Consistent Cutoff in Quantum Gravity

N.S. Baaklini
nsbqft@aol.com
http://www.vixra.org/author/n_s_baaklini

Abstract

The development of covariant quantum gravity in flat spacetime demands, for the sake of consistency, that any vacuum (or cosmological constant) terms that are generated from loop contributions must vanish. This consistency condition enables us, in the framework of the effective action with ultraviolet cutoff, to relate the cutoff parameter to the gravitational coupling, order by order in the perturbative scheme. We develop the formalism of covariant quantum gravity that enables us to compute the vacuum contributions up to the second loop order, dealing with the system describing the self-interactions of virtual gravitons, together with scalar and vector fields. We show how the vanishing of the resulting vacuum contributions gives a relation between the cutoff and the gravitational coupling. In a fundamental theory unifying gravity and the other interactions, this approach offers an ultimate solution to the problems of quantum field divergences.

1 Introduction

It is an old speculative idea[1], [2], [3] that quantum gravity may hold the key for an ultimate solution to the problem of the divergences that plague quantum field theory. Old attempts to implement this idea had taken shape[4], [5], [6] in a suggestive process of summing an infinite number of divergent contributions of a certain type. However such schemes are beset with difficulties and could not be applied in general. On the other hand, the divergences of quantum gravity are well-known[7]-[19], and all previous treatments, working within the ordinary framework of covariant quantization, along the conventional lines of non-Abelian gauge field quantization, have all demonstrated the obvious fact that such divergences cannot be eliminated in the manner of renormalizable theories.

In our effective action framework for quantum gravity[20], [21], we find that it is necessary to introduce a fundamental gauge-invariant cutoff which *can be related perturbatively to the gravitational coupling*. The condition for establishing such a relation is the requirement, which stems from the *consistency of quantization in a flat spacetime* background, that the vacuum (or cosmological constant) contributions that are generated by the quantum loop corrections should be put equal to zero in a perturbative manner. Our purpose in this article is to present computations of the vacuum contributions up to the second loop order. We shall deal with a system of self-interacting virtual gravitons that are also coupled to virtual scalar and vector fields. The extension to include fermionic fields will be the subject of other articles.

In §2, we shall introduce the pertinent Lagrangian densities and shall derive the necessary propagators and vertices that are needed in our computations. Whereas quantum gravitational computations are notoriously complicated due to the unwieldy numbers of indices and terms constituting the graviton vertices, we attempt to make some simplification in our computations. In the first place, in our powerful effective action framework, it is possible to construct a gauge-invariant effective action imposing a gauge-covariant condition like $\nabla_\mu \phi_{\mu\nu} = 0$ on the virtual graviton (gauge-covariance pertains to the effective metric), and a condition like $\nabla_\mu A_\mu = 0$ on vector fields, all without the need for using the horrid ghost technique of conventional quantization. In our derivation of the graviton vertices, this allows for dispensing with many terms. On the other hand, a judicious use of the symmetries of the indices involved permits a reduction of the number of terms that need be displayed for the public. In spite of all that, computations remain laborious and would require the use of computer-aided symbolic manipulations, something we are constantly extending[22] for the purpose of tackling such fundamental theoretical problems.

In §3 we use our derived Feynman rules for the computation of the quantum corrections to the vacuum, up to the second loop order. This is the minimal result needed to show how one can relate the cutoff length to the gravitational coupling constant in a perturbative manner.

2 Lagrangians and the Feynman Rules

In the following subsections, we shall consider the Lagrangians for pure gravity, and for the coupling of the gravitational field to bosonic scalar and vector matter. The extension that includes fermionic Dirac fields will be treated in another article.

2.1 Pure Gravity

The Lagrangian density that describes the propagative and the self-interactive nature of the gravitational field, via the metric field of general relativity $g_{\mu\nu}$, take the following form:

$$\sqrt{g} g^{\mu\nu} g^{\lambda\rho} g^{\alpha\beta} \begin{pmatrix} \frac{1}{4}\partial_\mu g_{\lambda\alpha}\partial_\nu g_{\rho\beta} - \frac{1}{4}\partial_\mu g_{\lambda\rho}\partial_\nu g_{\alpha\beta} \\ +\frac{1}{2}\partial_\mu g_{\nu\lambda}\partial_\rho g_{\alpha\beta} - \frac{1}{2}\partial_\mu g_{\lambda\alpha}\partial_\rho g_{\nu\beta} \end{pmatrix} \tag{1}$$

In the above, $g^{\mu\nu}$ is the inverse metric, and g is the positive determinant of $g_{\mu\nu}$. The above Lagrangian density follows from the Einstein term $\sqrt{g}R$, where $R = g^{\mu\nu}R_{\mu\nu}$, and $R_{\mu\nu}$ is the Ricci tensor

$$R_{\mu\nu} = \partial_\mu \Gamma^\lambda_{\nu\lambda} - \partial_\lambda \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\mu\nu}\Gamma^\rho_{\lambda\rho} + \Gamma^\lambda_{\mu\rho}\Gamma^\rho_{\nu\lambda} \tag{2}$$

and using the metric connection

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\rho} \left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\mu\rho} - \partial_\rho g_{\mu\nu} \right) \tag{3}$$

As we shall be computing vacuum contributions only, we shall split the metric field such as $g_{\mu\nu} = \eta_{\mu\nu} + \phi_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowskian metric of flat spacetime and $\phi_{\mu\nu}$ is the virtual graviton field (being symmetric in its indices). All expansions will be done to second order in ϕ since that would be enough for two-loop computations. The inverse metric $g^{\mu\nu}$ and the factor $\sqrt{g} = \exp(\frac{1}{2}\mathbf{tr}\ln(\eta+\phi))$ would have the corresponding expansions:

$$\begin{cases} g^{\mu\nu} \approx \eta_{\mu\nu} - \phi_{\mu\nu} + \phi_{\mu\lambda}\phi_{\nu\lambda} \\ \\ \sqrt{g} \approx 1 + \frac{1}{2}\phi_{\lambda\lambda} + \frac{1}{8}\phi_{\lambda\lambda}\phi_{\rho\rho} - \frac{1}{8}\phi_{\lambda\rho}\phi_{\lambda\rho} \end{cases} \tag{4}$$

Notice that after expanding about the flat metric $\eta_{\mu\nu}$, it will be convenient to put all indices on the same level, with the understanding that summations (over repeated indices) are done with the flat metric.

Expanding the foregoing Lagrangian density for the metric field $g_{\mu\nu}$, to second order in the virtual graviton field $\phi_{\mu\nu}$, we shall obtain the bilinear terms which give us a propagator, the cubic terms which give us the 3-leg vertex, and the quartic terms, which give us the 4-leg vertex. Notice that we can multiply each $\phi_{\mu\nu}$ by a (gravitational) coupling constant κ, and divide the whole Lagrangian by κ^2. However, we shall suppress κ in the followings, reinstating it in the end results of the loop computations.

2.1.1 Bilinear Terms and the Graviton Propagator

The bilinear terms in the virtual graviton field are:

$$\frac{1}{4}\left(\partial_\mu\phi_{\lambda\nu}\right)^2 - \frac{1}{4}\left(\partial_\mu\phi_{\lambda\lambda}\right)^2 - \frac{1}{2}\partial_\lambda\phi_{\mu\nu}\partial_\mu\phi_{\lambda\nu} + \frac{1}{2}\partial_\lambda\phi_{\lambda\mu}\partial_\mu\phi_{\nu\nu} \tag{5}$$

In our covariant development of the effective quantum gravitational action we impose the condition $\nabla_\mu\phi_{\mu\nu} = 0$ on the virtual graviton field. The latter condition is covariant with respect to the effective graviton field. Here, where we intend to compute the vacuum contributions, this means $\partial_\mu\phi_{\mu\nu} = 0$. We shall use the latter expression to simplify the above bilinears, as well as all later expressions that give us the vertices. Hence, we can drop the last two terms of the above bilinears. Then converting to momentum space, we obtain

$$\frac{1}{8}\phi_{\mu\nu}(p)\phi_{\lambda\rho}(-p)\mathcal{K}_{\mu\nu,\lambda\rho}[p] \tag{6}$$

with the bilinear kernel

$$\mathcal{K}_{\mu\nu,\lambda\rho}[p] = p^2\left(\eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda} - 2\eta_{\mu\nu}\eta_{\lambda\rho}\right) \tag{7}$$

The inverse of the above bilinear kernel would give us the following propagator for the virtual graviton field:

$$\mathcal{P}_{\mu\nu,\lambda\rho}[p] = \frac{1}{p^2}\left(\eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda} - \frac{2}{3}\eta_{\mu\nu}\eta_{\lambda\rho}\right) \tag{8}$$

Notice that $\mathcal{P}$ and $\mathcal{K}$ are inverses of each other in the following sense:

$$\frac{1}{2}\mathcal{K}_{\mu\nu,\alpha\beta}\mathcal{P}_{\alpha\beta,\lambda\rho} = \eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda} \tag{9}$$

2.1.2 Cubic Terms and the Graviton 3-Leg Vertex

Resulting from the foregoing expansions, the terms that are cubic in the virtual graviton field are (after dropping three terms that involve $\partial_\mu\phi_{\mu\nu}$):

$$\begin{cases} -\frac{1}{2}\phi_{\mu\nu}\partial_\lambda\phi_{\mu\rho}\partial_\lambda\phi_{\nu\rho} + \frac{1}{2}\phi_{\mu\nu}\partial_\lambda\phi_{\mu\nu}\partial_\lambda\phi_{\rho\rho} - \frac{1}{2}\phi_{\mu\nu}\partial_\lambda\phi_{\rho\rho}\partial_\mu\phi_{\lambda\nu} + \phi_{\mu\nu}\partial_\lambda\phi_{\mu\rho}\partial_\nu\phi_{\lambda\rho} \\ -\frac{1}{4}\phi_{\mu\nu}\partial_\mu\phi_{\lambda\rho}\partial_\nu\phi_{\lambda\rho} + \frac{1}{8}\phi_{\mu\mu}(\partial_\nu\phi_{\lambda\rho})^2 + \frac{1}{4}\phi_{\mu\nu}\partial_\mu\phi_{\lambda\lambda}\partial_\nu\phi_{\rho\rho} - \frac{1}{8}\phi_{\mu\mu}\partial_\nu\phi_{\lambda\lambda}\partial_\nu\phi_{\rho\rho} \\ +\frac{1}{2}\phi_{\mu\nu}\partial_\lambda\phi_{\mu\rho}\partial_\rho\phi_{\lambda\nu} - \frac{1}{4}\phi_{\mu\mu}\partial_\nu\phi_{\lambda\rho}\partial_\rho\phi_{\lambda\nu} \end{cases} \tag{10}$$

Converting to momentum space (with three momenta r, s, t), we obtain[1]

$$-\delta^4[s+t+u]\begin{Bmatrix} -\frac{1}{2}\phi_{\mu\nu}[s]\phi_{\mu\rho}[t]\phi_{\nu\rho}[u](t\cdot u) + \frac{1}{2}\phi_{\mu\nu}[s]\phi_{\mu\nu}[t]\phi_{\rho\rho}[u](t\cdot u) \\ -\frac{1}{2}\phi_{\mu\nu}[s]\phi_{\rho\rho}[t]\phi_{\lambda\nu}[u](t_\lambda u_\mu) + \phi_{\mu\nu}[s]\phi_{\mu\rho}[t]\phi_{\lambda\rho}[u](t_\lambda u_\nu) \\ -\frac{1}{4}\phi_{\mu\nu}[s]\phi_{\lambda\rho}[t]\phi_{\lambda\rho}[u](t_\mu u_\nu) + \frac{1}{8}\phi_{\mu\mu}[s]\phi_{\lambda\rho}[t]\phi_{\lambda\rho}[u](t\cdot u) \\ +\frac{1}{4}\phi_{\mu\nu}[s]\phi_{\lambda\lambda}[t]\phi_{\rho\rho}[u](t_\mu u_\nu) - \frac{1}{8}\phi_{\mu\mu}[s]\phi_{\lambda\lambda}[t]\phi_{\rho\rho}[u](t\cdot u) \\ +\frac{1}{2}\phi_{\mu\nu}[s]\phi_{\mu\rho}[t]\phi_{\lambda\nu}[u](t_\lambda u_\rho) - \frac{1}{4}\phi_{\mu\mu}[s]\phi_{\lambda\rho}[t]\phi_{\lambda\nu}[u](t_\nu u_\rho) \end{Bmatrix} \tag{11}$$

