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# On Andrica's Conjecture, Cramér's Conjecture, gaps Between Primes and Jacobi Theta Functions II: A Simple Proof of Asymptotic for Andrica's Conjecture

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## 1. PRELIMINARES

In [1, p. 185] states that the prime number theorem yields:

(1) $p_n \sim n \log n$,

that is,

(2) $\lim_{n\to\infty} \frac{p_n}{n \log n} = 1.$

LEMMA1. *The Andrica's conjecture is equivalent to*

(3) $p_{n+1} < 1 + 2\sqrt{p_n} + p_n.$

*Proof.* Step 1. In [2, p.___], we conclude that Andrica's conjecture is equivalent to

(4) $\sqrt{p_n} < \frac{\theta_2}{\theta_3 - \theta_2} \Leftrightarrow \frac{1}{\frac{\theta_2}{\theta_3}} < 1 + \frac{1}{\sqrt{p_n}},$

where $k := \frac{p_n}{p_{n+1}}$ is a $k$ modulus. In [3, p. 83], we encounter

(5) $\frac{\theta_2}{\theta_3} = k^{1/2}.$

Substituting (5) in (4) and considering $k = \frac{p_n}{p_{n+1}}$, we have

(6) $\sqrt{p_n} < \frac{\theta_2}{\theta_3 - \theta_2} \Leftrightarrow \frac{\sqrt{p_{n+1}}}{\sqrt{p_n}} < 1 + \frac{1}{\sqrt{p_n}},$

ergo, squaring both sides of the equation (6),

$\frac{p_{n+1}}{p_n} < 1 + \frac{2}{\sqrt{p_n}} + \frac{1}{p_n} \Leftrightarrow p_{n+1} < 1 + 2\sqrt{p_n} + p_n.$ □

## 2. THEOREM

THEOREM 1 (Asymptotic for Andrica's Conjecture). *Let $n \in \mathbb{N}$ and $n$ sufficiently large, then*

$\sqrt{p_{n+1}} - \sqrt{p_n} < 1.$

*Proof.* Henceforth, we will use the *reductio ad absurdum* to prove the Lemma 1. We assume that

(7) $p_{n+1} \geq 1 + 2\sqrt{p_n} + p_n.$

For $n$ sufficiently large, we set (1) in (7), as follows

(8) $$(n+1)\log(n+1) \geq 1 + 2\sqrt{n \log n} + n \log n.$$

Dividing (8) by $\log(n+1)$, we encounter

(9) $n + 1 \geq \frac{1}{\log(n+1)} + 2\frac{\sqrt{n \log n}}{\log(n+1)} + n\frac{\log n}{\log(n+1)}.$

On the other hand, it is easy to see that, as $n$ is sufficiently large, then

(10) $\frac{\log n}{\log(n+1)} \to 1, \qquad \frac{1}{\log(n+1)} \to 0, \qquad \frac{\sqrt{n \log n}}{\log(n+1)} \to \infty,$
namely,

$$\lim_{n\to\infty} \frac{\log n}{\log(n+1)} = 1, \qquad \lim_{n\to\infty} \frac{1}{\log(n+1)} = 0, \qquad \lim_{n\to\infty} \frac{\sqrt{n \log n}}{\log(n+1)} = \infty.$$

Substituting (10) in (9), we find

(9) $n + 1 \geq 0 + 2(\infty) + n \cdot 1 \Leftrightarrow 1 \geq 2(\infty),$

which is false. Therefore, for $n$ sufficiently large, $p_{n+1} < 1 + 2\sqrt{p_n} + p_n$. In face of Lemma 1, the asymptotic for Andrica's conjecture is proved.▯

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