedure to account for the peculiar nature of the spatial oscillations in the electromagnetic wave propagation.

Since this compacting is a mere mathematical construct one may presume that the spatial amplitude of the electromagnetic wave could be experimentally measured to the required accuracy. In the light of the above discussion we are tempted to conjecture that a coherent beam of light passing through a circular aperture of diameter less than $\frac{\lambda}{\pi}$ (being twice the spatial amplitude of the electromagnetic wave) should cut off transmission of waves through it thereby firmly establishing the existence of the spatial oscillations in the electromagnetic wave. But unfortunately the situation is not that simple as there is a cut off in the intensity of the transmitted wave when the aperture has a diameter of only $\frac{\lambda}{2}$ which is confirmed on the basis of the conventional approach [4]. Therefore, it will not be possible to observe the attenuation of the wave transmission at the aperture diameter of $\frac{\lambda}{\pi}$. In fact, this could be one of the reasons why the existence of the spatial oscillations in the electromagnetic wave has remained unnoticed.

V. INTRODUCING VECTOR POTENTIAL AS A REAL QUANTITY

In the conventional approach the vector potential is introduced as a mathematical construct devoid of physical reality. In spite of the fact that the $Aharanov-Bohm\ effect$ has been experimentally observed proving the physical reality of the vector potential, it appears that no success has been achieved in incorporating this aspect into the theory. We shall show that with the introduction of the spatial amplitude to the electromagnetic wave the physical reality of the vector potential can be explained. We know that the electric and magnetic fields can be expressed in terms of the vector potential as

$$\boldsymbol{\xi} = -\nabla\boldsymbol{\phi} - \frac{\partial \mathbf{A}}{\partial t} \tag{9a}$$

$$\mathbf{B} = \nabla \times \mathbf{A} \tag{9b}$$

Note that we have to ignore the gradient of the scalar potential in the expression for the electric field as we are dealing with vacuum which is a charge free state. Let us now examine how the vector potential will fit into the helical structure of the electromagnetic wave.

The best way to introduce vector potential is attribute to it a path along a solenoid which wraps tightly around the helical path. In other words, the path of the vector potential will be a solenoid which in turn forms a helix ($fig.2a$). But the problem with the figure given in $2(a)$ is that the direction of $\frac{\partial \mathbf{A}}{\partial t}$ will be always directed to the axis of the solenoid and therefore in terms of equations 9a and 9b the electric field (we may take $\boldsymbol{\phi}$ to be zero as we are dealing with the charge free case) will keep undergoing rotation at a higher frequency than the frequency of the helical wave with the result that the electric field cannot remain pointed in the radial direction of the helical wave continuously as demanded by Maxwell's equations. However, this problem can be resolved if we assume that the vector potential completes only one turn of the solenoid in one wavelength of the helical wave. In that case the path of the helical wave and the vector potential could be represented by a helical stair case ($fig.2b$).

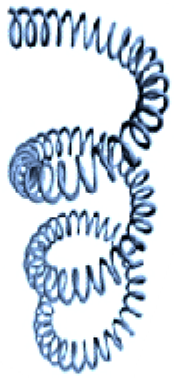

fig2a

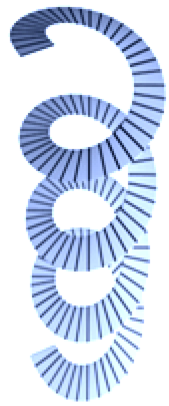

fig2b

The helix formed by the inner railing would represent the helical electromagnetic wave while the helix formed by the outer railing could represent the path of the vector potential. Note that $fig.(2b)$ is obtained from $fig.(2a)$ by taking the number of solenoid contained in one helix to be unity. The direction of the electric field will be parallel to the edges of the steps which are in the radial direction. It can be easily shown (see annexure B) that the curl of **A** which represents the magnetic field will be directed perpendicular to both the electric field and the direction of propagation and is therefore no different from what we have in the conventional solutions of Maxwell's equations.