In order to obtain the graviton 3-leg vertex, we must differentiate the above three times, with respect to $\phi_{\alpha\beta}[p]$, $\phi_{\sigma\tau}[q]$, and $\phi_{\kappa\omega}[r]$, with $(\alpha,\beta,\sigma,\tau,\kappa,\omega)$ being vectorial spacetime indices, and (p,q,r) being momenta. We must use the rule:

$$\frac{\partial\phi_{\mu\nu}[p]}{\partial\phi_{\lambda\rho}[q]} = (\eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda})\,\delta^4[p-q] \tag{12}$$

Now to reduce the size (8-fold) of the resulting expression of the 3-leg vertex (after renaming indices), we give it unsymmetrized with respect to (μ,ν), (λ,ρ), and (α,β). The following unsymmetrized expression can be used as it is in our loop computations since it will be multiplied by tensor propagators having the required symmetries. Dropping all terms that involve $(p_\mu, p_\nu, q_\lambda, q_\rho, r_\alpha, r_\beta)$ since they correspond to the condition[2] $\partial_\mu\phi_{\mu\nu} = 0$, then using the implications of the momentum-conserving delta function

[1] **Integration over all momenta in the action terms are implicit, but are suppressed throughout.**

[2] **Notice, however, that this can be done only since we are computing vacuum vertices. For the computation of effective vertices, the dropped terms must be replaced by effective field counterparts using the covariant condition $\nabla_\mu\phi_{\mu\nu}$. Otherwise we should not drop any terms in the derivation of the vertices whether vacuum or effective.**

$\delta^4(p+q+r)$, while suppressing the latter, we obtain

$$\mathcal{V}_{\mu\nu,\lambda\rho,\alpha\beta}[p,q,r] = \left\{ \begin{array}{l} -4r_\lambda r_\rho \eta_{\alpha\mu}\eta_{\beta\nu} - 4q_\mu q_\nu \eta_{\alpha\lambda}\eta_{\beta\rho} - 8q_\mu r_\lambda \eta_{\alpha\nu}\eta_{\beta\rho} \\ +4q_\mu q_\nu \eta_{\alpha\beta}\eta_{\lambda\rho} - 4q_\alpha q_\mu \eta_{\beta\nu}\eta_{\lambda\rho} + 2(2q^2 + q\cdot r + r^2)\eta_{\alpha\mu}\eta_{\beta\nu}\eta_{\lambda\rho} \\ +4r_\lambda r_\rho \eta_{\alpha\beta}\eta_{\mu\nu} - 4q_\alpha r_\lambda \eta_{\beta\rho}\eta_{\mu\nu} + 2(2q^2 + 3q\cdot r + 2r^2)\eta_{\alpha\lambda}\eta_{\beta\rho}\eta_{\mu\nu} \\ +4q_\alpha q_\beta \eta_{\lambda\rho}\eta_{\mu\nu} - 2(q^2 + q\cdot r + r^2)\eta_{\alpha\beta}\eta_{\lambda\rho}\eta_{\mu\nu} + 4q_\mu r_\lambda \eta_{\alpha\beta}\eta_{\nu\rho} \\ +8q_\alpha q_\mu \eta_{\beta\lambda}\eta_{\nu\rho} - 8(q^2 + q\cdot r + r^2)\eta_{\alpha\mu}\eta_{\beta\lambda}\eta_{\nu\rho} \\ +8q_\alpha r_\lambda \eta_{\beta\mu}\eta_{\nu\rho} - 4q_\alpha q_\beta \eta_{\lambda\mu}\eta_{\nu\rho} + 2(q^2 + q\cdot r + 2r^2)\eta_{\alpha\beta}\eta_{\lambda\mu}\eta_{\nu\rho} \end{array} \right\} \tag{13}$$

The above gives the expression of the vacuum vertex with 3 legs corresponding to virtual graviton having respective indices $(\mu\nu, \lambda\rho, \alpha\beta)$, and respective momenta (p, q, r), with p replaced by $-(q+r)$ due to momentum conservation.

2.1.3 Quartic Terms and the 4-Leg Vertex

Also, resulting from the foregoing expansions, the terms that are quartic in the virtual graviton field are (after dropping seven terms that involve $\partial_\mu \phi_{\mu\nu}$):

$$\left\{ \begin{array}{l} -\frac{1}{4}\phi_{\lambda\rho}\phi_{\mu\nu}\partial_\alpha\phi_{\lambda\rho}\partial_\alpha\phi_{\mu\nu} + \frac{1}{4}\phi_{\lambda\rho}\phi_{\mu\nu}\partial_\alpha\phi_{\lambda\mu}\partial_\alpha\phi_{\nu\rho} + \frac{1}{16}\phi_{\mu\nu}^2\partial_\alpha\phi_{\lambda\rho}\partial_\lambda\phi_{\alpha\rho} \\ -\frac{1}{16}\phi_{\mu\mu}\phi_{\nu\nu}\partial_\alpha\phi_{\lambda\rho}\partial_\lambda\phi_{\alpha\rho} + \frac{1}{2}\phi_{\lambda\rho}\phi_{\mu\nu}\partial_\alpha\phi_{\mu\nu}\partial_\lambda\phi_{\alpha\rho} - \frac{1}{32}\phi_{\mu\nu}^2\left(\partial_\lambda\phi_{\alpha\rho}\right)^2 \\ +\frac{1}{32}\phi_{\mu\mu}\phi_{\nu\nu}\left(\partial_\lambda\phi_{\alpha\rho}\right)^2 + \frac{1}{32}\phi_{\mu\nu}^2\partial\phi_{\alpha\alpha}\partial_\lambda\phi_{\rho\rho} - \frac{1}{32}\phi_{\mu\mu}\phi_{\nu\nu}\partial_\lambda\phi_{\alpha\alpha}\partial_\lambda\phi_{\rho\rho} \\ +\frac{1}{2}\phi_{\lambda\rho}\phi_{\mu\nu}\partial_\lambda\phi_{\mu\rho}\partial_\nu\phi_{\alpha\alpha} - \frac{1}{2}\phi_{\lambda\rho}\phi_{\mu\nu}\partial_\lambda\phi_{\alpha\mu}\partial_\nu\phi_{\alpha\rho} - \frac{1}{8}\phi_{\lambda\nu}\phi_{\mu\mu}\partial_\lambda\phi_{\alpha\rho}\partial_\nu\phi_{\alpha\rho} \\ +\frac{1}{4}\phi_{\lambda\mu}\phi_{\lambda\nu}\partial_\mu\phi_{\alpha\rho}\partial_\nu\phi_{\alpha\rho} + \frac{1}{8}\phi_{\lambda\nu}\phi_{\mu\mu}\partial_\lambda\phi_{\alpha\alpha}\partial_\nu\phi_{\rho\rho} - \frac{1}{4}\phi_{\lambda\mu}\phi_{\lambda\nu}\partial_\mu\phi_{\alpha\alpha}\partial_\nu\phi_{\rho\rho} \\ -\frac{1}{4}\phi_{\lambda\rho}\phi_{\mu\nu}\partial_\lambda\phi_{\mu\nu}\partial_\rho\phi_{\alpha\alpha} - \frac{1}{4}\phi_{\lambda\nu}\phi_{\mu\mu}\partial_\lambda\phi_{\nu\rho}\partial_\rho\phi_{\alpha\alpha} + \frac{1}{2}\phi_{\lambda\mu}\phi_{\lambda\nu}\partial_\mu\phi_{\nu\rho}\partial_\rho\phi_{\alpha\alpha} \\ -\phi_{\lambda\rho}\phi_{\mu\nu}\partial_\alpha\phi_{\lambda\mu}\partial_\rho\phi_{\alpha\nu} - \frac{1}{2}\phi_{\lambda\mu}\phi_{\lambda\nu}\partial_\alpha\phi_{\mu\rho}\partial_\rho\phi_{\alpha\nu} + \frac{1}{4}\phi_{\lambda\lambda}\phi_{\mu\nu}\partial_\alpha\phi_{\mu\rho}\partial_\rho\phi_{\alpha\nu} \\ +\frac{1}{2}\phi_{\lambda\rho}\phi_{\mu\nu}\partial_\lambda\phi_{\alpha\mu}\partial_\rho\phi_{\alpha\nu} + \frac{1}{4}\phi_{\lambda\nu}\phi_{\mu\mu}\partial_\lambda\phi_{\alpha\rho}\partial_\rho\phi_{\alpha\nu} - \phi_{\lambda\mu}\phi_{\lambda\nu}\partial_\mu\phi_{\alpha\rho}\partial_\rho\phi_{\alpha\nu} \\ +\frac{1}{4}\phi_{\lambda\lambda}\phi_{\mu\nu}\partial_\mu\phi_{\alpha\rho}\partial_\rho\phi_{\alpha\nu} + \frac{1}{2}\phi_{\lambda\mu}\phi_{\lambda\nu}\partial_\rho\phi_{\alpha\mu}\partial_\rho\phi_{\alpha\nu} - \frac{1}{4}\phi_{\lambda\lambda}\phi_{\mu\nu}\partial_\rho\phi_{\alpha\mu}\partial_\rho\phi_{\alpha\nu} \\ -\frac{1}{4}\phi_{\lambda\rho}\phi_{\mu\nu}\partial_\lambda\phi_{\alpha\alpha}\partial_\rho\phi_{\mu\nu} - \frac{1}{2}\phi_{\lambda\mu}\phi_{\lambda\nu}\partial_\rho\phi_{\alpha\alpha}\partial_\rho\phi_{\mu\nu} + \frac{1}{4}\phi_{\lambda\lambda}\phi_{\mu\nu}\partial_\rho\phi_{\alpha\alpha}\partial_\rho\phi_{\mu\nu} \end{array} \right. \tag{14}$$

