General Relativity as curvature of space

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Abstract

With the Planck 'constants' length, time, mass and acceleration will be shown, that a Quantum Gravity of the cosmos exists. This paper shows how Einstein’s Field Equations in Friedmann Robertson Walker Metric solves the Planck Era context.

1 The Planck 'constants'

Planck length \( \Delta x = \sqrt{\frac{Gh}{c^3}} \)

Planck time \( \Delta t = \sqrt{\frac{Gh}{c^5}} \)

Planck mass \( \Delta m = \sqrt{\frac{hc}{G}} \)

Planck acceleration \( \Delta a = \frac{c}{\Delta t} = \sqrt{\frac{c^5}{hG}} \)

2 Modern Cosmology

Within modern cosmology the Einstein’s Field Equations would be written with cosmological term \( \Lambda \) as follows (see [2] and [3]):

\[
R_{ik} - \frac{1}{2} g_{ik} R - \Lambda g_{ik} = \frac{8\pi G}{c^4} T_{ik} \quad (2.0)
\]

The solutions of the Field Equations in Friedmann-Roberson-Walker-Metric are:
Einstein abandoned the cosmological term $\Lambda$ as his "greatest blunder" after Hubble’s 1928 discovery that the distant galaxies are expanding away from each other. Within a Universe with ideal Quantum gas and without cosmological term $\Lambda$ and geometry factor $k = 0$ the equation (2.1) and (2.2) will become:

**FRW Equation (I)**

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G \rho}{3} (2.3)$$

and **FRW Equation (II)**

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3} \left( \frac{\rho + 3p}{c^2} \right) (2.4)$$

With the relation $p = \frac{\rho c^2}{3}$ (Quantum gas) will change (2.4) as follows: **FRW Equation (II)**

$$\frac{\ddot{R}}{R} = -\frac{8\pi G \rho}{3} (2.5)$$

Within Thermodynamics we assume $dE = TdS - pdV$ and an adiabatic process it holds $TdS = 0$. We become $d(\epsilon V) = d\epsilon V + \epsilon dV = -pdV$ and it follows:

With $d\epsilon = -(\epsilon + p) \frac{dV}{V}$ and the relation $p = \frac{\epsilon}{3}$ we assume:

$$\frac{d\epsilon}{\epsilon} = -\frac{4dV}{3V} \text{ or } \epsilon \sim V^{-4/3} \sim R^{-4} (2.6)$$

### 3 The Cosmic Background Radiation

Now we know the energy-density in the Planck-Era $\rho c^2 = \tilde{a} \Delta T^4 = \frac{3c^7}{8\pi^2\tilde{a}}$ ($\tilde{a} = \text{Radiation constant} = 7.5657 e^{-16}$). If we assume $c = h = G = k_B = 1$ we get:

$$\frac{3}{8\pi} = \tilde{a} \Delta T^4 = \frac{8\pi^5}{15} T^4 \text{ or } T^4 = \frac{45}{64\pi^6}$$
Now we get the Planck-Temperature $\Delta T = \left(\frac{45}{64\pi^2}\right)^{1/4} \frac{1}{6.08088337383}$

In Planck-Era the following 6 relations are valid:

\[ \Delta m \Delta x = \frac{\hbar}{c} \] (3.1)

\[ \Delta m \Delta t = \frac{\hbar}{c^2} \] (3.2)

\[ \frac{\Delta m}{\Delta a} = \frac{\hbar}{c^3} \] (3.3)

\[ \frac{\Delta m}{\Delta x} = \frac{c^2}{\alpha} \] (3.4)

\[ \frac{\Delta m}{\Delta t} = \frac{c^3}{\alpha} \] (3.5)

\[ \Delta F = \Delta m \Delta a = \frac{c^4}{\alpha} \] (3.6)

Now we could calculate the CBR ($T_\gamma$) as follows:

\[ E_\gamma = \frac{hc}{\lambda} = 6.08088337383kT_\gamma \text{ with } T_\gamma = 2.725K \]

We could calculate $m_\gamma = 2.5444e^{-39}kg$ and $x = \lambda_\gamma = 8.6828e^{-4}m$, also $t_\gamma = \frac{\lambda_\gamma}{c} = 2.8963e^{-12}s$.

For the CBR we receive:

\[ E_\gamma = m_\gamma ax = \frac{h\nu mc^2 c}{c^2 \frac{h}{\nu}} = m_\gamma c^2 \]
4 Gravitation as curvature of space

In macroscopic Scale the equation (3.1) til (3.6) will we rewritten as: (Entropieconstant
\( \zeta = \frac{\Delta T}{kT} = 2.1432e^{31} \))

\[ M \, R = \zeta^4 \frac{h}{c} \quad (4.1) \]

\[ M \, t = \zeta^4 \frac{h}{c^2} \quad (4.2) \]

\[ \frac{M}{a} = \zeta^4 \frac{h}{c^3} \quad (4.3) \]

\[ \frac{M}{R} = \frac{c^2}{G} \quad (4.4) \]

\[ \frac{M}{r} = \frac{c^3}{G} \quad (4.5) \]

\[ \Delta F = M \, a = \frac{c^4}{G} \quad (4.6) \]

With \( a = \frac{G \, M}{R^2} = \frac{M \, c^3}{\zeta^4 h} \) follows:

\[ \frac{G}{R^2} = \frac{c^3}{\zeta^4 h} \quad (4.7) \]

Furthermore we receive from GR:

\[ R = \zeta^2 \, \Delta x \Rightarrow \text{Radius of Universe} \, R = 1.861e^{28}m \]

\[ \frac{M}{R} = \frac{c^2}{G} = \frac{\Delta m}{\Delta x} \Rightarrow \text{Mass of Universe} \, M = 2.506e^{55}kg \]

\[ \frac{M}{t} = \frac{c^3}{G} = \frac{\Delta m}{\Delta t} \Rightarrow \text{Age of Universe} \, t = 6.207e^{19}s \]

For \( \dot{R}^2 = \frac{GM}{R^2} \) is with (4.7): \( \dot{R}^2 = M \, R \frac{c^3}{\zeta^4 h} = c^2 \)
The FRW Gleichung (1) (2.3) is as follows:

\[
\frac{c^2}{R^2} = \frac{8\pi G\rho}{3}
\]

or

\[
\frac{1}{R^4} = \frac{8\pi \rho c^2}{3\zeta^4 \hbar c}
\]

We become the \(R^4\) dependency of (2.6) as follows:

\[
\frac{3\zeta^4 \hbar c}{8\pi R^4} = \rho c^2 = \tilde{a} T^4
\]

## References

2. V. Sahni, The Case for a Positive Cosmological \(\Lambda\)-Term, astro-ph/9904398