VI. GAUGE INVARIANCE OF THE SPATIAL AMPLITUDE AND THE REAL NATURE OF THE VECTOR POTENTIAL

We know that Maxwell's equations possess the gauge freedom in terms of vector potential and scalar potentials which satisfy the relations

$$\mathbf{A}' = \mathbf{A} + \nabla \boldsymbol{\chi} \tag{10a}$$

$$\boldsymbol{\phi}' = \boldsymbol{\phi} - \frac{\partial \boldsymbol{\chi}}{\partial t} \tag{10b}$$

where $\boldsymbol{\chi}$ satisfies the wave equation

$$\frac{\partial^2 \boldsymbol{\chi}}{\partial x^2} - \left(\frac{1}{c^2}\right)\frac{\partial^2 \boldsymbol{\chi}}{\partial t^2} = 0 \tag{11}$$

Needless to say equations 10a and 10b are the direct results of the introduction of the Lorenz gauge condition

$$\nabla \cdot \mathbf{A} - \left(\frac{1}{c^2}\right)\frac{\partial \boldsymbol{\phi}}{\partial t} = 0 \tag{12}$$

It can be easily shown that the values of the electric field and the magnetic field given by equations 9a and 9b are independent of $\boldsymbol{\chi}$. In fact, till recently it was presumed that only $\boldsymbol{\xi}$ and $\mathbf{B}$ represent whatever is observable in an electromagnetic field. The vector potential $\mathbf{A}$ and the scalar potential $\boldsymbol{\phi}$ were not considered to be observable entities. However this assumption had to be changed after the discovery of $Aharonov - Bohm\ effect$ [6].

Let us now examine the gauge transformation given in 10a and 10b to find out what $\boldsymbol{\chi}$ stands for in the light of the helical wave structure of the electromagnetic wave. In the previous section we had taken $\mathbf{A}$ to be a vector with constant magnitude defined at every point on the solenoid which wraps around the helical path of the electromagnetic wave with its direction along its tangent. In the more general case we may attribute a normal component at every point on the solenoid. Therefore, we may resolve the vector potential $\mathbf{A}'$ as

$$\mathbf{A}' = \mathbf{A}_T + \mathbf{A}_N \tag{13}$$

where $\mathbf{A}_T$ is the component tangential to the solenoid while $\mathbf{A}_N$ is normal to it.Since $\mathbf{A}_N$ is in the direction of the radius vector $\boldsymbol{\eta}$, we may express it as a gradient in the form

$$\mathbf{A}_N = \nabla \boldsymbol{\chi} \tag{14}$$

There fore when we take the curl of $\mathbf{A}$ we obtain

$$\nabla \times \mathbf{A}' = \nabla \times \mathbf{A}_T \tag{15}$$

since

$$\nabla \times \mathbf{A}_N = 0 \tag{16}$$

As far as the electric field is concerned, the component$\mathbf{A}_N$ gets cancelled from the right hand side of equations 9a and 9b and therefore it will not contribute to the observable electric field. This means that whatever be its value, $\mathbf{A}_N$ does not alter the magnetic field. We may now re-express 13 as

$$\mathbf{A}^{'} = \mathbf{A}_T + \nabla\boldsymbol{\chi} \tag{17}$$

In the light of the above analysis we also observe that the second term of the Lorenz gauge condition given in equation 12 will always be zero because $\boldsymbol{\phi}$ is zero everywhere in the charge free situation. This means that $\nabla \cdot A = 0$. Such a result is possible only if the vector potential $\mathbf{A}$ has no contribution from $\mathbf{A}_N$ and is constituted entirely by $\mathbf{A}_T$. In the charge free case this is the essence of the Coulomb Gauge condition. Here we see that Coulomb Gauge condition and Lorenz Gauge conditions coincide. We know that the electric and the magnetic field are defined only on the helical path of the electromagnetic wave. But this does not mean that $\mathbf{A}$ and $\boldsymbol{\chi}$ are also defined only on the helical path of the wave. We shall show that even points removed from the helical path may be identified with specific values of $\mathbf{A}$ and $\boldsymbol{\chi}$. Here we should keep in mind that $\mathbf{A}$ and $\boldsymbol{\chi}$ possess directional symmetry in a plane transverse to the direction of propagation of the helical wave. Therefore, the value of $\mathbf{A}$ at a point could be taken as a function of r, where r is the radial distance of the point from the axis of the helical wave. In other words, if we take a transverse cross section of the helical electromagnetic wave, each point on the plane could possess vector potential $\mathbf{A}(r, \theta)$ where r and θ are the polar coordinates of the point. For the sake of convenience we shall take the transverse cross section to be in the $X - Y$ plane assuming that the electromagnetic wave is propagating in the $Z - direction$. In the case of $\boldsymbol{\chi}$, since it is scalar it will be a function of only x and y. We may now re-express equation **??**13) as