Converting to momentum space (with four momenta t, u, v, w), we obtain

$$
-\delta^4[t+u+v+w]\left(\begin{array}{l}
-\frac{1}{4}\phi_{\lambda\rho}[t]\phi_{\mu\nu}[u]\phi_{\lambda\rho}[v]\phi_{\mu\nu}[w](v\cdot w)+\frac{1}{4}\phi_{\lambda\rho}[t]\phi_{\mu\nu}[u]\phi_{\lambda\mu}[v]\phi_{\nu\rho}[w](v\cdot w)\\
+\frac{1}{16}\phi_{\mu\nu}[t]\phi_{\mu\nu}[u]\phi_{\lambda\rho}[v]\phi_{\alpha\rho}[w](v_\alpha w_\lambda)-\frac{1}{16}\phi_{\mu\mu}[t]\phi_{\nu\nu}[u]\phi_{\lambda\rho}[v]\phi_{\alpha\rho}[w](v_\alpha w_\lambda)\\
+\frac{1}{2}\phi_{\lambda\rho}[t]\phi_{\mu\nu}[u]\phi_{\mu\nu}[v]\phi_{\alpha\rho}[w](v_\alpha w_\lambda)-\frac{1}{32}\phi_{\mu\nu}[t]\phi_{\mu\nu}[u]\phi_{\alpha\rho}[v]\phi_{\alpha\rho}[w](v\cdot w)\\
+\frac{1}{32}\phi_{\mu\mu}[t]\phi_{\nu\nu}[u]\phi_{\alpha\rho}[v]\phi_{\alpha\rho}[w](v\cdot w)+\frac{1}{32}\phi_{\mu\nu}[t]\phi_{\mu\nu}[u]\phi_{\alpha\alpha}[v]\phi_{\rho\rho}[w](v\cdot w)\\
-\frac{1}{32}\phi_{\mu\mu}[t]\phi_{\nu\nu}[u]\phi_{\alpha\alpha}[v]\phi_{\rho\rho}[w](v\cdot w)+\frac{1}{2}\phi_{\lambda\rho}[t]\phi_{\mu\nu}[u]\phi_{\mu\rho}[v]\phi_{\alpha\alpha}[w](v_\lambda w_\nu)\\
-\frac{1}{2}\phi_{\lambda\rho}[t]\phi_{\mu\nu}[u]\phi_{\alpha\mu}[v]\phi_{\alpha\rho}[w](v_\lambda w_\nu)-\frac{1}{8}\phi_{\lambda\nu}[t]\phi_{\mu\mu}[u]\phi_{\alpha\rho}[v]\phi_{\alpha\rho}[w](v_\lambda w_\nu)\\
+\frac{1}{4}\phi_{\lambda\mu}[t]\phi_{\lambda\nu}[u]\phi_{\alpha\rho}[v]\phi_{\alpha\rho}[w](v_\mu w_\nu)+\frac{1}{8}\phi_{\lambda\nu}[t]\phi_{\mu\mu}[u]\phi_{\alpha\alpha}[v]\phi_{\rho\rho}[w](v_\lambda w_\nu)\\
-\frac{1}{4}\phi_{\lambda\mu}[t]\phi_{\lambda\nu}[u]\phi_{\alpha\alpha}[v]\phi_{\rho\rho}[w](v_\mu w_\nu)-\frac{1}{4}\phi_{\lambda\rho}[t]\phi_{\mu\nu}[u]\phi_{\mu\nu}[v]\phi_{\alpha\alpha}[w](v_\lambda w_\rho)\\
-\frac{1}{4}\phi_{\lambda\nu}[t]\phi_{\mu\mu}[u]\phi_{\nu\rho}[v]\phi_{\alpha\alpha}[w](v_\lambda w_\rho)+\frac{1}{2}\phi_{\lambda\mu}[t]\phi_{\lambda\nu}[u]\phi_{\nu\rho}[v]\phi_{\alpha\alpha}[w](v_\mu w_\rho)\\
-\phi_{\lambda\rho}[t]\phi_{\mu\nu}[u]\phi_{\lambda\mu}[v]\phi_{\alpha\nu}[w](v_\alpha w_\rho)-\frac{1}{2}\phi_{\lambda\mu}[t]\phi_{\lambda\nu}[u]\phi_{\mu\rho}[v]\phi_{\alpha\nu}[w](v_\alpha w_\rho)\\
+\frac{1}{4}\phi_{\lambda\lambda}[t]\phi_{\mu\nu}[u]\phi_{\mu\rho}[v]\phi_{\alpha\nu}[w](v_\alpha w_\rho)+\frac{1}{2}\phi_{\lambda\rho}[t]\phi_{\mu\nu}[u]\phi_{\alpha\mu}[v]\phi_{\alpha\nu}[w](v_\lambda w_\rho)\\
+\frac{1}{4}\phi_{\lambda\nu}[t]\phi_{\mu\mu}[u]\phi_{\alpha\rho}[v]\phi_{\alpha\nu}[w](v_\lambda w_\rho)-\phi_{\lambda\mu}[t]\phi_{\lambda\nu}[u]\phi_{\alpha\rho}[v]\phi_{\alpha\nu}[w](v_\mu w_\rho)\\
+\frac{1}{4}\phi_{\lambda\lambda}[t]\phi_{\mu\nu}[u]\phi_{\alpha\rho}[v]\phi_{\alpha\nu}[w](v_\mu w_\rho)+\frac{1}{2}\phi_{\lambda\mu}[t]\phi_{\lambda\nu}[u]\phi_{\alpha\mu}[v]\phi_{\alpha\nu}[w](v\cdot w)\\
-\frac{1}{4}\phi_{\lambda\lambda}[t]\phi_{\mu\nu}[u]\phi_{\alpha\mu}[v]\phi_{\alpha\nu}[w](v\cdot w)-\frac{1}{4}\phi_{\lambda\rho}[t]\phi_{\mu\nu}[u]\phi_{\alpha\alpha}[v]\phi_{\mu\nu}[w](v_\lambda w_\rho)\\
-\frac{1}{2}\phi_{\lambda\mu}[t]\phi_{\lambda\nu}[u]\phi_{\alpha\alpha}[v]\phi_{\mu\nu}[w](v\cdot w)+\frac{1}{4}\phi_{\lambda\lambda}[t]\phi_{\mu\nu}[u]\phi_{\alpha\alpha}[v]\phi_{\mu\nu}[w](v\cdot w)
\end{array}\right) \tag{15}
$$

In order to obtain the graviton 4-leg vertex, we must differentiate the above four times with respect to $\phi_{\mu\nu}[p]$, with appropriate indices and momenta. After renaming the indices, we can reduce the size of the resulting expression, 16-fold, by unsymmetrizing with respect to (μ,ν), (λ,ρ), (α,β), and (σ,τ). The resulting expression can be used as it is in our loop computations since it will be multiplied by tensor propagators having the required symmetries. Also, dropping all terms that involve $(p_\mu, p_\nu, q_\lambda, q_\rho, r_\alpha, r_\beta, s_\sigma, s_\tau)$ since they correspond to the condition $\partial_\mu\phi_{\mu\nu}=0$, then using the implications of the momentum-conserving delta function $\delta^4(p+q+r+s)$, while suppressing the latter, we obtain the following expression for the 4-leg vertex $\mathcal{V}_{\mu\nu,\lambda\rho,\alpha\beta,\sigma\tau}[p,q,r,s]$, (we give it here in 8 sections each containing 13 terms)

$$
\begin{array}{l}
-4\,(2\,q_\sigma\,q_\tau+2\,q_\tau\,r_\sigma-q_\sigma\,r_\tau+r_\sigma\,r_\tau)\,\eta_{\alpha,\mu}\,\eta_{\beta,\nu}\,\eta_{\lambda,\rho}-4\,(3\,q_\sigma\,r_\mu+r_\mu\,r_\sigma+2\,q_\sigma\,s_\mu+2\,r_\sigma\,s_\mu)\,\eta_{\alpha,\tau}\,\eta_{\beta,\nu}\,\eta_{\lambda,\rho}\\
+4\,(q_\sigma\,r_\mu+3\,r_\mu\,r_\sigma+2\,r_\sigma\,s_\mu)\,\eta_{\alpha,\nu}\,\eta_{\beta,\tau}\,\eta_{\lambda,\rho}-4\,(2\,r_\mu\,r_\nu+r_\nu\,s_\mu+2\,r_\mu\,s_\nu+2\,s_\mu\,s_\nu)\,\eta_{\alpha,\sigma}\,\eta_{\beta,\tau}\,\eta_{\lambda,\rho}\\
-4\,(2\,q_\sigma\,q_\tau+2\,q_\tau\,r_\sigma+q_\sigma\,r_\tau+2\,r_\sigma\,r_\tau)\,\eta_{\alpha,\lambda}\,\eta_{\beta,\rho}\,\eta_{\mu,\nu}+4\,(2\,q_\sigma\,r_\lambda+2\,r_\lambda\,r_\sigma+q_\sigma\,s_\lambda)\,\eta_{\alpha,\tau}\,\eta_{\beta,\rho}\,\eta_{\mu,\nu}+4\,(2\,r_\lambda\,r_\sigma\\
+q_\sigma\,s_\lambda+2\,r_\sigma\,s_\lambda)\,\eta_{\alpha,\rho}\,\eta_{\beta,\tau}\,\eta_{\mu,\nu}-\\
\quad 2\,(4\,r_\lambda\,r_\rho+3\,r_\rho\,s_\lambda+3\,r_\lambda\,s_\rho+4\,s_\lambda\,s_\rho)\,\eta_{\alpha,\sigma}\,\eta_{\beta,\tau}\,\eta_{\mu,\nu}+4\,(q_\sigma\,q_\tau+q_\tau\,r_\sigma+r_\sigma\,r_\tau)\,\eta_{\alpha,\beta}\,\eta_{\lambda,\rho}\,\eta_{\mu,\nu}-\\
\quad 4\,(2\,q_\sigma+r_\sigma)\,(2\,q_\alpha+s_\alpha)\,\eta_{\beta,\tau}\,\eta_{\lambda,\rho}\,\eta_{\mu,\nu}+2\,(q\cdot q+q\cdot r+q\cdot s+2\,r\cdot r+3\,r\cdot s+2\,s\cdot s)\,\eta_{\alpha,\sigma}\,\eta_{\beta,\tau}\,\eta_{\lambda,\rho}\,\eta_{\mu,\nu}+\\
\quad 8\,q_\tau\,(q_\sigma+r_\sigma)\,\eta_{\alpha,\mu}\,\eta_{\beta,\lambda}\,\eta_{\nu,\rho}+8\,(q_\sigma+3\,r_\sigma)\,s_\mu\,\eta_{\alpha,\tau}\,\eta_{\beta,\lambda}\,\eta_{\nu,\rho}
\end{array} \tag{16}
$$

$$8\,(q_\sigma\, q_\tau + q_\tau\, r_\sigma + 2\, r_\sigma\, r_\tau)\, \eta_{\alpha,\lambda}\, \eta_{\beta,\mu}\, \eta_{\nu,\rho} - 8\,(q_\sigma\, r_\lambda + r_\lambda\, r_\sigma + 2\, q_\sigma\, s_\lambda)\, \eta_{\alpha,\tau}\, \eta_{\beta,\mu}\, \eta_{\nu,\rho}$$
$$+ 8\,(2\, q_\sigma\, r_\mu + q_\sigma\, s_\mu - r_\sigma\, s_\mu)\, \eta_{\alpha,\lambda}\, \eta_{\beta,\tau}\, \eta_{\nu,\rho} - 8\, r_\lambda\,(q_\sigma + r_\sigma)\, \eta_{\alpha,\mu}\, \eta_{\beta,\tau}\, \eta_{\nu,\rho} + 4\,(3\, r_\lambda\, r_\mu$$
$$+ r_\mu\, s_\lambda + r_\lambda\, s_\mu + 3\, s_\lambda\, s_\mu)\, \eta_{\alpha,\sigma}\, \eta_{\beta,\tau}\, \eta_{\nu,\rho} - 4\,(q_\sigma\, q_\tau + q_\tau\, r_\sigma + 2\, r_\sigma\, r_\tau)\, \eta_{\alpha,\beta}\, \eta_{\lambda,\mu}\, \eta_{\nu,\rho}$$
$$+ 4\,(4\, q_\alpha\, q_\sigma + 2\, q_\alpha\, r_\sigma + 2\, q_\sigma\, s_\alpha + 3\, r_\sigma\, s_\alpha)\, \eta_{\beta,\tau}\, \eta_{\lambda,\mu}\, \eta_{\nu,\rho} - 2\,(q\cdot q + q\cdot r + q\cdot s + 4\, r\cdot r + 7\, r\cdot s + 4\, s\cdot s)\, \eta_{\alpha,\sigma}\, \eta_{\beta,\tau}\, \eta_{\lambda,\mu}\, \eta_{\nu,\rho}$$
$$+ 4\,(q_\sigma\, r_\mu + 3\, r_\mu\, r_\sigma - q_\sigma\, s_\mu + r_\sigma\, s_\mu)\, \eta_{\alpha,\beta}\, \eta_{\lambda,\tau}\, \eta_{\nu,\rho} - 8\,(q_\alpha\, q_\sigma + q_\alpha\, r_\sigma + 2\, r_\sigma\, s_\alpha)\, \eta_{\beta,\mu}\, \eta_{\lambda,\tau}\, \eta_{\nu,\rho}$$
$$+ 8\,(q\cdot q + q\cdot r + r\cdot r + s\cdot s)\, \eta_{\alpha,\sigma}\, \eta_{\beta,\mu}\, \eta_{\lambda,\tau}\, \eta_{\nu,\rho} - 16\, s_\alpha\, s_\mu\, \eta_{\beta,\sigma}\, \eta_{\lambda,\tau}\, \eta_{\nu,\rho}$$
$$+ 4\,(q\cdot r + q\cdot s + 2\, r\cdot r + 3\, r\cdot s + s\cdot s)\, \eta_{\alpha,\mu}\, \eta_{\beta,\sigma}\, \eta_{\lambda,\tau}\, \eta_{\nu,\rho} \tag{17}$$

