Reply to "Comments on "There is No Axiomatic System for the Quantum Theory""

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Jose Acacio de Barros discuss as follows [Jose Acacio de Barros, Int. J. Theor. Phys. **50**, 1828 (2011)]. Nagata claims to derive inconsistencies from quantum mechanics. [K. Nagata, Int. J. Theor. Phys. **48**, 3532 (2009)]. Jose Acacio de Barros discuss that the inconsistencies do not come from quantum mechanics, but from extra assumptions about the reality of observables. Here we discuss there is a contradiction within the quantum theory. We do not accept extra assumptions about the reality of observables. We use the actually happened results of quantum measurements (raw data). We use a single Pauli observable. We do not use the quantum predictions. We stress that we can use the quantum theory even if we give up the axiomatic system for the quantum theory.

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Jose Acacio de Barros discuss as follows [1]. Nagata claims to derive inconsistencies from quantum mechanics [2]. Jose Acacio de Barros discuss that the inconsistencies do not come from quantum mechanics, but from extra assumptions about the reality of observables. Here we discuss there is a contradiction within the quantum theory. We do not accept extra assumptions about the reality of observables. We use the actually happened results of quantum measurements (raw data). We use a single Pauli observable. We do not use the quantum predictions. We stress that we can use the quantum theory even if we give up the axiomatic system for the quantum theory.

First we discuss easy contradiction within the quantum theory as follows [3].

Matrix theory is not compatible with probability theory. Matrix theory has axioms. Probability theory has axioms. We consider joint set of such axioms. Does such joint set work as new set of axioms for matrix theory and for probability theory?

Let us consider joint probability. A is an observable. B is an observable. a, b are measurement outcome in a quantum state, respectively. A and B are not commutative. Thus,

$$[A, B] \neq 0. \tag{1}$$

We consider as follows: first we measure observable A and obtain the actually happened result of measurement a and next we measure observable B and obtain the actually happened result of measurement b. This joint event is different if we exchange A to B, in general. Hence

$$P(\overbrace{A = a}^{\text{first}} \cap \overbrace{B = b}^{\text{second}}) \neq P(\overbrace{B = b}^{\text{first}} \cap \overbrace{A = a}^{\text{second}}). \tag{2}$$

On the other hand, the joint probability is depictured in terms of conditional probability:

$$\begin{aligned} P(A = a|B = b)P(B = b) = P(A = a \cap B = b), \\ P(B = b|A = a)P(A = a) = P(B = b \cap A = a). \end{aligned} \tag{3}$$

From axioms of probability theory, we have

$$P(A = a \cap B = b) = P(B = b \cap A = a). \tag{4}$$

We cannot assign truth value "1" for the proposition (2) and for the proposition (4), simultaneously. We are in a contradiction. It turns out that the joint set of axioms does not work as new set of axioms for matrix theory and for probability theory. There is the contradiction within the quantum theory.

Next we discuss there is a contradiction within the quantum theory by using a single Pauli observable [4]. In this case, there is no argumentation concerning commuting observables or non-commuting observables. Especially, we systematically describe our assertion based on more mathematical analysis using raw data (the actually happened results of quantum measurements). In this case, there is no argumentation concerning the reality of observables. There exists raw data because we have seen it.

We consider the relation between double-slit experiment and projective measurement theory. We assume an implementation of double-slit experiment. There is a detector just after each slit. Thus interference figure does not appear, and we do not consider such a pattern. The actually happened results of measurements are ± 1 (in $\hbar/2$ unit). If a particle passes one side slit, then the value of the actually happened result of measurement is $+1$. If a particle passes another slit, then the value of the actually happened result of measurement is -1.

A. A wave function analysis

Let (σ_z, σ_x) be Pauli vector. We assume that a source of spin-carrying particles emits them in a state $|\psi\rangle$, which can be described as an eigenvector of Pauli observable σ_z. We consider a quantum expected value $\langle \sigma_x \rangle$ as

$$\langle \sigma_x \rangle = \langle \psi | \sigma_x | \psi \rangle = 0. \tag{5}$$

The above quantum expected value is zero if we consider only a wave function analysis.

We derive a necessary condition for the quantum expected value for the system in the pure spin-1/2 state $|\psi\rangle$ given in (5). We derive the possible value of the product $\langle\sigma_x\rangle \times \langle\sigma_x\rangle = \langle\sigma_x\rangle^2$. $\langle\sigma_x\rangle$ is the quantum expected value given in (5). We derive the following proposition

$$\langle\sigma_x\rangle^2 = 0. \tag{6}$$

B. Projective measurement theory

On the other hand, a mean value E satisfies projective measurement theory if it can be written as

$$E = \frac{\sum_{l=1}^{m} r_l(\sigma_x)}{m} \tag{7}$$

where l denotes a label and r is the actually happened result of projective measurement of the Pauli observable σ_x. We assume the actually happened value of r is ± 1 (in $\hbar/2$ unit).