$$\mathbf{A}^{'} = \mathbf{A}_T + \nabla_r\boldsymbol{\chi}(r) \tag{18}$$

This requirement $\nabla_r\boldsymbol{\chi}(r) = \mathbf{A}_N$ will be met if we express $\boldsymbol{\chi}(r)$ as

$$\boldsymbol{\chi}_0 = \pm\frac{g(t,z)}{r} = \pm g(t,z)\left(x^2 + v^2\right)^{-\frac{1}{2}} \tag{19}$$

where $g(t, z)$ is a function of t and z. Remember that $\boldsymbol{\chi}$ satisfies the wave equation 11 and therefore $g(t, z)$ represents a wave propagating in the $Z - direction$. It is interesting to note that $e\boldsymbol{\chi}$ has the dimension of action and therefore $\dfrac{eg(t,z)}{\hbar}$ has the dimension of length. In other words, $\dfrac{e\boldsymbol{\chi}}{\hbar}$ would represent the length of the arc of the circle with radius $r = (x^2 + y^2)^{\frac{1}{2}}$. It is obvious that in such a situation $\dfrac{e\boldsymbol{\chi}}{\hbar}$ could represent a rotation through a certain angle in the $X - Yplane$. The gauge freedom ultimately boils down to this invariance in an arbitrary rotation. Recall that we had compressed the spatial amplitude into the internal space. Therefore, the rotation here pertains to the internal coordinates. In the forthcoming paper we shall show that this freedom to choose any value for $\boldsymbol{\chi}$ (gauge freedom) actually translates into the invariance of the wave function in an arbitrary phase change.

VII. DISCUSSION

From the above analysis it becomes quite clear that the introduction of the spatial amplitude to the electromagnetic wave not only simplifies many aspects of the theory but also provide us with a consistent physical picture. We further observe that the vector and scalar potentials are no more just mathematical constructs, but real fields which are defined in the three dimensional space.

We observe that the approach based on the classical mechanics has to treat A as a mathematical construct for two reasons. The first one is that the limiting condition on the spatial amplitude cannot be introduced based on classical physics. We saw the limit to the spatial amplitude is imposed due to the fact that the velocity of light is the limiting velocity for any physical entity. The second requirement for a solution of the problem is the introduction of the concept of photons as the basic state of the electromagnetic waves. Without the concept of photons, it will be impossible to account for the oscillations of two waves at a given spatial point as their displacements do not add up.

The introduction of the spatial amplitude to the electromagnetic wave implies the existence of a new field. It will be shown in a separate paper that electron-positron pair can be created from the confinement of the electromagnetic waves and they could be attributed the structure of confined helical half wave. The generation of the rest (mass) energy of the particle is seen to be the direct result of the localization of the energy of the electromagnetic wave contained in the spatial oscillations. It is observed that the fine structure constant, is the ratio of the energy of the electromagnetic field of the electron to its rest mass. Since the fine structure constant represents the strength of the electromagnetic interactions in relation to some other basic field, we are tempted to conclude that the rest mass of electron obtains its contribution from some other field. This leads to the suggestion that the energy of the spatial oscillations belongs to new field. We should keep in mind that the existence of the Higgs field has been mooted for quite some time as the field that creates mass [6]. We shall use the term Higgs field in the limited sense as a field which creates mass. We do not propose to relate our approach to the standard model. If the proposed approach is to hold good, the electromagnetic wave will have to be treated as a composite wave having oscillations both in the Higgs field and in the electromagnetic field.

VIII. CONCLUSION

Although prima-facie the idea that the electromagnetic waves possess spatial oscillations goes against all long established concepts, its existence does not warrant any modification in the conventional approach. This is because the concept of the spatial amplitude has already been accounted for in the conventional approach by the introduction of the internal coordinates. That apart, the existence of the spatial amplitude of $\frac{\lambda}{2\pi}$ cannot be directly observed because even by the conventional approach where the electromagnetic wave is supposed to propagate along a straight line a circular aperture of $\frac{\lambda}{\pi}$ will cut off transmission of waves through it. In a series of papers we shall show that we could treat electron as a confined helical wave formed from the electromagnetic wave with its spatial amplitude playing a crucial role in explaining the spin and electric charge of the particle. Therefore, it is a comforting thought that the introduction of the spatial amplitude will in no way affect the results validated by the conventional approach. The idea that the major part of the energy of the electromagnetic wave is constituted by the oscillations in the Higgs field, if found acceptable, may result in a completely new way of looking at the basic structure of particles.