$$8\,(r_\lambda\, r_\sigma + q_\sigma\, s_\lambda + r_\sigma\, s_\lambda)\, \eta_{\alpha,\beta}\, \eta_{\mu,\tau}\, \eta_{\nu,\rho} - 8\,(2\, q_\sigma + 3\, r_\sigma)\, s_\alpha\, \eta_{\beta,\lambda}\, \eta_{\mu,\tau}\, \eta_{\nu,\rho}$$
$$- 8\,(q\cdot r - q\cdot s - s\cdot s)\, \eta_{\alpha,\sigma}\, \eta_{\beta,\lambda}\, \eta_{\mu,\tau}\, \eta_{\nu,\rho} + 8\,(q_\alpha\, r_\lambda - 2\, s_\alpha\, s_\lambda)\, \eta_{\beta,\sigma}\, \eta_{\mu,\tau}\, \eta_{\nu,\rho}$$
$$+ 4\,(2\, q\cdot r + q\cdot s + 2\, r\cdot r + 4\, r\cdot s + 2\, s\cdot s)\, \eta_{\alpha,\lambda}\, \eta_{\beta,\sigma}\, \eta_{\mu,\tau}\, \eta_{\nu,\rho} - 8\,(q_\beta\, s_\alpha - 2\, q_\alpha\, s_\beta - 2\, s_\alpha\, s_\beta)\, \eta_{\lambda,\sigma}\, \eta_{\mu,\tau}\, \eta_{\nu,\rho}$$
$$- 4\,(q\cdot s + 2\, r\cdot r + 2\, r\cdot s + 2\, s\cdot s)\, \eta_{\alpha,\beta}\, \eta_{\lambda,\sigma}\, \eta_{\mu,\tau}\, \eta_{\nu,\rho} + 8\,(2\, q_\sigma\, r_\lambda - r_\lambda\, r_\sigma + q_\sigma\, s_\lambda + r_\sigma\, s_\lambda)\, \eta_{\alpha,\rho}\, \eta_{\beta,\mu}\, \eta_{\nu,\tau}$$
$$+ 8\,(r_\lambda\, r_\rho + r_\lambda\, s_\rho + 2\, s_\lambda\, s_\rho)\, \eta_{\alpha,\sigma}\, \eta_{\beta,\mu}\, \eta_{\nu,\tau} - 4\,(2\, q_\sigma\, r_\mu - 2\, r_\mu\, r_\sigma + 3\, q_\sigma\, s_\mu + r_\sigma\, s_\mu)\, \eta_{\alpha,\lambda}\, \eta_{\beta,\rho}\, \eta_{\nu,\tau}$$
$$- 8\,(2\, q_\sigma\, r_\lambda + r_\lambda\, r_\sigma + 3\, q_\sigma\, s_\lambda + 3\, r_\sigma\, s_\lambda)\, \eta_{\alpha,\mu}\, \eta_{\beta,\rho}\, \eta_{\nu,\tau} + 8\,(r_\lambda\, r_\mu + r_\mu\, s_\lambda - r_\lambda\, s_\mu - 3\, s_\lambda\, s_\mu)\, \eta_{\alpha,\sigma}\, \eta_{\beta,\rho}\, \eta_{\nu,\tau}$$
$$+ 8\, r_\rho\,(r_\lambda + s_\lambda)\, \eta_{\alpha,\mu}\, \eta_{\beta,\sigma}\, \eta_{\nu,\tau} \tag{18}$$

$$-8\,(3\, r_\lambda\, r_\mu + r_\mu\, s_\lambda + r_\lambda\, s_\mu - s_\lambda\, s_\mu)\, \eta_{\alpha,\rho}\, \eta_{\beta,\sigma}\, \eta_{\nu,\tau} + 4\,(q_\sigma - r_\sigma)\,(2\, r_\mu + s_\mu)\, \eta_{\alpha,\beta}\, \eta_{\lambda,\rho}\, \eta_{\nu,\tau}$$
$$+ 8\,(2\, q_\alpha\, q_\sigma + q_\alpha\, r_\sigma + q_\sigma\, s_\alpha + r_\sigma\, s_\alpha)\, \eta_{\beta,\mu}\, \eta_{\lambda,\rho}\, \eta_{\nu,\tau} - 4\,(4\, q\cdot q + 3\, q\cdot r + 2\, q\cdot s + r\cdot r + r\cdot s + 2\, s\cdot s)\, \eta_{\alpha,\sigma}\, \eta_{\beta,\mu}\, \eta_{\lambda,\rho}\, \eta_{\nu,\tau}$$
$$- 8\,(q_\alpha\, r_\mu + q_\alpha\, s_\mu - s_\alpha\, s_\mu)\, \eta_{\beta,\sigma}\, \eta_{\lambda,\rho}\, \eta_{\nu,\tau} + 4\,(q\cdot r - r\cdot r - r\cdot s)\, \eta_{\alpha,\mu}\, \eta_{\beta,\sigma}\, \eta_{\lambda,\rho}\, \eta_{\nu,\tau} + 8\, r_\lambda\,(q_\sigma + r_\sigma)\, \eta_{\alpha,\beta}\, \eta_{\mu,\rho}\, \eta_{\nu,\tau}$$
$$- 8\,(2\, q_\alpha\, q_\sigma - r_\sigma\, s_\alpha)\, \eta_{\beta,\lambda}\, \eta_{\mu,\rho}\, \eta_{\nu,\tau} + 4\,(2\, q\cdot q + 4\, q\cdot r + q\cdot s + 2\, r\cdot r)\, \eta_{\alpha,\sigma}\, \eta_{\beta,\lambda}\, \eta_{\mu,\rho}\, \eta_{\nu,\tau}$$
$$- 8\, q_\alpha\,(3\, r_\lambda + 2\, s_\lambda)\, \eta_{\beta,\sigma}\, \eta_{\mu,\rho}\, \eta_{\nu,\tau} + 8\, q\cdot q\, \eta_{\alpha,\lambda}\, \eta_{\beta,\sigma}\, \eta_{\mu,\rho}\, \eta_{\nu,\tau} + 8\,(q_\alpha\, q_\beta + q_\beta\, s_\alpha - q_\alpha\, s_\beta)\, \eta_{\lambda,\sigma}\, \eta_{\mu,\rho}\, \eta_{\nu,\tau}$$
$$- 4\,(q\cdot q + 3\, q\cdot r + 2\, r\cdot r)\, \eta_{\alpha,\beta}\, \eta_{\lambda,\sigma}\, \eta_{\mu,\rho}\, \eta_{\nu,\tau} \tag{19}$$

$$-4\,(2\, r_\lambda\, r_\rho + r_\rho\, s_\lambda + s_\lambda\, s_\rho)\, \eta_{\alpha,\beta}\, \eta_{\mu,\sigma}\, \eta_{\nu,\tau} + 4\,(3\, q_\alpha\, r_\lambda + 2\, r_\lambda\, s_\alpha + 2\, q_\alpha\, s_\lambda + 4\, s_\alpha\, s_\lambda)\, \eta_{\beta,\rho}\, \eta_{\mu,\sigma}\, \eta_{\nu,\tau}$$
$$- 2\,(4\, q\cdot q + 7\, q\cdot r + q\cdot s + 4\, r\cdot r + r\cdot s + s\cdot s)\, \eta_{\alpha,\lambda}\, \eta_{\beta,\rho}\, \eta_{\mu,\sigma}\, \eta_{\nu,\tau} - 4\,(2\, q_\alpha\, q_\beta + q_\beta\, s_\alpha + s_\alpha\, s_\beta)\, \eta_{\lambda,\rho}\, \eta_{\mu,\sigma}\, \eta_{\nu,\tau}$$
$$+ 2\,(2\, q\cdot q + 3\, q\cdot r + q\cdot s + 2\, r\cdot r + r\cdot s + s\cdot s)\, \eta_{\alpha,\beta}\, \eta_{\lambda,\rho}\, \eta_{\mu,\sigma}\, \eta_{\nu,\tau} + 8\,(3\, q_\sigma + r_\sigma)\, s_\mu\, \eta_{\alpha,\nu}\, \eta_{\beta,\lambda}\, \eta_{\rho,\tau}$$
$$- 16\,(r_\nu\, s_\mu - r_\mu\, s_\nu - s_\mu\, s_\nu)\, \eta_{\alpha,\sigma}\, \eta_{\beta,\lambda}\, \eta_{\rho,\tau} - 8\,(2\, r_\mu\, r_\sigma + q_\sigma\, s_\mu + r_\sigma\, s_\mu)\, \eta_{\alpha,\lambda}\, \eta_{\beta,\nu}\, \eta_{\rho,\tau}$$
$$+ 4\,(2\, q_\sigma\, r_\lambda + 4\, r_\lambda\, r_\sigma + 3\, q_\sigma\, s_\lambda + 2\, r_\sigma\, s_\lambda)\, \eta_{\alpha,\mu}\, \eta_{\beta,\nu}\, \eta_{\rho,\tau} - 16\, s_\lambda\, s_\mu\, \eta_{\alpha,\sigma}\, \eta_{\beta,\nu}\, \eta_{\rho,\tau} + 16\, r_\nu\,(r_\mu + s_\mu)\, \eta_{\alpha,\lambda}\, \eta_{\beta,\sigma}\, \eta_{\rho,\tau}$$
$$- 16\, r_\mu\,(r_\lambda + s_\lambda)\, \eta_{\alpha,\nu}\, \eta_{\beta,\sigma}\, \eta_{\rho,\tau} - 4\,(3\, q_\sigma\, r_\mu + r_\mu\, r_\sigma + q_\sigma\, s_\mu + r_\sigma\, s_\mu)\, \eta_{\alpha,\beta}\, \eta_{\lambda,\nu}\, \eta_{\rho,\tau} \tag{20}$$

$$-8\, q_\alpha\,(q_\sigma + r_\sigma)\, \eta_{\beta,\mu}\, \eta_{\lambda,\nu}\, \eta_{\rho,\tau} + 4\,(2\, q\cdot q + q\cdot r + 3\, q\cdot s + r\cdot s + s\cdot s)\, \eta_{\alpha,\sigma}\, \eta_{\beta,\mu}\, \eta_{\lambda,\nu}\, \eta_{\rho,\tau}$$
$$+ 16\,(q_\alpha + s_\alpha)\,(r_\mu + s_\mu)\, \eta_{\beta,\sigma}\, \eta_{\lambda,\nu}\, \eta_{\rho,\tau} - 4\,(2\, r_\mu\, r_\nu + r_\mu\, s_\nu + s_\mu\, s_\nu)\, \eta_{\alpha,\beta}\, \eta_{\lambda,\sigma}\, \eta_{\rho,\tau}$$
$$- 4\,(3\, q_\alpha\, r_\mu + r_\mu\, s_\alpha + 2\, q_\alpha\, s_\mu - 2\, s_\alpha\, s_\mu)\, \eta_{\beta,\nu}\, \eta_{\lambda,\sigma}\, \eta_{\rho,\tau} - 2\,(4\, q\cdot q + q\cdot r + 7\, q\cdot s + r\cdot r + r\cdot s + 4\, s\cdot s)\, \eta_{\alpha,\mu}\, \eta_{\beta,\nu}\, \eta_{\lambda,\sigma}\, \eta_{\rho,\tau}$$
$$- 4\,(q_\sigma + 2\, r_\sigma)\,(2\, r_\lambda + s_\lambda)\, \eta_{\alpha,\beta}\, \eta_{\mu,\nu}\, \eta_{\rho,\tau} + 8\,(2\, q_\alpha\, q_\sigma + q_\alpha\, r_\sigma + q_\sigma\, s_\alpha + r_\sigma\, s_\alpha)\, \eta_{\beta,\lambda}\, \eta_{\mu,\nu}\, \eta_{\rho,\tau}$$
$$- 8\,(q\cdot q + 2\, q\cdot r + 2\, q\cdot s + r\cdot r + 2\, r\cdot s + 2\, s\cdot s)\, \eta_{\alpha,\sigma}\, \eta_{\beta,\lambda}\, \eta_{\mu,\nu}\, \eta_{\rho,\tau} + 8\,(q_\alpha\, r_\lambda + r_\lambda\, s_\alpha + q_\alpha\, s_\lambda + 2\, s_\alpha\, s_\lambda)\, \eta_{\beta,\sigma}\, \eta_{\mu,\nu}\, \eta_{\rho,\tau}$$
$$- 8\,(q\cdot q + q\cdot r + q\cdot s + r\cdot r + r\cdot s)\, \eta_{\alpha,\lambda}\, \eta_{\beta,\sigma}\, \eta_{\mu,\nu}\, \eta_{\rho,\tau} - 2\,(4\, q_\alpha\, q_\beta + 3\, q_\beta\, s_\alpha + 3\, q_\alpha\, s_\beta + 4\, s_\alpha\, s_\beta)\, \eta_{\lambda,\sigma}\, \eta_{\mu,\nu}\, \eta_{\rho,\tau}$$
$$+ 2\,(2\, q\cdot q + q\cdot r + 3\, q\cdot s + r\cdot r + r\cdot s + 2\, s\cdot s)\, \eta_{\alpha,\beta}\, \eta_{\lambda,\sigma}\, \eta_{\mu,\nu}\, \eta_{\rho,\tau} \tag{21}$$

