General Relativity as curvature of space

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Abstract

With the Planck 'constants' length, time, mass and acceleration will be shown, that a Quantum Gravity of the cosmos exists. This paper shows how Einstein’s Field Equations in Friedman Robertson Walker Metric solves the Planck Era context.

1 The Planck 'constants'

Planck length $\Delta x = \sqrt{\frac{Gh}{c^3}}$

Planck time $\Delta t = \sqrt{\frac{Gh}{c^5}}$

Planck mass $\Delta m = \sqrt{\frac{hc}{G}}$

Planck acceleration $\Delta a = \frac{c}{\Delta t} = \sqrt{\frac{c^7}{hc}}$

2 Modern Cosmology

Within modern cosmology the Einstein’s Field Equations would be written with cosmological term $\Lambda$ as follows (see [2] and [3]):

$$R_{ik} - \frac{1}{2}g_{ik}R - \Lambda g_{ik} = \frac{8\pi G}{c^4}T_{ik} \quad (2.0)$$
The solutions of the Field Equations in Friedman-Roberson-Walker-Metric are:

FRW Equation (I)
\[ \frac{\dot{R}^2}{R^2} = \frac{8\pi G \rho}{3} + \frac{\Lambda c^2}{3} - \frac{k c^2}{R^2} \quad (2.1) \]

FRW Equation (II)
\[ \frac{\ddot{R}}{R} = \frac{\Lambda c^2}{3} - \frac{4\pi G}{3} \left( \rho + \frac{3p}{c^2} \right) \quad (2.2) \]

Einstein abandoned the cosmological term \( \Lambda \) as his "greatest blunder" after Hubble’s 1928 discovery that the distant galaxies are expanding away from each other. Within a Universe with ideal Quantum gas and without cosmological term \( \Lambda \) and geometry factor \( k = 0 \) the equation (2.1) and (2.2) will become:

FRW Equation (I)
\[ \frac{\dot{R}^2}{R^2} = \frac{8\pi G \rho}{3} \quad (2.3) \]

and FRW Equation (II)
\[ \frac{\ddot{R}}{R} = -\frac{4\pi G}{3} \left( \rho + \frac{3p}{c^2} \right) \quad (2.4) \]

With the relation \( p = \frac{\epsilon c^2}{3} \) (Quantum gas) will change (2.4) as follows: FRW Equation (II)
\[ \frac{\ddot{R}}{R} = -\frac{8\pi G \rho}{3} \quad (2.5) \]

Within Thermodynamics we assume \( dE = TdS - pdV \) and an adiabatic process it holds \( TdS = 0 \). We become \( d(\epsilon V) = d\epsilon V + \epsilon dV = -pdV \) and it follows:

With \( d\epsilon = -(\epsilon + p) \frac{dV}{V} \) and the relation \( p = \frac{\epsilon}{3} \) we assume:
\[ \frac{d\epsilon}{\epsilon} = -\frac{4dV}{3V} \quad \text{or} \quad \epsilon \sim V^{-4/3} \sim R^{-4} \quad (2.6) \]
3 The Cosmic Background Radiation

Now we know the energy-density in the Planck-Era $\rho c^2 = \tilde{a}\Delta T^4 = \frac{3\bar{c}^7}{8\pi^5 G^2} (\tilde{a} = \text{Radiation constant} = 7.5657 e^{-16})$. If we assume $c = h = G = k_B = 1$ we get:

$$\frac{3}{8\pi} = \tilde{a}\Delta T^4 = \frac{8\pi^5}{15} T^4 \text{ or } T^4 = \frac{45}{64\pi^5}$$

Now we get the Planck-Temperature $\Delta T = \left(\frac{45}{64\pi^5}\right)^{1/4} = \frac{1}{6.08088337383} = 5.8404 e^{31} K$

In Planck-Era the following 6 relations are valid:

$$\Delta m \Delta x = \frac{h}{c} \quad (3.1)$$

$$\Delta m \Delta t = \frac{h}{c^2} \quad (3.2)$$

$$\frac{\Delta m}{\Delta a} = \frac{h}{c^3} \quad (3.3)$$

$$\frac{\Delta m}{\Delta x} = \frac{c^2}{G} \quad (3.4)$$

$$\frac{\Delta m}{\Delta t} = \frac{c^3}{G} \quad (3.5)$$

With Planck-Force $\Delta F = \Delta m \Delta a = \frac{\Delta a}{\Delta t}$ \quad (3.6)
4 Gravitation as curvature of space

In macroscopic scale the equation (3.1) til (3.6) will be rewritten as: (Entropy constant $\zeta = \sqrt{\frac{I}{\Delta M}} = 1.7952e^{30}$)

\[ M R = \zeta^4 \frac{h}{c} \quad (4.1) \]

\[ M t = \zeta^4 \frac{h}{c^2} \quad (4.2) \]

\[ \frac{M}{a} = \zeta^4 \frac{h}{c^3} \quad (4.3) \]

\[ \frac{M}{\mathcal{R}} = \frac{c^2}{G} \quad (4.4) \]

\[ \frac{M}{\mathcal{T}} = \frac{c^3}{G} \quad (4.5) \]

With Planck-Force $\Delta F = M a = \frac{c^4}{G}$ (4.6)

With Planck-Acceleration $a = \frac{GM}{R^2} = \frac{Mc^3}{\zeta^4h}$ follows:

\[ \frac{GM}{R^2} = \frac{c^3}{\zeta^4h} \quad (4.7) \]

Furthermore we receive from GR:

\[ \frac{M}{t} = \frac{c^3}{G} = \frac{\Delta m}{\Delta t} \Rightarrow \text{Age of Universe } t = 4.355e^{17}s \]

\[ R = \zeta^2 \Delta x \Rightarrow \text{Radius of Universe } R = 1.306e^{26}m \]

\[ \frac{M}{\mathcal{R}} = \frac{c^2}{G} = \frac{\Delta m}{\Delta x} \Rightarrow \text{Mass of Universe } M = 1.758e^{53}kg \]

For $\dot{R}^2 = \frac{GM}{R}$ is with (4.7): $\dot{R}^2 = MR \frac{c^3}{\zeta^4h} = c^2$

The FRW Equation (I) (2.3) is with $\dot{R} = c$ as follows:
\[ \frac{c^2}{R^2} = \frac{8\pi G \rho}{3} \]

or with (4.7):

\[ \frac{1}{R^4} = \frac{8\pi \rho c^2}{3\zeta^4 h c} \]

We become the \( R^4 \) dependency of (2.6) as follows:

\[ \frac{3\zeta^4 h c}{8\pi R^4} = \rho c^2 = \dot{a} T^4 \gamma \]

We get also:

\[ \frac{1}{R} = \frac{M c}{\zeta^4 \hbar} \text{ q.e.d.} \]

5 References


3. V.Sahni, The Case for a Positive Cosmological \( \Lambda \)-Term, astro-ph/9904398