IX. APPENDIX

Maxwell's equations in vacuum are given by

$$\nabla \cdot \boldsymbol{\xi} = 0, \tag{20a}$$

$$\nabla \times \boldsymbol{\xi} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{20b}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{20c}$$

$$c^2 \left(\nabla \times \mathbf{B}\right) = \frac{\partial \boldsymbol{\xi}}{\partial t}. \tag{20d}$$

We shall now solve these equations using Feynman's insightful approach to understand how the concept of the electromagnetic wave emerges from them [2] .

We know that the equation 20a could be expanded to obtain

$$\nabla \cdot \boldsymbol{\xi} = \frac{\partial \boldsymbol{\xi}_x}{\partial x} + \frac{\partial \boldsymbol{\xi}_y}{\partial y} + \frac{\partial \boldsymbol{\xi}_z}{\partial z} = 0 \tag{21}$$

here we assume that there are no variations in the field variables with respect to x and y so that the first two terms could be taken as zero. Hence we have

$$\frac{\partial \boldsymbol{\xi}_z}{\partial z} = 0 \tag{22}$$

This means that $\boldsymbol{\xi}_z$ is a constant in the z-direction. If we study Maxwell's equation 20d assuming that just as in the case of the electric field, the magnetic field also has no variation in x and y directions, then it can be seen that $\boldsymbol{\xi}_z$ is also a constant in time. Such a field could be conveniently taken as zero as we are interested in only dynamic fields. Therefore we may take $\boldsymbol{\xi}_z = 0$. In other words, the electric field exists only in the x and y directions. Now as a first step, for the sake of simplicity, we may assume that the electric field has a component only in the x-direction and obtain a solution on that basis. Later we may take up the case where the electric field has a component only in the y-direction and get the corresponding solutions. Then, the general solution could always be expressed as the superposition of the two cases.

Let us take the Maxwell's equation 20b and express the components along the three coordinate axes as

$$(\nabla \times \boldsymbol{\xi})_x = \frac{\partial \boldsymbol{\xi}_z}{\partial y} - \frac{\partial \boldsymbol{\xi}_y}{\partial z} \tag{23a}$$

$$(\nabla \times \boldsymbol{\xi})_y = \frac{\partial \boldsymbol{\xi}_x}{\partial z} - \frac{\partial \boldsymbol{\xi}_z}{\partial x} \tag{23b}$$

$$(\nabla \times \boldsymbol{\xi})_z = \frac{\partial \boldsymbol{\xi}_y}{\partial x} - \frac{\partial \boldsymbol{\xi}_x}{\partial y} \tag{23c}$$

Here $(\nabla \times \boldsymbol{\xi})_z$ will be zero because the derivatives with regard to x and y are zero. Note that from equation 21 we have already taken $\boldsymbol{\xi}_x$ as a constant while $\boldsymbol{\xi}_y$ is taken as zero. $(\nabla \times \boldsymbol{\xi})_x$ is zero because the first term which is a derivative of $\boldsymbol{\xi}_z$ is zero while the second term is zero for reasons already stated. The only component which is not zero is $(\nabla \times \boldsymbol{\xi})_y$ which is equal to $\frac{\partial \boldsymbol{\xi}_x}{\partial z}$. Setting the three components of $(\nabla \times \boldsymbol{\xi})$ equal to the corresponding components of $\dfrac{\partial \mathbf{B}}{\partial t}$, we obtain

$$\frac{\partial \mathbf{B}_z}{\partial t} = 0 \tag{24a}$$

$$\frac{\partial \mathbf{B}_x}{\partial t} = 0 \tag{24b}$$

$$\frac{\partial \mathbf{B}_y}{\partial t} = \frac{\partial \boldsymbol{\xi}_x}{\partial z} \tag{24c}$$

Since the z and x components of the magnetic field have zero time derivatives, they represent constant fields. Such a field could be conveniently taken as zero as we are interested in only dynamic fields. Therefore, we may take $\mathbf{B}_z = \mathbf{B}_x = 0$. The equation 24c shows that the electric field has only the x-component while the magnetic field has only the y-component. This means $\boldsymbol{\xi}$ and $\mathbf{B}$ are perpendicular to each other.