$$8\,(r_\lambda\, r_\mu + s_\lambda\, s_\mu)\, \eta_{\alpha,\beta}\, \eta_{\nu,\sigma}\, \eta_{\rho,\tau} + 16\,(q_\alpha\, r_\mu - s_\alpha\, s_\mu)\, \eta_{\beta,\lambda}\, \eta_{\nu,\sigma}\, \eta_{\rho,\tau}$$
$$+ 8\,(q\cdot q + 2\, q\cdot r + r\cdot r + r\cdot s + s\cdot s)\, \eta_{\alpha,\mu}\, \eta_{\beta,\lambda}\, \eta_{\nu,\sigma}\, \eta_{\rho,\tau} - 16\,(q_\alpha\, r_\lambda + r_\lambda\, s_\alpha + s_\alpha\, s_\lambda)\, \eta_{\beta,\mu}\, \eta_{\nu,\sigma}\, \eta_{\rho,\tau}$$
$$+ 8\,(q\cdot q + 2\, q\cdot s + r\cdot r + r\cdot s + s\cdot s)\, \eta_{\alpha,\lambda}\, \eta_{\beta,\mu}\, \eta_{\nu,\sigma}\, \eta_{\rho,\tau} + 8\, q_\beta\,(q_\alpha + s_\alpha)\, \eta_{\lambda,\mu}\, \eta_{\nu,\sigma}\, \eta_{\rho,\tau}$$
$$- 4\,(q\cdot q - q\cdot r + q\cdot s)\, \eta_{\alpha,\beta}\, \eta_{\lambda,\mu}\, \eta_{\nu,\sigma}\, \eta_{\rho,\tau} - 4\,(r_\lambda\, r_\rho + r_\lambda\, s_\rho + 2\, s_\lambda\, s_\rho)\, \eta_{\alpha,\mu}\, \eta_{\beta,\nu}\, \eta_{\sigma,\tau}$$
$$+ 4\,(2\, r_\lambda\, r_\mu + 3\, r_\lambda\, s_\mu + 3\, s_\lambda\, s_\mu)\, \eta_{\alpha,\rho}\, \eta_{\beta,\nu}\, \eta_{\sigma,\tau} - 4\,(r_\mu\, r_\nu - r_\nu\, s_\mu + 2\, r_\mu\, s_\nu + 2\, s_\mu\, s_\nu)\, \eta_{\alpha,\lambda}\, \eta_{\beta,\rho}\, \eta_{\sigma,\tau}$$
$$- 4\,(3\, r_\lambda + s_\lambda)\, s_\mu\, \eta_{\alpha,\nu}\, \eta_{\beta,\rho}\, \eta_{\sigma,\tau} + 4\,(r_\mu\, r_\nu + r_\mu\, s_\nu + s_\mu\, s_\nu)\, \eta_{\alpha,\beta}\, \eta_{\lambda,\rho}\, \eta_{\sigma,\tau}$$
$$+ 4\,(q_\alpha - s_\alpha)\,(r_\mu + 2\, s_\mu)\, \eta_{\beta,\nu}\, \eta_{\lambda,\rho}\, \eta_{\sigma,\tau} \tag{22}$$

$$
\begin{aligned}
&2\,(2\,q\cdot q+q\cdot r+3\,q\cdot s+r\cdot r+r\cdot s+2\,s\cdot s)\,\eta_{\alpha,\mu}\,\eta_{\beta,\nu}\,\eta_{\lambda,\rho}\,\eta_{\sigma,\tau}+2\,(2\,r_\lambda\,r_\rho+r_\rho\,s_\lambda+r_\lambda\,s_\rho+2\,s_\lambda\,s_\rho)\,\eta_{\alpha,\beta}\,\eta_{\mu,\nu}\,\eta_{\sigma,\tau}\\
&-4\,(q_\alpha+2\,s_\alpha)\,(r_\lambda+2\,s_\lambda)\,\eta_{\beta,\rho}\,\eta_{\mu,\nu}\,\eta_{\sigma,\tau}+2\,(2\,q\cdot q+3\,q\cdot r+q\cdot s+2\,r\cdot r+r\cdot s+s\cdot s)\,\eta_{\alpha,\lambda}\,\eta_{\beta,\rho}\,\eta_{\mu,\nu}\,\eta_{\sigma,\tau}\\
&+2\,(2\,q_\alpha\,q_\beta+q_\beta\,s_\alpha+q_\alpha\,s_\beta+2\,s_\alpha\,s_\beta)\,\eta_{\lambda,\rho}\,\eta_{\mu,\nu}\,\eta_{\sigma,\tau}-2\,(q\cdot q+q\cdot r+q\cdot s+r\cdot r+r\cdot s+s\cdot s)\,\eta_{\alpha,\beta}\,\eta_{\lambda,\rho}\,\eta_{\mu,\nu}\,\eta_{\sigma,\tau}\\
&-4\,(r_\lambda-s_\lambda)\,(r_\mu-s_\mu)\,\eta_{\alpha,\beta}\,\eta_{\nu,\rho}\,\eta_{\sigma,\tau}-8\,(q_\alpha\,r_\mu+q_\alpha\,s_\mu-s_\alpha\,s_\mu)\,\eta_{\beta,\lambda}\,\eta_{\nu,\rho}\,\eta_{\sigma,\tau}\\
&-4\,(q\cdot q+q\cdot r-q\cdot s)\,\eta_{\alpha,\mu}\,\eta_{\beta,\lambda}\,\eta_{\nu,\rho}\,\eta_{\sigma,\tau}+8\,(q_\alpha\,r_\lambda+r_\lambda\,s_\alpha+q_\alpha\,s_\lambda+2\,s_\alpha\,s_\lambda)\,\eta_{\beta,\mu}\,\eta_{\nu,\rho}\,\eta_{\sigma,\tau}\\
&-4\,(q\cdot q+q\cdot r+3\,q\cdot s+2\,r\cdot r+2\,r\cdot s+4\,s\cdot s)\,\eta_{\alpha,\lambda}\,\eta_{\beta,\mu}\,\eta_{\nu,\rho}\,\eta_{\sigma,\tau}-4\,(q_\alpha\,q_\beta+q_\alpha\,s_\beta+2\,s_\alpha\,s_\beta)\,\eta_{\lambda,\mu}\,\eta_{\nu,\rho}\,\eta_{\sigma,\tau}\\
&+2\,(q\cdot q+q\cdot r+q\cdot s+2\,r\cdot r+3\,r\cdot s+2\,s\cdot s)\,\eta_{\alpha,\beta}\,\eta_{\lambda,\mu}\,\eta_{\nu,\rho}\,\eta_{\sigma,\tau}
\end{aligned}
\tag{23}
$$

The above 104 terms give the (unsymmetrized) expression of the vacuum vertex with 4 legs corresponding to virtual gravitons having respective indices $(\mu\nu, \lambda\rho, \alpha\beta, \sigma\tau)$, and respective momenta (p, q, r, s), with p replaced by $-(q + r + s)$ due to momentum conservation.

2.2 Coupling to a Scalar Field

We shall consider the coupling of the metric field to a virtual scalar field A. This is given by the Lagrangian density

$$\sqrt{q}\frac{1}{2}g^{\mu\nu}\partial_\mu A\partial_\nu A \tag{24}$$

We do not include a mass term with the presumption that all masses are much smaller than the cutoff mass, and that we shall consider the most important cutoff-dependent vacuum contributions. Expanding the metric about flat spacetime, $g_{\mu\nu} = \eta_{\mu\nu} + \phi_{\mu\nu}$, where as before the $\phi_{\mu\nu}$ represents the virtual graviton field, we only keep the terms that are quartic or less in the virtual fields.

2.2.1 Bilinear Terms and the Scalar Propagator

From the scalar bilinears

$$\frac{1}{2}(\partial_\mu A)^2 \tag{25}$$

we obtain in momentum space

$$\frac{1}{2}A[p]A[-p]\mathcal{K}[p] \tag{26}$$

With the kernel $\mathcal{K}[p] = p^2$. The inverse of the latter gives the scalar propagator

$$\mathcal{P}[p] = \frac{1}{p^2} \tag{27}$$

2.2.2 Cubic Terms and the Graviton-Scalar 3-Leg Vertex

From the foregoing expansion of the Lagrangian density, we obtain the cubic terms

$$-\frac{1}{2}\phi_{\mu\nu}\partial_\mu A\partial_\nu A + \frac{1}{4}\phi_{\mu\mu}(\partial_\nu A)^2 \tag{28}$$

Converting to momentum space (with three momenta s, t, u), we obtain

$$-\delta^4[s+t+u]\left(-\frac{1}{2}\phi_{\mu\nu}[s]A[t]A[u](t_\mu u_\nu)+\frac{1}{4}\phi_{\mu\mu}[s]A[t]A[u](t\cdot u)\right) \tag{29}$$

In order to obtain the 3-leg vertex, we must differentiate with respect to $\phi_{\lambda\rho}[p]$, then with respect to $A[q]$ and $A[r]$. Simplifying and renaming indices, the result defines the 3-leg vertex

$$\mathcal{V}_{\mu\nu}[p,q,r]=-2r_\mu r_\nu-(q\cdot r)\eta_{\mu\nu} \tag{30}$$

In the above we have suppressed but used the implications of the momentum-conserving delta function $\delta^4[p+q+r]$. We also dropped terms that involve p_μ and p_ν since they correspond to the condition $\partial_\mu\phi_{\mu\nu}=0$.

2.2.3 Quartic Terms and the Graviton-Scalar 4-Leg Vertex

Again, from the foregoing expansion of the Lagrangian density, we obtain the quartic terms

$$\begin{cases} -\frac{1}{16}\phi_{\mu\nu}^2(\partial_\lambda A)^2+\frac{1}{16}\phi_{\mu\mu}\phi_{\nu\nu}(\partial_\lambda A)^2 \\ -\frac{1}{4}\phi_{\lambda\nu}\phi_{\mu\mu}\partial_\lambda A\partial_\nu A+\frac{1}{2}\phi_{\lambda\mu}\phi_{\mu\nu}\partial_\lambda A\partial_\nu A \end{cases} \tag{31}$$

Converting to momentum space (with four momenta t, u, v, w), we obtain

$$-\delta^4[t+u+v+w]\begin{pmatrix} -\frac{1}{16}\phi_{\mu\nu}[t]\phi_{\mu\nu}[u]A[v]A[w](v\cdot w)+\frac{1}{16}\phi_{\mu\mu}[t]\phi_{\nu\nu}[u]A[v]A[w](v\cdot w) \\ -\frac{1}{4}\phi_{\lambda\nu}[t]\phi_{\mu\mu}[u]A[v]A[w](v_\lambda w_\nu)+\frac{1}{2}\phi_{\lambda\mu}[t]\phi_{\mu\nu}[u]A[v]A[w](v_\lambda w_\nu) \end{pmatrix} \tag{32}$$

In order to obtain the 4-leg vertex, we must differentiate with respect to $\phi_{\alpha\beta}[p]$ and $\phi_{\sigma\tau}[q]$, then with respect to $A[r]$ and $A[s]$. Simplifying and renaming indices, the result defines the 4-leg vertex

$$\mathcal{V}_{\mu\nu,\lambda\rho}[p,q,r,s]=\begin{pmatrix} 2\left(r_\mu s_\nu\eta_{\lambda\rho}+r_\lambda s_\rho\eta_{\mu\nu}-2r_\mu s_\lambda\eta_{\nu\rho}-2r_\lambda s_\mu\eta_{\nu\rho}\right) \\ +(r\cdot s)\left(\eta_{\mu\lambda}\eta_{\nu\rho}-\eta_{\mu\nu}\eta_{\lambda\rho}\right) \end{pmatrix} \tag{33}$$

In the above we have suppressed but used the implications of the momentum-conserving delta function $\delta^4[p+q+r+s]$. We also dropped terms that involve p_μ and p_ν, as well as those involving q_λ and q_ρ, since all correspond to the condition $\partial_\mu\phi_{\mu\nu}=0$. We also reduced the size of the expression by unsymmetrizing with respect to (μ,ν) and with respect to (λ,ρ) (since eventually it will be multiplied by tensor propagators having these symmetries).