Assume the quantum mean value with the system in an eigenvector ($|\psi\rangle$) of Pauli observable σ_z given in (5) admits projective measurement theory. One has the following proposition concerning projective measurement theory

$$\langle\sigma_x\rangle(m) = \frac{\sum_{l=1}^{m} r_l(\sigma_x)}{m}. \tag{8}$$

We can assume as follows by Strong Law of Large Numbers,

$$\langle\sigma_x\rangle(+\infty) = \langle\sigma_x\rangle = \langle\psi|\sigma_x|\psi\rangle. \tag{9}$$

In what follows, we show that we cannot assign the truth value "1" for the proposition (8) concerning projective measurement theory.

Assume the proposition (8) is true. By changing the label l into l' and by changing the label m into m', we have same quantum mean value as follows

$$\langle\sigma_x\rangle(m') = \frac{\sum_{l'=1}^{m'} r_{l'}(\sigma_x)}{m'}. \tag{10}$$

An important note here is that the actually happened value of the right-hand-side of (8) is equal to the actually happened value of the right-hand-side of (10) because we only change the labels. We have

$$\begin{aligned}
&\langle\sigma_x\rangle(m) \times \langle\sigma_x\rangle(m') \\
&= \frac{\sum_{l=1}^{m} r_l(\sigma_x)}{m} \times \frac{\sum_{l'=1}^{m'} r_{l'}(\sigma_x)}{m'} \\
&= \frac{\sum_{l=1}^{m} r_l(\sigma_x)}{\sum_{l=1}^{m}} \times \frac{\sum_{l'=1}^{m'} r_{l'}(\sigma_x)}{\sum_{l'=1}^{m'}} \times \frac{\delta_{ll'}}{\delta_{ll'}} \\
&= \frac{\sum_{l=1}^{m}}{m} \cdot (r_l(\sigma_x))^2 = \frac{\sum_{l=1}^{m}}{m} = 1.
\end{aligned} \tag{11}$$

Here $\delta_{ll'}$ is a delta function. We use the following fact

$$(r_l(\sigma_x))^2 = 1 \tag{12}$$

and

$$\frac{\delta_{ll'}}{\delta_{ll'}} = 1. \tag{13}$$

Thus we derive a proposition concerning the quantum mean value under the assumption that projective measurement theory is true (in a spin-1/2 system), that is

$$\langle\sigma_x\rangle(m) \times \langle\sigma_x\rangle(m') = 1. \tag{14}$$

From Strong Law of Large Numbers, we have

$$\langle\sigma_x\rangle \times \langle\sigma_x\rangle = 1. \tag{15}$$

Hence we derive the following proposition concerning projective measurement theory

$$\langle\sigma_x\rangle^2 = 1. \tag{16}$$

We do not assign the truth value "1" for two propositions (6) (concerning a wave function analysis) and (16) (concerning projective measurement theory), simultaneously. We are in a contradiction.

We cannot accept the validity of the proposition (8) (concerning projective measurement theory) if we assign the truth value "1" for the proposition (6) (concerning a wave function analysis). In other words, such projective measurement theory does not meet the detector model for spin observable σ_x. There is the contradiction within the quantum theory.

In conclusions, Jose Acacio de Barros has discussed as follows. Nagata has claimed to derive inconsistencies from quantum mechanics. Jose Acacio de Barros has discussed that the inconsistencies do not come from quantum mechanics, but from extra assumptions about the reality of observables. Here we have discussed there is a contradiction within the quantum theory. We do not have accepted extra assumptions about the reality of observables. We have used the actually happened results of quantum measurements (raw data). We have used a single Pauli observable. We do not have used the quantum predictions. We have stressed that we can use the quantum theory even if we give up the axiomatic system for the quantum theory.

[1] Jose Acacio de Barros, Int. J. Theor. Phys. **50**, 1828 (2011).
[2] K. Nagata, Int. J. Theor. Phys. **48**, 3532 (2009).
[3] K. Nagata and T. Nakamura, Quantum Inf. Process. (23.August.2013 accepted).
[4] K. Nagata and T. Nakamura, Precision Instrument and Mechanology, **2**, 104 (2013).