Let us now take the last Maxwell's equation whose components along x, y and z directions could be written as

$$c^2\left(\frac{\partial \mathbf{B}_z}{\partial y} - \frac{\partial \mathbf{B}_y}{\partial z}\right) = \frac{\partial \boldsymbol{\xi}_x}{\partial t} \tag{25a}$$

$$c^2\left(\frac{\partial \mathbf{B}_x}{\partial z} - \frac{\partial \mathbf{B}_z}{\partial x}\right) = \frac{\partial \boldsymbol{\xi}_z}{\partial t} \tag{25b}$$

$$c^2\left(\frac{\partial \mathbf{B}_y}{\partial x} - \frac{\partial \mathbf{B}_x}{\partial y}\right) = \frac{\partial \boldsymbol{\xi}_y}{\partial t} \tag{25c}$$

On the left hand sides of the above equations, with the exception of $\dfrac{\partial \mathbf{B}_y}{\partial z}$ all terms are zero. Therefore

$$-c^2\frac{\partial \mathbf{B}_y}{\partial z} = \frac{\partial \boldsymbol{\xi}_x}{\partial t} \tag{26}$$

Now taking partial differentiation with regard to t and using the equation 24c, we obtain the wave equations

$$\frac{\partial^2 \boldsymbol{\xi}_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{\xi}_x}{\partial t^2} = 0 \tag{27}$$

and

$$\frac{\partial^2 \mathbf{B}_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \mathbf{B}_y}{\partial t^2} = 0 \tag{28}$$

Note that the above equations represent waves having polarization in one plane. Similarly, we can obtain the equations for waves having polarization in a perpendicular plane involving only $\boldsymbol{\xi}_y$ and $\mathbf{B}_x$ as

$$\frac{\partial^2 \boldsymbol{\xi}_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{\xi}_y}{\partial t^2} = 0 \tag{29}$$

and

$$\frac{\partial^2 \mathbf{B}_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \mathbf{B}_x}{\partial t^2} = 0 \tag{30}$$

The solutions for these equations can be written as

$$\boldsymbol{\xi}_x = \boldsymbol{\xi}_{x0} \sin(\boldsymbol{\omega} t - \mathbf{k} \cdot \mathbf{z}). \tag{31a}$$

$$\boldsymbol{\xi}_y = \boldsymbol{\xi}_{y0} \sin(\boldsymbol{\omega} t - \mathbf{k} \cdot \mathbf{z}). \tag{31b}$$

$$\mathbf{B}_y = \mathbf{B}_{y0} \sin(\boldsymbol{\omega} t - \mathbf{k} \cdot \mathbf{z}). \tag{31c}$$

$$\mathbf{B}_x = -\mathbf{B}_{x0} \sin(\boldsymbol{\omega} t - \mathbf{k} \cdot \mathbf{z}). \tag{31d}$$

where $\boldsymbol{\omega}$ is the angular frequency and $\mathbf{k}$ is the wave vector. Actually we could have as well taken cosine function or even a complex function of the type $\boldsymbol{\xi}_0 e^{-i(\boldsymbol{\omega} t - \mathbf{k}\cdot\mathbf{z})}$. It is a matter of convenience. However, the fact that the sine function could be expressed as a linear combination of two waves, one travelling forward in time and the other travelling reverse in time is an added advantage as the wave equations given by equations 29 and 30 possess functions representing both waves as its solutions. Combining both, the wave equation in a general direction will be given by

$$\boldsymbol{\xi} = \boldsymbol{\xi}_0 \sin(\boldsymbol{\omega} t - \mathbf{k} \cdot \mathbf{r}). \tag{32}$$

Similarly, we may obtain the wave equation for the magnetic component also which may be written as

$$\mathbf{B} = \mathbf{B}_0 \sin(\boldsymbol{\omega} t - \mathbf{k} \cdot \mathbf{r}). \tag{33}$$

where $\mathbf{B}_0$ will always be perpendicular to $\boldsymbol{\xi}_0$

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