2.3 Coupling to a Vector Field

We consider the coupling of gravity to a virtual vector field A_μ, with Lagrangian density

$$-\sqrt{g}\frac{1}{4}g^{\mu\lambda}g^{\nu\rho}F_{\mu\nu}F_{\lambda\rho} \qquad F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu \tag{34}$$

We shall expand the metric about flat spacetime, $g_{\mu\nu} = \eta_{\mu\nu} + \phi_{\mu\nu}$, to second order in the virtual graviton field $\phi_{\mu\nu}$, and obtain the rules for the vector propagator, the 3-leg graviton-vector vertex, and the 4-leg graviton-vector vertex.

2.3.1 Bilinear Terms and the Vector Propagator

The bilinear terms of the vector field are

$$-\frac{1}{2}(\partial_\mu A_\nu)^2 + \frac{1}{2}\partial_\mu A_\nu \partial_\nu A_\mu \tag{35}$$

Our virtual vector field will be subjected to the condition $\partial_\mu A_\mu = 0$, hence using integration by parts, we can drop the last term of the above. Then converting to momentum space, we obtain

$$\frac{1}{2}A_\mu[p]A_\nu[-p]\mathcal{K}_{\mu\nu}[p] \tag{36}$$

with the kernel $\mathcal{K}_{\mu,\nu}[p] = -p^2\eta_{\mu\nu}$. The inverse of the latter gives the vector propagator:

$$\mathcal{P}_{\mu,\nu}[p] = -\frac{1}{p^2}\eta_{\mu\nu} \tag{37}$$

2.3.2 Cubic Terms and the Graviton-Vector 3-Leg Vertex

From the foregoing expansion of the Lagrangian density, we obtain the cubic terms

$$\left\{\begin{array}{l} \frac{1}{2}\phi_{\mu\nu}\partial_\lambda A_\mu \partial_\lambda A_\nu - \phi_{\mu\nu}\partial_\lambda A_\mu \partial_\nu A_\lambda + \frac{1}{4}\phi_{\mu\mu}\partial_\lambda A_\nu \partial_\nu A_\lambda \\ +\frac{1}{2}\phi_{\mu\nu}\partial_\mu A_\lambda \partial_\nu A_\lambda - \frac{1}{4}\phi_{\mu\mu}(\partial_\nu A_\lambda)^2 \end{array}\right. \tag{38}$$

Converting to momentum space (with three momenta s, t, u) we obtain

$$-\delta^4[s+t+u]\left(\begin{array}{l} \frac{1}{2}\phi_{\mu\nu}[s]A_\mu[t]A_\nu[u](t\cdot u) - \phi_{\mu\nu}[s]A_\mu[t]A_\lambda[u](t_\lambda u_\nu) \\ +\frac{1}{4}\phi_{\mu\mu}[s]A_\nu[t]A_\lambda[u](t_\lambda u_\nu) + \frac{1}{2}\phi_{\mu\nu}[s]A_\lambda[t]A_\lambda[u](t_\mu u_\nu) \\ -\frac{1}{4}\phi_{\mu\mu}[s]A_\lambda[t]A_\lambda[u](t\cdot u) \end{array}\right) \tag{39}$$

In order to obtain the 3-leg vertex we differentiate the above expression, first with respect to $\phi_{\alpha\beta}[p]$, then with respect to $A_\sigma[q]$ and $A_\tau[r]$. Simplifying and renaming indices, we obtain the 3-leg vertex

$$\mathcal{V}_{\mu\nu,\lambda,\rho}[p,q,r] = \left(\begin{array}{l} 2q_\rho r_\mu \eta_{\lambda\nu} + 2r_\mu r_\nu \eta_{\lambda\rho} - q_\rho r_\lambda \eta_{\mu\nu} - 2r_\lambda r_\mu \eta_{\nu\rho} \\ +(q\cdot r)\left(\eta_{\mu\nu}\eta_{\lambda\rho} - 2\eta_{\lambda\mu}\eta_{\nu\rho}\right) \end{array}\right) \tag{40}$$

In the above we have suppressed but used the implications of the momentum-conserving delta function $\delta^4[p+q+r]$. We also dropped terms that involve p_μ and p_ν since they correspond to the condition $\partial_\mu\phi_{\mu\nu} = 0$. Likewise, we dropped terms involving q_λ and r_ρ since they correspond to the condition $\partial_\mu A_\mu = 0$. We also unsymmetrized with respect to (μ, ν).

2.3.3 Quartic Terms and the Graviton-Vector 4-Leg Vertex

Again, from the foregoing expansion of the Lagrangian density, we obtain the following quartic terms:

$$\left\{\begin{aligned}
&\tfrac{1}{16}\phi_{\mu\nu}^2(\partial_\lambda A_\rho)^2-\tfrac{1}{16}\phi_{\mu\mu}\phi_{\nu\nu}(\partial_\lambda A_\rho)^2+\tfrac{1}{4}\phi_{\lambda\rho}\phi_{\mu\nu}\partial_\lambda A_\mu\partial_\nu A_\rho\\
&+\tfrac{1}{8}\phi_{\lambda\nu}\phi_{\mu\mu}\partial_\lambda A_\rho\partial_\nu A_\rho-\tfrac{1}{4}\phi_{\lambda\rho}\phi_{\mu\nu}\partial_\mu A_\lambda\partial_\nu A_\rho-\tfrac{1}{2}\phi_{\lambda\mu}\phi_{\lambda\nu}\partial_\mu A_\rho\partial_\nu A_\rho\\
&+\tfrac{1}{8}\phi_{\lambda\lambda}\phi_{\mu\nu}\partial_\mu A_\rho\partial_\nu A_\rho-\tfrac{1}{16}\phi_{\mu\nu}^2\partial_\lambda A_\rho\partial_\rho A_\lambda+\tfrac{1}{16}\phi_{\mu\mu}\phi_{\nu\nu}\partial_\lambda A_\rho\partial_\rho A_\lambda\\
&-\tfrac{1}{4}\phi_{\lambda\rho}\phi_{\mu\nu}\partial_\lambda A_\mu\partial_\rho A_\nu-\tfrac{1}{4}\phi_{\lambda\nu}\phi_{\mu\mu}\partial_\lambda A_\rho\partial_\rho A_\nu+\tfrac{1}{4}\phi_{\lambda\rho}\phi_{\mu\nu}\partial_\mu A_\lambda\partial_\rho A_\nu\\
&+\phi_{\lambda\mu}\phi_{\lambda\nu}\partial_\mu A_\rho\partial_\rho A_\nu-\tfrac{1}{4}\phi_{\lambda\lambda}\phi_{\mu\nu}\partial_\mu A_\rho\partial_\rho A_\nu+\tfrac{1}{8}\phi_{\lambda\nu}\phi_{\mu\mu}\partial_\rho A_\lambda\partial_\rho A_\nu\\
&-\tfrac{1}{2}\phi_{\lambda\mu}\phi_{\lambda\nu}\partial_\rho A_\mu\partial_\rho A_\nu+\tfrac{1}{8}\phi_{\lambda\lambda}\phi_{\mu\nu}\partial_\rho A_\mu\partial_\rho A_\nu
\end{aligned}\right. \tag{41}$$

Converting to momentum space (with four momenta t, u, v, w), we obtain $\delta^4[t+u+v+w]$ multiplying the following terms:

$$\left(\begin{aligned}
&-\tfrac{1}{16}\phi_{\mu\nu}[t]\phi_{\mu\nu}[u]A_\rho[v]A_\rho[w](v\cdot w)+\tfrac{1}{16}\phi_{\mu\mu}[t]\phi_{\nu\nu}[u]A_\rho[v]A_\rho[w](v\cdot w)\\
&-\tfrac{1}{4}\phi_{\lambda\rho}[t]\phi_{\mu\nu}[u]A_\mu[v]A_\rho[w](v_\lambda w_\nu)-\tfrac{1}{8}\phi_{\lambda\nu}[t]\phi_{\mu\mu}[u]A_\rho[v]A_\rho[w](v_\lambda w_\nu)\\
&+\tfrac{1}{4}\phi_{\lambda\rho}[t]\phi_{\mu\nu}[u]A_\lambda[v]A_\rho[w](v_\mu w_\nu)+\tfrac{1}{2}\phi_{\lambda\mu}[t]\phi_{\lambda\nu}[u]A_\rho[v]A_\rho[w](v_\mu w_\nu)\\
&-\tfrac{1}{8}\phi_{\lambda\lambda}[t]\phi_{\mu\nu}[u]A_\rho[v]A_\rho[w](v_\mu w_\nu)+\tfrac{1}{16}\phi_{\mu\nu}[t]\phi_{\mu\nu}[u]A_\rho[v]A_\lambda[w](v_\lambda w_\rho)\\
&-\tfrac{1}{16}\phi_{\mu\mu}[t]\phi_{\nu\nu}[u]A_\rho[v]A_\lambda[w](v_\lambda w_\rho)+\tfrac{1}{4}\phi_{\lambda\rho}[t]\phi_{\mu\nu}[u]A_\mu[v]A_\nu[w](v_\lambda w_\rho)\\
&+\tfrac{1}{4}\phi_{\lambda\nu}[t]\phi_{\mu\mu}[u]A_\rho[v]A_\nu[w](v_\lambda w_\rho)-\tfrac{1}{4}\phi_{\lambda\rho}[t]\phi_{\mu\nu}[u]A_\lambda[v]A_\nu[w](v_\mu w_\rho)\\
&-\phi_{\lambda\mu}[t]\phi_{\lambda\nu}[u]A_\rho[v]A_\nu[w](v_\mu w_\rho)+\tfrac{1}{4}\phi_{\lambda\lambda}[t]\phi_{\mu\nu}[u]A_\rho[v]A_\nu[w](v_\mu w_\rho)\\
&-\tfrac{1}{8}\phi_{\lambda\nu}[t]\phi_{\mu\mu}[u]A_\lambda[v]A_\nu[w](v\cdot w)+\tfrac{1}{2}\phi_{\lambda\mu}[t]\phi_{\lambda\nu}[u]A_\mu[v]A_\nu[w](v\cdot w)\\
&-\tfrac{1}{8}\phi_{\lambda\lambda}[t]\phi_{\mu\nu}[u]A_\mu[v]A_\nu[w](v\cdot w)
\end{aligned}\right) \tag{42}$$

In order to obtain the desired 4-leg vertex, we first differentiate the above expression with respect to $\phi_{\alpha\beta}[p]$ and $\phi_{\sigma\tau}[q]$, then with respect to $A_\kappa[r]$ and $A_\omega[s]$. Simplifying and renaming indices, the result defines the following 4-leg vertex

$$\mathcal{V}_{\mu\nu,\lambda\rho,\alpha,\beta}[p,q,r,s] = \left(\begin{array}{l} 2r_\mu s_\nu(2\eta_{\alpha\lambda}\eta_{\beta\rho} - \eta_{\alpha\beta}\eta_{\lambda\rho}) + 2r_\lambda s_\rho(2\eta_{\alpha\mu}\eta_{\beta\nu} - \eta_{\alpha\beta}\eta_{\mu\nu}) \\ -4r_\mu s_\lambda(\eta_{\alpha\rho}\eta_{\beta\nu} - \eta_{\alpha\beta}\eta_{\nu\rho}) - 4r_\lambda s_\mu(\eta_{\alpha\nu}\eta_{\beta\rho} - \eta_{\alpha\beta}\eta_{\nu\rho}) \\ +2r_\beta s_\mu(\eta_{\alpha\nu}\eta_{\lambda\rho} - 2\eta_{\alpha\lambda}\eta_{\nu\rho}) + 2r_\beta s_\lambda(\eta_{\alpha\rho}\eta_{\mu\nu} - 2\eta_{\alpha\mu}\eta_{\nu\rho}) \\ +2r_\mu s_\alpha(\eta_{\beta\nu}\eta_{\lambda\rho} - 2\eta_{\beta\lambda}\eta_{\nu\rho}) + 2r_\lambda s_\alpha(\eta_{\beta\rho}\eta_{\mu\nu} - 2\eta_{\beta\mu}\eta_{\nu\rho}) \\ +r_\beta s_\alpha(-\eta_{\lambda\rho}\eta_{\mu\nu} + \eta_{\lambda\mu}\eta_{\nu\rho}) \\ +(r\cdot s)\left(\begin{array}{l} -2\eta_{\alpha\mu}\eta_{\beta\nu}\eta_{\lambda\rho} - 2\eta_{\alpha\lambda}\eta_{\beta\rho}\eta_{\mu\nu} + \eta_{\alpha\beta}\eta_{\lambda\rho}\eta_{\mu\nu} \\ +4\eta_{\alpha\mu}\eta_{\beta\lambda}\eta_{\nu\rho} + 4\eta_{\alpha\lambda}\eta_{\beta\mu}\eta_{\nu\rho} - \eta_{\alpha\beta}\eta_{\lambda\mu}\eta_{\nu\rho} \end{array} \right) \end{array} \right) \quad (43)$$

In the above we have suppressed but used the implications of the momentum-conserving delta function $\delta^4[p+q+r+s]$. We also reduced the size of the expression by unsymmetrizing with respect to (μ,ν) and with respect to (λ,ρ) (Since eventually it will be multiplied by tensor propagators having these symmetries).

3 Quantum Contributions to the Vacuum

In the preceeding section, we have derived the Feynman rules (propagators and vertices) that are pertinent to the computation of the quantum loop corrections to the vacuum coming from virtual scalar, vector, and tensor (graviton) fields. In the followings, we shall compute the one-loop and the two-loop contributions using these rules.

3.1 One-Loop Contributions

3.1.1 Scalar 1-Loop

The one-loop contribution from a virtual massless scalar field is given by

$$\frac{i}{2}\int \frac{\mathrm{d}^4p}{(2\pi)^4}\ln(-p^2) \quad (44)$$

Here p is the Minkowskian momentum of the virtual particle. Converting to Euclidean loop momentum[3], then replacing the logarithm by a cutoff-regularized counterpart

$$\ln(p^2) \to -\int_{a^2}^{\infty} \frac{\mathrm{d}x}{x} e^{-xp^2} \quad (45)$$

[3]The conversion from Minkowskian momenta to Euclidean counterparts will be done throughout using $(d^4p) \to i(d^4p)$ and $p^2 \to -p^2$.

where, a is a cutoff length parameter, we obtain

$$\int_{a^2}^{\infty} \frac{\mathrm{d}x}{x} \int \frac{\mathrm{d}^4p}{(2\pi)^4} e^{-xp^2} \tag{46}$$

Integrating over momentum p, then integrating over x, we obtain

$$\int_{a^2}^{\infty} \mathrm{d}x \left(\frac{1}{32\pi^2 x^3} \right) = \frac{1}{64\pi^2 a^2} \tag{47}$$

3.1.2 Vector 1-Loop

The one-loop contribution from a virtual massless vector field is given by

$$\frac{i}{2} \int \frac{\mathrm{d}^4p}{(2\pi)^4} \ln(-p^2)\, \eta_{\mu\nu}\eta_{\mu\nu} \tag{48}$$

Notice that one $\eta_{\mu\nu}$ comes from the bilinear kernel, and another is used for the vectorial trace. Contracting the eta tensors, then again, converting to Euclidean loop momentum and replacing the logarithm by a cutoff-regularized counterpart we obtain

$$2\int_{a^2}^{\infty} \frac{\mathrm{d}x}{x} \int \frac{\mathrm{d}^4p}{(2\pi)^4} e^{-xp^2} \tag{49}$$

Integrating over p, then over x, we obtain

$$\int_{a^2}^{\infty} \mathrm{d}x \left(\frac{1}{8\pi^2 x^3} \right) = \frac{1}{16\pi^2 a^2} \tag{50}$$

3.1.3 Tensor 1-Loop

The one-loop contribution from a virtual massless tensor field (graviton) is given by

$$\frac{i}{2} \int \frac{\mathrm{d}^4p}{(2\pi)^4} \ln(-p^2)\, \frac{1}{2}(\eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\nu\lambda}\eta_{\mu\rho})(\eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\nu\lambda}\eta_{\mu\rho} - 2\eta_{\mu\nu}\eta_{\lambda\rho}) \tag{51}$$

It should be clear what tensorial factor comes from the bilinear kernel of the graviton field, and what factor is used for the tensorial trace (together with an appropriate factor of $\frac{1}{2}$). Contracting the eta tensors, then again, converting to Euclidean loop momentum and replacing the logarithm by a cutoff-regularized counterpart we obtain

$$6\int_{a^2}^{\infty} \frac{\mathrm{d}x}{x} \int \frac{\mathrm{d}^4p}{(2\pi)^4} e^{-xp^2} \tag{52}$$

Integrating over p, then over x, we obtain

$$\int_{a^2}^{\infty} \mathrm{d}x \left(\frac{3}{8\pi^2 x^3} \right) = \frac{3}{16\pi^2 a^2} \tag{53}$$

3.1.4 Summary of Bosonic One-Loop

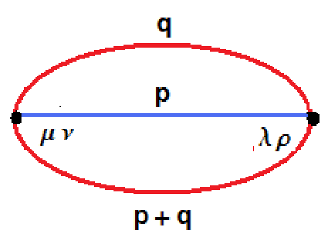
For a system consisting of one tensor graviton, N_s scalars and N_v vectors (all massless compared to cutoff), we obtain the one-loop result

$$\frac{1}{64\pi^2 a^4} \left(12 + N_s + 4N_v\right) \tag{54}$$

3.2 Two-Loop Contributions

3.2.1 Scalar 2-Loop with 3-Leg Vertices

Here we compute the vacuum 2-loop contribution coming from this graph[4]



This contribution is given by the expression

$$\int\int \frac{\mathrm{d}^4p}{(2\pi)^4} \frac{\mathrm{d}^4q}{(2\pi)^4} \frac{1}{16} \mathcal{V}_{\mu\nu}[p, q, -p-q] \mathcal{V}_{\lambda\rho}[-p, -q, p+q] \mathcal{P}_{\mu\nu,\lambda\rho}[p] \mathcal{P}[q] \mathcal{P}[p+q] \tag{55}$$

In the above, $\mathcal{V}_{\mu\nu}[p, q, r]$ is the vertex with one graviton leg and two scalar legs (obtained in §2.2.2), $\mathcal{P}_{\mu\nu,\lambda\rho}[p]$ is the virtual graviton propagator (obtained in §2.1.1), and $\mathcal{P}[p]$ is a scalar popagator (obtained in §2.2.1). Notice the implementation of momentum conservation at the vertices. Notice also that a factor of $\frac{1}{4}$ comes from the symmetries if the graph, and another $\frac{1}{4}$ comes from the symmetrical indices (μ, ν) and (λ, ρ). Substituting for the vertices and the graviton propagator, contracting the tensorial indices and simplifying, we obtain

$$\int\int \frac{\mathrm{d}^4p}{(2\pi)^4} \frac{\mathrm{d}^4q}{(2\pi)^4} \left\{ \frac{1}{6} \left(2p^4 + 9(p\cdot q)^2 + 9(p\cdot q)q^2 + 2q^4 + p^2(9(p\cdot q) + 5q^2)\right) \mathcal{P}[p]\mathcal{P}[q]\mathcal{P}[p+q] \right\} \tag{56}$$

[4] **Throughout this article, scalar propagators are depicted with red color, vector propagators with green, and tensor propagators with blue.**

Now replacing the scalar propagators in the above by their cutoff-regularized counterparts (with three respective integration parameters x, y, z), according to

$$\frac{1}{p^2} \to -\int_{a^2}^{\infty} \mathrm{d}x\, e^{xp^2} \tag{57}$$

we obtain for the foregoing expression (suppressing integration symbols and measures)

$$-\frac{1}{6}e^{(x+z)p^2+2z(p\cdot q)+(y+z)q^2}\left\{2p^4+9(p\cdot q)^2+9(p\cdot q)q^2+2q^4+p^2(9(p\cdot q)+5q^2)^2\right\} \tag{58}$$

Completing the square for q in the argument of the exponential, then making appropriate shift in that loop momentum, simplifying, and symmetrizing in q, we obtain

$$-\frac{1}{6(y+z)^4}e^{\frac{xy+xz+yz}{y+z}p^2+(y+z)q^2}\left\{\begin{array}{l} y^2(2y^2-yz+z^2)p^4-\frac{1}{4}(y+z)^2(29y^2 \\ +4yz-z^2)p^2q^2+2(y+z)^4q^4 \end{array}\right\} \tag{59}$$

Converting to Euclidean loop momenta p and q, then integrating over them, we obtain

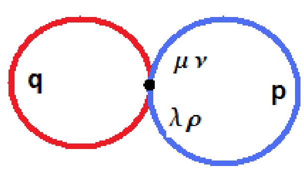
$$\int_{a^2}^{\infty}\mathrm{d}x\int_{a^2}^{\infty}\mathrm{d}y\int_{a^2}^{\infty}\mathrm{d}z\left\{\frac{12x^2+29xy+12y^2-xz-yz}{1536\pi^4(xy+xz+yz)^4}\right\} \tag{60}$$

Evaluating the above integrals, we obtain

$$\frac{-1+\ln\left(\frac{16384}{2187}\right)}{3072\pi^4a^6} \tag{61}$$

3.2.2 Scalar 2-Loop with a 4-Leg Vertex

Here we give the 2-loop contribution coming from this graph



This contribution is given by the expression[5]

$$-\frac{1}{16}\int\int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{\mathrm{d}^4q}{(2\pi)^4}\mathcal{V}_{\mu\nu,\lambda\rho}[p,-p,q,-q]\mathcal{P}_{\mu\nu,\lambda\rho}[p]\mathcal{P}[q] \tag{62}$$

[5] **Recall that the overall sign is determined by a factor of i for each vertex, a factor of i for each propagator, and an overall factor of $-i$.**

In the above, $\mathcal{V}_{\mu\nu,\lambda\rho}[p,q,r,s]$ is the vertex with two gravitons and two scalars (obtained in §2.2.3), $\mathcal{P}_{\mu\nu,\lambda\rho}[p]$ is the graviton propagator, and $\mathcal{P}[q]$ is the scalar propagator. Substituting for the vertex and the graviton propagator, then contracting the tensorial indices, and simplifying, we obtain

$$-\frac{13}{12}\int\int \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{\mathrm{d}^4q}{(2\pi)^4}\, q^2\mathcal{P}[p]\mathcal{P}[q] \tag{63}$$

Replacing the scalar propagators in the above by their cutoff-regularized counterparts (with respective two integration parameters x, y), according to

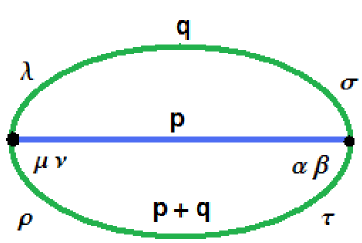
$$\frac{1}{p^2} \to -\int_{a^2}^{\infty} \mathrm{d}x\, e^{xp^2} \tag{64}$$

then converting to Euclidean loop momenta p and q, and integrating over them, we obtain

$$\int_{a^2}^{\infty}\mathrm{d}x\int_{a^2}^{\infty}\mathrm{d}y\left\{-\frac{13}{1536\pi^4x^2y^3}\right\} = -\frac{13}{3072\pi^4a^6} \tag{65}$$

3.2.3 Vector 2-loop with 3-Leg Vertices

Here we compute the vacuum 2-loop contribution coming from this graph



This contribution is given by the expression

$$\frac{1}{16}\int\int \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{\mathrm{d}^4q}{(2\pi)^4}\mathcal{V}_{\mu\nu,\lambda,\rho}[p,q,-p-q]\mathcal{V}_{\alpha\beta,\sigma,\tau}[-p,-q,p+q]\mathcal{P}_{\mu\nu,\alpha\beta}[p]\mathcal{P}_{\lambda,\sigma}[q]\mathcal{P}_{\rho,\tau}[p+q] \tag{66}$$

In the above $\mathcal{V}_{\mu\nu,\lambda,\rho}[p,q,k]$ is the vertex with one graviton leg and two vector legs (obtained in§2.3.2), $\mathcal{P}_{\mu\nu,\alpha\beta}[p]$ is the graviton propagator (obtained in §2.1.1), and $\mathcal{P}_{\mu,\nu}[p]$ is the vector propagator (obtained in §2.3.1). Replacing the vertices and the propagators by their expressions, contracting the tensorial indices, and simplifying, we obtain

$$\frac{1}{12}\int\int \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{\mathrm{d}^4q}{(2\pi)^4}\left\{\begin{array}{c} 21p^4+62(p\cdot q)^2+90(p\cdot q)q^2 \\ +36q^4+60p^2(p\cdot q)+31p^2q^2 \end{array}\right\}\mathcal{P}[p]\mathcal{P}[q]\mathcal{P}[p+q] \tag{67}$$

Replacing the scalar propagators in the above by their cutoff-regularized counterparts (with respective three integration parameters x, y, z), according to

$$\frac{1}{p^2} \to -\int_{a^2}^{\infty} \mathrm{d}x\, e^{xp^2} \tag{68}$$

we obtain for the foregoing expression (suppressing integration symbols and measures)

$$-\frac{1}{12} e^{(x+z)p^2+2z(p\cdot q)+(y+z)q^2} \left\{21p^4 + 62(p\cdot q)^2 + 90(p\cdot q)q^2 + 36q^4 + 60p^2(p\cdot q) + 31p^2q^2\right\} \tag{69}$$

Completing the square for q in the argument of the exponential, then making appropriate shift in that loop momentum, simplifying, and symmetrizing in q, we obtain

$$-\frac{1}{12(y+z)^4} e^{\frac{xy+xz+yz}{y+z}p^2+(y+z)q^2} \left\{ \begin{array}{l} 3y^2(7y^2+8yz+13z^2)p^4 + \frac{3}{2}(y+z)^2(31y^2 \\ -28yz+13z^2)p^2q^2 + 36(y+z)^4q^4 \end{array} \right\} \tag{70}$$

Converting to Euclidean loop momenta p and q, then integrating over them, we obtain

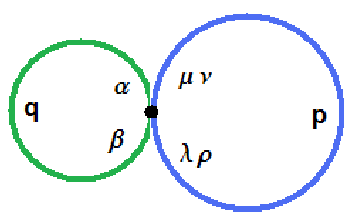
$$\int_{a^2}^{\infty} \mathrm{d}x \int_{a^2}^{\infty} \mathrm{d}y \int_{a^2}^{\infty} \mathrm{d}z \left\{ \frac{36x^2+31xy+21y^2+13xz+13yz}{512\pi^4(xy+xz+yz)^4} \right\} \tag{71}$$

Evaluating the above integrals, we obtain

$$\frac{19(-1+12\ln(2)-6\ln(3))}{6144\pi^4 a^6} \tag{72}$$

3.2.4 Vector 2-Loop with a 4-Leg Vertex

Here we give the 2-loop contribution coming from this graph



This contribution is given by the expression

$$-\frac{1}{16}\int\int \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{\mathrm{d}^4q}{(2\pi)^4} \mathcal{V}_{\mu\nu,\lambda\rho,\alpha,\beta}[p,-p,q,-q]\mathcal{P}_{\mu\nu,\lambda\rho}[p]\mathcal{P}_{\alpha,\beta}[q] \tag{73}$$

In the above, $\mathcal{V}_{\mu\nu,\lambda\rho,\alpha,\beta}[p,q,r,s]$ is the vertex with two gravitons and two vectors (obtained in §2.3.3), $\mathcal{P}_{\mu\nu,\lambda\rho}[p]$ is the graviton propagator, and $\mathcal{P}_{\alpha,\beta}[q]$ is the vector propagator. Substituting for the vertex, for the graviton propagator, and for the vector propagator, then contracting the tensorial indices, and simplifying, we obtain

$$-\frac{31}{4}\int\int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{\mathrm{d}^4q}{(2\pi)^4}\,q^2\mathcal{P}[p]\mathcal{P}[q] \tag{74}$$

Replacing the scalar propagators in the above by their cutoff-regularized counterparts (with respective two integration parameters x, y), according to

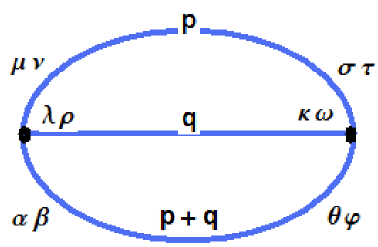
$$\frac{1}{p^2}\rightarrow -\int_{a^2}^{\infty}\mathrm{d}x\, e^{xp^2} \tag{75}$$

then converting to Euclidean loop momenta p and q, and integrating over them, we obtain

$$\int_{a^2}^{\infty}\mathrm{d}x\int_{a^2}^{\infty}\mathrm{d}y\left\{-\frac{31}{512\pi^4x^2y^3}\right\} = -\frac{31}{1024\pi^4a^6} \tag{76}$$

3.2.5 Tensor 2-loop with 3-Leg Vertices

Here we compute the vacuum 2-loop contribution coming from this graph



This contribution is given by the expression

$$\frac{1}{768}\int\int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{\mathrm{d}^4q}{(2\pi)^4}\mathcal{V}_{\mu\nu,\lambda\rho,\alpha\beta}[p,q,-p-q]\mathcal{V}_{\sigma\tau,\kappa\omega,\theta\varphi}[-p,-q,p+q]\mathcal{P}_{\mu\nu,\sigma\tau}[p]\mathcal{P}_{\lambda\rho,\kappa\omega}[q]\mathcal{P}_{\alpha\beta,\theta\varphi}[p+q] \tag{77}$$

In the above $\mathcal{V}_{\mu\nu,\lambda\rho,\alpha\beta}[p,q,r]$ is the vertex with three graviton legs(obtained in§2.1.2), while $\mathcal{P}_{\mu\nu,\alpha\beta}[p]$ is the graviton propagator (obtained in §2.1.1). Replacing the vertices and the propagators by their expressions, contracting the tensorial indices, and simplifying, we obtain

$$\frac{1}{162}\int\int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{\mathrm{d}^4q}{(2\pi)^4}\left\{\begin{array}{c}990p^4+1563(p\cdot q)^2+3210(p\cdot q)q^2\\ +1517q^4+2340p^2(p\cdot q)+2220p^2q^2\end{array}\right\}\mathcal{P}[p]\mathcal{P}[q]\mathcal{P}[p+q] \tag{78}$$

Replacing the scalar propagators in the above by their cutoff-regularized counterparts (with respective three integration parameters x, y, z), according to

$$\frac{1}{p^2}\rightarrow -\int_{a^2}^{\infty}\mathrm{d}x\, e^{xp^2} \tag{79}$$

we obtain for the foregoing expression (suppressing integration symbols and measures)

$$-\frac{1}{162}e^{(x+z)p^2+2z(p\cdot q)+(y+z)q^2}\left\{\begin{array}{l}990p^4+1563(p\cdot q)^2+3210(p\cdot q)q^2\\ +1517q^4+2340p^2(p\cdot q)+2220p^2q^2\end{array}\right\} \tag{80}$$

Completing the square for q in the argument of the exponential, then making appropriate shift in that loop momentum, simplifying, and symmetrizing in q, we obtain

$$-\tfrac{1}{162(y+z)^4}e^{\frac{xy+xz+yz}{y+z}p^2+(y+z)q^2}\times$$
$$\left\{\begin{array}{l}(990y^4+1620y^3z+2703y^2z^2+1296yz^3+740z^4)p^4\\ +\frac{3}{4}(y+z)^2(3481y^2+542yz+3129z^2)p^2q^2+1517(y+z)^4q^4\end{array}\right\} \tag{81}$$

Converting to Euclidean loop momenta p and q, then integrating over them, we obtain

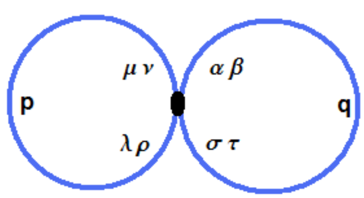
$$\int_{a^2}^{\infty}\mathrm{d}x\int_{a^2}^{\infty}\mathrm{d}y\int_{a^2}^{\infty}\mathrm{d}z\left\{\frac{3034x^2+3481xy+1980y^2+3129xz+2761yz+1480z^2}{13824\pi^4(xy+xz+yz)^4}\right\} \tag{82}$$

Evaluating the above integrals, we obtain

$$\frac{-370+21702\ln(2)-10851\ln(3)}{248832\pi^4a^6} \tag{83}$$

3.2.6 Tensor 2-Loop with a 4-Leg Vertex

Here we give the 2-loop contribution coming from this graph



This contribution is given by the expression

$$-\frac{1}{128}\int\int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{\mathrm{d}^4q}{(2\pi)^4}\mathcal{V}_{\mu\nu,\lambda\rho,\alpha\beta,\sigma\tau}[p,-p,q,-q]\mathcal{P}_{\mu\nu,\lambda\rho}[p]\mathcal{P}_{\alpha\beta,\sigma\tau}[q] \tag{84}$$

In the above, $\mathcal{V}_{\mu\nu,\lambda\rho,\alpha\beta,\sigma\tau}[p,q,r,s]$ is the vertex with four gravitons(obtained in §2.1.3), while $\mathcal{P}_{\mu\nu,\lambda\rho}[p]$ is the graviton propagator. Substituting for the vertex and for the propagator, then contracting the tensorial indices, and simplifying, we obtain

$$-\frac{5}{36}\int\int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{\mathrm{d}^4q}{(2\pi)^4}(132p^2+61q^2)\mathcal{P}[p]\mathcal{P}[q] \tag{85}$$

Replacing the scalar propagators in the above by their cutoff-regularized counterparts (with respective two integration parameters x, y), according to

$$\frac{1}{p^2} \to -\int_{a^2}^{\infty} \mathrm{d}x\, e^{xp^2} \tag{86}$$

then converting to Euclidean loop momenta p and q, and integrating over them, we obtain

$$\int_{a^2}^{\infty} \mathrm{d}x \int_{a^2}^{\infty} \mathrm{d}y \left\{ -\frac{5(61x+132y)}{4608\pi^4 x^3 y^3} \right\} = -\frac{965}{9216\pi^4 a^6} \tag{87}$$

3.2.7 Summary of Bosonic Two-Loop

For a system consisting of one tensor graviton, N_s scalars and N_v vectors (all massless compared to cutoff), we obtain the folowing two-loop vacuum contribution:

$$\frac{1}{497664\pi^4 a^6} \begin{pmatrix} 2(-26425 + 21702\ln(2) - 10851\ln(3)) \\ +162(-14 + \ln(16384/2187))N_s \\ +81(-205 + 228\ln(2) - 114\ln(3))N_v \end{pmatrix} \tag{88}$$

4 Discussion

Our computation for the quantum contribution to the vacuum, up to two loops, is given by the approximate expression:

$$\frac{1}{64\pi^2 a^4} \left\{ (12 + N_s + 4N_v) - \frac{(23303 + 972N_s + 6966N_v)}{3888\pi^2} \frac{\kappa^2}{a^2} + \ldots \right\} \tag{89}$$

Here N_s is the number of virtual scalars, N_v is the number of virtual vectors, κ is the (reinstated) gravitational coupling, and a is the cutoff length used to regularize the effective propagators. As we have pointed out before, the consistency of gravidynamic quantum field theory, developed around a flat spacetime background, requires the vanishing of the above series. Assuming that the latter equated to zero is invertible for κ, we can relate the gravitational coupling κ to the cutoff parameter a. To the order computed, we obtain

$$\frac{\kappa}{a} \approx \pi \sqrt{\frac{3888(12 + N_s + N_v)}{23303 + 972N_s + 6966N_v}} \tag{90}$$

Notice that for a pure gravity theory without scalars or vector, we have $\kappa/a \approx 4.5$, for an infinite number of scalars $\kappa/a \approx 2\pi$, and for an infinite number of vectors $\kappa/a \approx 4.7$. This means that whatever the number of matter fields, *the gravitational* *coupling is greater but always within an order of magnitude from the value of the cutoff length parameter*. Conversely, the energy cutoff is greater but always within an order of magnitude from the Planck mass.

The inclusion of fermionic matter in our computations is not likely to change the situation drastically. However, we shall present results for a theory including Dirac fermionic particles, and possibly for supergravitational fields, in other articles.

References

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